NETWORKING BOST COMPANY DEPOTS TO REDUCE COST OF TRANSPORTING PETROLEUM PRODUCTS.

By

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DECLARATION

I hereby declare that the thesis is my own work towards the Master of Science degree and that to the best of my knowledge, it contains no material which has previously been published or submitted by another person for the award of any other degree of this or any other University, except where due acknowledgement has been made.

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ABSTRACT

BOST Company established in 1993 is a Government Institution responsible for transporting and storing petroleum products in Ghana. It has six operational depots; Accra Plains Depot (APD), Kumasi depot situated at Kaase, Buipe depot, Bolgatanga depot, Akosombo depot and Mami-Water depot. APD is an originating Depot with storage capacity of 139,250 cubic meters which is connected by pipelines to Tema Oil Refinery (TOR) and Conventional Buoy Mooring (CBM). TOR is the only refinery in Ghana and has a production capacity of 45,000 barrels a day. CBM is a discharge port at Kpong near Tema where refined petroleum products imported pass through. Transportation of petroleum products from TOR and APD to other depots are mostly carried out by tanker trunks referred to as bulk road vehicles. Road transportation of petroleum products is a means of transporting petroleum products which is expensive and not reliable in terms of time of delivery and safety. The Transportation Algorithm was used to re-network the shipping schedule of BOST Company to reduce the total transportation cost from GHC 804,298.79 to GHC 661,114.19 a day. The Minimum Cost Method (MCM) which is an initial solution method of the transportation algorithm was used to obtain an initial transportation cost of GHC 698,551.23 and was improved to GHC 661,114.19 by the MODI Method.

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LIST OF ABBREVIATIONS

APD--Accra Plains Depot

BFS--Basic Feasible Solution

BOST--Bulk Oil Storage and Transportation

BRV--Bulk Road Vehicles

CBM---Conventional Buoy Mooring

LPG--Liquefied Petroleum Gas

MCM---Minimum Cost Method

MODI--Modified Distribution Method

NWCM---North-West Corner Method

TOR---Tema Oil Refinery

UPPF--Unified Petroleum Price Fund

VAM--Vogel's Approximation Method

GHC--Ghana cedi

DEDICATION

I dedicate this work to; my lovely husband Dennis Ziorklui, my spiritual fathers Rev. Sevor, Rev. Sosu, Rev. Adegle, my parents Godson Kofi Kporhor and Paulina Awume and also to my siblings especially Elorm Adegle.

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CHAPTER ONE

Introduction

1.0 The Transportation Problem

A transportation problem refers to a class of linear programming problems that involves selection of most economical shipping routes for transfer of a uniform commodity from a number of sources to a number of destinations. One of the earliest applications of the linear programming techniques is the formulation and solution of the transportation problems as a linear programming problem (Gupta, 1973).

The main concept of linear programming is that of optimization, which could mean maximizing profit or minimizing cost. This is common with most individuals and organizations who will from time to time want the best methods of achieving a stated target with least effort, least time or least cost.

Transportation problem is one aspect of linear programming technique. The most widely used method of solution for the transportation problem is the simplex method of linear programming (Hay). The simplex method is an algorithm invented for solving linear program by progressing from one extreme point of the feasible polyhedron to an adjacent one (Kumar). In the beginning the transportation problem was formulated for determining the optimal shipping pattern, hence the name transportation problem (Deepika, 2010).

The transportation problem was originally proposed by Hitchcock in 1941 and plays an important role in logistics and supply chain management for reducing cost and improving service. Lin and Tsai (2009) stated that the transportation problem can be extended to areas other than transportation of commodity, such as inventory control, employment scheduling, and personnel assignment. The transportation model is basically a linear program that can be solved by the regular simplex method.

However, its special structure allows the development of a solution procedure called the transportation technique that is computationally more efficient and simple (Badu, 2011).

1.1 Overview of Transportation

Transportation is the movement of people, animals, and goods from one location to another. The first means of transport for human beings was walking and swimming. The domestication of animals introduces a new way to lay the burden of transport on more powerful creatures, allowing heavier loads to be hauled, or humans to ride the animals for higher speed and duration.

Transportation system can be divided into infrastructure, vehicles and operations. Transport infrastructure consists of the fixed installations necessary for transport, and may be roads, railways, airways, canal and pipelines, and terminals such as airports, railway stations. Vehicle is living device that is used to move people and goods from one point to another. Bicycles, buses, trains, trucks, and aircrafts are examples of vehicles.

1.2 Mode of Transportation

Movement of people and goods can be done through different ways. The means of transport may depend on the development of the area and the resources available. The following are some means of transport; Human-power, Animal, Rail, Air, Water, Road, Pipeline, Spaceflight.

1.2.1 Human transport

This is the movement of people and goods from one place to another by means of human muscle power, in the form of walking, running and swimming. Human power transport remains popular because it saves money; it's always available and serves as a form of physical exercise to the body. It is the only means of transport especially in most undeveloped or inaccessible regions.

1.2.2 Animal powered transport

In this mode of transport, some special animals are used for the movement of people and goods. These animals are harnessed by human beings. Animals are superior to human beings in terms of speed, endurance and carrying capacity. Animals such as horses, donkeys, bulls and camels are used as a means of transport under this mode. In most less developed areas, animal power transports remain an important mode of transport.

1.2.3 Air transport

This is a means of transporting people and goods by airplane. The movement of the air in relation to the wings of the aircraft is used to generate its lift. The aircraft is the second fastest mode of transport after the rocket. Aviation is able to quickly transport people and cargo over longer distances, but incur high costs in energy used.

1.2.4 Rail transport

This is where a train runs along a set of two parallel steel rails, known as a railway or rail line. The rails are anchored perpendicular to ties of timber, concrete or steel, to maintain a consistent distance apart. A train consists of one or more connected vehicles that run on the rails. Rail vehicles move with much less friction than rubber tires on paved roads, making trains more energy efficient, though not as efficient as ships and are less expensive in terms of cost of transportation compare to road transport. Since 1960, container trains have become the dominant solution for general freight (Wikipedia, 2011).

1.2.5 Road transport

This means of transportation can transport people and goods very near their destinations. The vehicles can pack at the lorry station to load passengers or goods to their destinations or the vehicles move to where the passengers or the goods are to load. The most common road vehicles are buses, minivan, trucks, cargos, motorcycles and bicycles. Road is an identifiable route, way or path between two or more places. Roads are typically smoothed, paved made to allow easy travel, though; the story may differ in less developed areas. In urban areas, roads may pass through a city or village and be named as streets, serving a dual functions as urban space easement and route.

1.2.6 Water transport

Water transport is the process of transporting by a watercraft, such as large boat, canoes, barges, pontoon, ship or sailboat in a body of water, such as a sea, ocean, lake, or a river. In the 19th century, the first steam ships were developed using a steam engine to drive a paddle wheel or propeller to move the ship. Now, most ships have an engine using a slightly refined type of petroleum called bunker fuel.

Although, water transport is slow, modern sea transport is a highly effective means of transporting large quantities of none-perishable goods. Transportation by water is less costly than air transport for transcontinental shipping and short sea shipping. Ferries remain viable in the coastal areas (Wikipedia, 2011)

1.2.7 Pipeline

Pipelines transport goods through a pipe, most commonly liquids and gases. In some cases pneumatic tubes can also send solid capsules using compressed air. Short distance pipeline systems are mostly used to transport sewage, slurry, water and beer. Petroleum and natural gas are transported through long-distance pipelines networks.

1.2.8 Spaceflight

Spaceflight is a means of transport from the earth's atmosphere into outer space by means of a spacecraft. It is only used to put satellite into orbit, and conduct scientific experiments.

1.3 Importance and Impact of Transportation on Economic Development

Transport has throughout history been a spur to industrial development. It allows production and consumption of product to occur at different locations. Transportation brings about trade between countries and people to generate income, employment and tax revenues.

Modern society dictates a physical distinction between home and work, forcing people to transport themselves to places to work or study as well as temporally relocate for other daily activities (Wikipedia, 2011). Adequate, reliable and cost-effective transport system is essential although not sufficient, for the social and economic development of rural areas. This encourage investment in remote areas, rural communities get access to urban health facilities and there is an option to rural school going age to access education of their choice in urban areas.

Transportation establishes civilization and brings about infrastructure development. When transport systems are efficient, its provide economic and social opportunities and benefits such as better access to markets, employment, banking and other social amenities.

1.4 Means of Transportation in Ghana

Transportation in Ghana is accomplished by road, rail, air, water, and pipelines (Wikipedia, 2010). In the Northern part of Ghana, animals such as horses and donkeys serve as a means of transport for goods and human beings.

Currently, most of the railway systems in Ghana are not being used because they are undergoing rehabilitation. The road transportation system is the dominant means of transportation of people and freight in Ghana. The common vehicles used are the buses, trucks, cargos and minivans. Others are the bicycles and motorcycles.

Boats, barges, pontoon and canoes are the common crafts use in the Lakes, Rivers, Lagoons and sea.

Air transportation is not well patronized within as the road and water transportations. The four airports being used mostly in the country are the Kotoka International Airport at Accra, Sekondi-Takoradi airport, Kumasi airport and Tamale airport. The Kotoka International Airport is an International airport established in 1958 by the Ghana government. Pipelines are used to transport petroleum products in some parts of the country.

1.5 Petroleum and its products

Petroleum is a complex mixture of hydrocarbons derived from the geologic transformation and decomposition of plants and animals that lived hundreds of millions of years ago. It is separated into fractions including natural gas, gasoline, naphtha, kerosene, fuel and lubricating oils, paraffin wax and asphalt.

The petroleum industry is involved in the global processes of exploration, extraction, refining, transportation and marketing petroleum products. The largest products of the industry are fuel oil and petrol.

Petroleum and its products are used as fuels for heating; in land, air and sea transportation. It can also be used for electric power generation and as petrochemical sources and lubricants (Wikipedia, 2008). Petroleum is also the raw material for many chemical products including pharmaceuticals, solvents, fertilizers, pesticides and plastics.

It is recovered from drilled wells, transported by pipelines or tanker ships to refineries, and there converted to fuels and petrochemicals.

1.6 Modes of Transporting Petroleum Products

Crude oil is transported from production operations to a refinery by marine vessels, barges, rail tank cars, tank trucks, and pipelines. The refined products are conveyed to fuel marketing terminals and petrochemical industries by the same modes. The mode of transportation use depends on the resources available in the area.

Globally, pipelines are the most preferred method of transporting petroleum products (Manchie, 2010). They can run through any part of the earth, although the initial construction is difficult and expensive, once they are constructed, properly maintained and operated, they provide one of the fastest, safest and most economical means of transporting crude oil and petroleum products. Giant tanker ships are used for international transport of crude oil. Tank vessel transportation is generally a more cost-effective and energy-efficient means of transporting bulk commodities such as refined petroleum products than transportation by rail car or truck.

1.7 Transportation of Petroleum Products in Ghana

In Ghana, transportation of petroleum products is done mostly by road using tanker truck vehicles commonly known as Bulk Road Vehicles (BRV). Other means of transporting petroleum products are pipelines, rails and river barges. Most of the private bulk oil distribution companies like Chase Petroleum Ghana and Cirrus Oil mostly use pipelines in their distributions.

Government, through the Unified Petroleum Price Fund (UPPF), pays for haulage of petroleum products to all regions in Ghana. This is done to ensure uniform prices of products as well as maintain availability of petroleum products across the country.

To meet this target, Government collects taxes on every liter of Premium, Gas oil, Marine Gas oil, Premix, Kerosene and LPG.

1.8 Transportation and Storage of Petroleum Products in Ghana

Bulk Oil Storage and Transportation Company Limited (BOST) are responsible for transportation and Storage of Petroleum Products. It is an oil company established in 1993. It has six storage depots located across the country. These are, Accra Plains Depot, Kumasi Depot, Bolgatanga Depot, Buipe Depot, Mami Water Depot and Akosombo Depot in the Greater Accra, Ashanti, Upper East, Northern, Volta and Eastern Regions respectively. The Bulk Oil Storage and Transportation Company are responsible for bulk petroleum products transportation and storage. The company has been able to expand the total storage capacity from a level of about 20,000 cubic meters to 315,050 cubic meters for the six depots currently in operations.

