Optimal Loan Portfolio

(Case Study: Prudential Bank Ltd And Asante Akyem Rural Bank)

By

Augusina Adu (B. Ed Mathematics)

PG3006509

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DECLARATION

I Augustina Adu hereby declare that except for reference to other people’s work, which have duly been cited. This submission is my own work towards the Master of Science degree and that, it contains no material previously published by another person nor presented elsewhere.

Augustina Adu, PG3006509	………….	………..
Student’s Name & ID	Signature	Date

Certified By …………………
……….

Dr. S. K. Amponsah	Signature	Date
Supervisor		

Certified By	…………………	……….
Dr. S. K. Amponsah	Signature	Date
(Head of Department)		

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DEDICATION

This research work is dedicated to my husband Mike Osei-Owusu and Children Ama Serwaa Owusu-Achiaw and Yaw Osei-Owusu. For their love, support and understanding through the study of this course.

ABSTRACT

The Banking Industry in Ghana is now characterized by increasing competition and innovation. This phenomena has led to most banks adopting cutting edge technology to improve the quality of their Loan structure . The decline of relevant portfolio planning models especially in Ghana is attributed mainly to the evolving dynamics of the Ghanaian banking industry where the regulatory controls have changed with a high frequency. A lot of banks had suffered substantial losses from a number of bad loans in their portfolio due to the models used in allocating funds to loans. As a result, most banks are not able to maximize their profit margin due to poor allocation of funds. The purpose of this Study is to develop a linear programming model using the Simplex algorithm to help Prudential Bank Limited and Asante Akyem Rural Bank to maximize their profit margin. The results from the model showed that, Prudential Bank and Asante Akyem Rural Bank would be making annual profit of GH¢8003572.5 and GH¢176750 respectively if they are to stick to the model. From the study, it was realized that the scientific method used to develop the propose model can have a dramatic increase in the two banks profit margin if put into practice.

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CHAPTER ONE

INTRODUCTION

The banking sector of Ghana in the past could be divided into two groups- the elite foreign banks which concentrated on the rich of the society and the local banks mainly owned by the state.The latter served the interest of most working class people. The elite banks were Barclays Bank (formerly called the Colonial Bank) and Standard Chartered Bank (formerly,Bank of British West Africa). The second group of banks with state ownership include Ghana Commercial bank (GCB), Social Security Bank (now SG-SSB), Agricultural Development Bank (ADB), and the National Investment Bank (NIB).

The clients of the localy owned banks found business transations very frustration espercally during salary payments, for example, it was not uncommon observing long winding queus extending serveral meters outside the banking hall. The few foreign banks on the other hand, apply high charges and the initial deposit to open accounts was very high. The average Ghanaian could therefore not open accounts with these banks. Choices were very few and competition was virtually absent in the sector.

The Bank of Ghana (BOG) with the support of government undertook a process of financial sector restructuring which transformed the financial sector. Some of the initiatives that led this transformation is the movement to universal banking, the adoption of an open licensing system and the modernization of the payments systems. According to Acquah (2006) the governor of the BOG,´universal banking involves the removal of restrictions on banking activities which allow banks to choose the type of banking services that they would like to offer in line with their capital, risk appetite and their business orientation'(2006).

Universal banking creates room for diversification in the range of financial services that a bank can provide thus also allowing them to spread their risks. Universal banking could as a result lead to increasing banking penetration, branch network expansion and also competition for deposits to the benefit of both savers and borrowers in the economy. Along with the universal banking, the central bank adopted an open but selective licensing policy, which allows for the entry of new banks. The addtion of new banks was expected to induce competition and will encourage faster modernization of banking operations and efficiency of the financial system industry as a whole. The expected increasing competion could put downward presssure on the tariffs banks charge for their services, lower lending rates and increase accesibility of credit.

These initiatives paid dividends by transforming the financial sector landscape in Ghana such that by the time of the redenomination, there were twenty-five (25) licensed banks operating in the sector. (The report of the Ghana banking survey, (2009). Pricewaterhousecoopers and Ghana Association of Bankers). The financial sector of Ghana is now characterished by increasing competition and innovation because most of the banks are employing cutting edge technology to improve the quality of their services and to roll out new products for their clients.

One of the new development in the Ghanaian banking sector is the entry into the sector by Nigerian banks.´Nigeria has one of the largest banking sector in Africa with over eighty banks in operation'(George, and Bob-Mills, 2007). The sector in Nigeria is one of the most competitive among emerging market countries and it is known for its innovation. According to the African Business Magazine (2006), Nigerian banks make up five (5) of the twenty (20) largest banks in sub-saharan Africa by capital. Nigerian banks as a result are bringing some of their strengths into the Ghanaian banking sector. Even thought these banks may have their own peculiar problems their coming into the sector have heightened competition.

Another development in the Ghanaian banking sector, which cannot be overlooked is the expansion in branch network of most of the banks. According to the past deputy governor of the Bank of Ghana, (Bawumia, 2006) bank branches in Ghana increased by 1.3 percent from three hundred and nine to three hundred and fourty four from 2002 to 2004 and eighty-one (81) new branches sprang up between 2004 and 2006. One interesting development in the sector is that there are no more ’elite banks in operation’ as the banks that formerly had this status are also chasing the average Ghanaian income eaner for his or her business together with the other banks.The economic lanscape in Ghana at the time of the redenomination was favourable for financial intermediation by the banks. The country has moved from an economic environment of generally high inflation and large exchange rate swings. Heavy domestic borrowing by government in the past had crowded out private sector finance. Faced with relatively low-risk, high return government debts in the form of treasury bills, the banks had little incentive to lend to the private sector which was riskier. The consequence was very limited access by small and medium-sized enterprise and individuals to credit. Thus the banks had relatively low capacity to lend to the private sector and manage its associated risks. However, the economic and financial sector reforms that was adopted had reversed this trend years before the redenomination was implemmented. Government reduced its borrowing in the domestic market reducing the return on government securities and thus banks were forced to lend more to the private sector.

1.1 BACKGROUND TO THE STUDY

There are a number of reasons why banks may suddenly stop or slow lending activity. This may be due to an anticipated decline in the value of the collateral used by the banks to secure the loans; an exogenous change in monetary conditions (for example, where the central bank suddenly and unexpectedly raises reserve requirements or imposes new regulatory constraints on lending); the central government imposing direct credit controls on the banking system; or even an increased perception of risk regarding the solvency of other banks within the banking system.The result of inefficences in the management of a banks loan portfolio may result in a credit crunch. A credit crunch is often caused by a sustained period of careless and inappropriate lending, which results in losses for lending institutions and investors in debt when the loans turn sour and the full extent of bad debts becomes known. These institutions may then reduce the availability of credit, and increase the cost of accessing credit by raising interest rates. In some cases lenders may be unable to lend further, even if they wish, as a result of earlier losses.The crunch is generally caused by a reduction in the market prices of previously "overinflated" assets and refers to the financial crisis that results from the price collapse. This can result in widespread foreclosure or bankruptcy for those investors and entrepeneurs who came in late to the market, as the prices of previously inflated assets generally drop precipitously. In contrast, a liquidity crisis is triggered when an otherwise sound business finds itself temporarily incapable of accessing the bridge finance it needs to expand its business or smooth its cash flow payments. In this case, accessing additional credit lines and "trading through" the crisis can allow the business to navigate its way through the problem and ensure its continued solvency and viability. It is often difficult to know, in the midst of a crisis, whether distressed businesses are experiencing a crisis of solvency or a temporary liquidity crisis.

Ghana has a well-developed banking system that was used extensively by previous governments to finance and develop the local economy in the areas of lending. By the late 1980s, the banks had suffered substantial losses from a number of bad loans in their portfolios. In addition, cedi depreciation had raised the banks' external liabilities. In order to strengthen the banking sector, the government in 1988 initiated comprehensive reforms. In particular, the amended banking law of August 1989 required banks to maintain a minimum capital base equivalent to six (6) percent of net assets adjusted for risk and to establish uniform accounting and auditing standards. The law also introduced limits on risk exposure to single borrowers and sectors. These measures strengthened central bank supervision, improved the regulatory framework, and gradually improved resource mobilization and credit allocation. In 1989 the Bank of Ghana issued temporary promissory notes to replace non-performing loans and other government-guaranteed obligations to state-owned enterprises as of the end of 1988 and on private-sector loans in 1989. The latter were then replaced by interest-bearing bonds from the Bank of Ghana or were offset against debts to the bank. Effectively, the government stepped in and repaid the loans. By late 1989, some ¢62 billion worth of non-performing assets had been offset or replaced by central bank bonds totaling about ¢47 billionAs part of the regulatory framework,the cental bank prescribed minimum capital requirements for three types of banks:

- Banks with at least 60% Ghanaian ownership (i.e. Ghanaian banking business) the minimum paid-up capital not less than twenty thousand Ghana cedis (GH¢20,000).
- Banks with Ghanaian ownership less than 60% (i.e.foreign bbanking buisness) the minimum paid-up capital not less than fifty thousand Ghana cedis (GH¢50,000).
- In the case of development banking business,the minimum paid-up capital was hundred thousand ghana cedis (Gh¢100,000).

The much higher minimum paid-up requirement for development banks is presumaably based on the concept that as these banks undertake medium and long-term lending,they are exposed to greater loan-loss risk.

1.2 STATEMENT OF THE PROBLEM

The decline of relevant portfolio planning models especially in Ghana is attributed mainly to the evolving dynamics of the Ghanaian banking industry where the regulatory controls have changed with a high frequency. Other contributory factors include the emergence of some unconventional assets as well as an increase in treasury and foreign exchange activities.

The changing face of the Ghanaian banking industry coupled with the need to sustain and improve Bank performance (especially at this critical period when many of them are financially distressed) necessitates that the suitability and continued relevance of existing models be evaluated.

According to Cohen and Hammer (1967), what makes the task of allocation/selection difficult is the need to find an appropriate balance between three desirable objectives in loan portfolio management-These objectives are; profitability, liquidity and safety. Generally, four major factors influence the asset portfolio management behavior of banks. These factors are government regulations, safety of deposits, credit demand as well as income aspirations of shareholders.

However, some finance experts - Melnik (1968), Anderson and Burger (1969), Bradley and Crane (1976), Sealey (1977), Reed et al., (1984) and Lambo (1986) have argued that the demand for the safety of deposits and the and the income expectations of shareholders are, to a large extent, incompatible. This incompatibility, they further contended, is reflected in the unavoidable trade-off between desired profitability, necessary liquidity and acceptable safety that is present in virtually every financial transaction of a bank.

Consequently, this trade-off between profitability, liquidity and safety could be regarded as the central issue in the management of banks' loan portfolio.

It is from the above observations that this research work is skewed at developing a linear model with the specific purpose of providing an optimal solution of allocating funds to the various types of loan of bank with a case study on Prudential Bank and Asante Akyem Rural Bank.

1.3 OBJECTIVES OF THE STUDY

It is an indisputable fact that most banks operating in Ghana today are faced with the complex problem of how to manage their loan portfolios in such a manner that the goals of the bank are best achieved. For this purpose, the general objective of the study is to:

(i) select optimum loan portfolio adhering to the regulations governing the activities of Prudential Bank and Asante Akyem Rural bank.

(ii) design a linear programming model to optimize the loan given out using the financial loan records of Prudential Bank and Asante Akyem Rural Bank

(iii) explore ways of disbursing funds allocated for loans effectively and efficiently in order to optimize profit margin of the two banks.

(iv) determine the sectors that records higher loan portfolio for the two banks.

(v) make recommendations that can address the issue of loan portfolio in the industry of banking in Ghana.

1.4 SIGNIFICANCE OF THE STUDY

This project seeks to access the performance of the loan portfolio of Prudential Bank and Asante Akyem Rural Bank on the community especially its customers. It is hoped that the model designed in the course of this study based on empirical evidence, would go a long way in providing useful planning tool to Banks operating in Ghana.

Suggestions and recommendations would be given to strengthen any weakness of the loan portfolios of the two bank in that it would be exposed in the course of the study.

1.5 METHODOLOGY

The model to be used by this study seeks to find an optimal way of allocating funds to the various loans types at Prudential Bank and Asante Akyem Rural Bank so as to maximised profit. The linear programming optimization process was used to setup the allocation of loans.

Linear programming models deals with optimization problems that can be modelled with a linear objective function subject to a set of linear constraints. The model has three basic components, that is the objective function which is to optimized (maximised or minimized), the constraints or limitation and the negativity constraint. Linear programming is most used among all the mathematical optimization techniques. It is best understood by both the elite and the ordinary business man.

A set of questionnaires were developed and administered to Prudential Bank and Asante Akyem Rural bank to obtain information on the various types of loan policies of they operate. The data obtained was first tabulated and used to develop a linear programming model which was then solved by the simplex method.

The simplex method passes from vertex to vertex on the boundary of the feasible polyhedron, repeatedly increasing the objective function until either an optimal solution is found or, it is established that no solution exists.

The simplex method was considered an appropriate method for solving the linear programming problem developed as a result of its practical superiority and advantages over the other methods.

Again, the simplex method was considered the most appropriate method for the study in view of the fact that many computer software application programs for solving linear programming problems involving simplex method are available.

The computerized software application program called Quantitative Methods (QM) model for windows based on the simplex algorithm was used to facilitate the solution of the linear programming model developed.

