KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI

BANK LOAN PORTFOLIO EQUILIBRUIM MIX:

A MARKOV CHAIN APPROACH

BY

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DECLARATION

I hereby declare that this submission is my own work towards the award of Master of Philosophy in Applied Mathematics (Statistics) and that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the University, except where due acknowledgement has been made in the text.

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DEDICATION

This project work is dedicated to my son Barima Kofi Amoani Domsuro.

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I would like to express my profound gratitude to the Almighty God for making it possible for this dream to be a reality.

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ABSTRACT

Managing credit risk (loan) has been the priority of almost all Financial Institutions in recent years. The interest lies as to whether the financial institution will be able to meet the demands of their potential clients whereas clients are expected to meet their short term or long term loan obligation. In view of this an optimal loan allocation mix policy from the steady State distribution of loan disbursement process is presented in this study.

The objectives of the study are to (i) obtain an optimal loan allocation mix policy (ii) to estimate the transition matrix using time series data on loans.

Monthly data on actual loan Disbursement of four loan types for a period of twenty-four months is analyzed. An estimated Transition probability matrix of the movement of one loan type to another is obtained using an optimization technique. It is from this transition probability matrix that the steady state distribution of loan disbursement process is obtained. Opportunity International Savings and Loans in Ghana was used as our case study. Among the loan types offered are Agricultural, Susu, Small and Medium Enterprise (SME) and Salary loans.

The estimated transition matrix showed that the probability of loan switching from Agricultural to SME is the highest (0.54) while loan switching from Salary to Agric is the lowest (0.034). Probability of no loan switching for Susu is (0.380), Probability of no loan switching for SME is (0.52), and whiles that of Salary is (0.044).

From the estimated probability transition matrix, the steady state distribution indicated that in the long run, SME loan constitutes 52.36% of the total funds allocated for loans. This is followed by Agricultural loan 38.17%, salary loan 4.95% and Susu loan 3.76%, of the total loan amount.

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CHAPTER ONE

GENERAL OVERVIEW

1.0 INTRODUCTION

Managing credit risk (loan) has been the priority of almost all Financial Institutions in recent years. The interest lies as to whether the financial institution will be able to meet the demands of their potential clients whereas clients are expected to meet their short term or long term loan obligation.

1.1 BACKGROUND OF THE STUDY

Accessing a loan in a financial institution is on the increase due to the establishment of small and medium enterprises.

In the quest to make loans accessible to all, financial Institutions encounter series of financial crisis that create excessive loss in the banking industry.

Other financial analysts associate the evolutionary changes the Banking Industries go through to the world wide credit crunch. For this reason banks have to manage their finances efficiently in times like these. This can be achieved by

(i) being efficient and effective in their product offering and

(ii) Maintaining an appropriate fund portfolio for their survival and growth.(Gardiner, 2010)

The Bank of Ghana realizing these distortions as a result of the world financial crisis initiated the following policies;

(i) Computerization of banking operations and networking of branches have been earnestly pursued by banks in the last three years: While most banks have computerized all their operations nationwide, others have reached advanced stages in this regard. The Computerization is aimed at speeding up operations and information processing.

(ii) Product/service development: Banks in Ghana are engaged in healthy competition with regard to the introduction of well packaged and unique products/services designed to enhance deposit mobilization, effective credit delivery, funds transfer and customer care.

The Bank of Ghana initiated the process of replacing the existing Law, PNDCL225, 1989 with a new one to bring it in line with international standards and to advance global development in banking. The draft bills seek to focus on the following, among others:

- provide greater autonomy to the Bank of Ghana
- facilitate mandatory exit of problematic banks
- streamline the appeal and hearing procedures, and
- provide flexibility to the central bank in changing prudential provisions and requirements

Payment System: The Bank of Ghana has also initiated a process to transform the current payment system in Ghana, which is dominated by cash to electronic funds transfer/storage system. This is aimed at replacing the cash driven Ghanaian economy with an effective cashless system. Already, an automated cheque-clearing system and a cheque printer accreditation scheme have been put in place (Mbendi Information Services).

Meanwhile, the Malaysian Central Bank has envisaged a merger scheme to combat the crisis. These merged banks (anchor banks), are to accelerate economic growth. (Thyaragarajan et al., 2010).

Lending is one of the main activities of banks in Ghana and other parts of the world. This is evidenced by the volume of loans that constitute banks assets and the annual substantial increase in the amount of credit granted to borrowers in the private and public sectors of the economy. According to Comptroller (1998), lending is the principal business for most commercial banks. Loan portfolio is therefore typically the largest asset and the largest source of revenue for banks. In view of the significant contribution of loans to the financial health of banks through interest income earnings, these assets are considered the most valuable assets of banks.

A survey in 2006 on the Ghanaian banking sector revealed that loans accounted for about 50% of total bank assets which had increased from 41.5% in 2005 (Appertey and Arkaifie, 2006).
In 2007, the figure increased to 53% of the industry's total assets of GH¢ 7,795.6 million (Infodata Associates, 2009).

The reason why banks give much attention to the lending activity, especially in periods of a stable economic environment, is that a substantial amount of banks income is earned on loans, which contribute significantly to the financial performance of banks. A financial report of ADB in 2007, indicated that out of the total interest income of GH¢42,327,367.00 earned in that year, about 66.5% was earned on loans and in 2004, CAL Bank earned about 55.9% of its total interest income on loans and advances (CAL Bank Financial Statement, 2004). Thus, the figures point to the fact that loans contribute immensely to the financial performance of banks in Ghana.

The above literature gives ample evidence that healthy loan portfolios are vital assets for banks in view of their positive impact on the performance of banks.

The study seeks to device a loan allocation policy to different types of loans using Markov chain. Market share model and as well attempt to estimate the transition matrix using time series data on loan disbursements. This provides the probability of loan switching among its loan type (Yushkevic, 2001).

The Financial Institution chosen for the current study is one of the largest non -banking Financial Institutions in Ghana. Opportunity International Savings and Loans Limited is a non -banking financial institution licensed by the Central Bank of Ghana in 2001 to operate in savings and loans.

They are a partner member of Opportunity International Network an ecumenical Christian economic development organization with forty-seven (47) partners operating in Africa, Asia, Europe, Latin America and North America.

Their key objective is to offer small entrepreneurs with micro loans, deposits, and other financial services in the Greater Accra, Ashanti and Brong Ahafo regions of Ghana.

Loans offered by Opportunity International are Susu, Agricultural, Small and Medium Enterprise (SME) and Salary Loan.

1.2 STATEMENT OF PROBLEM

Loan portfolio is typically the largest asset and the predominant source of income for banks. In spite of the huge income generated from their loan portfolio, available literature shows that huge portions of banks loans usually go bad and therefore affect the financial performance of these institutions (Comptroller, 1998). The Bank of Ghana's classifications of advances of the Banking industry indicated that bad loans in the loss category increased from GH¢125, 196,732 in December 2007 to GH¢204, 978,569.00 in December 2008, indicating over 63% jump in bad loans. A report on the performance of banks in 2006 indicated that among other factors, higher loan loss provision accounted for a decline in the profitability of banks in 2005 (Bank of Ghana, 2006).

The issue of bad loans can fuel banking crisis and result in the collapse of some of these institutions with their attendant repercussions on the economy as a whole. (Kane and Rice)

(2001) stated that at the peak of the financial crisis in Benin, 80% of total bank loans portfolio which was about 17% of GDP, was non-performing in the late nineties.

Indeed bad loans can lead to the collapse of banks which have huge balances of these non-performing loans if measures are not taken to minimize the problem. In Ghana, the banking industry plays an important role in the development of the economy. Huge bad loans could therefore affect banks in the performance of this important role.

In view of the above, it is imperative to find out the extent of the impact of bad loans on banks performance and identify the causes of bad loans of banks in Ghana.

In other for the banks to be in good standing and as well break even, resources have to be managed efficiently and effectively.

This can be achieved by:

(i) Offering attractive products and services that will be welcomed by potential clients.

(ii) Maintaining an appropriate retention premium for their survival in times of crisis that may arise as a result of panic withdrawal.

1.3 OBJECTIVES OF THE STUDY

The objectives of the study are

(i) To obtain an optimal loan allocation mix policy this could be used as a guiding principle for future allocation purpose.

(ii) To estimate the transition matrix using time series data on loans.

1.4 METHODOLOGY

Monthly data on actual loan disbursement for a period of twenty-four (24) months beginning January 2010 to January 2012 for four types of loans are used as a basis to estimate the transition probability matrix.

The transition probability matrix gives the probability of loan switching from one type to another type.

The study aim to devise the determination of equilibrium loan allocation using Markov Chain.

Markov Probability Model**:** The probability of switching a loan disbursement from loan type *i* to loan type j is a conditional probability and can be represented by the transition matrix P=pij

Such that $\sum P_{ij}=1$ for (j=1…..m) (1.1)

Indices i refer to the number of loans where as j refer to the loan type. For example P_{21} represent the probability of a change in loan disbursement from A to B in the next period of time. While P_i Represents the probabilities of no change in loan disbursement for loan type *i*. The stochastic model used to explain the loan disbursement behavior is a Markov Chain with finite number of States $\{E\}$. Markov process $\{Xt\}$ with discrete time t such that Pij in general represents the probability of the process moving from state i at time (t-1) to state j at time t. In this study we assume that the loan disbursement for type i in the next period t (month) is only determined by the loan disbursement at the preceding period(t-1). In other words, the "history "of loan disbursement before time t-1 does not influence the future loan disbursement. This is known as a first order time dependency. In statistical notation it is represented as;

$$P\ (X_t = j\ |X_0, X_{1,\ldots}X_{t-1} = i\ ) = P\ (X_t = j|\ X_{t-1} = i) \qquad (1.2)$$

For i, j € E

Furthermore, it is assumed that the underlying variable that are responsible for the generation of loan disbursement do not change overtime, such that the transition probability has a stationary property i.e.

$$P\ (X_t = j/\ X_{t-1} = i) = P(\ t_{+1} = j/X_t = i\ ) \qquad (1.3)$$

for all t.

Also, the probability relations must be satisfied

$\sum P_{ij}=1$ and $0 \leq pij \leq 1$, for all i € E

1.5 SIGNIFICANCE OF THE STUDY

Loan portfolios form a greater portion of the total assets of banks in Ghana. These assets generate huge interest income for banks which to a large extent determines the financial performance of banks. It could therefore be concluded that a healthy loan portfolio has a direct bearing on the financial performance of banks. However, some of these loans usually fall into non-performing status and adversely affect banks' performance.

In view of the critical role banks play in the performance of the economy, it is essential to identify problems that affect the performance of these institutions. This is because these non-performing assets can affect the banks' ability to play their role in the development of the economy. In the light of the foregoing, the significance of the study includes the following:

• The findings would enable management of banking institutions come out with pragmatic policies for loan portfolio management aimed at improving the quality of their loan portfolios. The findings are expected to remind credit staff about the implications of their credit duties in creating quality loan portfolio for their banks.

• The findings of this study could be seen as a contribution to existing works on bad loans. Indeed, this would contribute immensely in building up academic knowledge in a wide range of issues.

• The study would also play a significant role of engineering further research into other aspects of the topic under consideration or other related topics in the banking sector. This would provide various solutions to some of the problems in banking institutions.

Thus through the above, the study would contribute significantly to the development of the banking industry which plays a pivotal role in the development of the economy. This is because the study also seeks to identify causes of bad loans in banks and recommend some measures that can solve these problems.

Generally, the study seeks to find out whether the loan portfolio provides adequate guidance to control the quality and quantity of credit risk. Also to determine whether risk can change or is likely to change because of portfolio changes.

1.6 LIMITATIONS OF THE STUDY

The research covers the loan portfolio selection of Opportunity International for the 2010 /2012 financial year due to unavailable data.

However, the problem of inadequate funds and time constraint compelled the researcher to be limited to one branch of the Financial Institution for all information.

1.7 ORGANIZATION OF THE STUDY

In this chapter, we considered the background, problem statement and objectives of the study.

The justification, methodology, scope and limitation of the study were also put forward. Chapter two presents relevant literature on the application of Markov Chain. Chapter three is devoted for the research methodology. In chapter four we shall put forward the data collection and analysis of the study. Chapter five, which is the final chapter of the study, presents conclusions and recommendations of the study.

