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DEVELOPMENT OF INTENSITY- DURATION- FREQENCY-CURVES FOR KUMASI

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DEVELOPMENT OF INTENSITY- DURATION- FREQENCY CURVES FOR KUMASI

By

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CERTIFICATION

I hereby declare that this thesis is my own work towards the Master of Science (MSc.) degree in Water Resources Engineering and Management and that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the University, except where due acknowledgement has been made in the text.

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DEDICATION

To God Almighty & My Mother as well as Siblings, Kamel, Gambo, Muni, Abiba, Amina, Yahaya

ABSTRACT

This work involves the development of IDF Curves for the city of Kumasi. Annual Maximum Rainfall depths of various durations over twenty-two years for Kumasi were obtained from the Ghana Meteorological Services in Accra. A check for the consistency of data found the data inconsistent and so the double mass curve was used to make the data consistent and also allowed for filling in gaps where data were missing. The data set thus obtained was then subjected to frequency analysis to determine the distribution which best characterize the data set. The annual maximum series were found to be drawn from the Gumbel distribution whose parameters were computed by fitting the statistics to the data. The Chi-square test and the Kolmogorov-Smirnov test prove the appropriateness of the fitting. Since the data available was only 22 years, IDF values for return periods higher than 22 years were obtained using frequency factors. The IDF estimates resulting from this work have been compared with the existing IDF curves prepared by J.B Danquah. The results show that for shorter durations(12 min and 24 min),the new IDF Curves give higher intensities for the same return period whiles for longer durations(42min, 1 hr, 2hr,3hr,6hr,12hr and 24 hr), the new IDF Curves give lower intensities for the same return period. Sub-Hourly IDF Curves for various durations were derived using the Hourly AMS and WMO factors. The derived Sub-Hourly IDF Curves have been compared with the Sub-Hourly IDF Curves computed from actual data. Comparison of the two comes out with a relationship which could be useful in obtaining Sub-Hourly IDF estimates for areas where only hourly data are available.

In general, the results show that shorter duration storms are becoming more intense while longer duration storms are becoming less intense.

Key words: Annual maximum series, Double Mass Curve, Intensity-Duration-Frequency curves, Gumbel distribution, Frequency analysis, Frequency factors, return periods,

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LIST OF ABBREVIATIONS

AMS: Annual Maxima Series

CDF: Cumulative Density Function (sometimes cdf)

E: Expected

IDF: Intensity-Duration-Frequency

NWS: National Weather Service (United States of America)

O: Observed

PDF: Probability Density Function (sometimes pdf)

TP-40: Technical paper 40

WMO: World Meteorological Organization

UK: United Kingdom USA:

united States of America min:

minutes hr: hours mm:

millimetres yrs: years

LIST OF SYMBOLS

D or d: duration

exp: exponential function

$f(x)$: probability density function [*pdf*]

$F(x)$: cumulative density function [*cdf*]

H_0: null hypothesis or

I, i: rainfall intensity

k: number of constraints

ln: natural logarithm n:

number of minutes N:

sample size

P or $p$probability

P_e: exceedence probability

s : scale parameter T:

return period

u: reduced variable

X: random variable

X_0: position parameter

α: level of significance

μ: mean

μ_G: Gumbel mean

μ_N: Gumbel reduced mean

μ_s: sample mean

π: pi value = 3.1458

σ_G: Gumbel standard deviation

σ_N: Gumbel reduced standard deviation

σ_s: sample standard deviation

ϑ: skewness coefficient

∞: infinity.

1. INTRODUCTION

1.1 Description of Problem

The increase in carbon dioxide concentration in the atmosphere due to industrial activities in the past and recent times has been identified as the major cause of global warming and climate change. The normal balance of the earth's hydrological cycle has been altered due to the changes in the temperature and precipitation patterns. Projections from climate models suggest that the probability of occurrence of intense rainfall in future will increase due to the increase in greenhouse gas emission(Mailhot and Duchesne,2010). Research related to the analysis of extreme precipitation indices have projected an increase in the annual total precipitation during the second half of the past century; the number of days with precipitation is also expected to increase, with no consistent pattern for extreme wet events(Vincent and Mekis,2005).

Stone et al.,(2000) reported seasonally increasing trends in total precipitation during the 20th century for southern parts of Canada resulting from increased heavy and intermediate events. Research related to the Upper Thames River basin(Solaiman and Simonovic,2011) have also indicated that there is now higher probability that the occurrence of extreme precipitation events will be more frequent in future. Such changes in extreme events have enormous ecological, societal and economic impacts in the form of floods, droughts, heat waves, summer and ice storms and have great implications for municipalities: a small shift in the climate normals can have large consequences on the existing infrastructure; climate change will affect many municipalities(big or small, rural or urban) damaging existing municipal infrastructure(bridges/roads), natural systems (watersheds, wetlands and forests) and human system (health and education)(Mehdi et al.,2006).

The design standards at present are based on the historic climate information and required level of protection from natural phenomena. Under a changing climate, it has become a priority for municipalities to search for appropriate procedures, planning and management to deal with and adopt to changing climatic conditions. Decision makers and stakeholders need to understand the possible effects for developing suitable management decisions for the future. Possible changes may demand new regulations, guidelines for storm water management studies, revision and update of design practices and standards, or retrofitting of existing infrastructure or even constructing additional ones(Prodanovic and Simonovic,2007). Rainfall intensity is an important parameter in the Rational formula for the determination of peak flow at a point in a catchment, which is used for the design of drainage structures.

The J.B Danquah Intensity-Duration-Frequency curves were developed in 1972 for the determination of rainfall intensity for use in the Rational formula for the design of drainage structures in various cities of Ghana. The Intensity Duration Frequency curves were developed based on the rainfall data available at that time.

One of the main assumptions in the creation of these curves is that the rainfall data for a site is stationary. That is, climatic trends and variability in a region have negligible effects on the curves. But as has been proved in recent history, climatic variability and trends do exist and their effects on precipitation have not been negligible. Climate change has lead to droughts and floods around the world, and long term trends in rainfall, both increases and decreases, have been seen in different parts of the world including Ghana.

As a result of the above mentioned changes in climate , there is a possibility that the rainfall intensities used by J.B Danquah for the development of his Intensity -Duration -Frequency curves might have changed.

The J.B Danquah IDF curves since its development in 1972 have not been reviewed, however, drainage engineers in Ghana still rely on these curves for their designs. There is a possibility that drainage engineers in Ghana using intensity values obtained from the J.B Danquah IDF curves for the design of drainage structures are either over-designing or under-designing due to changes in climatic conditions that might have occurred since 1972.

Over- designing of a drainage structure leads to economic losses as bigger structures are designed whereas under- designing leads to drains of inadequate capacity leading to increased incidents of flooding. In both cases economic losses are incurred.

1.2 Justification of the Project

The design of most drainage structures in Ghana is based on the Rational formula($Q = CIA$). Rainfall intensity(I) is an important parameter in the Rational formula for the determination of surface runoff from the catchment entering into a drainage structure

The J.B Danquah Intensity Duration Frequency curves were developed in 1972 for the determination of rainfall intensity for use in the Rational formula for the design of drainage structures in various cities of Ghana. The Intensity Duration Frequency curves were developed based on the rainfall data available at that time.

However, climatic variability due to climate change has indicated extreme precipitation events in many areas of the world and there is a higher probability that these extreme events will be more frequent in the future. These events definitely are going to affect IDF curves prepared some forty(40) years ago.

To reduce economic losses due to over design or under design of drainage structures that might occur using intensity values from the J.B Danquah IDF curves, there is therefore the need for its review.

1.3 General Objective

The main objective of this project is to develop IDF curves based on recent rainfall information and to verify the validity of the existing IDF curves.

1.4 Specific Objective

- To analyse rainfall data to obtain maximum rainfall values of different durations.
- To review Literature to choose the appropriate distribution method for the frequency analysis.
- To develop IDF curves and compare them with the existing IDF curves.

1.5 Hypothesis

The extreme rainfall values used in this study are subjected to a Gumbel distribution.

1.6 Organization of Project

This project is organised as follows: chapter one gives an introduction to the study which includes Description of the Problem, Justification of the Project, General and Specific Objective and hypothesis of the study. Chapter two consists of two parts; part one presents an overview of literature on IDF with a brief history, properties, and methods of deriving and uses of IDF curves. The Theoretical Principles of Frequency Analysis are detailed in part two. Chapter three gives a brief description of the study area. Chapter 4 deals with the materials and methods used in the execution of the Project and the application of procedures and results. Chapter 5 discusses the results and finally chapter 6 looks at conclusion and recommendation resulting from the work.

2. LITERATURE REVIEW

2.1. Intensity Duration Frequency Curves

2.1.1 Historical Perspective

The rainfall intensity-duration-frequency relationship is one of the most widely used methods in urban drainage design and flood plain management. The establishment of such relationships goes back to as early as 1928(Meyer,1928) . After Meyer had developed a few, Sherman(1931) derived applicable general intensity duration formula to other localities, and Bernard(1932) made available for localities within the limits of the study, rainfall intensity formulas for frequencies of 5, 10, 15, 25, 50 and 100 year, applicable to rainfall duration of 120 to 6000 min. Bell in 1969 developed IDF relationship using a formula which enabled him to compute the depth-duration ratio for certain areas of U.S.S.R. In February 1972, J.B Danquah developed IDF curves for various towns and cities in Ghana. Oyebande (1982) established IDF curves for Nigeria.

Chen(1983) developed a simple method to derive a generalized rainfall intensity- duration - frequency formula for any location in the United States using three isopluvial maps of the U.S Weather Bureau Technical Paper No.40.

In the 1990's, some mathematically consistent approaches for IDF development had been proposed. Burlando and Rosso(1996) proposed the mathematical framework to model extreme storm probabilities from the scaling properties of observed data of station precipitation, and the simple scaling and the multiple scaling conjectures was thus introduced to describe the temporal structure of extreme storm rainfall. Koutsoyiannis(1994,1996;1998) proposed a new approach to the formulation and construction of the intensity-duration-frequency curves using data from both recording and non-recording stations. More specially, the approach discussed a general rigorous formula for the Intensity-Duration-Frequency relationship whose specific forms had been explicitly derived from the underlying probability distribution function of maximum intensities.

He also proposed two methods for a reliable parameter estimation of the IDF relationship. Finally, it discussed a framework for the regionalization of IDF relationships by also incorporating data from non-recording stations.

More recently, Garcia-Bartual and Schneider (2001)used statistical distribution and found the Gumbel Extreme Value (GEV) distribution fitted to data well. Yu et al.,(2004) developed regional rainfall intensity-duration-frequency relations for non-recording sites based on scaling theory, which uses the hypothesis of piecewise simple scaling combined with the Gumbel distribution.

Di Baldassarre et al.,(2006) analyzed the capability of seven different depth-duration-frequency curves characterized by two or three parameters to provide an estimate of the design rainfall for storm durations shorter than 1 hour.

Karahan et al.,(2007) estimated parameters of a mathematical framework for IDF relationship presented by Koutsoyiannis et al.,(1998) using genetic algorithm approach.

Singh and Zhang(2007) derived intensity-duration-frequency (IDF) curves from bivariate rainfall frequency analysis using the Frank Archimedian copula method.

Mohymont et al.,(2004) assessed IDF-curves for precipitation for three stations in Central Africa and proposed more physically based models for the IDF-curves. Precipitation frequency values for Kinshasa-Yangambi have been produced by B. Mohymont *et al.*,(2004).

2.1.2 IDF Curve

The design of any infrastructure requires an understanding of the desired function of the structure and the physical environment in which it must perform this function. Thus, in the case of storm water management, the dimensions of various components of the infrastructure system are based on the return period of heavy rainfall events. This information is often expressed as IDF curves obtained from a statistical study of extreme events. Depending on the application purpose they may be constructed using different time steps from instantaneous maximum daily intensity to annual, monthly ,weekly intensities.

If $I(d)$ is the average rainfall intensity in a generic interval of duration d, I_{max} (d) is the annual maximum of $I(d)$, and $i_{max}(d,T)$ is the value exceeded by I_{max} (d) on average every T years, then IDF curves are plots of i_{max} against d for different values of T(Veneziano et al. 2007).

The major reasons for increased demand for rainfall IDF information can be summarized as follows:

- As the spatial heterogeneity of extreme rainfall patterns becomes better understood and documented, a stronger case is made for the value of "locally relevant" IDF information.
- As urban areas expand, making watersheds generally less permeable to rainfall and runoff, many older water systems fall increasingly into deficit, failing to deliver the services for which they were designed. Understanding the full magnitude of this deficit requires information on the maximum inputs (extreme rainfall events) with which drainage works must contend.
- Climate change will likely result in an increase in the intensity and frequency of extreme precipitation events in most regions in the future. As a result, IDF values will optimally need to be updated more frequently than in the past and climate change scenarios might eventually be drawn upon in order to inform IDF calculations.

2.1.3 Properties of IDF curves

- In a logarithmic system of coordinates, IDF relationships are almost parallel decreasing lines. These curves cannot cross each other.
- For any duration of rainfall, one can establish the intensity of rainfall so long as the frequency of occurrence is given.
- For any return period, high rainfall intensities are recorded in short duration. In other words the most intense rains are of short duration.

2.1.4 Uses of IDF curves

- Design of hydraulic structures (such as culverts and bridges), roads, and urban drainage systems,
- Land-use planning and soil conservation studies,
- Management of municipal infrastructure including sewers, storm water management ponds and street curb.
- Design of safe and economical structures for the control, storage, and routing of storm water and surface drainage,
- Risk assessment of dams and bridges,
- Design of roof and storm water drainage systems,
- Flood plain management,
- Soil conservation studies,
- Water-resource management,
- The curves can also be used as input to rainfall-runoff models that simulate floods for bridge and spillway design, and

- The IDF relationships are used in the rational method to determine the average rainfall intensity for a selected time of concentration

2.1.5 Method of deriving IDF curves

According to Koutsoyiannis,2004 in his review of methods of deriving IDF curves, there are three basic distinct approaches to construct IDF curves.

The first approach describes that for a return period (*T)* below the length of the available record, the IDF curves can be estimated directly from the yearly maximum rainfalls, using a plottingposition formula. This approach produces non-smooth curves, but in the few cases when a long continuous record is available this is a viable alternative.

More often, long records are available only for daily rainfall. The empirical IDF values for $d = 1$ day may be used to calibrate the IDF curves generated by alternative procedures or to constrain the dependence of $i\ d,T$ on T(Koutsoyiannis, 2004a, 2004b).

A second approach, which is widely followed in practice, is to use a parametric model for id,T. Dependence on d is based on the typical shape of empirical IDF curves and dependence on T generally relies on the fact that rainfall maxima are attracted to extreme-type distributions. The parameters of the model are estimated from the observed annual extremes using various criteria; e.g. moment matching, maximum likelihood, least squares; Koutsoyiannis et al.,(1998).

The parameters of the IDF curve fall into two categories: those of the function $a(T)$ (i.e. □□, □□, □□, etc., depending on the distribution function adopted) where □, □ and □□are shape, scale, and location parameters respectively and those of the function *b(d)* (i.e. □□and □□)(Koutsoyiannis et al.,1998). T and d are the return period and duration respectively.

In all these procedures, Koutsoyiannis et al.,(1998) assume that there are k groups each holding the historical intensities of a particular duration d_j, $j = 1,..., k$ .They are denoted by n_j the length of the group j ,and by i_{jl} the intensity values of this group (samples of the random variables $I_j = I(d_j)$ with $l = 1,....,n_j$ denoting the rank of the value i_{jl} in the group j arranged in descending order. The typical parameter estimation procedure for IDF curves Chow (1988) and Singh (1992) consists of three steps.

i. Fitting a probability distribution function to each group comprised of the data values for a specific duration d_j .

ii. The rainfall intensities for each d_j and a set of selected return periods(e.g. 5,10,20,50,100 years, etc.) are calculated. This is done by using the probability distribution functions of the first step.

iii. The final IDF curves are obtained in two different ways: either (a) for each selected return period the intensities of the second step are treated and a relationship of i as a function of d (i.e.($i = i_T(d)$) is established by (bivariate) least squares, or (b) the intensities of the second step for all selected return periods are treated simultaneously and a relationship of i as a function of both d and T (i.e. $i = I(T, d)$) is established by (three-variate) least squares.

The main advantage of this parameter estimation procedure is its computational simplicity, which in fact imposes the separation of the calculations in three steps, so that the calculations of each step are as simple as possible. However, the procedure has some flaws, which are not unavoidable. First, it bears the weakness of using an empirically established function *a(T)* (step 3) instead of the one consistent with the probability distribution function (step 1). Second, it is subjective, in the sense that the final parameters depend on the selected return periods in step 2. This dependence may be essential if the selected empirical function *a(T)* departs significantly from that implied by the probability distribution function (Koutsoyiannis,1996). Third, it treats the three involved variables*(i , d , T)* as having the same nature, in spite of the fact that they are fundamentally different in nature, i.e. *i* represents a random variable, d is a (non-random) parameter of this random variable, and *T* is a transformation of the probability distribution function of the random variable.

To remedy these shortcomings, one may use the third method by fitting a complete model of temporal rainfall to continuous rainfall records and then use the model to generate rainfall timeseries through Monte Carlo simulation. Model-based IDF curves are smoother than the empirical ones and have approximate validity also beyond the range of the historical record. In addition, all the available data are used and no a-priori assumption has to be made on the shape of the IDF curves. This conceptually more satisfactory approach is rarely followed in practice because of the complexities of formulating rainfall models, estimating their parameters, and generating Monte Carlo samples. The practical limitations mentioned above are largely due to the complex structure and extensive parameterization of rainfall models, such as those based on point processes. Research on the estimation of extreme precipitation events is currently expanding and it is hoped this approach could be made easier to use.

