

KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY,

KUMASI

COLLEGE OF SCIENCE

DEPARTMENT OF MATHEMATICS

TOPIC:

“FACILITY LOCATION SELECTION USING AHP/PROMETHEE II RANKING

METHOD

A THESIS SUBMITTED TO THE DEPARTMENT OF MATHEMATICS

IN PARTIAL FULFILLMENT OF REQUIREMENT FOR THE AWARD

OF MASTER OF SCIENCE IN INDUSTRIAL MATHEMATICS

BY

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(BEd. MATHEMATICS)

JUNE, 2013

Abstract

Facility location selection is the determination of a strategic site for institution operations. The facility location decision involves institutions seeking to locate, relocate or expand their operations. The facility location decision process encompasses the identification, analysis, evaluation and selection among alternatives with respect to criteria

Selecting the best location among many alternatives is a Multi Criteria Decision Making (MCDM) problem. The aim of this thesis is to demonstrate the implementation of Analytic Hierarchy Process (AHP)/Preference Ranking Method for Enrichment Evaluation (PROMETHEE) II ranking method for selecting the most suitable location for the financial institution in Obuasi Municipality. The related problem includes five possible alternatives and five criteria to evaluate them. The AHP is used to analyze the structure of the facility location selection problem and to determine weights of the criteria and the alternatives, and PROMETHEE II ranking method was finally used to obtain the complete ranking.

The location with the highest score is suggested as the best strategic site for the facility to be located. In accordance with the results generated by the proposed method, A5 has the highest net outranking flow, followed by A3, A4, A2 and the last rank is A1. Therefore, A5 is the best alternative for the facility to be located.

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Dedication

I dedicate this thesis to my wife Adu-Poku Rebecca and my lovely children Marfo Mavis, Lawrencia Darko, Addae Matilda, Oteng Evelyn, Benning Adwoa Charlotte and Benning-Darko Charles for their unfailing emotional support and understanding during my study.

Acknowledgment

First and foremost, I must acknowledge and thank The Almighty God for blessing, protecting and guiding me throughout this period. I could never have accomplished this without the faith I have in the Almighty.

I would like to express my profound gratitude to supervisor Mr. Kwaku Darkwah for his invaluable support, encouragement, supervision and useful suggestions throughout this study. His moral support and continuous guidance enabled me to complete my work successfully.

I would like to thank my family: my parents Mr. Felix Oheneba Darko and Madam Cecilia Nkansah, for giving birth to me at the first place and supporting me spiritually throughout my life. I am also grateful to Mr. and Mrs. Darko for their advice throughout the programme.

I am also highly thankful to my friends and study group members for their valuable suggestions throughout this study.

I express my deepest gratitude to the Mr. Richard Boakye Yiadom, Director of Statistical Department of Obuasi Municipality and the Entire Staff for giving me the needed assistance..

Lastly, to all those who contributed in diverse ways towards this thesis I say God richly bless you.

CHAPTER ONE

1.0 Introduction

Facility location decisions are observed to be of immense importance in long-term planning for the financial institutions. High cost related to property acquisition and facility construction make the facility location selection a long-term investment decision. The location selection decision may be required due to various reasons, like increase in service delivery, put up more branches or change in customer demand. Wrong selection of location may result in inadequate qualified work force, unavailability of power supply, insufficient transportation facility, increased operating expenses or even disastrous effect on the organization due to political and social interference. Thus, the decision maker must select the location for a facility that will not only perform well, but also it will be flexible enough to accommodate the necessary future changes. Various important qualitative and quantitative criteria, such as proximity to customers, nearness to market, community desirability, nearness of other facilities etc, are usually considered while selecting a facility location for a specific financial institution. The success or failure of a financial organization largely depends on the consideration of these criteria as they directly influence the institutional performance.

Selection of a proper location involves consideration of multiple feasible alternatives. It is also observed that the selection procedure involves several objectives and it is often necessary to make compromise among the possible conflicting criteria. For these reasons, Multi Criteria Decision-Making (MCDM) is found to be an effective approach to solve the location selection problems. In this paper, Analytic Hierarchy Process (AHP) and the Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE II)

are employed to obtain the best choice from a finite set of alternative facility locations. While applying the AHP/PROMETHEE II method to solve a real time facility location selection problem (Rao R. V., 2007), it is observed that this method proves its applicability and potentiality to solve such type of decision-making problems with multiple conflicting criteria and alternatives.

The decision regarding the selection of the most suitability facility location has become one of the most important and challenging issues in today's highly competitive business and manufacturing environment (Tompkins and White, 1984). The decision maker has to consider the facility location problems while expanding the existing production capacity, setting of a new production line or setting up of a new facility. The best location is to be selected, keeping in mind various criteria/attributes affecting the location selection decision-making problem and also the requirements of the organization. Hence, selecting the most appropriate facility location design from a finite set of possible alternatives for a given industrial application is really a difficult task. Usually, the problem of location selection aims at minimizing the total transportation cost between different facilities so that the necessary materials and services can move uninterrupted within the entire organization. It will be a major setback for the organization, if the problems, such as backtracking, congestion, disturbed flow pattern etc., start functioning within the organization due to poor and unplanned location. Thus, the facility location selection decision becomes a strategic issue for an organization and has significant impact on its overall effectiveness and performance. There are several quantitative and qualitative criteria, like material handling distance, adjacency score, shape ratio, flexibility, accessibility, maintenance etc., which directly affect the facility location selection

decision. Among these criteria, some are beneficial in nature which are to be maximized, whereas, others are non-beneficial whose minimum values are always preferable.

Selection of the most suitable facility location for a given financial use is considered as an example of Multi-Criteria Decision-Making (MCDM) problem, requiring the fulfillment of all the conflicting criteria. Various MCDM methods, like Simple Additive Weighting (SAW), Weighted Product Method (WPM), Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), Graph Theory and Matrix Approach (GTMA), Grey Relational Analysis (GRA), Data Envelopment Analysis (DEA), Elimination and Et Choice Translating Reality (ELECTRE) etc., are already available to give an effective framework for evaluating the alternatives and selecting the best one. Other combinatorial optimization techniques, such as Simulated Annealing (SA), Generic Algorithm (GA), Ant Colony Optimization (ACO), Tabu search etc., are also successfully used to solve the location selection problems. Suitable computer packages and expert systems are also developed to solve the location problems and graphically generate the best location.

1.1 Background of the study

The Obuasi Municipality is one of the 27 districts of the Ashanti Region and was created as part of the government's effort to further decentralized governance. It was carved out of the erstwhile Adansi West District Assembly on the strength of executive instruments (E. I.) 15 of December, 2003 and Legislative Instrument L. I. 1795 of 17th March, 2007.

The Municipality is located at the southern part of Ashanti Region between latitude 5.35N and 5.65N and longitude 6.35N and 6.90N. It covers a land area of 162.4sqkm. There are 53 communities in the Municipality which share 30 electoral

areas.

It is bounded to the east by Adansi South, west by Amansie Central and to the north by Adansi North, to the south by Upper Denkyira District in the Eastern Region. It has Obuasi as its Administrative Capital where the famous and rich Obuasi Gold Mines, now Anglo Gold Ashanti is located. The company currently employs over 6000 workers.

The Municipality has a rather undulating topography and the climate is of the semi-equatorial type with a double rainfall regime. Mean annual rainfall ranges between 125mm and 175mm. Mean average annual temperature is 25.5OC and relative humidity is 75% - 80% in the wet season.

The population of the Municipality is estimated at 205,000 using the 2010 Housing and Population Census as a base and applying a 4% annual growth rate. The vegetation is predominantly a degraded and semi-deciduous forest. The forest consists of limited species of hard wood which are harvested as lumber. The Municipality has nice scenery due to the hilly nature of the environment.

1.2 Statement of Problem

With the increase in infrastructural developments in Obuasi Municipality coupled with the large population of about 205,000, there would be a lot of stress on the various facilities. The banks in Obuasi Municipality are not enough and not well distributed compare to the number of customers they serve (i.e. AngloGold Ashanti workers, civil

servants, teachers, businessmen and women, farmers etc). This has led to widespread of unlawful mobile money (Susu) collectors who eventually ran away with the money they collected.

1.3 Objective of study

In spite of the existence of few banks in Obuasi, this work seeks to identify and evaluate a more strategic site within the Obuasi Township to locate the facility using AHP/PROMETHEE II ranking Method and also factors that influence location of banking facilities in Obuasi Municipality.

1.4 Significance of the study

It is strongly believed that this thesis will:

- make people have easy access to banking facility and therefore will help the economy grow.
- add to the knowledge of resources available to all managers or decision makers.
- give suggestions to future researchers as a base on which they can use to facilitate.
- facility location has a long-term impact on the banking service and must be part of the institution's strategy.

1.5 Methodology

Basically our research purpose have twofold aim: to propose the new methodology for evaluating the facility location and to determine the best location for facility based on the criteria proposed in the new methodology. This thesis examined the location of a selected

number of towns in Obuasi: Brahabebome, Tutuka, Wawasi, Gausu and Konka. This data of facility location will be taken from Statistical Department of Obuasi Municipal Assembly, questionnaire from respondents and special people from the selected towns will be consulted over a period of time. Using the proposed method Analytical Hierarchy Process (AHP)/Preference Ranking PROMETHEE Methods) the aim of this thesis will be explored.

1.6 Thesis organization

The thesis is divided into five (5) main chapters:

Chapter 1 gives a preview of the thesis topic under consideration. This chapter also gives a brief history of facility location selection. Chapter 2 looks at literature related to our scope of study in the thesis. Chapter 3 looks at solutions that have been employed in solving the defined problem. We continue with the use of AHP/PROMETHEE II methods in solving the facility location selection problem. Chapter 4 deals with data collection, analysis and results. Chapter 5 is conclusion and recommendation.

CHAPTER TWO

Literature Review

Several researchers have already applied different techniques to solve the facility location selection problems. But most of those techniques use complex mathematical formulations, while ignoring qualitative information about the considered criteria. In this chapter, we present a brief review of literature on facility location selection.

Calvo and Marks (1973) constructed p-median model to locate multi-level hierarchical health care facilities including central hospitals, community hospitals and local reception centers. The model minimized distance and user costs, and maximized distance and utilization. The hierarchical p-median model was later improved by Tien et al (1983).

Carson and Betta (1990) proposed a p-median model to find the dynamic ambulance positioning on the campus of the State University of New York at Buffalo in response to changing daily conditions. This is a particular problem on a large University Campus since the center of gravity of the population shift from dormitories to classrooms and offices over the course of the day. They did determine that modeling four different time periods would suffice. By relocating the ambulance for each period, they were able to reduce the predicted average response time by 30% from 3.38 minutes (with a single static location) to 2.28 minutes (with four periods of unequal duration). There was an actual decrease in travel time when solution was implemented by close to 6% with the difference attributed to the non-linear nature of travel times. Their work also emphasized the need for careful modeling of travel time relationship, particularly when the average time is likely to be small.

Carbone (1974) formulated a deterministic p-median model with the objective of minimizing the distance travelled by a number of users to fixed public facility such as banks, medical or day-care centers. There is uncertainty in recognizing the number of users at each demand node. He further extended the deterministic p-median model to a chance constraint model. The model seeks to minimize a threshold and meanwhile ensure the probability that the travel distance below the threshold is smaller than a specified level α .

Serra and Marianov (1999) implemented a p-median model and introduced the concept of regret and minmax objectives when locating fire station for emergency services in Barcelona. The authors explicitly addressed in their model the issue of locating facility when there are uncertainties in demand, travel time or distance. In addition, the model uses scenarios to incorporate the variation of uncertainties and seek to give a compromise solution by minimizing the maximum regret over the scenarios. P-median models have also been extended to solve emergency service location problems in a queuing theory context. An example is the Stochastic Queue Median (SQM) model due to Berman et al (1985). The SQM model seeks to optimally dispatch mobile servers such as emergency response units to demand points and locate the facilities so as to minimize average cost of response.

ReVelle and Swain (1970) formulated the p-median problem as a linear programme and used a branch and bound algorithm to solve the problem.

Paluzzi (2004) discussed and a p-median based heuristic location model for placing emergency service facilities for the city Carbondale, Illinois. The goal of his model was

to determine the optimal location for placing a new fire station by minimizing the total aggregate distance from the demand sites to the fire station. His results were compared with the results from other approaches and comparison validated the usefulness and effectiveness of the p -median based on location model. One major application of the p -median models is to dispatch Emergency Medical Service (EMS) units such as ambulances during emergencies.

Hakimi (1964) shows that minimizing the average distance to end users when placing a switching center in a communication network results in the minimum use of wire. The ability to consider distance minimization strategies for facility placements on discrete network intersections allows planners to find more optimal solutions for problems that, for example, involve travel along a road network. The problem of placing multiple facilities at node in a network was also explored in depth by ReVelle and Swain (1970) and termed the p -median problem. This work establishes a methodology for placing facilities on a network and has been used in myriad applications (Hale and Moberg 2003), ranging from computer server allocation and placement (Cameron et al. 2002, Liu et al. 2007) to locating retail stores (Drezner and Drezner 2002).