1.8 SUMMARY

In this chapter, we considered the background of the study, problem statement, objectives, and justification, methodology and limitations of the study.

In the next chapter, we shall put forward relevant literature on the need for loan, portfolio management, Markov chain and its related problem.

CHAPTER TWO

LITERATURE REVIEW

2.0 INTRODUCTION

This chapter presents relevant literature on the need for loan, portfolio management and Markov chain and its related problem.

2.1 THE NEED FOR LOAN

According to Burton (2002), cited in Offei (2011) engaging in loan gives a greater amount of money to fulfill ones project. Some clients find it difficult to pay for these loans but they still want to apply for it due to financial constraints.

Most people apply for loans because of the following underlying reasons:

(i)To start or develop an existing business

(ii) To pay for existing loan

(iii) To own a property such as car, house etc

(iv) For educational purposes and others.

(v) To cater for unexpected emergency such as car repair, medical expense etc.

(vi) For recurring everyday expense such as rent, food, utilities etc.

2.2 BENEFITS OF LOAN PORTFOLIO

Light et al., (2005) indicated that loan portfolio management is one of the responsibilities critical to the success of an institution.

It is the dynamic process of managing an institution primary earning asset to achieve the primary objectives of the board’s strategic business and capital plans.

Loan portfolio encompasses all systems and processes used by management to adequately plan,

Achieved whether such results will continue and how the institution can optimize its opportunity.

Loan portfolio encompasses all systems and processes used by management to adequately plan, direct, control and monitor the institution lending operations.

Loan portfolio also ensures that all material aspects of lending operations are adequately controlled relative to the institutions risk bearing capacity.

Loan portfolio helps management and decision makers in the analysis of how business results are achieved whether such results will continue and how the institution can optimize its opportunity and provide great benefit to its members.

Loan portfolio also helps decision makers to measure the portfolio risk both for short term returns and hold long term strategy.

Finally it helps managers to minimize the finding of cost while lending against the market risk. (Offei, 2011)

2.3 PORTFOLIO MANAGEMENT

It is essential for sound operations of banks and lending institutions to have models and analytic tools available by which they can measure the performance (or health status) associated with a certain loan portfolio as well as predict this status over time from prevailing macroeconomic factors. A Markov chain defined on different payment states of a mortgage loan allows one to define and calculate a health index on the loan portfolio which can be used as a performance measure of that portfolio.

A performance measure such as a health index measure, for a mortgage portfolio will be useful for a bank or lending institution in its loan or credit policy. It will help the management to monitor the performance of its portfolio over time.

An empirical model that can relate a health index to macroeconomic factors will be useful in forecasting performance level. In a previous study (Liu et al., 2010) a Markov chain approach was developed to determine the transitions among payment states of a mortgage loan. Based on the probabilities of transitions among states, a loan health index was defined as a measure of its performance.

Amponsah et al., (2010) presented an optimal loan allocation mix policy from the steady state distribution of loan distribution. Monthly data on actual loan disbursement of four loans types for a period of twenty-four months was considered by using a transition matrix. From the estimated probability transition indicated that in a long run trade loan should constitute 77.3% of the total loan ,10.3%,for service loan ,2.0% for production loan and 10.4%for susu loan.

Loan Classification and Provision

• *Loan Classification*

Loan portfolios of banks are classified into various classifications to determine the level of provisions to be made in line with banking regulations. Loans are classified into five categories including Current, Other Loans Especially Mentioned (OLEM), substandard, doubtful and loss (Bank of Ghana, 2008). The classifications indicate the level of provisions banks are required to make to reflect the quality of their loan portfolio. Indeed the various classifications clearly group loans into performing and nonperforming, in line with banking regulations. These categories further help banks to know the structure of their loan portfolio and for that matter their assets quality.

Loan provisioning

In Ghana, a major factor considered in making loans is the ability of the borrower to repay the loan. However, to mitigate the risk of default, banks ensure that loans are well secured. Though advances shall be granted on the basis of the borrower's ability to pay back the advance and not on the basis to pledge sufficient assets to cover the advance in case of default, it is highly desirable for all advances made to customers and staff to be well secured. This means that in the event of default the bank shall fall on the collateral used in securing the facility to mitigate the effect of loss of principal and interest (Banking Act, 2004).

TABLE 2.1: CATEGORIES OF LOANS AND THEIR PROVISION

CATEGORY	PROVISION (%)	NO. OF DELINQUENT
Current	1	0-less than 30
OLEM	10	30-less than 90
Substandard	25	90-lessthan 180
Doubtful	50	180-lessthan 360
Loss	100	360 and above

From table 2.1, banks take into account the assets used in securing the facility to determine the level of provision to be made. Bank of Ghana regulations indicate that certain amount of provisions are made on the aggregate outstanding balance of all current advances, and aggregate net unsecured balance of all other categories as shown in the Table 2.1

This constitutes huge cost to banks. In 2006, ADB made a total provision for bad and doubtful loans to the tune of GH¢35,080,800.00 which reduced the bank's loan portfolio from GH ¢186,004,100.00 to GH¢150,923,300.00.The bank's charge for bad debts also reduced its net interest income by about 25% (ADB, 2006)

Study of the financial statement of banks indicates that bad loans have a direct effect on profitability of banks. This is because charge for bad debts is treated as expenses on the profit and loss account and as such impact negatively on the profit position of banks. For example Barclays Bank Ghana Limited declared a loss in its 2008 financial statement partly due to the huge charge for bad debts which increased from GH¢5,540,000.00 in 2007 to GH¢46,890,000.00 in 2008 (Price Water-House Coopers, 2009). The annual report of ADB for 2007 showed that the bank had embarked on a five-year bad loan provisioning which affected its profitability during the period. The report indicated that the net profit for 2007 decreased by 13.81% which was attributed mainly to the non-performing loan provisions.

Some foreign literature indicates that bad loans can fuel banking crisis and subsequently result in the collapse of banks with huge non-performing loans. Demirgue-Kunt et al (1989), cited in Berger and De Young (1997) indicated that failing banks have huge proportions of bad loans prior to failure and that asset quality is a statistically significant predictor of insolvency As was indicated earlier in this research, Caprio and Klingebiel (2002), cited in Fofack (2005), reported that during the banking crisis in Indonesia, non-performing loans represented about 75% of total loan assets, which led to the collapse of over sixty banks in 1997. This means that Banks holding huge bad loans in their books can run into bankruptcy if such institutions are unable to recover their bad debts.

A possible effect of bad loans is on shareholders earnings. Dividends payments are based on banks performance in terms of net profit. Thus since bad loans have an adverse effect on profitability of banks, it can affect the amount of dividend to be paid to shareholders. The Banking Act of Ghana spells out that a bank shall not declare or pay dividend on its shares unless it has, among other things, made the required provisions for nonperforming loans and other erosions in assets value (Section 30 (1) of Banking Act, 2004)

The effect of bad loans on the amount of dividend paid to shareholders can also affect capital mobilization because investors will not invest in banks that have huge non-performing loans portfolio.

Elebute (2009) identified among other things, foreign direct investment and domestic capital mobilization as some of the options available to Ghanaian banks to source funds to meet the minimum capital requirement of Bank of Ghana which is pegged at GH¢60,000,000.00.

It is evident that non-performing loans with their attendant negative impact on investors' earnings can affect the Ghanaian banks in meeting the minimum capital requirement.

The foregoing discussions show the implications of bad loans on banks performance in Ghana and other parts of the world. To ensure a comprehensive study, the causes of these bad loans in Ghana were identified to enable the study offer some suggestions to reduce the problem. (Asamoah, 2009).

2.4 MARKOV CHAIN APPROACH

A Markov chain, named after Andre Markov, is a mathematical system that undergoes transitions from one state to another, between a finite or countable number of possible states. It is a random process usually characterized as memory less this is due to the fact that the next state depends only on the current state and not on the sequence of events that preceded it. This specific kind of "memorylessness" is called the Markov property. Markov chains have many applications as statistical models of real-world processes (Wikipedia)

Formally, a Markov chain is a random process with the Markov property. Often, the term "Markov chain" is used to mean a Markov process which has a discrete (finite or countable) state-space. Usually a Markov chain is defined for a discrete set of times (i.e., a discrete-time Markov chain,(Everitt,2002) although some authors use the same terminology where "time" can take continuous values.

The use of the term in Markov chain Monte Carlo methodology covers cases where the process is in discrete time (discrete algorithm steps) with a continuous state space. The following concentrates on the discrete-time discrete-state-space case. (Parzen, 1962).

Since the system changes randomly, it is generally impossible to predict with certainty the state of a Markov chain at a given point in the future. However, the statistical properties of the system's future can be predicted. In many applications, it is these statistical properties that are important. The changes of state of the system are called transitions, and the probabilities associated with various state-changes are called transition probabilities. The set of all states and transition probabilities completely characterizes a Markov chain. By convention, we assume all possible states and transitions have been included in the definition of the processes, so there is always a next state and the process goes on forever.

A famous Markov chain is the so-called "drunkard's walk", a random walk on the number line where, at each step, the position may change by +1 or −1 with equal probability. From any position there are two possible transitions, to the next or previous integer. The transition probabilities depend only on the current position, not on the manner in which the position was reached.

Thyagarajan et al., (2011) analyzed the actual loan sanctions with the non-documented method of loan allocation of the selected retail bank over a period of twenty-four (24) months revealed that there is a scope to their income earnings. Realizing its importance Markov Chain Market Share model was applied to inter temporal data of loan disbursements of the selected bank. By applying Estimate Transition Matrix, scope for probability of loan switching among its types was calculated to suggest the probable mix of loan portfolio. From the results it was suggested that the loan proportions among various types were as follows: Housing (32.0%), others (28.1%), Business (20.0%) and Education (19.7%). These proportions can be taken as guideline percentage within the government norms for the priority sector. Simulation studies were also done to calculate the expected income of interest using Markov proportions and compared with the actual interest earnings to prove the superiority of the model.

Mariano et al., (1970) used probability in constructing a monthly model for predicting currency crises in Southeast Asia. .

The approach was designed to avoid the estimation inconsistency that might arise from misclassification errors in the construction of crisis dummy.

Demiris, (2006) uses Markov chain in his Bayesian Inference for Stochastic Epidemic model. The researcher Used Susceptible-infective-Removed (SIR) epidemic models in his analysis.

Baik et al., (2006) stated that the accurate prediction of the current and future conditions of waste water systems using available assessment data is crucial for developing appropriate proactive maintenance and rehabilitation strategies for an aging waste water collection and conveyance system. The authors proposed a method to estimate the transition probabilities of different condition states in Markov chain based deterioration models for waste water system using an ordered probit model. The proposed model was applied and evaluated using the condition data of sewer pipes managed by the City of San Diego s metropolitan Wastewater Department. The developed model presented some advantages in estimating transition probabilities over the approaches developed in the past in terms of versatility in the implementation, precision of the estimated data, and appropriateness of the assumptions in the model adopts the Bayesian paradigm and developed suitably tailored Markov chain Monte Carlo (MCMC) algorithms. The focus is on methods that are easy to generalize in order to accommodate epidemic models with complex population structures.

Soyer and Feng, (2010) considered reliability models for assessing mortgage default risk. White (1993) presented several models employed in the banking industry. These included discriminant analysis, decision tree, expect system for static decision, dynamic programming, linear programming, and Markov chains for dynamic decision making. Markov chain modeling is a common approach used in the analysis of credit risk. As discussed by White (1993), Markov decision models have been used extensively to analyze real world data in (i) Finance and Investment, (ii) Insurance, and (iii) Credit area.

Cyert et al., (1962) developed a finite stationary Markov chain model to predict uncollectible amounts (receivables) in each of the past due category. The states of the chain were defined as normal payment, past due, and bad-debt states.

Grinold, (1983) used a finite Markov chain model to analyze a firm's market value.

Lee (1997) used an ARMA model to analyze the linkage between time-varying risk premier in the term structure and macroeconomic state variables.

This dissertation is about the Reversible-Jump Markov Chain Monte Carlo (RJMCMC) method in Bayesian statistics and its application to solving the change-points problem. A change-points problem is to determine the parameters of a step model with flexible number of steps. This model is widely used in different fields. Thus many applications, such as change-points in genes determining and flexible number's variables selection in regression, benefit from RJMCMC.