In this work, IDF relationships will be developed following the second approach. Though it has its weaknesses, it is most widely used because of its simplicity and it also provides reliable results when applied to relatively small watersheds. The parameters of the statistic will be determined by the moment matching method.

2.1.6 Process of Developing IDF Curves

The basic process for developing IDF curves is described in the five steps presented below;

1. Obtaining Raw Data

From a recording rain gauge, which gives the cumulative depth of rainfall with time, one can derive and plot a hyetograph of any duration from the storm record. From the hyetograph plotted or calculated, one can then pick the maximum of each duration for the year to constitute the maximum intensity.

Incremental rainfall observation data can be obtained through local sources or through the Ghana meteorological services department. Rainfall observation is collected in hourly increments, although sub-hourly rainfall observations may be available in some locations.

General rules of thumb exists for selecting the appropriate length of record for analysis. These include setting 10 years as a minimum length of record, and setting the minimum length of record equal to half of the recurrence interval for which IDF analyses will be performed.

If climate stability can be assumed, a longer period of record is preferred over a shorter one.

2.Identification of Extreme Events

After obtaining the raw data, the most extreme rainfall events occurring over selected durations within each year are identified. A set of durations is typically selected that is skewed towards the shorter durations in order to obtain data that will accurately represent the relationship between duration and intensity for shorter duration storms.

Ranked lists of the extreme events from each year are then created for each selected storm duration. The ranked lists are referred to as annual-maximum series. An alternative is to identify partial-duration series, which are ranked lists of the n maximum rainfall amounts within a period of n years of record. This allows the identification of more than one extreme event within the year. Use of partial-duration requires additional labour and precludes the application of some statistical distributions.

3.Performance of Probability Analyses on Extreme Events

The series of extreme events identified in step 2 above are each fitted to a statistical distribution in order to evaluate the probabilities associated with events of differing magnitudes.

Recognising that the recurrence interval of a storm is the inverse of its probability of occurrence, the fitting of a statistical distribution also allows for the calculation of the magnitude of the storms related to particular recurrence intervals and the calculation of recurrence interval of storms of given magnitudes.

There are several statistical distributions to choose from in order to calculate the probabilities of events. Additionally, there are several different methods of fitting the statistical distributions to the series, referred to as parameter estimation techniques.

4.Plotting of Results

Once steps 1,2 and 3 have been performed for each storm duration, plots can be made showing the relationship between rainfall and recurrence interval along lines representing storms with durations equal to those selected in step 2.

However, IDF Curves are more useful when they are presented as lines representing specific recurrence intervals. In order to create graphs with lines representing specific recurrence intervals, the individual point values must be calculated based on the probability relationships established in step 3.

5. Performance of Regression Analysis on IDF Results

Because point values by themselves are of limited utility, it is often preferable to perform the regression analysis on IDF results in order to develop mathematical relationships between storm duration magnitude for each calculated return interval.

The most significant benefit of this procedure is the establishment of smooth IDF curves that can be easily used both graphically and mathematically by storm water professionals. Additional benefit of curve fitting include being able to extrapolate IDF values for durations shorter than those available in the raw data.

There are several methods available to perform regression analysis which are based on the theoretical principles of frequency analysis. The best method used depends on the nature of the data and often several methods must be tried and compared to one another in order to identify the best choice.

2.2 Theoretical Principles of frequency analysis

2.2.1.Theory of fitting probability distribution to data.

Quantitative scientific data may be classified as either experimental/deterministic or historical/stochastic. The experimental data are measured through experiments and can usually be obtained repeatedly by experiments whenever required. The historical data, however, are collected from natural phenomena that can be observed only once and then will not occur again. Thus they cannot be reconstituted when they are lost. Most hydrological data are historical and can be treated as statistical variables.

It is difficult for hydrologists to collect and store all hydrological data such as rainfall and runoff. In most cases, the available amount of data is limited and it may also contain some gaps. When a decision is made to carry out a water resources project in a region, it is first necessary to collect all the information related to the region and then to analyze the collected data.

A frequency analysis of the data is the most commonly applied method.

A mathematical tool is used in characterizing the available data and also to fill the gaps in the observations or to extend it to a longer period. This is very necessary so as to ensure the consistency of the rainfall data available.

The gaps in the data can be filled using correlated data obtained from hydrological areas that are geographically closer and similar to the area under study and also by using historical mean values. The observed data can be extended by a mathematical equation (model). Although hydrological variables are of the continuous type, they are discretized and used as a discrete series.

Frequency analyses of hydrologic data use probability distributions to relate the magnitude of extreme events to their frequency of occurrence. It is generally assumed that a hydrological variable has a certain distribution type. Some of the most common and important probability distributions used in hydrology are the Normal, Log-Normal, Exponential, Gamma, Pearson Type I,II and III, Log Pearson, General Extreme Value I(Gumbel), General Extreme Value II, and General Extreme Value III(Weibull).

The normal distribution generally fits to the annual flows of rivers. The log-normal distribution is also used for the same purpose. In hydrology, the gamma distribution has the advantage of having only positive values, since hydrological variables such as rainfall and runoff are always positive (greater than zero) or equal to zero as a lower limit value.

The Gumbel and Weibull distributions are used for extreme (maxima and minima values) respectively of hydrological variables.

The Gumbel distribution is used in the frequency analysis of floods and the Weibull distribution in the analysis of low flow values observed in rivers.

The assumed distribution is fitted to the sample, i.e. the parameter of the statistic are determined from discrete data. Then a test of goodness of fit is conducted to assess the validity of the fitting.

Different techniques are used in estimating the parameters. These are listed here in ascending order of efficiency, from the least efficient to the most efficient: the graphical method, the leastsquares method, the method of moments, and the maximum likelihood method. The recently developed method of probability-weighted moments (L-moments) can also be used.

The graphical method for parameter estimation is particularly used for distributions which can be plotted as a straight line on a probability graph paper. The normal distribution is the most common example. It is not easy to use this method to estimate the parameters of distributions which cannot be plotted as a straight line on the probability graph paper, as in the case for gamma distribution. The sum of squares of differences between the coordinates of the observed values and their corresponding values on the fitted distribution should be the smallest in the least-squares method. Being one of the oldest and the most useful methods of parameter estimation, the method of moments uses relations between the central moments and parameters of the distribution.

In the maximum likelihood method, values maximizing the maximum likelihood function of the distribution are taken as estimates of the parameters. Developed in 1979 originally for distributions expressible in inverse form, the method of probability-weighted moments developed in 1997 can also be used for parameter estimation of the 2-parameter gamma distribution, a distribution which is non-expressible in inverse form. The method is based on relations between the probability-weighted moments or L-moments and parameters of the
distribution.

The parameters of the statistic will be determined by the method of moments. It has been used for IDF generation, by a lot of hydrological and meteorological services in the World, such as the Canadian Weather service, NWS National Weather Service (USA), United Kingdom and
Nigeria.

Many probability distributions are not a single distribution, but are in fact a family of distributions. This is due to the distribution having one or more shape parameters.

Shape parameters allow a distribution to take on a variety of shapes, depending on the value of the shape parameter. These distributions are particularly useful in modeling applications since they are flexible enough to model a variety of data sets.

2.2.2 Moments of probability distributions.

A probability function defines the relationship between the outcome of a random process and the probability of occurrence of that outcome. A probability function defined over a discrete sample space is called discrete probability distribution function otherwise the probability function is continuous (continuous probability distribution function). The following review will be limited to continuous probability distribution.

If X is a continuous random variable so that $P(a \leq x \leq b) = \int_a^b f(x)dx$ and if $f(x)$satisfies the following conditions: $f(x) \geq 0 \ \forall x$ and

$$\int_{-\infty}^{+\infty} f(x)dx = 1 \qquad (2.1)$$

$f(x)$ is the probability density function *(pdf)* of the random variable X.

Its cumulative distribution function *(cdf)* is defined as follow $F(x) = p(X < x)$

$$F(x) = \int_{-\infty}^{x} f(x)dx \qquad (2.2)$$

The expected value of a random function g is given by $< g >= \int_{-\infty}^{+\infty} g(x)f(x)dx \quad (2.3)$

a) Mean or first moment of the probability distribution

If $g(x) = x$; $< g >$is the mean of the random variable X $< x >= \mu = \int_{-\infty}^{+\infty} xf(x)dx \quad (2.4)$.

b)Variance or second moment of the probability distribution

If $(x) = (x-\mu)^2$; $< g >$ is the variance of the continous random variable X:

$$\sigma^2 = \int_{-\infty}^{+\infty} (x-\mu)^2 f(x)dx \qquad (2.5)$$

The average magnitude of deviation of the random variable from its mean is measured by the standard deviation $\sigma = \sqrt{\sigma^2}$ $\qquad (2.5')$.

c) Skewness or third moment of the probability distribution

The skewness or the skewness coefficient measures the symmetry of the *pdf* about the mean; it is given by the expected value of the function $g(x) = \frac{(x-\mu)^3}{\sigma^3}$

$$\vartheta = \frac{1}{\sigma^3} \int_{-\infty}^{+\infty} (x-\mu)^3 f(x)dx \qquad (2.6)$$

The variables μ, σ *and* ϑ are measures of the average, variability about the average and the symmetry about the average respectively of a statistic population.

2.3 Probability Distributions for Extreme Hydrologic Variables

a) Exponential Distribution

Some sequences of hydrologic events, such as the occurrence of precipitation, may be considered as poisson processes, in which events occur instantaneously and independently on a time horizon, or along a line.

The time between such events or interarrival time, is described by the exponential distribution whose parameter λ is the mean rate of occurrence of the events. The exponential distribution is used to describe the interarrival times of random shocks to hydrologic systems such as slugs of polluted runoff entering streams as rainfall washes the pollutants off the land surface.

The advantage of the exponential distribution is that it is easy to estimate λ from the observed data and the exponential distribution lends itself well to theoretical studies, such as a probability model for the linear reservoir $\left(\lambda = {}^{1}/_{k}\right)$, where k is the storage constant of the linear reservoir.

Its disadvantage is that it requires the occurrence of each event to be completely independent of its neighbours, which may not be a valid assumption for the process under study.

b) Gamma Distribution

The time taken for a number β of events to occur in a Poisson process is described by the gamma distribution, which is the distribution of a sum of β independent and identical exponentially distributed random variables. The gamma distribution has a smoothly varying form like the typical probability density function and is useful for describing skewed hydrologic variables without the need for log transformation. It has been applied to describe the distribution of depth of precipitation in storms. The gamma distribution involves the gamma function $\Gamma(\beta)$, which is

given by $\Gamma(\beta) = (\beta - 1)! = (\beta - 1)(\beta - 2)$……(2.7)

$$\Gamma(\beta) = \int_0^\infty u^{\beta-1} e^{-u} du \qquad (2.8.)$$

The two- parameter gamma distribution (parameters β *and* λ has a lower bound at zero, which is a disadvantage for application to hydrologic variables that has a lower bound larger than zero. Maximum intensity values used in IDF analyses have a lower bound larger than zero and hence the gamma distribution has a limitation in IDF analyses.

c) Pearson Type III Distribution

The Pearson Type III distribution, also called the three-parameter gamma distribution, introduces a third parameter, the lower bound ϵ, so that by the method of moments, three sample moments (the mean, the standard deviation, and the coefficient of skewness) can be transformed into three parameters λ, β *and* ϵ of the probability distribution.

This is a very flexible distribution, assuming a number of different shapes as $\lambda, \beta\ and\ \epsilon$ vary. The Pearson system of distributions includes seven types; they are all solutions of $f(x)$ in an equation of the form;

$$\frac{d[f(x)]}{dx} = \frac{f(x)(x-d)}{C_0 + C_1 x + C_2 x^2} \qquad (2.9)$$

where d is the mode of the distribution(the value of x for which $f(x)$ is maximum) and

$C_0, C_1, and\ C_2$ are coefficients to be determined. When $C_2 = 0$, the solution of eqn.(2.9) is a Pearson Type III distribution.

The Pearson Type III distribution was first adopted in hydrology by Foster in 1974 to describe the probability distribution of annual maximum flood peaks. It has a limitation when the data is very positively skewed.

d) Log- Pearson Type III distribution

If log X follows a Pearson Type III distribution, then X is said to follow a log-Pearson Type III distribution. This distribution is the standard distribution for frequency analysis of annual maximum floods in the United States.

The location of the bound ϵ in the log-Pearson Type III distribution depends on the skewness of the data. The disadvantage of the log-Pearson Type III distribution is that if the data are positively skewed, then $\log X \geq \epsilon$ and ϵ is the lower bound and if the data is negatively skewed, and is an upper bound. $\log X \leq \epsilon$ ϵ

The log transformation reduces the skewness of the transformed data and may produce transformed data which are negatively skewed from the original data which are positively skewed. In that case, the application of the log-Pearson Type III distribution would impose an artificial upper bound on the data. Again, the log-Pearson Type III distribution requires a lot data to fix the value of the shape parameters(λ, β) and the bound (ϵ) of the distribution.

e) Extreme Value Distribution of Gumbel

Extreme values are selected maximum and minimum values of sets of data. For example, the annual maximum discharge of a given location is the largest recorded discharge value during a year, and the annual maximum discharge values for each year of historical record make up a set of extreme values that can be analysed statistically.

Distributions of the extreme values selected from sets of samples of any probability distribution converge to one of the three forms of extreme value distributions called Types I,II and III respectively, when the number of selected extreme values is large. The three limiting forms are special cases of a single distribution called the General Extreme Value (GEV) distribution. The probability distribution function for the GEV is;

$$F(x) = exp\left[-\left(1 - k\frac{x-u}{\alpha}\right)^{1/k}\right] \qquad (2.10.),$$

where k, u and α are parameters to be determined. The three limiting cases are

- For $k = 0$, the Extreme Value Type I distribution,
- For $k < 0$, the Extreme Value Type II distribution for which (2.10) applies for $(u + \alpha/k) \leq x \leq \infty$
- For $k > 0$, the extreme Value Type III distribution for which (2.10) applies for $-\infty \leq x \leq u + \alpha/k$.

In all three cases, α is assumed to be positive. For the EV I distribution x is unbounded, while for EV II, x is bounded from below by $u + \alpha/k$, and for the EV III distribution x is bounded from above by $u + \alpha/k$.

The EV I and EV II distributions are known as the Gumbel and Frechet distributions respectively. If a variable x is described by the EV III distribution, then $-x$ is said to have a Weibull distribution.

There are three asymptotic forms of the distributions of extreme values, named Type I, Type II, and Type III respectively. The extreme value Type I (EV I) probability distribution function is;

$$F(x) = exp\left[-exp\left(-\frac{x-u}{\propto}\right)\right] \quad -\infty \leq x \leq \infty \qquad (2.11.)$$

The parameters are estimated as;

$$\propto = \frac{\sqrt{6}s}{\pi} \qquad (2.12)$$

$$u = \bar{x} - 0.5772 \propto \qquad (2.13)$$

The parameter u is the mode of the distribution(point of maximum probability density). A reduced variate y can be defined as;

$$y = \frac{x-u}{\propto} \qquad (2.14)$$

Substituting the reduced variate into(2.11) yields $F(x) = exp[-exp(-y)]$ (2.15). Solving for y:

$$y = -\ln\left[\ln\left(\frac{1}{F(x)}\right)\right] \qquad (2.16)$$

Equation (2.16) is used to define y for the Type II, and Type III distributions.
The values of x and y can be plotted. For the EV I distribution, the plot is a straight line while, for large values of y, the corresponding curve for the EV II distribution slopes more steeply than for EV I, and the curve for the EV III distribution slopes less steeply, being bounded from above. Extreme value distributions have been widely used in hydrology. They form the basis for the standardised method of flood frequency analysis in Great Britain. Storm rainfalls are commonly modelled by the extreme value Type I (Gumbel) distribution and drought flows by the Weibull distribution, that is the EV III distribution applied to $-x$.

The extreme rainfall values used in this study are subjected to a Gumbel distribution. For maxima extremes frequency analysis the Gumbel and the log-Pearson distribution functions are recommended. As the former is a two-parametric function it is more advantageous than the latter for it does not require a lot of data to determine all the parameters. Gumbel distribution is recommended when frequency analysis is performed on an individual gauge records because data are not enough to determine the shape parameter.

2.3.1 Gumbel Distribution

Gumbel distribution is a statistical method often used for predicting extreme hydrological events such as floods. The Extreme Value type I (Gumbel) distribution is used extensively in flood studies in the UK and in many other part of the world.

A random variable(X) is said to follow a Gumbel distribution if its cumulative distribution function and probability density function are defined as follows:

$$F(x) = \exp\left(-exp\left(-\frac{x - x_0}{s}\right)\right) \quad s \neq 0 \qquad (2.17)$$

$$f(x) = \frac{1}{s}\exp\left\{-\frac{x - x_0}{s} - exp\left[-\frac{x - x_0}{s}\right]\right\} \qquad (2.18)$$

$-\infty < x < +\infty$; $-\infty < x_0 < +\infty$; $s > 0$,

$s$and x_0 are scale parameter and position parameter respectively.

The mean, variance and the skewness coefficient for the extreme-value type I distribution are defined by

$$\mu_G = x_o + 0.577216 \times s \qquad (2.19)$$

$$\sigma_G^2 = \frac{\pi^2}{6} \times s^2 \qquad (2.20)$$ and

$$\vartheta = 1.1396 \qquad (2.21)$$

By using the following transformation $u = \frac{x - x_0}{s}$ the Gumbel distribution can be written in form

$F(u) = \exp[-exp(-u)]$ (2.22) - cumulative distribution function(cdf)

$f(u) = \exp[-u - exp(-u)]$ (2.23) -probability distribution function(pdf)

This cdf is most useful in determining the quantile for a given frequency or return period of extreme events such as flood flows (annual maximum flows), maximum rainfall.