Garfinkel et al (1977) examined the fundamental properties of the P -center problem. He modeled the P -center problem using integer programming and the problem was successfully solved by using a binary search technique and a combination of exact tests and heuristics.

ReVelle and Hogan (1989b) formulated a P -center problem to locate facilities so as to minimize the maximum distance within which EMS is available with reliability. System

congestion is considered and a derived server busy probability is used to constrain the service reliability level that must be satisfied for all demands. Stochastic P-center models have also been formulated for Emergency Medical Service (EMS) location problems.

Hochbaum and Pathria (1998) considered the emergency facility location problem that must minimize the maximum distance on the network across all time periods. The authors used k underlying networks to represent different periods and provided a polynomial time 3-approximation algorithm to obtain the solution for each problem.

Talwar (2002) utilized a P-center model to locate and dispatch three emergency rescue helicopters to serve the growing EMS demands from accidents of tourist activities such as skiing, hiking and climbing at the north and south end of the Alphine mountain ranges. One of the model's aim is to minimize the maximum (worst) response time and the author used effective heuristic to solve the problem.

According to Deskin et al (1988) there are circumstances where the provision of a service needs more than one "covering" facility, this occurs when facilities may not always be available. For example, assume that ambulances are being located at dispatching points in order to serve demand across an urban area, and the nearest ambulance is busy, then the next closest available ambulance will need to be assigned to a call when it is received. If the closest available ambulance is farther than the service standard then that demand or call for the service is not provided within the coverage standard. To handle such issues, models have been developed that seek multiple-coverage. Two examples of multiple-coverage exist, stochastic or probabilistic and deterministic.

Daskin (1983) formulated a probabilistic multiple cover model called the maximal expected coverage model. Hogan and ReVelle(1986) also formulated the simple back up covering model as a good example of a deterministic cover model that involve maximizing second-level coverage. Toregas (1970, 1971) was the first to recognize the possible need for multi-level coverage. Toregas defined the multi-level Location Set Covering Problem (ML-LSCP) as a search for the smallest number of facility needed to cover each demand, a preset number of times, where the need for coverage might vary between demands.

ReVelle and Hogan (1989a) later developed the maximum availability location problem (MALP) which distributed a fixed number of servers to maximize the population covered with a server available within the response-time standard with reliability. They presented two versions of MALP, one with a system wide busy probability which is somewhat similar to MEXCLP, and the other version computed the local busy fractions for servers assuming that the immediate area of interest is isolated from the rest of the region (Haldun and Saydam, 2002).

Saydam et al (1994) compared the accuracy of the predicted expected coverage of adjusted MEXCLP and found that MEXCLP provides optimal or near optimal set of locations, but, that there can be a significant over or underestimation of coverage.

Batta et al (1989) suggested adjustments to the MEXCLP to improve the accuracy of the expected coverage predicted by it. They proposed a two step heuristic that utilizes Larson hypercube optimization procedure (Larson, 1974)

Chiyoshi et al (2003a) analyzed non-homogeneous servers and compared MEXCLP and the adjusted maximum expected covering location problem, AMEXCLP (Batta et al 1989)

Saydam and Aytug (2003) develop a genetic algorithm that combines MEXCLP with a hypercube approximation algorithm developed by Jarvis (1975) in order to solve MEXCLP with increased accuracy. Gendreau et al (1997), (Laporte et al., 2001) used tabu search in a similar context. Galvao et al (2005), applied simulated annealing in the solution of MEXCLP and the maximum availability location problem (MALP) (ReVelle and Hogan, 1989a), Widmer et al (2007).

(Doumpos and Zopounidis, 2010) presented a case study on the implementation of a multi criteria approach to bank rating, especially in Greece. Their proposed methodology was based on the PROMETHEE II method. A rich set of evaluation criteria was used in the analysis and was selected in accordance with widely accepted bank rating principles.

Special emphasis was put on the sensitivity of the results with regard to the relative importance of the evaluation criteria and the parameters of the PROMETHEE method such as the criteria, weights and parameters of the preference functions. Analytic and Monte Carlo simulation techniques were used for this purpose. The data involved detailed information for all Greek banks during the period of 2005-2007. Overall, sixteen (16) banks were considered. The banks were evaluated on a set of thirty one (31) criteria. The criteria had been selected in close co-operation with expert analysts of the bank of Greece, who were responsible for monitoring and evaluating the performance of the banks. The criteria were organized into six (6) categories (capital, assets, management,

earnings, liquidity, sensitivity to market risks), in accordance with the camels frame work. Overall, seventeen (17) quantitative and fourteen (14) qualitative criteria were used. All qualitative criteria were evaluated on an interval 0.5-5.5 scale, defined by the analysts of the bank of Greece, with lower values indicating higher performance. The weights of each category of criteria and the criteria therein had been defined by the expert analysts of the bank of Greece. The quantitative criteria were assigned a weight of 70%, with the remaining 30% involving qualitative criteria. In all, evaluation results from both the relative assessment procedure and the absolute evaluation process were similar. The results indicated that most banks achieved a rating grade of 2 or 3, each corresponding to performance scores in (1.5, 2.5] and (2.5, 3.5] respectively. There was no bank in the first (best) grade (score nor in the highest (5th) risk grade (score >4.5). the dynamics of the performance scores of the banks, indicated that no significant changes were observed between the 5(five) years of the analysis.

PROMETHEE II method had been used to solve a facility location problem in which there were eight (8) criteria against four (4) alternative solutions (Athawale and Chakraborty, 2010). In the end, the most cost-effective and highest yielding location alternative was identified and selected. They remarked that the PROMETHEE method as a multi criteria decision making approach is a viable tool in solving the location selection decision problems and that it allows the decision maker to rank the candidate alternatives more efficiently and easily.

(Maragoudaki and Tsakiris, 2005) identified PROMETHEE methodology as one of the most efficient multi criteria decision analysis (MCDA) outranking techniques that could be used to arrive at the optimal flood mitigation plan for a river basin. The criteria used to

rank alternatives consisted of the cost of flood defense works and their maintenance cost (quantitative assessment) together with environmental and socioeconomic factors representing flood impacts to the environment and the society of the river basin district (qualitative assessment). Alternative scenarios were formulated and evaluated by different stakeholders. The PROMETHEE method was used for aggregating the various criteria and various stakeholder evaluations and proposing the final ranking of the alternative plans.

Four alternative irrigation projects for the east Macedonia-Thrace district – had been evaluated using AHP and PROMETHEE multi criteria methods (Anagnostopouls et al., 2005). The projects goal was the rational water resources management of Nestos River in relation to the operation of two recently constructed dams. They proposed that the management of the water supply system should balance the needs for irrigation, the needs of the public electrical corporation for hydropower generation, as well as environmental requirements given the presence of valuable natural ecosystems in the area.

A preventive maintenance decision model based on integrating PROMETHEE method and the Bayesian approach was developed to help decision makers establish replacement intervals (Ferreira et al., 2007). Finally, a numerical application was given to illustrate the proposed decision model and showed the effectiveness of the model in terms of the decision maker's preferences.

In multi criteria decision making (MCDM) problems dealing with qualitative criteria and uncertain information, the use of linguistic values is suitable for the experts in order to express their judgments (Halouani et al., 2009) . To them, it was common that the group

of experts involved in such problems had different degrees of knowledge about the criteria, so they proposed a multi-granular linguistic frame work such that each expert could provide his/her evaluations in different linguistic term sets according to his or her knowledge. The authors were concerned about developing tools and operators for the PROMETHEE method to deal with multigranular linguistic information. They later presented an investment scenario to show the integration between the aggregation operators of PROMETHEE method and the linguistic hierarchies. In this scenario, an investment company wanted to invest a sum of money in the best option. There was a panel with four possible alternatives $A = \{a_1, a_2, \dots, a_4\}$ of investment possibilities. a_1 was a car industry, a_2 was a food company, a_3 was a computer company and a_4 was an arms industry. The investment company chose four experts $E = \{e_1, \dots, e_4\}$ from four consultancy departments: risk analysis, growth analysis, social-political analysis, and environmental impact analysis departments respectively, to construct a decision group throughout a set of three criteria $C = \{c_1, c_2, c_3\}$ where c_1 was profit, c_2 denoting pollution and c_3 denoting employment. These experts used different linguistic term sets from the linguistic hierarchy (LH) to provide their preferences over the set of alternatives. In the end, based on the ranking of the alternatives by the experts, the company was advised to choose alternative, a_4 (an arms industry) for its investment.

(Albadvi, 2004) formulated national information technology strategies: a preference ranking model using PROMETHEE method. The sole purpose of his research was to define a national strategy model for information technology (IT) development in developing countries and to apply the model in a real case of Iran. Albadvi research was structured around a three dimensional configuration of strategy development process.

These dimensions were key technologies (a set of technology clusters, which have high impact on the development of IT), socio-economic sectors (major economic and social sectors with potential use of IT opportunities); and applications (IT application flagships to provide different strategic choices). The model was a multi-criteria decision making and in order to solve it and select a set of IT application flagships in different budgeting levels, they used the PROMCALC and GAIA decision support system. Finally, it was discovered that by allocating 1% of GDP, four major IT applications for investing were identified. e-education, e-research, e-office and e-information services were ranked as highly important for the realization of long term objectives in the economical, social and cultural development of the country.

The PROMETHEE technique had been applied to determine depression outlet location and flow direction in Digital Elevation Model (DEM) in northern Taiwan (Chou et al., 2004). In their study, the authors proposed depression water shed method coupled with the Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEES) theory to determine the optimal outlet and calculate the flow direction in depressions. The method therein developed was used to delineate the Shihmen reservoir water shed located in northern Taiwan. The results they had, however, showed that the depression watershed method could effectively solve the shortcomings such as depression outlet differentiating and looped flow direction between depressions. The suitability of the proposed approach was verified.

A fuzzy based pipe condition assessment model using PROMETHEE II was developed by (Zhou et al., 2010). This method was used to calculate pipe breakage risk to reflect the condition assessment in order to enable them rehabilitate the deteriorated pipes in a

planned and proactive way. The numerous influential factors they identified as responsible for pipe breakage included ground load, pipe material, soil corrosion, pipe age, construction quality, pipe length, soil condition, breakage history etc. They argued that the proposed model was different from previous model being used in that it only required usually available data, and that it gave an insight into expert opinion's uncertainty and preference that had a pipe breakage signification in each criterion. The model developed was meant to apply as a new method to some pipes in a water distribution system. This application demonstrated both the stability of the new method and its ability to generate results that will greatly assist decision makers in the development of their rehabilitation strategies.

A PROMETHEE based uncertainty analysis of UK police force performance rank improvement was designed for a periodic comparison of the police forces in the UK with each other in terms of performance by both government and non-government bodies (Barton and Baynon, 2009). The study demonstrated the employment of PROMETHEE in an investigation of the targeted performance rank improvement of individual UK police forces. The graphical representations presented offered an insight into the implications of such a PROMETHEE based series of perceived improvement analysis. The goals of their study were two folds, namely to exposit PROMETHEE based uncertainty analysis in rank improvement and secondly, how the subsequent results could form part of the evidence to aid in their performance strategies.

A new sorting method (Flow Sort) based on the ranking methodology of PROMETHEE for assigning actions to completely ordered categories, defined either by limiting profiles

or by central profiles was established by (Nemery and Lamboray, 2007) . The Flow Sort assignment rules were based on the relative position of an action with respect to the reference profiles in terms of the incoming, leaving and/or net flows. The authors added that for a better understanding of the issues involved, a graphical representation was given. An explicit relationship between the assignments obtained when working either with limiting or central profiles was formalized. Finally, an empirical comparison with ELECTRE-TRI was made to compare the resulting assignments.

(Grau et al., 2010) proposed a mathematical model to select the optimal alternative for an integral plan to desertification and erosion control for the Chaco area in Salta province (Argentina). They used three multi criteria decision methods – Elimination and Et Choice Translating Reality (ELECTRE), Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE) and Analytic Hierarchy Process (AHP) for different sub zones which were established based on previous studies. In the development of the model, they took into consideration economical, environmental, cultural and sociological criteria. Their multi-criteria model to select among different alternatives to prepare an integral plan to ameliorate or /and solve this problem in each area has been elaborated taking into account eight criteria and six alternatives. Their results indeed, showed a high level of consistency among the three different multi criteria methods in spite of the complexity of the system studied.

(Manzano et al., 2005) conducted an economic evaluation of the Spanish port system using the PROMETHEE multi criteria decision method. The work established an ordering relationship among twenty-seven Spanish port authorities at different strategically considered time points. They developed various ratios to evaluate the

different port authorities. These ratios were referred to economic management, port traffic and labor productivity. Overall, they used six criteria: Economic Yield, Dynamism of Port Activity, Specialization in Containers, Capitalization, Harbor Business and Productivity of the Labor Factor –to order the ports under consideration.