Jialin, (2011) introduced Markov chain Monte Carlo (MCMC) and the Metropolis-Hastings (MH) Algorithm. The concept "reversibility", and the relationship between "reversible" and "stationary" was explained, and then prove that the Markov chain generated by Metropolis-Hastings is reversible and stationary.

The author introduced Bayesian estimation and realized it with M-H algorithm. He gave a simple and clear example and implemented it in R with Bayesian estimates algorithm. Next, he introduced the subspaces assumption of reversible-jump Markov chain, and the basic form of the acceptance probability jumping between subspaces of differing dimensionality based on Green (1995).

He solved a change-points problem of coal-mining disasters data and calculated the acceptance probabilities of different movement types; combine the results and the Metropolis Hastings algorithm into an R program.

Liu et al., (2009), said Markov chain has been widely applied in the disciplines of natural science, engineering, economics and management. This approach has also been widely used in drought forecasting, Lohani and Loganathan, (1997); Lohani et al., (1998).

Paulo and Pereira (2007) stated that the Markov chain modeling approach is useful in understanding the stochastic characteristics of droughts and rainfall through the analysis of Probabilities for each severity class, times for reaching the non-drought class from any drought severity state, and residence times in each drought class were then analyzed. They found that the approach can be satisfactorily used as a predictive tool for forecasting transitions among drought severity classes up to 3 months ahead.

Lohani and Loganathan (1997) and Lohani et al., (1998) developed an early warning system for drought management using the Markov chain, in two climatic areas of Virginia (U.S.A.). The same approach was also adopted for developing a meteorological drought/rainfall forecasting model by Liu et al., (2009) in Laohahe catchment in northern China. In their study, spatio-temporal distributions were analyzed and forecasted by Markov chain.

Steinemann (2003) adopted six classes of severity, from wet to dry conditions, similar to those in PDSI, and used the Markov chain to characterize probabilities for drought class and duration in a class. The results obtained were used to propose triggers for early-activating of the drought preparedness plans at the basin scale.

Liu et al., (2009) demonstrated two advantages of the Markov chain technique for forecasting drought and rainfall conditions. They were: (i) the predictive performance increased greatly as the severity of drought increased, and (ii) the predictive performance was always satisfactory for drought state transitions, and the prediction performance was acceptable for the successive and smooth states.

For accounting dependence in any time series, often a first order Markov Chain is used for modeling. For instance, large variety of weather events modeling and simulation were studied through Markov Chain.(Gringorten1996).

Racsko et al., (1991) had achieved long time series of weather data generations also**.** For rainfall data, many authors have demonstrated that Markov Chain model is used to synthesize rainfall time series.

Gabriel and Neumann (1962) started the study on the sequence of daily rainfall occurrence. They found that the daily rainfall occurrence for the Tel Aviv data was successfully fitted with the first-order Markov chain model.

Markov chain Monte Carlo algorithms

Markov Chain Monte Carlo (MCMC) methods are a class of algorithms which are used for simulating samples from a posterior distribution that has the desired true posterior distribution as its stationary distribution.

Briey, (1998) described Markov chain as a large number of samples from the distribution. Since the samples are Markov generated two successive samples are usually strongly dependent. One often samples only every mth iteration since the large amount of output that is generated if sampling every generation may be unpractical due to memory constraints of the computer. This procedure is called thinning of the chain. Notice that since successive samples are dependent, one does not lose nearly as much information as discarding independent observations. By the Monte Carlo property we then get our desired posterior distribution by observing the relative frequencies of the samples generated from this chain. (Kristoffer, 2011)

The MCMC algorithm has been known in statistics for many decades (Metropolis et al., 1953), however, it took many years before statisticians understood the full potential of the algorithm. Step by step, the MCMC algorithm was applied to many applications which required approximations of difficult distributions, such as the posterior distribution of phylogenetic trees. Many applications for the MCMC algorithm are known, such as statistical physics, molecular simulation, dynamic system analysis, computer vision and Bayesian phylogenetic inference. In 1996, the first Markov chains for sampling from phylogenetic trees were introduced (Rannala and Yang, 1996; Mau and Newton, 1997; Li et al., 2000).

Since that time, the tree proposal distributions have not changed much although the literature is full of more complex concepts of transition Kernels for other applications (Gilks et al., 1996; Brooks, 1998; Liu, 2001).

The transition Kernels need to be adapted for the specific problem domain to achieve a high efficiency.

However, not much is known about the efficiency of the tree proposal operators. Some Theoretical discussions have been initiated, but an empirical study of real data is missing

(Mossel andVigoda, 2005; Ronquist et al., (2006).

Lakner et al., (2002) provided a first evaluation of the tree proposal operators in Bayesian phylogenetic inference; however, they used unrooted, unconstrained trees. The efficiency of the current tree proposal operators for rooted, clock-constrained trees remains unknown. The first goal of this research is to study the performance of the currently used tree proposal operators. The focus of the study is to identify the best operator available and provide an analysis of its shortcomings and their causes. This knowledge should help to design new tree proposal operators which are more efficient. The current tree proposal Operators are simplistic compared to the variety of transition kernels discussed in the Statistics literature, however, the problem domain of phylogenetic inference is complex and renders many methods difficult or impossible. A major problem with current MCMC approaches is their unreliability (Huelsenbeck et al., 2002). Although many techniques for determining the convergence of an MCMC Run exist in theory, it is difficult to determine with high confidence that the MCMC run has converged. Therefore, there is a great need to increase the reliability of an MCMC.

Watanabe, et al., (2000) proposed an approximation method of the expected State for homogeneous Markov chains, named as pseudo expectation. It is based on simple Probabilistic recurrence formulas. The aim of this work is to give a bound on the approximation error of the pseudo expectation. Two bounding techniques are explained. Both are given as recurrence formulas. The first one uses statistical properties of the process. It requires at least a bound on the 2nd moment, e.g. the variance.

The second one, the so called linearization error, which captures the non-linearity of the process, is expressed in terms of the function f used to iteratively calculate the pseudo expectation. The use of these techniques is demonstrated by an example, giving an explicit error formula (in closed form) for the considered process.

Markov Decision Processes

Markov Decision Processes (MDPs) provide a mathematical framework for modeling decision making in situations where outcomes are partly random and partly under the control of the decision maker. They originated in the study of stochastic optimal control in the 1950s and have remained of key importance in that area ever since. Their theory has continued to develop over the last decades to fit a broader spectrum of problems and has led to a wealth of common algorithmic ideas and theoretical analysis. Today MDPs are used in a variety of areas, including robotics, automated control, planning, economics and manufacturing. An MDP consists of an agent and an environment that the agent interacts with. These interactions happen over a sequence of discrete time steps t; at each time step the agent perceives the state of the environment s_t and selects an action at to perform. The environment reacts to the action by making a transition to a new state st+1 and returns a scalar reward. The agent's goal is to maximize the total amount of reward it receives from its interactions with the environment. The dynamics of the environment tare stationary and the state signal must contain all relevant information but is otherwise unconstrained(Wikipedia).

Markov Decision Processes provide a mathematical framework for modeling decision making in situations where outcomes are partly random and partly under the control of the decision maker. MDPs are used in a variety of areas including robotics, automated control, planning, economics and manufacturing. For the solving of Markov Decision Processes (MDP) various different approaches exists. Value Iteration is an algorithm which falls under the class of Dynamic Programming methods and can be used to solve MDPs. In recent years Graphics Processing Units have been evolving rapidly from being very limited processing devices with the sole purpose of accelerating certain parts of the graphics pipeline into fully programmable and powerful parallel processing units. In the autumn of 2007 Network Value Iteration Divide Iteration Algorithm (NVIDIA) introduced CUDA, a hardware and software architecture for utilizing the Graphics Processing (GPU) for general purpose calculations. In this thesis we introduce two parallel CUDA based implementations of the Value Iteration algorithm: Block Divided Iteration and Result Divided Iteration. We discuss the different approaches each algorithm takes for utilization of the parallel processing power of the CUDA device. We also present a framework we implemented which enables researchers to easily apply the parallel algorithms to MDPs within C or C++ applications. Empirical results are also presented which show a substantial performance improvement achieved by the parallel algorithms compared to a sequential implementation running on a Computer Processing Unit (CPU) .(ÁrsællÞór 2009)

Lukosz (2006) presents the theory applicable to the option pricing and short fall risk minimization problem. The market is arbitrage-free without transaction costs and the underlying asset price process is assumed to possess a Markov chain structure. Under these assumptions, stochastic dynamic programming is exploited to price the European type option. By using the utility Concept, the Fundamental Theorem of Asset Pricing is proved via portfolio optimization. Furthermore, it is shown how to use dynamic programming to control the risk related to the future payoff of the option. The approach extends to the case when there is restricted information on the underlying asset price evolution. The methods deal with both complete and incomplete markets.

Osain's (2006) paper on Software Reliability Using Markov Chain Usage Model revealed that Statistical testing gives us opportunity to have statistical inferences such as reliability, mean time to failure (MTTF) etc. for software systems and Markov chain usage model gains its credibility in this field. Markov chain usage model has several benefits. It allows generating test sequences from usage probability distributions, assessing statistical inferences based on analytical results associated with Markov chains and also to derive stopping criterion of the test process. But the main problem in this process is to model software behavior in a single Markov chain. For large software systems the model size i.e. the number of sates become unwieldy and it becomes infeasible to apply this method in generating test cases as well as measuring reliability.

Two Markov models called usage chain and testing chain are developed from the example software. The discriminant value of the two chains is determined to analyze software reliability. As the software becomes more complex the model size grows quickly, which is known as state explosion problem. To overcome this problem a technique is presented to measure software reliability by combining the ideas drawn from stochastic modeling, statistical testing using Markov chain usage model and component based software testing. The researcher took example from database based application software, find its modules, in this case forms, and measure reliability of each forms using Markov chain usage model. The system reliability from those form's reliabilities according to their usage probabilities is then analyzed. Our experimental efforts lead us to a more practical and effective approach for software reliability and quality assurance.

Markov Chain Model for Statistical Software Testing

Statistical testing of software establishes a basis for statistical inferences about a software systems expected field quality. They described a method for statistical testing based on a Markov chain model of software usage. The significance of the Markov chain is twofold. First, it allows test input sequences to be generated from multiple probability distributions, making it more general than many existing techniques. Analytical results associated with Markov chains facilitate informative analysis of the sequence before they are generated, indicating how the test is likely to unfold. Second, the test input sequences generated from the chain and applied to the software are themselves a stochastic model and are used to create a second Markov chain to encapsulate the history of the test, including any observed failure information. The influence of the failures is assessed through analytical computations of the chain. We also derive a stopping criterion for the Testing process based on a comparison of the sequence generating properties of the two Chains. (Wikipedia)

Statistical testing process can be carried out in three major steps.

Step 1: Construct the statistical models based on actual usage scenarios and related frequencies.

Step 2: Use these models for test case generation and execution.

Step 3: Analyze the test results for reliability assessment and predictions, and help with decision-making.

In Markov chain based statistical testing software usage behavior is modeled as a finite state, discrete parameter, time homogeneous Markov chains. It is known as usage Markov *chain* or in short *usage model.* The usage model consists of elements from *d*, the domain of the intended function, and a probabilistic relationship defined on these elements. A test input is a finite sequence of inputs form domain *d* probabilistically generated forms the usage model. The statistical properties of the model lend insight into the expected makeup of the sequences for test planning purposes.

As the test sequences are applied to the software, the results are incorporated into a second model. This *testing model* or the *testing Markov chain* consists of the inputs executed in the test sequences, plus any failures discovered while applying the sequences to the software *P*. in other words; it is a model of what has occurred during testing. The testing model also allows analysis of the test data in terms of random variables appropriate for the application. For example, we may measure the evolution of the testing model and decide to stop testing when it has reached some suitable "steady state". (Wikipedia)

The Usage Markov Chain

A usage chain for a software system consists of states, i.e., externally visible modes of operation that must be maintained in order to predict the application of all system inputs, and state transitions that are labeled with system inputs and transition probabilities. To determine the state set, one must consider each input and the information necessary to apply the input. It may be that certain software modes cause an input to become more or less probable (or even illegal). Such a mode represents a state or set of states in the usage chain. Once the states are identified, we establish a start state, a terminate state (for bookkeeping purposes), and draw a state transition diagram by considering the effect of each input from each of the identified states. The Markov chain is completely defined when transition probabilities are established that represent the best estimate of real usage.