BOST take delivery of refined petroleum products for the various depots from Tema Oil Refinery (TOR) and Convention Buoy Mooring (CBM) floated thirty-two kilometers (32km) offshore, being operated by Marine Services at Kpong near Tema. The Accra Plains Depot (APD) is linked by a twin 6-inch pipelines to TOR and 18-inch pipeline to CBM (BOST, 2012).

The APD serve as a major storage depot from which other depots get their delivery and has a capacity of 139,250 cubic meters (m^3). Kumasi depot gets its delivery by road tankers and has a storage capacity of 87,900 cubic meters (m^3). The Bolgatanga depot is supplied by pipelines and road tankers and has a capacity of 46,500 cubic meters (m^3). Buipe depot has a capacity of 42,500 cubic meters (m^3) and is supplied by river barges and road tanker trucks. The Akosombo depot which has a capacity of 80,000 cubic meters (m^3) gets its delivery by pipeline and road vehicles.

TOR refines crude oil into various products needed in the country and has a production capacity of 45,000 bpd. The petroleum products requirement for the country exceed the capacity of TOR hence the remaining products are imported from the neighboring oil producing countries at high cost than when produce in the country. The short fall as at 2011 is between 26% - 30%. (Energy Commission, Ghana, 2011)

1.9 Statement of the problem

Most of the operational depots of BOST take their delivery through the road truck tankers which happens to be the most expensive means of transporting petroleum products.

Moreover, road truck tankers are not energy-efficient, they delayed because of traffic situations faced on the road and other problems vehicles face leading to perennial shortages facing the country. These make BOST to incur an extra cost of transportation in order to supply to all its depots in operations.

Using a Mathematical approach, an effective way of distributing petroleum products is found to help reduce total cost of transportation and also solve the problem of shortage of products.

1.10 Objectives

The objective of this thesis is to use the Information collected from BOST to design a Mathematical model for the operations of transportation of petroleum products and use a simple algorithm to find a way distribution of products will be done so that the company will not incur much lost. The main objectives of the thesis are;

1. To model the transportation activities of BOST as transportation problem.
2. To solve the transportation problem in order to minimize total cost of transporting petroleum products in BOST using Transportation Algorithm.

1.11 Methodology

Transportation of petroleum products by road truck tankers is very expensive. Currently, most of the petroleum products are being transported by road to the various depots in the country. Problems associated with transportation by road, the high cost with which petroleum products are transported using the road and the shortages experience all over the country most often are the serious problems the company is facing at the moment.

Finding a better way of distributing petroleum products to the various depots is a way to minimize cost of transportation and also address the shortage problems. With the various depots and the sources of getting the petroleum products, the transportation activities can be model as a transportation problem and using the transportation algorithm a better means of distributing the products in order to cut down the cost of transportation.

The transportation algorithm will be used because it is simple and easy to solve. The data collected is from January, 2011 up to November, 2011. It is a primary data from BOST Company Limited, Accra and will be computed manually. Other Information are accessed on the internet and from books obtained from the main KNUST Library.

1.12 Justification

In Ghana, inadequate distribution, storage and transportation infrastructures as well as limited number of retail outlets have led to ineffective distribution of petroleum products (CommerceGhana.com).

The road transportation of petroleum products is accepted worldly as the most expensive means of transporting petroleum products. At the moment, major transportation of petroleum products in the country is through the road by truck tankers which is an indication that the company will spend so much on the cost of transportation.

Using the transportation algorithm, the best means of distributing the products will be found so as to minimize the total transportation cost of the company. The findings of the thesis are to give more insight into how best to distribute petroleum products to the depots currently in operations so that cost of transportation will be reduced.

Finally, the result of the thesis if adopted by the company is to provide the best and efficient means to use to transport petroleum products in order to minimize cost of transportation of transporting petroleum products which will go a long way to help the company to make more revenue to improve the socio - economic status of the people in the society especially in the areas of their operations and also, the company will be able to increase the number of depots to curtail the perennial shortage problems.

1.13 Thesis Organization

Chapter one is the Introduction of the thesis. In the Introduction, the general transportation problem was reviewed. In the same chapter we discussed the problem of BOST Company and the methodology to use to solve the problem. In Chapter two, other people's work which is related to the objectives of the thesis is review. Chapter three discussed the general transportation model. In Chapter four, the transportation algorithm is used to solve the transportation problem of BOST Company. Chapter five discusses the findings, conclusions and recommendations.

CHAPTER TWO

Literature Review

2.1 Introduction

Transportation planning recognizes the critical links between transportation and other societal goals. The planning process is more than merely listing highway and transits capital projects. It requires developing strategies for operating, managing, maintaining, and financing the areas of transportation system in such a way as to advance the areas long-term goals.

Fair et al, (1981) discussed that transportation plays a connective role among the several steps that result in the conversion of resources into useful goods in the name of the ultimate consumer. It is the planning of all these functions and sub-functions into a system of goods movement in order to minimize cost, maximize service to the customers that constitute the concept of business logistics.

Echols (2003) stated that people everywhere need to get somewhere else and are dependent on automobiles, buses, trains, subways or airplanes to arrive at a final destination. Most people would like to arrive at a destination with lowest cost. These transportation issues concern individuals, companies and governments as well. Networks are an important part of everyday lives and analysis of these networks improves the move of people, goods, services and flow of resources cited as (Lo and Yeung, 2002).

Esirgen (2010) said a road network can be used for transporting goods from a source to a sink and can be modeled as a network. It continued that in the net work, the cities are represented by nodes, and each link represents a section of a road that connects a city to another.

Transportation problem is concerned with finding minimum cost of transporting a single commodity from a given sources to a given number of destinations. The data of the model include; the unit transportation cost of the commodity from each sources to each destination, so far as there is only one commodity, and the level of supply at each source and the amount of demand at each destination. A destination can receive its demand from more than one source. The objective is to determine how much should be shipped from each source to each destination so as to minimize the total transportation cost. Mathematically, a transportation problem is nothing but a special linear programming problem in which the objective function is to minimize the cost of transportation subjected to the demand and supply constraints. (www.callwey-shop.deltopos)

Barry and Jay (2004) (cited in Jay) said the transportation modeling is an iterative procedure for solving problems that involve minimizing the cost of shipping goods from a series of sources to a series of destinations.

Carlberg (2009) in his lecture note presented that transportation is the application of optimization which involve minimizing cost by choosing routes to transport goods between warehouses and outlets.
Farnalskiy (2006) discussed in his work that, cost minimization has become as one of the important issues in business activities which have achieved a high priority especially today, when the economic slowdown has hit most business and production sectors. Cost rationalization has become an imperative for many companies to survive. Transportation modeling is one of these techniques that can help to find an optimum solution and save the costs in transportation activities.

The transportation problem of linear programming has an early history that can be traced to the work of Kantorovich, Hitchcock, Koopmans and Dantzig.

Dantzig (1951) used the simplex method to solve the transportation problem which was earlier formulated by Hitchcock and Koopmans.

2.2 Transportation Problem Solution Methods

Anderson et al. (2008) (cited in Samari, 2010) discussed a simulation model of a barge as a means of transporting petroleum within inland waterway. The simulation was used as an evaluation model within a decision support system. This also involved a criterion model, represented as a decision maker's utility function, and an optimization procedure which employed scatter search. They also used variance reduction techniques in order to improve the accuracy of the estimates of the performance measures associated with the system.

Leavengood and Reeb (2002) solved the XYZ sawmill company transportation problem using the North-West Corner rule Method to obtained the initial basic feasible solution which was $5760 and happen to be the optimal solution or the minimum cost to ship logs from the various site to the sawmill. This they knew because LINDO was first used to obtain the solution.

Hussain (2010) used the North-West Corner Method (NWCM), Least Cost Method (LCM) and Vogel's Approximation Method (VAM) to find the basic feasible solutions for the transportation of the Raw Material Company. When the methods for obtaining the basic feasible solution was applied, NWCM gave BFS as 34982, LCM gave BFS as 34854 and VAM resulted in 34718. It has been realized that the BFS to the problem was obtained using the VAM. The original cost of transporting the raw materials was 35854. The Method of Multiplier was applied to each resulting BFS matrix. The Least Cost Method after the second iteration realized the optimal result as 34718, North-West Corner Method had the optimal solution after the third iteration and Vogel's Approximation Method had 34718 after the fourth iteration. The Vogel's Approximation Method had the same solution which was 34718 for the basic feasible solution. Though the VAM produce good basic feasible solution, it does that after much iteration.

Elsharawy et al. (2009) used the North-west Corner Method, Minimum Cost Method, Vogel's Approximation method, Row Minimum Method and Column Minimum Method to obtain the starting solution for a Sunray Transportation Company which supposed to ship truckloads of grain from its three silos to four mills. The Minimum Cost Method and the Vogel's Approximation Method produced the same starting solution which was the minimum starting transportation cost. The authors discussed the advantage of each of the methods. To them, the North-west Corner Method give quick solution because computation take a few time but give a bad result which is very far from the optimal solution. Vogel's Approximation Method and the Minimum Cost Method gave the shortest distance from the silos to the mills implying that these two methods obtained the minimum cost of transportation for the starting solution. The two methods mostly yield the best starting solution, because they give initial solution which is most often very near the optimal solution. The Minimum Cost Method is considered the best method because the least cost of transporting are satisfied before moving to the high cost. By the time they get to the high cost demand point the goods would have been distributed.

Wang (2008) presented how in a production schedule, supply was more than demand destination. In order to make the production schedule problem fit into the transportation model, a dummy destination was created for the extra supply. In real life situation involving transportation of goods from sources to warehouses, demand may not be equal to supply. To be able to model such problems, a dummy cell(s) depending on where the shortage is located.

Mathirajan and Meenakshi (2003) (cited in Kirca and Satir 1990) developed a heuristic called TOM (Total Opportunity-cost Method), for obtaining an initial basic feasible solution for the transportation problem. They put forward that; the Total Opportunity-Cost Method (TOM) is an effective application of the 'best cell method' along with some tie-breaking features on the total opportunity cost (TOC) matrix. They discussed how the Total opportunity matrix was obtained.

Arsham (2011) discussed how transportation models play an important role in logistics and supply chain management for reducing cost and improving service. The aim of the transportation model is to find the most effective way to transport the goods. A linear programming formulation for minimizing total transport cost was formulated from a transportation problem where the Algebraic Method was used to obtain the minimum cost of transportation.

Balan et al. (2008) used S-N algorithm which gives optimal solution directly without finding initial basic feasible solution. In the algorithm there would not be any need to bother about looping as well as degeneracy. According to the paper when compared to methods most commonly used in finding optimal solution it is the best. Apart from all these, they talk of the algorithm being used to solved problems involving unbalanced and maximization cases. The algorithm was used to solved the transportation problem, the algorithm was used to find the optimal solution directly by using Mathematical suffix average, without finding the initial basic solution. The algorithm they said gives best optimal solution for all types of transportation problems. This implies that it gives overall feasibility and optimality with few steps.

Nunkaew and Phruksaphanrat (2009) used the goal programming to solve the multi-objective transportation problem. This method comprises of two well-known methods, the Weight Goal Programming (WGP) and the Lexicographic Goal Programming (LGP). They adopted LGP for the formulation of the problem, but in the transportation problem, the demand for a customer was to be met by only one depot if the capacity of one depot is sufficient but the conventional model does not considered this condition, hence not included in the constraint of the method. The result obtained by using this method is the same as that of a single objective optimization problem that has a minimization of total transportation cost as an objective. They have optimized the second priority which yielded a result best solution from all possible solutions.