The Quantitative Methods (QM) model was considered the best option for the project because the spreadsheet offers a very convenient data entry and editing features which allows for a greater understanding of how to construct linear programs.

Again, the Quantitative Methods (QM) model for windows software application programme was selected and used among the numerous computer programmes in view of the fact that it is a popular programme used by the operational researchers.

1.6 SCOPE OF THE STUDY

The will cover the loan portfolio management policies Prudential Bank and Asante Akyem Rural Bank for the 2010 financial year.

1.7 LIMITATION OF THE STUDY

The constrains encountered include finance considerations, inaccessibility of data, limited time and unpreparedness and unreadiness of personnel to give out information necessary for the study.

1.8 ORGANISATION OF THE STUDY

The research is organized into five chapters. Chapter one which is the introduction, gives the background information of the study, statement of the problem, objectives of the study, research questions, and significance of the study, scope of the study, methodology description and organisation of the study.

Chapter two looks at review of related literature, which covers application of linear programming to portfolio selection, types of loan portfolio and risk associated with loans.

Chapter three describes the methodology used for the study. It looks at the method of data collection, organizational profiles of the selected banks for the research.

Chapter four discusses and analyses the data collected. Chapter five summarizes the various findings, conclusions and recommendations.

CHAPTER TWO

LITERATURE REVIEW

In every field of study, it is possible to look back and identify a person or event that caused a major change in the direction or development of the field. In the field of "investments" in general and "portfolio management" in particular, it is an indisputable fact that the work by Markowitz on Portfolio Theory changed the field more than any other single event

The doctoral thesis written by Markowitz (1952) at the University of Chicago dealt with portfolio selection and in it he developed the basic portfolio model.

Because of this work, Markowitz is often referred to as the "father of modern portfolio theory", and much subsequent research had been based on this effort (Sharpe, 1963, Fama, 1965 and Melnik, 1970).

The basic model, developed by Markowitz, derived the expected rate of return for a portfolio of assets and an expected risk measure. Markowitz showed that the variance of the rate of return was a meaningful measure of risk under a reasonable set of assumptions and derived the formula for computing the variance of the portfolio.

This portfolio variance formulation indicated the importance of diversification for reducing risk, and showed how to properly diversify. The Markowitz model is based on certain assumptions. Under these assumptions, a single asset or portfolio of assets is considered to be efficient if no other asset or portfolio of assets offers higher expected return with the same (or lower) risk, or lower risk with the same (or higher) expected return (Markowitz, 1952 and 1959)

2.1 THE EFFICIENT FRONTIER

The Full Variance Model developed by Markowitz is based on the assumption that the purpose of portfolio management is to minimize variance for every possible combination of the expected yield (Best and Grauer, 1991:980). It has been argued by Blume (1970) and Hodges and Brealey (1972) that the concept of the efficient frontier is basic to the understanding of portfolio theory. Assume that in the market place, there are a fixed number of common stocks in which a businessman can invest. Each of the securities has its own expected yield and standard deviation; others have the same standard deviation but vary in expected yield.

The investor will select the security that offers the highest yield for a chosen level of risk exposure as presented by the standard deviation. It is assumed that investors try to minimize risk by minimizing the deviation from the expected yield, and this is done by means of portfolio diversification.

2.2 LINEAR PROGRAMMING IN FINANCIAL MANAGEMENT

The use of linear and other types of mathematical programming techniques has received extensive coverage in the banking literature. Chambers and Chames (1961), as well as Cohen and Hammer (1967, 1972), developed a series of sophisticated linear programming models for managing the balance sheet of larger banks, while Waterman and Gee (1963) and Fortson and Dince (1977) proposed less elegant formulations which were better suited for the small-to medium-sized bank. Several programming models have also been proposed for managing a bank's investment security portfolio, including those by Booth (1972).

Baldirer et al., (1981) used linear programming model to solve fundamental issues facing senior bank management of Central Carolina Bank and Trust Company in structuring the bank's balance sheet of approximately $360 million.

2.3 LINEAR PROGRAMMING FOR BANK PORTFOLIO MANAGEMENT

Various portfolio theories have been propounded for the management of bank funds. Ronald I Robinson secondary reserves (1961) proposed four priorities of the use of banks funds. These include primary reserves, (or protective investment), loans and advances (customer credit demand) and investment account(open market investment for income) in descending order of priority. His assessment has been fully supported in other works by Sheng-Yi and Yong(1988).

A bank has to place primary reserves at the top of the priority in order to comply with the minimum legal requirement, to meet any immediate withdrawal demand by depositors and to provide a means of clearing cheques and credit obligations among banks.

Secondary reserves include cash items from banks, treasury bills and other short-term securities. Bank should have to satisfy customers' loan demand before allocating the balance of the funds in the investment market.

Loans and investment are in fact complementary. According to Robinson, (1961) investment should be tailored to the strength, seasonality and character of loan demand. He reiterated that banks that experience sharp seasonal fluctuations in loan demand need to maintain more liquidity in their investment programmme. Moreover, during a boom when loan demand is high and credit-worthy customers are available, banks should allocate more funds to loans and less funds to investment, and vice versa during recession when loan demand is low.

According to Robinson, (1961) the crucial banking problem is to resolve the conflict between safety and profitability in the employment of bank funds. The conflict is essentially the problem between liquidity and the size of the earning assets. Robinson suggested that where there is a conflict between safety and profitability, it is better to err on the side of safety.

The best practice is identifying procedures that can bring out the optimal mixture of management of banks funds. According to Tobin (1965) portfolio theory can be applied to bank portfolio management in that a bank would maximize the rates of return of its portfolio of assets, subject to the expected degree of risk and liquidity. Chambers and Charnes (1961) applied linear programming analysis on the consolidated balance sheets of commercial banks in Singapore for the period 1978-1983. The results show that that by a large banks do not try to maximize the returns of their portfolios, subject to legal, policy, bounding and total assets constraints, which denote riskiness and liquidity of the portfolio of assets. In a direct way, banks conform to the portfolio choice theory; they have to balance yield and liquidity against security. The pointed out that although the computer cannot replace a manager, linear programming can serve as a useful guide.

2.4 RISK MEASURES IN LOAN PORTFOLIO MANAGEMENT

Given that a bank wants to find the optimal way of funding its Loan portfolio without taking on too much risk, a good risk measure is essential. The risk measure must be easy for management to interpret and suitable to act as a part of an optimization problem.

2.4.1 Conditional Value at Risk

A popular and widely used risk measure is Value at Risk (VaR). VaR is a measure that is defined as the lowest amount ζ such that with probability α the loss will not exceed ζ during a specified time period. For example, if you choose the probability level α to be 0.95 and the time period to be one week, VaR states the maximum loss that you can expect over a one-week period with 95% certainty.

Value at Risk (VaR) has a role in the approach , but the emphasis is on Conditional Value at Risk (CVaR) which is known also as Mean Excess Loss, Mean Shortfall, or Tail VaR.

Conditional Value at Risk (CVaR) is defined as the conditional expected loss above VaR. by definition with respect to a specific probability β, the loss will not exceed α, whereas the β-CVaR is the conditional expectations of losses above the amount α. Three values of β are commonly considered: 0.90, 0.95, and 0.99. The definitions ensure that the β-CVaR is never more than the β-CVaR , so portfolios with low CVaR must have low VaR as well.

Rockafellar and Uryasev (1999) and Uryasev (2000) showed that VaR has undesirable mathematical features. For instance, VaR has a lack of subadditivity, resulting in the fact that the sum of the VaR of two different portfolios can be greater than the VaR of the combination of the two portfolios. Under most circumstances, the portfolios are not perfectly correlated and therefore the VaR of the combination of the two portfolios should not be greater than the sum of the individual portfolios' risk measures. In addition, Rockafellar and Uryasev (1999) clarify that it is problematic to optimize a problem where VaR is used as a risk measure. Difficulties arise, for example, from the fact that VaR then will be non-convex. Convexity is a key property in optimization since it assures that a local optimum is also a global optimum. The major drawback of using VaR as a risk measure is the fact that tail events are not considered. In other words, great losses that might be devastating for a company are not taken into account by using VaR as the risk measure of choice. For more information on the difficulties regarding VaR as a risk measure in an optimization problem, we refer to Rockafellar and Uryasev (1999).

According to Uryasev (2000), VaR can be restricted by constraining CVaR because of the fact that CVaR always will be greater than VaR. This means that portfolios with low CVaR also will have low VaR. Uryasev (2000) also shows how to optimize a problem with the CVaR risk measure as a constraint while calculating VaR at the same time. Since the Division has expressed that they want to know the VaR of the Combined portfolio, this is a very useful feature of the CVaR approach presented by Rockafellar and Uryasev (1999) and Uryasev (2000).

2.5 PROBABILITY OF LOSS ON LOAN PORTFOLIO

According to Klaus Rheinberger and Martin Summer in their credit risk portfolio models, three parameters drives loan losses: The probability of default by individual obligors (PD), the loss given default (LGD) and the exposure at default (EAD). While the standard credit risk models focus on modelling the PD for a given LGD, a growing recent literature has looked closer into the issue of explaining LGD and of exploring the consequences of dependencies between PD and LGD. This literature is surveyed in Altman et al; (2003). Most of the papers on the issue of dependency between PD and LGD have been written for US data and usually find strong correlations between these two variables. The first papers investigating the consequences of these dependencies for credit portfolio risk analysis were Frye (2000a) and Frye (2000b) using a credit risk model suggested by Finger [1999] and Gordy (2000). The authors used a different credit risk model in the tradition of actuarial portfolio loss models and focus directly on two risk factors: an aggregate PD and an aggregate API as well as their dependence. The authors used this approach because their interest was to investigate the implications of some stylized facts on asset prices and credit risk that have frequently been found in the macro economic literature for the risk of collateralized loan portfolios. The authors also believe that the credit risk model we use gives us maximal flexibility with assumptions about the distribution of systematic risk factors.

There are a variety of models that try to capture the dependence between PD and LGD. These models are developed in the papers of Jarrow (2001), Jokivuolle and Peura (2003), Carey and Gordy (2003), Hu and Perraudin (2002), Bakshi et al. (2001), G¨urtler and Heithecker (2005) and Altman et al., (2004). Most of these papers look at bond data but some also cover loans. There is a literature that looks in some detail into the determinants of LGD. Acharya et al., (2003) investigated defaulted bonds, Duellmann and Trapp (2004) look into recoveries of US corporate credit exposures, Grunert and Weber (2005) investigated recoveries of German bank loans and Schuermann (2004) summarizes existing knowledge about recoveries. While these papers show a nuanced picture of the determinants of recoveries that consists of many microeconomic and legal features such as the industry sector in which exposures are held or the seniority of a claim all papers find that macroeconomic conditions play a key role.

2.6 CREDIT RISK AND ASSET PRICES

2.6.1 Boom and Bust Cycles

The close relationship between macroeconomic cycles and boom and bust cycles in bank lending and asset prices has been described as a stylized fact by several authors dealing with financial stability. Two recent examples are Borio (2002) and Goodhart et al.,

(2004). Borio (2002) provided evidence about the cyclical co-movements between credit, asset prices and the macro-economy. Goodhart et al., (2004) analyze this dependency in the context of banking system liberalization and banking regulation during the last two decades. While these authors focus mainly on the past two decades, Bordo et al., (2001) point out that financial accelerator mechanisms and boom and bust cycles in a long term perspective were the rule rather than the exception.

When banking systems were liberalized after the break down of the Bretton Woods System in the early 1970s banks were suddenly confronted with volatile exchange rates and interest rates, tighter margins and increased competition from financial markets.

Due to this disintermediation process, banks often lost their biggest and safest borrowers in industry to the capital market. As a consequence banks began to increase lending to smaller and also riskier borrowers such as small and medium sized enterprises and persons. Banks also increasingly engaged in mortgage lending to households. Since such a larger and more dispersed pool of borrowers makes information acquisitions and monitoring more costly (compared to a small pool of large industry customers), the weight of collateralized lending increased. Goodhart et al., (2004) pointed out that this increasing weight of collateral as basis for bank lending automatically accentuates financial accelerator mechanisms described in the literature by Bernanke and Gilchrist (1999) or Kyotaki and Moore (1997).

Borio (2002) has described a stylized pattern of such an accelerator mechanism or a financial cycle as we have observed it repeatedly in the past. The buildup of imbalances that trigger a crisis usually starts with booming economic conditions. This boom is accompanied by a climate of overly optimistic risk assessment, the gradual weakening of financing and credit constraints and hiking asset prices (in particular property and real estate prices). In this climate financial and real imbalances are building up. At some point an essentially unpredictable trigger like an asset price drop or the interruption of an investment boom causes a sudden run down of financial buffers and once these buffers are exhausted and the contraction exceeds a certain threshold a full scale financial crisis occurs.

2.7 Data on Asset Prices and Default

Since loan quality and asset prices both depend on the general macroeconomic conditions, these variables tend to be highly correlated. For instance, in the terminology of quantitative risk management, probabilities of default (credit risk) of individual borrowers are high at the same time when asset prices (market risk) are depressed. The distinction between market and credit risk, which has been common standard in the regulatory and supervisory community, has frequently been criticized by economists in the past (see for instance Hellwig (1995). Academic research on quantitative risk management as well as risk management practitioners are currently undertaking substantial efforts to include these dependencies into their risk models and the integration of credit and market risk is an active field of research.