Nottenbelt, (1997) presented an analytical method for assessing the Generalized Markovian Analysis of Timed transition System. The performance of concurrent systems showed focus on the efficient generation and solution of large Markov chains, which are derived from models of unrestricted time transition system.

Timed transition systems may be described using several high level formalisms, including generalized Stochastic Petrinets, queuing networks and queuing Petrinets.

A system modeled with one of these formalisms may be mapped onto a Markov chain through a process known as state space generation.

The Markov chain thus generated can then be solved for its steady state distribution by numerically determining the solution to a large set of sparse linear equations known as the steady state equations. Existing techniques of state space–saving probabilistic dynamic state management technique is proposed and analyzed in terms of its reliability and space complexity.

(Wikipedia).

State space reduction technique s involving on the fly elimination of vanishing states are also considered. Linear equation solvers suitable for solving large sparse sets of linear equations are surveyed, including direct methods, classical iterative methods, Krylov substance techniques and decomposition based techniques.

Emphasis is placed on Krylov subspace technique and the Aggregation-Isolation technique which is a recent decomposition based algorithm applicable to solving general Markov chains. Since Markov chains derive from real life models may have very large state spaces, it is desirable to automate the performance analysis sequence.

Consequently, the new state management technique and several linear equation solvers have been implemented in the Markov chain analyzer De-Ox ribonucleic Acid (DNA) maca. DNA maca accepts a high level model description of a timed transition system generate the state space, derives and solves the steady state equations and produces performance statistics.

Hidden Markov Models

Hidden Markov Models (HMMs) are a popular probabilistic framework for modeling processes that have structure in time. They have a clear Bayesian semantics, efficient algorithms for state and parameter estimation, and they automatically perform dynamic time warping for signals that are locally squashed amid stretched. An HMM is essentially a quantization of a system's configuration space into a small number of discrete states, together with probabilities for transitions between states. A single finite discrete variable indexes the current state of the system. Any information about the history of the process needed for future inferences must be reflected in the current value of this state variable. (Wikipedia)

Coupled Hidden Markov Models

In a regular Markov Model, the conditional probability distribution of the hidden variable *St* at time *t*, given the value of the hidden variable S_{t-1}, depends only on the value of the hidden variable S_{t-1}: the values at time $(t - 2)$ and before have no influence. Any information about the history of the process needed for future inferences must be reflected in the current value of this state variable.

However, many interesting systems are composed of multiple interacting processes, and thus merit a compositional representation of two or more variables. This is typically the case for systems that have structure both in time and space. With a single state variable, Markov models are ill-suited to these problems. In order to model these interactions a more complex architecture is needed (Wikipedia).

Esbitt, (1986) provided empirical evidence that a bank's portfolio quality has close relationship with the macroeconomic situation. Examples include the state-chartered banks' failure and the Great Depression in Chicago between 1930 and 1932.

McNulty et al.,(2001) proposed an empirical regression modeling approach to study the hypothesis that small community banks have an information advantage in evaluating and monitoring loan quality.

Hauswald and Marquez (2004) studied the relationship between the current regulative policy and the loan quality, or risks encountered by a financial institution.

Gambera, (2000) used a vector-autoregressive (VaR) model to predict the loan quality in business cycles. D'Amico et al., (2005) applied Semi-Markov reliability models to the study of credit risk management.

Douglas et al., (1996) proposed the use of non-stationary Markov and logistic modeling approaches to predict the performance of credit home mortgage portfolios. Pennington-Cross (2008) used a multinomial logit model to study the duration of foreclosure in the subprime mortgage market.

Burkhard and De-Giorgi (2006) used a non-parametric approach to model the probability distribution of defaults in residential mortgage portfolios. Hayre et al., (2008) presented a model that forecasts default rates as a function of economic variables and mortgage and borrower characteristics.

Green and Shoven (1986) used a proportional hazard model to study the effects of interest rate on mortgage prepayment.

Deng et al., (2000) used the option theory approach to predict mortgage termination by prepayment or default. They showed that the model performed well, but was not sufficient by itself. Heterogeneity among homeowners must be taken into account in estimating or predicting the prepayment behavior.

Schwartz and Tourous (1993) applied a Poisson regression to estimate the proportional hazard model for prepayment and default decisions in a sample of single-family fixed rate mortgages.

Amponsah et al., (2006) modeled a banking policy for Atwima Kwawoma Rural bank by using Linear Programming Method.

The bank's policy of granting loans were modeled as a linear programming problem with respect to profit and budget constraints on the loan portfolios.

Their research showed greater profit and expansion of service if recommendation were to be implemented .The bank found their policy proposal suitable for implementation.

Analysis of a simple method to approximate the expected state of a Markov chain was put forward by Johannes Schneider (2004).

The goal of this thesis is to estimate and bound the error of pseudo expectations. For this goal, we obtained the following main results. Two general bounding techniques are explained. Both are given as recurrence formulas. The first one, given in chapter 4, uses statistical properties of the Markov chain, e.g. requires at least a bound on the 2nd moment of the stochastic process. The second one, the so called linearization error, which captures the non-linearity of the process, is expressed in terms of the function f, which is used to iteratively calculate the pseudo expectation. Apart from that a technique for computing a lower and upper bound for the convergence speed of the pseudo expectation is given. For the considered example process, a tight bound for the steady state error has been obtained.

Henry et al., (2011) evaluated an investor's ability to choose a specific risk profile for their portfolio among three optimization strategies. A mean-variance, min-variance, and 1/N rebalancing out-of-sample strategies for randomly constructed portfolios of liquid S&P500 assets between 2006 and 2011 were evaluated. The reduction in variance between these three portfolio strategies was shown to be significant providing the investor a low risk, medium risk, and high risk portfolio with the chosen assets. Further, we show that the returns of the three portfolios are statistically significant and confirm that higher risk leads to higher rewards. Lastly, it was shown that no strategy is dominant as measured by Sharpe Ratio because of estimation error in the mean-variance optimization. However, there is clear dominance in risk reduction strategies. It was then concluded that investors can select a risk profile optimization strategy after choosing the desired assets. This has strong implications for the traditional financial services which currently choose assets based on the desired risk profile.

Amir et al., (2007) did an empirical study on Harry Markowitz work on Modern Portfolio Theory model introduced by him assumes the normality of assets' return. The OMX Large Cap List by mathematical and statistical method for normality of assets' returns was examined and the effect of the parameters, Skewness and Kurtosis for different time series data was as well studied. The research figured out which data series was better to construct a portfolio and how these extra parameters could make us better informed in our investments.

2.5 SUMMARY

In this chapter we considered relevant literature on the need for loan, portfolio management, the markov chain approach, benefits of loan portfolio and other related areas. The next chapter presents an introduction to Markov chain, portfolio theory and selection.

CHAPTER THREE

METHODOLOGY

3.0 INTRODUCTION

In this chapter, we shall put forward the research methodology of the study. The first section of the chapter shall be devoted for the profile of Opportunity International Savings and Loans Limited.

3.1 PROFILE OF OPPORTUNITY INTERNATIONAL

The Financial Institution chosen for the study is one of the largest non- Banking Financial Institutions in Ghana. Opportunity International Savings and Loans Limited is a non- banking financial institution licensed by the Central Bank of Ghana in 2001 to operate in savings and loans. Its asset position crosses more than seventy billion Ghana cedis with over twelve branches in the Ashanti, Brong Ahafo and the Greater Accra Regions of Ghana.

3.1.1 VISION

To see the lives of micro and small entrepreneurs transformed through a partnership in which we provide customer-focused financial and transformational services.

3.1.2 MISSION

To serve micro and small entrepreneurs with loans, deposits and other financial services that enable them to increase their income and help transform their lives while earning appropriate returns for our shareholders.

3.1.3 CORE VALUES

The core values of Opportunity International include;

(i) C**ommitment** to the highest quality of care.

(ii) Humility towards clients.

(iii) Respect to all clients.

(iv) Integrity in product prices and fees that are clearly shown; that clients are not overburdened with debt and

(v) Transformation in supportive behavior at all times.

3.1.4 CORE PRODUCTS

The core products of Opportunity International include**;**

(i) Susu loan (SL)

(ii) Small and medium Enterprise loan (SME)

(iii) Agricultural Loan (AL) and

(iv) Salary Loan (SL)

3.2 RESEARCH METHODOLOGY

Monthly data on actual loan disbursement for a period of twenty-four (24) months beginning January 2010 (t=1) for four types of loans are used as a basis to estimate the transition probability matrix. The appropriate vectors and matrix are defined so as to be able to apply an optimization routine. Since $y_j(t)$ is defined as a proportion of loan type j at time t the actual loan disbursement for each period t.

The study aimed to devise the determination of equilibrium loan allocation using Markov Chain Market Share Model. The main objective of the study is to obtain optimal loan allocation mix policy, which would be used as guiding principle on future allocation purpose. Time series data on four loan disbursements are used as a basis of estimating a Transition Probability Matrix. (TPM). Transition Probability Matrix gives the probability of loan switching from one type to another type. Markov chain model will then be applied to the set of time series on the actual loan disbursement proportions to calculate the estimated TPM using a quadratic programming technique. Further, several statistical tests will be conducted to investigate the suitability of using macro data to test the homogeneity, communicability, periodicity and absorption status of the process. Forecast of loan disbursement for a period of twenty-four (24) months will then be made, in order to forecast the future allocation of each type of loan.

3.3 MARKOV CHAIN.

A Markov chain is a random process usually characterized as memory less, this is due to the fact that the next state depends only on the current state and not on the sequence of events that preceded it. In other words,

A Markov chain is a sequence of random variables X_1, X_2, X_3, ... with the Markov property, namely that, given the present state, the future and past states are independent. Formally,

$$P\,(X_{n+1}=x/X_1=x_1, X_2=x_2 \ldots, X_n=x_n)=P(X_{n+1}=x/X_n=x_n) \qquad (3.1)$$

The possible values of Xi form a countable set S called the state space of the chain.

Markov chains are often described by a directed graph, where the edges are labeled by the probabilities of going from one state to the other states.

STATE SPACE

Assume we have a set of *states*, $S = \{S_1, S_2, S_3, \ldots S_R\}$ The process starts in one of these states and moves successively from one state to another. Each move is called a *step.* If the chain is currently in state s_i, thenit moves to state$s_j$at the next step with a probability denoted by p_{ij}, and this probability does not depend upon which states the chain was in before the current state. The probabilities p_{ii} are called *transition probabilities.* The process can remain in the state it is in, and this occurs with probability p_{ii}. An initial probability distribution, defined on *S*, specifies the starting state. Usually this is done by specifying a particular state as the starting state

MARKOV PROPERTY

A Markov chain is a sequence of random variables X_1, X_2, X_3, ... with the Markov property, namely that, given the present state, the future and past states are independent. Formally,

$P\ (X_{n+1}=x/X_1=x_1, X_2=x_2 \ldots, X_n=x_n)=P(X_{n+1}=x/X_n=x_n)$

TRANSITION MATRIX

Let **P** be the transition matrix of a Markov chain, and let **u** be the probability vector which represents the starting distribution. Then the probability that the chain is in state *Si* after *n* steps is the *i*th entry in the vector $\mathbf{U}^n = \mathbf{U}P^n$

Figure 1.2 represents the transition states of a matrix from states A to state D with some probability P.

ABSORBING MARKOV CHAIN

The subject of Markov chains is best studied by considering special types of Markov Chains. The first type that we shall study is called an *absorbing Markov chain.*

A state S_i of a Markov chain is called *absorbing* if it is impossible to leave it (i.e., $P_{ii}= 1$). A Markov chain is *absorbing* if it has at least one absorbing state, and if from every state it is possible to go to an absorbing state (not necessarily in one step).

In an absorbing Markov chain, a state which is not absorbing is called *transient.*

$\mathbf{N} = (\mathbf{I} - Q)^{-1}$.

For an absorbing Markov chain the matrix $(\mathbf{I} - \mathbf{Q})$ has an inverse $\mathbf{N}$ and $\mathbf{N} = \mathbf{I} + \mathbf{Q} + \mathbf{Q}^2 + \ldots$

The ij-entry n_{ij} of the matrix $\mathbf{N}$ is the expected number of times the chain is in state S_j, given that it starts in state S_i. The initial state is counted if $i = j$.

