# KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY

## INSTITUTE OF DISTANCE LEARNING

## TOPIC:

# MODELLING QUEUING SYSTEM IN THE BANKING INDUSTRY: CASE STUDY

## GHANA COMMERCIAL BANK, SUAME, KUMASI
## ECOBANK, ASHTOWN, KUMASI AND
## BARCLAYS BANK OF GHANA, TANOSO BRANCHES



By

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In partial fulfilment of the requirement for the degree of**

## MASTER OF SCIENCE

**Department of Mathematics**

**Institute of Distance Learning**

# DECLARATION

I hereby declare that this submission is my own work towards the MSc. and that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the University, except where due acknowledgment has been made in the text.



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## DEDICATION

This work is dedicated to my wife, Dorcas; daughter, Nhyira; parents, Very Rev & Mrs Kyei Baffour, and all my siblings, teachers and all those who contribute to the betterment of the well being of humankind.

# ABSTRACT

The issue of queuing is a bother to both management and customers in the delivery of service in the banking industry. The main idea or pivot of the queuing system is a teller who provides some services to a population of customers. This study therefore determined whether the present capacity level in the banking industry strike a balance between waiting and service time using Barclays Bank, Tanoso Branch, Ghana Commercial Bank, Suame Branch and Ecobank, Ashtown Branch as a case of interest. Primary data on five hundred and fifty-eight (558) customers arriving at the case of study throughout the selected hours and days were collected, taken into consideration; the arrival, processing and departure times of each customer. By using the queuing rule First-come, First-serve as practiced by the case study and $M/M/s$ queuing model, the performance measures were calculated uncovering the applicability and extent of usage of queuing models in achieving customer satisfaction at lowest cost by minimizing waiting times, idle times, capacity utilization and queues at the bank. The study shows how the data collected at the respective dates possesses the Markovian properties, that is, Poisson and Exponential Distributions, hence the use of two "M's" in the $M/M/s$ queuing model. It determined the probabilistic analysis that the teller(s) is idle and also determined the probability of certain number of arrivals occurring at a given time. It was observed that any time the number of tellers at BBG an Ecobank were reduced to two during a peak day, the queue elongated resulting in high capacity utilization factor and hence a high waiting time. It is therefore recommended that management makes better decisions relating to number of tellers that would be necessary to serve customers during peak and off-peak hours, days and weeks.

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| ACRONYMS | MEANING |
|---|---|
| GCB | Ghana Commercial Bank |
| BBG | Barclays Bank of Ghana |
| ECO | Ecobank. |
| i.i.d | Independent and identically distributed |
| r.v | Random Variable |

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Kyei Baffour, Yaw Anokye.

# CHAPTER 1

# INTRODUCTION

## 1.1 BACKGROUND OF THE STUDY

The banking sector is the largest and most competitive segment of Ghana's financial sector. The liberalisation of the banking sector in the 90's led to an increase in the number of banks in the country. The universal banking concept introduced in 2003 by the Central Bank of Ghana also opened wide the doors of the Ghanaian banking industry to investors who decided to establish as many as banks. Most of these banks are operating in the country under African Ownership and management. Currently, the number of banks operating in Ghana is 28 (BoG, 2010), with numerous emerging and existing Rural Banks, Credit Unions and Micro Finance, but the aggressive competition has forced long-operating banks to reconfigure their strategy and business to sustain or improve their competitive advantage. Consequently, the banking industries have largely implemented service delivery technology as a way of augmenting the services traditionally provided by bank personnel.

Competition has become keener due to regulatory imperatives of universal banking, technological inventions, quality of customers and globalisation of the market and also due to customers' awareness of their rights. Customers have become increasingly demanding, as they require high quality, low priced and immediate service delivery. They want additional improvement of value from their chosen banks (Olaniyi, 2004). This has resulted in a paradigm shift in the banking business. The modern banking in Ghana has shifted from a seller's market to buyer's market.

The buyer's market puts emphasis on customer needs, wants and satisfaction. Competition is making customer retention more important than ever. The bank that is unable to satisfy its customers is likely to lose them. Actually, all banks suffer from voluntary churn – the decision by the customer to switch to another bank. This is particularly true for Ghanaian banks. Ankomah (2008) estimates annual churn rate for banks to be between 10 and 18 per cent on average.

In this emerging market, customers are not that loyal to one particular bank and managers are greatly aware that providing quality customer satisfaction is a key retention component that results in the industry profitability. Hence, the major banks have been forced to consider how to create a loyal customer base that will not be eroded even in the face of fierce competition. Therefore, these banks must realize the necessity of studying and understanding various antecedents (viz. service quality and customer satisfaction) of the customer retention which might help them to develop a loyal customer base. The banking industry, whose service depends on building long term relationship need to concentrate on maintaining customer's loyalty. In this respect, retention is greatly influenced by service quality. As such, banks often invest in managing their relationships with customers and maintaining quality to ensure that customers whose loyalty is in the short term will continue to be loyal in the long term, the need has also arise to bring sanity in the baking hall to avoid unnecessary long queues. High quality service helps to generate customer satisfaction and customer retention by soliciting new customers, and improved productivity and financial performance.

As information technology becomes more sophisticated, banks in many parts of the world are adopting a multiple-channel strategy. Changes in banking industry as a result of challenges posed by technologically innovative competitors such as those resulting from deregulation, rapid global networking, and the rise in personal wealth have thus made the implementation of sophisticated delivery systems. The technology of electronic banking, for examples; Automated Teller Machines (ATMs), Internet, Mobile and Telephone Banking, etc, are aimed to render faster and convenient banking services to customers in anywhere  and at anytime, a strategy necessary in many cases. These electronic banking has already advanced in the developed countries, but only few customers in Ghana subscribe to it. However, it, cannot entirely replace the more traditional channels. Research indicates that a substantial portion of the customer base may always demand the type of personal interaction that can only be provided by individual branch personnel (Lewis et al., 1994). In other relation, it has been observed that, as the country population increases, the customer population also increases. Hence, queue(s) may be witness in the banking halls. At time, the subsidiary channel like, the ATMs do witness queues by the customers, how much less queues at the respective banking halls in Ghana?

Since service delivery in banks is personal, customers are either served immediately or join a queue if the system is busy. A queue occurs when facilities are limited and cannot satisfy demand made against them at a particular period. However, most customers are not comfortable with waiting or queuing (Olaniyi, 2004). The danger of keeping customers in a queue is that their waiting time may amount to or could become a cost to them (i.e. bank customers).

When we encounter a queue, we often make a quick estimate of the expected waiting time and decide whether to join the queue based on the amount of time we are willing to wait. Basically, queuing is a core of almost every transaction undertaken by customers within banking industry. However, several concerns are being raised by customers over the unsatisfactory service condition as a result of queue(s) delay. There may be several factors arising to this fact, but not withstanding those accessions, the ultimate aim for every customer is to spend limited time at the banking halls. As a result of these constraints, most managers attempt to solve queuing problems procuring additional facilities or hiring more workers to reduce the waiting time.

Almost all banks are opening more branches and some are even forming mergers to have a bigger capital base and also maximise profit. The most recent is Ecobank and TTB in April, 2012 has increased their branches to forty-two (42).

## 1.2 PROBLEM STATEMENT

In this era of shrinking markets, banks can gain competitive edge over rivals by devoting resources to improve customer satisfaction in order to retain existing customers instead of wasting useful time and resource to win new customers. Excellent customer satisfaction has a positive correlation with profitability, growth and customer retention and on the other hand, poor customer service causes considerable damage to goodwill of the banking industry and this eventually leads to financial loss and dwindling market share. In this regards, customers must be absolutely satisfied at the banking hall in order not to be stressed upon receiving and paying in their own resources as a result of long queues, poor service quality and unnecessary long waiting time.

Queue is a social phenomenon and if managed well, it would be beneficial to the society, especially to both the unit that waits and the one that serves. Queue is a line of customers waiting their turn for service. Queuing occurs when customers arrive faster than they can be served and the system temporarily buffers them in queues. Queuing system is a system that includes the customer population source, a queuing discipline as well as the service system.

The ultimate objective of this analysis of queuing systems is to determine whether customer satisfaction can be enhanced through managing the queuing systems in the banking industry using selected cases of study so that informed and intelligent decisions can be made in their management. This study is based on a mathematical building process as well as designing and implementing of an appropriate experiment involving that model. This encompasses the study of arrival, behaviour of customers, service times, service discipline, service capacity and the departure of customers at the case of interest, which is the marketing strategy of many banks as well. The problems are categorised into three main parts and can be identified as:

1. *Arrival Problems*. Usually, there is an assumption that service times are independent and identically distributed, and that they are independent of the inter arrival times. For example, the service times can be deterministic or exponentially distributed. It can also occur that service times are dependent of the queue length. Most queuing models assume that the inter arrival times are independent and have a common distribution. In many practical situations customers arrive according to a Poisson stream (i.e. exponential inter arrival times).

If the occurrence of arrivals and service delivery are strictly according to schedule, a queue can be avoided. But in practice, this does not happen. In most cases the arrivals are the product of external factors. Therefore, the best one can do is to describe the input process in terms of random variables which can represent either the number arriving during a time interval or the time interval between successive arrivals. Customers may arrive one by one, or in batches. If customers arrive in groups, their size can be a random variable as well. An example of batch arrivals is the Customs Office at Aflao where travel documents of bus passengers have to be checked.

Customer population can be considered as finite or infinite. When potential new customers for the queuing system are affected by the number of customers already in the system, the customer population is finite. When the number of customers waiting in line does not significantly affect the rate at which the population generates new customers, the customer population is considered infinite. Banking customer population is infinite since they do not restrict a specific number of customers to be serviced each day and work strictly with time.

2. *Behavioural problems.* The study of behavioural problems of queuing at banks is intended to understand how customers behave under various conditions. This normally affects the smooth service delivery irrespective of the facilities that a bank may have if much attention is not given to it. Customer's behaviour at the banking hall may destruct attention of the one that serves and can bring serving to a halt. A customer may balk, renege, or jockey and their queuing analysis is based on behavioural problem researches. Balking occurs when the customer decides not to enter the queue. Reneging occurs when the customer enters the queue but leaves

before being serviced. Jockeying occurs when a customer changes from one line to another, hoping to reduce the waiting time. A good example of this is picking a line at GCB, Suame Kumasi Branch and changing to another line with the hope of serving quicker. The models used in this supplement assumed that customers are patient; they do not balk, renege, or jockey; and the customers come from an infinite population.

3. *Operational problems.* The operational system is characterized by the number of queues, the number of tellers, the arrangement of the tellers, the arrival and service patterns, and the service priority rules. Under this heading, all problems that are inherent in the operation of queuing systems are included. How many customers can wait at a time if a queuing system is a significant factor for consideration? There may be a single teller or a group of tellers helping the customers. If the banking hall is large, one can assume that for all practical purposes, it is infinite. In many queues, it is useful to determine various waiting times and queue sizes for particular components of the system in order to make judgments about how the system should be run. Some of such problems are statistical in nature. Others are related to the design, control, and the measurement of effectiveness of the systems. In many situations customers in some classes get priority in service over others. Also, there are other factors of customer behaviour such as balking, reneging, and jockeying that require consideration as well. The slow pace and the number of tellers also affect the time a customer waits in the queuing system.