Following the work of Borio et al., (1994) the Bank for International Settlements (BIS) has constructed an aggregate API for several of the major industrial countries (Arthur (2004). The aggregate application programming interface (API) of the BIS is a geometric weighted average of equity, commercial and residential real estate, the most important asset classes used in collateralized bank lending. The weights represent estimates of the shares of those assets in the total private sector wealth. While the aggregate API provides only a broad brush perspective on the risk of collateral values – and therefore LGD in a collateralized loan portfolio – we think that these data provide an excellent starting point to explore the order of magnitude by which credit risk measures are underestimated when collateral values (an thus recovery rates) are taken to be fixed or independent from credit risk. According to Crouhy et al., (2000), this assumption is currently used in most of the standard portfolio credit risk models used in the banking industry.

2.8 STOCHASTIC PROGRAMMING

Stochastic programming is an approach intended for finding optimal solutions to problems including random variables such as interest and exchange rates. It is an approach that can be used for practical decision making under uncertainty. The solution should be derived with respect to the problem's objective function (the Division's preference functional), which will encompass a utility function, and given constraints regarding for example the amount of risk that the Division deems acceptable.

2.8.1 Expected Utility Theory

Gollier (2001) states that "Before addressing any decision problem under uncertainty, it is necessary to build a preference functional that evaluates the level of satisfaction of the decision maker who bears risk. If such a functional is obtained, decision problems can be solved by looking for the decision that maximizes the decision maker's level of satisfaction."

The maximization of a decision maker's satisfaction is one of the fundamental tenets of expected utility theory (EUT). According to Mongin (1997), EUT "states that the decision maker chooses between risky or uncertain prospects by comparing the expected utility values, i.e., the weighted sums obtained by adding the utility values of outcomes multiplied by their respective probabilities." By choosing the prospect with the greatest expected utility value, the decision maker maximizes his level of satisfaction.

Utility values are calculated with a utility function. Gustafsson and Salo (2004) explained the distinction between a preference functional and a utility function. They assert that a preference functional is $U[X] = E[u(X)]$ where u is the investor's utility function and X is an act that can result in many different outcomes. To each outcome, we can apply u and calculate the decision maker's utility associated with the outcome.

Kall and Wallace (1994) explained that attitudes towards risk can be characterized by utility functions. A utility function can be regarded as a function that describes one's happiness or utility from a certain wealth. The function is used in order to determine if one outcome is better or more preferable than another. We could e.g. choose to participate in a game where we can win or lose a certain amount of money with given probabilities and level of initial wealth. Given that we participate in the game, with a utility function we can quantify our satisfaction (utility) with each of the possible outcomes. With a preference functional, we can quantify our expected satisfaction from participating in the game. We can also choose not to participate in the game. A preference functional can then determine which of the two alternatives, to participate or not, is preferable.

From the perspective of an investor, the distinction between a preference functional and a utility function can be explained as follows. The preference functional lets the investor compare risky portfolios and rank them according to the degree of his satisfaction with the portfolios. In essence, the investor can compare portfolios head-to-head and decide which one is best. The utility function lets the investor compare different outcomes, given that he has already selected a portfolio. By changing the utility function, the investor can adapt it so that it mirrors his tolerance for losses.

Kall and Wallace (1994) show an example of a game where we can win or lose δw with equal probabilities (50%). The initial wealth is *w0* and it costs nothing to participate in the game. After the outcome of the game is known, we will have a wealth of ($w0 + \delta w$ or $w0 - \delta w$) depending on whether we win or lose. If we choose to maximize the expected value of the total wealth, we would consider the decision maker to be risk-neutral, meaning that the decision maker would accept to participate in a fair game. A fair game is one where the expected payoff is zero, as the case is for the game mentioned above.

2.9 MODELS OF LOAN PORTFOLIO MANAGEMENT

2.9.2 Multiple Discriminant Models

This model is statistical technique used to evaluate financial decisions that proposes a set of alternatives, such as different shares of stock in a portfolio. An analyst takes multiple factors into account, such as different financial ratios, when choosing between stocks in order to design an efficient portfolio. This model was proposed by Altman (1968). Banks and the corporate world nowadays use this model in predicting bankruptcy Halim (2008).

Previous bankruptcy research had identified many ratios that were important in predicting bankruptcy. However, there was no conclusive agreement of which ratios were most useful to assess the likelihood of failure. Altman (1993) noted that ratios measuring profitability, liquidity, solvency and cash flow were the most significant indicators of bankruptcy.

The development of bankruptcy prediction model started with the use of univariate analysis by Beaver (1966), followed by multivariate discriminant analysis (MDA) by Altman in 1968. Beaver's (1966) univariate analysis used individual financial ratios to predict distress. By using 79 failed and non-failed companies that were matched by industry and assets size in 1954 to 1964, his results from the prediction error tests suggested that cash flow to total debt, net income to total asset and total debt to total assets have the strongest ability to predict failure. These ratios differed from the MDA model proposed by Altman (1968). By utilizing 33 bankrupt companies and 33 nonbankrupt companies over the period 1946 to 1964, five variables were selected on the basis that they did the best overall job in predicting bankruptcy. These were working capital to total assets, retained earnings to total assets, earnings before interest and taxes to total assets, market value of equity to book value of total debt and sales to total assets. Z-Score was determined and those companies with a score greater than 2.99 fall into the non-bankrupt group, while those companies having a Z-Score below 1.81 were in the bankrupt group. The area between 1.81 and 2.99 is defined as the zone of ignorance or the gray area. The cut-off index that made the most accurate prediction of bankruptcy one year before filing for bankruptcy was 2.675. The MDA model was able to provide a high predictive accuracy of 95 % one year prior to failure.

2.9.3 Portfolio Manager by Moody's KMV

Portfolio Manager, which was the first portfolio credit risk model, was developed by KMV in 1993. It implements the Merton model in its commercial credit risk model for loan portfolios.

Based on the option price model, KMV estimates expected asset value and asset volatility as a function of the existing capital structure of the firm, equity value (or stock price), and its volatility. It measures the number of standard deviations between the mean of the future distribution of the asset and a critical threshold, the default point, which is called distance-to-default (DD). The probability of default (or expected default frequency, EDF) for each individual loan is directly calculated by the predetermined relationship between the distance-to-default and historical default or bankruptcy frequencies. The relationship[15] is developed from the database managed by KMV, which contains the firm's stock price and balance sheet.

To estimate credit risk at the portfolio level, Portfolio Manager uses asset return correlations between all pairs of obligors as a proxy of asset correlation, which takes the effect of portfolio diversification into account. It is derived from a multi-factor structural model[16] to avoid computational problems expected from a huge correlation matrix in a large loan portfolio. In the multi-factor model, asset return is assumed to be generated by systematic factors and idiosyncratic factors, and its correlations between two borrowers are only explained by the common systematic factors to all firms.

In the banking environment, KMV seeks to estimate an efficient frontier for loans and thus the optimal or best proportions (X_i) in which to hold loans made to different borrowers needs to determine and measure three things:

- the expected return on a loan to borrower i (R_i),
- the risk of a loan to borrower i (σ_i), and
- the correlation of default risks between loans made to borrowers i and j (ρ_{ij}).

KMV measures each of these as follows:

$R_i = AIS_i - E(L_i) = AIS_i - [EDF_i * LGD_i]$

$\sigma_i = UL_i = \sigma_{Di} * LGD_i = [EDF_i (1 - EDF_i)]^{1/2} * LGD_i$

where

AIS = All-in-spread = Annual fees earned on the loan plus The annual spread between the loan rate paid by the borrower and the FI's cost of funds - The expected loss on the loan [$E(L_i)$].

[$E(L_i)$] = The Expected Loss = (The expected probability of the borrower defaulting over the next year or its expected default frequency (EDF_i)) * (The amount lost by the FI if the borrower defaults [the loss given default or LGDi]).

Return on the Loan (Ri):

Measured by the so-called annual all-in-spread (AIS), which measures annual fees earned on the loan by the FI plus the annual spread between the loan rate paid by the borrower and the FI's cost of funds. Deducted from this is the expected loss on the loan [$E(L_i)$].

This expected loss [$E(L_i)$] is equal to the product of the expected probability of the borrower defaulting over the next year, or its expected default frequency (EDF_i) times the amount lost by the FI if the borrower defaults [the loss given default or LGDi].

Risk of the Loan (σ_i):

The risk of the loan reflects the volatility of the loan's default rate (σ_{Di}) around its expected value times the amount lost given default (LGDi).

The product of the volatility of the default rate and the LGD is called the unexpected loss on the loan (ULi) and is a measure of the loan's risk or σ_i.

To measure the volatility of the default rate, assume that loans can either default or repay (no default); then defaults are "binomially" distributed, and the standard deviation of the default rate for the ith borrower (σ_{Di}) is equal to the square root of the probability of default times 1 minus the probability of default $[(EDF) * (1-EDF)]^{1/2}$.

Correlation of Loan Defaults (ρ_{ij}):

To measure the unobservable default risk correlation between any two borrowers, the KMV Portfolio Manager model uses the systematic return components of the stock or equity returns of the two borrowers and calculates a correlation that is based on the historical co movement between those returns.

According to KMV, default correlations tend to be low and lie between .002 and .15. This makes intuitive sense. For example, what is the probability that both IBM and General Motors will go bankrupt at the same time? For both firms, their asset values would have to fall below their debt values at the same time over the next year!

A number of large banks are using the KMV model (and other similar models) to actively manage their loan portfolios. Nevertheless, some banks are reluctant to use such models if it involves selling or trading loans made to their long-term customers. In the view of some bankers, active portfolio management harms the long-term relationships bankers have built up with their customers. As a result, gains from diversification have to be offset against loss of reputation.

2.9.4 Loan Volume-Based Models

This model was designed by Saunders and Marcia Cornett and in their work they explained how this model work with the use of two(2) banks and a National Bank as benchmark as illustrated in Table 2.1

Table 2.1: Allocation of the Loan Portfolio to Different Sector

	National	Bank A	Bank B
Real estate	10%	15%	10%

C&I	60	75	25
Individuals	15	5	55
Others	15	5	10

To calculate the extent to which each bank deviates from the national benchmark, Saunders and Cornett used the standard deviation of bank A's and bank B's loan allocations from the National benchmark. They calculated the relative measure of loan allocation deviation as

$$\sigma j = \frac{[\Sigma (X_{ij} - X_i)^2]^{1/2}}{N}$$

TABLE 2.2: Measures of Loan Allocation Deviation from the National Benchmark Portfolio

	Bank A	Bank B
$(X_{1j} - X_1)^2$	$(.05)^2 = .0025$	$(0)^2 = 0$
$(X_{2j} - X_2)^2$	$(.15)^2 = .0225$	$(.05)^2 = .0025$
$(X_{3j} - X_3)^2$	$(-.10)^2 = .01$	$(.4)^2 = .16$
$(X_{4j} - X_4)^2$	$(-.10)^2 = .01$	$(-.05)^2 = .0025$
$\Sigma (X_{jj} - X_i)^2$	$\Sigma = .045$	$\Sigma = .285$

$\sigma_A = 10.61\%$ $\sigma_B = 26.69\%$

The results from the analysis after calculating the standard deviations of the two banks showed that;

- Bank B deviates significantly from the national benchmark due to its heavy concentration in individual loans.
- The standard deviation simply provides a manager with a measure of the degree to which a bank's loan portfolio composition deviates from the national average or benchmark.
- This partial use of modem portfolio theory provides an FI manager with a feel for the relative degree of loan concentration carried in the asset portfolio.

2.9.5 Loan Loss Ratio-Based Models

This model involves estimating the systematic loan loss risk of a particular sector relative to the loan loss risk of a bank's total loan portfolio. This systematic loan loss can be estimated by running a time series regression of quarterly losses of the ith sector's loss rate on the quarterly loss rate of a bank's total loans:

(Sectoral losses in the ith sector/Loans to the ith sector) = α + β (Total Loan Losses/Total Loans)

Where β measures the systematic loss sensitivity of the ith sector loans.

The implication of this model is that sectors with lower βs could have higher concentration limits than high β sectors--since low β loan sector risks (loan losses) are less systematic, that is, are more diversifiable in a portfolio

CHAPTER 3

METHODOLOGY

3.0 INTRODUCTION

This chapter takes a critical look at the methodology adopted for the study.We shall discuss Linear programming with particular emphases on the Simplex method.

3.1 LINEAR PROGRAMING

Linear programming, sometimes known as linear optimization, is the problem of maximizing or minimizing a linear function over a convex polyhedron specified by linear and non-negativity constraints. Simplistically, linear programming is the optimization of an outcome based on some set of constraints using a linear mathematical model.

A linear programming model helps the business community to maximize the profit by using the available resources or to minimize the cost of expenses. The linear programming model is designed as a model in the following ways:

- An objective function of linear function is created which is to be maximized or to be minimized.
- The objective function depends on certain constraints which will be represented in the form of inequalities. The constraints equations will be represented in "≤" for maximization model and for the minimization model it will have "≥".

A linear programming problem is one in which one finds the maximum or minimum value of a linear expression

$ax + by + cz + \ldots$

(called the objective function), subject to a number of linear constraints of the form

$Ax + By + Cz + \ldots \leq N$ for maximization problems.

or

$Ax + By + Cz + \ldots \geq N$ for minimization problems.