CANONICAL FORM

Consider an arbitrary absorbing Markov chain. Renumber the states so that the transient states come first. If there are r absorbing states and t transient states, the transition matrix will have the following *canonical form*

$$\mathbf{P} = \begin{array}{c} \\ \textbf{TR} \\ \textbf{ABS} \end{array} \begin{array}{c} \begin{array}{cc} \textbf{TR} & \textbf{ABS} \end{array} \\ \left(\begin{array}{c|c} \mathbf{Q} & \mathbf{R} \\ \hline \mathbf{0} & \mathrm{I} \end{array}\right) \end{array}$$

Here **I** is an r-by-r identity matrix, **0** is an r-by-t zero matrix, **R** is a nonzero t-by-r matrix and **Q** is an t-by-t matrix. The first t states are transient and the last r states are absorbing ij of the matrix $\mathbf{P}^n$ is the probability of being in the state S_j after n steps, when the chain is started in state Si. A standard matrix algebra argument shows that $\mathbf{P}^n$ is of the form

$$\mathbf{P}^n = \begin{array}{cc} & \begin{array}{cc} \mathbf{TR} & \mathbf{ABS} \end{array} \\ \begin{array}{c} \mathbf{TR} \\ \text{ABS} \end{array} & \left(\begin{array}{c|c} \mathbf{Q} & * \\ \hline O & I \end{array}\right) \end{array}$$

PROBABILITY OF ABSORPTION

In an absorbing Markov chain, the probability that the process will be absorbed is 1.From each non absorbing state S_j it is possible to reach an absorbing state. Let M_j be the minimum number of steps required to reach an absorbing state, starting from S_j. Let P_j be the probability that, starting from S_j, the process will not reach an absorbing state in M_j steps. Then p_j<1. Let m be the largest of the M_j and let p be the largest of P_j. The probability of not being absorbed in m steps is less than or equal to p, in 2n steps less than or equal to P^2, etc. Since p <1 these probabilities tend to 0.

Let b_{ij} be the probability that an absorbing chain will be absorbed in the absorbing state$S_j$if it starts in the transient state S_i Let **B** be the matrix with entries b_{ij}.Then **B** is an t-by-r matrix, and

B = **NR**

Where **N** is the fundamental matrix and **R** is as in the canonical form.

TIME TO ABSORPTION

We now consider the question: Given that the chain starts in state S_i, what is the

Expected number of steps before the chain is absorbed?

Let t_i be the expected number of steps before the chain is absorbed, given that the chain starts in state S_i, and let **t** be the column vector whose *i*th entry t_i. Then **t** = **Nc**

Where **c** is a column vector with all entries being 1.

ERGODIC MARKOV CHAIN

A Markov chain is called an *ergodic* chain if it is possible to go from every state to every state (not necessarily in one move).

EQUILIBRUIM

Let **P** be the transition matrix for a regular chain. Then, as $n \longrightarrow \infty$ the powers P^n

Approaches a limiting matrix **W** with all its rows having the same vector **w**.

The vector **w** is a strictly positive probability vector (i.e., the components are all positive)

We also obtain a new interpretation for **w**. Suppose that our starting vector picks

State S_i as a starting state with probability w_i, for all *i*. Then the probability of being in the various states after *n* steps is given by $\mathbf{wP}^n = \mathbf{w}$, and is the same on all steps. This method of starting provides us with a process that is called stationary. "The fact that **w** is the only probability vector for which $\mathbf{wP} = \mathbf{w}$ shows that we must have a starting probability vector of exactly the kind described to obtain as stationary process. Many interesting results concerning regular Markov chains depend only on the fact that the chain has a unique fixed probability vector which is positive. This property holds for all ergodic Markov chains.

3.4 MARKOV PROBABILTY MODEL

The probability of switching a loan disbursement from loan type *i* to loan type *j* is a conditional probability and can be represented by the transition matrix ***P*= [**P_{ij}**]** such that

$$\sum_{i=1}^{m} P_{ij} = 1$$

Indices *i* refer to the loan type. For example P_{21} represents the probability of a Change in loan disbursement from A to B in the next period of time. While P_i Represents the probabilities of no change in loan disbursement for loan type *i*. The stochastic model used to explain the loan disbursement behavior is a Markov Chain with finite number of States $\{E\}$ Markov process $\{Xt\}$ with discrete time *t* such that Pij in general represents the probability of the process moving from state *i* at time (*t*-1) to state *j* at time *t*. In this study we assume that the loan disbursement for type *i* in the next period *t* (month) is only determined by the loan disbursement at the preceding period (t-1). In other words, the "history "of loan disbursement before time t-1 does not influence the future loan disbursement. This is known as a first order time dependency. In statistical notation it is represented as;

$$P\ (X= j\ |X_0, X_{1,\ldots}X_{t-1}= i\ )\ =P\ (X_t= j|\ X_{t-1}= i) \qquad (3.2)$$

For $i,\ j \in E$

Furthermore, it is assumed that the underlying variable that are responsible for the generation of loan disbursement do not change overtime, such that the transition probability has a stationary property i.e.

$$P(X_t = j / X_{t-1} = i) = P(_{t+1} = j / X_t = i) \qquad (3.3)$$

For all t.

Also, the probability relations must be satisfied

$\sum P_{ij} = 1$ and $0 \leq p_{ij} \leq 1$, for all i € E

3.5 PORTFOLIO THEORY

The fundamental concept behind Modern Portfolio Theory is that the asset in an investment portfolio should not be selected individually, each on its own merits. Rather, it is important to consider how each asset changes in price relative to how every other asset in the portfolio changes in price. Investing is a tradeoff between risk and expected return.

In general, assets with higher expected returns are riskier. For a given amount of risk, Modern Portfolio Theory describes how to select a portfolio with the highest possible expected return. Or, for a given expected return, Modern Portfolio Theory explains how to select a portfolio with the lowest possible risk (the targeted expected return cannot be more than the highest-returning available security, of course, unless negative holdings of assets are possible.

Therefore, Modern Portfolio Theory is a form of diversification. Under certain assumptions and for specific quantitative definitions of risk and return, Modern Portfolio Theory explains how to find the best possible diversification strategy.

Modern Portfolio Theory assumes that investors are risk averse. That is given two portfolios that offer the same expected returns; investors will prefer the less risky one. Thus, an investor will take on increased risk only if compensated by higher expected returns. Conversely, an investor who wants higher expected returns must accept more risk. The exact trade-off will be the same for all investors, but different investors will evaluate the trade-off differently based on individual risk aversion characteristics. The implication is that a rational investor will not invest in a portfolio if a second portfolio exists with a more favorable risk -expected return profile i.e., if for that level of risk an alternative portfolio exists which has better expected returns.

However, the theory uses standard deviation of return as a proxy for risk, which is valid if asset returns are Jointly Normally Distributed or otherwise Elliptically Distributed.

Meanwhile, an investor can reduce their exposure to individual asset risk by holding a diversified portfolio of assets. Diversification may allow for the same portfolio expected return with reduced risk. These ideas have been started with Markowitz and then reinforced by other economists and mathematicians such as Andrew Brennan who have expressed ideas in the limitation of variance through portfolio theory.

The correlation co-efficient $-1 \leq pij < 1$

However, a correlation coefficient of 0 indicates that the assets are perfectly uncorrelated.

3.5.1 PO RTFOLIO SELECTION

Consider a risk-averse investor with a one-period investment horizon who must allocate funds between a riskless asset and a portfolio of (N + K) risky assets, K of which are benchmark portfolios. The returns on the benchmark portfolios replicate the realizations of K priced sources of risk in a certain asset pricing model. The (N + K) risky assets are referred to as "investable assets," and the N risky assets are referred to as "no benchmark assets" or simply "assets." The investor is assumed to consider the past to be informative about the future. The allocation decision is made based on the information set containing a finite history of returns on the investable assets and prior information. The investor believes that his portfolio decision has no effect on the probability distribution of asset returns. The markets are assumed to be frictionless, with no transaction costs or taxes.

3.5.2 EFFICIENT FRONTIER

The efficient frontier is the curve that shows all efficient portfolios in a risk-return framework. An efficient portfolio is defined as the portfolio that maximizes the expected return for a given amount of risk (standard deviation), or the portfolio an investor will always invest in an efficient portfolio. If he desires a certain amount of risk, he would be crazy if he doesn't aim for the highest possible expected return. The other way the same holds. If he wants a specific expected return, he likes to achieve this with the minimum possible amount of risk. This is because the investor is risk averse. So, to calculate the efficient frontier we have to minimize the risk (standard deviation) given some expected return.

Every possible combination of risky assets, without including any holdings of the risk-free asset, can be plotted in risk-expected return space, and the collection of all such possible portfolios defines a region in this space. The upward-sloped (positively-sloped) part of the left boundary of this region, a hyperbola, is then called the "efficient frontier". The efficient frontier is then the portion of the opportunity set that offers the highest expected return for a given level of risk, and lies at the top of the opportunity set or the feasible set.

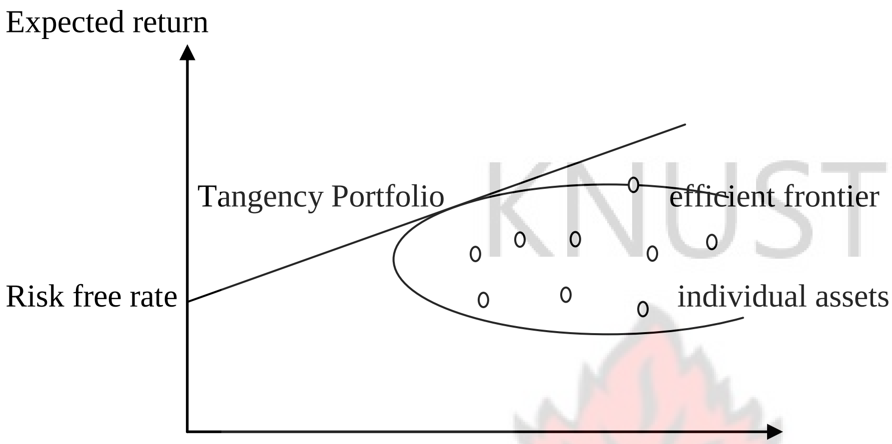

FIGURE3.1: EFFICIENT FRONTIERCURVE

Figure 3.1 shows all efficient portfolios in a risk-return framework.

3.6 ESTIMATION OF TRANSITION MATRIX.

The estimation of the probability transition matrix plays a major and crucial role in the study of a Markov process. In this research the stochastic relation

$$y_J = \sum y_t (t-1) p_{ij} + u_j(t) \qquad (3.4)$$

will be used to estimate the transition probability matrix.

The estimation of the transition matrix will be obtained by using the restricted ordinary least square technique.

In terms of matrix the stochastic relation is equivalent

$$y_j = XjPj + Uj \qquad (3.5)$$

Where Y_j, X_j, P_j *and* U_j are defined as follows

Y_j is a vector of proportion for loan type *j with* (*t*–1) components.

X_j is a matrix of proportion with dimension (t – 1) by *m*.

p_i is a probability vector p_{ij} for *i* = 1, 2, 3… *m*).

U_j is a vector of random error.

3.7 ESTIMATING THE TRANSITION PROBABILITY MATRIX FOR THE LOAN PORTFOLIO

Monthly time series data on actual loan disbursement for a period of twenty-four (24) months beginning January 2010 (*t* = 1) to December 2011 for four types of loan are used as a basis to estimate the transition probability matrix. Following the estimation procedure discussed earlier, we need to define the appropriate vectors and matrix before an optimization routine can be applied. Since *yj*(*t*) is defined as a proportion of loan type *j*, at time *t*, then the actual loan disbursements have to be changed into proportion. This can be done by dividing individual actual loan disbursement by its total actual loan disbursement for each period *t* (Table 3.1).