## 1.3 OBJECTIVES OF THE STUDY

The primary objective of this study in line with the identified problems is to determine whether the present capacity level in the banking industry, strikes a balance between waiting and service time using Barclays Bank of Ghana, Tanoso branch, Ecobank, Ashtown branch and Ghana Commercial Bank, Suame branch as case study. This would be carried out by measuring customers;

a) The arrival time.
b) The processing time.
c) The departure time.



The study specifically aims to determine:

i) The probabilistic analysis that the teller(s) will be idle.
ii) The amount of waiting time a customer is likely to experience in a system;
iii) How the waiting time will be affected if there are changes in the facilities and
iv) Make policy recommendation base on the findings from the study.

## 1.4 METHODOLOGY

In an observational way, primary data would be collected at the same time and date at the two cases of study in some selected days within January and February, 2010. Microsoft Office Excel 2007 would be used to assess and interpret data with the help of various method of queuing analysis. Secondary data used to execute the literature review of the study was gathered from the internet, professional magazines, research papers, journals, and textbooks.

## 1.5 JUSTIFICATION

Queues are so commonplace in society that it is highly worthwhile to study them, even if one waits in the check line for a few seconds. It may take some creative thinking, but if there is any sort of scenario where time passes before a particular event occurs, there is probably some way to develop it into a queuing model.

## 1.5 LIMITATION

It was a big task in observing and taking customers arrival, processing and departure times, especially at GCB, Suame branch which use several single teller with single stage queues arranged in parallel.

## 1.7 ORGANIZATION OF THE STUDY

Chapter one highlights on the background of study, the objectives, some of the challenges faced during the study and the organisation of the study. Chapter two shows the origin of queuing theory and also highlight on the works of some of its contributors that lead to the study of this thesis topic. In chapter three, the methodology used to achieve the objectives under study would be clearly stated.

Component of the basic queuing system, fundamental queuing relations and various assumptions related to model development are explicitly mentioned, and overall model structure would be explained. Analysis and interpretation of data collected would be done in chapter four. Summary of findings, recommendation and conclusion would also appear in chapter five.



# CHAPTER 2

## LITERATURE REVIEW

## 2.0 INTRODUCTION

This chapter shows the origin of queuing theory and also highlights on the works of some of its contributors that lead to the study of this thesis topic.

## 2.1 EVOLUTION OF QUEUING THEORY

Queuing Theory had its beginning in the research work of a Danish engineer named A. K. Erlang. In 1909 Erlang experimented with fluctuating demand in telephone traffic. Eight years later he published a report addressing the delays in automatic dialing equipment. At the end of World War II, Erlang's early work was extended to more general problems and to business applications of waiting lines.

In Erlang's work, as well as the work done by others in the twenties and thirties, the motivation has been the practical problem of congestion. Notable example is the works of (Molina, 1927; Fry, 1928). During the next two decades several theoreticians became interested in these problems and developed general models which could be used in more complex situations. Some of the authors with important contributions are Crommelin, Feller, Jensen, Khintchine, Kolmogorov, Palm, and Pollaczek. A detailed account of the investigations made by these authors may be found in books by (Syski, 1960; Saaty, 1961). Kolmogorov and Feller's study of purely discontinuous processes laid the foundation for the theory of Markov processes as it developed in later years.

Noting the inadequacy of the equilibrium theory in many queue situations, (Pollaczek, 1934) began investigations of the behaviour of the system during a finite time interval. Since then and throughout his career, he did considerable work in the analytical behavioural study of queuing systems, (Pollaczek, 1965). The trend towards the analytical study of the basic stochastic processes of the system continued, and queuing theory proved to be a fertile field for researchers who wanted to do fundamental research on stochastic processes involving mathematical models.

To this day a large majority of queuing theory results used in practice are those derived under the assumption of statistical equilibrium. Nevertheless, to understand the underlying processes fully, a time dependent analysis is essential. But the processes involved are not simple and for such an analysis sophisticated mathematical procedures become necessary.

Queuing theory as an identifiable body of literature was essentially defined by the foundational research of the 1950's and 1960's. The queue *M/M/*1 (Poisson arrival, exponential service, single server) is one of the earliest systems to be analyzed. The first of such solution was given by (Bailey, 1954) using generating functions for the differential equations governing the underlying process, while Lederman and Reuter (1956) used spectral theory in their solution. Laplace transforms were later used for the same problem, and their use together with generating functions has been one of the standard and popular procedures in the analysis of queuing systems ever since.

A probabilistic approach to the analysis was initiated by Kendall (1951, 1953) when he demonstrated that imbedded Markov chains can be identified in the queue length process in systems M/G/1 and GI/M/s. Lindley (1952) derived integral equations for waiting time distributions defined at imbedded Markov points in the general queue GI/G/1. These investigations led to the use of renewal theory in queuing systems analysis in the 1960's. Identification of the imbedded Markov chains also facilitated the use of combinatorial methods by considering the queue length at Markov points as a random walk.

Mathematical modelling is a process of approximation. A probabilistic model brings it a little bit closer to reality; nevertheless it cannot completely represent the real world phenomenon because of involved uncertainties. Therefore, it is a matter of convenience where one can draw the line between the simplicity of the model and the closeness of the representation. In the 1960's several authors initiated studies on the role of approximations in the analysis of queuing systems. Because of the need for useable results in applications various types of approximations have appeared in the literature (Bhat et al., 2002).

By the end of 1960's most of the basic queuing systems that could be considered as reasonable models of real world phenomena had been analyzed and the papers coming out dealt with only minor variations of the systems without contributing much to methodology. There were even statements made to the effect that queuing theory was at the last stages of its life. But such predictions were made without knowing what advances in the computer technology would mean to queuing theory. At later instance, in the seventies, its application has been extended to computer performance evaluation and manufacturing. The need to analyze traffic processes in the rapidly growing computer and communication industry is the primary reason for the resurgence of queuing theory after the 1960's. Research on queuing networks by (Jackson, 1957; Coffman and Deming, 1973; Kleinrock 1975) laid the foundation for a vigorous growth of the subject. Some of the special queuing models of the 1950's and 1960's have found broader applicability in the context of computer and communication systems and some other real life problems such as banking, manufacturing systems, to mention but a few.

In any theory of stochastic modelling statistical problems naturally arise in the applications of the models. Identification of the appropriate model, estimation of parameters from empirical data and drawing inferences regarding future operations involve statistical procedures. These were recognized even in earlier investigations in the studies by Erlang. The first theoretical treatment of the estimation problem was given by (Clarke, 1957) who derived maximum likelihood estimates of arrival and service rates in an M/M/1 queuing system. Billingsley's (1961) treatment of inference in Markov processes in general and Wolff's (1965) derivation of likelihood ratio tests and maximum likelihood estimates for queues that can be modelled by birth and death processes are other significant advances that have occurred in this area.

The first paper on estimating parameters in a non-Markovian system is by (Goyal and Harris, 1972), who used the transition probabilities of the imbedded Markov chain to set up the likelihood function. Since then significant progress has occurred in adapting statistical procedures to various systems. For instance, (Basawa and Prabhu, 1988) considered the problem of estimation of parameters in the queue GI/G/1; (Rao, et al. 1984) used a sequential probability ratio technique for the control of parameters in M/Ek/1 and Ek/M/1; and Armero (1994) used Bayesian techniques for inference in Markovian queues, to identify only a few. More recent investigations are by (Bhat and Basawa, 2002) who use queue length as well as waiting time data in estimating parameters in queuing systems.

Hillier's (1963) paper on economic models for industrial queue problems is, perhaps, the first paper to introduce standard optimization techniques to queuing problems. While Hillier considered an M/M/1 queue, (Heyman, 1968) derived an optimal policy for turning the server on and off in an M/G/1 queue, depending on the state of the system. Since then, operations researchers trained in mathematical optimization techniques have explored their use in much greater complexity to a large number of queuing systems. These have paved way for us to study the queuing systems in some selected banks by using queuing theory.

Queuing Discipline, that is to say, when an arrival occurs, it is added to the end of the queue and service is not performed on it until all of the arrivals that came before it are served in the order they arrived. Although this is a very common method for queues to be handled, it is far from the only way. Bank queues are typical example that outlines a first-come-first-serve discipline, or an FCFS discipline. Barrer (1957) compared this with a situation where the customers are served at random, and found that the steady state probability of service is slightly less for random selection. Another situation of interest has two classes of customer with different priorities. Other possible disciplines include last-come-first-served or LCFS, and service in random order, or SIRO. While the particular discipline chosen will likely greatly affect waiting times for particular customers (nobody wants to arrive early at an LCFS discipline), the discipline generally doesn't affect important outcomes of the queue itself, since arrivals are constantly receiving service regardless.

A probabilistic approach to the queuing capacity analysis was initiated by (Kendall, 1951, 1953) when he demonstrated that embedded Markov chains can be identified in the queue length process in systems *M/G/*1 and *GI/M/s*. Lindley (1952) derived integral equations for waiting time distributions defined at embedded Markov points in the general queue *GI/G/*1.

Models with dependencies between inter arrival and service times have been studied by several authors. Models with a linear dependence between the service time and the preceding inter arrival time have been studied in (Cidon, et. al, 1991). Mitchell, et al (1977) analyzes the M/M/1 queue where the service time and the preceding inter arrival time have a bivariate exponential density with a positive correlation. The linear and bivariate exponential cases are both contained in the correlated M/G/1 queue studied by (Borst,et al, 1993). The correlation structure considered in Borst,et al (1993) arises in the following framework: customers arrive according to a Poisson stream at some collection point, where after exponential time periods, they are collected in a batch and transported to a service system.

Single-server queues with Markovian Arrival Processes (MAP), MAP/G/1 queue provides a powerful framework to model dependences between successive inter arrival times at the bank, but typically for the case where the arrival process is Poisson and the service times are independent and identically distributed (i.i.d.) random variables with a general distribution function G, has been investigated by (Newel, 1966). The present study concerns single-server queues where the inter arrival times and the service times depend on a common discrete time Markov Chain; i.e., the so-called semi-Markov queues. As such, the model under consideration is a generalization of the MAP/G/1 queue, by also allowing dependencies between successive service times and between inter arrival times and service times. The more challenging problem of customers' behaviour on the waiting time was first addressed by (Barrer 1957; Gnedenko and Kovalenko, 1968) in the *M/M/*1 setting. There are also works on the similarly challenging problem of customers' behaviour on waiting plus service time, including (Loris-Teghem 1972; Gavish and Schweitzer, 1977; Hokstad; 1979; Van Dijk, 1990).

In order to serve the customers at faster rates, there must be good customer advisors, faster computers and better networks provided the computers are networked to avoid queuing or jamming networks. The need to analyze service mechanism in the rapidly growing computer and communication industry is the primary reason for the strengthening of queuing theory after the 1960's. Research on queuing networks and books such as (Coffman and Deming, 1973; Kleinrock, 1976) laid the foundation for a vigorous growth of the subject. In tracking this growth, one may cite the following survey type articles from the journal *Queuing Systems*: (Coffman and Hoffri, 1986), describing important computer devices and the queuing models used in analyzing their performance.

Traffic processes in computers and computer networks have necessitated the development of mathematical techniques to analyse them. The first article on queuing networks is by (Jackson, 1957). Mathematical foundations for the analysis of queuing networks are due to (Whittle, 1968; Kingman, 1969), who treated them in the terminology of population processes. Complex queuing network problems have been investigated extensively since the beginning of the 1970's.