The largest or smallest value of the objective function is called the optimal value, and a collection of values of $x, y, z, \ldots$ that gives the optimal value constitutes an optimal solution. The variables $x, y, z, \ldots$ are called the decision variables.

The Objective Function is a linear function of variables which is to be optimized i.e., maximised or minimize. The objective function may be expressed as a linear expression. In other words, it represents the goal of the decision maker and should be related to the decision variables and constrains are expressions that combains variables to express limits on the possible solution.

Generally, there are there are four main steps that need to be followed in formulating a linear programming model.

STEP 1: Identify the decision variables and assign symbols x and y to them. These decision variables are those quantities whose values we wish to determine.

STEP 2: Identify the set of constraints and express them as linear equations/inequations in terms of the decision variables. These constraints are the given conditions.

STEP 3: Identify the objective function and express it as a linear function of decision variables. It might take the form of maximizing profit or production or minimizing cost.

STEP 4: Add the non-negativity restrictions on the decision variables, as in the physical problems, negative values of decision variables have no valid interpretation.

3.2 FORMS OF LINEAR PROGRAMMING PROBLEMS

A linear programing may take one of the following forms:

(i) Matrix form

(ii) General form and

(iii) Standard form

3.2.1 A Linear Programming in the Matrix Form

Linear programs are problems that can be expressed in canonical form as:

Maximize $c^T x$

Subject to $Ax \leq b$

Where x represents the vector of variables (to be determined), c and b are vectors of (known) coefficients and A is a (known) matrix of coefficients. The expression to be maximized or minimized is called the objective function ($c^T x$ in this case).

The inequalities $Ax \leq b$ are the constraints which specify a convex poly-tope over which the objective function is to be optimized.

A Linear programming model may simply be presented in the matrix vector form as;

Maximize (Minimize) $\quad c^T x$

Subject to: $\quad Ax \leq b, x \geq 0$

3.2.2 A Linear Program in the General Form

A linear programming in the general form may be presented as;

Maximize (Minimize) $\sum_{j=1}^{a} C_I X_I$

Subject to: $\sum_{i=1}^{n} X_I \leq b, \quad 1 \leq i \leq p$

$$\sum_{j=1}^{n} a_{ij} = b_i, \quad p+1 \leq i \leq k$$

$$\sum_{j=1}^{n} a_{ij} x_i \leq b, \quad k+1 \leq i \leq m$$

3.2.3 Linear Program in the Standard Form

Standard form is the usual and most intuitive form of describing a linear programming problem. It consists of the following four parts:

- A linear function to be maximized

  e.g., Maximize: $c_1 x_1 + c_2 x_2$

- Problem constraints of the following form

  e.g.,

$$a_{11} x_1 + a_{12} x_2 \leq b_1$$
$$a_{21} x_1 + a_{22} x_2 \leq b_2$$
$$a_{31} x_1 + a_{32} x_2 \leq b_3$$

- Non-negative variables

  e.g.,

  $x_1 \geq 0$

  $x_2 \geq 0$

- Non-negative right hand side constant
- $b_i \geq 0$

Other forms, such as minimization problems, problems with constraints on alternative forms, as well as problems involving negative variables can always be rewritten into an equivalent problem in standard form.

3.3 SIMPLEX ALGORITHM

The simplex algorithm, developed by George Dantzig in 1947, solves LP problems by constructing a feasible solution at a vertex of the polytope and then walking along a path on the edges of the polytope to vertices with non-decreasing values of the objective function until an optimum is reached. Many pivots are made with no increase in the objective function.

The simplex algorithm is quite efficient and has been proved to solve "random" problems efficiently.

To solve a standard maximization problem using the simplex method, take the following steps:

STEP 1: Convert to a system of equations by introducing slack variables to turn the constraints into equations, and rewriting the objective function in standard form.

STEP 2: Write down the initial tableau.

STEP 3: Select the pivot column: Choose the negative number with the largest magnitude in the bottom row (excluding the rightmost entry). Its column is the pivot column. (If there are two candidates, choose either one.) If all the numbers in the bottom row are zero or positive (excluding the rightmost entry), then you are done: the basic solution maximizes the objective function (see below for the basic solution).

STEP 4: Select the pivot in the pivot column: The pivot must always be a positive number. For each positive entry b in the pivot column, compute the ratio a/b, where a is the number in the Answer column in that row. Of these test ratios, choose the smallest one. The corresponding number b is the pivot.

STEP 5: Use the pivot to clear the column by using Gauss Elimination, and then relabel the pivot row with the label from the pivot column. The variable originally labeling the pivot row is the departing or exiting variable and the variable labeling the column is the entering variable.

STEP 6: Go to Step 3.

3.3.1 Basic Solution

To get the basic solution corresponding to any tableau in the simplex method, set to zero all variables that do not appear as row labels (these are the inactive variables).

The value of a variable that does appear as a row label (an active variable) is the number in the rightmost column in that row divided by the number in that row in the column labeled by the same variable.

3.3.2 Non-standard Constraints

To solve a linear programming problem with constraints of the form Ax + By + . . .≥ N with N positive, subtract a surplus variable from the left-hand side. The basic solution corresponding to the initial tableau will not be feasible since some of the active variables will be negative

To solve a minimization problem using the simplex method, convert it into a maximization problem. If you need to minimize c, instead maximize p = -c.

The terms used in the tableau are defined as follows:

c_j = Objective function coefficients for variable $_j$

b_i= Right-hand side coefficients (value) for constraint $_i$

a_{ij} = coefficients of variable $_j$ in constraint $_i$

c_b = Objective function coefficients of the basic variables.

From Table 3.1, the top row of the table presents the c_j, the objective function coefficients.

The next row gives the headings for the various columns which are then followed by constraints coefficients. The z_j row and the $(c_j - z_j)$ row which provides the current value of the objective function and the net contribution per unit of the jth variable respectively are presented.

The leftmost column in the tableau indicates the values of the objective function coefficients associated with the basic variable, with a set of constraints.

Table 3.1 The simplex tableau

			Decision variables				Slack variables					
C_j		C_1	C_2	……	c_n		0	0	…….	0	Solution (n)	Objective function coefficients
c_B	Basic variable	X_1	X_2	….	x_n		1		….	s_m		Headings
0	S_1	$a_{1\,1}$	$a_{1\,2}$	….	$a_{1\,n}$		1	0	….	0		
…	S_2	$a_{2\,1}$	$a_{2\,2}$	….	$a_{2\,n}$		0	1	….	0		
0	…….	….	…..	…..	……		…..	…..	…..	…..		
	s_m	$a_{m\,1}$	$a_{m\,2}$	…..	$a_{m\,n}$		0	0	….	1		
	Z_j	Z_1	Z_2	….	$Z_{m\,n}$	…	$Z_{1\,1}$	$Z_{1\,2}$	….	$Z_{1\,m}$	Current value of objective function	
	C_j- Z_j	C_1- Z_1	C_2- Z_2	….	C_n- $Z_{m\,n}$	… .	$C_{1\ 1}$- Z_{11}	$C_{1\,2}$- Z_{12}	..	C_{1N}- Z_{1N}		Reduced cost(net contribution /unit)

From Table 3.1, the top row of the table presents the c_j, the objective function coefficients.

The next row gives the headings for the various columns which are then followed by constraints coefficients. The z_j row and the $(c_j - z_j)$ row provides the current value of the objective function and the net contribution per unit of the jth variable respectively are presented.

The leftmost column in the tableau indicates the values of the objective function coefficients associated with the basic variable, with a set of constraints.

3.4 TYPES OF SIMPLEX METHOD SOLUTIONS

The simplex method will always terminate in a finite number of steps with an indication that a unique optimal solution has been obtained or that one of three special cases has occurred. These special cases are:

(i) Alternative optimal solutions

(ii) Unbounded solutions

(iii)Infeasible solutions

3.4.1 Alernatve Optmal Solutions

The simplex method provides a clear indication of the presence of alternative or multiple, optimal solutions upon its termination. These alternative optimal solutions can be recognized by considering the $(c_j - z_j)$ row. Assume that we are maximizing and remember that when all

$(c_j - z_j)$ values are all negative, we know that an optimal solution has been obtained. Now, the presence of an alternative optimal solution will be indicated by the fact that for some variable not in the basis, the corresponding $(c_j - z_j)$ value will equal zero.

Thus, this variable can be entered into the basis, the appropriate variable can be removed from the basis, and the value of the objective function will not change. In this manner, the various alternative optimal solutions can be determined.

3.4.2 Unbouded Solutions

In the case of an unbounded solution, the simplex method will terminate with the indication that the entering basic variable can do so only if it is allowed to assume a value of infinity. Specifically, for a maximization problem we will encounter a simplex tableau having a non basic variable whose $(c_j - z_j)$ row value is strictly greater than zero.And for this same variable all of the a. elements in its column will be zero or negative value (i.e. every coefficient in the pivot column will be either negative or zero). Thus, in performing the ratio test for the variable removal criterion, it will be possible only to form ratios having negative numbers or zeros as denominators. Negative numbers in the denominators cannot be considered since this will result in the introduction of a basic variable at a negative level Zeros in the denominator will produce a ratio having an undefined value and would indicate that the entering basic variable should be increased indefinitely (i.e. infinitely) without any of the current basic variables being driven from the basis.

Therefore, if we have an unbounded solution, none of the current basic variables can be driven from solution by the introduction of a new basic variable, even if that new basic variable assumes an infinitely large value.

Generally, arriving at an unbounded solution indicates that the problem was originally misformulated within the constraint set and needs reformulation.

3.4.3 Infesasble Solution

An indication that no feasible solution is possible will be given by the fact that at least one of the artificial variables, which should be driven to zero by the simplex method will be present as a positive basic variable in the solution that appears to be optimal. For example, assuming one wish to solve a maximization problem in which artificial variables are required. Then, at some iteration one achieve a solution in which all the $(c_j - z_j)$ values are zero or negative, but which has one or more artificial variables as positive basic variables.

When an infeasible solution is indicated the management science analyst should carefully reconsider the construction of the model, because the model is either improperly formulated or two or more of the constraints are incompatible.

Reformulation of the model is mandatory for cases in which the no feasible solution condition is indicated. consider the following linear programming problem as an example of simplex algorithm.

The slack variables form the initial solution mix. The initial solution assumes that all available hours are unused i.e. the slack variables take the largest possible values. Variables in the solution mix are called basic variables. Each basic variable has a column consisting of all 0's except for a single 1. All variables not in the solution mix take the value 0.

The simplex method uses a four step process (based on the Gauss Jordan method for solving a system of linear equations) to go from one tableau or vertex to the next. In this process, a basic variable in the solution mix is replaced by another variable previously not in the solution mix. The value of the replaced variable is set to 0.

3.5 DUALITY

Every linear programming problem, referred to as a primal problem, can be converted into a dual problem, which provides an upper bound to the optimal value of the primal problem. In matrix form, we can express the primal problem as:

Maximize $c^T x$ subject to $Ax \leq b$, $x \geq 0$;

There are two ideas fundamental to duality theory.

- The dual of a dual linear program is the original primal linear program.
- Every feasible solution for a linear program gives a bound on the optimal value of the objective function of its dual. The duality theorem states that if the primal has an

optimal solution, x^*, then the dual also has an optimal solution, y^*, such that

$c^Tx^*=b^Ty^*$.

Duality theory states that if the primal is unbounded then the dual is infeasible. Likewise, if the dual is unbounded, then the primal must be infeasible. However, it is possible for both the dual and the primal to be infeasible .

3.6 SUMMARY

This chapter discussed the Simplex method and its variants. In the next chapter, we shall put forward the data collected and its Analysis.

CHAPTER 4

DATA COLLECTION AND ANALYSIS

4.0 INTRODUCTION

In this chapter we shall analyze the data taken from Prudential Bank Limited and Asante Akyem Rural Bank. A model is proposed and solved to help these two banks maximise its profit.

Prudential Bank Limited (PBL) was incorporated in May 1993. The bank was opened to the public for business on August 15, 1996. PBL is a commercial/development bank with a strategic focus on the development and financing of industry and export. The bank currently has 10 branches in Ghana with 4 correspondent banks outside Ghana. The products and services of PBL include: Domestic Banking Services, International Banking Services, Project Financing, Export Development, Funds management and Cash Collection Services.

Prudential Bank Limited is in the process of formulating a loan policy involving GH¢ 406,491,393 for the year 2011. Being a full-service facility .The bank is obligated to grant loans to different clientele.

Table 4.1 provides the type of loans, the interest rate charged by the bank and the probability of bad debt as estimated from past experience.

Table 4.1: Loans available to the Prudential Bank Limited.

Type of loan	Interest rate	Probability of bad debt
Export	0.28	0.02
Industry or manufactory	0.30	0.12
Agriculture	0.31	0.2
Commence	0.31	0.1
Construction	0.32	0.1
Consumer	0.32	0.2
Fuel dealers	0.28	0.1

Bad debts are assumed unrecoverable and hence produce no interest revenue. For policy reasons, there are limits on how the bank allocates its funds. Competition with other financial institutions in the city requires that the bank

- Allocate at least 40% of the total funds to consumer loans and Industry loans.
- To assist agriculture production i n the region, agriculture loans must at least be greater than 50% of Export, Industry and fuel dealers loans
- The sum of consumer loans and construction loans must be at least greater than 40% of Export, commence and fuel dealers loans
- The sum of consumer loans and agriculture loans must be at least 25% of the total funds
- The bank also stated that the total ratio for bad debt on all loans must not exceed 0.08.