Call quality measurement for telecommunication network and proposition of tariff rates research was conducted by (Aburas et al., 2009). The idea of their research was basically the measurement of call quality from the end users perspective and could be used by both end user and operator to benchmark the network. The call quality was measured based on certain call parameters as average signal strength, the successful call rate, drop rate, handover success rate, handover failure rate, and Location Area Code (LAC). The quality parameters were derived from active calls and the results were analyzed and plotted for detailed analysis and benchmarking as well as used as a base for charging the customer by the operators. They suggested the charging rates in work based on the signal quality and the call statistics recorded.

(Michailidis and Chatzitheodoridis, 2006) proposed a model based on PROMETHEE – a multi-criteria decision aid – to be used to evaluate and rank three tourism destinations, located in the northern and central Greece. Additionally, innovatory elements were the incorporation of differing levels of socioeconomic data (destination image and destination personality) within the decision frame work and the direct determination of the PROMETHEE II preference thresholds. According to them, the developed methodology provides a user- friendly approach, promotes the synergy between different stakeholders, and could pave a way towards consensus. They identified the act of describing the design implementation and use of a Decision Support System (D.S.S),

which applied new methodological approaches for the evaluation and ranking of several tourism destinations as the main focus of their study.

PROMETHEE I was introduced to cope with interval criteria introduced for the evaluation of the environmental quality of building products through Life Cycle Assessment (LCA). Of course, this procedure could be applied to any situation where the decision matrix is an interval matrix.

(Téno and Mareschal, 1998) developed an interval version of PROMETHEE for the comparison of building products“ design with ill-defined data on environmental quality. They observed that the Life Cycle Assessment (LCA) is a powerful technique used to calculate total input and output flows of materials and energy from and to the environment during every step of a product life. They added that a measure of a product Environmental Quality (EQ) could then be derived and helped in the selection and in the design of more environmentally friendly design alternatives.

EQ is a multi criteria measurement. In the construction field, LCA flows could not be known with precision without loss of realism. Hence, intervals were introduced to model them. Thus, different designs were characterized by interval multi criteria measures. According to (Mareschal et al., 1997) manipulation of such environmental performances called for a multi criteria decision analysis method which;

- (i) did not allow for trade – offs between criteria
- (ii) preserved as much information as possible and
- (iii) was simple enough to be understood by non – specialist users.

PROMETHEE I was considered as the most suitable method introduced to cope with interval criteria incorporated into the model for the evaluation of the environmental quality of building products through Life Cycle Assessment (LCA).

(Pirdashti and Behzadian, 2009) applied AHP and PROMETHEE to the selection of the best module design for Ultra Filtration (UF) membrane in dairy industry. The authors noted that membrane with a type module had been expressed one of the key areas of interest in dairy industry. According to them, although recent publications had given a chance to academic and practitioners to prove successful applications of membrane process to the vast areas; a small number of publications had been devoted to the problem of capital equipment decision making. To facilitate the process of decision – making process in the membrane separation, their study focused on the application of Analytical Hierarchy Process (AHP) and preference ranking organization method for Enrichment Evaluations (PROMETHEE), from a group decision – making view point. They use the Delphi technique to evaluate available alternatives according to the criteria elicited from expert’s opinions. A real case study on the ultra filtration membrane area was put forward to determine the best module design based on the five (5) criteria expressed by decision makers: Sanitation design, clean – in – place, packing density, resistance to faulting and shear stress, and relative cost. Finally, expert choice and DECISION LAB soft wares were utilized to facilitate calculations.

The PROMETHEE methodology has been identified as the most sophisticated multi-criteria evaluation methods with deep intrinsic logic and wide flexibility; capable of

transforming values of criteria via so – called preference functions (Podvezko and Podvezko, 2010). The authors focus was on the use and choice of preference functions for evaluation of characteristics of socio-economical processes. According to them, all given alternative courses of action were mutually compared pair wise for each criterion. Choice of preference functions and their parameters was important, since it has influence on results of ranking, in which ranks of alternatives may considerably differ. Various preference functions were analyzed, their features described and applications were shown for various socio-economical characteristics.

The institute curie which is a hospital located in Paris, France, with its specialty in oncology seeking enhanced continuity of care inside and outside its walls by using computerized applications relies on two e - health tools as the heart of the institutes ICT systems – Elios and PROMETHEE (Electronic-Business Watch, 2001).

Elios is a comprehensive electronic patient record system, allowing patient data access during consultations, diagnosis and treatment. PROMETHEE to them is a sophisticated, yet simple to use search engine that enabled the health care professionals to classify medical questions across the hospital`s databases, including Elios. They added that Elios and PROMETHEE together fundamentally transformed health care processes. They improved the continuity and quality of care but offering access to patient data anytime from anywhere in the hospital and from outside. Both tools could be accessed by all members of the health care team involved in their treatment.

In the case of Elios this included external partners such as other hospitals or general practitioners. Both tools also led to considerable economic benefits. Some of the main benefits they identified from PROMETHEE included:

- (i) prompt answers to questions on demand
- (ii) activity reporting
- (iii) faster completion of research and evaluation studies leading to earlier implementation
- (iv) rapid evaluation of medical procedures reducing the cost of studies and
- (v) audits permitting faster adjustments of the hospital's organization.

These benefits were achieved through evaluation of medical practices, medical pathways, and medical information quantity.

(Wen-jun et al., 2008) appraised enterprise technology innovation project method based on PROMETHEE. In view of the question on the choice of the iron and steel enterprise technology innovation project, their research established the technology innovation project appraisal index system on the iron and steel enterprise. As mentioned, they used the PROMETHEE method – a class of outranking methods in multi criteria analysis, and it ranked various projects reasonable with the indefinite weight information. When compared with the TOPSIS method, it illuminated that the conclusion of this method was valid and credible.

The collaborative environmental planning in river management in the white river watershed in Vermont adopted the PROMETHEE as a multi criteria decision analysis methodology (Hermans et al, 2006). Their research presented the frame work and results

of a structured decision process using the PROMETHEE. The PROMETHEE was used to frame multi- stakeholder discussions of river management alternatives for the upper white river of central Vermont, in the North eastern United States. Stakeholders met over ten (10) months to create a shared vision of an ideal river and its services to communities, develop a list of criteria by which to evaluate river management alternatives, and elicit preferences to rank and compare individual and group preferences. The MCDA procedure helped to frame a group process that made stakeholder preferences explicit and substantive discussions about long – term river management possible.

(Kodikara, 2008) in his thesis on multi-objective optional operation of urban water supply systems made an appropriate use of the PROMETHEE methodology. Kodikara's study attempted to develop and assess the potential of a generic decision support framework to assist in evaluating alternative operating rules for multi-purpose, and multi-reservoir urban water supply systems.

The multi-objective outranking approach (PROMETHEE), which facilitated the incorporation of stakeholder preferences in the decision making process was a main focus area in his study. The main elements of the framework were illustrated on a case study of the Melbourne water supply system, demonstrating its capabilities for evaluating alternative operating rules under single or group decision-making situations. Eight (8) Performance Measures (PMS) were identified under four main objectives to evaluate the system performance related to sixteen pre – selected alternative operating rules. Three (3) major stakeholder groups: resource managers, water users and environmental interest groups were represented in hypothetical decision making situations. An interview-assisted questionnaire survey was used to derive stakeholder preferences on PMS in

terms of preference functions and weights as required by the PROMETHEE / GAIA method and its computer software tool – decision lab 2000. A total of ninety-seven (97) personnel selected from Melbourne water and Victoria University participated in the survey expressing their preferences on the eight performance measures. Finally, an overall ranking for alternative operating rules was obtained together with other output results, which focused on the best compromises between the objectives considered.

According to the author, the method yielded reliable and robust results in terms of varying group compositions considered in the study. The authors added that the major innovation of this project was the development of a transparent and intuitive multi – objectives decision support framework that has the potential to be developed for evaluating alternative operating rules for urban water supply systems.

(Mani et al., 2008) adopted the PROMETHEE method in their streamlined life cycle analysis of biomass densification process. They considered mechanical densification to be the process of transforming loose biomass into dense pellets. In their study, a wood pelleting plant was chosen to evaluate the total energy consumption, environmental emissions and cost of pellet production using different alternative fuel for the drying process. The fuels compared were natural gas, coal, dry and wet saw dust, and ground wood pellets. The process models were developed and applied to predict the energy consumption and emissions during combustion process. A streamlined life cycle analysis approach was used to quantify emissions. The authors used average emission factors from published literature to estimate the emissions of trace metals and toxic pollutants. The environmental impacts of the emissions were evaluated based on greenhouse gases, acid rain formation, smog formation and human toxicity impact potentials. A detailed

engineering cost analysis was conducted to estimate the pellet production cost using different process options and fuel sources. The PROMETHEE methodology was used to rank fuel alternatives. The best fuel source was selected based on four main criteria – energy, environmental impacts, economics and fuel quality. Their results showed that wood pellet or dry sawdust might be the best alternative when compared to natural gas, followed by coal and wet sawdust, when all the criteria were weighed equally. If the weighing factor for cost was doubled, coal ranked highest followed by dry sawdust, wet sawdust, wood pellet and natural gas respectively.

(Schwartz and Göthner, 2009) applied for the first time the multi-criteria outranking technique PROMETHEE in incubator evaluations. Based on data from four hundred and ten (410) graduate firms, their evaluation procedure was aimed at comparing the long-term effectiveness of five technology-oriented Business Incubators (BI"s) in Germany. In particular, they investigated whether PROMETHEE was a well-suited methodological approach for the evaluation and comparisons in the specific context of business incubation. In the end, they arrived at the conclusion that in using PROMETHEE for incubator comparisons required a set of incubators with sufficient homogeneity regarding major objectives, a set of multiple criteria that cover both incubator and incubator-incubatee dimension of BI performance and, ultimately, a strong participation of the local decision makers to avoid a black-box effect.

(Tzeng et al., 1992) applied two multi-criteria decision-making methods; AHP and Promethee to the evaluation of new energy system development in Taiwan. The energy crisis in the 1970s and the recent rise in environmental protectionism had heightened interest in the introduction of new energy systems and the development of techniques to

ensure the stability of the energy supply in Taiwan, where more than 90% of the supply was imported. In their study, multi criteria evaluation methods as mentioned above were employed to evaluate comprehensively the alternatives for new energy–system development. Energy technology, environmental impacts, sociology and economic factors were evaluated and development directions and strategy for future energy systems in Taiwan were proposed.

(Martel, 1998) proposed a multi criteria approach for selecting a portfolio manager. The PROMETHEE II method was applied for the selection of a portfolio manager. According to Martel, such application involved four main steps:

- (i) Defining the list of potential actions or solutions to the problem
- (ii) Defining the list of relevant criteria
- (iii) Evaluating the performance of each action based on each criteria
- (iv) Aggregating these performances with the multi criteria method PROMETHEE II.

The author underscored the appropriateness of the use of a multi criteria approach to this problem as multiple criteria seemed to be used by decision–makers in the selection of a portfolio manager. The criteria applied to this model were derived from a set of depth interviews with managers of the twelve (12) major pension funds in the province, of Quebec. They ended up with nine criteria that turned out to be heterogeneous and conflicting in their nature. These criteria were then grouped into four: Past performance, Investment philosophy, Staff criteria and Organizational criteria.

The richness of data collected through the interview allowed them to specify accurately the decision-maker's preference functions. It was thus possible to choose an outranking technique as a multi criteria aggregating procedure. The choice was limited to one technique of the ELECTRE family and one of the PROMETHEE family of methods. The PROMETHEE II was thus used because the interview revealed that no veto thresholds were applicable to the model. Furthermore, the application was a ranking problem where it was necessary to prioritize a set portfolio managers of from "best" to "worse". Finally, they concluded the analysis by applying their proposed model to the selection of a small capitalization stock portfolio manager.

Plazibat et al., (2006) adopted a multi criteria approach to credit risk assessment in a significant area of financial management which demands of credit/financial analysts to investigate a large number of financial indicators of firms and make crucial decisions regarding the financing of firms. The focus of their study was on the ranking of firms according to the credit risk assessment using the PROMETHEE method and Analytic Hierarchy Process (AHP). The PROMETHEE method was used for final ranking of great member of Croatian firms and AHP to determine the importance of the eleven criteria from the three main criteria groups: profitability, liquidity and solvency of the firms.

The lean improvement of the chemical emissions of motor vehicles based on preference ranking PROMETHEE uncertainty analysis has been considered (Baynon and wells, 2006). The authors observed that the motor vehicle had provided mobility and individual freedom for millions of people. Vehicles embodied the dilemma of contemporary

industrialization in that the environmental costs of automobility were equally large. Their non – country specific study undertook a PROMETHEE-based preference ranking of a small set of motor vehicles based on constituents of their exhaust emissions. As a model of an interested party's preference ranking of the motor vehicles, the subsequent uncertainty (sensitivity) analysis considered here, related to what minimal (lean) changes would be necessary to the emissions of a vehicle so that their preference ranking is improved. For a particular manufacturer, it could identify the necessary engineering performance modification to be made to improve their perceived consumer based ranking. This was compounded by a further consideration of different levels of importance conferred on the criteria (vehicle emissions) and analogue analysis undertaken. The visual elucidation of the results rankings and changes to criteria values, offered a clear presentation of the findings to the interested parties.