Customers finally leave one by one after being served. This is because each teller can serve one person at a time. The interest of many researchers has not been in customers' departure, resulting in little or no research on departure.

## 2.2 QUEUING RESEARCHES IN THE BANKING INDUSTRY

Queuing-based teller staffing models was introduced into the financial industry in the late 1960s and early 1970s primarily to control increasing labour expenses (Brewton, 1989). Transactions such as deposits, withdrawals, and cash-checking were handled exclusively by human tellers until the introduction of Automated Teller Machines (ATMs). Although automated banking and on-line banking have decreased the need for human tellers, many retail banks still rely on them to provide timely and personalized customer service. Agboola and Salawa (2008) identified various Information and Communication (ICT) in use and determined how they could be utilized for optimal performance on business transactions in the banking industry.

Price Waterhouse Cooper's publication (1999) indicates that the primary aim of the introduction of modern electronic delivery channels was to cut costs and congestions by attracting lower-value customers to non-branch channels. However, the result was quite different in the experimented areas from that expected: high-value customers started to use these channels, while the lower-value customers continued to use branches. This is never the same in a developing country like Ghana where every customer want to correspond directly with the personal bankers.

Wenny and Whitney (2004) determined bank teller scheduling using simulation with arrival rates. They investigated scheduling of banks at a branch in Indonesia and the model accounts for real system behaviour including changing arrival rates, customer balking and reneging for randomly selected hours in the day. Travis and Michael (2007) assume that all servers at retail banking have the same service time distribution and that this distribution is exponential. These assumptions are motivated more by operational and analytical convenience than supported by data.

Oladapo (1988) study conducted in Nigeria revealed a positive correlation between arrival rates of customers and bank's service rates. He concluded that the potential utilization of the banks service facility was 3.18% efficient and idle 68.2% of the time. However, Ashley (2000) asserted that even if service system can provide service at a faster rate than customer's arrival rate, queues can still form if the arrival and service processes are random. Emuoyibofarhe, et al., (2005) studied the queuing problem of banks in Nigeria, taking First Bank plc, Marina branch Lagos as a case of interest, and apply queuing theory to solve the multiple server problem ($M/M/s/./\infty/\infty$ queuing system) which yielded results upon which the management of the bank could optimality distribute servers (cashier) to minimize waiting times, idle times and queues in the bank.

One week survey conducted by Elegalam, (1978) revealed that 59.2% of the 390 persons making withdrawals from their accounts spent between 30 to 60 minutes while 7% spent between 90 and 120 minutes. Baale (1996) while paraphrasing Alamatu and Ariyo (1983) observed that the mean time spent was 53 minutes but customers prefer to spend a maximum of 20 minutes. Their study revealed worse service delays in urban centres (average of 64.32 minutes) compared to (average of 22.2 minutes) in rural areas. To buttress these observations, Juwah (1986) found out that customers spend between 55.27 to 64.56 minutes making withdrawal from their accounts.

Efforts in this study are directed towards application of queuing models in capacity planning to reduce customer waiting time and total operating costs.

# CHAPTER 3

# METHODOLOGY OF THE STUDY

## 3.0 INTRODUCTION

In this chapter, the methodology used to achieve the objectives under study is clearly stated. Component of the basic queuing system, fundamental queuing relations and various assumptions related to model development are explicitly mentioned, and overall model structure is explained.

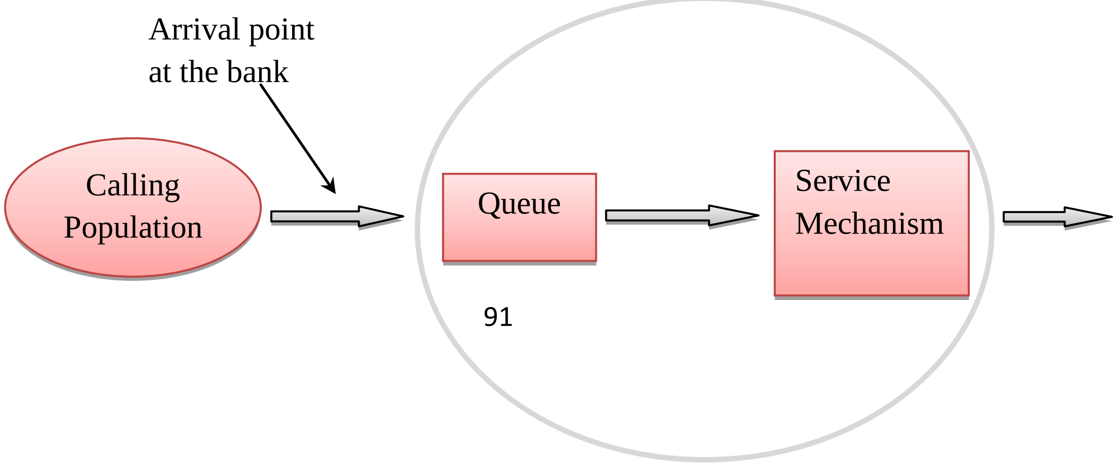
## 3.1 PRIMARY DATA COLLECTION

Data was collected at an even time and date at the three cases of study. The data of most fundamental importance were arrival time, service time and departure time of customers. These data were randomly collected within an hour interval between the period of 8:30am to 4:30 pm on 27th and 30th of April and 2nd May, 2012. These days are assumed to be some of the busiest days of the mouth. Summary of the data collected are shown in Table 4.1 and 4.2. Monday and Fridays are assumed to be the peak days for the case study, as customer's business proceeds from weekend are deposited in banks on Mondays and also the closing week's proceeds on Fridays. Most customers do withdraw for the weekends on Fridays. Series of interviews and observations of customer's attitude survey was also carried out to know the causes of some customers reneging, balking or jockeying.



## 3.2 DESCRIPTION OF QUEUING SYSTEM

A queuing system can be described as customers arriving for service, waiting for service if not immediate, utilizing the service, and leaving the system after being served. Figure 3.1 shows the basic components of a queuing system and identify them into four main elements namely: customer population, the queue, the service mechanism and departure.



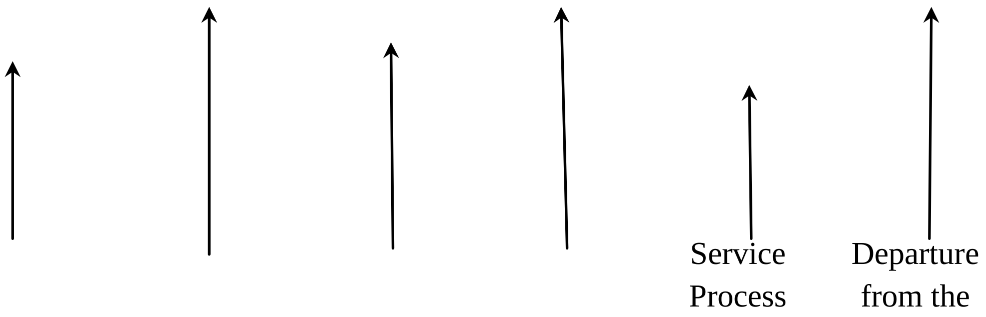



Figure 3.1 Hypothetical structure of a Queuing system in banks

The queue discipline is the order or manner in which customers from the queue are selected for service. There are a number of ways in which customers in the queue are served. Some of these are:

Customers can be served one by one or in batches. In this context, the rules such as;

i) Dynamic queue disciplines are based on the individual customer attributes in the queue. Few of such disciplines are:

   a) Service in Random Order (SIRO) - Under this rule customers are selected for service at random, irrespective of their arrivals in the service system. In this every customer in the queue is equally likely to be selected. The time of arrival of the customers is, therefore, of no relevance in such a case.

   b) Priority Service - Under this rule customers are grouped in priority classes on the basis of some attributes such as service time or urgency or according to some identifiable characteristic, and FCFS rule is used within each class to provide service. Treatment of VIPs in preference to other patients in a hospital is an example of priority service.

ii) Static queue disciplines are based on the individual customer's status in the queue. Few of such disciplines are:

a) If the customers are served in the order of their arrival, then this is known as the first-come, first-served (FCFS) service discipline. Prepaid taxi queue at airports where a taxi is engaged on a first-come, first-served basis is an example of this discipline.

b) Last-come-first-served (LCFS) - Sometimes, the customers are serviced in the reverse order of their entry so that the ones who join the last are served first. For example, assume that letters to be typed, or order forms to be processed accumulate in a pile, each new addition being put on the top of them. The typist or the clerk might process these letters or orders by taking each new task from top of the pile. Thus, a just arriving task would be the next to be serviced provided that no fresh task arrives before it is picked up. Similarly, the people who join an elevator last are the first ones to leave it.

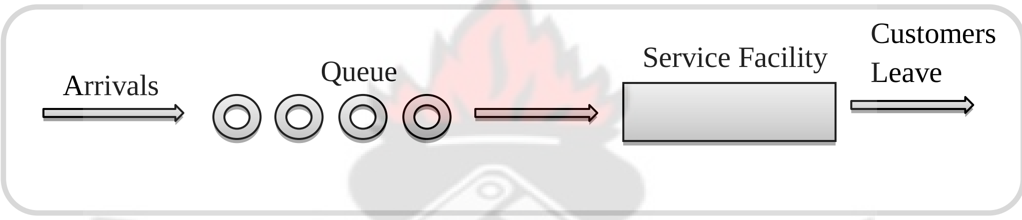
For the queuing models, the assumption that customers are serviced on the first-come-first-served basis would be considered.

The service mechanism can involve one or several queues consisting of more than one teller arranged in series and / or parallel. But for these cases of study, the tellers are arranged in parallel. The uncertainties involved in the queuing model's service mechanism are the number of tellers, the number of customers getting served at any time, and the duration and mode of service. Random variables are used to represent service times, and the number of tellers, when appropriate.

### 3.2.1 QUEUING SYSTEMS IN THE BANKING INDUSTRY

The central element of the system is a teller, who provides services to customers. Customers from some population of customers arrive at the bank to be served. Some of the services may include; cash deposit, cash withdrawal, cheque deposit and withdrawal, foreign currency deposit and withdrawal, and enquires. Below are three main queuing systems used in the banking industry:

1. *Single Teller, Single Queue*. The models that involve one queue – one service facility are called single teller models where customer waits till the service point is ready to take him for servicing.




2. Figure 3.2 Single Tellers Queuing System

3. *Several (Parallel) Tellers, Single Queue* – In this type of model there is more than one teller and each teller provides the same type of facility. The customers wait in a single queue until one of the service channels is ready to take them in for servicing.