4.1 PROPOSED MODEL PRUDENTIAL BANK LIMITED

The variables of the model can be defined as follows:

x_1= Export loans (in thousands of cedis)

x_2=Industry or Manufactory loans

x_3=Agriculture loans

x_4= commence

x_5= consumer loans

x_6= Construction loans

x_7= Fuel Dealers Loans

The objective of the Prudential Bank is to maximize its net returns comprised of the difference between the revenue from interest and lost funds due to bad debts.

Objective function:

Maximize Z = 0.28(0.98x_1) + 0.30(0.88x_2) + 0.31(0.80x_3) + 0.31(0.90x_4) +

0.32(0.90x_5) + 0.32(0.80x_6) + 0.28(0.90x_7) – 0.02x_1 – 0.12x_2 –0.2x_3 – 0.1x_4 –

0.1x_5 – 0.2x_6 – 0.1x_7

This function simplifies to

Maximize Z = 0.2544x_1 + 0.144x_2 + 0.048x_3 + 0.179x_4 +0.188x_5 +0.056x_6 +0.152x_7

The problem has seven constrains:

- Limits on total funds available

  $x_1+x_2+x_3+x_4+x_5+x_6+x_7 \leq$ 406,491,393

- Limits on industry, commence and consumer loans

$x_2 + x_4 + x_6 \geq 203245696.5$

- Limits on Agriculture loan compared to Export, industry and fuel dealers loan

  $x_1 \geq 0.5(x_2+x_7)$

  $x_1 - 0.5x_2 - 0.5x_7 \geq 0$

- Limits on consumer loans and construction loans compared to Export, commence and loan fuel dealer loans

$x_5+x_6 \geq 0.4(x_4+x_7)$

$-0.4x_4 + x_5 + x_6 - 0.4x_7 \geq 0$

- Limits on consumer and commence loans

$x_4+x_6 \geq 0.25(406491393)$

$x_4 + x_6 \geq 101622848.25$

- Limits on fuel dealers and industry or Manufactory loans

$x_2 + x_7 \geq 0.29(406491393)$

$x_2 + x_7 \geq 117882503.97000$

Limits on bad debts

$$\frac{0.02x_1 + 0.12x_2 + 0.2x_3 + 0.1x_4 + 0.1x_5 + 0.2x_6 + 0.1x_7}{x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7} \leq 0.08$$

Or

$-0.06x_1 + 0.04x_2 + 0.12x_3 + 0.02x_4 + 0.02x_5 + 0.12x_6 + 0.02x_7 \leq 0$

8. Non-negativity

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0, x_5 \geq 0, x_6 \geq 0, x_7 \geq 0$$

The Asante-Akyem Rural Bank . Juansa in the Asnate Akyem District give loans to the active-rural poor to help reduce poverty and improve living standards of the rural folks. Asante Akyem Rural Bank is in the process of formulating a loan policy involving GH¢ 700,000 the year 2011.Being a full-service facility the bank is obligated to grant loans to different clientele.

Table 4.2 depicts the type of loans, the interest rate charged by the bank and the probability of bad debt as estimated from past experience.

Table 4.2: Loans available to the Asante Akyem Rural bank

Type of loan	Interest rate	Probability of bad debt
Salary Loan	0.28	0.01
Trading and Transport Loans	0.30	0.1
Agriculture	0.30	0.15
Micro Finance Loans	0.32	0.15
Susu Loans	0.30	0.1

Adwuma Nkosoo	0.30	0.5
Trading Overdrafts Loans	0.30	0.1
Salary Overdraft	0.31	0.01

Bad debts are assumed unrecoverable and hence produce no interest revenue. For policy reasons, there are limits on how the bank allocates its funds. Competition with other financial institutions in the city requires that the bank

- Allocate at least 50% of the total funds to Salary loans and Micro Finance loans.
- To assist agriculture production in the region, agriculture loans must at least be 5% of The total fund.
- The sum of Susu loans and Adwuma Nkosoo loans must be at least 25% of Micro Finance loans and Salary Overdraft loans
- The sum of Trading and Transport loans and agriculture loans must be at least 25% of the total funds
- The bank also stated that the total ratio for bad debt on all loans must not exceed 0.05.

4.2 PROPOSED MODEL FOR ASANTE AKYEM RURAL BANK

The variables of the model can be defined as follows:

x_1= Salary loans (in thousands of cedis)

x_2=Trading and Transport loans

x_3=Agriculture loans

x_4= Micro Finance Loans

x_4= Micro Finance Loans

x_5= Susu loans

x_6= Adwuma Nkosoo loans

x_7= Trading Overdraft Loans

x_8 = Salary Overdraft Loans

The objective of the Asante Akyem Rural Bank is to maximize its net returns comprised of the difference between the revenue from interest and lost funds due to bad debts.

Objective function:

Maximize Z = $0.28(0.99x_1) + 0.30(0.90x) + 0.30(0.85x) + 0.32(0.85x) +$

$0.30(0.90x_5) + 0.30(0.80x_6) + 0.30(0.90x_7) + 0.31(0.99x_8) - 0.01x_1 - 0.1x_2$

$- 0.15x_3 - 0.15x_4 - 0.1x_5 - 0.2x_6 - 0.1x_7 - 0.01x_8$

This function simplifies to

Maximize Z=0.2672x_1 + 0.17x_2 + 0.105x_3 + 0.122x_4+0.17x_5+0.04x_6 +0.17x_7+0.3x_8

The problem has seven constrains:

1. Limits on total funds available

   $x_1+x_2+x_3+x_4+x_5+x_6+x_7+x_8 \leq 700{,}000$

2. Limits on Salary and micro finance loans

   $x_1+x_4 \geq 0.5\times700{,}000$

   $x_1+x_4 \geq 350000$

3. Limits on Agriculture

   $x_1 \geq 0.05(700{,}000)$

   $x_1 \geq 35000$

4. Limits on Susu loans and Adwuma Nkosoo loans compared to Micro Finance Loan and Salary Overdraft loans

   $x_5+x_6 \geq 0.25(x_4+x_8)$

$$-0.25x_4 + x_5 + x_6 - 0.25x_8 \geq 0$$

5. Limits on Trading and Overdrafts Loans and Susu Loans

$x_5 + x_7 \geq 0.25(700000)$

$x_5 + x_5 \geq 175000$

6.Limits on bad debts

$$\frac{0.01x_1 + 0.1x_2 + 0.15x_3 + 0.15x_4 + 0.1x_5 + 0.2x_6 + 0.1x_7 + 0.01x_8}{x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8} \leq 0.05$$

Or

$$-0.04x_1 + 0.05x_2 + 0.1x_3 + 0.1x_4 + 0.05x_5 + 0.15x_6 + 0.05x_7 - 0.04x_8 \leq 0$$

7. Non-negativity

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0, x_5 \geq 0, x_6 \geq 0, x_7 \geq 0, x_8 \geq 0$$

The allocation of funds to the various clientele as formulated above was transferred onto the Quantitative Methods (QM) for windows model by first selecting the linear programming option from the module, create data set was used to select the number of variables involve in each case and the number of constraints.

Solve problem icon was used to display the problem results including linear programming results, ranging, solution list, as well as the iterations.The results given by the (QM) is surmrised below.

4.3 OPTIMAL SULUTION FOR PRUDENTIAL BANK LIMITED

Optimal Value (Z)=80035725.5

Variable	Status	Value
x_1	Basic	146337000
x_2	Basic	117882500
x_3	NONBasic	0
x_4	Basic	101622800
x_5	Basic	40649120
x_6	NONBasic	0
x_7	NONBasic	0
slack 1	NONBasic	0
surplus 2	Basic	1625960
surplus 3	Basic	87395740
surplus 4	NONBasic	0

surplus 5	NONBasic	0
surplus 6	NONBasic	0
slack 7	Basic	1219481

Variable	Value	Reduced Cost
x_1	146337000.	0
x_2	117882500.	0
x_3	0.	0.206
x_4	101622800.	0
x_5	40649120.	0
x_6	0.	0.03
x_7	0.	0.0184

Constraint	Dual Value	Slack/Surplus
Constraint 1	.2544	0
Constraint 2	0	16259600
Constraint 3	0	87395740
Constraint 4	-0.0664	0
Constraint 5	-.10196	0
Constraint 6	-.1104	0
Constraint 7	0	1219481

4.5 OPTIMAL SULUTION FOR ASANTE AKYEM RURAL BANK

Optimal Value (Z) = 176750

Variable	Status	Value
x_1	Basic	350000
x_2	NONBasic	0
x_3	NONBasic	0
x_4	NONBasic	0
x_5	Basic	43750
x_6	NONBasic	0
x_7	Basic	131250
x_8	Basic	175000
slack 1	NONBasic	0
surplus 2	NONBasic	0
surplus 3	Basic	284375
surplus 4	NONBasic	0
surplus 5	NONBasic	0
slack 6	Basic	12250

Variable	Value	Reduced Cost
x_1	350000.	0
x_2	0.	0.13
x_3	0.	0.19
x_4	0.	0.15
x_5	43750.	0
x_6	0.	0.26

x_7	131250.	0
x_8	175000.	0
Constraint	Dual Value	Slack/Surplus
Constraint 1	.3	0
Constraint 2	-.03	0
Constraint 3	0	284375
Constraint 4	0	0.0011
Constraint 5	-.13	0
Constraint 6	0	12250

4.5 SUMMARY

In this chapter data collected from Prudential Bank and Asante Akyem Rural Bank were used to formulate the proposed model. The next chapter presents the summary, conclusion and recommendation for the study.

CHAPTER FIVE

SUMMARY,CONCLUSIONS AND RECOMMENDATIONS

5.0 INTRODUCTION

This chapter presents the summary, conclusions drawn from the study and make recommendations to help Prudential Bank and Asante Akyem Rural Bank to optimize their profit margin.

5.1 SUMMARY OF RESULTS (PRUDENTIAL BANK LIMITED ALLOCATION)

The problem went through ten iterations before an optimal solution was found.The reduced cost for $x_1, x_2, x_4, and\ x_5$ are zero. Since $x_1, x_2, x_4, and\ x_5$ are at basic at the

optimum, The reduced cost for $x_3, x_6\ and\ x_7\ are$ 0.206,0.03 and 0.0184 respectively are non-basic at the optimum.the dual prices for constraints (1), (4), (5) and (6) are nonzero at the optmum because they correspond to the four constraints at optmum,hence their slack variables are nonbasic.the Optimal solution gives the optimal Value (Z)=GH¢80035725.5 which occures at x_1=GH¢146337000, x_2=GH¢ 117882500, $x_4 = GH¢$101622800 and x_5 = GH¢40649120. From the results, the value of $x_3, x_6 and x_7$ *are* zero at optimum because theprobability of debt (see Table 4.1) of these these three variables are high and hence they do not make any returns.

Refer to appendix B, C, D, Eand F for tables showing the solution list, ranging values, linear programming results and the iterations as displayed by the QM for windows model

5.2 SUMMARY OF RESULTS (ASANTE AKYEM RURAL BANK LOAN ALLOCATION)

The problem went through nine iteration before an optimal solution was reached. The variables x_1, x_5, x_7 *and* x_8 were in basic at optimum and had their reduced cost to be zeros. The variables x_2, x_3, x_4 *and* x_6 were nonbasic variables and had their reduced cost to be nonzero. The optimal value (Z) = GH¢176750 occurs at x_1=GH¢350000, x_5= GH¢43750, x_7= GH¢131250, and x_8= GH¢175000.With the values of x_2, x_3, x_4 *and* x_6 being zero. Since these variables high probability of bad debt (as shown in table 4.2)

Refer to appendix G,H,I,J and K for tables showing the solution list, ranging values, linear programming results and the iterations as displayed by the QM for windows model.

From the results given by the (QM) on the allocation of funds to the various loan types by Prudential Bank and Asante Akyem Rural Bank , it could be seen in general that loan types that has high probability of bad debt do not yield any returns.

5.3 CONCLUSIONS

The results from the study has shown that most banks in the country do not have any scientific method to give out loans. As a result of this most banks are unable to optimize their profit margins. The model propose for Prudential bank and Asante Akyem Rural Bank would help these two banks to disburse their available funds for loans more effectively and profitably. The results from the model showed that, Prudential Bank and Asante Akyem Rural would be making annual profit of GH¢176750 and GH¢80035725.5 respectively if they are to stick to the model. Hence we conclude that the scientific method we used to developed the propose model can have a dramatic increase in the two banks profit margin if put into practise.

5.4 RECOMMENDATIONS

It was realised from the conclusion that the use of scientific methods to give out loans help banks to avoid giving out loans that do not yield profit there by allocating funds to areas they are sure to get returns. We therefore recommend Prudential Bank and Asante Akyem Rural Bank to adapt this model in their allocation of funds for loans.

We also recommend that Banks and other financial institutions be educated to employ scientific methods such as the use of mathematical models to help them disburse funds of the bank more effectively and profitably.

We again recommend the use of other Mathematical tools to do further research into this wor

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APPENDIX A

QUESTIONNAIRES

The questionnaire used to gather information on allocation of funds to loan types from Prudential Bank Limited and Asante Akyem Rural Bank.

You are kindly requested to complete this questionnaire as frankly as possible; your response will be kept confidential and shall be used only for this thesis.