Kopfer and Krajewska (2007) in their work considered four main aspects in the transportation activities; the attributes of requests that must be fulfilled by a freight forwarder, the main objectives of the transportation company expressed by the objective function, the definition of the planning problem for self-fulfillment and the types of subcontracting that occur in the integrated problem. In the requests aspect it was believed in the integrated operational transportation problem that a customer's request is assume to be a delivery request. This they mean that a single transportation demand resulting in direct transportation process without transshipment. In this situation splitting the loading is not allowed unless the volume of a single request is more than the vehicle's capacity. Delays are not tolerated in all the approaches. For the objective function, the aim of the freight forwarder is to maximize between the transaction volume and its costs. The main objective are to find a partition which splits the request portfolio into fulfillment clusters and to solve the planning problems inside the clusters so that the total execution cost are minimized. Moreover, the self-fulfillment cluster may be a homogeneous or heterogeneous set of vehicles. When it is a homogeneous fleet, the tariff rate per distance or time unit as well as the fixed costs per vehicle is equal for all vehicles; all vehicles are stationed in the same depot and have the same predefined maximal capacity. For the other approaches, a heterogeneous fleet is considered. Finally, the sub-contraction cluster in contrast to the self-fulfillment cluster, there is on standardized ways to calculate the costs for the external execution of requests in the sub-contraction cluster. The easiest form of sub-contraction is a simple shifting of requests, which means selling a single request independently from all the other requests to an external freight carrier. The requests are forwarded on uniform conditions, based on a linear distance-dependent function.

Altiparmak et al. (2006) for them Supply Chain Management (SCM) describes the discipline of optimizing the delivery of goods, services and information from supplier to customer. In the paper they consider an extension version of two-stage transportation problem ($t_{s.}$TP) to minimize the total logistic cost including the opening costs of distribution centers (DC_s) and shipping cost from plants to DC_s and from DC_s to customers. They developed a priority-based Genetic Algorithm (pb-GA), in which new decoding and encoding procedures were used to adapt to the characteristic of $t_{s.}$TP, and proposed a new crossover operator called Weight mapping crossover (WMX).

Sreenivas and Srinivas (2008) study approximations of optimization problems with probabilistic constraints in which the original distribution is replaced with an empirical distribution obtained from a random sample. They display how such a sample approximation-problem with risk level larger than the required risk level of the original distribution will produce a lower bound to the true optimal value with probability approaching one exponentially fast. They continue to provide conditions under which solving a sample approximation problem with a risk level smaller than the required risk level will yield feasible solutions to the original problem with high probability. The study show that using sample approximation problems allow a choice of which sampled constraints to satisfy can produce good quality feasible solutions.

Edokpia and Ohikhuare (2011) recommend the Linear Programming method for a beverage producing company in Nigeria to employ to use to make their policies. This they did after they have used the Linear Programming method to reduce the total transportation cost which was a problem for the company.

Ibrahim (2008) developed the Integer Mathematical Programming models to solve the crude palm oil and the palm kernel transportation problems for Northern Peninsular Malaysia. The two transportation problems were solved to get the mills-to-refineries and mill-to-crushers optimal assignments using distance minimization as the objective function. The solutions revealed similar mills-to-refineries and mills-to-crushers assignments because crude palm oil and palm kernel are proportionate in quantity, truck capacities are homogeneous for each product, and refineries and crushers are located at identical locations.

Adlakha and Arsham (1998) put forward a single unified algorithm that solves both transportation problem and assignment problem and also provides useful information to perform cost-sensitivity analysis to a decision maker. The proposed solution algorithm is pivotal and is similar to the simplex algorithm and its variants. Again, the solution algorithm facilitates incorporation of side-constraints which are mostly encountered in real-life applications. It makes available the full power of LP's SA extended to handle to handle optimal degenerate solution. Their preliminary computational results demonstrate that the algorithm is more efficient than the simplex method in terms of the number of iterations and size of tableaux.

Chu and Ji (2002) presented the dual-matrix approach. According to the paper the dual-matrix approach is a very efficient in terms of computation and it comprises of the simplex method and the stepping-stone method. It was said that the simplex method is not suitable for the transportation problem especially, large-scale transportation problems. The approach considers the dual of the transportation model instead of the primal, and obtains the optimal solution of the dual by the use of matrix operations. The dual has $m+n$ variables and m*n constraints. The dual obtained the feasible solution to the dual problem and its corresponding matrix. The duality theory is used to check the optimal condition and to get the leaving cell. All non-basic cells are evaluated in order to get the entering cell. The entering cell replaces the leaving cell.

Samuel and Venkatachalapathy (2011) proposed an algorithm called Modified Vogel's Approximation Method for solving fuzzy transportation problems, which to them is more efficient than other existing algorithms, because it requires least iterations to reach optimality. The Vogel's Approximation Method was improved by using Total Opportunity Cost (TOC) matrix by considering alternative allocation costs. They suggested that the method could be improved if they add the following steps for breaking ties;

1. If there is a tie in penalty or minimum transportation cost, choose the largest penalty for allocation.

2. If there is a tie in penalty and minimum transportation cost then calculate their corresponding row opportunity cost value or column opportunity cost value, and select the one with that is maximum.

Numerical examples indicate that MVAM is better than MMM and VAM for solving the fuzzy transportation problem. Again the solution of fuzzy transportation problem given by MVAM is very near to the optimal solution.

Fegade et al. (2011) proposed a method called separation method to find an optimal solution to a problem where transportation cost, supply and demand are intervals. They developed this method without using the midpoint and width of the interval in the objective function of the fully interval transportation problem which is a non-fuzzy method. This method is based on zero point method. It was recommended for decision makers as an important tool for handling various types of logistic problems having interval parameters.

Sen et al. (2010) they presented the distribution of rice that is demand, supply and cost of transportation from supply to demand point in Mizoram in special transportation table and used the following methods; Northwest corner Method, Vogel Approximation Method, Least cost Method, Row minima Method and Column minima Method to obtained the initial feasible solution. Vogel Approximation method gave the least total cost of transportation which is 12, 73000/.A test to check optimality was carried out using Modified Distribution Method (MODI). The test was done based on the basic feasible solution obtained from Vogel Approximation Method and an optimum solution of 12, 46000 was arrived at.

Barry and Jay in their paper said the Modified Distribution Method allowed computation of improvement indices to be performed quickly for each unused square without drawing all of the closed paths. This according to the paper often provided considerable time savings over other method for solving transportation problems. Moreover, MODI provides a new means of finding the unused route with the largest negative improvement index. The Arizona plumbing problem was solved using the MODI. The Vogel Approximation Method which is a method for finding basic feasible solution was noted as a method for obtaining a good initial solution and was applied to the Arizona plumbing problem for the basic feasible solution. The solution obtained happened to be the total minimum cost.

Taylor (2006) presented in his paper that the transportation problem could easily be manipulated when the basic solution is gotten using the special table. According to the paper, an algorithm exist which as expected, iteratively improved upon the objective by moving from one basic feasible solution to another. Moreover, when the technique is modified could take care of the unbalanced transportation problems or to exclude routes through addition of dummy sinks and sources and 'pricing out' certain entries.

Judge and Takayama (1971) developed the classical transportation model and using an example, solved the problem of interregional shipments of homogeneous commodities by minimizing the total transportation cost. The model was extended to take care of unbalanced situations.

Basirzadeh (2011) presented a very simple algorithm called parametric method for solving transportation problem. Though simple, it is very efficient method.

Moreover, he put forward that the method could be used for all types of transportation problem that is whether maximum or minimum objective function.

Pandian and Anuradha (1992) proposed an algorithm called floating point method which determined an optimal solution to transportation problems which have additional constraints. The proposed algorithm was used to layout two real life problems which coal are shipping problem and Iron making company. The optimal solution obtained using the floating point method is not necessary to have m+n-1 allotted entries which is the main merit of this algorithm.

Arunsankar et al. (2012) introduced a method called the zero suffix method which was met for finding an optimal solution to a transportation problem. They outlined the procedures involved in using the method and used a numerical problem to illustrate the method. The method produced an optimal value to the objective function for the transportation problem. Also, the procedure to the method is systematic and very easy to comprehend. The method is very efficient and can serve as an important tool for decision makers.

Lin (2010) introduces differential evolution to solve the transportation problem with fuzzy coefficients. According to the paper, differential evolution is simple and effective in using to solve numerical optimization problems. Numerical simulation results show that differential evolution is as much efficient as genetic algorithms for solving fuzzy transportation problems.

Hwang et al. (1995) developed a model which combined fuzzy multi-objective programming and multi-index transportation problems to solve allocation of coal planning of Taipower. The model according to the presentation is more flexible than the conventional transportation problems. Moreover, it can offer more information to the decision maker for reference.

Blumenfeld et al (1985) developed an analytic method for minimizing the sum of transportation and inventory costs for suppliers who distributes goods to many customers. The method focuses on the spatial density of customers and also on the distribution of customer demand, rather than on the demands of specific customers in precise locations. The proposed approach simplifies the analysis of distribution problems by eliminating the need to specify a detailed network and corresponding flows.

Dwyer and Galler (1956) discussed that the method of reduced matrices can be used to replace the simplex method in order to avoid the idea of finding the initial basic feasible solution. The paper discussed that the reduced matrices method can be used to obtain the exact solution to transportation problems. The complication normally encountered in the simplex method is avoided in this method. The method of reduced matrices is based on the fact that a constant may be subtracted from each element of any row of the cost matrix, where the final allocations should be made.

Ananya and Chakraborty (2010) modeled the time and cost minimization problem as a multi-objective linear programming problem with imprecise parameters. Fuzzy parametric programming was used to address the impreciseness of the problem and the resulting multi-objective problem was solved using prioritized or preemptive goal programming approach. According to the presentation, in real life, supply, demand and cost per unit of the quantities are generally not specified precisely meaning the parameters are fuzzy in nature. The proposed technique was illustrated with a case study of transportation of raw coal from different collieries of a regional coal company situated in Jharia.

Rowse (1981) suggested a mathematical programming approach for solving the classical transportation problem. It was generalized to cater for non-linear demand and supply functions and also the non-linear transportation costs. The approach according to the paper was made possible by the recent public availability of a powerful optimization system called MINOS. The program was put forward for the solution of large programming problems for which the constraint set is linear and spatial.

Mohd and Suprajitno (2010) proposed the modified simplex method for solving transportation problems with interval numbers as coefficients. According to their findings parameters in the real world are mostly not known as it should have been according to conventional transportation problem, and would have to be estimated hence the interval transportation problems. With this model some or all its parameters including variables are in the interval form.

Deepika (2010) as cited in Arsham and Kahn (1989) introduced a new algorithm for solving the transportation problem. The proposed method used only one operation, the Gauss Jordan pivoting used in simplex method. The resulting table according to the proposed algorithm can be used for the post-optimality analysis of the transportation problem. They concluded that the algorithm was faster than the simplex method, more general than the stepping stone and simpler than both in solving general transportation problem.

CHAPTER THREE

Methodology

3.1 Introduction

Transportation problem was one of the original applications of linear programming models which deal with the determination of a minimum-cost plan for transporting a homogeneous commodity from a number of sources to a number of destinations, satisfying both supply and demand needs.

For instance, a company may have three factories producing the same commodity, and four depots or warehouses that stock or demand the products. The purpose of the transportation model is to find the distribution network that will result into a minimum cost of transportation.

3.2 The General Representation of the Transportation Problem

The figure 3.1 is a network representation of the transportation problem with m sources and n destinations. The sources are from 1 to m and the destinations are also from 1 to n. Each source is represented by a node and each destination is also represented by a node. The supply available at each source is denoted as a_i and the demand at each destination is denoted by b_j where $i = 1,...,m$ and $j = 1,...,n$, the routes from supply node to demand node are represented by the arrows.

In the figure 3.1 below, source 1 supply to destinations 1, 2 up to n, source 2 supply to destinations 1, 2 up to n and so on. This means a particular source can supply to all destinations depending on the availability of product and the amount of quantity demanded at each destination.