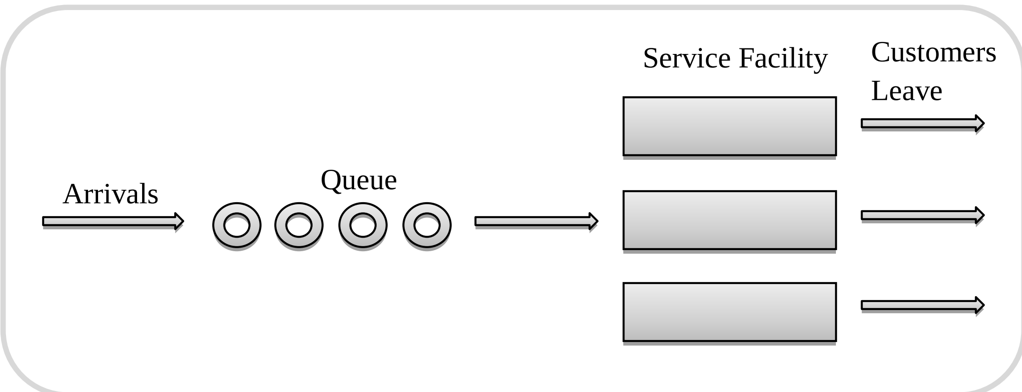


Figure 3.3 Multiple Tellers, Single-Stage Queuing System

4. *Several Servers, Several Queues* – This type of model consists of several tellers where each of the tellers has a different queue.

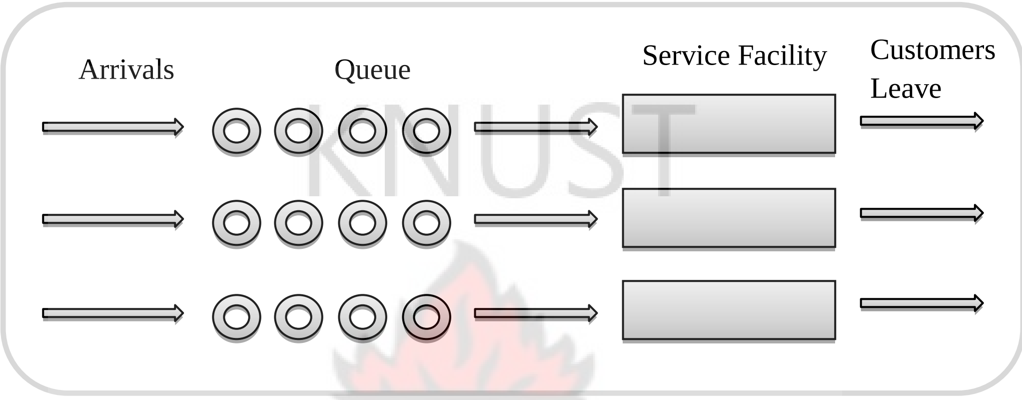



Figure 3.4 Several, Parallel Servers-Several Queues Model

## 3.3 FUNDAMENTAL QUEUING RELATIONS

Suppose customers arrive at time $t_1, t_2, \dots t_j$, random arrivals $T_j = t_j - t_{j-1}$ are inter arrival times. Let $S_j$ be the service time for the $j^{th}$ customer. Let $D_j$ be the time when the $j^{th}$ customer departs. Let $X$ be a possible number of customers. Take $X(t)$ to be the number of customers in the system at time $t$ and $X_j$ as the number of customers in the system just after $j^{th}$ customer departs. The waiting time $W$ is the time that a customer spends in a queuing system. Finally, by using $W_j$ as the waiting time of the $j^{th}$ customer and $W(t)$ as the total time it would take to serve all the customers in the waiting queue at time $t$ (the total remaining workload at time $t$), the intermediate queuing relations such as Lambda (λ) and Mu ($\mu$) are derived.

Lambda (λ) is the expected number of customers per whatever units $t$ is measured in. $\lambda = \frac{Number\ \ of\ customers}{Total\ \ time\ \ Involved}$

$\lambda t$ is therefore the average number of customer arrivals during $t$ amount of time.

Mu ($\mu$) is the average number of customers a teller can handle per unit time.

$$\mu = \frac{Number\ \ of\ customers\ \ a\ teller\ \ can\ handle}{Total\ \ time\ \ Involved}$$

## 3.4 CUSTOMER ARRIVAL AND INTER ARRIVAL DISTRIBUTIONS

There are many possible assumptions for distribution of the $T_j$:

*Assumption 1:* The customer inter-arrival times, i.e. the time between arrivals, are independent and identically distributed (usually written as "i.i.d"). Independent means, for example, that: customers do not come in groups, and tellers do not work faster when the queue is longer. All arriving customers enter the queuing system if there is room to wait. Also all customers wait till their service is completed in order to depart.

*Assumption 2:* The service times are independent and identically distributed random variables. Also, the tellers are stochastically identical, i.e. the service times are sampled from a single distribution. In addition, the tellers adopt a work-conservation policy, i.e. the teller is never idle when there are customers in the system and perform identical task.

### 3.4.1 PROBABILITY DISTRIBUTION

Probability of the number of customers in the system $P_j$ is often possible to describe the behaviour of a queuing system by means of estimating the probability distribution or pattern of the arrival times between successive customer arrivals. The mean values of most of the other interesting performance measures can be deduced from $P_j$: $P_j = P[there\ are\ j\ customers\ in\ the\ system]$.

### 3.4.2 EXPONENTIAL DISTRIBUTION

The most commonly used queuing models are based on the assumption of exponentially distributed service times and inter arrival times. The exponential distribution with parameter $\mu$ is given by $\mu\, e^{-\mu\, t}$ for $t \geq 0$. If $T$ is a random variable that represents inter-service times with exponential distribution, then $P(T \leq t) = 1 - e^{-\mu\, t}$ and $P(T > t) = e^{-\mu\, t}$. It has the interesting property that its mean is equal to its standard deviation $E[T] = \frac{1}{\mu}$.

### 3.4.3 POISSON DISTRIBUTION

The Poisson distribution is used to determine the probability of a certain number of arrivals occurring at a given time.

*Definitions*

1. The counting process $\{X(t_j),\ t_j \geq 0\}$ is said to be a *Poisson process* having random selection of inter arrival time, $T$ in a time interval$(t_{j-1}, t_j)$, $T = \Delta t = t_j - t_{j-1}$ from the probability of having $k$ points with an arrival rate $\lambda,\ \lambda > 0$, if:
   a. $X(0) = 0$. That is, when arrival enters an empty queue.
   b. The process has independent increments.
   c. The number of arrivals in any interval of length $t$ is Poisson distributed with mean $\lambda t$. That is for all $t_{j-1}, t \geq 0$,

$$P\{X(t_j) - X(t_{j-1}) = k\} = \frac{[\lambda(t_j - t_{j-1})]^k}{k!} e^{-[\lambda(t_j - t_{j-1})]} \qquad k = 0, 1, 2, \ldots$$

Given a time interval $(0, t)$, $t = t - 0$ from a probability having $k$ number of customers. $P[t = k] = \frac{(\lambda t)^k}{k!} e^{-(\lambda t)}$

Where $\lambda$ is an average customer arrival rate, $k$, the number of customers arriving at time interval $t$ (number of events) and $\lambda t$, therefore the average number of customers arriving during $t$ amount of time. The random time between arrival of customers, $t$, in the random arrival time $t_j$, $j = 1, 2, \ldots$ are i.i.d and follows exponential distribution with parameter $\lambda$, is $E[t] = \frac{1}{\lambda}$.

Thus, the mean inter arrival time $t$ is the reciprocal of the arrival rate. The Poisson process has stationary increments and that expected number of customers to arrive in time interval $t$ is $E(k) = \lambda t$. Arrivals occurring according to a Poisson process are often referred to as random arrivals. This is because the probability of arrival of a customer in a small interval is proportional to the length of the interval and is independent of the amount of elapsed time since the arrival of the last customer, $t_j$. That is, when customers are arriving according to poisson process, a customer is likely to arrive at one instant as any other, regardless on the instants at which the other customer arrives.

2. The counting process $\{X(t), t \geq 0\}$ is said to be a *Poisson process* having rate $\lambda$, $\lambda > 0$, if:
   a. $\mathrm{X}(0) = 0$;
   b. $\{X(t), t \geq 0\}$ has stationary independent increments
   c. Probability of 0 customer arriving at time interval $t$;

$$P(X(0) = 0) = e^{-(\lambda t)} = 1 - \lambda t + \frac{(\lambda t)^2}{2!} \dots = 1 - \lambda t + 0(t)$$

$$\longrightarrow \; P(0) = 1 - \lambda t$$

   d. Probability of exactly 1 customer arriving at time interval $t$;

$$P(X(t) = 1) = \lambda e^{-(\lambda t)} = \lambda t[1 - \lambda t + \frac{(\lambda t)^2}{2!} \dots] = \lambda t + 0(t) \; \longrightarrow \; P(1) = \lambda t$$

   e. Probability of more than 1 customer arriving at time interval $t$;

$$P(X(t) \geq 2) = \cdots = 0$$

## 3.5 TELLER UTILIZATION FACTOR (ρ)

If the queuing system consists of a single teller, then the utilization $(\rho)$ is the fraction of the time in which the teller is busy, i.e., occupied. In case when the source is infinite and there is no limit on the number of customers in the single teller queue, the teller utilization is given by: $\rho = \frac{arrival\ \ rate}{service\ \ rate} = \frac{\lambda}{\mu}$

If there are more than one teller serving, the utilization factor becomes: $\rho = \frac{\lambda}{s\,\mu}$, where $s$ is the number of tellers. $\rho$ is used to formulate the condition for stability of the queuing system. The condition for stability is always between zero and one, $0 \leq \rho \leq 1$. If the utilization exceeds this range then the situation is unstable and would need additional teller(s). That is, on average the number of customers that arrive in a unit time must be less than the number of customers that can be served.

## 3.6 LITTLE'S LAW

The relation between $L$ and $W$ is given by Little's Law. Letting $L$ to be the average number of customers in the queuing system at any moment of time assuming that the steady-state has been reached, $L$ can be broken down into $L_q$, the average number of customers waiting in the queue, and $L_s$, the average number of customers in service. Since customers in the system can only be either in the queue or in service, it goes to show that $L = L_q + L_s$. Likewise, $W$ being the average time a customer spends in the queuing system. $W_q$ is the average amount of time spent in the queue itself and $W_s$ is the average amount of time spent in service. As was the similar case before, $W = W_q + W_s$. It should be noted that all averages in the above definitions are the steady-state averages.

Defining $\lambda$ as the arrival rate into the system, that is, the number of customers arriving in the system per unit of time, it can be shown that

$$L = \lambda W, \qquad L_q = \lambda W_q, \qquad L_s = \lambda W_s$$

## 3.7 QUEUING MODEL DESCRIPTION

Kendall shorthand notation is used to identify and describe the queuing systems. The basic representation introduced by David Kendall has the form $A/B/s/K/m/Z$, where $A$ is the arrival time distribution, $B$ is the service time distribution, $s$ is the number of tellers, $K$ is the largest possible number of customers in the system (the capacity of the system), $m$ is the number of customers in the source, and $Z$ is the queuing discipline.

This notation can be shortened to $A/B/s$ where $K$ and $m$ are assumed to be infinite and $Z$ is assumed to be first-come, first-serve (FCFS). Banking queues possess the Markovian Memoryless properties and hence $A$ and $B$ are replaced by the two $M's$ to obtain the $M/M/s$ model that would be used to estimate the number of tellers needed during each time interval.

Modelling the three main queuing systems in the case of study, that is, Single Teller with Single-Stage Queue, Multiple Tellers with Single-Stage Queue, and Multiple single tellers with Single-Stage Queues in Parallel, would be possible by using $M/M/1$ and $M/M/s$ models. These models arrival's occur according to Poisson process and has exponential distribution. These two assumptions are often called Markovian properties, hence the use of the two "$M,s$" in the notation used for the models

### 3.7.1 $M/M/1$ MODEL

It is the simplest realistic queue model which assumes the arrival rate follows a Poisson distribution and the time between arrivals follows an exponential distribution, has only one teller, with infinite system capacity and population, and with First-In, First Out (FIFO) as its queuing discipline.