2.3.2 Frequency factor

When the Gumbel statistic has been fitted to the sample, any extreme value related to a return period greater than or equal to two years ($T \geq 2$ years), is found by the formula

$$X_T = \mu_G + K_T \times \sigma_G \quad (2.24), (chow, 1964)$$

Where μ_G and σ_G are Gumbel mean and standard deviation of the population, K_T is the coefficient factor depending on a certain return period T.

$$K_T = -\frac{\sqrt{6}}{\pi}\left\{0.577216 + \ln\left[\ln\left(\frac{T}{T-1}\right)\right]\right\} \qquad (2.25)$$

For $T < 2$ years, $K_T = -\frac{\sqrt{6}}{\pi} \times \left[0.5772 - lnT + \frac{1}{2 \times T} + \frac{1}{24 \times T^2} + \frac{1}{8 \times T^3}\right] \quad (2.26)$

2.3.3. Statistical test of hypothesis

A statistical hypothesis test is a method of making statistical decisions using experimental data. The decisions are made using null hypothesis tests. There are two types of tests; parametric test and non-parametric test.

It is often difficult to describe an appropriate theoretical model for the outcome of a random phenomenon. A model is often proposed(such as a linear model in regression with normality assumptions) that reasonably approximates the true model. If the assumptions are not violated, then an exact solution for the approximation of the problem is provided.

Parametric statistical procedures (t-test, ANOVA, least-squares regression, etc) depend on distributional assumptions about hydrologic events. Because hydrologic events do not always meet the assumptions for parametric tests, we need inferential procedures whose validity does not depend on the assumptions. In these cases, nonparametric methods can provide a potential solution to an exact problem. Statistical inferences not concerned with parameter values are called nonparametric.

The advantages of non-parametric test over the parametric tests are;

- Because most non parametric tests depend on fewer assumptions than parametric methods there is smaller risks of non parametric methods being improperly used.
- Many nonparametric methods are computationally simple.
- The distributional theory behind many nonparametric methods is not as complex as the theory used in parametric methods.
- Nonparametric methods and concepts are often easier for most people to understand than parametric methods and concepts.
- Most nonparametric methods are now available in most statistical computing packages.
- Nonparametric methods exist on all measurement scales making analysis of data measure on a weak scale possible.
- Nonparametric methods are often more powerful than the parametric methods if the assumptions behind the parametric model are not true. Power is the probability that the experimenter will correctly reject the null hypothesis when it is false.

Some of the disadvantages of Nonparametric Methods are:

- When a parametric model is appropriate, using a nonparametric test is less efficient (that is, it wastes information). For example, if data are a simple random sample from a normal distribution, then using a t-test for hypotheses about the population mean (μ) is appropriate.
- Some nonparametric methods are computationally demanding and are not available in statistical computing packages.
- Nonparametric methods do not exist for many research problems or data sets due to the complexity of the research problem or how the data was collected.

Non-parametric test is chosen over parametric test in this work for the test of goodness of fit due to its simplicity.

2.3.4 Test of goodness of fit

To account for the validity of the fitting of the probability distribution, a test of goodness of fit is required. Various criteria could be employed to evaluate the suitability of a probability distribution for describing a set of data. Statistical goodness-of-fit tests as well as graphical display such as probability plots are effective way to determine whether the fitted distributions are consistent with the given set of observations. In addition, the predictive ability of a model is important in applying the model for prediction of future events.

Often in selecting a particular distribution, one may be tempted to select a distribution with large number of parameters. Generally, the more parameters a distribution has, the better it will fit to the data. However, difficulty in the parameter estimation arises, and the distribution may be too rigid to accurately extrapolate beyond the range of the available data.

Test of goodness of fit is classified into three major groups, namely, Regression type tests, Probability transformation tests and Special features tests.

The most common regression test is probability plotting, where the ordered sample data is plotted on a graph whose axes are transformed so that if the data conforms to the selected distribution, they lie on a straight line. The deviation of the data from linearity may be assessed visually. It is strongly recommended that other tests of goodness fit should always be augmented by a probability plot.

An informal visual test requires that the relative frequencies of the sample data and the fitted probability density function be plotted on the same graph and visually compared. This visual test is a variation of the probability plotting test.

The chi-square test, the oldest of all goodness-of-fit tests is a less subjective comparison of frequency histograms with fitted distributions. In this procedure, the range of the sample data is divided into a discrete number of intervals and the number of data points falling in each interval is compared with the expected number predicted by the fitted distribution. The expected number is obtained by integrating the fitted probability distribution between the interval boundaries and multiplying by the number of data points in the sample.

Although this test can be used for continuous random variables, its use is recommended for categorical and numerically discrete random variables because for a continuous random variable, there are an infinite number of ways to partition the support (the set of possible values) of the variable, and the choice of the number of intervals is not unique.

Although it is less subjective than visual assessment, the chi-square test is not entirely objective. The test statistic is dependent on the number and lengths of the intervals. There is no single accepted rule for choosing either. It is recommended to use equi-probable intervals with the expected number in each interval being five or more.

Probability transformation tests are based on the fact that if a set of data conforms to a probability distribution, *f(x)*, then the transformed variable, y_i, given by

$y_i = \int_{-\infty}^{x_i} f(x)\,dx$ conforms to a uniform distribution. The test statistic of these tests, which include the Kolmogorov-Smirnov, Kuiper, Cramer-von Mises and the Anderson-Darling tests among others, are measures of the sample deviation from uniformity. In the KolmogorovSmirnov procedure, the test statistic is the maximum deviation of the ordered transformed variable either above or below the uniform line, while for the Kuiper procedure the test statistic is the sum of the maximum deviations both above and below the uniform line. The Cramer-von Mises test statistic is essentially the sum of the squared deviations from the uniform line while the Anderson-Darling statistic is a weighted sum of deviations, with more weight given to observations in the tails of the distribution.

2.3.5 Procedure for testing Hypothesis

1.Formulate hypotheses.

2.Select the appropriate statistical model (theorem) that identifies the test statistic and its distribution.

3.Specify the level of significance, which is a measure of risk.

4.Collect a sample of data and compute an estimate of the test statistic.
5.Obtain the critical value of the test statistic, which defines the region of rejection.

6. Compare the computed value of the test statistic (step 4) with the critical value (step 5) and make a decision by selecting the appropriate hypothesis.

2.3.6 Description of Types of Goodness-of-Fit Test

a)Chi-Square Test $\chi^2(\nu)$

The steps hereafter will be followed to make the chi-square test:

Step 1: put the observed data (O) and expected (E) values into intervals so as to determine the frequency of both variables in each class. This can be well express by a histogram of frequencies. Step 2: rearrange the classification so that the minimum expected frequency in each class becomes 5 or great. The classes with low frequency should be merged to this end. Step 3: compute the chi-square value for all intervals

$$\chi^2(\nu) = \sum_i^n \frac{(O_i - E_i)^2}{E_i} \qquad (2.27)$$

ν is the degree of freedom and equals to $n-k-1$, where *n* is the number of intervals and *k* is the number of distribution parameter obtained from the sample statistics; they are constraints imposed to the fitting process.

Step 4: compare the value so obtained to the chi-square value $\chi^2_{0.95}$ from tables. The null hypothesis will be accepted if $\chi^2 < \chi^2_{0.95}$ (2.28) and rejected if otherwise.

b) Kolmogorov-Smirnov Test

This statistical test defers from the chi-square test in the sense that it is done on cumulative distribution function whereas the chi-square is done on probability distribution. This test is only applicable to continuous distribution functions. The procedure for implementing the

Kolmogorov-Smirnov test is as follows:

1. Let $F_0(x)$ be the sample cumulative distribution function based on *N* observations. For any observed $x,\ F_0(x) = \frac{j}{N}$, where *j* is the number of observations less than or equal to *x*.
2. Let $F_t(x)$ be the specified theoretical cumulative distribution function under the null hypothesis.
3. Determine the maximum deviation, D, defined by

$$D = \max|F_0(x) - F_t(x)| \qquad (2.29)$$

The hypothesis is rejected if, for the chosen significance level, the observed value of D is greater than or equal to the critical value of the Kolmogorov-Smirnov statistic.

c) Anderson-Darling test

The Anderson-Darling test is a member of a group of goodness-of-fit statistics which has come to be known as empirical distribution statistics(EDF) because they measure the discrepancy between the empirical distribution function of a given sample and the theoretical distribution to be tested.

It is designed to tests that the random variable(X) has a continuous cumulative distribution function $F_x(x,\theta)$; θ is the vector of one or more parameters entering into the distribution function. Thus for a normal distribution, the vector$(\theta = \mu, \sigma^2)$. Where there is no ambiguity, $F(x,\theta)$ or $F(x)$ will be written for $F_x(x,\theta)$. The empirical distribution

function(EDF) is defined as $F_n(x)$=(proportion of sample $< x$) and a family of statistics measuring the discrepancy between $F_n(x)$ and $F(x,\theta)$ is the Cramer–von Mises family.

$W_n^2 = n\int_{-\infty}^{\infty}[F_n(x) - F(x,\theta)]^2\psi(x)dx$ (2.30);

The function $\psi(x)$ gives a weighting to the squared difference between $F_n(x)$ and $F(x,\theta)$. The Anderson-Darling test is W_n^2 with

$\psi(x) = [F(x,\theta)(1 - F(x,\theta)]^{-1}$ (2.31);Where $\psi(x)$ is the weight function.

The weight function has the effect of giving greater importance to observations in the tail than do other EDF statistics, counteracting the fact that $F_n(x) - F(x,\theta)$ approaches zero in each tail. Since tests of fit are often needed implicitly or explicitly to guard against wayward observations in the tails, the statistic is a recommended one with generally good power over a wide range of alternative distributions when $F(x,\theta)$ is not the true distribution.

The numerical calculation of Anderson-Darling test statistic A^2 is done by the following two steps: Calculate

$Z_i = F\left(x_{(i)},\theta\right)$ $(i = 1,\ldots,n)$ (2.32) then,

$A^2 = \sum_{i=1}^{n}\{(2i-1)[\ln z_i + \ln(1 - z_{n+1-i})]\}/_{n} - n$ (2.33), $x_{(i)}$ and z_i are in ascending order.

For the calculations above the tested distribution, $F(x,\theta)$ must be completely specified i.e, the parameters in θ must be known. When this is the case, the situation is described as CASE O. The statistic A^2 was introduced by Anderson-Darling and for CASE O, they gave the asymptotic distribution and a table of percentage points. Large values of A^2 will indicate a bad fit. The distribution of A^2 for a finite sample rapidly approaches the asymptotic distribution and for practical purposes, this distribution can be used for sample sizes greater than 5.

The percentage points are given in tables. To make the goodness –of –fit test, A^2 is calculated using equations (2.32 and 2.33) and compared with the percentage points from the tables. The null hypothesis that the random variable X has the distribution $F(x,\theta)$ is rejected at the level α(significance level) if A^2 exceeds the appropriate percentage point at this level.

Asymptotic theory of the Anderson-Darling statistic

The distribution of A^2 for case O is the same for all distributions tested. This is because the probability integral transformation is made at eqn.2.33 and the values of z_i are ordered values from a uniform distribution with limits 0 and 1. A^2 is therefore a function of ordered uniform random variables.

When θ contains unknown components, the z_i given by the transformation in eqn.2.33. using θ^* instead of θ will not be ordered uniform random variables and the distribution theory of A^2 (as for all other EDF statistics) becomes substantially more difficult. In general, the distribution of A^2 will depend on n and also on the values of the unknown parameters. Fortunately, an important simplication occurs when unknown components of θ are the location and scale parameters only. Then the distribution of an EDF statistic with an approximate estimate of θ will depend on the distribution tested but not on the specific values of the unknown parameters. The simplication makes it worthwhile to calculate the asymptotic theory and percentage points for A^2 for special distributions with location and scale parameters and this has been done for normal and exponential, extreme value Weibull distribution, logistic distribution and gamma distribution with unknown scale parameter but known shape parameter.

The first step in testing goodness-of-fit test for any of the distributions(Cramer-Van Mises and Anderson-Darling test) is to estimate the unknown parameters. This should be done by maximum likelihood for the modifications and asymptotic theory to hold. Suppose that θ^* is the vector of the parameters, with any unknown parameters estimated as above. Vector θ^* replaces θ in eqn.2.32. to give the z_i, and A^2 is always calculated from eqn.2.33. It is then compared with the percentage points from tables. The null hypothesis that the random variable X has the distribution $F(x,\theta^*)$ is rejected at the level α(significance level) if A^2 exceeds the appropriate percentage point at this level.

d) Cramer-Van Mises Test

The Cramer-Van Mises statistic W_n^2 is generally defined to be the statistic,

$$W_n^2 = n\int_{-\infty}^{\infty}[F_n(x) - F_o(x)]^2\,dF_o(x) \quad (2.34)$$

,where $F_o(x)$ is the hypothesized cumulative distribution function (CDF) and $F_n(x)$ is the sample or empirical CDF based on the sample $x_1, \ldots . x_n$. It is used to test the null hypothesis $H_o: F(x) = F_o(x)$ where it is assumed that the sample comes from a population with CDF $F(x)$.

This statistic can be used as an alternative to the chi-squared goodness-of-fit test which requires the data to be grouped before calculation. For the evaluation of the statistic, let $t_i = F_o(x_i')$ where $x_1' < x_2' < \cdots < x_n'$ are the original ordered observations, then;

$$W_n^2 = \sum_{i=1}^{n}\left[t_i - \frac{2i-1}{2n}\right]^2 + \frac{1}{12n} \qquad (2.35)$$

The null hypothesis H_o is rejected in favour of $H_1 \neq F_o(x)$ for large values of W_n^2. If F_o is continuous, the null distribution of W_n^2 is independent of F_o. Small percentage points of W_n^2 are found in tables.

None of the two tests(Anderson-Darling test and Cramer-Van Mises test) has reached a broad consensus in the hydrologic community, possibly due to some complications that inevitably arise when the parameters of the hypothetical distribution are unknown. This is the most common case in hydrology and for the case when the parameters are fully specified a priori, the distribution of EDF test statistics depend on the so-called null hypothesis H_o, i.e., on the probability distribution that is being tested. This means that the percentage points, i.e., the 100 ($1-\alpha$) percentiles of the distributions of the test statistics (α is the significance level of the test), have to be recalculated for any H_o . The method of parameter estimation, the presence of a shape parameter, and the sample size also have an influence on percentage points, and this further complicates the analysis. In this study, the chi-square and Kolmogorov -Smirnov tests are adopted due to the simplicity in their usage.

3. MATERIALS AND METHODS

3.1.Study Area

Kumasi is a city in southern central Ghana's Ashanti region. Kumasi is also the capital of the Ashanti region and of the Kumasi Metropolitan District. It is about 250 kilometres (160 mi) (by road) northwest of Accra. Kumasi is approximately 480 km (300 miles) north of the Equator and160 km(100 miles) north of the Gulf of Guinea.

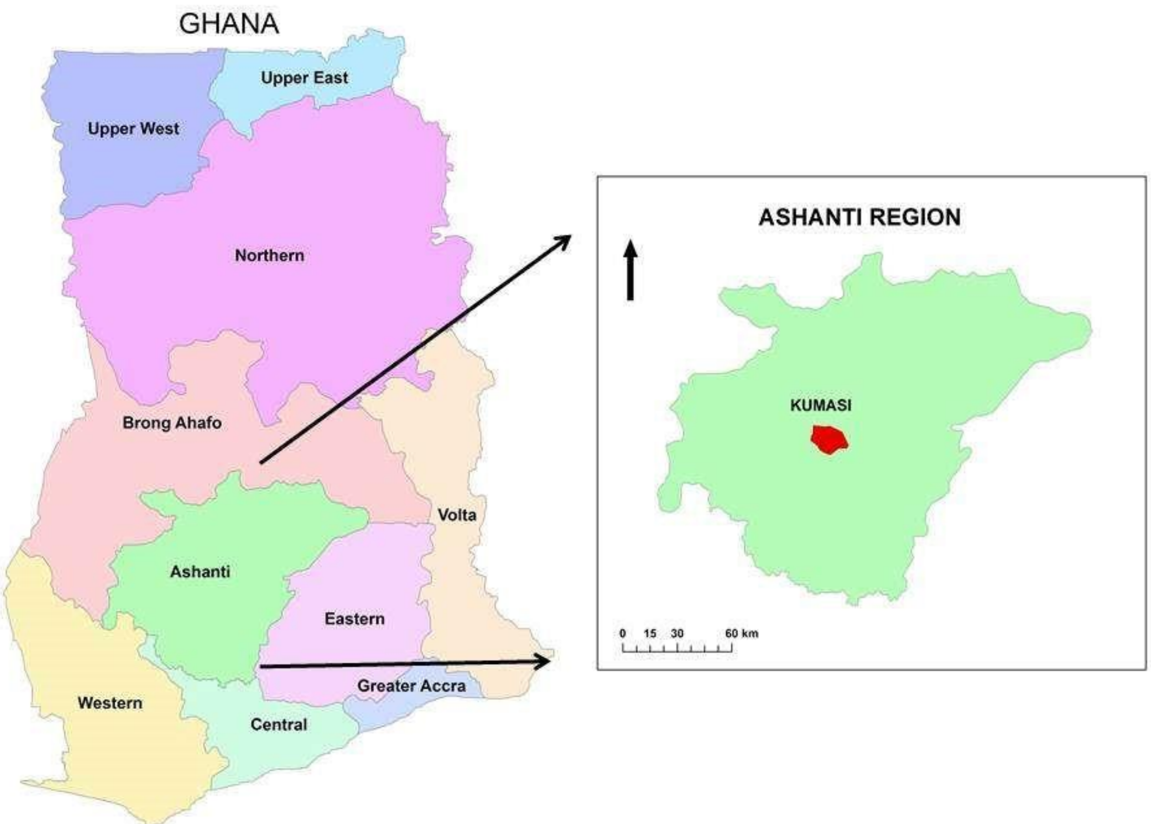

Fig 3.1. Map of Kumasi

(Source: Author's Map)

3.1.1 Temperature

Average maximum and minimum temperatures are about 35 C and 23 C which occurs in February and August respectively.