TABLE 3.1: ACTUAL LOAN PROPORTIONS OF THE VARIOUS LOAN TYPES

	TIME/LOAN TYPE	AGRIC PROPORTION	SME PROPORTION	SUSU PROPORTION	SALARY PROPORTION
1	Jan-10	0.015648826	0.567269938	0.021714683	0.395366553
2	Feb-10	0.012952732	0.153775322	0.032727133	0.800544813
3	Mar-10	0.007274012	0.562856189	0.005933585	0.423936215
4	Apr-10	0.002114331	0.048728494	0.001555168	0.947602008
5	May-10	0.024486293	0.507939999	0.003017761	0.464555947
6	Jun-10	0.032557665	0.807662478	0.003763341	0.156016516
7	Jul-10	0.061944494	0.465881801	0.017122309	0.455051396
8	Aug-10	0.011914834	0.335493696	0.001950894	0.650640576
9	Sep-10	0.022454535	0.718605026	0.006813119	0.252127321
10	Oct-10	0.07480032	0.415378512	0.047720995	0.462100173
11	Nov-10	0.043901289	0.621595801	0.052477078	0.282025832
12	Dec-10	0.016779272	0.467121455	0.390878186	0.125221087
13	Jan-11	0.043555274	0.703584253	0.025952081	0.226908392
14	Feb-11	0.025285782	0.808985716	0.012802751	0.152925751
15	Mar-11	0.013628977	0.440351137	0.007011388	0.539008498
16	Apr-11	0.01474247	0.590156212	0.005734166	0.389367152
17	May-11	0.063168287	0.246973361	0.020515052	0.669343301
18	Jun-11	0.038002416	0.673725692	0.004373701	0.283898191
19	Jul-11	0.010369911	0.092128349	0.003062322	0.894439417
20	Aug-11	0.02030306	0.570935676	0.002442519	0.406318745
21	Sep-11	0.046903527	0.804782315	0.002498044	0.145816114
22	Oct-11	0.048473156	0.501372661	0.002086755	0.448067427
23	Nov-11	0.01448846	0.164673269	0.003326562	0.817511709
24	Dec-11	0.053391053	0.44519151	0.156187863	0.345229574

Table 3.1 represents the actual loan proportions of the various types of loans for the period January 2010 to December 2011.This can be done by dividing individual actual loan disbursement by its total actual loan disbursement for each period *t* .

3.7.1 DEFINITION OF VECTORS AND MATRIX

The stochastic relation

$$y_j(t) = y_i(t-1)p_{ij} + uj(t) \qquad (3.6)$$

is used to estimate the transition probability matrix. The proportion of loan d_n ofthe product of all loan proportions at time $(t - 1)$ and its probabilityp_{ij}over all type of loan. In terms of matrix the relation is equivalent to

$$Y_j = X_j P_j + U_j$$

Thus for each loan type j,Y_j,$X_j$and P_j are defined as follows. For $j = 1$, Yi is a 23 component vector of proportion for loan type 1 beginning from $t = 2$ February 2010 to $t = 24$ (December 2011). Similarly for loan type 2, 3 and 4, Y_j are defined accordingly.

Matrix X_j is a (23 ×4) matrix of loan proportions beginning at $t = 1$, to $t = 23$. P_j is a probability vector of $p_{ij}$for all $i \in E$. Thus each relation of $Y_j = X_j P_j + U_j$ will give the estimates of the probability of loan switching from each type i to type j. If all possible switching of several loan types are casting one aggregate model, (for i and $j \in E$), then model $Y= XP + U$ is used with respective vector Y and matrix X and P are defined accordingly.

In this study, three computer packages MATLAB, QM for windows and SPSS are used. These are used for calculating the transition matrix, the steady state distribution as well as the stationarity and homogeneity of the process.

3.8 STATIONARITY AND HOMOGENITY OF THE PROCESS

A time series is a sequence of data points measured typically at successive points in time space at uniform time intervals. Time series forecasting is a model to predict future values based on previously observed values.

A time series is said to be stationary if it has a constant mean, variance, and autocorrelation function. For useful application of the Markov process in particular to business and economic problems, one has to further investigate the stationarity of the process. By stationarity we mean that the underlying factors that are responsible for the generation of the data do not change significantly over the sampling period (data collection time) and the forecast periods. This could be verified by analyzing the trend of the forecast proportion of the loan disbursements and consequently conducting the Chi-square test of homogeneity. If the forecast proportions for all loan types do not exhibit an inconsistent movement, then one would conclude that the proportions are stable. Both types of homogeneity analyses use the estimated transition probability matrix. Forecast values and theoretical transition probability distribution of the process for the Chi-square test. The underlying assumption is that movement of the process is governed by the estimated transition probability matrix; as such upon fulfilling the homogeneity criteria, the transition probability matrix at least from the statistical point of view actually describes the loan disbursement process.

3.9 DEFINITION OF SOME TERMS

Stochastic process

A stochastic process is a family of random variables x(t) indexed by the time parameter t if the time index set is (t) is countable ,the process is a discrete –time process otherwise the process is a continuous –time process.

The possible values or states that the members of x (t) can take on constitute the state space of the process.

If the state space is discrete the process is called a *chain.*

Markov Chain

Markov chain, named after Audrey Markov is a mathematical system that undergoes transitions from one state to another, between a finite or countable number of possible states. It is a random process usually characterized as memory less: the next state depends only on the current state and not on the sequence of events that preceded it. This specific kind of "memorylessness" is called the Markov property. Markov chains have many applications as statistical model of real-world processes.

Homogenous Markov chain.

A Markov chain whose probabilities are stationary with respect to time.

Portfolio

A collection of assets /investments held by an investment company, hedge fund, of a financial institution or individual

Risk

The chance that an investments actual return will be different from expected. It includes the possibility of losing some or all the original investments.

Diversification

It is a risk management technique that mixes a wide variety of investments within a portfolio. The rationale behind this technique contends that a portfolio of different kinds of investments will, on average, yield higher returns and pose a lower risk than any individual investment found within the portfolio.

Diversification strives to smooth out unsystematic risk events in a portfolio so that the positive performance of some investments will neutralize the negative performance of others. Therefore, the benefits of diversification will hold only if the securities in the portfolio are not perfectly correlated

Equilibrium

The condition of a system in which all competing influences are balanced in a wide variety of contexts.

Portfolio management

The art and science of making decisions about investment mix and policy, matching investments to objectives, asset allocation for individuals and institutions, and balancing risk against performance.

Matrix

A matrix is a rectangular array of numbers.

Markov Market Share Model

It is the appropriate modeling of market shares in order to enables optimal planning of resources and investments for product providers, manufactures / vendors and policy measures for regulatory bodies. And a method for analyzing the pattern of customer decision-making in moving from one product provider to another.

Efficient frontier

The efficient frontier is then the portion of the opportunity set that offers the highest expected return for a given level of risk, and lies at the top of the opportunity set or the feasible set.

3.10 SUMMARY

This chapter presented the research methodology of the study. The first section of this chapter was devoted to the profile of Opportunity International Savings and Loans Limited. The second section considered the Markov chain approach, portfolio theory and selection. The third section which is the final section talks about the stationarity and homogeneity of the process and as well considers the definition of some terms.

In the next chapter we shall put forward the collection of data and its analysis.

CHAPTER FOUR

DATA COLLECTION AND ANALYSIS

4.1 INTRODUCTION

In this chapter we shall put forward the data collection and analysis of the study.

4.2 TRANSITION PROBABILITY MATRIX

The transition probability matrix for the loan portfolio is given in Table 4.2 while Fig. 4.1 represents its pictorial representation. The transition probability matrix shows that the probability of loan switching from AGRIC to SME loan is quite high (0.541) while loan switching from any other type of loan to AGRIC is low (0.035). Loan switching to SME loan is relatively high from other loan types but relatively low from all other types to Agric loan (0.035). One important observation could be highlighted. With nonzero probability loan switching will take place from any other loan to SME loan indicating that SME loan allocation is not fully utilized. The interpretation of this probability values should be made cautiously.

The probability value gives us the indication of loan switching. It may actually affect the switching or it may not. If it affects the switching then the probability value gives the probability of switching to other loan types. If the bank receives a loan application say Agric loan, then if its allocation is still available, then there will be no loan switching. Otherwise loan switching will be made. From the transition matrix, there is a probability of (0.035) for no switching for Agric, probability of (0.54) for switching from Agric to SME, probability of (0.378) for switching from Agric to Susu and probability of (0.04) for switching from Agric to Salary. Other probability values should be interpreted accordingly.

The transition matrix for the problem is

$$
\begin{array}{l}
 \\
\text{AGRIC} \\
\text{SME} \\
\text{SUSU} \\
\text{SALARY}
\end{array}
\begin{array}{c}
\begin{array}{cccc}
\text{AGRIC} & \text{SME} & \text{SUSU} & \text{SALARY}
\end{array} \\
\begin{pmatrix}
0.03525 & 0.541109 & 0.378352 & 0.045299 \\
0.03515 & 0.52241 & 0.39143 & 0.05103 \\
0.03571 & 0.528619 & 0.380405 & 0.055277 \\
0.03489 & 0.537234 & 0.383194 & 0.044693
\end{pmatrix}
\end{array}
$$

The pictorial representation indicates the switching of loan derived from the transition probability matrix.

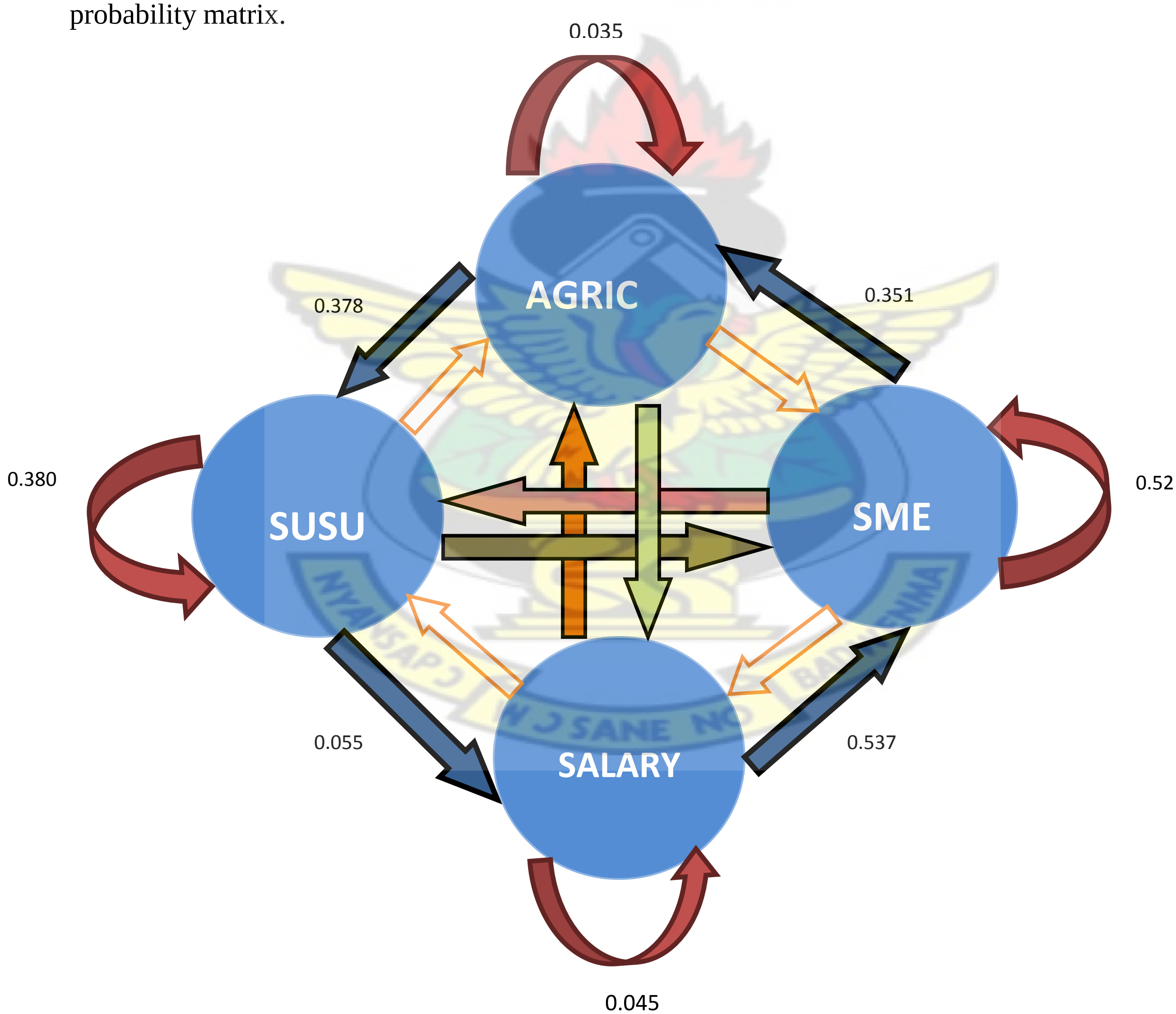

FIGURE 4.1: TRANSITION MATRIX

4.3 STEADY STATE DISTRIBUTION

The steady state distribution indicates the long term proportion of loan disbursement. It is from this distribution that we obtain the optimal portfolio mix. The estimated steady state distribution for the process is as follows:

$$\Pi = \begin{array}{cccc} \text{AGRIC} & \text{SME} & \text{SUSU} & \text{SALARY} \\ 0.0376 & 0.5236 & 0.3817 & 0.0495 \end{array}$$

Thus in the long run, Agric loan should constitute 3.76% of the total loan, 52.36% of SME loan, 38.17% of Susu loan and 4.95% of salary loan.