Consider a single stage queuing system where the arrivals are according to a Poisson process with average arrival rate λ per unit time. That is, the time between arrivals is according to an exponential distribution with mean $1/\lambda$. For this system the service times are exponentially distributed with mean $1/\mu$ and there is a single teller. The service rate must be greater than the arrival rate, that is, $\mu > \lambda$. If $\mu \leq \lambda$, the queue would eventually grow infinitely large. Figure 3.5 shows transition diagram for a single teller processing of a $M/M/1$ model.

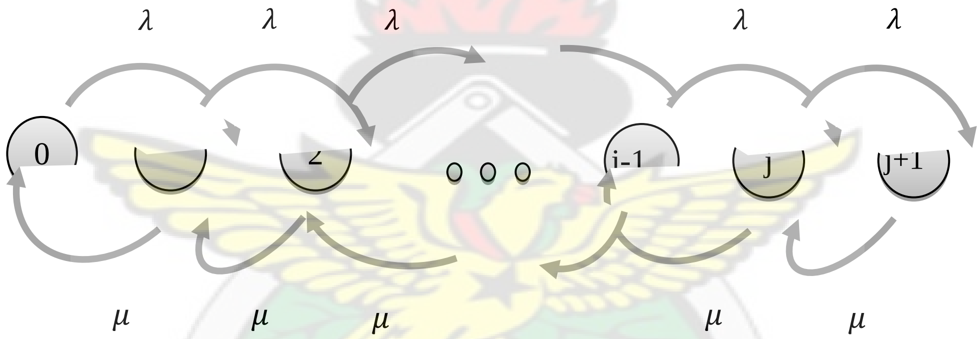



Figure 3.5 Transition diagram for a single teller processing.

Steady state probability $P_j$ of being in state $j = 0, 1, 2, \ldots j + 1$ is:

$$
\begin{array}{ll}
j = 0 & \lambda P_0 = \mu P_1 \\
j = 1 & (\lambda + \mu) P_1 = \lambda P_0 + \mu P_2 \\
j = 2 & (\lambda + \mu) P_2 = \lambda P_1 + \mu P_3 \\
\vdots & \vdots \\
jth\ equation & (\lambda + \mu) P_j = \lambda P_{j-1} + \mu P_{j+1}
\end{array}
$$

Putting it all together:

$$\lambda P_0 = \mu P_1, \qquad \lambda P_1 = \mu P_2, \qquad \dots, \qquad \lambda P_j = \mu P_{j+1}$$

$$P_1 = \frac{\lambda}{\mu} P_0, \qquad P_2 = \left(\frac{\lambda}{\mu}\right)^2 P_0, \quad \dots, \qquad P_j = \left(\frac{\lambda}{\mu}\right)^j P_0$$

Since

$$\sum_{j=0}^{\infty} P_j = 1, \quad \Rightarrow \quad P_0 \sum_{j=0}^{\infty} \left(\frac{\lambda}{\mu}\right)^j = 1 \quad \Rightarrow \quad P_0 = \frac{1}{\sum_{j=0}^{\infty} \left(\frac{\lambda}{\mu}\right)^j}$$

$$Let \quad \rho = \frac{\lambda}{\mu}, \quad then \quad \sum_{j=0}^{\infty} \left(\frac{\lambda}{\mu}\right)^j = \sum_{j=0}^{\infty} \rho^j = \frac{1-\rho^{\infty}}{1-\rho} = \frac{1}{1-\rho}, \quad \forall\, \rho < 1$$

Therefore, the probability of any given number of customers being in the system is given by

$$P_0 = \frac{1}{\sum_{j=0}^{\infty} \rho^j} = 1 - \rho$$

and $P_j = \rho^j (1-\rho)$ as the steady-state probability of state $j$. If $\rho \geq 1$, then it must be that $\lambda \geq \mu$, and if the arrival is greater than the service rate, then the state of the system will grow without end.

The steady-state probability for this system is known, $L$ can now be solved. Assuming $L$ is the average number of customers present in the system, waiting in queue or being served, the formula is represented as;

$$L = \sum_{j=0}^{\infty} jP_j \quad = \quad (1-\rho)\sum_{j=0}^{\infty} j\rho^j \quad = \quad (1-\rho)\rho \sum_{j=0}^{\infty} j\rho^{j-1}$$

$$= \quad (1-\rho)\rho \frac{d}{d\rho}\left(\sum_{j=0}^{\infty} \rho^j\right) \quad = \quad (1-\rho)\rho \frac{d}{d\rho}\left(\frac{1}{1-\rho}\right)$$

$$= \quad (1-\rho)\rho\left(\frac{1}{(1-\rho)^2}\right) \quad = \quad \frac{\rho}{(1-\rho)} \quad = \quad \frac{\lambda/\mu}{1-\lambda/\mu} = \frac{\lambda}{\mu-\lambda}$$

The average time customers spend in system, waiting plus being served $W$ is

$$W = \frac{L}{\lambda} = \frac{\lambda}{\mu-\lambda} \cdot \frac{1}{\lambda} = \frac{1}{\mu-\lambda}$$

The average time customers spend in waiting in queue before service starts $W_q$ is

$$W_q = W - \frac{1}{\mu} = \frac{1}{\mu-\lambda} - \frac{1}{\mu} = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{\rho}{\mu-\lambda}$$

The average time a costumer spent in service $W_s$ is

$$W_S = W - W_q = \frac{1}{\mu-\lambda} - \frac{\rho}{\mu-\lambda} = \frac{1-\rho}{\mu-\lambda}$$

The average number of customers waiting in the queue $L_q$ is

$$L_q = \lambda W_q = \lambda \cdot \frac{\lambda}{\mu(\mu-\lambda)} = \frac{\lambda^2}{\mu^2(1-\lambda/\mu)} = \frac{\rho^2}{1-\rho}$$

$L_s$ can be solved to determine the average number of customers that are in service at any given moment. In this particular system, there will be one customer in service except for when there are no customers in the system. Thus, it can be calculated as

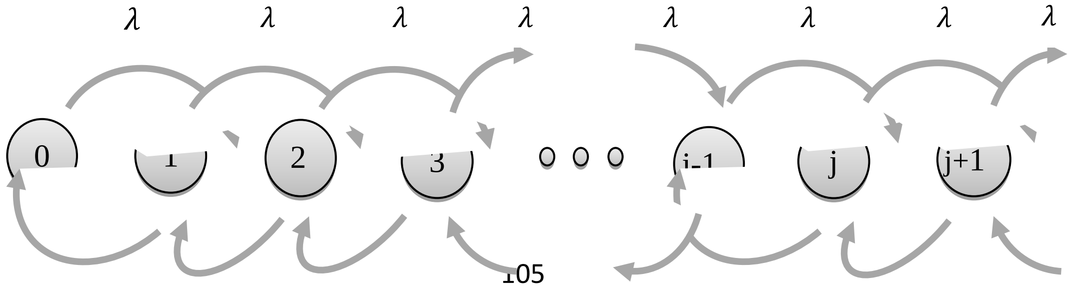
$$L_S = L - L_q = \frac{\rho}{(1-\rho)} - \frac{\rho^2}{(1-\rho)} = \frac{\rho(1-\rho)}{(1-\rho)} = \rho$$

These formulas imply that it can be easy to tell how busy the teller is by just watching the length of the queue.



### 3.7.2 $M/M/s$ MODEL

The description of an $M/M/s$ queue is similar to that of the classic $M/M/1$ queue except that there are $s$ tellers. By letting $s = 1$, all the results for the $M/M/1$ queue can be obtained. The number of customers in the system at time $t$, $X(t)$, in the $M/M/s$ queue can be modelled as a continuous Time Markov Chain. The condition for stability is $\rho = \frac{\lambda}{s\mu} < 1$ where $\rho$ is called the teller utilization factor, the proportion of time on average that each teller is busy. The total service rate must be greater than the arrival rate, that is, $s\mu > \lambda$. If $s\mu \leq \lambda$, the queue would eventually grow infinitely large. Figure 3.6 shows a transition diagram for a multiple teller processing of a $M/M/s$ model.



$\mu$ $2\mu$ $3\mu$ $(j-1)\mu$ $j\mu$ $j\mu$ $j\mu$

Figure 3.6 Transition diagram for a multiple teller processing

Steady state probability $P_j$ of being in state $j = 0, 1, 2, \dots j+1$ is:

$$\begin{array}{ll}
j = 0 & \lambda P_0 = \mu P_1 \\
j = 1 & (\lambda + \mu)P_1 = \lambda P_0 + 2\mu P_2 \\
j = 2 & (\lambda + 2\mu)P_2 = \lambda P_1 + 3\mu P_3 \\
j = 3 & (\lambda + 3\mu)P_3 = \lambda P_2 + 4\mu P_4 \\
\vdots & \vdots \\
jth\ equation & (\lambda + j\mu)P_j = \lambda P_{j-1} + (j+1)\mu P_{j+1}
\end{array}$$



Putting them together to obtain,

$$\lambda P_0 = \mu P_1, \quad \lambda P_1 = 2\mu P_2, \quad \lambda P_2 = 3\mu P_3, \quad \cdots, \quad \lambda P_j = (j+1)\mu P_{j+1}$$

$$P_1 = \frac{\lambda}{\mu} P_0, \quad P_2 = \frac{\lambda}{2\mu} P_1, \quad P_3 = \frac{\lambda}{3\mu} P_2, \quad \cdots, \quad P_{j+1} = \frac{\lambda}{(j+1)\mu} P_j,$$

$$P_1 = \frac{\lambda}{\mu} P_0, \quad P_2 = \frac{\lambda}{2\mu}\left(\frac{\lambda}{\mu}\right) P_0, \quad P_3 = \left(\frac{\lambda}{3\mu}\right)\left(\frac{\lambda}{2\mu}\right)\left(\frac{\lambda}{\mu}\right) P_0, \quad \cdots, \quad P_{j+1} = \left(\frac{\lambda}{(j+1)\mu}\right) \dots \left(\frac{\lambda}{\mu}\right) P_0$$

$$P_1 = \frac{\lambda}{\mu} P_0, \quad P_2 = \frac{1}{2!}\left(\frac{\lambda}{\mu}\right)^2 P_0, \quad P_3 = \frac{1}{3!}\left(\frac{\lambda}{\mu}\right)^3 P_0, \quad \cdots, \quad P_{j+1} = \frac{1}{(j+1)!}\left(\frac{\lambda}{\mu}\right)^{j+1} P_0,$$

$$\cdots, \quad P_{j+k} = \frac{1}{(j+k)!}\left(\frac{\lambda}{\mu}\right)^{j+k} P_0, \quad k = 1, 2, 3, \dots$$

And in general: $P_j = \frac{1}{j!}\left(\frac{\lambda}{\mu}\right)^j P_0$

Since

$$\sum_{j=0}^{\infty} P_j = 1, \quad \Rightarrow \quad P_0 \sum_{j=0}^{\infty} \left(\frac{\lambda}{j!\,\mu}\right)^j = 1 \quad \Rightarrow \quad P_0 = \frac{1}{\sum_{j=0}^{\infty} \left(\frac{\lambda}{j!\,\mu}\right)^j}$$