3.1.2 Precipitation

Most precipitation (rainy season) is seen in March, April, May, June, July, September and October.June is the wettest month and January is the driest month with precipitation values of about 200mm and 30mm respectively.

3.1.3 Sunshine Hours

The maximum and minimum sunshine hours occur in December and August with sunshine hours of about 170hrs and 75hrs respectively.

3.1.4 Vegetation

The vegetation of the metropolis falls within the moist semi-deciduous section of the South-East Ecological zone. Predominant species of trees found are Ceiba, Triplochlon, Celtis with Exotic Species

3.1.5 Humidity

The average humidity is about 84.16% at 09.00 GMT and 60% at 15.00 GMT.

3.1.6 Soil

The major soil type is the Forest Ochrosol. The detailed soil associations are the following: Kumasi - Offin Compound Association; Bomso – Offin Compound Association; Nhyanao - Tinkong Association; Bomso – Suko Simple Association; Bekwai – Oda Compound Association and Bekwai – Akumadan – Oda Compound Association. It is a very rich type of soil suitable for agriculture.

3.1.7 Topography

Kumasi is dominated by the middle Precambrian Rock. It is within the plateau of the South-West physical region which ranges between 250-300 meters above sea level. The topography is generally undulating.

3.1.8 Drainage

The major rivers and streams in Kumasi include Subin, Wewe, Susan, Aboabo, Oda, Owabi, Suntreso, Akrubu, Acheamponmene and Asuoyeboa.

3.2 Collection of Data

3.2.1 Data gathered

The data used in this work have been provided by the Ghana meteorological Services Department which has the responsibility of measuring, analysing and storing meteorological data and forecasting the weather in Ghana. The data consists of annual maximum series (AMS) of rainfall depth over a period of twenty-two years for nine(9) laps of time: 12min,24min,42min,1hour,2hours,3hours,6hours,12 hours, and 24 hours. Is not clear how these values were obtained from the original charts. However, if the original charts which are the cumulative rainfall depth with time were available, it would have been easier to have obtained any duration from the charts.

3.2.2. Filling in Missing Data

The data collected contained a lot of missing data which had to be filled. The table 3.1 below gives a breakdown of the data collected. Several procedures were applied in order to fill in the gaps. First, correlation analysis was made between the data and the average rainfall data for Kumasi and it was found that there was not a good correlation between them. Observing the data, it was seen that the 24 min AMS contained less number of missing data and hence an attempt was made to find the correlation between the 24 min AMS and the rest of the data.

When the correlation analysis was performed, it was found that there was a good correlation between the 24 min,12 min,42 min,1 hr, and 2 hr AMS but the correlation between the 24 min,3 hrs,6 hrs,12 hrs and 24 hrs AMS was not satisfactory.

Table 3.3 below gives the correlation between the 24 min and the rest of the AMS. **Table 3.1: Summary of Data Gathered**

Duration(hrs)	No. of years with some records
0.2	22

0.4	22
0.7	22
1.0	22
2.0	22
3.0	21
6.0	20
12.0	19
24.0	18

Table 3.2: Correlation between AMS and Average Rainfall depths for Kumasi

Duration of Rainfall	12 min	24 min	42 min	1 hr	2 hrs	3 hrs	6hrs	12hrs	24hrs
Correlation Values	0.4093	0.4218	0.3650	0.3250	0.2700	0.2950	0.2700	0.3820	0.2520

Table 3.3: Correlation between 24 min AMS and the other AMS

Duration of Rainfall	12 min	42 min	1 hr	2 hrs	3 hrs	6hrs	12hrs	24hrs
Correlation Values	0.6640	0.8510	0.7460	0.5470	0.4820	0.4020	0.3720	0.2760

Since the 24 min AMS did not give a consistent good correlation with the AMS, the Double Mass-Curve (which is used for consistency test) was applied between the 24 min AMS and the AMS of other durations.

In the application of the Double Mass-Curve, the following steps were taken: ➢ The 24 min AMS was taken as Y_{old} and the other AMS was referred to as X_{old}

- $\sum \boldsymbol{X_{old}}$ and $\sum \boldsymbol{Y_{old}}$ were determined.
- $\sum \boldsymbol{X_{old}}$ and $\sum \boldsymbol{Y_{old}}$ were plotted on a graph sheet which clearly exhibited a linear relationship.
- A line of best fit was drawn through the points.
- All points($\sum \boldsymbol{X_{old}}$ and $\sum \boldsymbol{Y_{old}}$) not lying on the line of best fit were brought perpendicularly onto the line using a 90^0 set square.
- When this was done, the original position of ($\sum X_{old}$ and $\sum Y_{old}$) changed and a new point ($\sum X_{new}$ and $\sum Y_{new}$) was obtained.
- X_{new} and Y_{new} were obtained from $\sum X_{new}$ and $\sum Y_{new}$ using the expressions;

$\sum X_{new} = X_{new} + X_{new-1} + X_{new-2} + X_{new-3} + \cdots$ and similarly,
$\sum Y_{new} = Y_{new} + Y_{new-1} + Y_{new-2} + Y_{new-3} + \cdots$

$X_{new-1}, X_{new-2}, X_{new-3} \ldots .\ are\ the\ preceeding\ X_{new}\ values\ and\ \ Y_{new-1}, Y_{new-2}, Y_{new-3} \ldots$ $are\ the\ preceeding\ Y_{new}\ values.$

- This procedure was then applied to the rest of the data and a good correlation coefficient was obtained between all the $X_{new}\ and\ Y_{new}\ values.$
- The $X_{new}\ values\ obtained\ represented\ the\ new\ AMS\ values.$

Table 3.4: Correlation between New 24 min AMS and the New AMS

Duration of Rainfall	12 min	24 min	42 min	1 hr	2 hrs	3 hrs	6hrs	12hrs	24hrs
Correlation Values	0.8166	1.0000	0.8807	0.9094	0.8381	0.9129	0.8625	0.9533	0.9042

3.2.3 Determination of n-min extremes from hourly extremes

The n-min extremes can be deducted from hourly extremes using the coefficients suggested by the World Meteorological Organisation; they can be used to estimate rainfall-frequency with a relative error less than 10 percent.

After a frequency analysis on hourly data, the obtained IDF estimates is multiplied by the coefficients in the table below to determine the n-min IDF estimates.

Table 3.5: Ratio of n-min to 60-min.

Duration [min]	5	10	15	30	60
Ratio [n-min to 60min]	0.290	0.450	0.570	0.790	0.999

Source: World Meteorological Organisation: *Operational Hydrology*, report no.1,

(Manual for Estimation of Probable Maximum Precipitation. Second Edition. Secretariat of the World Meteorological Organisation-Geneva-Switzerland, 1986.)

3.3 Methods and procedures.

3.3.1 Procedure for selecting the appropriate Probability distribution.

From literature review, Gumbel and Log Pearson Type III distributions were identified to be the most suitable for IDF construction. To select which of the two distributions to use, the following steps were followed:

- For each duration, the Gumbel and Log Pearson Type III distributions were fitted to the Annual maximum Series using the Easy Fit software.
- Best of fit for each distribution was determined using the Chi-square and Kolmogorov-Smirnov test-of-goodness-of-fits at 5% significance level for each duration.

- After the fitting, both distributions were ranked to determine the appropriateness of the fitting.
- A rank of 1 was assigned to the distribution with the better fit and 2 to the distribution with the less satisfactory fit.
- Gumbel distribution provided a better fit than the Log Pearson Type III distribution for eight(8) out of the nine(9) durations.
- Hence Gumbel distribution was selected for the frequency analysis.

3.3.2 Procedure for Fitting Gumbel distribution to sample data.

The fitting of a statistical distribution to data series aims to find out the parameters of the distribution from the sample; and then verify if really the sample are drawn from that statistic.

As it has been hypothesized that extreme rainfall events are Gumbel distributed, the steps below describe how the fitting is done. After that a statistical test of goodness fit is performed to assess the validity of the fitting. When the process succeeds, as for the case of extreme rainfall depth, events of very low probability of occurrence or very high return period can be approximated from the distribution.

Step 1: Rank AMS values from highest to lowest; assign a rank m to each value with the highest value having a rank of 1(one) and the lowest value a rank of n. They constitute the observed data.

Step 2: Calculate the exceedence probability. The Gringorten and Weibull formula could be used for estimating the cumulative probability distribution. The main drawback of the Weibull formula is that it is asymptotically exact (as the number of observations approaches infinity) only for a population with an underlying uniform distribution, which is relatively rare in nature. To address this shortcoming, Gringorten proposed that the exceedance probability of observed data be estimated using the relation: $p = \frac{m-0.44}{N+0.12} \quad (3.1)$;

$p = P(X \geq x)$;

Note that $p = 1 - F(x)$ and $F(x) = P(X < x)$

Step 3: Determine the reduced variable from p by the formula $u = -\ln[-\ln(1-p)]$ (3.2)

Step 4: Compute the sample mean μ_s and standard deviation σ_s; this is easily done with Excel program.

Step 5: Find the position parameter x_0 and the scale parameter s of the Gumbel distribution with the following formulae:

$$x_0 = \mu_s - \frac{\mu_N}{\sigma_N}\sigma_s \quad (3.3)$$

μ_N is the mean of the reduced variable

$$s = \frac{\sigma_s}{\sigma_N} \quad (3.4)$$

σ_N is the standard deviation of the reduced variable

Step 6: use the formulae [formulae (2.9) and (2.10)] below to derive the Gumbel mean μ_G and Standard deviation σ_G:

$$\mu_G = x_0 + 0.5772 \times s$$

$$\sigma_G = 1.2825 \times s$$

Step 7: for each rank, the Gumbel variable is obtained by use of the formula

$$X_G = x_0 + u \times s \quad (3.5)$$

They constitute the expected data.

3.3.3 Procedure for testing the null hypothesis

A test of a hypothesis is a rule that assigns one of the inferences: *"the hypothesis is accepted"* or *"the hypothesis is rejected"* to each foreseeable result of an experiment.

The null hypothesis will be H_0:"the observed data are drawn from a Gumbel distribution."

In other words the sample is supposed to be extracted from a Gumbel distributed population.

Level of significance

Level of significance is the probability of committing type I error which is the error of rejecting the null hypothesis when it is true.

$$\alpha = Prob(rejecting\ H_o/H_o\ is\ true) \qquad (3.6)$$

The test is conducted at a significance level of 5% ($\alpha = 0.05$).

3.4 Application of Procedures and Results

3.4.1 Selecting Appropriate Probability Distribution for 12 min(0.2 hr) Duration

The results obtained after fitting the Chi-square and Kolmogorov-Smirnov tests of goodness of fits to Gumbel and Log Pearson Type III distributions for 12 min(0.2 hr) duration are shown in tables 3.6,3.7,and 3.8 below. The results of the fitting for the other durations are found in the appendices. Table 3.9 gives the ranking for the nine rainfall durations after the fitting process.

Table 3.6 Goodness-of-Fit for Gumbel Distribution(0.2 hrs)

Kolmogorov-Smirnov					
Sample Size	22				
Statistic	0.18771				
P-Value	0.37337				
Rank	1				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	0.22115	0.25283	0.28087	0.31394	0.33666
Reject?	No	No	No	No	No
Chi-Squared					

Deg. of freedom Statistic P-Value Rank	2 1.292 0.52413 1				
□	0.2	0.1	0.05	0.02	0.01
Critical Value	3.2189	4.6052	5.9915	7.824	9.2103

Table 3.7: Goodness-of-Fit for Log Pearson Type III Distribution(0.2 hrs)

Kolmogorov-Smirnov					
Sample Size Statistic P-Value Rank	22 0.20126 0.29376 2				
□	0.2	0.1	0.05	0.02	0.01
Critical Value	0.22115	0.25283	0.28087	0.31394	0.33666
Reject?	No	No	No	No	No
Chi-Squared					
Deg. of freedom Statistic P-Value Rank	2 3.6866 0.1583 2				
□	0.2	0.1	0.05	0.02	0.01
Critical Value	3.2189	4.6052	5.9915	7.824	9.2103

Table 3.8: Summary of Goodness of Fit for 12 min(0.2 hrs)

#	Distribution	Kolmogorov Smirnov		Chi-Squared	
		Statistic	Rank	Statistic	Rank

1	Gen. Extreme Value	0.18771	1	1.292	1
2	Log-Pearson 3	0.20126	2	3.6866	2

Table 3.9. Ranking for Gumbel and Log Pearson Type III for the Nine Durations

Duration(hrs)	Chi-Square		Kolmogorov-Smirnov	
	Gumbel	Log-Pearson III	Gumbel	Log-Pearson III
0.2	1	2	1	2
0.4	1	2	1	2
0.7	1	2	1	2
1.00	1	2	1	2
2.00	1	2	1	2
3.00	1	2	1	2
6.00	2	1	1	2
12.00	1	2	2	1
24.00	1	2	1	2

3.4.2 12 Minute(0.2hrs) Annual Maxima Series (AMS) analysis

Sample Mean(μ_s)	26.545mm
Sample St. Dev.(σ_s)	7.494mm
Position Parameter(x_o)	23.214mm
Scale Parameter(s)	6.071mm
Gumbel Mean(μ_G)	26.719mm
Gumbel St. Dev.(σ_G)	7.786mm
Mean of Reduced Variable(μ_N)	0.549mm
St. Dev of reduced variable.(σ_N)	1.234mm

All the computations in the tables below have been done by use of Microsoft Excel program functions. They have been executed according to step 1 through step 7 as described in point 3.3.2. μ_N and σ_N are found to be equal to *0.5487* and *1.2344* respectively for *N=22*.

In the second column of table 3.11,sampled extreme rainfall depth in mm are ordered in descending order. Column four are exceedence probabilities obtained with Gringorten formula.

Table 3.10: 12 Minute(0.2hrs) AMS Sample and distribution parameters

In column 5, the reduced variable *u* is computed using expression (3.2) and column 6 of table 3.11 are the expected values. They were directly generated by Gumbel distribution in accordance with formula (3.5). Column 6 of table 3.11 gives the intensities which are obtained by dividing the rainfall depths in column(2) by their respective duration i.e.(0.2hrs).

The mean and standard deviation of the sample and the parameters of the distribution are given above in table 3.10; they have been derived with respect to formulae (2.4) and (2.5') The histograms in fig 3.2 give an idea of the distribution of data. It is quite obvious that the statistic might be skewed. The distribution is tailed to the right.

Tables 3.13 and 3.14 both show the tests- of- goodness-of-fit done on the measured data i.e. the chi-square test and the Kolmogorov-Smirnov test respectively. The data of table 3.13 have been rearranged, before proceeding to the chi-square test

Intervals of low frequencies have been merged so that the minimum frequency becomes greater than or equal to five. The computed chi-square value is equal to $\chi^2 = 5.00, M = 6$ number of intervals and since the mean and standard deviation were estimated from the measured data, n=2,the χ^2 statistic has M-1-n=3 degrees of freedom, where M=(number of intervals), and n=2(number of population parameters estimated from measured data).

Table 3.11: 0.2 Hours(12mins) AMS analysis

Year	0.2hrs/12mm(X_O)	Rank(m)	P	u	X_G
2002	44	1	0.0253	3.664	66.2415
1972	38	2	0.0705	2.6041	53.8099
1974	36	3	0.1157	2.0956	48.7226
1982	34	4	0.1609	1.7403	44.5653
1972	33	5	0.2061	1.4659	41.8999
1978	32	6	0.2514	1.2396	39.5259
1977	29	7	0.2966	1.0447	35.3428
1971	27	8	0.3418	0.8718	32.2927
1973	27	9	0.3870	0.7147	31.3388
1975	27	10	0.4322	0.5692	30.4558
2004	27	11	0.4774	0.4324	29.6253
2005	27	12	0.5226	0.3019	28.8329
1979	25	13	0.5678	0.1757	26.0665
2003	25	14	0.6130	0.0519	25.3154
1981	24	15	0.6582	-0.0710	23.5688
2008	22	16	0.7034	-0.1951	20.8152
2009	21	17	0.7486	-0.3227	19.0407
1970	20	18	0.7939	-0.4569	17.2262
1980	18	19	0.8391	-0.6025	14.3420
1999	18	20	0.9295	-0.9752	12.0793
1983	16	21	0.9295	-0.9752	10.0793
2006	14	22	0.9747	-1.3019	6.0960

Table 3.12: Observed and Expected frequencies repartition

Range(mm)	O freq	E freq
< 20	4	6
20-25	6	2
25-30	6	4
30-35	3	3

35-40	2	2
>40	1	5
	SUM=22	SUM=22

O: Observed Frequency; E: Expected Frequency.

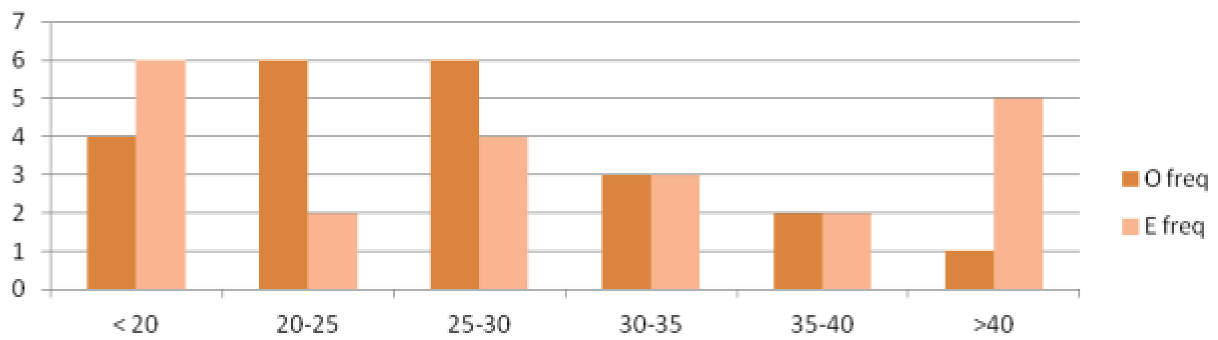

Fig 3.2 Distribution of Observed and Expected Frequencies

Hence $\chi^2 = 6 - 2 - 1$ degrees of freedom. Using a 5% significance level, the hypothesis that the observations are drawn from a Gumbel distribution is accepted if $0 \leq 5.00 \leq \chi^2_{0.05}$.From Appendix 2, we read that for 3 degrees of freedom and 0.050 significance, the $\chi^2 = 7.815$. Since $0 \leq 5.00 \leq 7.815$, the hypothesis that the annual data is drawn from a Gumbel distribution is accepted.