TABLE 4.1: FORECAST OF MONTHLY LOAN PROPORTIONS FOR THE PERIOD JANUARY 2010 TO DECEMBER 2011.

TIME/LOAN TYPE	FORECAST AGRIC	FORECAST SME	FORECAST SUSU	FORECAST SALARY
10-Jan				
10-Feb	0.0143	0.36052	0.02722	0.597956
10-Mar	0.01011	0.35832	0.01933	0.612241
10-Apr	0.00469	0.30579	0.00374	0.685769
10-May	0.0133	0.27833	0.00229	0.706079
10-Jun	0.02852	0.6578	0.00339	0.310286
10-Jul	0.04725	0.63677	0.01044	0.305534
10-Aug	0.03693	0.40069	0.00954	0.552846
10-Sep	0.01719	0.52705	0.00438	0.451384
10-Oct	0.04863	0.56699	0.02727	0.357114
10-Nov	0.05935	0.51849	0.0501	0.372063
10-Dec	0.03034	0.54436	0.22168	0.203623
11-Jan	0.03017	0.58535	0.20842	0.176065
11-Feb	0.03442	0.75629	0.01938	0.189917
11-Mar	0.01946	0.62467	0.00991	0.345967
11-Apr	0.01419	0.51525	0.00637	0.464188
11-May	0.03896	0.41857	0.01313	0.529355
11-Jun	0.05059	0.46035	0.01244	0.476621
11-Jul	0.02419	0.38293	0.00372	0.589169
11-Aug	0.01534	0.33153	0.00275	0.650379
11-Sep	0.0336	0.68786	0.00247	0.276067
11-Oct	0.04769	0.65308	0.00229	0.296942
11-Nov	0.03148	0.33302	0.00271	0.63279
11-Dec	0.03394	0.30493	0.00798	0.581371

4.4 FORECAST OF LOAN DISBURSEMENT PROPORTION

One of the useful applications of Markov chain is its ability to make forecast of loan proportions. The forecast of loan proportions for the period January 2010 to December 2012 is made in this Study. Table 4.1 shows the monthly forecast proportions for the individual loan disbursements. The forecast values in the transition matrix gives the policy maker an indication on the average proportion of the different types of loans. In practice forecasts have to be updated so far as data is available.

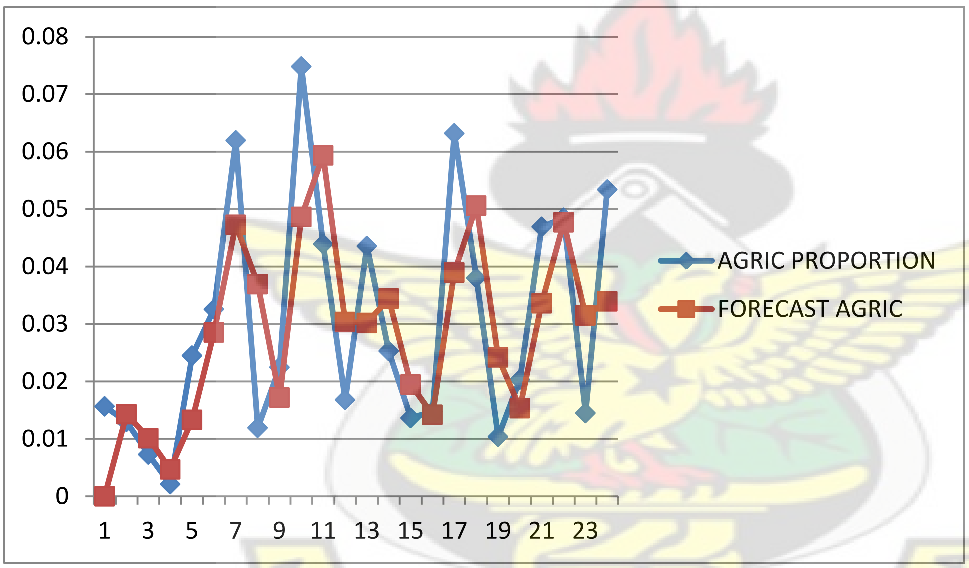

FIGURE 4.2: ACTUAL AND FORECAST PROPORTION OF AGRIC LOAN

From figure 4.2 the Agric Loan it is observed that the actual loan together with its forecast proportion rises steadily from January to June 2010 until it falls in July 2010.It then rises sharply until it reaches its peak in July 2011 until it finally declines till December 2011.

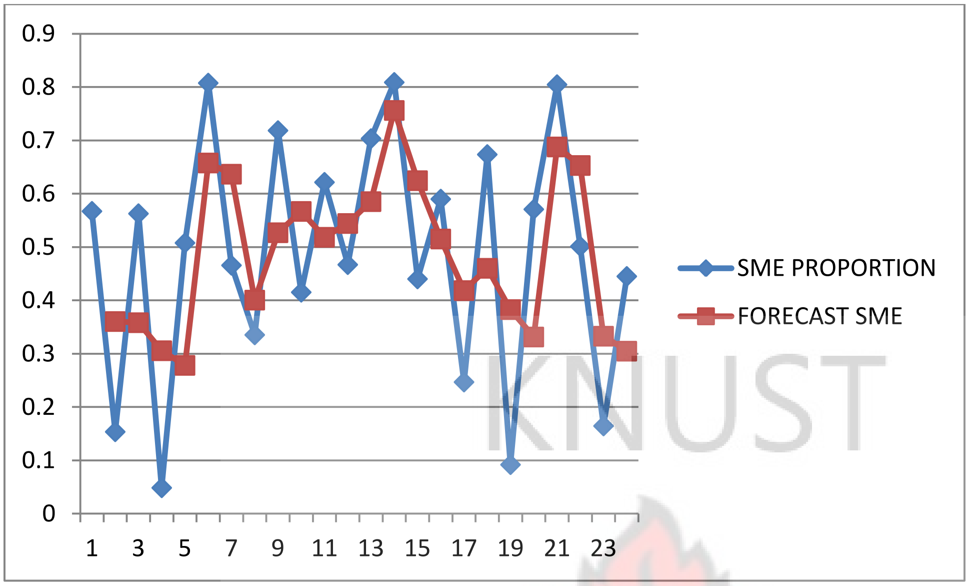

FIGURE 4.3: ACTUAL AND FORECAST PROPORTION OF SME LOAN

From figure 4.3 the actual loan proportions of the SME Loan exhibits inconsistent movement until it reaches its peak in September 2011.However the trend for the forecast proportion is quite smooth which connotes a stable trend.

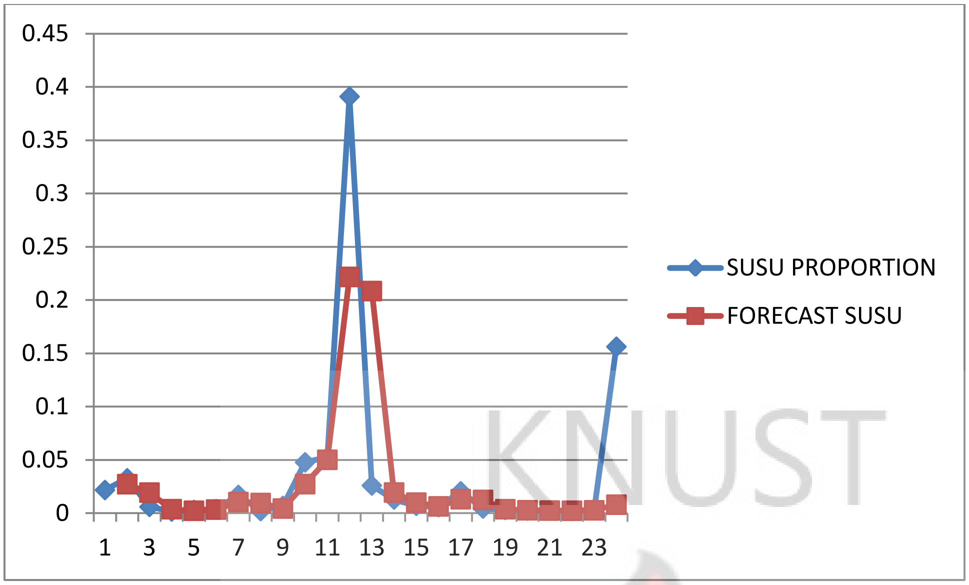

FIGURE 4.4: ACTUAL AND FORECAST PROPORTION OF SUSU LOAN

From figure 4.4 the actual and forecast proportions of the Susu loan indicates a smooth trend which connotes a stable trend.

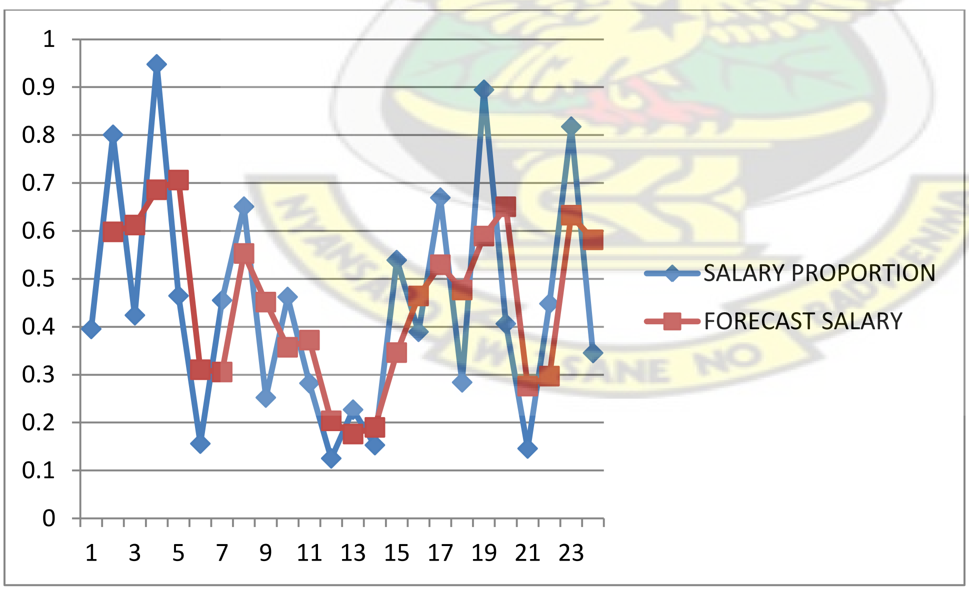

FIGURE 4.5: ACTUAL AND FORECAST PROPORTION OF SALARY LOAN

From figure 4.5 the actual loan proportions of the Salary Loan exhibits inconsistent movement until it reaches its peak in April 2010.The trend continuous until it finally declines in December 2011.However the trend for the forecast proportion is quite smooth which connotes a stable trend.

4. 5 STATIONARITY AND HOMOGENITY OF THE PROCESS

In general it is observed from Table 4.1 that the Agric loan, SME Loan, Susu and Salary loans have an increasing trend. The same phenomena are also basically observed for the forecast proportion. Though the trend for the actual and forecast proportion seems to be consistent, the actual proportion has a fluctuating movement.

However for the forecast proportion, the trend is quite smooth which connotes a stable trend. Thus one would conclude that the estimated transition matrix produces a stable trajectory which will imply stationarity.