Let

$$\rho = \frac{\lambda}{s\mu}, \quad then \quad \sum_{j=0}^{\infty} \frac{1}{j!}\left(\frac{\lambda}{\mu}\right)^j = \sum_{j=0}^{\infty} \frac{(s\rho)^j}{j!} \;, \quad \forall\, \rho < 1$$

The probability of any given number of customers being in the system is given $P_0$ follows from normalization, yielding

$$P_0 = \left(\sum_{j=0}^{s-1} \frac{(s\rho)^j}{j!} + \frac{(s\rho)^s}{s!} . \frac{1}{1-\rho}\right)^{-1}$$

The probability that at any given time there are no customers waiting or being served at steady state:

$$P_0 = \left(\sum_{j=0}^{s-1} \frac{(s\rho)^j}{j!} + \frac{(s\rho)^s}{s!\,(1-\rho)}\right)^{-1}$$

The probability that at any given time there are $j$ customers in the system;

If $j \le s$ then Probability ($j$ in system) $= P_0 \frac{(s\rho)^j}{j!}$

If $j \ge s$ then Probability ($j$ in system) $= P_0 \frac{s^s \rho^j}{s!}$

The expression for the average waiting time and queue lengths are fairly complicated and depend on the probability of the average number of customers waiting in queue to be served,

$$L_q = \frac{P_0(\lambda/\mu)^S \rho}{s!\,(1-\rho)^2} = P_0 \frac{S^S \rho^{S+1}}{S!\,(1-\rho)^2}, \quad where \quad \rho = \frac{\lambda}{s\mu}$$

The average number of customers in service $L_s$,

$$L_s = \sum_{j=1}^{s-1} jP_j + \sum_{j=s}^{\infty} sP_j = s\rho$$

Now, the average number of customers in the system becomes

$$L = L_q + L_s = L_q + s\rho = L_q + \frac{\lambda}{\mu}$$

The average time customers spend in waiting in queue before service starts $W_q$ is $W_q = \frac{L_q}{\lambda}$.

The average time customers spent waiting in the system, including service $W$ is

$$W = \frac{L}{\lambda} = \frac{L_q + \frac{\lambda}{\mu}}{\lambda} = \frac{L_q}{\lambda} + \frac{1}{\mu} = W_q + \frac{1}{\mu}$$

The average time a customer spent service $W_s$ is

$$W_s = W - W_q = \left(W_q + \frac{1}{\mu}\right) - W_q = \frac{1}{\mu}$$

## 3.8 DATA SHEET PROCESSING

For the best results, estimates of the arrival rate, service rate, departure rate and current number of tellers should be obtained. With this information available, data collected can be analysed from the respective banks by first keying them onto the Microsoft Excel 2010 data table. Simple Microsoft Excel functions are the basis for creating data tables and graphs which show the effects of customers' arrival rate, service time on utilization, waiting times and times customers spend in the system.

# CHAPTER 4

# ANALYSIS AND DISCUSSION OF RESULTS

## 4.0 INTRODUCTION

The $M/M/s$ queuing model is used to calculate the performance measures that are necessary in analysing data collected. The following assumptions are made when modelling in this environment.

1) The customers are patient (no balking, reneging, or jockeying) and come from a population that can be considered infinite.
2) Customer arrivals are described by a poisson distribution with an average arrival rate of $\lambda$ (lambda). This means that the time between successive customer arrivals follows an exponential distribution with an average of $1/\lambda$.
3) The customer services are described by a poisson distribution with an average service rate of $\mu$ (mu). This means that the service time for one customer follows an exponential distribution with an average of $1/\mu$.

4) The waiting line priority rule used is first-come, first-serve.

The poisson distribution is a limiting case of the binomial distribution and therefore, these two important assumptions would be necessary when dealing with poisson process.



1) $\lambda$ is constant. Each of the expected arrival rate does not change over the hour period.
2) Independence, which means no memory and no groups. Customers do not tend to arrive in groups. The fact that a customer just walked into the banking hall does not make it any more or less likely that a different customer is coming soon. If nobody comes or if ten times $\lambda$ customers come, during the last time period, the expected number of customers during the next time period is still $\lambda$.

Poisson and exponential distributions are often called the Markovian properties, hence the use of the two "M's" in the notation used for this model. The operating characteristics of a queuing system are calculated using $M/M/s$ queuing model proved earlier.

## 4.1 PRIMARY DATA EXAMINATION

Using Microsoft 2010, raw data graphs for respective days depicting the arrival, service and departure times of customers are drawn and shown in Appendix 1.0.

These graphs are plotted with number of customers against time. Customer number starts from 0 $to\ j$, where $j = 1, 2, 3, \dots$ and time begins from zero second to one hour. From left to right part of each graph in figures 4.1 below, and all figures in Appendix 1.0, show the departure time curve followed by service time and finally, the arrival time. The gap between arrival time and service time are wider than that of service time and departure time except data collected on 27th April, 2012 morning that the arrivals are not all that wider. But as the time increases, the gab between arrival and service times widens although that of the service and departure times remain partially constant. The graph below shows raw data graph for GCB on Wednesday, 2nd May, 2012.

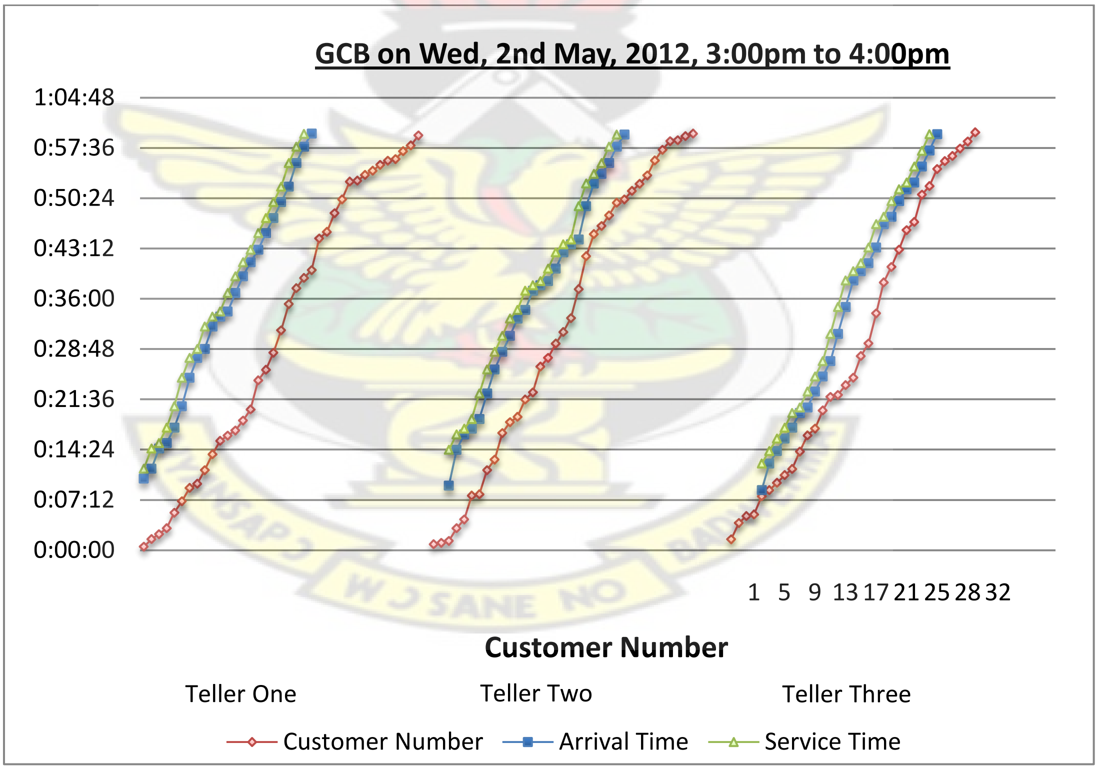



Figure 4.1 Raw Data Graph for GCB on 2nd May, 2012

Before the data collected is analysed to see if they possess the Markovian properties, the behaviour of customers is now studied from the graphs in figures 4.1, and all figures in Appendix 1.0, taking into consideration some of the comments raised by customers during observations. Some customers attributed the reasons why they jockey, renege or balk from queue to long queues, long waiting time, network problem, the slowness of a teller and bad attitude on the side of a staff. The tellers also testified to the fact that some customers are impatient, assumed that they are loyal customers and there is no need to join the queue.

Looking at customer's behaviour from the data collection point of view, the queue population was unlimited; there were no collusion, jockeying, reneging and balking behaviour of customers at all the branches. There were preferential treatments for some "special people" who receive quick services at the expense of others. Tellers especially from Barclays and Ecobank easily vacate their cubicles at will. This allows customers make complains after realising a long waiting time than expected.

The table below shows summary of primary data that were randomly selected within an hour on 27th and 30th April, and 2nd May, 2012.

Table 4.1 Summary of Primary Data Collected on 27th and 30th April and 02nd May, 2012

| **DATE** | **27th April** | | | **30th April** | | | **02nd May** | | |
|---|---|---|---|---|---|---|---|---|---|
| **TIME RANGE** | *8:30am - 9:30am* | | | *12:00pm – 1:00pm* | | | *3:00pm – 4:00pm* | | |
| **BANK** | **BBG** | **ECO** | **GCB** | **BBG** | **ECO** | **GCB** | **BBG** | **ECO** | **GCB** |
| **Number of Customers** | 64 | 30 | 33,35,32 | 48 | 26 | 26,31,30 | 68 | 31 | 37,35,32 |
| ***Total*** | | | 100 | | | 87 | | | 104 |

| Number of Tellers | 3 | 2 | 3 | 3 | 2 | 3 | 3 | 2 | 3 |
|---|---|---|---|---|---|---|---|---|---|
| Number of Customers served | 55 | 23 | 27,26,25 | 45 | 25 | 25,28,27 | 55 | 28 | 26,26,28 |
| ***Total*** | | | 78 | | | 80 | | | 80 |

All tellers were present throughout the data collection exercise. GCB uses multiple single tellers with single-stage Queues arrange in Parallel whilst BBG and Ecobank use single queue to multiple tellers. 2nd May receives the highest number of customers arriving even at the later part of the closing period of the working hours. It was made clear by the employees that Mondays and Fridays are peak days and since Monday, 1st May was Workers Day holiday, the following day was busier than expected.

## 4.2 DISCUSSION OF FINDINGS

All data are collected within an hour. In order to have simple average calculations throughout this analysis, an hour $(01:00:00)$ would be taken as $1.00$. The rate of customer arrival at the banks fluctuates throughout the day and there might be differences in arrivals from day to day, but it is assumed that they are independent and identically distributed.

Changes in banking industry as a result of challenges posed by technologically innovative competitors such as those resulting from deregulation, has seen to it that BBG, Ecobank and GCB are fully equipped with rapid global networking, induction of electronic banking and the rise in personal wealth have thus provided good customer service as a key strategy for firm profitability. Hence customers from these banks can transact business at any branch of their choice. Therefore, customers arriving rate can be attributed to the size of the banking hall and the location of these branches but not their year of establishment (i.e. whether it is an old or new branch).