(Maragoudaki and Tsakiris, 2005) developed an effective flood mitigation plan using PROMETHEE. The research indeed demonstrated the application of PROMETHEE, one of the most efficient Multi-Criteria Decision Analysis (MCDA) outranking techniques in order to achieve the optimal flood mitigation plan for a river basin. The criteria they used to rank alternatives consisted of the cost of flood defense works and their maintenance cost (quantitative assessment) together with environmental and socio economic factors representing flood impacts to the environment and the society of the river basin district (qualitative assessment). Alternative scenarios were formulated and evaluated by different stakeholders. The PROMETHEE method was used for aggregating the various

criteria and various stakeholder evaluations and proposing the final ranking of the alternative plans.

(Ayoko et al., 2004) applied multi criteria decision making methods – PROMETHEE and GAIA to air quality in the micro environment of residential houses in Brisbane, Australia. Their study centered on the application of the multi criteria decision making methods, PROMETHEE and GAIA, to indoor and outdoor air quality data. Fourteen (14) residential houses in a suburb of Brisbane, Australia were investigated for twenty-one (21) air quality – influencing criteria, which included the characteristics of the houses as well as the concentrations of volatile organic compounds, fungi, bacteria, sub micrometer, and super micrometer particles in their indoor and outdoor air samples. Ranking information necessary to select one house in preference to all others and to assess the parameters influencing the differentiation of the houses was found with the aid of PROMETHEE and GAIA. The outcome of their analysis showed that there was no correlation between the rank order of each house and the health complaints of its occupants. Patterns in GAIA plots showed that indoor air quality in these houses was strongly dependent on the characteristics of the houses (construction materials, distance of the house from a major road, and the presence of an in – built garage). Also, marked similarities were observed in the patterns obtained when GAIA and factor analysis were applied to the data. This to the authors underscored the potential of PROMETHEE and GAIA to provide information that could assist source apportionment and elucidation of effective remedial measures for indoor air pollution.

(Kalogeras et al; 2005) used the multi criteria decision aid approach –PROMETHEE method to determine whether or not the ownership structure of cooperative firms drive their financial success. According to these authors, research in finance regarding the impact of ownership structure on the performance of the competing forms of firm organization was scarce. In their study, the ownership structures of co-operatives (co-ops) were analyzed in order to examine whether new models of co-op ownership perform better than the more traditional ones. The assessment procedure introduces a newly developed financial decision – aid approach, which was based on data analysis techniques in combination with a preference ranking organization method for enrichment evaluations (PROMETHEE II). The application of this multi-criteria decision – aid approach allowed the rank ordering of the co-operatives on the basis of the most prominent financial ratios. The authors selected the financial ratios using principal components analysis. This analytical procedure reduced the dimensionality of large number of interrelated financial performance measures. The authors assessed the financial success of fourteen (14) Dutch agribusiness co-ops for the period 1999-2007. The outcome of the research showed that there was no clear-cut evidence that co-op models used to attract outside equity performed better than the more traditional models. This suggested that ownership structure of co-ops was not a decisive factor for their financial success.

(Khiabani, 2006) adopted PROMETHEE to aid him in his studies of business-to-business

E-commerce attributes and adoption. Khiabani observed that understanding intention of businesses to adopt e-commerce was important for researchers and firms. This could be

studied with different research strategies and from different perspectives. The authors study was conducted on business-to-business relationship (B2B) e-commerce adoption at firm level from the business-to-business relationship point of view. The respondents were asked to validate and assess the importance of attributes identified for business-to-business relationships. The second part of the study investigated the impacts of adoption of e-commerce on business-to-business relationship. Three different relationships validated the findings of the collected data by using PROMETHEE. The results were showing that business-to-business e-commerce would have certain impacts, with different magnitudes, on the relationship of businesses with each other. It also could guide businesses on how to prioritize their e-commerce projects roll out in the business-to-business context. This according to the author would help businesses to maximize their investment on their relationships deploy an effective business-to-business e-commerce and increase their business-to-business relationship efficiency by enabling electronic aspects in their relationships.

(Rao and Rajesh, 2009) suggested an effective decision making framework for software selection in manufacturing industries using a multiples criteria decision making method, preference ranking organization method for enrichment evaluations (PROMETHEE). The method was improved in that work by integrating with analytical hierarchy process (AHP) and the fuzzy logic. The fuzzy logic, however, was introduced to handle the imprecision of the human decision making process. The proposed decision making framework was practical for ranking competing software product in terms of their overall performance with respect to multiple criteria.

The methodology to be used for this thesis is the PROMETHEE methodology and it will largely depend on the work of (Podvezko and Podvezko, 2009) on the dependence of multi-criteria evaluation results on choice of preference functions and their parameters. According to them, a considerable usage increase of multi criteria methods was recently observed in the area of quantitative analysis of social or economical phenomena. The PROMETHEE methods were discerned from other multi-criteria methods by depth of their intrinsic logic and by using preference functions, which make up a foundation of the methods. Shapes of functions and their parameters were chosen by decision-makers thus exerting clear advantages and features of the methods. This work revealed the influence of the choice of preference functions and the corresponding parameters on the outcome of evaluation. Along with already recently described by the authors PROMETHEE I method the other PROMETHEE II method was described and examples of its application were provided. New types of preference functions were as well proposed.

Due to its reach acceptance and capability to share information, the World Wide Web has become an important tool for business (Villota, 2009). According to Villota, millions of websites had been developed and so inherently they could come across every kind of website from easy to hard-to-use. The authors added that there were some so-called usability criteria, which should be respected by web designers in order to make websites useful. As a result, using a multicriteria decision making approach, they evaluated the performance, based on seven (7) usability criteria, of five (5) websites from which one could be books online.

They explained that the complexity of multicriteria decision making was based on the fact that those multiple criteria were often contradicting with each other, and so a solution that optimizes every criterion simultaneously, or an ideal solution, was generally unfeasible. In that situation making a decision implied giving an answer which without being optimal was still satisfactory.

Considering usability as a subjective matter, they used two well-known methodologies that deal with this issue: Analytic Hierarchy Process (AHP) and PROMETHEE. Through PROMETHEE they related the preference of a decision maker with specially defined criterion functions.

CHAPTER THREE

METHODOLOGY

3.0 Introduction

In this chapter, we shall put forward the factors that influence facility location, a brief history of AHP/PROMETHEE as well as the details of the AHP/PROMETHEE II ranking methodology for the facility location selection.

3.1 Factors that influence facility location

The suitability of a specific location for proposed facility operations depends largely on what location factors are selected and evaluated, as well as their potential impact on corporate objectives and operations. There are a large number of location factors that have an influence on location decisions. Facility location attribute is defined as a factor that influences the selection of facility location for a financial institution. In the case of facility location selection in Obuasi Municipality, factors chosen include: proximity to customers, nearness to market, business climate, community desirability and other facilities.

Proximity to customers: A location close to the customer is important because of the ever-increasing need to be customer-responsive. This enables faster delivery of service to customers. In addition, it ensures that customers' needs are included in the decisions taken by the decision makers. Population characteristics provide a basis for decision making on these criteria.

Nearness to market: Locating a bank close to the market will help its customers have easy access to the facility without carry huge sums of money to travel from a long

distance to bank. Closeness to markets plays an important role in the location decisions of banks. The nearness to market is a major concern in the location decision of an institution. The proximity to markets is considered as an important factor in the location decision.

Business climate: A favourable business climate condition contributes meaningfully to the facility location decisions. Business climate indicates how regional and local policies, relationships and local communities support business development. Eventually, a good business climate allows businesses to conduct their affairs with minimal interference while accessing quality high inputs and customers at flexible terms of service. While no business climate is perfect for every kind of business, certain attributes of the regional or local economy allow investors to find fewer risks and higher returns when compared to other places.

Community desirability: The host community's interest in having the facility in its midst is a necessary part of the evaluation process. Community attitudes towards the facility (i.e. whether the people in the community are ready for the facility to be located). There are many environmental concerns associated with facility location. Therefore, decision makers consider it important to have a positive community attitude. Hence, this assumes that community attitude will have a positive impact on the facility location.

Other facilities: The location of other facilities or service centers of the similar financial institution may influence a new facility location in the network. Institutions will be aware of the extent of the competition in an area when they are looking to locate their business.

If there are several other similar businesses in the locality, it might influence their decision about location.

3.2 Facility Location Models

A rich literature has been developed and several models have been formulated and applied to the facility location problems over the last few years. The complexity of these problems is due to the multitude of quantitative and qualitative factors influencing location choices. However, investigators have focused both algorithms formulation in diverse setting in the private sector (e.g. industrial plants, retail facilities, telecommunication mast, etc) and the public sectors (e. g. banks, schools, health centers, ambulances, clinics, etc). In this work, our interest is in one of the public sector facility location selection problem, that is, bank location problem. But providing these facilities effectively is a complex issue that specially depends on some factors and most especially on the geographical location of the facility. The aim of this thesis therefore is to use the combination of Analytic Hierarchy Process (AHP)/ Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE II) ranking method to select the strategic location in Obuasi Municipality for the facility.

Facility location models concern the provision of a service to satisfy a spatially dispersed demand. A demand for the service exists at a large number of widely dispersed sites. It is impossible to provide the service anywhere. For instance, every household needs a source of groceries, but impossible to provide a grocery store at each household. Therefore, for reasons of cost, the service must be provided a few, centralized locations. Examples of

location models are the P-Median Problem (PMP), P-Center Problem (PCP), Location Set Covering Problem (LSCP) and Maximal Covering Location Problem (MCLP).

3.3 The P-Median Problem

The development of facility location models as they are thought of today traces back to research most notably done by Alfred Weber, who derived a method for placing a facility at a location that minimized the distance traveled by some set of customers (Weber, 1909). He considered the environment to be a continuous two-dimensional plane, where the facility could be placed anywhere on this surface. In many ways this transformed how planners thought of and executed the placement of facilities throughout regions. Weber's technique provides the means to quantitatively represent the efficiency of a placement and through this introduced strategic location and the framework for many other location models geared toward the world of business and public service.

This model may be formulated as follows:

$$\text{Minimize } \sum_{j \in J} \sum_{i \in I} h_i d_{ij} y_{ij} \quad (3.3a)$$

$$\begin{array}{ll} \text{Subject} & \text{to:} \\ \sum_{j \in J} x_j = P & (3.3b) \end{array}$$

$$\sum_{j \in J} y_{ij} = 1 \quad \forall i \in I \quad (3.3c)$$

$$y_{ij} - x_j \leq 0 \quad \forall i \in I, j \in J \quad (3.3d)$$

$$x_j \in \{0,1\} \quad \forall j \in J \quad (3.3e)$$

$$y_{ij} \in \{0, 1\} \quad \forall i \in I, j \in J \quad (3.3f)$$

The objective function (3.3a) minimizes the demand-weighted total distance traveled. Constraint set (3.3b) through (3.3d) are identical to (3.3a) through (3.3c) of the p-center problem. Constraint sets (3.3e) and (3.3f) are identical to (3.3a) and (3.3b). Constraint set (3.3b) can be eliminated following the same arguments as were used for constraint set (3.3a). Toregas and ReVelle (1972) show that this formulation also minimizes the average travel distance between the sited are nodes on the network would not reduce total travel cost. Consequently, this formulation will yield an optimal solution, even if the facilities could be located anywhere on an arc. Like the p-center problem, the p-median problem can be solved in polynomial time for fixed values of p, but is NP-hard for variable values of p (Garey and Johnson, 1979).

3.4 P-center Problem

The p-center problem (Hakimi, 1964, 1965) addresses the problem of minimizing the maximum distance that demand is from its closet facility given that we are siting a pre-determined number of facilities. There are several possible variations of the basic model. The vertex p-center problem restricts the set of candidate facility sites to the nodes of the network while the absolute p-center problem permits the facilities to be anywhere along the arcs. Both versions can be either weighted or unweighted. In the unweighted problem, all demand nodes are treated equally. In the weighted model, the distance between demand nodes and facilities are multiplied by a weight associated with the demand node.

For example, this weight might represent a node's importance or, more commonly, the level of its demand.

Given our previous definition and the following decision variables

W = the maximum distance between a demand node and the facility to which it is assigned

$$y_{ij} = \begin{cases} 1 & \text{if demand node } i \text{ is assigned to a facility at node } j \\ 0 & \text{if not} \end{cases}$$

The p-center problem can be formulated as follows:

Maximize W (3.4a)

Subject to:

$$\sum_{j \in J} x_j = P \quad (3.4b)$$

$$\sum_{j \in J} y_{ij} = 1 \quad \forall i \in I \quad (3.4c)$$

$$y_{ij} - x_j \leq 0 \quad \forall i \in I, j \in J \quad (3.4d)$$

$$W - \sum_{j \in J} h_j d_{ij} y_{ij} \geq 0 \quad \forall i \in I \quad (3.4e)$$

$$x_j \in \{0, 1\} \quad \forall j \in J \quad (3.4f)$$

$$y_{ij} \in \{0, 1\} \quad \forall i \in I, j \in J \quad (3.4g)$$

The objective function (3.4a) minimizes the maximum demand-weighted distance between each demand node and its closest open facility. Constraint (3.4b) stipulates that p facilities are to be located. Constraint set (3.4c) requires that each demand node be assigned to exactly one facility. Constraint set (3.4d) restricts demand node assignment only to open facilities. Constraint (3.4e) defines the lower bound on the maximum demand-weighted distance, which is being minimized. Constraint set (3.4f) established the siting decision variable as binary. Constraint set (3.4g) can be replaced by $y_{ij} \geq 0 \forall i \in I; j \in J$ because constraint set (3.4d) guarantees that $y_{ij} \leq 1$. If some y_{ij} are fractional, we can simply assign node I to its closest open facility.