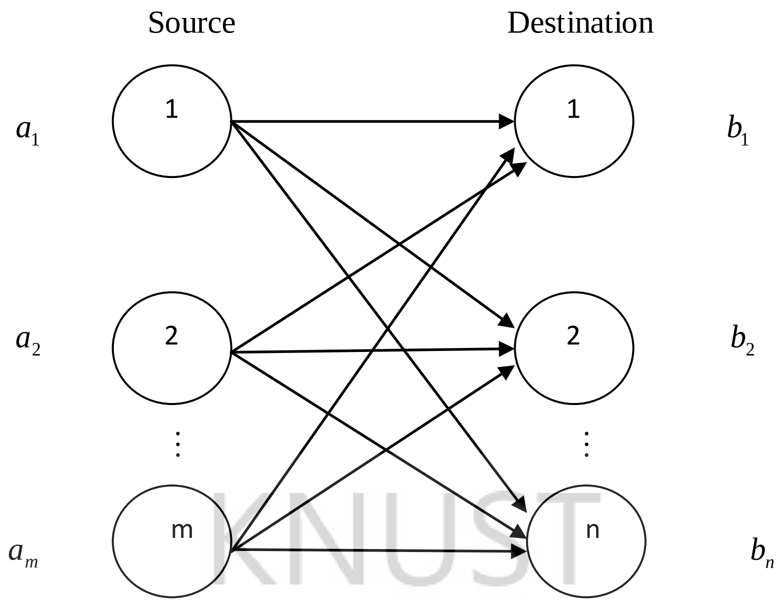

Figure 3.1: Network Representation of the Transportation Problem

3.2.1 Mathematical Representation of the General Transportation Problem

If x_{ij} represent the quantity of commodity transported from source i to destination j, and the cost of transporting a unit of product from source i to destination j is c_{ij}, then the cost involved in moving a unit quantity from source i to destination j is given as; Cost $= c_{ij} * x_{ij} = c_{ij}x_{ij}$.

The cost of transporting products from source i to all destinations is given as; Cost $= \sum_{j=1}^{n} c_{ij}x_{ij}$.

The total cost of transporting products from all the sources to all the destinations is given as; Minimize Z $= \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$, where $i = 1,...,m$ and $j = 1,...,n$.

If a_i and b_j are supply and demand respectively then, the objective function (Z) is;

$$\text{Minimize Z} = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 1$$

Subject to,

$$\sum_{j=1}^{n} x_{ij} \leq a_i , \qquad \text{for } i = 1,...,m \quad \ldots\ldots\ldots\ldots\ldots\ldots 2$$

$$\sum_{i=1}^{m} x_{ij} \geq b_j \qquad \text{For } j = 1,...,n \quad \ldots\ldots\ldots\ldots\ldots\ldots 3$$

$$x_{ij} \geq 0 \qquad \text{For all } i \text{ and } j \quad \ldots\ldots\ldots\ldots\ldots\ldots 4$$

The first constraint or equation two means the sum of all shipments from a source cannot exceed the available supply and the second constraint means the sum of all shipments to a destination should be at least as large as the demand. Equation four simply means shipment value should not be negative. From the two constraints we can say that the total supply is either greater or equal to the demand.

3.3 Definition of Terms

3.3.1 Feasible Solution

A feasible solution is a set of non-negative allocations, $x_{ij} \geq 0$ which satisfies the row and column restrictions that is the supply and demand constraints. A transportation problem will have feasible solutions if and only if $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$, a_i = supply, b_j = demand.

3.3.2 Objective Function

This is a mathematical expression that can be used to determine the best values of the decision variables. For instance, the objective function can maximize profits or minimize costs.

3.3.3 Constraints

These are limitations that restrict the alternatives available to decision makers.

3.3.4 Cell

A cell is a square or rectangle shape in the transportation table where a unit of commodity transported from source to destination and unit cost of transport are indicated.

3.3.5 Basic Feasible Solution (BFS)

A feasible solution of (m*n) transportation problem is said to be basic feasible solution, when the total number of allocations is equal to (m+n-1), the non-negativity constraints are satisfied and the allocations are independent and do not form a loop.

3.3.6 Optimal Solution

A feasible solution is said to be optimal when the total transportation cost is the minimum cost.

3.3.7 Unit Transportation Cost

It is the cost of transporting one unit of a commodity from an origin to a destination mostly denoted as c_{ij} .

3.3.8 Destination

It is the location to which shipments are transported to, these could be depots, warehouses or factories.

3.3.9 Origin

This is also called source, it is a location from which commodities are transported from.

3.3.10 Loop

A loop is made up of cells of the balanced transportation tableau which form a sequence of cells such that it starts and ends with the same cell. Each cell in the sequence can be connected to the next member of the sequence by a horizontal or vertical line in the tableau but there should not be three consecutive cells lying in the same row or column.

3.4 Balanced Transportation Problem

In a transportation problem if the total supply available at all sources is equal to total demand required at all the destinations, then the transportation problem is called a balanced transportation problem. This is represented mathematically as, $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ where a_i is supply available at the source and b_j is the demand require at the destination , $a_i \geq 0 \quad and \quad b_j \geq 0$. Where; $i = 1,...,m$, and $j = 1,...,n$, m is number of sources and n the number of destinations.

The objective function of the balanced transportation problem is;

Minimize $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$

Subject to $\sum_{j=1}^{n} x_{ij} = a_i$, where $i = 1,...,m$

$$\sum_{i=1}^{m} x_{ij} = b_j \text{ , and } j = 1,\dots,n$$

$$x_{ij} \geq 0 \text{ For all } i \text{ and } j$$

This situation is not mostly experienced in real life. For a feasible solution to exist, it is necessary that total supply be equal to total demand and there should be m+n-1 basic independent variables out of m*n variables, where m is the number of sources and n the number of destinations.

3.5 Unbalanced Transportation Problem

A situation in the transportation problem where total supply is not equal to the total demand is known as unbalanced transportation problem. This situation of the transportation problem is mostly encountered in everyday life, where there is a high demand of a particular commodity than a factory can supply or demand is less than what a factory can produce.

There are two situations, one is where total supply is greater than the total demand, represented mathematically as, $\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j$. In this situation, demand is made to be equal to the surplus by creating a dummy destination. The other situation is where total demand is greater than the total supply stated mathematically as $\sum_{j=1}^{n} b_j > \sum_{i=1}^{m} a_i$.

A dummy source is created with a supply equal to the excess of the demand.

3.6 Solving the Balanced Transportation Problem

The transportation problem can be solved using the simplex algorithm which is a linear programming technique; however an alternative solution method which requires less mathematical manipulation than the simplex method called the Transportation problem would be discussed.

Some of the methods used for solving the transportation problem are the Stepping-Stone Method and the Modified Distribution Method (MODI). But just as in the simplex method an initial solution is developed in the initial simplex tableau, so it is with the transportation problem a basic feasible solution would be needed.

A basic feasible solution is mostly obtained using any of these methods; NWCM, MCM and VAM. The solution given by these methods satisfied the row and column constraints, there is exactly m+n-1 allocations, the allocations are independent and do not form a loop and also the non-negativity constraints is satisfied. To establish a basic feasible solution the following methods are used;

1. North-West Corner Method (NWCM)
2. Minimum Cost Method (MCM)
3. Vogel's Approximation Method (VAM)

3.6.1 Setting up the Transportation Table

Using the problem; a steel company has three Machines, M_1, $M_{2,}$ and M_3, which can produce 150, 175, and 275 kilotons of steel each month. Three customers C_1, C_2 and C_3 have requirements of 200, 100, and 300 kilotons respectively in the same period. The cost, in units of $1000, of transporting a kiloton of steel from each machine to each customer is shown in Table 3.1.

1. Summarize the above information in a table form.
2. Determine the minimum-cost shipping schedule between the mills and the Customers

Table 3.1: Unit cost of shipping from the Machines to the Customers.

$M_1 \to C_1 = 6$	$M_1 \to C_2 = 8$	$M_1 \to C_3 = 10$
$M_2 \to C_1 = 7$	$M_2 \to C_2 = 11$	$M_2 \to C_3 = 11$
$M_3 \to C_1 = 4$	$M_3 \to C_2 = 5$	$M_3 \to C_3 = 12$

In Table 3.6.1.2, the transportation problem is represented in tabular form, the first column indicates the three machines with their corresponding products available in the last column. The first row represents the three customers and their demands in the last row. The unit cost of transporting a kiloton of steel from a particular machine to a particular customer is represented at the top right corner of each cell.

Table 3.2: Transportation Problem of the steel company in Table form

To / From	C_1	C_2	C_3	Supply (kiloton)
M_1	6	8	10	150
M_2	7	11	11	175
M_3	4	5	12	275
Demand (Kiloton)	200	100	300	Total=600

The unit cost of transporting is in thousands. For example, from table3.6.1.2 the cost of transporting a kiloton of steel from Machine one (M_1) to customer two (C_2) is \$8ooo. If there are m sources and n destinations, and the amount of supply at source i is a_i and the demand required at destination j is b_j where $i = 1,\ldots, m$ and

$j = 1,...,n$. The cost of transporting a unit product from source i to destination j is c_{ij} and x_{ij} also known as the decision variables represents the quantity transported from source i to destination j. The cost, c_{ij} of transporting a unit of commodity from source to destination is indicated at the top right corner of each cell and the quantity, x_{ij} of commodity transported from a particular source to a particular destination is indicated in the cell. Table3.6.1.3, illustrate these information.

Table 3.3: The General Transportation model in Table form

To / From	1	2	…………	n	Supply a_i
1	c_{11} x_{11}	c_{12} x_{12}	…………	c_{1n} x_{1n}	a_1
2	c_{21} x_{21}	c_{22} x_{22}	…………	c_{2n} x_{2n}	a_2
$\vdots$	$\vdots$	$\vdots$	…………	$\vdots$	$\vdots$
m	c_{m1} x_{m1}	c_{m2} x_{m2}	…………	c_{mn} x_{mn}	a_m
Demand b_j	b_1	b_2	……….	b_n	

3.6.2 Developing the Initial solution

The initial solution methods to the transportation model discuss in this section give basic feasible solution. Two of these methods; MCM and VAM consider the cost before allocating, which is a good factor for minimizing transport cost.

Transportation problem from Table 3.4.1.2 will be used to illustrate the three initial basic solution methods.

3.6.2.1 North-West Corner Method

Procedures for developing basic feasible solution using the NWCM Method;

1. The first cell that is as the name implies the north-west corner cell is allocated as much as possible subject to the supply and demand constraints.
2. Adjusting the supply and demand numbers in the respective rows and columns.
3. Cross out the row or column with zero supply or demand to indicate that no further allocations can be made in that row or column.
4. Allocate as much as possible to the next adjacent feasible cell
5. Repeat the second, third and the fourth procedure until the demand and supply constraints are met.

Table 3.4: Illustrating the solution Method of NWCM

To / From	C_1	C_2	C_3	Supply
M_1	6 **150**	8	10	150
M_2	7 **50**	11 **100**	11 **25**	175
M_3	4	5	12 **275**	275
Demand	200	100	300	Total=600

The value 150 in cell $X_{M_1C_1}$ is the quantity of steel transported from machine one (M_1) to customer one (C_1), 25 in cell $X_{M_2C_3}$ is the quantity of steel shipped from machine two (M_2) to customer three (C_3) and so on. The initial basic feasible cost of NWCM is calculated as; total Cost = 150 * 6 + 50 * 7 + 100 * 11 + 25 * 11 + 275 * 12 = $5,925

3.6.2.2 Minimum Cost Method (MCM)

Procedure for developing basic feasible solution using the Minimum Cost Method;

1. Allocate as much as possible to the cell with the smallest unit cost, meeting supply and demand constraints.
2. The satisfied row or column should be crossed out and the amounts of supply and demand adjusted.
3. Look for the next smallest unit cost and repeat the procedures above until exactly one row or column is left uncrossed out with which allocation is made without any other choice.

Table 3.5: Illustrating the solution Method of MCM

To / From	C_1	C_2	C_3	Supply
M_1	6	8 **25**	10 **125**	150
M_2	7	11	11 **175**	175
M_3	4 **200**	5 **75**	12	275
Demand	200	100	300	Total=600

The least cost which is in cell $X_{M_3C_1}$ is fully supplied 200 kilotons as demanded and the next least cost in cell $X_{M_3C_2}$ is also supplied 75 kilotons. Customer three (C_3) had 125 kilotons from Machine one (M_1) and 175 kilotons from Machine two (C_2). This last customer did not have the choice but is restricted to only where there are quantities of steel.

The initial basic feasible cost of MCM is; total cost = 200 * 4 + 25 * 8 +75 * 5 + 125 * 10 + 175 * 11 =$ 4,550

3.6.2.3 Vogel's Approximation Method (VAM)

This is an improved version of the minimum cost method and it gives a better starting solution in most cases. It is based on regret or penalty cost. Procedures for obtaining basic feasible solution using the Vogel's Approximation Method;

1. For each row and column determine a penalty for not allocating to the least cost and next least cost by subtracting the least cost from the next least cost.
2. Identify the row or column with the largest penalty cost and allocate as much as possible to the least cost cell. Ties should be broken arbitrary or choosing a least cost cell.
3. Adjust supply and demand numbers and cross out the satisfied row or column.
4. Calculate new penalty cost and repeat the procedures one and two. When it is left with two cells to be allocated, there are no penalties to be calculated; in that case allocation is done considering minimum cost.