Table 3.13: Chi-square test (0.2hrs analysis)

Interval	O	E	$O-E$	$(O-E)^2$	$(O-E)^2/E$
5-25	10	8	2	4	0.50
25-30	6	4	2	4	1.00
30-35	3	3	0	0	0.00
35-40	2	2	0	0	0.00
40-45	1	2	-1	1	0.50
>45	0	3	-3	9	3.00
				Σ	**5.00**

To perform the Kolmogorov-Smirnov test, the upper bounds of intervals from table 3.12 are considered and the respective cumulated frequency (number of sample points smaller than the considered rainfall depth).

Table 3.14: Kolmogorov-Smirnov test (12mins analysis)

1	2	3	4	5	6	7	8
Interval	X_0	O freq	Cum freq	$F_0(x)$	u	$F_t(x)$	$Dn = \lvert F_t(x) - F_0(x) \rvert$
<20	20	4	4	0.182	-0.529	0.183	0.001
20-25	25	4	8	0.364	0.294	0.475	**0.111**

25-30	30	8	16	0.727	1.118	0.721	0.006
30-35	35	3	19	0.864	1.941	0.866	0.002
35-40	40	2	21	0.955	2.765	0.939	0.016

0.111<0.284, ***null hypothesis is accepted at 0.05 level of significance***.

In column 5 of table 3.14, the $F_0(x) = \frac{j}{N}$ values are the ratio of cumulated frequency to 22(number of sample points). The reduced variable is yielded by $u = \frac{X_0 - x_0}{s}$.

The Gumbel cumulative probability distribution prompts theoretical cumulated probabilities

$$F_t(x) = F(u) = e^{-e^{-u}} \qquad (3.7)$$

The last column contains the Kolmogorov differences $D_n = |F_t(x) - F_0(x)|$.

The highest difference is equal to 0.111 and the critical value for Kolmogorov-Smirnov given in Appendix I for 5 intervals and a 5% significant level, is 0.563. Since 0.111<0.565 the null hypothesis is accepted at 0.05 significant level.

Both tests confirm that the 22 sample observations are drawn from a Gumbel distribution whose mean is 26.719mm and standard deviation is 7.786mm

Rainfall Annual Maxima depth of return periods longer than 22 years can be obtained with the following formula (2.24)

$$X_T = \mu_G + K_T \times \sigma_G (chow, 1964)$$

Where μ_G and σ_G are Gumbel distribution's mean and standard deviation; and K_T is a frequency factor [*formula (2.25)*].

For return period equal to one year, the formula of K_T (2.25) does not stand. The following formula(2.26) will be used

$$K_T = -\frac{\sqrt{6}}{\pi} \times \left[0.5772 - lnT + \frac{1}{2 \times T} + \frac{1}{24 \times T^2} + \frac{1}{8 \times T^3}\right]$$

Table 3.15: 12 minutes(0.2hrs) estimates

T	K(T)	X(T)	

Years		Mm	I(mm/hr)
1	-0.970	19.83	99.15
2	-0.164	25.44	127.2
5	0.719	32.32	161.6
10	1.305	36.88	184.4
15	1.635	39.45	197.3
20	1.866	41.25	206.3
30	2.189	43.76	218.8
40	2.416	45.53	227.7
50	2.592	46.90	234.5
22	**1.942**	**41.84**	**209.2**
25	2.044	42.63	213.2

For $T = 1$, $K_T = -0.970$ and the corresponding hourly rainfall depth is equal to $X_G = 19.83\ mm$. In the table above some estimates are provided.

Similar computations have been done for the other durations and they are found in the appendices. The hourly estimates are used to determine the extreme values for duration shorter than one hour by multiplying with coefficients provided by WMO in table 3.5 and represented in table 3.16. The Intensity-Duration-Frequency estimates are deduced from table 3.16. by division of rainfall depth(mm) with their respective duration. They are represented in the table 3.17.

Table 3.16: Depth Duration Frequency estimates Using WMO Coefficients

	1 Year	2	5	10	20	30	40	50
5 min	12.6	16.5	20.7	23.5	26.2	27.8	28.9	29.7
10 min	19.6	25.6	32.1	36.5	40.7	43.1	44.8	46.1
15 min	24.8	32.4	40.7	46.2	51.5	54.5	56.7	58.4
30 min	34.4	44.9	56.4	64.1	71.4	75.6	78.6	80.9

60 min	43.5	56.7	71.3	81.0	90.3	95.6	99.4	102.3

Table 3.17: Intensity Duration Frequency estimates Using WMO Coefficients

	1 Year	2	5	10	20	30	40	50
5 min	151.3	198.1	248.5	282.1	314.5	333.7	346.9	356.5
10 min	117.6	153.6	192.6	219.0	244.2	258.6	268.7	276.5
15 min	99.2	129.6	162.8	184.8	206.0	218.0	226.8	233.6
30 min	68.8	89.8	112.8	128.2	142.8	151.2	157.2	161.8
60 min	43.5	56.7	71.3	81.0	90.3	95.6	99.4	102.3

The above values are plotted in a scatter diagram whose X-axis contains duration in *min* and Yaxis intensities in *mm/hr*. A trend-line is drawn to join the points of same return period. The curves so obtained are Sub-Hourly IDF curves for Kumasi using WMO coefficients.

The same procedure described above is used to plot IDF curves for the sub- durations of the data analysed(Table 3.18) below.

	1 Year	2	5	10	20	30	40	50
12min	99.2	127.2	161.6	184.4	206.3	218.8	227.7	234.5
24min	69.9	94.2	120.7	138.3	155.2	164.9	171.7	177.0
42min	48.3	61.2	75.4	84.8	93.9	99.0	102.7	105.5
60 min	43.5	56.7	71.3	81.0	90.3	95.6	99.4	102.3

Table 3.18: Sub-Hourly IDF Estimates for Kumasi

Rainfall intensities are in mm/hr

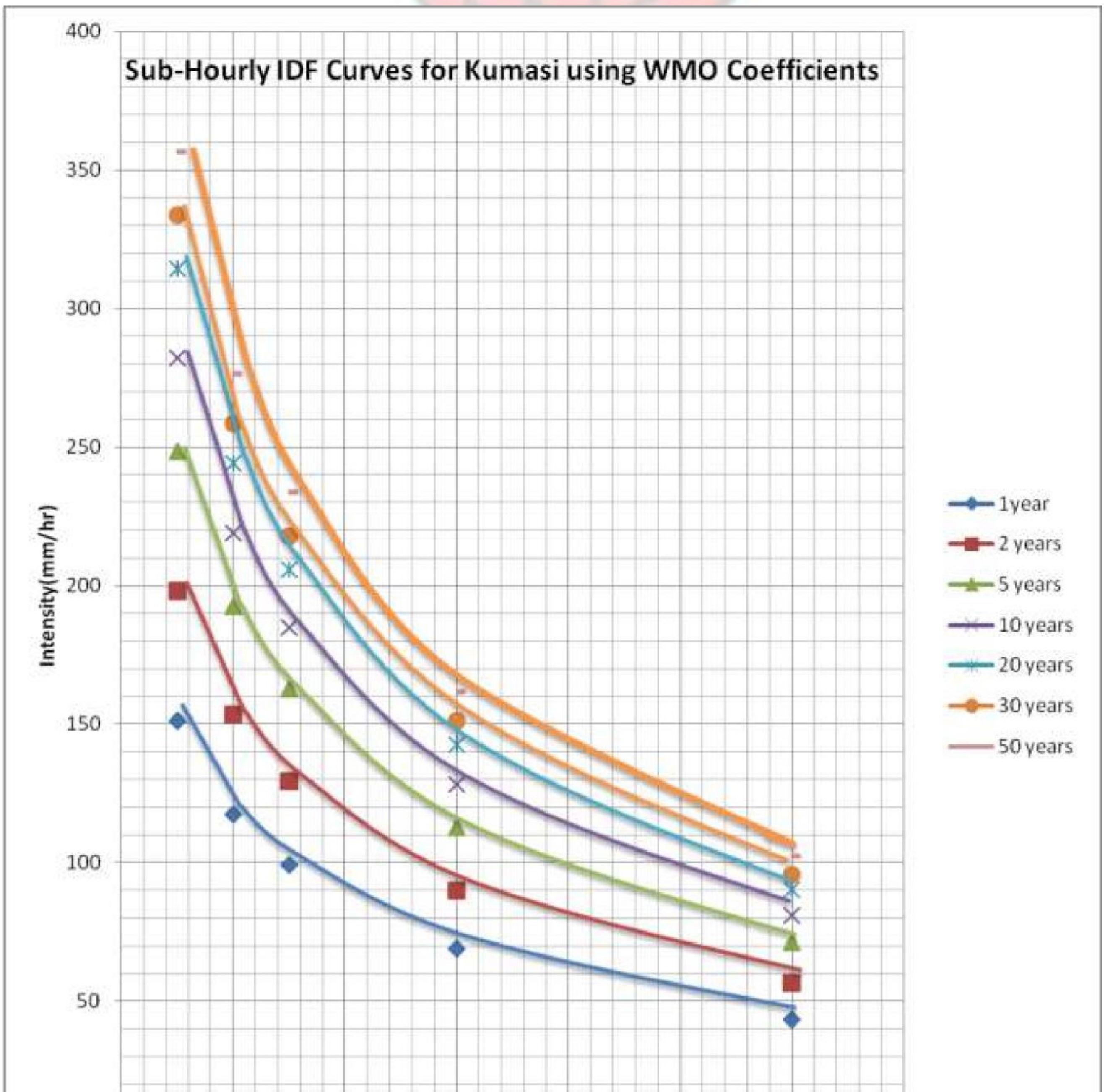

Fig.3.3 *Sub-Hourly IDF Curves for Kumasi using WMO Coefficients*

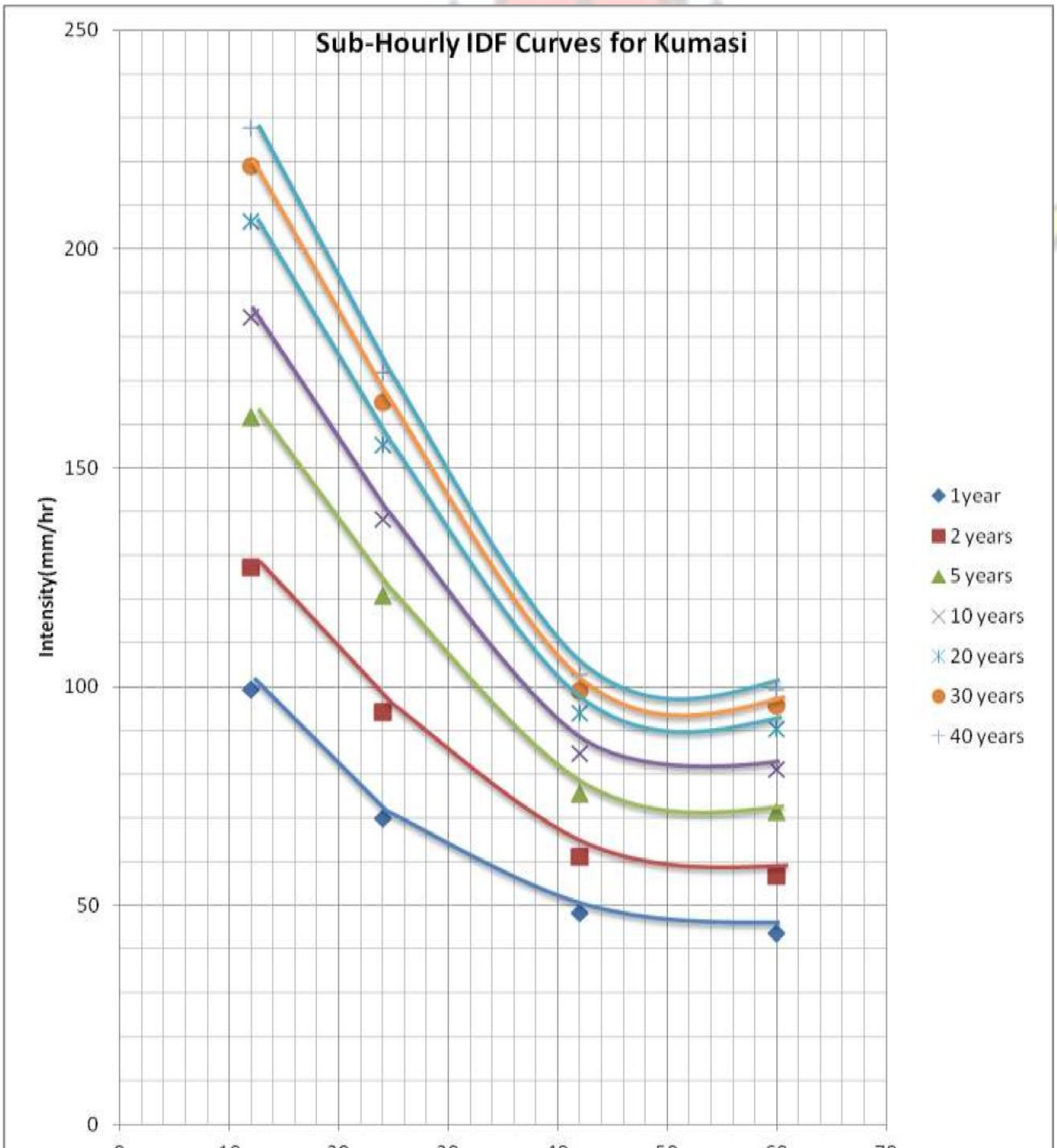

Fig.3.4 Sub-Hourly *IDF Curves for Kumasi*

Table 3.19 Intensity-Duration-Frequency estimates(complete table)

	1 Year	2	5	10	20	30	40	50
12min	99.2	127.2	161.6	184.4	206.3	218.8	227.7	234.5
24min	69.9	94.2	120.7	138.3	155.2	164.9	171.7	177.0
42min	48.3	61.2	75.4	84.8	93.9	99.0	102.7	105.5
60 min	43.5	56.7	71.3	81.0	90.3	95.6	99.4	102.3
2hrs	23.6	31.9	41.0	47.0	52.8	56.1	58.5	60.3
3hrs	16.1	21.1	26.6	30.2	33.7	35.7	37.1	38.2
6hrs	8.4	10.6	13.1	14.7	16.3	17.2	17.9	18.3
12hrs	3.8	5.3	6.8	7.8	8.8	9.4	9.8	10.1
24hrs	1.7	2.3	3.1	3.5	4.0	4.3	4.4	4.6

Rainfall Intensities are in mm/hr

These results are represented in the figure hereafter. Only the estimates of duration less or equal to six hours (360 min) have been plotted just for better (accurate) representation on the diagram.

Table 3.25: Relationship between WMO and Sub-Hourly IDF estimates for 40, and 50 Year Return Periods.