For fixed values of p , the vertex p -center problem can be solved in $O(N^p)$ time since we can enumerate each possible set of candidate locations in this amount of time. Clearly, even for moderate values of N and p , such enumeration is not realistic and more sophisticated approaches are required. For variable values of p , the problem is NP-hard (Garey and Johnson, 1979)

If integer-valued distances can be assumed, the unweighted vertex or absolute p -center problem is most often solved using a binary search over a range of coverage distance (Handler and Mirchandani, 1979; Handler, 1990) for each coverage distance, a set covering distance is the solution to the p -center problem. Deskin (2000) has recently shown how the maximal covering model can be used effectively in place of the set covering as a sub-problem in solving the unweighted vertex p -center problem.

3.5 Covering Models

Covering models are the widely used location models for solving the emergency facility location problem. The objective here is to provide covering to the demand points. A demand point is considered as covered only if a facility is available to service the demand point within a coverage distance limit which normally referred to as a critical distance. At the heart of the set covering and maximal covering model is the notion of covering.

3.5.1 Location Set Covering Model

The set covering problem is to find a set of facilities with minimum cost from among a finite set of candidate facilities so that every demand node is covered by at least one facility. According to Toregas (1970), location set covering problem involves finding the smallest number of facilities and their locations so that each demand is covered by at least one facility. The location set covering problem does not specify a prior distance covering within which a demand is covered.

However, the Maximal Covering Location problem finds the facilities and their locations such that each demand is not farther than a pre-specified distance or time from its closest facility. A demand is covered if one or more facilities are located within the maximum distance or time.

The formulated of the model is as follows:

$$\text{Minimize: } Z = \sum_{j \in J} x_j \quad (3.5.1a)$$

Subject to

$$\sum_{j \in N_i} x_j \quad \forall i \in I \quad (3.5.1b)$$

$$x_j = 0, 1 \quad \forall j \in J \quad (3.5.1c)$$

Where

J = set of eligible facility sites (indexed by j);

I = set of demand nodes (index by i);

$$x_j = \begin{cases} 1 & \text{if facility is located at node } j \\ 0 & \text{otherwise} \end{cases}$$

$N_i = \{j \mid d_{ji} \leq S\}$; with d_{ji} = shortest distance from potential facility location j to demand node i , and S = distance standard for coverage.

Here, N_i is the set of all those sites that are candidates for potential location of facilities, that are within distance S of the demand node i . If a facility is located in any of them, demand node i becomes covered. The objective (1) minimizes the number of facilities required. Constraints (2) state that the demand at each node i must be covered by at least one server located within the time or distance S .

The solution to this model can be easily found solving its linear programming relaxation, with occasional branch and bound applications. Before solving, its size can be reduced by successive row and column reductions, as proposed by Toregas and ReVelle (1973).

3.5.2 Maximum Covering Location Model (MCLP)

The set covering has associated problems, one of which is that the number of facilities that are needed to cover all demand nodes is likely to exceed the number that can actually be

built due to budget constraints and other related issues. Furthermore, the set covering model treats all demand nodes identical. Under certain conditions and budgetary constraints it is appropriate to fix the number of facilities that are to be located and then maximize the number of covered demands.

Church and ReVelle (1974) formulated a Maximum Covering Model as follows:

Let h_i = demand at node i

P = number of facilities to locate

Decision Variables be

$$Z_i = \begin{cases} 1, & \text{if node } i \text{ is covered} \\ 0, & \text{if not} \end{cases}$$

The Maximum Covering Location Model is formulated as follows;

$$\text{Maximize } \sum_j h_i z_i \quad (3.5.2a)$$

Subject to;

$$Z_i \leq \sum_j a_{ij} x_j \quad \forall i \quad (3.5.2b)$$

$$\sum_j x_j \leq P \quad (3.5.2c)$$

$$x_j = 0.1 \quad (3.5.2d)$$

$$z_j = 0.1 \quad (3.5.2e)$$

The objective function 3.5.2a maximizes the number of covered demands. Constraints 3.5.2b state that demand node i cannot be covered unless at least one of the facility sites that cover node i is selected. But, the right-hand side of constraints 3.5.2c which is $\sum a_{ij}x_j$ is identical to the left-hand side of constraints 3.5.2c $\sum_j a_{ij} x_j$ gives the number of selected facilities that can cover node i , the constraint 3.5.2c stipulates that we locate not more than p facilities. Constraint 2.5.2c will be binding in the optimal solution. Constraints 3.5.2d and 3.5.2e are the integrality constraints on the decision variables.

3.5.3 Maximum Expected Covering Location Model (MEXCLP)

Daskin (1983) proposed MEXCLP as extension to the Maximal Covering Location Problem (MCLP) formulated by Church and ReVelle (1974), (Chiyoshi et al 2003b). This was mainly to account for possibility of unavailability due to a congested system. The interest here is for demand to be covered by a located facility that is available when a demand for service arises. The approach attempted to maximize expected coverage given that the servers are busy and unavailable with a calculable system wide probability, P (Daskin, 1983).

Daskin (1983) made three (3) simplifying assumptions when he formulated the MEXCLP (Chiyoshi et al., 2003b)

- Server operate independently
- Each server has the same busy probability
- Server busy probabilities are invariant with respect to their location.

Daskin (1983) developed a substitution heuristic which he tested on a fifty-five (55) node network problem.

The MEXCLP maximized the expected value of population coverage within the time standard, given that p facilities are to be located on the network. Daskin computed the increase in the expected coverage of a demand, when a k^{th} server is added to its neighbourhood, which turns out to be just $(1 - q)q^{k-1}$. Then, the expected coverage for all possible number of servers k at each neighbourhood, and for all demand nodes weighted by their demand, is maximized:

$$\text{Minimize } Z = \sum_{i \in I} \sum_{k=1}^{n_i} a_i (1 - q) q^{k-1} y_{ik} \quad (3.5.3a)$$

Subject to

$$\sum_{k=1}^{n_i} y_{ik} \leq \sum_{j \in N_i} x_j \quad \forall i \in I \quad (3.5.3b)$$

$$\sum_{j \in J} x_j = p \quad (3.5.3c)$$

$$y_{ik} = 0, 1 \quad \forall i, k$$

$$x_j = \text{integers } \forall j$$

Where

y_{ik} is one if node i has at least k servers in its neighbourhood, zero otherwise,

x_j is the number of servers at site j , and

n_i is the maximum number of servers in N_i

The first constraint says that the number of servers covering demand i is bounded above by the number of servers sited in the neighbourhood. The second constraint limits the number of servers to be deployed. Declining weights $(1 - q)q^{k-1}$ on the variables y_{ik} make unnecessary any ordering constraints for these variables, and help to the integrality of these variables in the solution, if the linear relaxation of the model is solved. Daskin proposed a heuristic method of solution of the MEXCLP, which gives solution for the system for different ranges of values of q .

3.6 A Brief History of AHP and PROMETHEE

3.6.1 Analytic Hierarchy Process (AHP)

Prof. Thomas L. Saaty (1980) originally developed the Analytic Hierarchy Process (AHP) to enable decision making in situations characterized by multiple attributes and alternatives. AHP is one of the Multi Criteria decision making techniques. AHP has been applied successfully in many areas of decision-making. In short, it is a method to derive ratio scales from paired comparisons.

Analytic Hierarchy Process (AHP) provides a comprehensive framework for structuring a decision problem to represent and quantifying its elements. The outcome of AHP is a prioritized weighting of each decision alternative. The AHP converts these evaluations to numerical values that can be processed compared over the entire range of the problem. A numerical weight or priority is derived for each element of the hierarchy, allowing diverse and often incommensurable elements to be compared to one another in a rational and consistent way.

The first step in the Analytic Hierarchy Process is to model the problem as a hierarchy. The hierarchy is a structured means of describing the problem at hand. It consists of an

overall goal at the top level, a group of options or alternatives for reaching the goal and a group of factors or criteria that relate the alternatives to the goal.

3.6.2 Preference Ranking Organization Method for Enrichment Evaluation

(PROMETHEE)

The PROMETHEE methodology is a family of six outranking methods, which are the PROMETHEE I, PROMETHEE II, PROMETHEE III, PROMETHEE IV, PROMETHEE V and PROMETHEE VI (Behzadian et al., 2010).

The first two – PROMETHEE I and PROMETHEE II, which respectively deal with partial and complete ranking of alternatives were propounded by Brans and presented for the first time in 1982 at a conference organized by Nadeau and Landry at the University Laval, Quebec, Canada (Brans, 1982).

Few years afterwards, PROMETHEE III for ranking based on interval, PROMETHEE IV for complete or partial ranking of alternatives when the set of viable solutions is continuous was developed (Brans et al., 2011). The remaining two – PROMETHEE V for multicriteria problems involving segmentation constraints and PROMETHEE VI for the representation of the human brain were proposed between 1992 and 1994 (Brans et al., 2010). Other multicriteria decision aids (MCDA) such as the PROMETHEE group decision support system (GDSS) for group decision-making (Brans et al., 2010), and the visual interactive module GAIA (Geometrical Analysis for Interactive Aid) for pictorial representation to complement the algebraic methodology were developed to facilitate the analysis of more complex decision-making problems (Brans et al, 2010).

Two extensions of PROMETHEE have recently been proposed as PROMETHEE TRI for multicriteria decision-making problems involving sorting and the PROMETHEE CLUSTER for problems dealing with nominal classification (Figueira et al., 2004).

3.7 OVERVIEW OF METHODOLOGIES

3.7.1 The AHP method

AHP, developed by Saaty (1980), addresses how to determine the relative importance of a set of activities in a multi-criteria decision problem. The process makes it possible to incorporate judgments on tangible qualitative criteria alongside tangible quantitative criteria (Badri 2001). The AHP method is based on three principles: first, structure of the model; second, comparative judgment of the alternatives and the criteria; third, synthesis of the priorities.

The first step, a complex decision problem is structured as a hierarchy. AHP initially breaks down a complex MCDM problem into a hierarchy of interrelated decision elements (criteria, decision alternatives). With the AHP, the objectives, criteria and alternatives are arranged in a hierarchical structure similar to a family tree. A hierarchy has at least three levels: overall goal of the problem at the top, multiple criteria that define alternatives in the middle, and decision alternatives at the bottom as shown in the figure 3.1

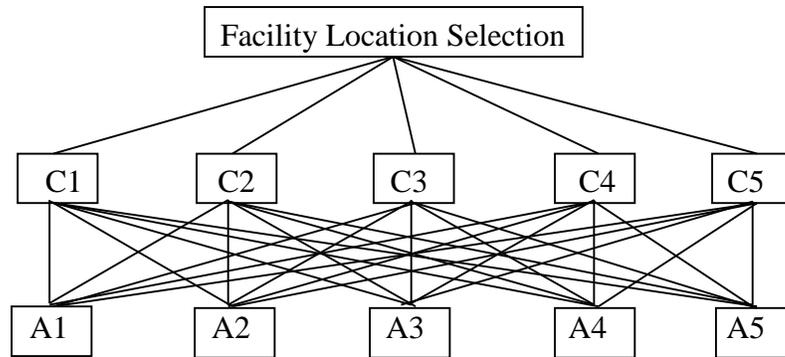


Figure 3.1: *Hierarchy for the facility location selection*

The second step, once the hierarchy is built, the decision makers systematically evaluate its various alternatives elements by comparing them to one another two at a time, with respect to their impact on criteria element above them in the hierarchy. The AHP converts these evaluations to numerical values that can be processed and compared over the entire range of the problem. A numerical weight or priority is derived for each alternative element of the hierarchy, allowing diverse and often incommensurable alternative elements to be compared to one another in a rational and consistent way.

In the final step of the process, numerical priorities are calculated for each of the decision alternatives. These numbers represent the alternatives' relative ability to achieve the decision goal, so they allow a straightforward consideration of the various courses of action.

Each of these judgments is then assigned an integer on a scale. In this thesis, the original definition of scale given by Saaty (1980) was adopted. The scale and their relative importance are explained in Table 3.1.