Table 3.6: Illustrating the solution method of VAM

To / From	C_1	C_2	C_3	Supply	Row Penalty
M_1	6	8	10	150	2
M_2	7	11	11	175	4
M_3	4	5	12	275	1
Demand	200	100	300	Total=600	
Column Penalty	2	3	1		

The row penalties are calculated by subtracting the lowest unit cost from the next lowest unit cost in each row, the column penalties are also calculated in the same way but this is done within each column. The results are indicated at the end of the row that is if it is a row penalty and column if it is column penalty. For example, in the M_1 row the penalty value is calculated by subtracting `6` which is the least cost in that row from the next least cost '8' to get `2`. The highest penalty value is in the M_2 row which is `4`.

The step by step calculations of the penalties and allocation to cells in the transportation table are indicated in the Appendices.

Table 3.7: The summary of the solution method of VAM Illustration

To / From	C_1	C_2	C_3	Supply	Row Penalty		
M_1	6	8	10 **150**	150	2	2	4
M_2	7 **175**	11	11	175	4	-	-
M_3	4 **25**	5 **100**	12 **150**	275	1	1	8
Demand	200	100	300	Total=600			
Column Penalty	2	3	1				
	2	3	1				
	2	-	2				
	-	-	2				

The first column in the row penalty and the first row in the column penalty is the first penalty calculated, and the second column of the row penalty and the second row of the column penalty is the second penalty calculated and so on. A dash indicated in a row or a column indicates that row or the column is satisfied. In the first penalty cost calculated, the least cost in the highest penalty row that is `4` is allocated with 175kilotons only because of the supply available. In the next penalty cost calculated `3` which is the highest in C_2 column, have the least cost to be `5` and is allocated 100kilotons as demanded. In the third penalty, `8` is the highest penalty number in M_3 row and have the least cost `4` which is allocated 25kilotons. The last penalty calculated for C_3 column is restricted to cell $x_{M_1C_3}$ and $x_{M_3C_3}$ due to supply available.

Table 3.8: The final solution method Illustration of VAM

To / From	C_1	C_2	C_3	Supply
M_1	6	8	10 **150**	150
M_2	7 **175**	11	11	175
M_3	4 **25**	5 **100**	12 **150**	275
Demand	200	100	300	Total = 600

The initial basic feasible Cost of VAM = 150 * 10 + 175 * 7 + 25 * 4 + 100 * 5 + 150 * 12 = $5125. It has been realized that the highest penalties did not give the least cost, this make the initial basic feasible cost of VAM not close to the minimum cost as it is in some cases. The least cost was only allocated 25kilotons compare to the MCM which is 200kilotons and the highest cost of VAM is allocated 150kilotons whiles in the MCM there is no allocation.

3.6.3 Discussing the Basic Feasible Solution (BFS) methods

Looking at the North-West Corner Method, the procedure for developing basic feasible solution is quick and the computations take just a few minutes but the solution it produces is far from the optimal solution.

The Minimum Cost Method produces the least BFS. This is so because the highest allocations are made to the least cost cells first. The VAM gave the next lowest BFS, this is due to the reason that some of the highest penalties row or column did not give the least cost cell.

Minimum Cost Method and the Vogel's Approximation methods are used to obtain the shortest road. This is because they try to satisfy cells with the least cost before moving to those with the high cost. Vogel's method produces the best basic feasible solution because it gives a basic solution very close to the optimal solution but the computations take a long time. In most cases for a basic feasible solution, the minimum cost method or the Vogel's method are used because they give a solution which is mostly close to the optimal solution.

3.7 Solving the Unbalanced Transportation Problem

The unbalanced transportation problem is solved by first changing the unbalanced problem into a balanced transportation problem by creating a dummy column or row depending on where the shortage is coming from, that is either from supply or demand. The solution procedure for balanced transportation problem is followed. The dummy row or column created in the unbalanced Transportation problem is just a row or column with zero (0) as a unit cost of transportation. The cost are zero because the quantity are not actually transported but was just excess supply or extra demand. Unbalanced transportation problems are mostly experienced in real life situations.

3.7.1 Total Supply Greater Than Total Demand

When total supply is more than the total demand, an additional column referred to as a dummy destination is created which indicate the surplus supply with a zero transportation cost. Mathematically, if a_i = supply and b_j = demand then, $\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j$, and the requirement is; $\sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j$, for $i = 1,\dots,m$ $and \;\; j = 1,\dots,n.$

Table 3.9: Transportation Problem of Total Supply greater than Total Demand

To / From	C_1	C_2	C_3	Supply
M_1	6	8	10	150
M_2	7	11	11	175
M_3	4	5	12	375
Demand	200	100	300	$\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j$

The total supply is equal to 700 = 150 + 175 + 375 and the total demand is equal to 600 = 200 + 100 + 300. It is clear that supply is more than the demand require in this transportation problem by 100.

Table 3.10: Correcting Total Supply greater than Total Demand

To / From	C_1	C_2	C_3	Dummy	Supply
M_1	6	8	10	0	150
M_2	7	11	11	0	175
M_3	4	5	12	0	375
Demand	200	100	300	100	Total=700

Because supply is more than the demand a Dummy column is created with zeros as the unit cost of transportation. The value hundred (100) indicated in the demand row of the dummy column is not actually a demand at any destination point but just created for the excess supply. Now the transportation problem in Table3.7.1.2, is balanced, that is total demand is equal to total supply.

3.7.2 Total Demand Greater Than Total Supply

When total demand is more than the total supply, an additional row called a dummy source is created in the transportation table, which represents unsatisfied demand with a zero transportation cost. Mathematically, this is stated as; $\sum_{j=1}^{n} b_j > \sum_{i=1}^{m} a_i$, and the requirement is written as , $\sum_{j=1}^{n} b_j - \sum_{i=1}^{m} a_i$.

Table 3.11: Transportation Problem of Total Demand greater than Total Supply.

To / From	C_1	C_2	C_3	Supply
M_1	6	8	10	150
M_2	7	11	11	175
M_3	4	5	12	275
Demand	200	100	350	$\sum_{j=1}^{n} b_j > \sum_{i=1}^{m} a_i$

Total demand is equal to 650 = 200 + 100 + 350 and the total supply is also equal to 600 = 150 + 175 + 275. Demand is greater than the supply by fifty (50). To balance this transportation problem, a Dummy row is created in with unit cost as zeros (0) and the extra requirement of 50 indicated as a supply which is not actually available.

Table 3.12: Correcting Total Demand greater than Total Supply

To / From	C_1	C_2	C_3	Supply
M_1	6	8	10	150
M_2	7	11	11	175
M_3	4	5	12	275
Dummy	0	0	0	50
Demand	200	100	350	Total = 650

The value fifty (50) put in the supply column of the dummy row is not actually available but is just an indication that fifty more is needed. From Table3.7.2.2, the total demand is equal to the total supply.

3.8 Degeneracy in the Transportation Model

The number of occupied cells at any stage of the solution to the transportation problem must be equal to the number of rows in the table plus the number of columns minus one. If a solution to transportation problem with m origins and n destinations at any stage of the solution has less than the number of rows plus the number of column minus one ($m + n – 1$), the solution is called DEGENERATE.

Degeneracy occurs when the shipping routes are fewer than ($m + n – 1$). This situation makes it impossible to trace a closed path for some empty cells when using the Stepping-Stone method to improve the solution. Table3.8.1 below illustrates a situation of degeneracy in a transportation model.

Table 3.13: Illustrating Degeneracy in the Transportation Model

From \ To	C_1	C_2	C_3	Supply
M_1	6	8 **100**	10 **50**	150
M_2	7	11	11 **250**	250
M_3	4 **200**	5	12	200
Demand	200	100	300	Total = 600

The number of rows plus the number of columns minus one is five (5); m + n - 1 = 5, but the cells allocated to are only four which indicates degeneracy.

3.8.1 Solving Degeneracy in the Transportation Model

To resolve degeneracy Epsilon (ε) or any symbol is put in one of the empty cells; this is regarded as allocation to that cell. In order to calculate the total cost the Epsilon is considered as zero (0).

The Epsilon is a very small and does not have any effect on the total cost. It should be placed in the cell with which a closed path can be formed when using the Stepping-Stone method or when using the MODI method so that computation of all the cij – (ui + vj) could be completed.

Table 3.14: Solving Degeneracy in the Transportation Model

To / From	C_1	C_2	C_3	Supply
M_1	6 ε	8 **100**	10 **50**	150
M_2	7	11	11 **250**	250
M_3	4 **200**	5	12	200
Demand	200	100	300	Total=600

The symbol Epsilon is considered as an allocation to the cell $X_{M_1C_1}$, to calculate the total cost the Epsilon is considered as zero. The basic variables are now five that is the m + n – 1 allocation have been met.

3.9 Optimality Test

To test a solution for optimality, the basic variables must satisfy the $m+n-1$ condition that is if m is the number of sources and n the number of destinations. The Stepping-Stone method and the MODI method will be used for optimality test in this section. Each of these methods work towards finding out if there is a variable not allocated to but can help reduce cost when allocated to.

3.9.1 Improving the BFS Using the Stepping-Stone Method

The stepping-Stone Method is used to improve upon the solution obtained from any of the basic feasible solution methods. The idea behind using this method is to determine if a non-basic variable would result in a lower total cost if it is introduce into the basics. To apply the stepping-stone method, the basic variables must be m +

n - 1 at any point in the solution. Table3.6.2.2.1 of MCM is used to illustrate the stepping-Stone Method.

To improve the basic feasible solution using the Stepping-Stone Method;

1. Select any unused cell to evaluate
2. Beginning with this cell, trace a closed path back to the original cell through the cells that are occupied, only horizontal and vertical moves are allowed. However, one can step over either an empty or an occupied cell.
3. Beginning with a plus (+) sign at the unused cell, place alternating minus signs and plus signs at each corner of the cell of the closed path just traced.
4. Calculate an improvement index by first adding the unit cost numbers found in each cell containing a plus sign and then by subtracting the unit costs in each cell containing a minus sign.
5. Repeat the above procedures until the entire improvement indices for all unused cells have been calculated. If all the improvement indices are greater than or equal to zero then stop and compute the current feasible solution as the optimal solution, but if the improvement indices is less than zero then the current solution need to be improved by going through the above steps. This will continue until optimal solution is reached.

Table 3.15: Illustrating the Stepping-stone Method, evaluating cell $X_{M_1C_1}$

From \ To	C_1	C_2	C_3	Supply
M_1	+ 6	- 8 **25**	10 **125**	150
M_2	7	11	11 **175**	175
M_3	- 4 **200**	+ 5 **75**	12	275
Demand	200	100	300	Total=600

A plus (+) sign is placed in the empty cell with the unit cost \`6\` follow by a negative in the cell with unit cost \`8\`,that is $X_{M_1C_1} \rightarrow X_{M_1C_2} \rightarrow X_{M_3C_2} \rightarrow X_{M_3C_1}$ = + 6 – 8 + 5 – 4 = - 1. The Improvement Index calculated resulted in a negative number which means if allocation is made to cell $X_{M_1C_1}$ there will be a reduction in the total cost.

Table 3.16: Evaluating cell $X_{M_2C_1}$

From \ To	C_1	C_2	C_3	Supply
M_1	6	- 8 **25**	+ 10 **125**	150
M_2	+ 7	11	- 11 **175**	175
M_3	- 4 **200**	+ 5 **75**	12	275
Demand	200	100	300	Total=600

The plus (+) and the negative (-) signs are indicated in the cells as follows $X_{M_2C_1} \rightarrow X_{M_2C_3} \rightarrow X_{M_1C_3} \rightarrow X_{M_1C_2} \rightarrow X_{M_3C_2} \rightarrow X_{M_3C_1}$. The Improvement Index is calculated as + 7 - 11 + 10 - 8 + 5 - 4 = $ - 1. The value of the Improvement Index is negative which means the total cost can be reduced if allocation is made to cell $X_{M_2C_1}$

Table 3.17: Evaluating cell $X_{M_2C_2}$

To / From	C_1	C_2	C_3	Supply
M_1	6	- 8 **25**	+ 10 **125**	150
M_2	7	+ 11	- 11 **275**	175
M_3	4 **200**	5 **75**	12	275
Demand	200	100	300	Total=600

$X_{M_2C_2} \rightarrow X_{M_2C_3} \rightarrow X_{M_1C_3} \rightarrow X_{M_1C_2}$ = +11 -11 +10 -8= $ 2. The Improvement index is positive which indicates that allocating to this cell will not reduce the total cost.