1) What is the name of your bank?

……………………………………………………………………………………………

2) What are the various types of loans your bank give to its customers?

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3) What percentage do you bank charge on each loan above as interest?

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4) What is the probability of bad debt on each of the loans stated above?

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5) State the total amount of money given out as loans.

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6) Is there any condition under which loan is given out to customers?

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APPENDIX B

PRUDENTIAL BANK LOAN ALLOCATION PROBLEM									
	X1	X2	X3	X4	X5	X6	X7		RHS
Maximize	0.2544	0.144	0.048	0.179	0.188	0.056	0.152		
Constraint 1	1.	1.	1.	1.	1.	1.	1.	<=	406,491,400
Constraint 2	0.	1.	0.	1.	0.	1.	0.	>=	203,245,700
Constraint 3	1.	-0.5	0.	0.	0.	0.	-0.5	>=	0
Constraint 4	0.	0.	0.	-0.4	1.	1.	-0.4	>=	0
Constraint 5	0.	0.	0.	1.	0.	1.	0.	>=	101,622,800
Constraint 6	0.	1.	0.	0.	0.	0.	1.	>=	117,882,500
Constraint 7	-0.06	0.04	0.12	0.02	0.02	0.12	0.02	<=	0

Created by QM for Windows

APPENDIX C

SOLUTION TO PRUDENTIAL BANK LOAN ALLOCATION

Variable	Status	Value
X1	Basic	146,337,000.
X2	Basic	117,882,500.
X3	NONBasic	0.
X4	Basic	101,622,800.
X5	Basic	40,649,120.
X6	NONBasic	0.
X7	NONBasic	0.
slack 1	NONBasic	0.
surplus 2	Basic	16,259,600.
surplus 3	Basic	87,395,740.
surplus 4	NONBasic	0.
surplus 5	NONBasic	0.
surplus 6	NONBasic	0.
slack 7	Basic	1,219,481.
Optimal Value (Z)		80,035,730.

Created by QM for Windows

APPENDIX D

SOLUTION TO PRUDENTIA BANK LOAN ALLOCATION

Variable	Value	Reduced Cost	Original Val	Lower Bound	Upper Bound
X1	146,337,000.	0.	0.25	0.21	0.28
X2	117,882,500.	0.	0.14	0.13	0.25
X3	0.	0.2064	0.05	-Infinity	0.25
X4	101,622,800.	0.	0.18	0.15	0.28
X5	40,649,120.	0.	0.19	0.17	0.23
X6	0.	0.03	0.06	-Infinity	0.09
X7	0.	0.0186	0.15	-Infinity	0.17
Constraint	Dual Value	Slack/Surplus	Original Val	Lower Bound	Upper Bound
Constraint 1	0.2544	0.	406,491,400.	386,166,700.	Infinity
Constraint 2	0.	16,259,600.	203,245,700.	-Infinity	Infinity
Constraint 3	0.	87,395,740.	0.	-Infinity	Infinity
Constraint 4	-0.0664	0.	0.	-Infinity	Infinity
Constraint 5	-0.102	0.	101,622,800.	85,363,200.	Infinity
Constraint 6	-0.1104	0.	117,882,500.	101,622,900.	Infinity
Constraint 7	0.	1,219,481.	0.	-Infinity	Infinity

Created by QM for Windows

APPENDIX E

LINEAR PROGRAMMING RESULTS

SOLUTION TO PRUDENTIAL BANK LOAN ALLOCATION PROBLEM										
	X1	X2	X3	X4	X5	X6	X7		RHS	Dual
Maximize	0.2544	0.144	0.048	0.179	0.188	0.056	0.152			
Constraint 1	1.	1.	1.	1.	1.	1.	1.	<=	406,491,400.	0.2544
Constraint 2	0.	1.	0.	1.	0.	1.	0.	>=	203,245,700.	0.
Constraint 3	1.	-0.5	0.	0.	0.	0.	-0.5	>=	0.	0.
Constraint 4	0.	0.	0.	-0.4	1.	1.	-0.4	>=	0.	-0.0664
Constraint 5	0.	0.	0.	1.	0.	1.	0.	>=	101,622,800.	-0.102
Constraint 6	0.	1.	0.	0.	0.	0.	1.	>=	117,882,500.	-0.1104
Constraint 7	-0.06	0.04	0.12	0.02	0.02	0.12	0.02	<=	0.	0.
Solution ->	146,337,000.	117,882,500.	0.	101,622,800.	40,649,120.	0.	0.		80,035,725.59	

Created by QM for Windows

APPENDIX F

ITERATIONS OF PRUDENTIAL BANK LOAN ALLOCATION

Cj	Basic Variables	.2544 X1	.144 X2	.048 X3	.179 X4	.188 X5	.056 X6	.152 X7	0 slack 1	0 artfcl 2	0 surplus 2	0 artfcl 3	0 surplus 3	0 artfcl 4	0 surplus 4	0 artfcl 5	0 surplus 5	0 artfcl 6	0 surplus 6	0 slack 7	Quantity
Iteration 1																					
0	slack 1	1.	1.	1.	1.	1.	1.	1.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	406,491,392.
0	artfcl 2	0.	1.	0.	1.	0.	1.	0.	0.	1.	-1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	203,245,696.
0	artfcl 3	1.	-0.5	0.	0.	0.	0.	-0.5	0.	0.	0.	1.	-1.	0.	0.	0.	0.	0.	0.	0.	0.
0	artfcl 4	0.	0.	0.	-0.4	1.	1.	-0.4	0.	0.	0.	0.	0.	1.	-1.	0.	0.	0.	0.	0.	0.
0	artfcl 5	0.	0.	0.	1.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	1.	-1.	0.	0.	0.	101,622,800.
0	artfcl 6	0.	1.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	-1.	0.	117,882,496.
0	slack 7	-0.06	0.04	0.12	0.02	0.02	0.12	0.02	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.
	zj	-0.7456	-1.356	0.048	-1.421	-0.812	-2.944	0.052	0.	0.	1.	0.	1.	0.	1.	0.	1.	0.	1.	0.	422,750,992.
	cj-zj	1.	1.5	0.	1.6	1.	3.	0.1	0.	0.	-1.	0.	-1.	0.	-1.	0.	-1.	0.	-1.	0.	
Iteration 2																					

0	slack 1	0.	1.5	1.	1.	1.	1.	1.5	1.	0.	0.	-1.	1.	0.	0.	0.	0.	0.	0.	0.	406,491,392.
0	artfcl 2	0.	1.	0.	1.	0.	1.	0.	0.	1.	-1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	203,245,696.
0.2544	X1	1.	-0.5	0.	0.	0.	0.	-0.5	0.	0.	0.	1.	-1.	0.	0.	0.	0.	0.	0.	0.	0.
0	artfcl 4	0.	0.	0.	-0.4	1.	1.	-0.4	0.	0.	0.	0.	0.	1.	-1.	0.	0.	0.	0.	0.	0.
0	artfcl 5	0.	0.	0.	1.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	1.	-1.	0.	0.	0.	101,622,800.
0	artfcl 6	0.	1.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	-1.	0.	117,882,496.
0	slack 7	0.	0.01	0.12	0.02	0.02	0.12	-0.01	0.	0.	0.	0.06	-0.06	0.	0.	0.	0.	0.	0.	1.	0.
	zj	0.2544	-1.856	0.048	-1.421	-0.812	-2.944	-0.448	0.	0.	1.	1.	0.	0.	1.	0.	1.	0.	1.	0.	422,750,992.
	cj-zj	0.	2.	0.	1.6	1.	3.	0.6	0.	0.	-1.	-1.	0.	0.	-1.	0.	-1.	0.	-1.	0.	
Iteration 3																					
0	slack 1	0.	1.5	1.	0.	3.5	3.5	0.5	1.	0.	0.	-1.	1.	2.5	-2.5	0.	0.	0.	0.	0.	406,491,392.
0	artfcl 2	0.	1.	0.	0.	2.5	3.5	-1.	0.	1.	-1.	0.	0.	2.5	-2.5	0.	0.	0.	0.	0.	203,245,696.
0.2544	X1	1.	-0.5	0.	0.	0.	0.	-0.5	0.	0.	0.	1.	-1.	0.	0.	0.	0.	0.	0.	0.	0.
0.179	X4	0.	0.	0.	1.	-2.5	-2.5	1.	0.	0.	0.	0.	0.	-2.5	2.5	0.	0.	0.	0.	0.	0.
0	artfcl 5	0.	0.	0.	0.	2.5	3.5	-1.	0.	0.	0.	0.	0.	2.5	-2.5	1.	-1.	0.	0.	0.	101,622,800.
0	artfcl 6	0.	1.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	-1.	0.	117,882,496.
0	slack 7	0.	0.01	0.12	0.	0.07	0.17	-0.03	0.	0.	0.	0.06	-0.06	0.05	-0.05	0.	0.	0.	0.	1.	0.
	zj	0.2544	-1.856	0.048	0.179	-4.812	-6.944	1.152	0.	0.	1.	1.	0.	-4.	5.	0.	1.	0.	1.	0.	422,750,992.
	cj-zj	0.	2.	0.	0.	5.	7.	-1.	0.	0.	-1.	-1.	0.	4.	-5.	0.	-1.	0.	-1.	0.	
Iteration 4																					
0	slack 1	0.	1.2941	-1.4706	0.	2.0588	0.	1.1176	1.	0.	0.	-2.2353	2.2353	1.4706	-1.4706	0.	0.	0.	0.	-20.5882	406,491,392.
0	artfcl 2	0.	0.7941	-2.4706	0.	1.0588	0.	-0.3824	0.	1.	-1.	-1.2353	1.2353	1.4706	-1.4706	0.	0.	0.	0.	-20.5882	203,245,696.
0.2544	X1	1.	-0.5	0.	0.	0.	0.	-0.5	0.	0.	0.	1.	-1.	0.	0.	0.	0.	0.	0.	0.	0.
0.179	X4	0.	0.1471	1.7647	1.	-1.4706	0.	0.5588	0.	0.	0.	0.8824	-0.8824	-1.7647	1.7647	0.	0.	0.	0.	14.7059	0.
0	artfcl 5	0.	-0.2059	-2.4706	0.	1.0588	0.	-0.3824	0.	0.	0.	-1.2353	1.2353	1.4706	-1.4706	1.	-1.	0.	0.	-20.5882	101,622,800.
0	artfcl 6	0.	1.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	-1.	0.	117,882,496.
0.05	X6	0.	0.0	0.7	0.	0.4	1.	-	0.	0.	0.	0.3	-	0.2	-	0.	0.	0.	0.	5.88	0.

6			588	059		118		0.1765				529	0.3529	941	0.2941					24	
	zj	0.2544	-1.4442	4.9892	0.179	-1.9296	0.056	-0.0833	0.	0.	1.	3.4706	-2.4706	-1.9412	2.9412	0.	1.	0.	1.	41.1765	422,750,992.
	cj-zj	0.	1.5882	-4.9412	0.	2.1176	0.	0.2353	0.	0.	-1.	-3.4706	2.4706	1.9412	-2.9412	0.	-1.	0.	-1.	-41.1765	
Iteration 5																					
0	slack 1	0.	1.6667	3.	0.	0.1429	0.	1.8095	1.	0.	0.	0.	0.	-1.1905	1.1905	-1.8095	1.8095	0.	0.	16.6667	222,602,517.7692
0	artfcl 2	0.	1.	0.	0.	0.	0.	0.	0.	1.	-1.	0.	0.	0.	0.	-1.	1.	0.	0.	0.	101,622,897.5991
0.2544	X1	1.	-0.6667	-2.	0.	0.8571	0.	-0.8095	0.	0.	0.	0.	0.	1.1905	-1.1905	0.8095	-0.8095	0.	0.	-16.6667	82,266,075.8299
0.179	X4	0.	0.	0.	1.	-0.7143	0.	0.2857	0.	0.	0.	0.	0.	-0.7143	0.7143	0.7143	-0.7143	0.	0.	0.	72,587,714.5445
0	surplus 3	0.	-0.1667	-2.	0.	0.8571	0.	-0.3095	0.	0.	0.	-1.	1.	1.1905	-1.1905	0.8095	-0.8095	0.	0.	-16.6667	82,266,075.8299
0	artfcl 6	0.	1.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	-1.	0.	117,882,496.
0.056	X6	0.	0.	0.	0.	0.7143	1.	-0.2857	0.	0.	0.	0.	0.	0.7143	-0.7143	0.2857	-0.2857	0.	0.	0.	29,035,086.3081
	zj	0.2544	-1.856	0.048	0.179	0.188	0.056	-0.848	0.	0.	1.	1.	0.	1.	0.	2.	-1.	0.	1.	0.	219,505,395.1983
	cj-zj	0.	2.	0.	0.	0.	0.	1.	0.	0.	-1.	-1.	0.	-1.	0.	-2.	1.	0.	-1.	0.	
Iteration 6																					
0	slack 1	0.	0.	3.	0.	0.1429	0.	1.8095	1.	-1.6667	1.6667	0.	0.	-1.1905	1.1905	-0.1429	0.1429	0.	0.	16.6667	53,231,025.2601
0.144	X2	0.	1.	0.	0.	0.	0.	0.	0.	1.	-1.	0.	0.	0.	0.	-1.	1.	0.	0.	0.	101,622,897.9284
0.2544	X1	1.	0.	-2.	0.	0.8571	0.	-0.8095	0.	0.6667	-0.6667	0.	0.	1.1905	-1.1905	0.1429	-0.1429	0.	0.	-16.6667	150,014,676.4679
0.179	X4	0.	0.	0.	1.	-0.7143	0.	0.2857	0.	0.	0.	0.	0.	-0.7143	0.7143	0.7143	-0.7143	0.	0.	0.	72,587,714.5445
0	surplus 3	0.	0.	-2.	0.	0.8571	0.	-0.3095	0.	0.1667	-0.1667	-1.	1.	1.1905	-1.1905	0.6429	-0.6429	0.	0.	-16.6667	99,203,225.9894
0	artfcl 6	0.	0.	0.	0.	0.	0.	1.	0.	-1.	1.	0.	0.	0.	0.	1.	-1.	1.	-1.	0.	16,259,598.0716
0.056	X6	0.	0.	0.	0.	0.7143	1.	-0.2857	0.	0.	0.	0.	0.	0.7143	-0.7143	0.2857	-0.2857	0.	0.	0.	29,035,086.3081
	zj	0.2544	0.144	0.048	0.179	0.188	0.056	-0.848	0.	2.	-1.	1.	0.	1.	0.	0.	1.	0.	1.	0.	16,259,599.3415
	cj-zj	0.	0.	0.	0.	0.	0.	1.	0.	-2.	1.	-1.	0.	-1.	0.	0.	-1.	0.	-1.	0.	
Iteration 7																					