Table 3.1: The Saaty (1980) Rating Scale

Scale	The relative importance of the element	Explanation
1	Equally important	i and j are equally important
2	—	
3	Moderately important	i is moderately more important than j
4	—	
5	Strongly important	i is strongly more important than j
6	—	
7	Very strongly important	i is very strongly more important than j
8	—	
9	Extremely important	i is extremely more important than j
2,4,6,8	Intermediate values	used when a compromise is needed

Pairwise Comparison Matrix

Given is a set of A alternatives: $A_1, A_2, A_3, \dots, A_n$ and a set of C decision criteria $C_1, C_2, C_3, \dots, C_n$ and the data of a decision matrix of $a_{11} = (A_1, A_1); a_{12} = (A_1, A_2); a_{13} = (A_1, A_3); a_{14} = (A_1, A_4); \dots, a_{1n} = (A_1, A_n); a_{21} = (A_2, A_1); a_{22} = (A_2, A_2); a_{23} = (A_2, A_3); a_{24} = (A_2, A_4); \dots, a_{2n} = (A_2, A_n)$, as the one shown below.

The pairwise comparison table is mathematically expressed in the form of square matrix $n \times n$, where n is the number of alternatives or criteria. The elements of the matrix are the estimated judgment weights, the relative importance among alternatives or criteria as explained earlier. For example, the pair wise comparison matrix A , in which the element a_{ij} of the matrix is the relative importance of the i^{th} factor with respect to the j^{th} factor and reciprocals are assigned automatically as

$$A = \begin{bmatrix} 1 & a_{12} & \dots & a_{1n} \\ 1/a_{12} & 1 & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 1/a_{1n} & \dots & \dots & 1 \end{bmatrix}$$

For example, consider the following A1, A2, A3

(1) Compare **A1** to **A2**

– Which is more important?

Say A1

– By how much? Say moderately → 3

(2) Compare **A1** to **A3**

– Which is more important?

Say A1

– By how much? Say strongly important → 5

(3) Compare **A2** to **A3**

– Which is more important?

Say A2

– By how much? Say moderately → 3

This set of comparison gives the following matrix

Table 3.2 Pairwise Comparison Matrix

	A1	A2	A3
A1	1	3	5
A2	1/3	1	3
A3	1/5	1/3	1

Calculating the weights and determine the consistency for each level

Weights are calculated from the pairwise comparison matrices. The first step would be to sum up the values of each row in the comparison matrix. The row sums are then added to give the total sum. The row sum is then divided by the total sum. The weight for each row is given by the formula below:

$$\text{Weight} = \frac{\text{row sum}}{\text{Total sum}}$$

Table 3.3 Weight of Pairwise Comparison Matrix

	A1	A2	A3	ROW SUM	WEIGHT
A1	1	3	5	9.000	0.6054
A2	1/3	1	3	4.333	0.2914
A3	1/5	1/3	1	1.533	0.1031
			TOTAL	14.866	

This step is to find the relative priorities of criteria or alternatives implied by these comparisons. The relative priorities are worked out using the theory of eigenvector. And the consistency check should be done at each stage of the selection process. To evaluate the consistency of the obtained results three components are needed from the analysis namely Consistency Index (CI), Random Consistency Index (RI) and Consistency Ratio (CR). The following techniques are used to determine the above said elements of calculation.

$$CI = \frac{(\lambda_{\max} - n)}{(n - 1)} \quad (3.1)$$

where λ_{\max} is the maximum eigenvalue and n is the size of the pairwise comparison matrix.

Table 3.4: Saaty (1980) Random consistency Index (RI)

N	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

The obtained CI value is compared with the random index RI given in Table 3.2. The Table 3.2 had been calculated as an average of CI's of many thousand matrices of the same order whose entries were generated randomly from the scale 1 to 9 with reciprocal effect. The simulation results of RI for matrices of size 1 to 10 had been developed by

Saaty (1980) and are given in Table 3.2. The ratio of CI and RI for the same order matrix is called the consistency ratio CR.

Thus the consistency ratio (CR) is obtained by using,

$$CR = \frac{CI}{RI} \quad (3.2)$$

Thus the $CR \leq 0.1$

In general, a consistency ratio of 10% (0.1) or less is usually acceptable. If inconsistency of judgments within the matrix has occurred then evaluation process should be reviewed and improved upon. At the final step of the calculation, the overall preference matrix would be constructed by multiplying all the weights with the factors, therefore the results are added to get the composite score of each factor.

3.7 .2 PROMETHEE II method

Preference function based outranking method is a special type of MCDM tool that can provide a ranking ordering of the decision options. The PROMETHEE (preference ranking organization method for enrichment evaluation) method was developed by Brans and Vincke in 1985. The PROMETHEE I method can provide the partial ordering of the decision alternatives, whereas, PROMETHEE II method can derive the full ranking of the alternatives. In this thesis, the combination of AHP/PROMETHEE II methods is employed to obtain the full ranking of the alternative facility location for a given financial application.

The procedural steps as involved in PROMETHEE II method are enlisted as below:

Step 1: Normalize the decision matrix using the following equation:

$$R_{ij} = \frac{[X_j - \min(X_j)]}{[\max(X_j) - \min(X_j)]} \quad (i = 1, 2, \dots, n, j = 1, 2, \dots, m) \quad (3.3a)$$

where X_{ij} is the performance measure of i^{th} alternative with respect to j^{th} criterion.

For non-beneficial criteria, Eqn. (3.3a) can be rewritten as follows:

$$R_{ij} = \frac{[\max(X_j) - X_j]}{[\max(X_j) - \min(X_j)]} \quad (3.3b)$$

Step 2: Calculate the evaluative differences of i^{th} alternative with respect to other alternatives. This step involves the calculation of differences in criteria values between different alternatives pairwise.

Step 3: Calculate the preference function, $P_j(A_k, A_i)$.

There are mainly six types of generalized preference functions as proposed by Brans and Mareschal (1994). But these preference functions require the definition of some preferential parameters, such as the preference and indifference thresholds. However, in real time applications, it may be difficult for the decision maker to specify which specific form of preference function is suitable for each criterion also to determine the parameters involved. To avoid this problem, the following simplified preference function is adopted here:

$$P_{ij}(A_k, A_i) = 0 \text{ if } R_{kj} \leq R_{ij} \quad (3.4a)$$

$$P_{ij}(A_k, A_i) = (R_{kj} - R_{ij}) \text{ if } R_{kj} \geq R_{ij} \quad (3.4b)$$

Step 4: Calculate the aggregated preference function taking into account the criteria weights.

Aggregated preference function,

$$\pi(A_k, A_i) = \frac{[\sum_{j=1}^m W_j \times P_j(A_k, A_i)]}{\sum_{j=1}^m W_j} \quad (3.5)$$

where w_j is the relative importance (weight) of j^{th} criterion.

Step 5: Determine the leaving and entering outranking flows as follows:

From the preference function $\pi(A_k, A_i) \forall A_k, A_i \in A$ where A is a finite set of alternatives indicates the degree of preference expressed by the decision maker for the alternative A_k over alternative A_i for all the criteria. On the other hand, there are some criteria too in which the alternative A_i may be preferred to the alternative A_k giving rise to the preference function $\pi(A_i, A_k)$. These shows how two alternatives have a comparative advantage over each other over a given finite criteria.

These two indices $\pi(A_k, A_i)$ and $\pi(A_i, A_k)$ connect every pair of alternatives say A_k, A_i , to each other. Such a connection or relation is known as the outranking relation.

Graphically, the relation is often represented by two nodes denoting the two alternatives linked to each other by a corresponding two arcs each for a preference index as presented in Figure 3.2

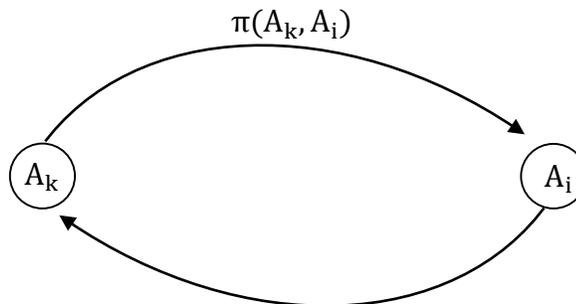


Figure 3.2: Outranking Flow Relation $\pi(A_k, A_i)$

From the figure above, the alternatives A_k and A_i in rings are the nodes. The preference index $\pi(A_k, A_i)$ which links node A_k to node A_i as indicated by the arrow of the upper arc of Figure 3.2, shows the magnitude of the preference of the alternative A_k over A_i . The preference function $\pi(A_i, A_k)$ on the other hand, connects node A_i to A_k and is indicated

by the arrow of the lower arc of Figure 3.2 showing the magnitude of preference of the alternative to A_i to A_k .

Here, each alternative faces $(n - 1)$ number of other alternatives. The leaving flow expresses how much an alternative dominates the other alternatives, while the entering flow shows how much an alternative is dominated by the other alternatives. Based on these outranking flows, the PROMETHEE I method can provide a partial preorder of the alternatives, whereas, the PROMETHEE II method can give the complete preorder by using a net flow, though it loses much information of preference relations.

Leaving (or positive) flow for A_k alternative,

$$\varphi^+(A_k) = \frac{1}{n-1} \sum_{A_i \in A}^n \pi(A_k, A_i) \quad (3.6)$$

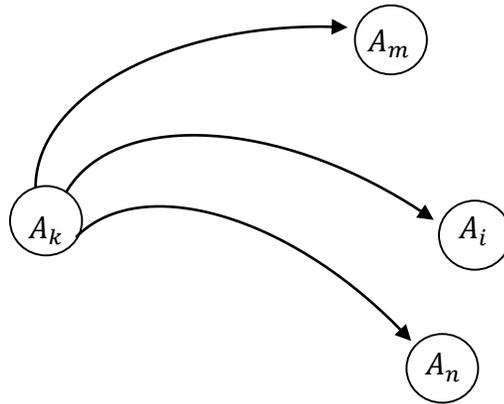


Figure 3.3: Positive outranking flow (Leaving flows) ($\varphi^+(A_k)$)

From the above Figure, the arrows directed at nodes A_i, A_m, A_n from node A_k show how the alternative A_k outranks all other alternatives. These directed arrows from A_k are called the positive outranking flows (leaving flows) denoted by $\varphi^+(A_k)$ as shown above.

Entering (or negative) flow for A_k alternative,

$$\varphi^-(A_k) = \frac{1}{n-1} \sum_{A_i \in A}^n \pi(A_i, A_k) \quad (3.7)$$

where n is the number of alternatives.

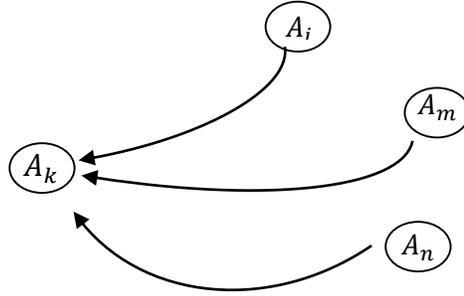


Figure 3.4: the negative outranking flow (Entering flows) ($\varphi^-(A_k)$)

The negative outranking flow is represented graphically as shown by Figure 3.4:

From the figure above, the arrows from nodes A_i, A_m, A_n etc. directed at node A_k are called the negative outranking (entering) flows and they show how the alternative A_k is outranked by the other alternatives.

Step 6: Calculate the net outranking flow for each alternative.

$$\varphi(A_k) = \varphi^+(A_k) - \varphi^-(A_k) \quad (3.8)$$

Step 7: Determine the ranking of all the considered alternatives depending on the values of $\varphi(A_k)$. The higher value of $\varphi(A_k)$, the better is the alternative. Thus the best is the one having the highest $\varphi(A_k)$ value.

The PROMETHEE method is an interactive multi-criteria decision-making approach designed to handle quantitative as well as qualitative criteria with discrete alternatives. In the method, pair-wise comparison of the alternatives is performed to compute a preference function for each criterion. Based on this preference function, a preference index A_k over A_i is determined. This preference index is the measure to support the hypothesis that alternative A_k is preferred to A_i .

CHAPTER FOUR

DATA COLLECTION, ANALYSIS, RESULTS AND DISCUSSION

4.1 Data Collection

A questionnaire was constructed by the researcher to tap perception of public and stakeholders towards factors associated with facility location. The questionnaire was administered to one hundred and twenty respondents with the aim to identify their perceptions of factors associated with facility location selection. Again, Statistical Department of Obuasi Municipal Assembly was consulted for information on each town. The age of respondents were varied from 20 to 65 years old. The data for the questionnaire were collected in the area of Obuasi Municipality. Respondents need to judge the relative comparison between criteria and the relative comparison between alternative with respect to criterion in linguistic scales. Each of these judgments is then assigned an integer on a scale. In this thesis, the original definition of scale given by Saaty (1980) was adopted. The scale and their relative importance are explained in Table 4.1.

A financial institution in Obuasi Municipality wants to select the best location. Five (5) alternatives/towns were identified. Alternatives are Brahabebome (A1), Konka (A2), Wawasi (A3), Gausu (A4) and Tutuka (A5). During the evaluation, five (5) main criteria/factors (C1: proximity to customers, C2: nearness to market, C3: community desirability, C4: business climate, C5: other facilities) have been selected. Finally, the best location selection among five (5) alternatives has been investigated.