Table 3.18: Evaluating cell $X_{M_3C_3}$

To / From	C_1	C_2	C_3	Supply
M_1	6	+ 8 **25**	- 10 **125**	150
M_2	7	11	11 **175**	175
M_3	4 **200**	- 5 **75**	+ 12	275
Demand	200	100	300	T0tal=600

The plus (+) and minus (-) signs are indicated as $X_{M_3C_3} \rightarrow X_{M_1C_3} \rightarrow X_{M_1C_2} \rightarrow X_{M_3C_2}$; = + 12 - 10 + 8 - 5 = 5. The positive Improvement Index value indicates that allocating to this cell will not reduce the total cost.

From the evaluations done so far, cell $X_{M_1C_1}, X_{M_2C_1}$ can reduce the cost by \$ 1 for each unit. One of these cells is selected arbitrarily to be allocated to. Selecting $X_{M_1C_1}$, allocation is made as much as possible to this cell adjusting to meet the row and column constraints, 25 units are allocated and the cells in that row and column are adjusted as such. Table3.9.1.5 illustrates this information.

Table 3.19: Transportation model after the first Iteration

From \ To	C_1	C_2	C_3	Supply
M_1	[6] **25**	[8]	[10] **125**	150
M_2	[7]	[11]	[11] **175**	175
M_3	[4] **175**	[5] **100**	[12]	275
Demand	200	100	300	Total = 600

The following empty cells $X_{M_2C_1}, X_{M_1C_2}, X_{M_2C_2}, X_{M_3C_3}$ are evaluated in the second Iteration.

Table 3.20: Evaluating cell $X_{M_2C_1}$

From \ To	C_1	C_2	C_3	Supply
M_1	- [6] **25**	[8]	+ [10] **125**	150
M_2	+ [7]	[11]	- [11] **175**	175
M_3	[4] **175**	[5] **100**	[12]	275
Demand	200	100	300	Total=600

The signs are indicated as follows $X_{M_2C_1} \to X_{M_2C_3} \to X_{M_1C_3} \to X_{M_1C_1}$. The Improvement Index value is + 7 -11 +10 -6 = \$ 0. The zero Improvement Index means the cost cannot be reduced if the cell evaluated is introduced into the basic variables. It also signifies an alternate solution. The alternate solution simply means existence of another route which can result in the same total cost of transportation.

Table 3.21: Evaluating cell $X_{M_1C_2}$

To / From	C_1	C_2	C_3	Supply
M_1	- [6] **25**	+ [8]	[10] **125**	150
M_2	[7]	[11]	[11] **175**	175
M_3	+ [4] **175**	- [5] **100**	[12]	275
Demand	200	100	300	Total=600

$X_{M_1C_2} \to X_{M_3C_2} \to X_{M_3C_1} \to X_{M_1C_1}$ The Improvement Index is calculated as; + 8 - 5 + 4 - 6 = \$ 1. The positive value indicates that the evaluated cell cannot help the total cost to decrease. Which means no allocation can be made to this cell.

Table 3.22: Evaluating cell $X_{M_2C_2}$

To / From	C_1	C_2	C_3	Supply
M_1	- [6] **25**	[8]	+ [10] **125**	150
M_2	[7]	+ [11]	- [11] **175**	175
M_3	+ [4] **175**	- [5] **100**	[12]	275
Demand	200	100	300	Total=600

The alternate signs of positive (+) and negative (-) are indicated in the cells $X_{M_2C_2} \rightarrow X_{M_3C_2} \rightarrow X_{M_3C_1} \rightarrow X_{M_1C_1} \rightarrow X_{M_1C_3} \rightarrow X_{M_2C_3}$. The Improvement Index value is + 11 - 5 + 4 - 6 + 10 - 11 = \$3. The total cost cannot be reduced if allocation is made to cell $X_{M_2C_2}$ because of the positive Improvement Index value.

Table 3.23: Evaluating cell $X_{M_3C_3}$

From \ To	C_1	C_2	C_3	Supply
M_1	+ [6] **25**	[8]	- [10] **125**	150
M_2	[7]	[11]	[11] **175**	175
M_3	- [4] **175**	[5] **100**	+ [12]	275
Demand	200	100	300	Total=600

$X_{M_3C_3} \rightarrow X_{M_1C_3} \rightarrow X_{M_1C_1} \rightarrow X_{M_3C_1}$ = + 12 - 10 + 6 - 4 = \$ 4. The evaluations of the four routes indicate the cost cannot be reduced anymore, because all the empty cells evaluated resulted in a positive Improvement Index which signifies optimality. The final transportation model is illustrated in Table3.9.1.10.

Table 3.24: Final Transportation Model of the Steeping-Stone method

From \ To	C_1	C_2	C_3	Supply
M_1	[6]	[8]	[10] **150**	150
M_2	[7] **25**	[11]	[11] **150**	175
M_3	[4] **175**	[5] **100**	[12]	275
Demand	200	100	300	Total=600

Total minimum cost is 25 * 7 + 175 * 4 + 100 * 5 + 150 * 10 + 150 * 11 = $ 4,525

3.9.2 Improving the BFS using the MODI Method

The Modified Distribution Method is a modified version of the Stepping-Stone Method. In the MODI method, the individual cell cost changes are determined by mathematical formulas, without identifying the stepping-stone paths for all the empty cells. It begins with the initial solution obtained from the three basic feasible solution methods. The procedures below lead to optimal solution using the MODI method;

1. Letting u_i representing the value assigned to the row i and v_j representing the value assigned to the column j where $i = 1,...,m$ and $j = 1,...,n$, m and n are the number of rows and columns respectively. The cost of transporting from source i to destination j is c_{ij}, where $i = 1,...,m$ and $j = 1,...,n$, m and n are the number of sources and destinations respectively.
2. Setting $u_1 = 0$, calculate the rest of u_i and v_j values using the c_{ij} values of the occupied cells with the formula $X_{ij} : c_{ij} = u_i + v_j$ where X_{ij} denote the cell.
3. Compute the improvement index for each of the unused cells or the non-basic variables by the relation $I_{ij} = c_{ij} - (u_i + v_j)$. I_{ij} Denote the improvement index value and the rest of the variables have their usual meaning.
4. Allocate as much as possible to the non-basic variable with the greatest negative value or the improvement index with the most negative value adjusting to meet the demand and supply constraints.
5. Repeat the above procedures until the improvement index values are all positive or zero (0) then the current solution is taken as optimal.

Table.3.6.2.2.1 obtained from MCM is used to illustrate the MODI Method below.

Table 3.25: Calculating the Rows and Columns values, First Iteration.

		$V_1=7$	$V_2=8$	$V_3=10$	
	To / From	C_1	C_2	C_3	Supply
$U_1= 0$	M_1	6	8 **25**	10 **125**	150
$U_2 = 1$	M_2	7	11	11 **175**	175
$U_3 = -3$	M_3	4 **200**	5 **75**	12	275
	Demand	200	100	300	Total=600

Setting M_1, M_2, M_3 rows as u_1, u_2, u_3 respectively and C_1, C_2, C_3 columns as v_1, v_2, v_3 respectively, letting $u_1 = 0$ we calculate the rest of the rows and columns values with the formula; $X_{ij}: \quad C_{ij} = U_i + V_j$

$$X_{M_1C_2}: \quad c_{M_1C_2} = u_1 + v_2 \Rightarrow 8 = 0 + v_2 \therefore v_2 = 8$$

$$X_{M_1C_3}: \quad c_{M_1C_3} = u_1 + v_3 \Rightarrow 10 = 0 + v_3 \therefore v_3 = 10$$

$$X_{M_2C_3}: \quad c_{M_2C_3} = u_2 + v_3 \Rightarrow 11 = u_2 + 10 \therefore u_2 = 1$$

$$X_{M_3C_2}: \quad c_{M_3C_2} = u_3 + v_2 \Rightarrow 5 = u_3 + 8 \therefore u_3 = -3$$

$$X_{M_3C_1}: \quad c_{M_3C_1} = u_3 + v_1 \Rightarrow 4 = -3 + v_1 \therefore v_1 = -3$$

From Table3.9.2.2, we calculate the Improvement Indices of the unoccupied cells using the relation, $X_{ij} : I_{ij} = c_{ij} - (u_i + v_j)$

Table 3.26: Calculating the Improvement Index of the empty cells.

	To / From	C1 ($V_1 = 7$)	C2 ($V_2 = 8$)	C3 ($V_3 = 10$)	Supply
$U_1 = 0$	M_1	**-1** [6]	**25** [8]	**125** [10]	150
$U_2 = 1$	M_2	**-1** [7]	**+2** [11]	**175** [11]	175
$U_3 = -3$	M_3	**200** [4]	**75** [5]	**+5** [12]	275
	Demand	200	100	300	Total=600

$$X_{M_1C_1} : I_{M_1C_1} = c_{M_1C_1} - (u_1 + v_1) \Rightarrow \quad I_{M_1C_1} = 6 - (0 + 7) = -1$$

$$X_{M_2C_1} : I_{M_2C_1} = c_{M_2C_1} - (u_2 + v_1) \Rightarrow \quad I_{M_2C_1} = 7 - (1 + 7) = -1$$

$$X_{M_2C_2} : I_{M_2C_2} = c_{M_2C_2} - (u_2 + v_2) \Rightarrow \quad I_{M_2C_2} = 11 - (1 + 8) = 2$$

$$X_{M_3C_3} : I_{M_3C_3} = c_{M_3C_3} - (u_3 + v_3) \Rightarrow \quad I_{M_3C_3} = 12 - (-3 + 10) = 5$$

From the calculations, some of the Improvement Indices are negative which indicate we have not reached optimal solution. In Table3.9.2.3, allocation is made to one of the cells with the negative values adjusting to meet demand and supply constraints.

Table 3.27: Calculating the Rows and Columns values in the Second Iteration.

	To / From	C1 ($V_1=6$)	C2 ($V_2=7$)	C3 ($V_3=10$)	Supply
$U_1 = 0$	M_1	**25** [6]	[8]	**125** [10]	150
$U_2 = 1$	M_2	[7]	[11]	**175** [11]	175
$U_3 = -2$	M_3	**175** [4]	**100** [5]	[12]	275
	Demand	200	100	300	Total=600

From Table3.9.2.3, 25kilotons is allocated to cell $X_{M_1C_1}$ and cells $X_{M_3C_1}$ and $X_{M_3C_2}$ are adjusted to meet supply and demand constraints.