These make it obvious that, the customer arriving rate at GCB, Suame Branch may be greater than that of BBG, Tanoso Branch and Ecobank, Ash-town Branch respectively. Throughout the three hours in three days of data collection at the respective branches, five hundred and fifty-eight (558) customers arrived at the selected branches, (194) in 27th April, (161) – 30th April and (203) in 2nd May, 2012. 32.26% of customers arrived at BBG, 15.59% at Ecobank whilst 52.15% at GCB. The lowest arriving customers were recorded on 30th April for all selected branches. It was noted that 84.05% of the arriving customers were served.

Consider the Poisson model for data collected on 30th April, 2012 in Table A2.0 in Appendix 2.0. Lambda ($\lambda$) for BBG is 48.00, Ecobank, 52.00 and that of GCB is 87.00 expected arriving customers per hour, taken one hour to be 1.00. Using a probability limit of 0.95, by looking down the running total of the probabilities, the first number that is bigger than or equal to 0.95 is selected. On the side of BBG from Table A2.0, the probability, 0.9508, is in the row where number of customers ($X_j$) is 77, hence 0.9508 is the probability that 77 or fewer customers would have arrived at BBG on that faithful hour and day. The table below shows the number of customers that should have arrived at the respective banks and their probability using a probability limit of 0.95.

Table 4.2 Probability of the number of customers or fewer that would have arrived by 0.95 probability limit.

| DATE | BANK | $X_j$ | $Prob\ of\ \leq X_j$ |
|---|---|---|---|
| 27TH APRIL | BBG | 77 | 0.9508 |
| | ECOBANK | 48 | 0.9514 |
| | GCB | 117 | 0.9572 |
| 30TH APRIL | BBG | 60 | 0.9605 |
| | ECOBANK | 64 | 0.9547 |
| | GCB | 103 | 0.9586 |
| 2 ND MAY | BBG | 82 | 0.9573 |
| | ECOBANK | 67 | 0.9504 |
| | GCB | 121 | 0.9542 |

The Poisson distribution provides a realistic model for many random phenomena and it is characterised by the mean (average arrival rate) and variance being equal. Distribution with two different average arrival rate ($\lambda$) of BBG, Ecobank and GCB for each day are plotted on the graphs in Appendix 2.0. Poisson distribution is skewed. They may seem to be symmetrical about their average arrival rates, but are not. The degree of skewness depends on $\lambda t$. The larger $\lambda t$ is, the more the Poisson distribution approaches a symmetrical bell shape, the shape of the normal distribution. The smaller $\lambda t$ is, the more the peak is to the left of centre.

Consider, the poisson distribution graph of data collected on 27$^{th}$ April, 2012 in Figure 4.2 below, the $\lambda t$ of GCB is greater than that of Ecobank and BBG respectively and all their shape approach a symmetrical bell shape.

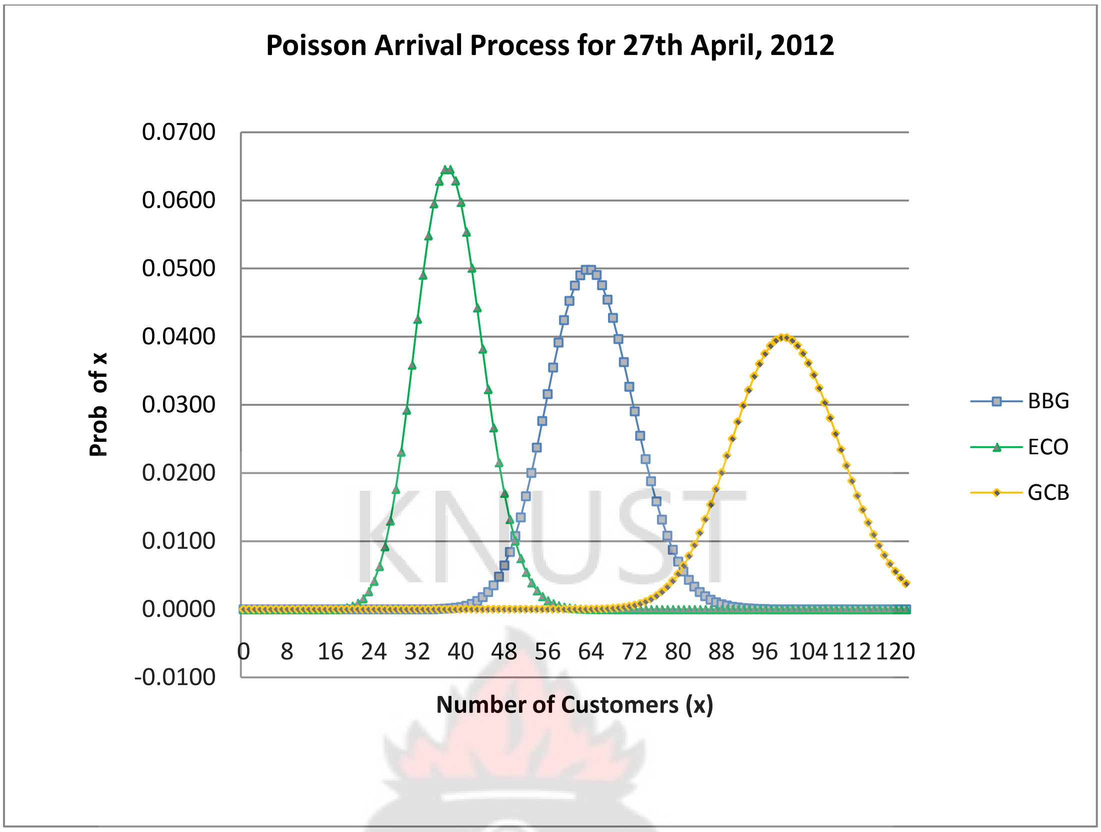



Figure 4.2 Poison Arrival Process for 27th April, 2012

As $X_j$ increases from 0, the probability of exactly $X_j$ arrivals rises at first. The highest probability is at $X_j = \lambda t - 1$ and $X_j = \lambda t$. According to Table A2.0 in Appendix 2.0, for Ecobank, it was $X_j = 37$ and $X_j = 38$ with probability of $X_j$ being 0.5430 and $X_j = 99$ and $X_j = 100$ with probability of $X_j$ being 0.5266 for GCB. After attaining a higher value of $X_j$, the probability of $X_j$ diminishes, approaching zero (0) as $X_j$ gets very large. Table 4.3 below shows the probability of $X_j$, where the curve starts to turn for all data collected.

Table 4.3 Probability of the highest turning points of the Poisson Distribution Graphs in Appendix 2.0

| DATE | BANK | $X_j$ | $Prob\ of\ \le X_j$ |
|---|---|---|---|
| 27TH APRIL | BBG | 63 and 64 | 0.5332 |
| | ECOBANK | 37 and 38 | 0.5430 |
| | GCB | 99 and 100 | 0.5266 |
| 30TH APRIL | BBG | 47 and 48 | 0.5383 |
| | ECOBANK | 51 and 52 | 0.5368 |
| | GCB | 86 and 87 | 0.5285 |
| 2 ND MAY | BBG | 67 and 68 | 0.5322 |
| | ECOBANK | 54 and 55 | 0.5358 |
| | GCB | 103 and 104 | 0.5260 |



## 4.3 QUEUING MODEL ANALYSIS

Presentation of result on queuing analysis on data collected within April and May, 2012 is shown in Table A3.0 in Appendix 3.0. On Table A3.0, the Total Time involved $(t)$, Number of Customers Arrived $(X_j)$, Number of Customers Served and the Number of Tellers $(s)$ are entered into the input section. The 'Intermediate Calculations' calculates the Average Arrival Rate $(\lambda)$ and Average Service Rate $(\mu)$. 'Performance Measures' also calculates: the Average Teller Utilization - $Rho$ $(\lambda/s\mu)$, Probability that the system is empty - $P_0$, Average number of customers in the system – $L$, Average number of customers in the queue - $L_q$, Average number of customers waiting in the service - $L_s$, Average time spent in the system – $W$, Average time spent in the queue - $W_q$ and finally, the Average time a customer is served - $W_s$. The extreme column of the table shows the unit of measurements.

### 4.3.1 PERFORMANCE ANALYSIS

One advantage of using the $M/M/s$ model is that given an arrival rate, average service duration and the number of tellers, formulae for performance measures such as the probability of a positive waiting time or the average waiting time is easily obtained and implemented on a spreadsheet.

All average arriving rates are greater than the average service rate. The closer the average service rate to the average arriving rate, the smaller utilization factor becomes. The lowest capacity utilization of 0.35556 was recorded by BBG on 30th April, 2012 with its arriving rate of 48.00 against its average service rate of 45.00 at time where all tellers were available. Ecobank lowest utilisation factor of 0.59375 was also recorded on 27th April, 2012 whilst that of GCB, 0.36250 was also recorded on 30th April, 2012. All tellers were available during data collection and this implies that the tellers were at their best and managed to serve many customers as possible.

All utilization factors for the data in Table A3.0 were stable. Generally, there are two items of interest; utilization - how much time is the tellers busy or working, and waiting time - how long do customers have to wait for service? These two items usually conflict. As utilization ($\rho$) increases, waiting time increases and vice versa. The highest utilisation factor of 0.68750 was recorded by Ecobank on 2nd May, followed by 0.61905 on 30th April, 2012. It was observed that the very day that the highest utilization factor was recorded, the highest waiting time of 0.04741 was also obtained.

The waiting time is measured from the time of the customer demands for service to the time at which transaction begins (e.g. a teller is available to transact business). It is important to note that the model's waiting time predictions pertained only to waiting times due to teller unavailability and do not include any other possible waiting times prior to seeing a teller such as filling savings and debit forms and seeing personal banker, which would have to been estimated independently. As the number of tellers decreases, the closer the utilization approaches one. That is the main reason why the highest utilization factors were recorded at Ecobank on 2nd May and 30th April, 2012 respectively, when there were only two tellers working.

The greater the capacity utilization factor, the smaller the probability of the system to be empty and as the probability increases, the more probability of the number of customers in the entire queuing system approaches zero. Table 4.4 shows the highest probability that a customer and three customers are in the Queuing system as depicted in Figure 4.5 below and all figures in Appendix 3.0. The probability that there are no customers in the system is given in Table A3.0.

Table 4.4 Probability that a customer and five customers are in the Queuing system

| | | Highest Probability that a customer is in the system | | Probability that three customers are in the system | |
|---|---|---|---|---|---|
| **DATE** | **BANK** | $X_j$ | $Prob\ of\ X_j$ | $X_j$ | $Prob\ of\ X_j$ |
| **27TH APRIL** | BBG | 1 | 0.30584 | 3 | 0.08032 |
| | ECOBANK | 1 | 0.25490 | 3 | 0.10671 |
| | GCB | 1 | 0.26902 | 3 | 0.09448 |
| **30TH** | BBG | 1 | 0.33905 | 3 | 0.06858 |

| | | | | | |
|---|---|---|---|---|---|
| **APRIL** | ECOBANK | 1 | 0.23529 | 3 | 0.11164 |
| | GCB | 1 | 0.33167 | 3 | 0.07109 |
| **2ND MAY** | BBG | 1 | 0.28277 | 3 | 0.08907 |
| | ECOBANK | 1 | 0.18519 | 3 | 0.12035 |
| | GCB | 1 | 0.26377 | 3 | 0.09658 |

Considering 2nd May, 2012 from Table A3.0, it was observed that there was a huge difference between the capacity utilization factors of BBG and that of Ecobank, that is, 0.41212 and 0.68750, and the probability of the number of customers in the system to be zero was 0.28277 and 0.18519 respectively. This means that, there would be 1.34257 and 2.60741 average number of customers in the system and would take 0.01974 and 0.04741 hour(s) average time to be in the system respectively. From Figure 4.4, the probability that only one customer is in the system are 0.33589, 0.30270 and 0.34490 respectively. The probability that only three customers are in the system are 0.08032, 0.10671 and 0.09448 respectively.