0	slack 1	0.	0.	3.	0.	0.1429	0.	0.1429	1.	0.	0.	0.	0.	-1.1905	1.1905	-1.8095	1.8095	-1.6667	1.6667	16.6667	26,131,695.8746
0.144	X2	0.	1.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	-1.	0.	117,882,495.9473
0.2544	X1	1.	0.	-2.	0.	0.8571	0.	-0.1429	0.	0.	0.	0.	0.	1.1905	-1.1905	0.8095	-0.8095	0.6667	-0.6667	-16.6667	160,854,408.8036
0.179	X4	0.	0.	0.	1.	-0.7143	0.	0.2857	0.	0.	0.	0.	0.	-0.7143	0.7143	0.7143	-0.7143	0.	0.	0.	72,587,714.5445
0	surplus 3	0.	0.	-2.	0.	0.8571	0.	-0.1429	0.	0.	0.	-1.	1.	1.1905	-1.1905	0.8095	-0.8095	0.1667	-0.1667	-16.6667	101,913,159.0733
0	surplus 2	0.	0.	0.	0.	0.	0.	1.	0.	-1.	1.	0.	0.	0.	0.	1.	-1.	1.	-1.	0.	16,259,598.019
0.056	X6	0.	0.	0.	0.	0.7143	1.	-0.2857	0.	0.	0.	0.	0.	0.7143	-0.7143	0.2857	-0.2857	0.	0.	0.	29,035,086.3081
	zj	0.2544	0.144	0.048	0.179	0.188	0.056	0.152	0.	1.	0.	1.	0.	1.	0.	1.	0.	1.	0.	0.	1.3226
	cj-zj	0.	0.	0.	0.	0.	0.	0.	0.	-1.	0.	-1.	0.	-1.	0.	-1.	0.	-1.	0.	0.	
Iteration 8																					
0	slack 1	0.	0.	3.	0.	0.1429	0.	0.1429	1.	0.	0.	0.	0.	-1.1905	1.1905	-1.8095	1.8095	-1.6667	1.6667	16.6667	26,131,695.8746
0.144	X2	0.	1.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	-1.	0.	117,882,495.9473
0.2544	X1	1.	0.	-2.	0.	0.8571	0.	-0.1429	0.	0.	0.	0.	0.	1.1905	-1.1905	0.8095	-0.8095	0.6667	-0.6667	-16.6667	160,854,408.8036
0.179	X4	0.	0.	0.	1.	-0.7143	0.	0.2857	0.	0.	0.	0.	0.	-0.7143	0.7143	0.7143	-0.7143	0.	0.	0.	72,587,714.5445
0	surplus 3	0.	0.	-2.	0.	0.8571	0.	-0.1429	0.	0.	0.	-1.	1.	1.1905	-1.1905	0.8095	-0.8095	0.1667	-0.1667	-16.6667	101,913,159.0733
0	surplus 2	0.	0.	0.	0.	0.	0.	1.	0.	-1.	1.	0.	0.	0.	0.	1.	-1.	1.	-1.	0.	16,259,598.019
0.056	X6	0.	0.	0.	0.	0.7143	1.	-0.2857	0.	0.	0.	0.	0.	0.7143	-0.7143	0.2857	-0.2857	0.	0.	0.	29,035,086.3081
	zj	0.2544	0.144	-0.5088	0.179	0.1302	0.056	0.1428	0.	0.	0.	0.	0.	0.215	-0.215	0.3498	-0.3498	0.3136	-0.3136	-4.24	72,515,608.8374
	cj-zj	0.	0.	0.5568	0.	0.0578	0.	0.0092	0.	0.	0.	0.	0.	-0.215	0.215	-0.3498	0.3498	-0.3136	0.3136	4.24	
Iteration 9																					
0	slack 7	0.	0.	0.18	0.	0.0086	0.	0.0086	0.06	0.	0.	0.	0.	-0.0714	0.0714	-0.1086	0.1086	-0.1	0.1	1.	1,567,901.7479
0.144	X2	0.	1.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	-1.	0.	117,882,495.9473
0.2544	X1	1.	0.	1.	0.	1.	0.	0.	1.	0.	0.	0.	0.	0.	0.	-1.	1.	-1.	1.	0.	186,986,106.5957
0.179	X4	0.	0.	0.	1.	-0.7143	0.	0.2857	0.	0.	0.	0.	0.	-0.7143	0.7143	0.7143	-0.7143	0.	0.	0.	72,587,714.5445

0	surplus 3	0.	0.	1.	0.	1.	0.	0.	1.	0.	0.	-1.	1.	0.	0.	-1.	1.	-1.5	1.5	0.	128,044,856.8655
0	surplus 2	0.	0.	0.	0.	0.	0.	1.	0.	-1.	1.	0.	0.	0.	0.	1.	-1.	1.	-1.	0.	16,259,598.019
0.056	X6	0.	0.	0.	0.	0.7143	1.	-0.2857	0.	0.	0.	0.	0.	0.7143	-0.7143	0.2857	-0.2857	0.	0.	0.	29,035,086.3081
	zj	0.2544	0.144	0.2544	0.179	0.1665	0.056	0.1791	0.2544	0.	0.	0.	0.	-0.0879	0.0879	-0.1105	0.1105	-0.1104	0.1104	0.	79,163,512.6373
	cj-zj	0.	0.	-0.2064	0.	0.0215	0.	-0.0271	-0.2544	0.	0.	0.	0.	0.0879	-0.0879	0.1105	-0.1105	0.1104	-0.1104	0.	
Iteration 10																					
0	slack 7	0.	0.	0.18	0.	0.	-0.012	0.012	0.06	0.	0.	0.	0.	-0.08	0.08	-0.112	0.112	-0.1	0.1	1.	1,219,480.6097
0.144	X2	0.	1.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	-1.	0.	117,882,495.9473
0.2544	X1	1.	0.	1.	0.	0.	-1.4	0.4	1.	0.	0.	0.	0.	-1.	1.	-1.4	1.4	-1.	1.	0.	146,336,985.5107
0.179	X4	0.	0.	0.	1.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	1.	-1.	0.	0.	0.	101,622,799.3031
0	surplus 3	0.	0.	1.	0.	0.	-1.4	0.4	1.	0.	0.	-1.	1.	-1.	1.	-1.4	1.4	-1.5	1.5	0.	87,395,735.7805
0	surplus 2	0.	0.	0.	0.	0.	0.	1.	0.	-1.	1.	0.	0.	0.	0.	1.	-1.	1.	-1.	0.	16,259,598.019
0.188	X5	0.	0.	0.	0.	1.	1.4	-0.4	0.	0.	0.	0.	0.	1.	-1.	0.4	-0.4	0.	0.	0.	40,649,121.085
	zj	0.2544	0.144	0.2544	0.179	0.188	0.086	0.1706	0.2544	0.	0.	0.	0.	-0.0664	0.0664	-0.102	0.102	-0.1104	0.1104	0.	80,035,725.5864
	cj-zj	0.	0.	-0.2064	0.	0.	-0.003	-0.0186	-0.2544	0.	0.	0.	0.	0.0664	-0.0664	0.102	-0.102	0.1104	-0.1104	0.	

Created by [QM for Windows]

APPENDIX G

ASANTE AKYEM RURAL BANK ALLOCATION OF LOAN PROBLEM										
	X1	X2	X3	X4	X5	X6	X7	X8		RHS
Maximize	0.27	0.17	0.11	0.12	0.17	0.04	0.17	0.3		
Constraint 1	1.	1.	1.	1.	1.	1.	1.	1.	<=	700,000
Constraint 2	1.	0.	0.	1.	0.	0.	0.	0.	>=	350,000
Constraint 3	1.	-0.5	0.	0.	0.	0.	-0.5	0.	>=	0
Constraint 4	0.	0.	0.	-0.25	1.	1.	0.	-0.25	>=	0
Constraint 5	0.	0.	0.	0.	1.	0.	1.	0.	>=	175,000
Constraint 6	-0.04	0.05	0.1	0.1	0.05	0.15	0.05	-0.04	<=	0

Created by QM for Windows

APPENDIX H

Solution To Asante Akyem Rural Bank loan Allocation					
Variable	Value	Reduced Cost	Original Val	Lower Bound	Upper Bound
X1	350,000.	0.	0.27	0.12	0.3
X2	0.	0.13	0.17	-Infinity	0.3
X3	0.	0.19	0.11	-Infinity	0.3
X4	0.	0.15	0.12	-Infinity	0.27
X5	43,750.	0.	0.17	0.05	0.17
X6	0.	0.26	0.04	-Infinity	0.3
X7	131,250.	0.	0.17	0.17	0.27
X8	175,000.	0.	0.3	0.27	Infinity
Constraint	Dual Value	Slack/Surplus	Original Val	Lower Bound	Upper Bound
Constraint 1	0.3	0.	700,000.	525,000.	1,225,000.
Constraint 2	-0.03	0.	350,000.	25,000.	525,000.
Constraint 3	0.	284,375.	0.	-Infinity	284,375.
Constraint 4	0.	0.0011	0.	-43,750.	131,250.
Constraint 5	-0.13	0.	175,000.	70,000.	311,111.1
Constraint 6	0.	12,250.	0.	-12,250.	Infinity

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APPENDIX I

SOLUTION TO ASANTE AKYEM RURAL BANK LOAN ALLOCATION		
Variable	Status	Value
X1	Basic	350,000.
X2	NONBasic	0.
X3	NONBasic	0.
X4	NONBasic	0.
X5	Basic	43,750.
X6	NONBasic	0.
X7	Basic	131,250.
X8	Basic	175,000.
slack 1	NONBasic	0.
surplus 2	NONBasic	0.
surplus 3	Basic	284,375.
surplus 4	NONBasic	0.
surplus 5	NONBasic	0.
slack 6	Basic	12,250.
Optimal Value (Z)		176,750.

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APPENDIX J

LINEAR PROGRAMMING RESULTS

d) Solution											
	X1	X2	X3	X4	X5	X6	X7	X8		RHS	Dual
Maximize	0.27	0.17	0.11	0.12	0.17	0.04	0.17	0.3			
Constraint 1	1.	1.	1.	1.	1.	1.	1.	1.	<=	700,000.	0.3
Constraint 2	1.	0.	0.	1.	0.	0.	0.	0.	>=	350,000.	-0.03
Constraint 3	1.	-0.5	0.	0.	0.	0.	-0.5	0.	>=	0.	0.
Constraint 4	0.	0.	0.	-0.25	1.	1.	0.	-0.25	>=	0.	0.
Constraint 5	0.	0.	0.	0.	1.	0.	1.	0.	>=	175,000.	-0.13
Constraint 6	-0.04	0.05	0.1	0.1	0.05	0.15	0.05	-0.04	<=	0.	0.
Solution->	350,000.	0.	0.	0.	43,750.	0.	131,250.	175,000.		176,750.01	

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APPENDIX

ITERATIONS OF ASANTE AKYEM RURAL BANK LOAN ALLOCATION

Solution																				
Cj	Basic Variables	.27 X1	.17 X2	.11 X3	.12 X4	.17 X5	.04 X6	.17 X7	.3 X8	0 slack 1	0 artfcl 2	0 surplus 2	0 artfcl 3	0 surplus 3	0 artfcl 4	0 surplus 4	0 artfcl 5	0 surplus 5	0 slack 6	Quantity
Iteration 1																				
0	slack 1	1.	1.	1.	1.	1.	1.	1.	1.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	700,000.
0	artfcl 2	1.	0.	0.	1.	0.	0.	0.	0.	0.	1.	-1.	0.	0.	0.	0.	0.	0.	0.	350,000.
0	artfcl 3	1.	-0.5	0.	0.	0.	0.	-0.5	0.	0.	0.	0.	1.	-1.	0.	0.	0.	0.	0.	0.
0	artfcl 4	0.	0.	0.	-0.25	1.	1.	0.	-0.25	0.	0.	0.	0.	0.	1.	-1.	0.	0.	0.	0.
0	artfcl 5	0.	0.	0.	0.	1.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	1.	-1.	0.	175,000.
0	slack 6	-0.04	0.05	0.1	0.1	0.05	0.15	0.05	-0.04	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.
	zj	-1.73	0.67	0.11	-0.63	-1.83	-0.96	-0.33	0.55	0.	0.	1.	0.	1.	0.	1.	0.	1.	0.	525,000.
	cj-zj	2.	-0.5	0.	0.75	2.	1.	0.5	-0.25	0.	0.	-1.	0.	-1.	0.	-1.	0.	-1.	0.	
Iteration 2																				