	40 Year			50 Year		
	WMO	Actual	Relationship	WMO	Actual	Relationship
12min	251.9	227.7	**Actual=0.90WMO**	259.3	234.5	**Actual=0.90WMO**
24min	185.0	171.7	**Actual=0.93WMO**	190.5	177.0	**Actual=0.93WMO**
42min	134.1	102.7	**Actual=0.77WMO**	138.0	105.5	**Actual=0.76WMO**
1hr(60mins)	99.4	99.4	**Actual=WMO**	102.3	102.3	**Actual=WMO**

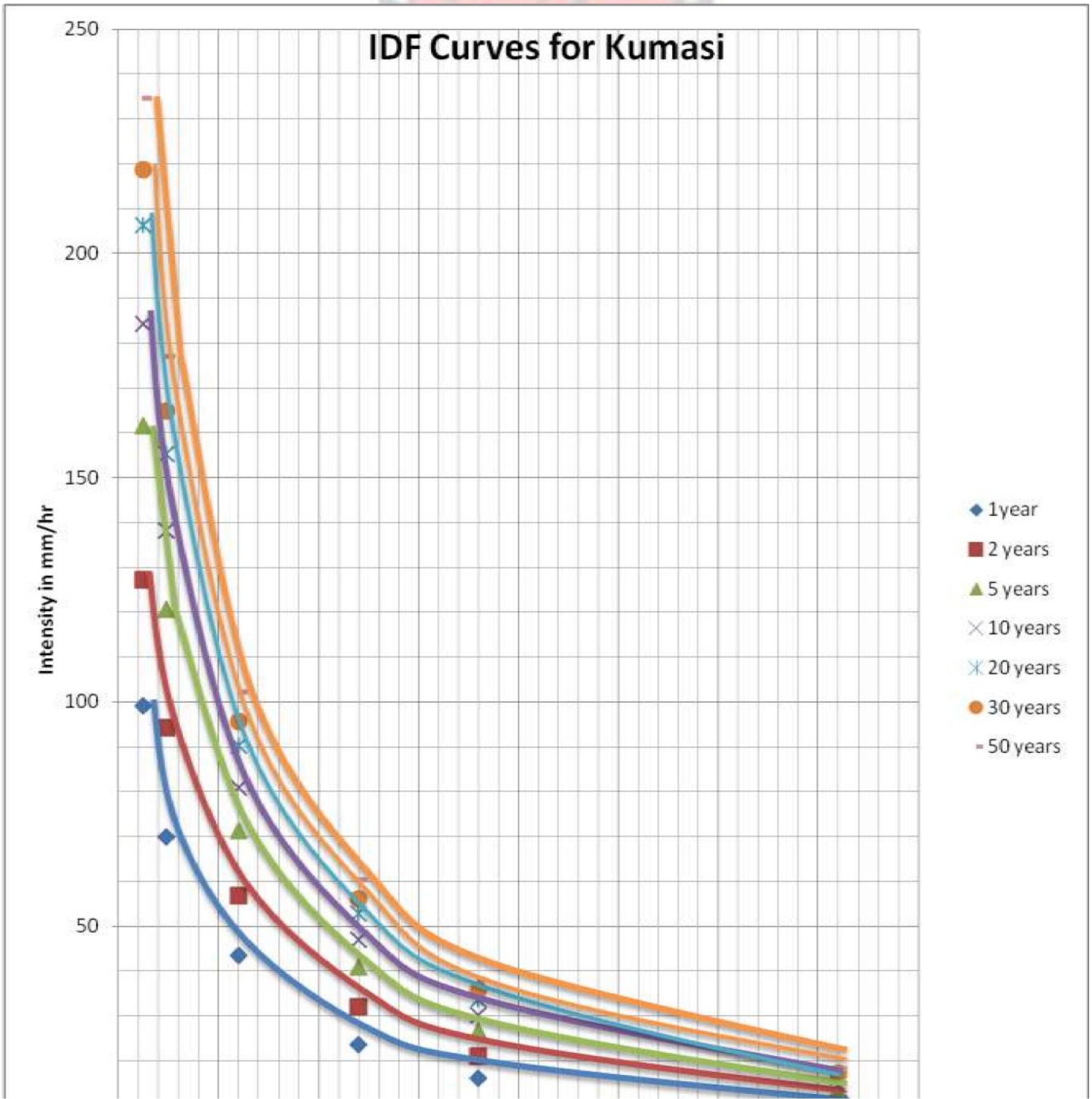
IDF Curves for Kumasi
Intensity in mm/hr
250
200
150
100
50
1year
2 years
5 years
10 years
20 years
30 years
50 years

Fig 3.5 Intensity Duration Frequency Curves for Kumasi

3.4.2 Relationship Between Sub hourly and WMO Sub-Hourly IDF Estimates for Kumasi

The 12min,24min and 42min duration can be interpolated from table 3.17 and this is shown in table 3.20 below.

Table 3.20. Sub-Hourly IDF estimates from WMO Coefficients

	1 Year	2	5	10	20	30	40	50
12min	110.2	144.0	180.7	205.3	228.9	242.4	251.9	259.3
24min	81.0	105.7	132.8	150.8	168.1	177.9	185.0	190.5
42min	58.7	76.6	96.2	109.3	121.8	129.0	134.1	138.0
60 min	43.5	56.7	71.3	81.0	90.3	95.6	99.4	102.3

Rainfall Intensities are in mm/hr

For each return period a relationship can be obtained between the sub-hourly IDF estimates from the WMO coefficients (table 3.20) and the sub-hourly IDF estimates(table 3.18) for Kumasi. The relationship between the WMO and the Actual IDF estimates for each duration for selected return periods are presented in tables 3.21,3.22 and 3.23,3.24 and 3.25.

Table 3.21: Relationship between WMO and Sub-Hourly IDF estimates for 1 and 2 Year Return Periods.

	1 Year	2 Year

	WMO	Actual	Relationship	WMO	Actual	Relationship
12min	110.2	99.2	**Actual=0.90WMO**	144.0	127.2	**Actual=0.88WMO**
24min	81.0	69.9	**Actual=0.86WMO**	105.7	94.2	**Actual=0.89WMO**
42min	58.7	48.3	**Actual=0.82WMO**	76.6	61.2	**Actual=0.80WMO**
1hr(60mins)	43.5	43.5	**Actual=WMO**	56.7	56.7	**Actual=WMO**

Table 3.22. Relationship between WMO and Sub-Hourly IDF estimates for 10 and 20, Year Return Periods.

	10 Year			20 Year		
	WMO	Actual	Relationship	WMO	Actual	Relationship
12min	205.3	184.4	**Actual=0.90WMO**	228.9	206.3	**Actual=0.90WMO**
24min	150.8	138.3	**Actual=0.92WMO**	168.1	155.2	**Actual=0.92WMO**
42min	109.3	84.8	**Actual=0.78WMO**	121.8	93.9	**Actual=0.77WMO**
1hr(60mins)	81.0	81.0	**Actual=WMO**	90.3	90.3	**Actual=WMO**

Table 3.23: Relationship between WMO and Sub-Hourly IDF estimates for 5 Year Return Period.

	5 Year		
	WMO	Actual	Relationship
12 min	180.7	161.6	**Actual=0.89WM0**
24 min	132.8	120.7	**Actual=0.91WMO**
42 min	96.2	75.4	**Actual=0.78WMO**
1 hr	71.3	71.3	**Actual=WMO**

30 Year

Table 3.24: Relationship between WMO and Sub-Hourly IDF estimates for 30 Year Return Period.

	WMO	Actual	Relationship
12 min	242.4	218.8	**Actual=0.90WM0**
24 min	177.9	164.9	**Actual=0.93WMO**
42 min	129.0	99.0	**Actual=0.77WMO**
1 hr	95.6	95.6	**Actual=WMO**

In general, it is observed that the actual sub-hourly intensity values for all durations could be obtained from the sub-hourly intensity values obtained from applying the WMO factors by multiplying these values by a constant ranging from 0.93 to 0.76 with an average value of 0.88.

Table 3.26 below shows the relationship between the calculated WMO and the Sub-hourly IDF estimates for 1 year return period.

The relationship between the calculated WMO and the Sub hourly IDF estimates for other return periods are provided in the appendices.

Table 3.26: Relationship between Calculated WMO and Sub-Hourly IDF estimates for 1 Year Return Period.

Duration/min.	WMO	Calculated Value	Actual Value	% error(absolute)
12	110.2	0.88*110.2=96.98	99.20	2.2
24	81.0	0.88*81.0=71.28	69.90	2.0

42	58.7	0.88*58.7=51.66	48.30	7.0
60(1hr)	43.5	43.50	43.50	0

4. DISCUSSION OF RESULTS

4.1 IDF estimates

The IDF estimates resulting from this work, Table 3.19 are in accordance with the general properties of Intensity Duration Frequency curves, given in point 2.1.3. For the same return period, high intensities are related to short durations. The curves of Fig. 3.3, Fig. 3.4 and Fig. 3.5 are parallel; they do not cross each other.

4.2. Comparison of New IDF estimates and Existing IDF estimates

Comparing these results to the existing (J.B Danquah),there are variations in the intensities.

- The New IDF estimates are higher for shorter durations. The percentage increase ranges between 2% and 25%.This might be as a result of low precipitation trends for shorter durations in 1970s.
- New IDF estimates are lower for longer durations. The percentage decrease ranges between 3% and 49%. This might be as a result of high precipitation trends for longer durations in 1970s.

In order to picture the degree of variation, the new IDF estimates and the existing IDF estimates and their incremental rate of change are presented in table 4.1. below.

4.3.Comparison of Sub-Hourly and WMO Sub-Hourly IDF Estimates for Kumasi

The WMO sub-hourly IDF estimates are slightly higher than the sub-hourly IDF estimates obtained from the analysis of rainfall data for all the return periods selected(1,2, 5 Year, 10,20,30, 40, and 50) as shown in tables 3.21,3.22 and 3.23,3.24,3.25.

The relationship is **Sub-hourly=0.88 WMO Sub-hourly**.

This relationship will be useful in obtaining sub-hourly IDF estimates for Kumasi when only hourly data for Kumasi is available

Table 4.1. Comparison of New IDF estimates and J.B Danquah IDF estimates

	5 Year			10 Year			25 Year			50 Year		
	J.B Danq.	New	% incr.	J.B Danq	New	% incr.	J.B Danq	New	% incr.	J.B Danq	New	% incr.
12 min	137.2	161.6	**+17.8**	153.7	184.4	**+20.0**	176.5	213.2	**+20.8**	191.8	234.5	**+22.3**
24 min	118.1	120.7	**+2.2**	127.0	138.3	**+8.9**	154.9	160.5	**+3.6**	170.2	177.0	**+4.0**
42 min	93.7	75.4	**-19.6**	104.1	84.8	**-18.5**	122.9	96.7	**-21.3**	134.9	105.5	**-21.8**
1hr	77.2	71.3	**-7.7**	83.8	81.0	**-3.3**	101.9	93.3	**-8.4**	112.0	102.3	**-8.7**
2hrs	45.7	41.0	**-10.3**	53.3	47.0	**-11.8**	64.8	54.6	**-15.7**	73.7	60.3	**-18.2**
3hrs	33.0	26.6	**-19.4**	38.6	30.2	**-21.8**	45.2	34.8	**-23.0**	50.3	38.2	**-24.1**

6hrs	19.3	13.1	**-32.1**	22.9	14.7	**-35.8**	27.4	16.8	**-38.7**	30.7	18.3	**-40.4**
12hrs	10.4	6.8	**-34.7**	12.4	7.8	**-37.1**	15.2	9.1	**-40.1**	17.3	10.1	**-41.6**
24hrs	5.3	3.1	**-41.8**	6.4	3.5	**-45.3**	7.9	4.1	**-48.1**	8.9	4.6	**-48.3**

5. CONCLUSION AND RECOMMENDATION

5.1. Conclusion

The results show that:

- For shorter durations(12 min and 24 min),the new IDF Curves give higher intensities for the same return period whiles for longer durations(42min,1hr, 2hr,3hr,6hr,12hr and 24 hr), the new IDF Curves give lower intensities for the same return period.
- The sub-hourly IDF estimates are slightly lower than that of the WMO estimates. Thus:
  Sub-hourly=0.88 WMO Sub-hourly

 The percentage error between the calculated value and the actual value for all sub-hourly durations and for all return periods, ranges between 0% and 15.1%

5.2. Recommendations.

I recommend that:

- There should be regular updating of Intensity-Duration-Frequency curves for all the major cities and towns in Ghana to take into account the effect of climate change.
- There should be new regulations, revision and update of design practices and standards.

- For scientific work, it is recommended that the actual(original) charts removed from recording rain gauges are kept for references.

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APPENDIX

Appendix 1 Critical values of Kolmogorov-Smirnov test.

Sample size (n)	Critical differences $D_{\alpha}(n)$				
	α = .20	α = .15	α = .10	α = .05	α = .01
1	.900	.925	.950	.975	.995
2	.684	.726	.776	.842	.929
3	.565	.597	.642	.708	.828
4	.494	.525	.564	.624	.733
5	.446	.474	.510	.565	.669
6	.410	.436	.470	.521	.618
7	.381	.405	.438	.486	.577
8	.358	.381	.411	.457	.543
9	.339	.360	.388	.432	.514
10	.322	.342	.368	.410	.490
11	.307	.326	.352	.391	.468
12	.295	.313	.338	.375	.450
13	.284	.302	.325	.361	.433
14	.274	.292	.314	.349	.418
15	.266	.283	.304	.338	.404
16	.258	.274	.295	.328	.392
17	.250	.266	.286	.318	.381
18	.244	.259	.278	.309	.371

19	.237	.252	.272	.301	.363
20	.231	.246	.264	.294	.356
25	.210	.220	.240	.270	.320
30	.190	.200	.220	.240	.290
35	.180	.190	.210	.230	.270
> 35	1.07 / √n	1.14 / √n	1.22 / √n	1.36 / √n	1.63 / √n

Appendix 2 Chi-square Distribution Table

df	$\chi^2(0.995)$	$\chi^2(0.990)$	$\chi^2(0.975)$	$\chi^2(0.950)$	$\chi^2(0.900)$	$\chi^2(0.100)$	$\chi^2(0.050)$	$\chi^2(0.025)$	$\chi^2(0.010)$	$\chi^2(0.005)$
1	0.000	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
Sam10ple	Mean2.156	(μ_s)2.558	3.247	3.940	4.86539.	5515.987mm	18.307	20.483	23.209	25. 188
Sam11ple	St. De2.603	v.(σ_s)3.053	3.816	4.575	5.57811.	7617.275mm	19.675	21.920	24.725	26. 757
12Position	Para3.074	neter(x_o)3.571	4.404	5.226	6.30434.	1718.549mm	21.026	23.337	26.217	28. 300
13Scale	Paramet3.565	er(s)4.107	5.009	5.892	7.0429.3	619.812mm	22.362	24.736	27.688	29. 819
14Gu	mbel Mean4.075	(μ_G)4.660	5.629	6.571	7.79039.	2921.064mm	23.685	26.119	29.141	31. 319
15 Gu	mbel St.De4.601	v.(σ_G)5.229	6.262	7.261	8.54712.	222.3075mm	24.996	27.488	30.578	32. 801
										267

16	5.142 5.697	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.
17	6.265 6.844	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.7
18	7.434	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.1
19		7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.5
20		8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034 8.643	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	9.260	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796
23	9.886	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	10.520	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25		11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169

df: degree of freedom

Appendix 3 24 Minute AMS Sample and distribution parameters

Appendix 4 24 minute (0.4 hrs) AMS analysis

Year	0.4hrs/15mm(X_O)		Rank(m)	P		u	X_G	
1982	64		1	0.0253		3.664	98.3496	
1979	55		2	0.0705		2.6041	79.5229	
2004	53		3	0.1157		2.0956	72.6488	
1972	51		4	0.1609		1.7403	67.3170	
1975	50		5	0.2061		1.4659	63.7450	
1981	47		6	0.2514		1.2396	58.6230	
Range(mm)			) freq			E freq		
2008	45		7	0.2966	1.	0447	54.7958	
< 20		1				3		

1978	43		8	0.3418	0	.8718	51.1740
20-25 2002	43	2	9	0.3870	0	1 .7147	49.7008
251977-30	42	2	10	0.4322	0	.5692 2	47.3371
301976-35	40	4	11	0.4774	0	.4324 2	44.0545
351970-40	38	3	12	0.5226	0	.30192	40.8307
197440-45	38	4	13	0.5678	0	.172 57	39.6471
199945-50	35	2	14	0.6130	0	.05192	35.4870
200550-55	35	3	15	0.6582	-	).07102	34.3340
200655-60	35	0	16	0.7034	-	).19511	33.1703
1973	33		17	0.7486	-0.3227		29.9741
1980	30		18	0.7939	-0.4569		25.7161
1983	28		19	0.8391	-0.6025		22.3507
1971	25		20	0.9295	-0.9752		17.7947
2003	20		21	0.9295	-0.9752		10.8560
2009	18		22	0.9747	-1.3019		5.7931

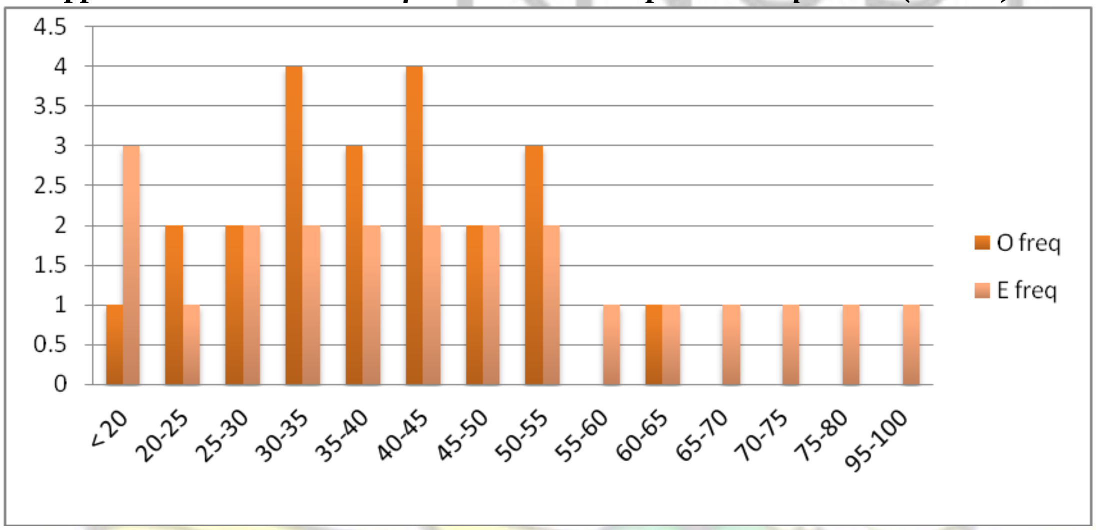
Appendix 5 Observed and Expected frequencies repartition

60-65	1	1
65-70	0	1
70-75	0	1

75-80	0	1
95-100	0	1
	SUM=22	SUM=22

O: Observed Frequency; E: Expected Frequency

Appendix 6 Distribution of Observed and Expected Frequencies(0.4 hrs)



Appendix 7 Chi-square test 0.4 Hours(24mins) AMS

Interval	$Observed(O)$	$Expected(E)$	$(O-E)$	$(O-E)^2$	$(O-E)^2/E$
<20	1	3	-2	4	1.33
20-25	2	1	1	1	1.00
25-30	2	2	0	0	0.00
30-35	4	2	2	4	2.00
35-40	3	2	1	1	0.50
40-45	4	2	2	4	2.00
45-50	2	2	0	0	0.00
50-60	3	3	0	0	0.00
60-65	1	1	0	0	0.00
65-70	0	1	-1	1	1.00
70-75	0	1	-1	1	1.00
75-80	0	1	-1	1	1.00
95-100	0	1	-1	1	1.00
					Σ 10.83