We use Table3.9.2.3, to find new rows and columns values with X_{ij} : $C_{ij} = U_i + V_j$

$$X_{M_1C_1}: \quad c_{M_1C_1} = u_1 + v_1 \Rightarrow 6 = 0 + v_2 \therefore v_2 = 6$$
$$X_{M_1C_3}: \quad c_{M_1C_3} = u_1 + v_3 \Rightarrow 10 = 0 + v_3 \therefore v_3 = 10$$
$$X_{M_2C_3}: \quad c_{M_2C_3} = u_2 + v_3 \Rightarrow 11 = u_2 + 10 \therefore u_2 = 1$$
$$X_{M_3C_1}: \quad c_{M_3C_1} = u_3 + v_1 \Rightarrow 4 = u_3 + 6 \therefore u_3 = -2$$
$$X_{M_3C_2}: \quad c_{M_3C_2} = u_3 + v_2 \Rightarrow 5 = -2 + v_2 \therefore v_2 = 7$$

Table3.9.2.4 reflects the new Improvement Indices calculated using the relation, $X_{ij} : I_{ij} = c_{ij} - (u_i + v_j)$ and also indicates the final transportation model for MODI.

$$X_{M_1C_2} : I_{M_1C_2} = c_{M_1C_2} - (u_1 + v_2) \Rightarrow I_{M_1C} = 8 - (0 + 7) = +1$$
$$X_{M_2C_1} : I_{M_2C_1} = c_{M_2C_1} - (u_2 + v_1) \Rightarrow I_{M_2C_1} = 7 - (1 + 6) = 0$$
$$X_{M_2C_2} : I_{M_2C_2} = c_{M_2C_2} - (u_2 + v_2) \Rightarrow I_{M_2C_2} = 11 - (1 + 7) = +3$$
$$X_{M_3C_3} : I_{M_3C_3} = c_{M_3C_3} - (u_3 + v_3) \Rightarrow I_{M_3C_3} = 12 - (-2 + 10) = +4$$

Table 3.28: The final Solution Table of the MODI Method

		$V_1=6$	$V_2=7$	$V_3=10$	
	From \ To	C_1	C_2	C_3	Supply
$U_1 = 0$	M_1	[6] **25**	**+1** [8]	[10] **125**	150
$U_2 = 1$	M_2	**0** [7]	**+3** [11]	[11] **175**	175
$U_3 = -2$	M_3	[4] **175**	[5] **100**	**+4** [12]	275
	Demand	200	100	300	Total=600

From the calculation of the improvement indices, none of the values are negative which indicate optimality. The total cost = 25 * 6 + 175 * 4 + 100 * 5 + 125 * 10 +

175 * 11 =$ 4,525. The zero improvement index value in cell $X_{M_2C_1}$ is an indication that there is an alternate solution as discussed in the Stepping-Stone Method.

CHAPTER FOUR

Analysis of Data

4.1 Introduction

In this chapter, the Transportation Algorithm is used to analyze the data from BOST Company. BOST is a government institution responsible for transporting and storing of petroleum products. It has six operational depots; these are the Accra Plains Depot (APD), Kumasi Depot, Bolgatanga Depot, Buipe Depot, Mami-Water Depot and Akosombo Depot. The Accra Plains Depot has a total storage capacity of 139,250 cubic meters (m^3) and serves as an originating depot. It is connected to Tema Oil Refinery which has a production capacity of 45,000 bpd and Conventional Buory Mooring by pipelines. CBM is a channel at Kpong near Tema. The remaining depots are supplied by TOR and APD.

The storage capacity of each depot is the demand in cubic meters (m^3). The cost for transporting a liter of products from source that is APD or TOR to each of the depot is represented in Table 4.2.1. The sources or the supply points are TOR and APD and the demand points or the destinations are Buipe depot, Bolgatanga depot, Akosombo depot and Kumasi depot. The source from which a particular depot gets its supply depends on the situation at the sources, TOR and APD. Table4.1.1 illustrates the current schedule of BOST Company.

The following abbreviations will be used in the Transportation table in this chapter, Bu, Bo, Ak, Ku, and Du for Buipe, Bolgatanga, Akosombo, Kumasi and Dummy respectively. Again, Lit, D, and S will also be used to mean Liters, Demand and Supply respectively.

Table 4.1: Current Schedule of Petroleum Products Transportation

To / From	Bu	Bo	Ak	Ku	S (lit)
TOR	0.15020 **1,370,968**	0.17975 **1,500,000**	0.03305 **1,580,645**	0.09024 **2,835,484**	7,154,415
APD	0.10764	0.12833	0.02064 **1,000,000**	0.09274	4,491,935
D (lit)	1,370,968	1,500,000	2,580,645	2,835,484	

The transportation cost for the current schedule is 0.15020 * 1,370,968 + 0.17975 * 1,500,000 + 0.09024 * 2,835,484 + 0.03305 * 1,580,645 + 0.02064 * 1,000,000 = GHC 804298.79

4.2 Formulating the Transportation Problem

In Table4.2.1, the demands at each depot are in liters and the supplies are also in liters. The cost of transporting a liter of petroleum products from a source to a depot is in Ghana cedis indicated at the top right corner of the cell.

Table 4.2: Unit cost Transportation Table

To / From	Bu	Bo	Ak	Ku	S (lit)
TOR	0.15020	0.17975	0.03305	0.09024	7,154,415
APD	0.10764	0.12833	0.02064	0.09274	4,491,935
D (lit)	1,370,968	1,500,000	2,580,645	2,835,484	

4.3 Finding the Initial Transportation cost

In Table4.3.1, allocations start from the North-West Corner cell and proceed as suggested by the procedure of the North-West Corner method.

Table 4.3: Finding the Initial Transportation Cost using NWCM

To / From	Bu	Bo	Ak	Ku	Du	S (Lit)
TOR	0.15020 **1,370,968**	0.17975 **1,500,000**	0.03305 **2,580,645**	0.09024 **1,702,802**	0	7,154, 415
APD	0.10764	0.12833	0.02064	0.09274 **1,132,682**	0 **3,359,253**	4,491, 935
D (lit)	1,370,968	1,500,000	2,580,645	2,835,484	3,359,253	

From Table4.3.1, Buipe is allocated all its supply from TOR so as Bolgatanga and Akosmbo. Kumasi depot is allocated 1,702,802 liters from TOR and 1,132,682 liters from APD. The Initial total Transportation cost of NCWM = 1370968 * 0.15020 + 1500000 * 0.17975 + 2580645 * 0.03305 + 0.09024 * 1702802 + 0.09274 * 1132682 +0 * 3,359,253 = GH 819,540.49

4.4 Testing for Optimal Solution

Calculating the Rows and Columns values, First Iteration;

TOR and APD rows are label u_1 and u_2 respectively and Buipe, Bolgatanga, Akosombo, Kumasi and Dummy columns are label v_1, v_2, v_3, v_4 and v_5 respectively Letting $u_1 = 0$, we calculate the rows and the columns values using $C_{ij} = U_i + V_j$

$i = u_1$ and u_2 and $j = v_1, v_2, v_3, v_4$ and v_5 for the occupied cells. From Table4.3.1;

$U_1 + V_1 = 0.15020$, but $u_1 = 0$, $\Rightarrow$ $0 + v_1 = 0.15020 \because v_1 = 0.15020$

$U_1 + V_2 = 0.17975$, but $u_1 = 0 \Rightarrow 0 + v_2 = 0.17975 \because v_2 = 0.17975$

$U_1 + V_3 = 0.03305$, but $u_1 = 0 \Rightarrow 0 + v_3 = 0.03305 \because v_3 = 0.03305$

$U_1 + V_4 = 0.09024$, but $u_1 = 0 \Rightarrow 0 + v_4 = 0.09024 \because v_4 = 0.09024$

$U_2 + V_4 = 0.09274$, but $v_4 = 0.09024 \Rightarrow u_2 + 0.09024 = 0.09274 \because u_2 = 0.0025$ calculating the Improvement Indices for the empty cells in Table4.3.1 using $X_{ij} = c_{ij} - (u_i + v_j)$. First Iteration;

$x_{21} = 0.10764 - (0.0025 + 0.15020) = -0.04506$

$x_{22} = 0.12833 - (0.0025 + 0.17975) = -0.05392$

$x_{23} = 0.02064 - (0.0025 + 0.09024) = -0.01491$

The three cells checked can help reduce the total transportation cost but the cell x_{22} with the highest negative Improvement Index value can reduce the cost more than the others. Allocation is made to it adjusting the other appropriate cells in order to meet supply and demand constraints.

Table 4.4: Allocating to Cell x_{22}

To / From	Bu	Bo	Ak	Ku	Du	S (Lit)
TOR	0.15020 **1,370,968**	0.17975	0.03305 **2,580,645**	0.09024 **2,835,484**	0 **367,318**	7,154, 415
APD	0.10764	0.12833 **1,500,000**	0.02064	0.09274	0 **2,991,935**	4,491, 935
D (lit)	1,370,968	1,500,000	2,580,645	2,835,484	3,359,253	

New rows and column values are calculated in the Second Iteration;

$U_1 + V_1 = 0.15020$, but $u_1 = 0$, $\Rightarrow$ $0 + v_1 = 0.15020 \because v_1 = 0.15020$

$U_1 + V_3 = 0.03305$, but $u_1 = 0 \Rightarrow 0 + v_3 = 0.03305 \because v_3 = 0.03305$

$U_1 + V_4 = 0.09024$, but $u_1 = 0 \Rightarrow 0 + v_4 = 0.09024 \because v_4 = 0.09024$

$U_1 + V_5 = 0$, but $u_1 = 0 \Rightarrow 0 + v_5 = 0 \because v_5 = 0$

$U_2 + V_5 = 0$, but $v_5 = 0 \Rightarrow u_2 + 0 = 0 \because u_2 = 0$

$U_2 + V_2 = 0.12833$, but $u_2 = 0 \Rightarrow 0 + v_2 = 0.12833 \because v_2 = 0.12833$

Calculating new Improvement Indices in the 2nd Iteration;

x_{12} = 0.17975 – (0 + 0.12833) = 0.5142

x_{21} = 0.10764 – (0 + 0.15020) = - 0.04256

x_{23} = 0.02064 – (0 + 0.03305) = - 0.0124

x_{24} = 0.09274 – (0 + 0.09024) = 0.0025.

Cell x_{21} gives the highest negative value which means we have not reached the optimal solution. Much quantity is allocated to this cell in Table4.4.2 adjusting the appropriate cells to meet supply and demand constraints.

Table 4.5: Allocating to cell x_{21}

To / From	Bu	Bo	Ak	Ku	Du	S (Lit)
TOR	0.15020	0.17975	0.03305 **2,580,645**	0.09024 **2,835,484**	0 **1,738,286**	7,154,415
APD	0.10764 **1,370,968**	0.12833 **1,500,000**	0.02064	0.09274	0 **1,620,967**	4,491,935
D(lit)	1,370,968	1,500,000	2,580,645	2,835,484	3,359,253	

New rows and columns values are calculated in the Third Iteration;

$U_1 + V_3 = 0.03305$, but $u_1 = 0 \Rightarrow 0 + v_3 = 0.03305 \because v_3 = 0.03305$

$U_1 + V_4 = 0.09024$, but $u_1 = 0 \Rightarrow 0 + v_4 = 0.09024 \because v_4 = 0.09024$

$U_1 + V_5 = 0$, but $u_1 = 0 \Rightarrow 0 + v_5 = 0 \because v_5 = 0$

$U_2 + V_5 = 0$, but $v_5 = 0 \Rightarrow u_2 + 0 = 0 \because u_2 = 0$

$U_2 + V_1 = 0.10764$, but $u_2 = 0, \Rightarrow 0 + v_1 = 0.10764 \because v_1 = 0.10764$

$U_2 + V_2 = 0.12833$, but $u_2 = 0, \Rightarrow 0 + v_2 = 0.12833 \because v_2 = 0.12833$

Calculating the Improvement Indices in the 3nd Iteration;

$x_{11} = 0.15020 - (0 + 0.10764) = 0.04256$

$x_{12} = 0.17975 - (0 + 0.12833) = 0.0514$

$x_{23} = 0.02064 - (0 + 0.03305) = - 0.01241$

$x_{24} = 0.09274 - (0 + 0.09024) = 0.0025$

Cell x_{23} indicates there would be a reduction in transportation cost when allocated some quantity of petroleum products. In Table4.4.3 some quantity of petroleum products is allocated to cell x_{23} adjusting to meet demand and supply constraints.