4.1.1 Organization of Data

The decision-makers individually express their opinions regarding the relative importance of the criteria and preferences among pairs of alternatives using pairwise comparison and Saaty Rating Scale in Table 4.1 was used ranging from 1 to 9. If however, one criterion is preferred less than the comparison criterion, the reciprocal of the preference score is assigned as shown in Table 4.2.

The matrix components on the diagonal of this matrix take 1 value, since they are equally important. A basic, but very reasonable, assumption is that if C1 is strongly important than C3 and is rated at 5, then C3 must be extremely less important than C1 and is valued at 1/5.

Table 4.1: The Saaty (1980) Rating Scale

Scale	The relative importance of the element	Explanation
1	Equally important	i and j are equally important
2	—	
3	Moderately important	i is moderately more important than j
4	—	
5	Strongly important	i is strongly more important than j
6	—	
7	Very strongly important	i is very strongly more important than j
8	—	
9	Extremely important	i is extremely more important than j
2,4,6,8	Intermediate values	used when a compromise is needed

Questionnaire results

Out of 120 questionnaires administered to the respondents, 113 were returned. The details of the returned questionnaires are summarized in the pairwise comparison matrices from Table 4.2 – 4.7. The questionnaire sample is shown in Appendix 2.

4.1.2 Constructing the pairwise comparison matrix

From the questionnaires, decision-makers determined relative values for the criteria and each alternative using Saaty (1980) rating scale of table 4.1. Information gathered from the questionnaires of the criteria with respect to the objectives show that C1 in the first row and C1 in the first column are equally important and have been assigned a value 1; C1 in the first row is slightly important than C2 and the value assigned is 2; C1 in the first row is strongly important than C3 in the first column, the value assigned is 5; C1 in the first row and C4 in the first column are equally important and assigned 1; C1 in the first row is moderately important than C5, value assigned is 3; C2 in the second row and C2 in the column are equally important, value assigned is 1 and so on. On the other hand, C2 in second row is slightly less important than C1 in the first column; C3 in the third row is strongly less important than C1, therefore, a reciprocal value is assigned to them. This can be expressed mathematically as $a_{11} = (C1, C1)=1$; $a_{12} = (C1, C2) =2$; $a_{13} = (C1, C3) = 5$; $a_{14} = (C1, C4) = 1$; $a_{15} = (C1, C5) = 3$; $a_{31} = (C3, C1) = 1/5$. The Pairwise comparison matrix for criteria of the above is displayed in Table 4.2. Similar explanation holds for all the alternatives as shown in Table 4.3 – 4.7.

Table 4.2 Pairwise comparison matrix for criteria with respect to objectives

	C1	C2	C3	C4	C5
C1	1	2	5	1	3
C2	1/2	1	3	2	3
C3	1/5	1/3	1	1/4	2
C4	1	1/2	4	1	4
C5	1/3	1/3	1/2	1/4	1

Table 4.3 Pairwise comparison matrix of alternatives with respect to C1

	A1	A2	A3	A4	A5
A1	1.000	0.200	0.500	0.333	0.143
A2	5.000	1.000	1.000	0.333	0.200
A3	2.000	1.000	1.000	0.500	0.200
A4	3.000	3.000	2.000	1.000	0.333
A5	7.000	5.000	5.000	3.000	1.000

Table 4.4 Pairwise comparison matrix of alternatives with respect to C2

	A1	A2	A3	A4	A5
A1	1.000	2.000	0.500	0.333	1.000
A2	0.500	1.000	0.250	0.143	0.333
A3	2.000	4.000	1.000	0.500	2.000
A4	3.000	7.000	2.000	1.000	3.000
A5	1.000	3.000	0.500	0.333	1.000

Table 4.5 Pairwise comparison matrix of alternatives with respect to C3

	A1	A2	A3	A4	A5
A1	1.000	3.000	5.000	3.000	7.000
A2	0.333	1.000	5.000	1.000	9.000
A3	0.200	0.200	1.000	1.000	3.000
A4	0.333	0.200	1.000	1.000	3.000
A5	0.143	0.111	0.333	0.333	1.000

Table 4.6 Pairwise comparison matrix of alternatives with respect to C4

	A1	A2	A3	A4	A5
A1	1.000	3.000	3.000	7.000	9.000
A2	0.333	1.000	1.000	3.000	7.000
A3	0.333	1.000	1.000	2.000	5.000
A4	0.143	0.333	0.500	1.000	3.000
A5	0.111	0.143	0.200	0.333	1.000

Table 4.7 Pairwise comparison matrix of alternatives with respect to C5

	A1	A2	A3	A4	A5
A1	1.000	0.500	0.333	0.200	0.143
A2	2.000	1.000	0.500	0.250	0.200
A3	3.000	2.000	1.000	0.200	0.143
A4	5.000	4.000	5.000	1.000	0.333
A5	7.000	5.000	7.000	3.000	1.000

4.2 Data analysis

As already mentioned, the data used in this study is the perception of public and stakeholders towards factors associated with facility location. The analysis of the data obtained is done using AHP /PROMETHEE II ranking method.

4.2.1 Computation of Analytic Hierarchy Process (AHP)

Calculation of Analytic Hierarchy Process involves a series of computations that are done in the following steps:

Step one: Calculating the weight for each level

Weights are calculated from the pairwise comparison matrices of Table 4.2 – 4.7. The first step would be to sum up the values of each row in the comparison matrix. The row sums are then added to give the total sum. The row sum is then divided by the total sum. However, the computation for weight is shown in Appendix 1, Table 4.2. The weight for each row is given by the formula below and the weight for criteria and each alternative are shown in Table 4.8 and 4.9 respectively:

$$\text{Weight} = \frac{\text{row sum}}{\text{Total sum}}$$

Table 4.8: Weight (W) for the criteria matrix

Criteria	C1	C2	C3	C4	C5
Weight	0.3141	0.2487	0.0990	0.2749	0.0632

Table 4.9: Weight (W) for each alternative against the criteria

Alternative	Criteria				
	C1	C2	C3	C4	C5
A1	0.0496	0.1259	0.3943	0.4472	0.0428
A2	0.1521	0.0580	0.3390	0.2398	0.0778
A3	0.1071	0.2474	0.1121	0.1815	0.1249
A4	0.2127	0.4168	0.1148	0.0968	0.3018
A5	0.4785	0.1519	0.0399	0.0348	0.4527

Step three: Calculating the Eigenvalue

Power method was used to obtain the maximum eigenvalues (λ_{max}) of the comparison matrices. The computation for the λ_{max} of the criteria and the alternative matrices is shown in the appendix 1 and the results are shown in Table 4.10

Table 4.10: Computed eigenvalues for main criteria and alternatives

	C	A1	A2	A3	A4	A5
λ_{max}	5.28185	5.24719	5.02366	5.02372	5.08708	5.27266

Step four: Computing the Consistency Index and Consistency Ratio

The formulas below were used to determine the consistency index and ratio of Table 4.2 – 4.7 and it is summarized in Table 4.11.

$$\text{Consistency Index (C. I.)} = \frac{\lambda_{max} - n}{n - 1} \tag{4.1}$$

where λ_{max} is the maximum eigenvalue and $n=5$ is the size of the pairwise comparison matrix. Using $\lambda_{max} = 5.28185$ for the criteria matrix.

$$C.I. = \frac{5.28185-5}{5-1} = 0.0704625$$

Saaty (1980) has calculated Random Index (R.I.) corresponding to the size of square matrix as shown in table 3.2. Consistency Ratio (C.R.) is calculated by dividing the Consistency Index by the Random Index for the corresponding size of the matrix. Since n =5, the R. I. = 1.12 and the computation is shown in equation 4.2

$$\text{Consistency Ratio (C. R.)} = \frac{C.I.}{R.I.} = \frac{0.0704625}{1.12} = 0.06291 \quad (4.2)$$

Consistency Index (C. I.) and Consistency Ratio (C. R.) for criteria and all the alternatives are shown in Table 4.11

Table 4.11: Consistency Index and Consistency Ratio for criteria and alternatives

	C	A1	A2	A3	A4	A5
C. I.	0.07046	0.06180	0.00591	0.00593	0.02177	0.06817
C. R.	0.06291	0.05518	0.00528	0.00529	0.01944	0.06086

All the Consistency Ratios are acceptable, since the values are less than 10% (0.1).

4.2.2 Computation by PROMETHEE II Ranking Method

The procedural steps as involved in PROMETHEE II method are shown below:

Step five: Normalize the decision matrix using the following equation

The decision matrix was obtained by multiplying the weight of the first criteria (C1) in Table 4.8 by each alternative in the first column of Table 4.9, weight of the second criteria (C2) by each alternative in the second column and so on as shown below in Table 4.12

The computed decision matrix of alternatives with respect to the criteria is as follows:

Entries in decision matrix are x_{ij} and are called performance measure.

Table 4.12 DECISION MATRIX

LOCATION	C1	C2	C3	C4	C5
A1	0.0156	0.0313	0.0390	0.1229	0.0027
A2	0.0478	0.0144	0.0336	0.0659	0.0049
A3	0.0336	0.0615	0.0111	0.0499	0.0079
A4	0.0668	0.1036	0.0114	0.0266	0.0191
A5	0.1503	0.0378	0.0039	0.0096	0.0286

$$R_{ij} = \frac{[x_j - \min(X_j)]}{[\max(X_j) - \min(X_j)]} \quad (i = 1, 2, \dots, n, j = 1, 2, \dots, m) \quad (4.3)$$

where X_{ij} is the performance measure of i^{th} alternative with respect to j^{th} criterion.

$$R_{ij} = \frac{0.0156 - 0.0156}{0.1503 - 0.0156} = 0$$

Calculation for each element is done using equation 4.3 and the normalized values of decision matrix are shown in Table 4.12. Entries are R_{ij} and it is denoted by $A_{ij} = R_{ij}$

Table 4.13 NORMALIZED DECISION MATRIX

LOCATION	C1	C2	C3	C4	C5
A1	0.0000	0.3584	1.0000	1.0000	0.0000
A2	0.2391	0.0000	0.8439	0.4972	0.0852
A3	0.1341	0.5281	0.2037	0.3557	0.2001
A4	0.3802	1.0000	0.2115	0.1503	0.6318
A5	1.0000	0.2619	0.0000	0.0000	1.0000

Step six: Calculation of the preference function, $P_{ij}(A_k, A_i)$.

To avoid the definition of some preferential parameters, such as the preference and indifference thresholds, the following simplified preference function is adopted here using Table 4.13:

$$P_{ij}(A_k, A_i) = 0 \text{ if } A_{kj} \leq A_{ij} \quad (4.4a)$$

$$P_{ij}(A_k, A_i) = (A_{kj} - A_{ij}) \text{ if } A_{kj} \geq A_{ij} \quad (4.4b)$$

where $P_{ij}(A_k, A_i)$ is a number between 0 and 1 which increases if $(A_{kj} - A_{ij})$ is large and equals zero if $A_{kj} \leq A_{ij}$. Now, the preference function is the difference between the pairs of alternatives are calculated for all the pairs of alternatives, using equations (4.4a) and (4.4b), and are given in Table (4.14)

Table 4.14 PREFERENCE FUNCTIONS FOR ALL PAIRS OF ALTERNATIVES

LOCATION PAIR	C1	C2	C3	C4	C5
A1,A2	0.0000	0.3584	0.1561	0.5028	0.0000
A1,A3	0.0000	0.0000	0.7963	0.6443	0.0000
A1,A4	0.0000	0.0000	0.7885	0.8497	0.0000
A1,A5	0.0000	0.0965	1.0000	1.0000	0.0000
A2,A1	0.2391	0.0000	0.0000	0.0000	0.0852
A2,A3	0.1050	0.0000	0.6401	0.1414	0.0000
A2,A4	0.0000	0.0000	0.6323	0.3468	0.0000
A2,A5	0.0000	0.0000	0.8439	0.4972	0.0000
A3,A1	0.1341	0.1697	0.0000	0.0000	0.2001
A3,A2	0.0000	0.5281	0.0000	0.0000	0.1149
A3,A4	0.0000	0.0000	0.0000	0.2054	0.0000
A3,A5	0.0000	0.2662	0.2037	0.3557	0.0000
A4,A1	0.3802	0.6416	0.0000	0.0000	0.6318
A4,A2	0.1411	1.0000	0.0000	0.0000	0.5466
A4,A3	0.2461	0.4719	0.0078	0.0000	0.4317
A4,A5	0.0000	0.7381	0.2115	0.1503	0.0000
A5,A1	1.0000	0.0000	0.0000	0.0000	1.0000
A5,A2	0.7609	0.2619	0.0000	0.0000	0.9148
A5,A3	0.8659	0.0000	0.0000	0.0000	0.7999
A5,A4	0.6198	0.0000	0.0000	0.0000	0.3682

Step seven: Aggregated Preference Function

The aggregated preference function is derived by summing the entire individual preference index and the results are summarized in the table below.