$\mathbf{10.83} < 18.307$ ***null hypothesis is accepted at 0.05 level of significance***

Appendix 8 Kolmogorov-Smirnov test 0.4 Hours(24 mins) AMS

Interval	X_o	Observed freq	Cum freq	$F_O(x)$	u	$F_t(x)$	$\lvert F_t(x) - F_o(x) \rvert$
20-25	25	2	2	0.0909	-0.98	0.0696	0.0213
25-30	30	2	4	0.1818	-0.45	0.2084	0.0266
30-35	35	4	8	0.3636	0.08	0.3630	0.0006
35-40	40	3	11	0.5000	0.62	0.3973	**0.1027**
Sample Mean40-45	(μ_s) 45	4	15	44 5450.6818mm	1.15	0.7286	0.0468
Sample St. De	v.(σ_s)			10 734mm			
45-50	50	2	17	0.7727	1.68	0.8300	0.0573
50-60	60	3	20	0.9091	2.75	0.9381	0.029
60-65	65	1	21	0.9545	3.28	0.9631	0.0086

$\mathbf{0.1027} < 0.284$ ***null hypothesis is accepted at 0.05 level of significance***

Appendix 9 24 minutes(0.4hrs) estimates

T Years	K(T)	X(T) Mm	I(mm/hr)
1	-0.970	27.96	69.9
2	-0.164	37.66	94.2
5	0.719	48.27	120.7
10	1.305	55.32	138.3
15	1.635	59.29	148.2
20	1.866	62.07	155.2
30	2.189	65.95	164.9
40	2.416	68.68	171.7
50	2.592	70.80	177.0
22	**1.942**	**62.98**	**157.5**
25	2.044	64.21	160.5

Position Parameter(x_o)	39.647mm
Scale Parameter(s)	8.770mm

Gumbel Mean(μ_G)	44.709mm
Gumbel St.Dev.(σ_G)	11.248mm

Appendix 10 (0.7hrs) Annual Maxima Series(AMS) Sample and distribution parameters

Appendix 11 0.7 Hours(42mins) AMS analysis

Year	0.7hrs/20mm(X_O)	Rank(m)	P	U	X_G
1971	60	1	0.0253	3.664	71.7757
2008	59	2	0.0713	2.6041	62.5842
1982	57	3	0.1157	2.0956	58.0252
2002	55	4	0.1609	1.7403	54.9089
1978	53	5	0.2061	1.4659	52.5031
1972	51	6	0.2514	1.2396	50.5182
1973	51	7	0.2966	1.0447	48.8092
1976	51	8	0.3418	0.8718	47.2923
1975	48	9	0.3870	0.7147	45.9143
1977	46	10	0.4322	0.5692	44.6388
2004	46	11	0.4774	0.4324	43.4391
1974	45	12	0.5226	0.3019	42.2944
1979	44	13	0.5678	0.1757	41.1873
1981	44	14	0.6130	0.0519	40.1023
2003	44	15	0.6582	-0.0710	39.0238
2005	42	16	0.7034	-0.1951	37.9353
1970	38	17	0.7486	-0.3227	36.8164
1980	35	18	0.7939	-0.4569	35.6398
1999	34	19	0.8391	-0.6025	34.3626
1983	32	20	0.9295	-0.9752	32.9072
2009	30	21	0.9295	-0.9752	31.0939
2006	15	22	0.9747	-1.3019	28.2290

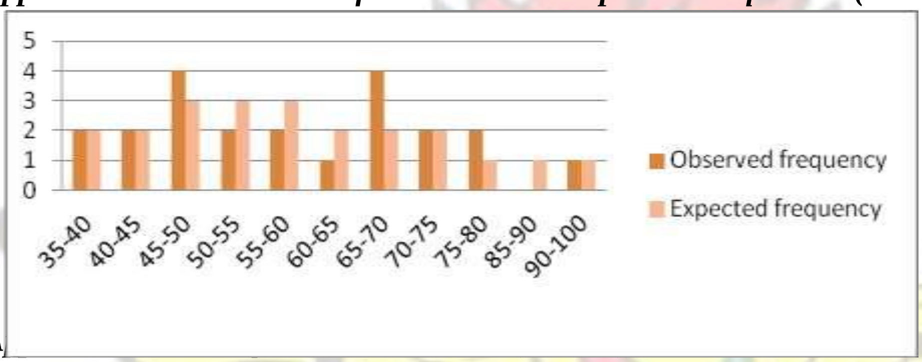
Appendix 12 Observed and Expected frequencies repartition(0.7 hrs)

Range(mm)	O freq	E freq
< 20	1	0
20-25	0	0
25-30	1	1
30-35	3	3
35-40	1	4
40-45	5	5

45-50	3	3
50-55	5	3
55-60	3	1
60-65	0	1
>70	0	1
	SUM=22	SUM=22

O: Observed Frequency; E: Expected Frequency.

Appendix 13   Distribution of Observed and Expected Frequencies(0.7 hrs)



A

Interval	$Observed$	$Expected$	$(O-E)$	$(O-E)^2$	$(O-E)^2/E$
10-30	2	1	1	1	1.00
30-35	3	3	0	0	0.00
35-45	6	9	-3	9	1.00
45-50	3	3	0	0	0.00
50-55	5	3	2	4	1.33
55-75	3	3	0	0	0.00
					Σ 3.33

$\mathbf{3.33} < 7.815$ ***null hypothesis is accepted at 0.05 level of significance***

Appendix 15   Kolmogorov-Smirnov test 0.7 Hours(42mins) AMS

Interval	X_o	O freq	Cum freq	$F_O(x)$	u	$F_t(x)$	$\lvert F_t(x) - F_o(x) \rvert$
25-30	30	1	1	0.0450	-1.10	0.0498	0.0048
30-35	35	3	4	0.1820	-0.53	0.1829	0.0009
35-40	40	1	5	0.2270	0.04	0.3826	**0.1556**
40-45	45	5	10	0.4550	0.61	0.5808	0.1258

45	50	50	3	13	0.591	0	1.18	0.7354
0.1444								
50	55	55	5	18	0.818	0	1.75	0.8405
0.0225								
55	75	60	3	21	0.955	0	2.32	0.9064
0.0486								

Sample Mean(μ_s) 59.273mm

Sample St. Dev. (σ_s) 15.791mm

Position Parameter (x_o) 52.066mm

Scale Parameter(s) 12.902mm

Gumbel Mean(μ_G) 59.513mm

Gumbel St.Dev.(σ_G) 16.547mm

$\mathbf{0.1556} < 0.284$

null hypothesis is accepted at 0.05 level of significance.

Appendix 16 42 minutes(0.7hrs) estimates

T Years	K(T)	X(T) Mm	I(mm/hr)
1	-0.970	33.80	48.3
2	-0.164	42.86	61.2
5	0.719	52.80	75.4
10	1.305	59.39	84.8
15	1.635	63.10	90.1
20	1.866	65.70	93.9
30	2.189	69.33	99.0
40	2.416	71.88	102.7
50	2.592	73.86	105.5
22	**1.942**	**66.55**	**95.1**
25	2.044	67.70	96.7

Appendix 17 1hr Annual Maxima Series(AMS) Sample and distribution parameters

Year	0.2hrs/12mm(X_O)	Rank(m)	P	u	X_G
1982	100	1	0.0253	3.664	99.3325
2004	77	2	0.0705	2.6041	85.8105
1979	76	3	0.1157	2.0956	79.1034
2003	75	4	0.1609	1.7403	74.5188
2002	71	5	0.2061	1.4659	70.9796
1972	68	6	0.2514	1.2396	68.0596

1977	68	7	0.2966	1.0447	65.5453
1978	68	8	0.3418	0.8718	63.3137
1975	66	9	0.3870	0.7147	61.2865
1976	65	10	0.4322	0.5692	59.4100
1974	59	11	0.4774	0.4324	57.6451
2008	57	12	0.5226	0.3019	55.9611
1981	55	13	0.5678	0.1757	54.3324
2005	53	14	0.6130	0.0519	52.7361
2006	49	15	0.6582	-0.0710	51.1495
1980	47	16	0.7034	-0.1951	49.5481
1970	46	17	0.7486	-0.3227	47.9021
1973	46	18	0.7939	-0.4569	46.1711
1983	43	19	0.8391	-0.6025	44.2921
1999	43	20	0.9295	-0.9752	42.1510
2009	38	21	0.9295	-0.9752	39.4834
1971	34	22	0.9747	-1.3019	35.2687

Appendix 18 1 Hour AMS analysis

Appendix 19 Observed and Expected frequencies repartition

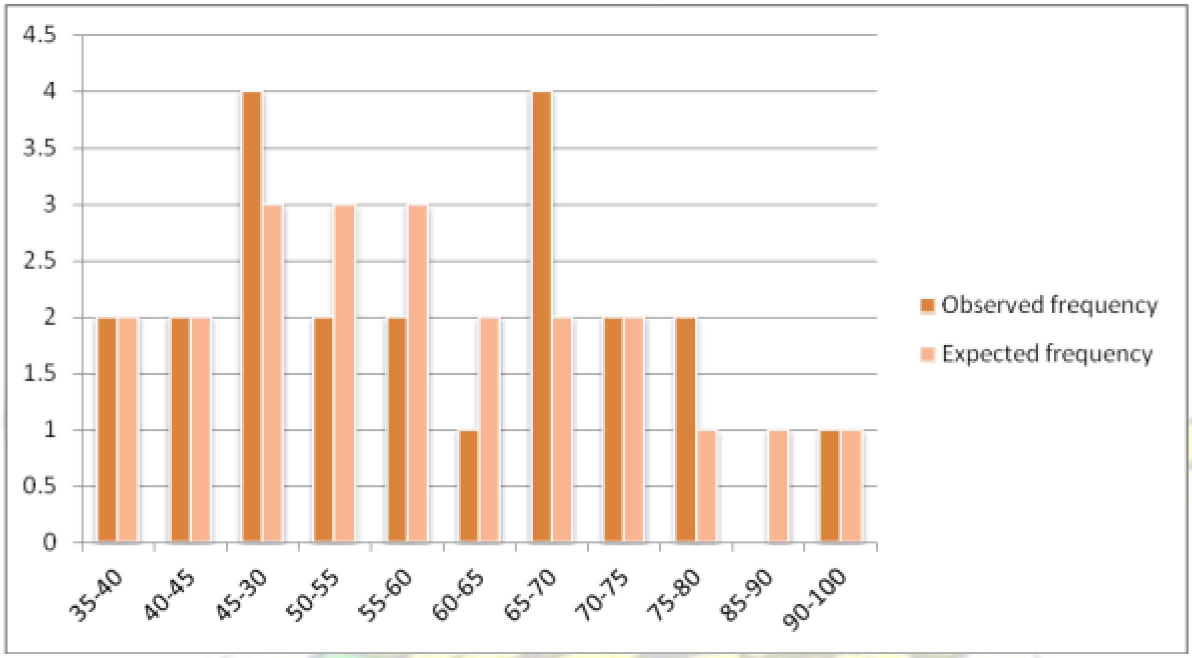
Range(mm)	O freq	E freq
35-40	2	2
40-45	2	2
45-50	4	3
50-55	2	3
55-60	2	3
60-65	1	2
65-70	4	2
70-75	2	2
75-80	2	1
85-90	0	1
90-100	1	1

	SUM=22	SUM=22

O: Observed Frequency; E: Expected Frequency.

Interval	O	E	$(O-E)$	$(O-E)^2$	$(O-E)^2/_E$
35-40	2	2	0	0	0.000
40-45	2	2	0	0	0.000

Appendix 20 Distribution of Observed and Expected Frequencies(1 hr)



Appendix 21 Chi-square test 1 Hour AMS

45-50	4	3	1	1	0.333
50-55	2	3	-1	1	0.333
55-60	2	3	-1	1	0.333
60-65	1	2	-1	1	0.500
65-70	4	2	2	4	2.000
70-75	2	2	0	0	0.000
75-80	2	1	1	1	1.000
85-90	0	1	-1	1	1.000
90-100	1	1	0	0	0.000
					Σ 5.499

$5.499 < 15.507$ ***null hypothesis is accepted at 0.05 level of significance***

Appendix 22 Kolmogorov-Smirnov test 1 Hour AMS

Interval	X_o	O freq	Cum freq	$F_O(x)$	u	$F_t(x)$	$\lvert F_t(x) - F_o(x) \rvert$
35-40	40	2	2	0.09	-0.94	0.0773	0.0127
40-45	45	2	4	0.18	-0.55	0.3021	**0.1221**
45-50	50	4	8	0.36	-0.16	0.3093	0.0507
50-55	55	2	10	0.45	0.23	0.4518	0.0018
55-60	60	2	12	0.55	0.61	0.5808	0.0308
60-65	65	1	13	0.59	1.00	0.6922	0.1022
65-70	70	4	17	0.77	1.39	0.7795	0.0095
70-75	75	2	19	0.86	1.78	0.8448	0.0152
75-80	80	2	21	0.95	2.17	0.8921	0.0579

$\mathbf{0.1221} < 0.284$ ***null hypothesis is accepted at 0.05 level of significance***

Appendix 23 1 Hour estimates

T Years	K(T)	X(T) mm
1	-0.970	43.46
2	-0.164	56.80
5	0.719	71.41
10	1.305	81.11
15	1.635	86.57
20	1.866	90.39
30	2.189	95.73
40	2.416	99.49
50	2.592	102.40
22	**1.942**	**91.65**
25	2.044	93.34

Appendix 24 2 hr Annual Maxima Series(AMS) Sample and distribution parameters

Sample Mean(μ_s)	66.864
Sample St. Dev.(σ_s)	19.648mm
Position Parameter(x_o)	57.897mm
Scale Parameter(s)	16.053mm
Gumbel Mean(μ_G)	67.162mm
Gumbel St.Dev.(σ_G)	20.588mm

Appendix 25 2 Hour AMS analysis

Year	2hrs/25mm(X_O)	Rank(m)	P	u	X_G
1982	118	1	0.0253	3.664	116.7075
2003	89	2	0.0705	2.6041	99.8829
1972	85	3	0.1157	2.0956	91.5377
2006	81	4	0.1609	1.7403	85.8334
1976	80	5	0.2061	1.4659	81.4297
2002	80	6	0.2514	1.2396	77.7966
2004	80	7	0.2966	1.0447	74.6682
1979	75	8	0.3418	0.8718	71.8915
1975	69	9	0.3870	0.7147	69.3692
1978	69	10	0.4322	0.5692	67.0344
1981	69	11	0.4774	0.4324	64.8384
2008	69	12	0.5226	0.3019	62.7431
1977	67	13	0.5678	0.1757	60.7167
2005	62	14	0.6130	0.0519	58.7305
1974	61	15	0.6582	-0.0710	56.7564
1970	53	16	0.7034	-0.1951	54.7638
1980	53	17	0.7486	-0.3227	52.7159
1973	52	18	0.7939	-0.4569	50.5620
1999	52	19	0.8391	-0.6025	48.2242
1971	39	20	0.9295	-0.9752	45.5601
1983	39	21	0.9295	-0.9752	42.2409
2009	29	22	0.9747	-1.3019	36.9969

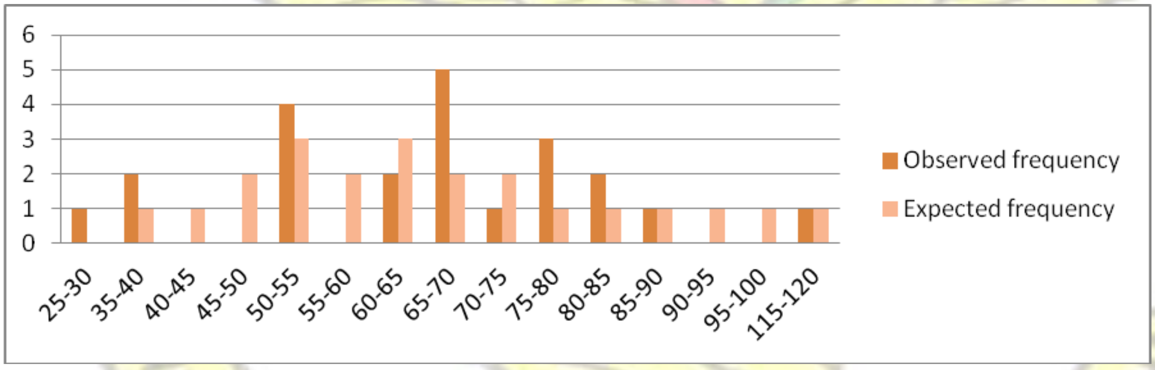
Appendix 26 Observed and Expected frequencies repartition

Range(mm)	Observed freq	Expected freq

25-30	1	0
30-35	0	0
35-40	2	1
40-45	0	1
45-50	0	2
50-55	4	3
55-60	0	2
60-65	2	3
65-70	5	2
70-75	1	2
75-80	3	1
80-85	2	1
85-90	1	1
90-95	0	1
95-100	0	1
100-105	0	0
105-110	0	0
110-115	0	0
115-120	1	1
	SUM=22	SUM=22

O: Observed Frequency; E: Expected Frequency

Appendix 27 Distribution of Observed and Expected Frequencies(2 hr)



Appendix 28 Kolmogorov-Smirnov test 2 Hour AMS

Interval	X_o	O freq	Cum freq	$F_O(x)$	u	$F_t(x)$	$\lvert F_t(x) - F_o(x) \rvert$
25-45	45	3	3	0.14	-0.80	0.1081	0.0319
45-55	55	4	7	0.32	-0.18	0.3021	0.0179
55-70	70	7	14	0.64	0.75	0.6235	0.0165
70-80	80	4	18	0.82	1.38	0.7776	**0.0424**
80-100	100	3	21	0.95	2.62	0.9298	0.0202

$\mathbf{0.0424} < 0.284$ ***null hypothesis is accepted at 0.05 level of significance***