Table 4.6: Allocating to cell x_{23}

To / From	Bu	Bo	Ak	Ku	Du	S (Lit)
TOR	0.15020	0.17975	0.03305 **959,678**	0.09024 **2,835,484**	0 **3,359,253**	7,154,415
APD	0.10764 **1,370,968**	0.12833 **1,500,000**	0.02064 **1,620,967**	0.09274	0	4,491,935
D (lit)	1,370,968	1,500,000	2,580,645	2,835,484	3,359,253	

Fourth Iteration;

$U_1 + V_3 = 0.03305$, but $u_1 = 0$, $\Rightarrow 0 + v_3 = 0.03305 \because v_3 = 0.03305$

$U_1 + V_4 = 0.09024$, but $u_1 = 0 \Rightarrow 0 + v_4 = 0.09024 \because v_4 = 0.09024$

$U_2 + V_3 = 0.02064$, $\Rightarrow u_2 = 0.02064 - 0.03305 = -0.01241$

$U_2 + V_1 = 0.10764$, but $u_2 = -0.01241 \Rightarrow v_1 + (-0.01241) = 0.10764 \because v_1 = 0.12005$

$U_2 + V_2 = 0.12833$, but $u_2 = -0.01241 \Rightarrow -0.01241 + v_2 = 0.12833 \Rightarrow v_2 = 0.14074$

Calculating the improvement indices in the 4th Iteration;

$x_{11} = 0.15020 - (0 + 0.12005) = 0.0315$

$x_{12} = 0.17975 - (0 + 0.14074) = 0.03901$

$x_{24} = 0.09274 - (-0.01241 + 0.09024) = 0.01491$

The Improvement Indices calculated are all positive. This indicates the Transportation cost cannot be reduced any further. The total minimum cost is calculated from Table 4.4.4. The total minimum cost is 0.10764 * 1,370,968 + 0.12833 * 1,500,000 + 0.02064 * 1,620,967 + 0.03305 * 959,678 + 0.09024 * 2,835,484 + 0 * 3,359,253 + 0 * 3,359,253 = GHC 661,114.19

4.5 Finding the Initial Transportation Cost using MCM

Table 4.7: Finding Initial Solution using MCM

To / From	Bu	Bo	Ak	Ku	Du	S (Lit)
TOR	0.15020	0.17975 **959,678**	0.03305	0.09024 **2,835,484**	0 **3,359,253**	7,154, 415
APD	0.10764 **1,370,968**	0.12833 **540,322**	0.02064 **2,580,645**	0.09274	0	4,491, 935
D (lit)	1,370,968	1,500,000	2,580,645	2,835,484	3,359,253	

Allocation is done to cell x_{23} from APD because it has the least cost, follow by x_{14} from TOR and x_{21} from APD. Cell x_{22} got only 450,322 liters from APD which is the next least cost cell compare to cell x_{12} but supply is finished from APD. The total cost is, 0.15020 * 1370968 + 0.12833 * 540322 + 0.17975 * 959678 + 0.02064 * 2580645 + 0.09024 * 2835484 + 0 * 3359253 = GHC 698,551.23

4.6 Testing for Optimal Solution using MODI

Calculating the Rows and Columns values in the First Iteration; TOR and APD rows are label u_1 and u_2 respectively and Buipe, Bolgatanga, Akosombo, Kumasi and Dummy columns are label v_1, v_2, v_3, v_4 and v_5 respectively Letting $u_1 = 0$, we calculate the rows and the columns values using $C_{ij} = U_i + V_j$ i = u_1 and u_2 and j = v_1, v_2, v_3, v_4 and v_5 for the occupied cells. From Table4.5.1;

$U_1 + V_2 = 0.17975$, but $u_1 = 0$, $\Rightarrow$ $0 + v_2 = 0.17975$ $\because$ $v_2 = 0.17975$

$U_1 + V_4 = 0.09024$, but $u_1 = 0$ $\Rightarrow$ $0 + v_4 = 0.09024$ $\because$ $v_4 = 0.09024$

$U_2 + V_2 = 0.12833$, but $v_2 = 0.17975$ $\Rightarrow u_2 + 0.17975 = 0.12833$ $\because$ $u_2 = -0.05142$

$U_2 + V_1 = 0.10764$, but $u_2 = -0.05142 \Rightarrow v_1 + (-0.05142) = 0.10764 \because$ $v_1 = 0.15906$

$U_2 + V_3 = 0.02064$, $\Rightarrow$ $v_3 = 0.02064 - (-0.05142) = 0.07206$

The improvement indices in the First Iteration;

$x_{11} = 0.15020 - (0 + 0.15906) = -0.00886$

$x_{13} = 0.03305 - (0 + 0.07206) = -0.03901$

$x_{24} = 0.09274 - (-0.05142 + 0.09024) = 0.05392$

The two cells x_{11} and x_{13} indicate we have not reached the optimal solution, they can help reduce the total transportation cost, x_{13} with the highest negative Improvement Index value can reduce the cost more than the other one. In Table4.5.2, allocation is made to cell x_{13} adjusting to meet supply and demand constraints.

Table 4.8: Allocating to Cell x_{13}

To / From	Bu	Bo	Ak	Ku	Du	S (Lit)
TOR	0.15020	0.17975	0.03305 **959,678**	0.09024 **2,835,484**	0 **3,359,253**	7,154, 415
APD	0.10764 **1,370,968**	0.12833 **1,500,000**	0.02064 **1,620,967**	0.09274	0	4,491, 935
D (lit)	1,370,968	1,500,000	2,580,645	2,835,484	3,359,253	

Second Iteration;

New rows and new column values are calculated, u_1 is steel taken as zero (0),

$U_1 + V_3 = 0.03305$, but $u_1 = 0$, $\Rightarrow$ $0 + v_3 = 0.03305$ $\because$ $v_3 = 0.03305$

$U_1 + V_4 = 0.09024$, but $u_1 = 0$ $\Rightarrow$ $0 + v_4 = 0.09024$ $\because$ $v_4 = 0.09024$

$U_2 + V_3 = 0.02064$, $\Rightarrow$ $u_2 = 0.02064 – 0.03305 = - 0.01241$

$U_2 + V_1 = 0.10764$, but $u_2 = - 0.01241 \Rightarrow v_1 + (-0.01241) = 0.10764 \because$ $v_1 = 0.12005$

$U_2 + V_2 = 0.12833$, but $u_2 = - 0.01241 \Rightarrow$ $v_2 = 0.14074$

Calculating the improvement indices in the 2nd Iteration;

x_{11} = 0.15020 – (0 + 0.12005) = 0.0315

x_{12} = 0.17975 – (0 + 0.14074) = 0.0390

x_{24} = 0.09274 – (- 0.01241 + 0.09024) = 0.01491.

From the values of the Improvement Indices calculated, the transportation cost cannot be reduced any further because all the Improvement Indices values are positive which is an indication that the optimal solution has been reached.

Table 4.9: The final Transportation Table

From \ To	Bu	Bo	Ak	Ku	Du	S (Lit)
TOR	0.15020	0.17975	0.03305 **959,678**	0.09024 **2,835,484**	0 **3,359,253**	7,154, 415
APD	0.10764 **1,370,968**	0.12833 **1,500,000**	0.02064 **1,620,967**	0.09274	0	4,491, 935
D (lit)	1,370,968	1,500,000	2,580,645	2,835,484	3,359,253	

The final total minimum cost is, 0.10764 * 1,370,968 + 0.12833 * 1,500,000 + 0.02064 * 1,620,967 + 0.03305 * 959,678 + 0.09024 * 2,835,484 + 0 * 3,359,253

= GH₵ 661,114.19

4.7 Discussion of Solution

The total transportation cost for the current shipping schedule to the depots is GHC 804, 298.79 a day. After using the transportation algorithm; that is MCM for the basic feasible solution and MODI for the optimal solution, the cost has been reduced to GHC 661,114.19 a day which is a 17.8% reduction.

From table4.7.6, for the company to see reduction in total cost of transportations, Kumasi depot should be supplied by TOR only, Buipe and Bolgatanga depots should also take their delivery from APD and Akosombo should take 959,668 liters from TOR and 1,620,967 liters from APD.

The two basic feasible solution methods were used so as to ascertain which is best in terms of number of iterations before arriving at the BFS, the closeness of the BFS obtain to the optimal solution and also the reliability in general. It has been realized that for the NWCM, the placement of TOR and APD affect the BFS. That is when TOR is place in row one and APD in row two the BFS is different from When APD is place in row one and TOR in row two. This makes NWCM not reliable in this work hence MCM used. Again, improving the BFS of NWCM to optimal involve a lot of iterations because the BFS obtain is far from the optimal solution obtained.

CHAPTER FIVE

Conclusions and Recommendations

5.1 Conclusions

The Transportation Problem was used in addressing the transportation problem of BOST Company. The Minimum Cost Method was used to obtain the basic feasible solution to the transportation problem; the MODI method was used to improve the solution to optimal.

The outcome of the study suggest that for the cost of distributing petroleum products to the depots be minimized, Kumasi depot which is situated at Kaase should be supplied from TOR, Buipe and Bolgatanga should take their supply from APD and Akosombo should take 959,678 liters from TOR and 1,670,967 liters from APD. The total transportation cost for the current transportation schedule, is GHC 804298.79. When the transportation algorithm was used to network shipping activities; the total transportation cost became GHC 661,114.19 which represent 17.8% reduction in transportation cost.

5.2 Recommendations

The North-West Corner Method gave a BFS very far from what is obtained as the optimal solution and because of that four Iterations were used before obtaining the optimal solution while the Minimum Cost Method arrive at the same optimal solution after going through only two Iterations after obtaining the BFS.

Looking at the unit cost of transporting petroleum products from APD to Akosombo, it is suggested that the storage capacity of APD be increased so that Akosombo will be able to take all its supply from APD so that supply to the North through Akosombo will be continues. This will not only minimize the total cost further but also curtail the shortages of petroleum products being experienced mostly in the Northern part of the country.

The company with conjunction with the Government should take pragmatic decisions by constructing pipelines to connect all the current depots in the country and also build more depots so that cost of transportation may be reduced. This can help reduce the shortage of petroleum products and also reduce transportation cost. Again, measures should be taken to protect the Lake from drying up so that barges can be used to transport petroleum products from Akosombo to the Northern region specifically Buipe.

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APPENDICES

A. Table 1: Allocating to the Least cost cell

To / From	C_1	C_2	C_3	Supply	Row Penalty
M_1	6	8	10	150	2
M_2	7 **175**	11	11	~~175~~	4
M_3	4	5	12	275	1
Demand	~~200~~ 25	100	300	Total=600	
Column Penalty	2	3	1		

After calculating the penalties, allocation is made to the least cost cell in the highest penalty row. To satisfy the row two constraints, only `175` are allocated to that cell. Row two is crossed out and column one demand is adjusted.

In table `2`, the second column got the highest penalty value. Allocation is made to the least cost cell which is cell x_{32}. The demand for that row is met so column two is crossed out and row three is adjusted.

B. Table 2: Allocating to Least cost cell after the new penalties calculations.

From \ To	C_1	C_2	C_3	Supply	Row Penalty
M_1	6	8	10	150	2
M_2	7 **175**	11	11	~~175~~	
M_3	4	5 **100**	12	~~275~~ 175	1
Demand	~~200~~ 25	~~100~~	300	Total=600	
Column Penalty	2	3	2		

The next least cell allocated to is in cell x_{31}, which is in table3. The highest penalty is row three and cell x_{31} which contain the least cost is allocated to. The demand for column one is met so it's crossed out and row three is adjusted.

C. Table 3: Allocating to the next least cell after calculating new penalties.

From \ To	C_1	C_2	C_3	Supply	Row Penalty
M_1	6	8	10	150	4
M_2	7 **175**	11	11	~~175~~	
M_3	4 **25**	5 **100**	12	~~275~~ ~~175~~ 150	8
Demand	~~200~~ ~~25~~	~~100~~	300	Total=600	
Column Penalty	2		2		

In table4, it has been realized that column three is the only column with penalty value. In this case allocation is made depending on which cell could be supplied. Where there is an availability of supply, the cell corresponding to that supply is the one being allocated to. Cell x_{13} qualify and is supplied with everything.

D. Table 4: Allocating to the next least cell after calculating the next penalties.

To / From	C_1	C_2	C_3	Supply	Row Penalty
M_1	6	8	10 **150**	~~150~~	-
M_2	7 **175**	11	11	~~175~~	
M_3	4 **25**	5 **100**	12	~~275~~ ~~175~~ 150	-
Demand	~~200~~ ~~25~~	~~100~~	~~300~~ 150	Total=600	
Column Penalty			2		

The final transportation table indicated the basic variables or the cells that are allocated quantity of steel and how the supply values have been used and also how the demand values have been met.

E. Table 5: The final Transportation table with demand and supply cancelled.

To / From	C_1	C_2	C_3	Supply	Row Penalty
M_1	6	8	10 **150**	~~150~~	-
M_2	7 **175**	11	11	~~175~~	
M_3	4 **25**	5 **100**	12 **150**	~~275~~ ~~175~~ ~~150~~	-
Demand	~~200~~ ~~25~~	~~100~~	~~300~~ ~~150~~	Total=600	
Column Penalty			2		