Table 4.14 exhibits the aggregated preference function values for all the paired of alternatives, as calculated using equation:

$$\pi(A_k, A_i) = \frac{[\sum_{j=1}^m W_j \times P_j(A_k, A_i)]}{\sum_{j=1}^m W_j} \quad (4.5)$$

Table 4.15: Aggregated preference function

Location	A1	A2	A3	A4	A5
A1	–	0.03142	0.03938	0.01788	0.00352
A2	0.04796	–	0.03136	0.02985	0.01060
A3	0.03376	0.06175	–	0.04837	0.01312
A4	0.06704	0.10400	0.00850	–	0.01049
A5	0.15085	0.03792	0.00725	0.07204	–

Step eight: Computation of Positive (Leaving) and Negative (Entering) Flow values

From the aggregate preference function in Table 4.15, the following analysis can be made:

The leaving and the entering flows for different facility location alternatives are done by computing:

1. The positive (leaving) flow measures the average degree to which an action is preferred to the other ones.

2. The negative (entering) flow measures the average degree to which the other actions are preferred to that action. The preference flow formulae are stated below:

Positive Outranking Flow:

$$\varphi^+(A_k) = \frac{1}{n-1} \sum_{A_i \in A}^n \pi(A_k, A_i) \quad (4.6a)$$

Negative Outranking Flow:

$$\varphi^-(A_k) = \frac{1}{n-1} \sum_{A_i \in A}^n \pi(A_i, A_k) \quad (4.6b)$$

The values of the positive and negative flows are shown in Table 4.16

Table 4.16: Leaving and entering flows for different locations

Location	Leaving flow	Entering flow	Difference
A1	0.0185800	0.0749000	-0.0563200
A2	0.0299425	0.0587725	-0.0288300
A3	0.0392500	0.0212623	0.0179877
A4	0.0475075	0.0420350	0.0054725
A5	0.2680600	0.0094325	0.2586275

Step nine: Calculation of the net outranking flow for each alternative. The net outranking flow is obtained by finding the difference between the leaving flow and the

entering flow using the equation below. $\varphi(A_k) = \varphi^+(A_k) -$

$$\varphi^-(A_k) \quad (4.7)$$

The net outranking flow values for different alternative location and their relative rankings are given in Table 4.17.

Table 4.17: Net outranking flow values for different location alternatives

Location	Net outranking flow	Rank
A5	0.2586275	1
A3	0.0179877	2
A4	0.0054725	3
A2	-0.02883	4
A1	-0.05632	5

From the table 4.17, it is observed that $A5 > A3 > A4 > A2 > A1$ where ‘>’ means ‘alternative better than’. Therefore, A5 is the best alternative to be selected for the facility location.

Step ten: Partial Ranking

We obtain the Partial Ranking of our finite set of alternatives through the equations (4.6a) and (4.6b).

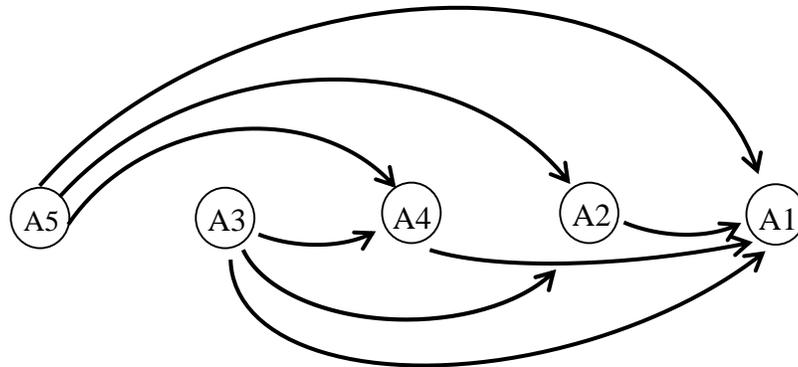


Figure 4.1: Graph of partial ranking

From Figure 4.1, it is realized that there is no connection between A5 and A3, A4 and A2. Showing that the two alternatives are incomparable. For this reason, we apply the complete ranking method.

Step eleven: Complete Ranking

In Complete Ranking, we consider pairs of alternatives using their net flows ($\varphi(A_k)$). This is achieved using equation (4.7). The net flows for the five (5) alternatives are displayed in Table 4.17.

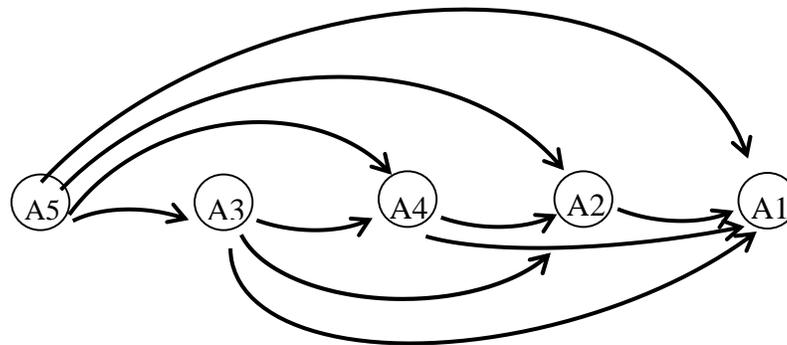


Figure 4.2: Graph of complete ranking

From Table 4.17, the ranking is done based on the difference between leaving flow and entering flow such that the best alternative is the one with the highest number.

4.3 Result

The location with has the highest score is suggested as the best location by the AHP/PROMETHEE II ranking method. In accordance with the results generated Tutuka has the highest net outranking flow of 0.2586275 in comparison with the rest of the locations. The AHP/PROMETHEE II method ranking for the facility location is: (A5) Tutuka (score: 0.2586275), (A3) Wawasi (score: 0.179877), (A4) Gausu (score:

0.0054725), (A2) Konka (score: -0.0288300), and the last rank is (A1) Brahabebome (score: -0.0563200).

4.4 Discussion

From results obtained, incomparability existed when using partial ranking to rank all alternatives from best to worst, hence complete ranking was used since there was no incomparability. A5 (Tutuka) is the best alternative to be selected for the facility location. Since, it has the highest net outranking flow of 0.2586275 in comparison with the rest of the locations.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATION

In this chapter, we highlight the conclusions and recommendations that are derived from the method presented in this thesis.

5.1 CONCLUSION

From the Facility Location Selection using the AHP/PROMETHEE II ranking Method the following conclusions can be drawn:

1. AHP/PROMETHEE II ranking methodology has been employed successfully to provide consistent evaluation (weighting and ranking) of location alternatives.
2. In accordance with the results generated A5 has the highest net outranking flow of 0.258675 in comparison with the rest of the alternatives. Therefore A5 was selected as the best alternative for the facility to be located.
3. The AHP/PROMETHEE II ranking method for the facility location is: A5 has the highest net outranking flow, followed by A3, A4, A2 and the lowest rank is A1.

5.2 RECOMMENDATIONS

Based on the research the following are recommended:

1. It is recommended that this AHP/PROMETHEE II ranking method should be used by the Obuasi Municipal Assembly and banks in Obuasi to select the best facility location.

2. It is recommended that more people should be trained to understand pair wise comparison used in the AHP/PROMETHEE II ranking method which makes implementation of this method very simple.
3. It is also recommended that this approach can be helpful in making any multi criteria decision in any industrial field.
4. It is recommended that further studies be made using other methods.

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APPENDICES

APPENDIX 1

Computation of Eigenvalues and Weight

Power method was used to obtain the eigenvalue (λ_{\max}). A Series of iterations were performed by MS Excel and finally the principal value was taken.

Table 4.1 Pairwise comparison matrix for criteria

	C1	C2	C3	C4	C5
C1	1.000	2.000	5.000	1.000	3.000
C2	0.500	1.000	3.000	2.000	3.000
C3	0.200	0.333	1.000	0.250	2.000
C4	1.000	0.500	4.000	1.000	4.000
C5	0.333	0.333	0.500	0.250	1.000

Table 4.2 Pairwise Comparison matrix of main criteria with weight

	C1	C2	C3	C4	C5	TOTAL	WEIGHT	AW	λ_{\max}	
C1	1.000	2.000	5.000	1.000	3.000	12.000	0.3141	1.77133	5.63858	
C2	0.500	1.000	3.000	2.000	3.000	9.500	0.2487	1.44237	5.79968	
C3	0.200	0.333	1.000	0.250	2.000	3.783	0.0990	0.43989	4.44185	
C4	1.000	0.500	4.000	1.000	4.000	10.500	0.2749	1.3625	4.95676	
C5	0.333	0.333	0.500	0.250	1.000	2.416	0.0632	0.36891	5.83278	
						TOTAL	38.199		26.6697	5.33393

Table 4.3a ITERATION 1

1.000	2.000	5.000	1.000	3.000	9.32476	5.26427	
0.500	1.000	3.000	2.000	3.000	7.47944	5.18553	
0.200	0.333	1.000	0.250	2.000	2.35291	5.34882	
1.000	0.500	4.000	1.000	4.000	7.09022	5.20385	
0.333	0.333	0.500	0.250	1.000	1.99964	5.4204	
					TOTAL	26.4229	5.28457

Table 4.3b ITERATION 2

1.000	2.000	5.000	1.000	3.000	49.1373	5.26956	
0.500	1.000	3.000	2.000	3.000	39.3799	5.26509	
0.200	0.333	1.000	0.250	2.000	12.4804	5.30422	
1.000	0.500	4.000	1.000	4.000	37.5649	5.29813	
0.333	0.333	0.500	0.250	1.000	10.5445	5.27317	
					TOTAL	26.4102	5.28203

Table 4.3c ITERATION 3

1.000	2.000	5.000	1.000	3.000	259.497	5.28106	
0.500	1.000	3.000	2.000	3.000	208.153	5.28576	
0.200	0.333	1.000	0.250	2.000	65.9015	5.28042	
1.000	0.500	4.000	1.000	4.000	198.491	5.28396	
0.333	0.333	0.500	0.250	1.000	55.6521	5.27786	
					TOTAL	26.4091	5.28181

Table 4.3d ITERATION 4

1.000	2.000	5.000	1.000	3.000	1370.76	5.28236	
0.500	1.000	3.000	2.000	3.000	1099.55	5.28239	
0.200	0.333	1.000	0.250	2.000	348.043	5.28126	
1.000	0.500	4.000	1.000	4.000	1048.28	5.28123	
0.333	0.333	0.500	0.250	1.000	293.953	5.28198	
					TOTAL	26.4092	5.28184

Table 4.3e ITERATION 5

1.000	2.000	5.000	1.000	3.000	7240.2	5.2819	
0.500	1.000	3.000	2.000	3.000	5807.47	5.2817	
0.200	0.333	1.000	0.250	2.000	1838.32	5.28188	
1.000	0.500	4.000	1.000	4.000	5536.79	5.28179	
0.333	0.333	0.500	0.250	1.000	1552.66	5.28198	
					TOTAL	26.4092	5.28185

Table 4.3f ITERATION 6

1.000	2.000	5.000	1.000	3.000	38241.5	5.28183	
0.500	1.000	3.000	2.000	3.000	30674.1	5.28183	
0.200	0.333	1.000	0.250	2.000	9709.76	5.28187	
1.000	0.500	4.000	1.000	4.000	29244.6	5.28187	
0.333	0.333	0.500	0.250	1.000	8200.89	5.28185	
					TOTAL	26.4092	5.28185

From the above calculations, the dominance eigenvalue is $\lambda_{\max} = 5.28185$

Similar calculation holds for the rest of the alternatives

APPENDIX 2

Questionnaire used to tap decision makers opinion about the facility location selection in the Obuasi Municipality is shown below.

KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI
INSTITUTE OF DISTANCE LEARNING (IDL)

QUESTIONNAIRE FOR PEOPLE IN THE OBUASI MUNICIPALITY

This questionnaire seeks to gather data on the facility location selection in the Obuasi Municipality for analysis and recommendations of the best location. Any information given will be treated as confidential. Thank you for being a part of this study.

Note: You are kindly requested to tick in the space or on the number of options provided.

Section 1: Characteristics of Respondents

1. Sex
 - (i) Male ()
 - (ii) Female ()
2. Age:.....
3. **The Format of Scale:** To express the relationship between two criteria with respect to objectives as well as alternatives with respect to criteria, the following format the Saaty (1980) Rating Scale 9-point scale as shown below, is proposed to score the items for respondents.

Note: 1 – Equally important; 3 – Moderately important; 5 – Strongly important; 7 Very strongly important; 9 – Extremely important

2,4,6,8 – Intermediate values between the two adjacent judgments

Tick “√” the corresponding score in the symbol “□”

During the evaluation, five (5) main criteria/factors (C1: proximity to customers, C2: nearness to market, C3: community desirability, C4: business climate, C5: other facilities) have been selected. Alternatives are Brahabebome (A1), Konka (A2), Wawasi (A3), Gausu (A4) and Tutuka (A5).

Compare two criteria with respect to objectives

For example, compare C1 to C2, which more important? By how much

C1	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	C2
C1	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	C3
C1	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	C4
C1	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	C5
C2	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	C1
C2	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	C3
C2	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	C4
C2	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	C5
C3	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	C1
C3	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	C2
C3	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	C4
C3	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	C5
C4	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	C1
C4	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	C2
C4	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	C3
C4	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	C5
C5	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	C1
C5	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	C2
C5	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	C3
C5	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	C4

Suggestions:.....