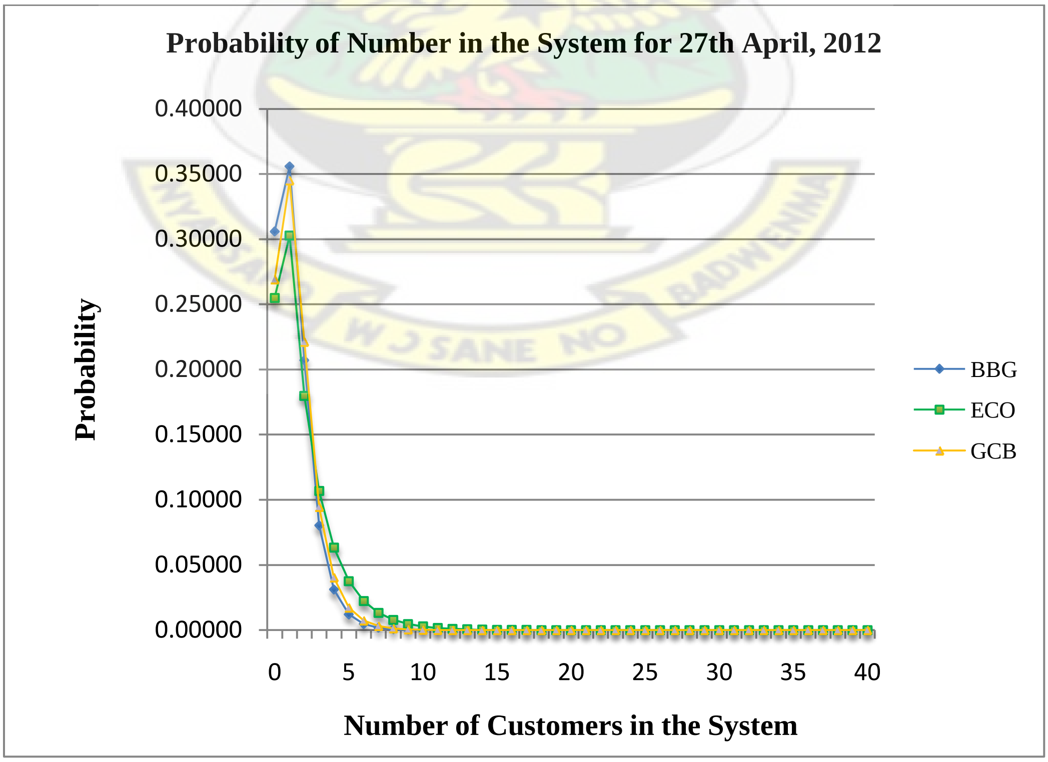


Figure 4.4 Probability of a number of customers in the System for 27th April, 2012

From the results obtained, the highest capacity utilization of 0.68750 obtained at Ecobank on 2nd May, 2012 has an average number of customers waiting in queue to be 1.23241 customers, 1.37500 average number of customers in the service at a point in time, with an average waiting time in the queue and in the queuing system to be 0.02241 hour(s) and 0.04741 hour(s) respectively.

Assuming there is one teller, two tellers, three tellers and four tellers, comparing their performance measures and to making policy recommendations, taking into consideration, the average arrival rate, $\lambda = 52.00$ and service rate, $\mu = 36.00$, Table 4.5 presents the results for considering one to four tellers at a given time.

Table 4.5 Presentation of results for considering one to four tellers at a given point in time

| Model Type | M / M / 1 | M / M / 2 | M / M / 3 | M / M / 4 |
|---|---|---|---|---|
| **Performance Measures** | | | | |
| *Rho* $(\rho)$ | 1.44444 | 0.72222 | 0.48148 | 0.36111 |
| *Probability the system is empty,* $P_0$ | -0.44444 | 0.16129 | 0.22440 | 0.23398 |
| *Average number in the system, L* | -3.25000 | 3.01935 | 1.64629 | 1.48199 |
| *Average number waiting in the queue, Lq* | -4.69444 | 1.57491 | 0.20185 | 0.03755 |
| *Average time in the system, W* | -0.06250 | 0.05806 | 0.03166 | 0.02850 |
| *Average time in the queue,* $Wq$ | -0.09028 | 0.03039 | 0.00388 | 0.00072 |
| *Average time a customer is served, Ws* | 0.02778 | 0.02778 | 0.02778 | 0.02778 |

| | | | | |
|---|---|---|---|---|
| *Average number waiting in the service,Ls* | 1.44444 | 1.44444 | 1.44444 | 1.44444 |

From the Table 4.5, it was observed that, as the number of tellers' $(s)$ increase, the teller utilization factor $(\rho)$ increases, increasing the probability that a system is empty $(P_0)$ and hence the good the performance measures. The average number of customers waiting in the service and time they are served remain constant throughout all the model types. The inappropriateness of a single teller model $(M/M/1)$ to solve customer-waiting problems becomes apparent as the utilization factor of 1.4444 becomes unstable as a result of the arrival rate $(\lambda)$ being greater than the service rate $(\mu)$. The remaining performance indicators under the model type $(M/M/1)$ revealed negative times and number of customers. But there is no negative number of customers waiting at a negative time in the queue.

It is therefore much prudent to increase the number of tellers to have a stable utilization factor. However, multi-teller models were compared and it was observed that;

a) Using a model type with three-teller system is better than a two-teller system in all complicating result. For instance, assuming that there are three tellers serving the arriving customers, there will be 1.64629 customers waiting in queue instead of 3.01935 customers; a customer will spend 0.03166 hour(s) in the system instead of 0.05806 hour(s). A three-teller system has a high probability of being idle 0.22440 than two-teller system.

b) If all facilities are available, then a four-teller system will be preferable than three and two-teller system.

# CHAPTER 5

# SUMMARY, RECOMMENDATIONS AND CONCLUSION



## 5.0 INTRODUCTION

This chapter concludes the analysis and gives recommendation that would be necessary to reduce capacity utilization of facilities, enhancing free movement of customers and reduction of time a customer spends at the banking hall.

## 5.1 SUMMARY OF FINDINGS

Out of five hundred and fifty-eight (558) customers arriving at the two banks, 84.05% customers were served, 52.15% of them transacted business with GCB, 32.26% with BBG whilst 15.59% with Ecobank. The average arriving number of customers per hour each day for the three day data collection period was 60, 29 and 97 customers for BBG, Ecobank and GCB respectively.

Network constraints and tellers are considered to be the cause of long queuing especially at GCB as some tellers work at a slow pace and easily vacate their cubicles to do other business within the bank while customers keep waiting. Since the study concentrated on queues within the bank only, it was very difficult to identify customers balking but there were some jockeying behaviour of customers recorded with no reneging.

The data collected possess the Markovian properties and was capable to be analysed with the queuing model $M/M/s$. All average arriving rates (λ) are greater than the average service rate (μ). The closer the average service rate to the average arriving rate, the smaller utilization factor becomes. The greater the capacity utilization factor, the smaller the probability of the system to be empty and as the probability increases, the more the probability of the number of customers in the entire queuing system approaches zero. 2nd May,2012, was the busiest day among the selected data collection days at all selected branches.

Considering one-teller system up to four-teller system, the average number of customers waiting in the system and time they are served remain constant from one teller to four tellers. The inappropriateness of a single teller model for solving customer-waiting problems becomes apparent as it shows negative figures for all performance criteria except $W_s$ and $L_s$. If facilities are available, then a four-teller system would be preferable than three and two-teller system.

## 5.2 CONCLUSION

With the knowledge of probability theory and stochastic process, it is possible to derive many different queuing models, including but not limited to the ones we observed in this study.

It was observed that the trade-offs between utilization of the present capacity level in the banking industry  was easily seen by making better decisions relating to tellers and potential waiting times for customers. It also uncovered the applicability and extent of usage of queuing models in achieving customer satisfaction at lowest cost.

There were consistency in the number of tellers at GCB but there were several times that the number of tellers at BBG and Ecobank were reduced to either two or one, resulting in long queues and the recording of the highest capacity utilization factors at Ecobank. Based on the data collected, Monday and Fridays confirm to be the peak days of the week.

## 5.2 RECOMMENDATION TO STAKEHOLDERS

Based on the conclusion of this study, the following recommendations are suggested for efficiency improvement and quality of service to customers in the selected branches and the banking industry as a whole:

1. It is clear that during peak times, the moment the number of tellers is reduced to less than three; it exerts unnecessary pressure in the banking hall, and on the entire queuing system and staff of the bank. Hence, increasing the capacity utilization on the banking facilities. It is recommended that any time

a teller is at break or is on leave, especially at peak periods, managers should replace them to avoid time delays due to long queue of customers.

2. Tellers must be well equipped with fast computers, uninterrupted network service and must be trained to work fast as much as possible. It was observed that as the average time a customer is served by a teller ($W_s$) decreases, the average number of customers in the system ($L_s$) also decreases and hence a good performance measures.
3. The queue characteristics should be viewed from a strategic point by managers or director of operations as to whether the waiting time is reasonable and acceptable by increasing the number of tellers to reduce the capacity utilization factor and unnecessary pressure at the banking hall if and only if the facilities are available.

   For instance, since the banking halls of BBG, Tanoso and Ecobank, Ash-town Branches are very small, the management should always increase the number of tellers to four or three respectively to reduce the average time a customer spends in the banking hall.
4. Introduction to banking ethics, communication skills and regular training of employees to enable them to eliminate unnecessary delays when serving a customer for a better improvement of staff-customers relationship.
5. Conjecture of customers on queues seems to be very common at GCB. The manger should provide queuing guards in order to prevent the rampant walking around and jockeying of customers inside the banking hall.

## 5.3 RECOMMENDATION TO FUTURE RESEARCHERS

It is important to note that the model's waiting time predictions pertain only to waiting time due to teller unavailability and do not include any other possible waiting times prior to seeing a teller such as filling savings and debit forms and enquiries times, which would have to be estimated separately. I therefore recommend to future researchers to work on the total time a customer spends in the banking hall, taking into account time the customer enters the bank but not the queue and the time he/she departs from the banking hall irrespective of what transactions the customer deals at the bank.



The challenges posed by customers on technologically innovative subsidiary channels such as the Automated Teller Machines (ATMs), Internet, Mobile and Telephone banking cannot be left out and need thorough queuing analysis on their queuing systems.

The applications of queuing theory extend well beyond waiting line at the banking industry. It may take some creative thinking, but if there is any sort of scenario where time passes before a particular event occurs, there is probably some way to develop it into a queuing model. Queues are so commonplace in society that it is highly worthwhile to study them, even if only a few seconds off one's wait in the check line.

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