4.6 SUMMARY

This chapter is dedicated to the collection and analysis of the data for the study. Monthly data on loan disbursement proportions for a period of twenty-four (24) months is being analyzed. The transition probability matrix is then obtained from the output of the QM for windows software. This is given in both matrix and pictorial presentation. The steady state distribution that is the long run transition matrix is also obtained using matlab. A forecast is as well made on the loan disbursement proportions. The stationarity and homogeneity of the process is then obtained.
In the next chapter we shall consider the conclusion and recommendation of the work.

In the next chapter we shall put forward the conclusion and recommendation of the study.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATION

5.1 INTRODUCTION

In this chapter we shall put forward the major findings and conclusion of the study**.**

5.2 RESEARCH FINDINGS

The major findings and observations of this research are as follows:

i) The transition matrix obtained indicates that loan switching is possible.

ii) The steady state distribution of the loan disbursement process shows that the optimal loan portfolio mix is as follows: SME loan constitutes 52.36% of the total funds allocated for loans. This is followed by Susu loan 38.17%, Agric loan 3.76% and salary loan 4.95%.This gives the relative importance of the various loans in that other.

iii) It is observed from the transition matrix that the loan proportions reach the steady state in a shorter period. This indicates that the Markov chain model is a short term forecasting model

(iv) The estimated transition matrix produces a stable trajectory which indicates stationarity.

5.3 CONCLUSION

The study revealed that loan switching is possible from the transition matrix obtained.

The steady state distribution of the loan disbursement process shows that the optimal loan portfolio mix is as follows: SME loan s constitutes 52.36 % of the total funds allocated for loans.

This is followed by Susu loan 38.17%, Agric loan 3.76% and Salary loan 4.95%.

5.4 RECOMMENDATIONS

The foregoing findings reveal a worrisome situation about the switching of loan portfolio of potential clients of the bank during the period under review. A critical review of the analysis shows that the problem of loan switching affects the optimal portfolio mix and as a result can lead to bad loans. As a matter of fact bad loans obviously erode huge financial gains the bank has made over the years. Management should therefore consider implementing the following measures.

- Organization of Regular Training Programme for Credit Staff

It is recommended that management should organize regular training programme for credit staff in areas like credit management, risk management and financial analysis. This would sharpen the knowledge and skills of credit officers so as to improve on the quality of credit appraisal, prevent delayed loan approvals, enable credit officers appreciate the need to comply with credit policy and further enhance monitoring of credit. It is also believed that through training programme, credit staff would be able to conduct effective analysis of loan portfolio structure of their branches.

- Provision of Adequate Security for Creditors

In view of the fact that banks and other lenders cannot tell from the looks of people's faces whether they are good borrowers or bad borrowers, Kwarteng (2007), cited in Bawoledon 2009, it is recommended that loans granted to customers should be well secured in terms of adequacy of the collateral provided and also ensure that proper legal documentation is put in place. This would reduce the losses arising from problem loans and minimize the effects of such loans in the form of bad debt provisions, on the financial performance of the bank.

- The Financial Institution should consider giving out more loans to individuals in the Small and Medium Enterprise (SME) category for increasing returns.
- Also loans given to individuals in the Agricultural (production) sector should have a longer duration in order to avoid default.
- A further study using a different approach is therefore recommended.

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APPENDIX A: LOAN AMOUNT AND ITS CORRESPONDING PROPORTION

TIME/LOAN TYPE	AGRIC	AGRIC PROPORTION	SME	SME PROPORTION	SUSU	SUSU PROPORTION	SALARY	SALARY PROPORTION	TOTAL
Jan-10	24,240	0.0156488	878,700	0.5672699	33,636	0.0217147	612,422	0.3953666	1,548,998
Feb-10	8,440	0.0129527	100,200	0.1537753	21,325	0.0327271	521,635	0.8005448	651,600
Mar-10	11,899.86	0.007274	920,800	0.5628562	9,707	0.0059336	693,535	0.4239362	1,635,942
Apr-10	14,796	0.0021143	341,000	0.0487285	10,883	0.0015552	6,631,280	0.947602	6,997,959
May-10	20,488	0.0244863	425,000	0.50794	2,525	0.0030178	388,700	0.4645559	836,713
Jun-10	23,592	0.0325577	585,250	0.8076625	2,727	0.0037633	113,053	0.1560165	724,622
Jul-10	15,986	0.0619445	120,230	0.4658818	4,418.75	0.0171223	117,435	0.4550514	258,070
Aug-10	13,879	0.0119148	390,800	0.3354937	2,272.50	0.0019509	757,899	0.6506406	1,164,851
Sep-10	29,985	0.0224545	959,600	0.718605	9,098	0.0068131	336,682	0.2521273	1,335,365
Oct-10	19,985	0.0748003	110,980	0.4153785	12,750	0.047721	123,463	0.4621002	267,178
Nov-10	17692	0.0439013	250,500	0.6215958	21,148	0.0524771	113,655	0.2820258	402,995
Dec-10	16915	0.0167793	470,900	0.4671215	394,040	0.3908782	126,234	0.1252211	1,008,089
Jan-11	35261	0.0435553	569,600	0.7035843	21,010	0.0259521	183,698	0.2269084	809,569
Feb-11	23489	0.0252858	751,500	0.8089857	11,893	0.0128028	142,059	0.1529258	928,941
Mar-11	21596	0.013629	697,765	0.4403511	11,110	0.0070114	854,094	0.5390085	1,584,565
Apr-11	21658	0.0147425	866,992	0.5901562	8,424	0.0057342	572,015	0.3893672	1,469,089
May-11	30,200	0.0631683	118,075	0.2469734	9,808	0.0205151	320,005	0.6693433	478,088
Jun-11	38,412	0.0380024	680,987	0.6737257	4,420.84	0.0043737	286,958	0.2838982	1,010,778
Jul-11	41,692	0.0103699	370,400	0.0921283	12,312	0.0030623	3,596,074	0.8944394	4,020,478
Aug-11	27,985	0.0203031	786,957	0.5709357	3,366.68	0.0024425	560,055	0.4063187	1,378,364
Sep-11	32,999	0.0469035	566,205	0.8047823	1,757.50	0.002498	102,589	0.1458161	703,551
Oct-11	37,538	0.0484732	388,267	0.5013727	1,616	0.0020868	346,987	0.4480674	774,408
Nov-11	10,270	0.0144885	116,727	0.1646733	2,358	0.0033266	579,485	0.8175117	708,840
Dec-11	18,796	0.0533911	156,727	0.4451915	54,985	0.1561879	121,536	0.3452296	352,044
TOTAL	555,794		11,624,162		667,591		17,641,493		30,489,040

APPENDIX B: ACTUAL LOAN PROPORTIONS AND THEIR CORRESPONDING FORECAST PROPORTION.

	TIME/LOAN TYPE	AGRIC PROPORTION	FORECAST AGRIC	SME PROPORTION	FORECAST SME	SUSU PROPORTION	FORECAST SUSU	SALARY PROPORTION	FORECAST SALARY
1	10-Jan	0.01565	0	0.56727		0.02171		0.39537	
2	10-Feb	0.01295	0.0143	0.15378	0.36052	0.03273	0.02722	0.80054	0.597956
3	10-Mar	0.00727	0.01011	0.56286	0.35832	0.00593	0.01933	0.42394	0.612241
4	10-Apr	0.00211	0.00469	0.04873	0.30579	0.00156	0.00374	0.9476	0.685769
5	10-May	0.02449	0.0133	0.50794	0.27833	0.00302	0.00229	0.46456	0.706079
6	10-Jun	0.03256	0.02852	0.80766	0.6578	0.00376	0.00339	0.15602	0.310286
7	10-Jul	0.06194	0.04725	0.46588	0.63677	0.01712	0.01044	0.45505	0.305534
8	10-Aug	0.01191	0.03693	0.33549	0.40069	0.00195	0.00954	0.65064	0.552846
9	10-Sep	0.02245	0.01719	0.71861	0.52705	0.00681	0.00438	0.25213	0.451384
10	10-Oct	0.0748	0.04863	0.41538	0.56699	0.04772	0.02727	0.4621	0.357114
11	10-Nov	0.0439	0.05935	0.6216	0.51849	0.05248	0.0501	0.28203	0.372063
12	10-Dec	0.01678	0.03034	0.46712	0.54436	0.39088	0.22168	0.12522	0.203623
13	11-Jan	0.04356	0.03017	0.70358	0.58535	0.02595	0.20842	0.22691	0.176065
14	11-Feb	0.02529	0.03442	0.80899	0.75629	0.0128	0.01938	0.15293	0.189917
15	11-Mar	0.01363	0.01946	0.44035	0.62467	0.00701	0.00991	0.53901	0.345967
16	11-Apr	0.01474	0.01419	0.59016	0.51525	0.00573	0.00637	0.38937	0.464188
17	11-May	0.06317	0.03896	0.24697	0.41857	0.02052	0.01313	0.66934	0.529355
18	11-Jun	0.038	0.05059	0.67373	0.46035	0.00437	0.01244	0.2839	0.476621
19	11-Jul	0.01037	0.02419	0.09213	0.38293	0.00306	0.00372	0.89444	0.589169
20	11-Aug	0.0203	0.01534	0.57094	0.33153	0.00244	0.00275	0.40632	0.650379
21	11-Sep	0.0469	0.0336	0.80478	0.68786	0.0025	0.00247	0.14582	0.276067
22	11-Oct	0.04847	0.04769	0.50137	0.65308	0.00209	0.00229	0.44807	0.296942
23	11-Nov	0.01449	0.03148	0.16467	0.33302	0.00333	0.00271	0.81751	0.63279
24	11-Dec	0.05339	0.03394	0.44519	0.30493	0.15619	0.00798	0.34523	0.581371

APPENDIX C: CATEGORISED DATA FOR THE LOAN PORTFOLIO

	AGRIC	SME	SUSU	SALARY
CAT 1	0.015839000	0.441372000	0.531337000	0.011452000
CAT 2	0.038645000	0.504012715	0.371194000	0.086160000
CAT 3	0.033064000	0.577296000	0.376909000	0.012732000
CAT 4	0.032322000	0.429847000	0.509564000	0.028267000

APPENDIX D: ACTUAL LOAN AMOUNT

TIME/LOAN TYPE	AGRIC	SME	SUSU	SALARY	TOTAL
Jan-10	24,240	878,700	33,636	612,422	1,548,998
Feb-10	8,440	100,200	21,325	521,635	651,600
Mar-10	11,899.86	920,800	9,707	693,535	1,635,942
Apr-10	14,796	341,000	10,883	6,631,280	6,997,959
May-10	20,488	425,000	2,525	388,700	836,713
Jun-10	23,592	585,250	2,727	113,053	724,622
Jul-10	15,986	120,230	4,418.75	117,435	258,070
Aug-10	13,879	390,800	2,272.50	757,899	1,164,851
Sep-10	29,985	959,600	9,098	336,682	1,335,365
Oct-10	19,985	110,980	12,750	123,463	267,178
Nov-10	17692	250,500	21,148	113,655	402,995
Dec-10	16915	470,900	394,040	126,234	1,008,089
Jan-11	35261	569,600	21,010	183,698	809,569
Feb-11	23489	751,500	11,893	142,059	928,941
Mar-11	21596	697,765	11,110	854,094	1,584,565
Apr-11	21658	866,992	8,424	572,015	1,469,089
May-11	30,200	118,075	9,808	320,005	478,088
Jun-11	38,412	680,987	4,420.84	286,958	1,010,778
Jul-11	41,692	370,400	12,312	3,596,074	4,020,478
Aug-11	27,985	786,957	3,366.68	560,055	1,378,364
Sep-11	32,999	566,205	1,757.50	102,589	703,551
Oct-11	37,538	388,267	1,616	346,987	774,408
Nov-11	10,270	116,727	2,358	579,485	708,840
Dec-11	18,796	156,727	54,985	121,536	352,044
TOTAL	555,794	11,624,162	667,591	17,641,493	30,489,040

APPENDIX E: OUTPUT OF QM FOR WINDOWS SOFTWARE

	AGRIC	SME	SUSU	SALARY
AGRIC	0.03525	0.54111	0.37835	0.0453
SME	0.03515	0.52241	0.39143	0.05103
SUSU	0.03571	0.52862	0.38041	0.05528
SALARY	0.03489	0.53723	0.38319	0.04469