0	slack 1	0.	1.5	1.	1.	1.	1.	1.5	1.	1.	0.	0.	-1.	1.	0.	0.	0.	0.	0.	700,000.
0	artfcl 2	0.	0.5	0.	1.	0.	0.	0.5	0.	0.	1.	-1.	-1.	1.	0.	0.	0.	0.	0.	350,000.
0.27	X1	1.	-0.5	0.	0.	0.	0.	-0.5	0.	0.	0.	0.	1.	-1.	0.	0.	0.	0.	0.	0.
0	artfcl 4	0.	0.	0.	-0.25	1.	1.	0.	-0.25	0.	0.	0.	0.	0.	1.	-1.	0.	0.	0.	0.
0	artfcl 5	0.	0.	0.	0.	1.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	1.	-1.	0.	175,000.
0	slack 6	0.	0.03	0.1	0.1	0.05	0.15	0.03	-0.04	0.	0.	0.	0.04	-0.04	0.	0.	0.	0.	1.	0.
	zj	0.27	-0.33	0.11	-0.63	-1.83	-0.96	-1.33	0.55	0.	0.	1.	2.	-1.	0.	1.	0.	1.	0.	525,000.
	cj-zj	0.	0.5	0.	0.75	2.	1.	1.5	-0.25	0.	0.	-1.	-2.	1.	0.	-1.	0.	-1.	0.	
Iteration 3																				
0	slack 1	0.	1.5	1.	0.	5.	5.	1.5	0.	1.	0.	0.	-1.	1.	4.	-4.	0.	0.	0.	700,000.
0	artfcl 2	0.	0.5	0.	0.	4.	4.	0.5	-1.	0.	1.	-1.	-1.	1.	4.	-4.	0.	0.	0.	350,000.
0.27	X1	1.	-0.5	0.	0.	0.	0.	-0.5	0.	0.	0.	0.	1.	-1.	0.	0.	0.	0.	0.	0.
0.12	X4	0.	0.	0.	1.	-4.	-4.	0.	1.	0.	0.	0.	0.	0.	-4.	4.	0.	0.	0.	0.
0	artfcl 5	0.	0.	0.	0.	1.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	1.	-1.	0.	175,000.
0	slack 6	0.	0.03	0.1	0.	0.45	0.55	0.03	-0.14	0.	0.	0.	0.04	-0.04	0.4	-0.4	0.	0.	1.	0.
	zj	0.27	-0.33	0.11	0.12	-4.83	-3.96	-1.33	1.3	0.	0.	1.	2.	-1.	-3.	4.	0.	1.	0.	525,000.
	cj-zj	0.	0.5	0.	0.	5.	4.	1.5	-1.	0.	0.	-1.	-2.	1.	3.	-4.	0.	-1.	0.	
Iteration 4																				
0	slack 1	0.	1.1667	-0.1111	0.	0.	-1.1111	1.1667	1.5556	1.	0.	0.	-1.4444	1.4444	-0.4444	0.4444	0.	0.	-11.1111	700,000.
0	artfcl 2	0.	0.2333	-0.8889	0.	0.	-0.8889	0.2333	0.2444	0.	1.	-1.	-1.3556	1.3556	0.4444	-0.4444	0.	0.	-8.8889	350,000.
0.27	X1	1.	-0.5	0.	0.	0.	0.	-0.5	0.	0.	0.	0.	1.	-1.	0.	0.	0.	0.	0.	0.
0.12	X4	0.	0.2667	0.8889	1.	0.	0.8889	0.2667	-0.2444	0.	0.	0.	0.3556	-0.3556	-0.4444	0.4444	0.	0.	8.8889	0.
0	artfcl 5	0.	-0.0667	-0.2222	0.	0.	-1.2222	0.9333	0.3111	0.	0.	0.	-0.0889	0.0889	-0.8889	0.8889	1.	-1.	-2.2222	175,000.
0.17	X5	0.	0.0667	0.2222	0.	1.	1.2222	0.0667	-0.3111	0.	0.	0.	0.0889	-0.0889	0.8889	-0.8889	0.	0.	2.2222	0.
	zj	0.27	0.0033	1.2211	0.12	0.17	2.1511	-0.9967	-0.2556	0.	0.	1.	2.4444	-1.4444	1.4444	-0.4444	0.	1.	11.1111	525,000.
	cj-zj	0.	0.1667	-1.1111	0.	0.	-2.1111	1.1667	0.5556	0.	0.	-1.	-2.4444	1.4444	-1.4444	0.4444	0.	-1.	-11.1111	
Iteration 5																				
0	slack	0.	0.91	0.83	0.	0.	-	0.91	1.29	1.	-	1.06	0.	0.	-	0.91	0.	0.	-	327,049.

	1		8	61			0.1639	8	51		1.0656	56			0.918	8			1.6393	1835
0	surplus 3	0.	0.1721	-0.6557	0.	0.	-0.6557	0.1721	0.1803	0.	0.7377	-0.7377	-1.	1.	0.3279	-0.3279	0.	0.	-6.5574	258,196.7238
0.27	X1	1.	-0.3279	-0.6557	0.	0.	-0.6557	-0.3279	0.1803	0.	0.7377	-0.7377	0.	0.	0.3279	-0.3279	0.	0.	-6.5574	258,196.7238
0.12	X4	0.	0.3279	0.6557	1.	0.	0.6557	0.3279	-0.1803	0.	0.2623	-0.2623	0.	0.	-0.3279	0.3279	0.	0.	6.5574	91,803.2741
0	artfcl 5	0.	-0.082	-0.1639	0.	0.	-1.1639	0.918	0.2951	0.	-0.0656	0.0656	0.	0.	-0.918	0.918	1.	-1.	-1.6393	152,049.1815
0.17	X5	0.	0.082	0.1639	0.	1.	1.1639	0.082	-0.2951	0.	0.0656	-0.0656	0.	0.	0.918	-0.918	0.	0.	1.6393	22,950.8185
	zj	0.27	0.252	0.2739	0.12	0.17	1.2039	-0.748	0.0049	0.	1.0656	-0.0656	1.	0.	1.918	-0.918	0.	1.	1.6393	152,049.1835
	cj-zj	0.	-0.082	-0.1639	0.	0.	-1.1639	0.918	0.2951	0.	-1.0656	0.0656	-1.	0.	-1.918	0.918	0.	-1.	-1.6393	
Iteration 6																				
0	slack 1	0.	1.	1.	0.	0.	1.	0.	1.	1.	-1.	1.	0.	0.	0.	0.	-1.	1.	0.	175,000.0055
0	surplus 3	0.	0.1875	-0.625	0.	0.	-0.4375	0.	0.125	0.	0.75	-0.75	-1.	1.	0.5	-0.5	-0.1875	0.1875	-6.25	229,687.5017
0.27	X1	1.	-0.3571	-0.7143	0.	0.	-1.0714	0.	0.2857	0.	0.7143	-0.7143	0.	0.	0.	0.	0.3571	-0.3571	-7.1429	312,500.0024
0.12	X4	0.	0.3571	0.7143	1.	0.	1.0714	0.	-0.2857	0.	0.2857	-0.2857	0.	0.	0.	0.	-0.3571	0.3571	7.1429	37,499.9956
0.17	X7	0.	-0.0893	-0.1786	0.	0.	-1.2679	1.	0.3214	0.	-0.0714	0.0714	0.	0.	-1.	1.	1.0893	-1.0893	-1.7857	165,625.0013
0.17	X5	0.	0.0893	0.1786	0.	1.	1.2679	0.	-0.3214	0.	0.0714	-0.0714	0.	0.	1.	-1.	-0.0893	0.0893	1.7857	9,374.9989
	zj	0.27	0.17	0.11	0.12	0.17	0.04	0.17	0.3	0.	1.	0.	1.	0.	1.	0.	1.	0.	0.	0.0055
	cj-zj	0.	0.	0.	0.	0.	0.	0.	0.	0.	-1.	0.	-1.	0.	-1.	0.	-1.	0.	0.	
Iteration 7																				
0	slack 1	0.	1.	1.	0.	0.	1.	0.	1.	1.	-1.	1.	0.	0.	0.	0.	-1.	1.	0.	175,000.0055
0	surplus 3	0.	0.1875	-0.625	0.	0.	-0.4375	0.	0.125	0.	0.75	-0.75	-1.	1.	0.5	-0.5	-0.1875	0.1875	-6.25	229,687.5017
0.27	X1	1.	-0.3571	-0.7143	0.	0.	-1.0714	0.	0.2857	0.	0.7143	-0.7143	0.	0.	0.	0.	0.3571	-0.3571	-7.1429	312,500.0024
0.12	X4	0.	0.3571	0.7143	1.	0.	1.0714	0.	-0.2857	0.	0.2857	-0.2857	0.	0.	0.	0.	-0.3571	0.3571	7.1429	37,499.9956
0.17	X7	0.	-0.0893	-0.1786	0.	0.	-1.2679	1.	0.3214	0.	-0.0714	0.0714	0.	0.	-1.	1.	1.0893	-1.0893	-1.7857	165,625.0013
0.17	X5	0.	0.0893	0.1786	0.	1.	1.2679	0.	-0.32	0.	0.0714	-0.07	0.	0.	1.	-1.	-0.08	0.0893	1.7857	9,374.9989

									14			14					93			
	zj	0.27	-0.0536	-0.1071	0.12	0.17	-0.1607	0.17	0.0429	0.	0.2271	-0.2271	0.	0.	0.	0.	0.2236	-0.2236	-1.0714	118,625.0037
	cj-zj	0.	0.2236	0.2171	0.	0.	0.2007	0.	0.2571	0.	-0.2271	0.2271	0.	0.	0.	0.	-0.2236	0.2236	1.0714	
Iteration 8																				
0	slack 1	0.	1.	1.	0.	0.	1.	0.	1.	1.	-1.	1.	0.	0.	0.	0.	-1.	1.	0.	175,000.0055
0	surplus 3	0.	0.5	0.	0.875	0.	0.5	0.	-0.125	0.	1.	-1.	-1.	1.	0.5	-0.5	-0.5	0.5	0.	262,499.9978
0.27	X1	1.	0.	0.	1.	0.	0.	0.	0.	0.	1.	-1.	0.	0.	0.	0.	0.	0.	0.	349,999.9975
0	slack 6	0.	0.05	0.1	0.14	0.	0.15	0.	-0.04	0.	0.04	-0.04	0.	0.	0.	0.	-0.05	0.05	1.	5,249.9994
0.17	X7	0.	0.	0.	0.25	0.	-1.	1.	0.25	0.	0.	0.	0.	0.	-1.	1.	1.	-1.	0.	175,000.0001
0.17	X5	0.	0.	0.	-0.25	1.	1.	0.	-0.25	0.	0.	0.	0.	0.	1.	-1.	0.	0.	0.	0.0001
	zj	0.27	0.	0.	0.27	0.17	0.	0.17	0.	0.	0.27	-0.27	0.	0.	0.	0.	0.17	-0.17	0.	124,250.0035
	cj-zj	0.	0.17	0.11	-0.15	0.	0.04	0.	0.3	0.	-0.27	0.27	0.	0.	0.	0.	-0.17	0.17	0.	
Iteration 9																				
0.3	X8	0.	1.	1.	0.	0.	1.	0.	1.	1.	-1.	1.	0.	0.	0.	0.	-1.	1.	0.	175,000.0041
0	surplus 3	0.	0.625	0.125	0.875	0.	0.625	0.	0.	0.125	0.875	-0.875	-1.	1.	0.5	-0.5	-0.625	0.625	0.	284,374.9983
0.27	X1	1.	0.	0.	1.	0.	0.	0.	0.	0.	1.	-1.	0.	0.	0.	0.	0.	0.	0.	349,999.9975
0	slack 6	0.	0.09	0.14	0.14	0.	0.19	0.	0.	0.04	0.	0.	0.	0.	0.	0.	-0.09	0.09	1.	12,249.9994
0.17	X7	0.	-0.25	-0.25	0.25	0.	-1.25	1.	0.	-0.25	0.25	-0.25	0.	0.	-1.	1.	1.25	-1.25	0.	131,249.9991
0.17	X5	0.	0.25	0.25	-0.25	1.	1.25	0.	0.	0.25	-0.25	0.25	0.	0.	1.	-1.	-0.25	0.25	0.	43,750.0011
	zj	0.27	0.3	0.3	0.27	0.17	0.3	0.17	0.3	0.3	-0.03	0.03	0.	0.	0.	0.	-0.13	0.13	0.	176,750.0068
	cj-zj	0.	-0.13	-0.19	-0.15	0.	-0.26	0.	0.	-0.3	0.03	-0.03	0.	0.	0.	0.	0.13	-0.13	0.	

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