Appendix 29 Chi-square test 2 Hour AMS

Interval	O	E	$(O-E)$	$(O-E)^2$	$(O-E)^2/E$
25-45	3	2	1	1	0.500
45-55	4	5	-1	1	0.200
55-70	7	7	0	0	0.000
70-80	4	3	1	1	0.333
80-100	3	4	-1	1	0.250
115-120	1	1	0	0	0.000
					Σ 1.283

$1.283 < 7.815$ ***null hypothesis is accepted at 0.05 level of significance***

Appendix 30 **2 Hour estimates**

T Years	K(T)	X(T) mm
1	-0.970	47.19
2	-0.164	63.79
5	0.719	81.96
10	1.305	94.03
15	1.635	100.82
20	1.866	105.58
30	2.189	112.23
40	2.416	116.90
50	2.592	120.53
22	**1.942**	**107.14**
25	2.044	109.24

Appendix 31 **3 hr Annual Maxima Series(AMS) Sample and distribution parameters**

Sample Mean(μ_s)	66.091mm
Sample St. Dev.(σ_s)	17.720mm
Position Parameter(x_o)	58.004mm
Scale Parameter(s)	14.478mm
Gumbel Mean(μ_G)	66.360mm
Gumbel St.Dev.(σ_G)	18.568mm

Appendix 32 3 Hour AMS analysis

Year	3hrs/30mm(X_O)	Rank(m)	P	u	X_G
1982	100	1	0.0253	3.664	111.0442
2002	92	2	0.0705	2.6041	95.8704
1978	87	3	0.1157	2.0956	88.3440
1972	85	4	0.1609	1.7403	83.1994
1975	80	5	0.2061	1.4659	79.2278
1977	77	6	0.2514	1.2396	75.9511
2004	74	7	0.2966	1.0447	73.1297
1976	72	8	0.3418	0.8718	70.6255
1979	72	9	0.3870	0.7147	68.3506
1981	68	10	0.4322	0.5692	66.2449
2008	67	11	0.4774	0.4324	64.2644
2003	66	12	0.5226	0.3019	62.3747
1970	65	13	0.5678	0.1757	60.5471
1974	63	14	0.6130	0.0519	58.7558
1980	61	15	0.6582	-0.0710	56.9754
1999	58	16	0.7034	-0.1951	55.1783
1971	54	17	0.7486	-0.3227	53.3313
1973	53	18	0.7939	-0.4569	51.3888
2006	50	19	0.8391	-0.6025	49.2803
2005	49	20	0.9295	-0.9752	46.8777
1983	34	21	0.9295	-0.9752	43.8842
2009	27	22	0.9747	-1.3019	39.1546

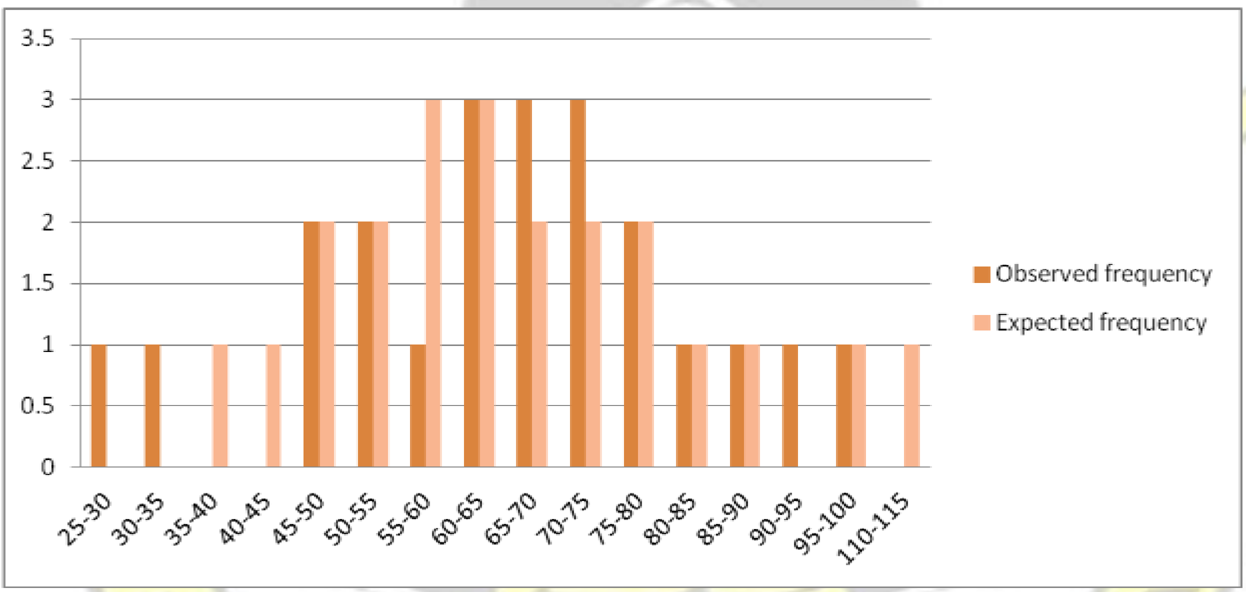
Appendix 33 Observed and Expected frequencies repartition(3 hrs)

Range(mm)	O freq	E freq
25-30	1	0
30-35	1	0
35-40	0	1
40-45	0	1
45-50	2	2
50-55	2	2

55-60	1	3
60-65	3	3
65-70	3	2
70-75	3	2
75-80	2	2
80-85	1	1
85-90	1	1
90-95	1	0
95-100	1	1
100-105	0	0
105-110	0	0
110-115	0	1
	SUM=22	SUM=22

O: Observed Frequency; E: Expected Frequency

Appendix 34 Distribution of Observed and Expected Frequencies(3 hr)



Appendix 35 Chi-square test (3 hrs analysis)

Interval	O	E	$O-E$	$(O-E)^2$	$(O-E)^2/E$
25-40	2	1	1	1	1.00
40-55	4	5	-1	1	0.20
55-70	7	8	-1	1	0.13
70-90	7	6	1	1	0.17
90-115	2	2	0	0	0.00
				Σ	**1.50**

$\mathbf{1.500} < 5.991$ ***null hypothesis is accepted at 0.05 level of significance***

Appendix 36 Kolmogorov-Smirnov test 3 Hour AMS

Interval	X_o	O freq	Cum freq	$F_O(x)$	u	$F_t(x)$	$\lvert F_t(x) - F_o(x) \rvert$
25-30	30	1	1	0.05	-1.93	0.0010	0.049
30-35	35	1	2	0.09	-1.59	0.0074	0.083
45-50	50	2	4	0.18	-0.55	0.1768	0.003
50-55	55	2	6	0.27	-0.21	0.2913	0.021
55-60	60	1	7	0.32	0.14	0.4193	0.099
60-65	65	3	10	0.45	0.48	0.5386	0.089
65-70	70	3	13	0.59	0.83	0.6466	**0.183**
70-75	75	3	16	0.73	1.17	0.7332	0.003
75-80	80	2	18	0.82	1.52	0.8035	0.017
80-85	85	1	19	0.86	1.86	0.8558	0.004
85-90	90	1	20	0.91	2.21	0.8961	0.014
90-95	95	1	21	0.95	2.56	0.9256	0.024

$\mathbf{0.183} < 0.284$ ***null hypothesis is accepted at 0.05 level of significance***

***Appendix 37* 3 Hour estimates**

T Years	K(T)	X(T) Mm
1	-0.970	48.35
2	-0.164	63.31
5	0.719	79.71
10	1.305	90.59
15	1.635	96.72
20	1.866	101.01
30	2.189	107.01
40	2.416	111.22
50	2.592	114.49
22	**1.942**	**102.42**

25	2.044	104.31

Appendix 38 6 hr Annual Maxima Series(AMS) Sample and distribution parameters

Sample Mean(μ_s)	66.318mm
Sample St. Dev.(σ_s)	16.022mm
Position Parameter(x_o)	59.006mm
Scale Parameter(s)	13.091mm
Gumbel Mean(μ_G)	66.562mm
Gumbel St.Dev.(σ_G)	16.789mm

Appendix 39 6 Hour AMS analysis

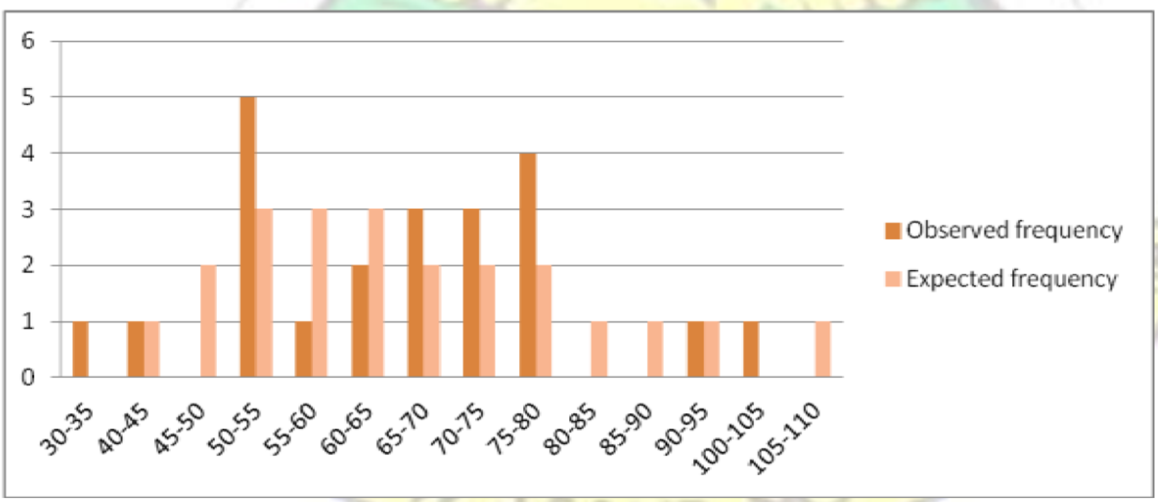
Year	3hrs/30mm(X_O)	Rank(m)	P	u	X_G
1982	105	1	0.0253	3.664	106.9642
1972	92	2	0.0705	2.6041	93.2443
1979	79	3	0.1157	2.0956	86.4391
2004	78	4	0.1609	1.7403	81.7874
1978	77	5	0.2061	1.4659	78.1964
1976	76	6	0.2514	1.2396	75.2336
1975	73	7	0.2966	1.0447	72.6825
2002	73	8	0.3418	0.8718	70.4183
1977	72	9	0.3870	0.7147	68.3614
1981	69	10	0.4322	0.5692	66.4575
2003	69	11	0.4774	0.4324	64.6667
2008	68	12	0.5226	0.3019	62.9580
1974	64	13	0.5678	0.1757	61.3055
1999	62	14	0.6130	0.0519	59.6858
2006	59	15	0.6582	-0.0710	58.0760
1970	54	16	0.7034	-0.1951	56.4512
1971	54	17	0.7486	-0.3227	54.7811
1980	54	18	0.7939	-0.4569	53.0248
2005	54	19	0.8391	-0.6025	51.1183
1973	52	20	0.9295	-0.9752	48.9459
1983	42	21	0.9295	-0.9752	46.2392
2009	33	22	0.9747	-1.3019	41.9628

Appendix 40 Observed and Expected frequencies repartition(6 hr)

Range(mm)	O freq	E freq
30-35	1	0
35-40	0	0
40-45	1	1
45-50	0	2
50-55	5	3
55-60	1	3
60-65	2	3
65-70	3	2
70-75	3	2
75-80	4	2
80-85	0	1
85-90	0	1
90-95	1	1
95-100	0	0
100-105	1	0
105-110	0	1
	SUM=22	SUM=22

O: Observed Frequency; E: Expected Frequency

Appendix 41 Distribution of Observed and Expected Frequencies(6 hr)



Appendix 42 Chi-square test (6 hrs analysis)

Interval	O	E	$O-E$	$(O-E)^2$	$(O-E)^2/E$
30-45	2	2	0	0	0.00
45-55	5	5	0	0	0.00
55-75	9	10	-1	1	0.10
75-85	4	3	1	1	0.33
85-95	1	2	-1	1	0.50
95-110	1	1	0	0	0.00
				Σ	**0.93**

$\mathbf{0.930} < 7.815$ ***null hypothesis is accepted at 0.05 level of significance***

Appendix 43 Kolmogorov-Smirnov test 6 Hour AMS

Interval	X_o	O freq	Cum freq	Fo(x)	u	$F_t(x)$	$\lvert F_t(x) - F_o(x) \rvert$
40-45	45	1	1	0.05	-1.07	0.0542	0.0042
50-55	55	5	6	0.27	-0.31	0.2558	0.0142
55-60	60	1	7	0.32	0.08	0.3973	0.0773
60-65	65	2	9	0.41	0.46	0.5319	**0.1219**
65-70	70	3	12	0.55	0.84	0.6494	0.0994
70-75	75	3	15	0.68	1.22	0.7444	0.0644
75-80	80	4	19	0.86	1.60	0.8172	0.0428
90-95	95	1	20	0.91	2.75	0.9381	0.0281
100-105	105	1	21	0.95	3.51	0.9705	0.0205

$\mathbf{0.1219} < 0.284$ ***null hypothesis is accepted at 0.05 level of significance***

Appendix 44 6 Hour estimates

T Years	K(T)	X(T) mm
1	-0.970	50.28
2	-0.164	63.81
5	0.719	78.63
10	1.305	88.47
15	1.635	94.01
20	1.866	97.89
30	2.189	103.31
40	2.416	107.12
50	2.592	110.08
22	**1.942**	**99.17**
25	2.044	100.88

Appendix 45 12 hr Annual Maxima Series(AMS) Sample and distribution parameters

Sample Mean(μ_s)	66.273mm
Sample St. Dev.(σ_s)	20.091mm
Position Parameter(x_o)	57.103mm
Scale Parameter(s)	16.415mm
Gumbel Mean(μ_G)	66.578mm
Gumbel St.Dev.(σ_G)	21.053mm

Appendix 46 12 Hour AMS analysis

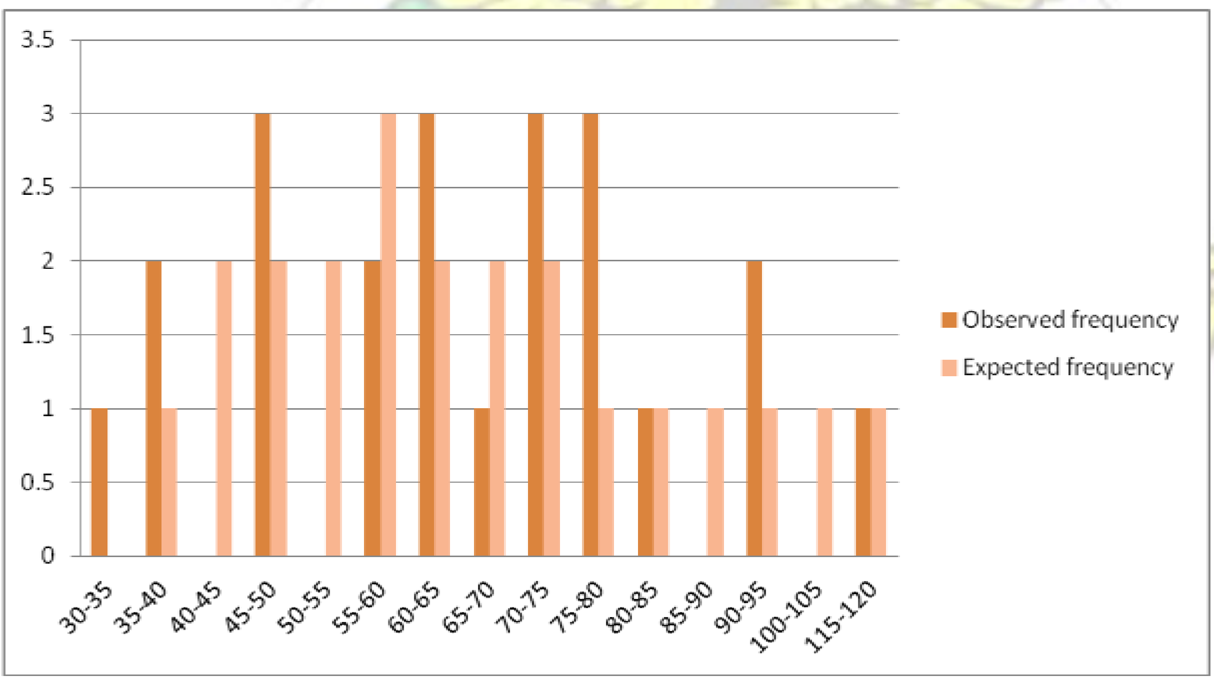
Year	12hrs/30mm(Xo)	Rank(m)	P	u	X_G
1982	116	1	0.0253	3.664	117.2407
1972	93	2	0.0705	2.6041	100.0367
1979	93	3	0.1157	2.0956	91.5033
2008	81	4	0.1609	1.7403	85.6703
1978	78	5	0.2061	1.4659	81.1673
2002	77	6	0.2514	1.2396	77.4522
1977	76	7	0.2966	1.0447	74.2533
1976	74	8	0.3418	0.8718	71.4140
1981	72	9	0.3870	0.7147	68.8348
2004	72	10	0.4322	0.5692	66.4474
1971	69	11	0.4774	0.4324	64.2019
1970	65	12	0.5226	0.3019	62.0593
1980	61	13	0.5678	0.1757	59.9871
2006	61	14	0.6130	0.0519	57.9561
2005	60	15	0.6582	-0.0710	55.9375
1999	56	16	0.7034	-0.1951	53.9001
1975	50	17	0.7486	-0.3227	51.8059
2009	48	18	0.7939	-0.4569	49.6035
1983	47	19	0.8391	-0.6025	47.2129
1974	38	20	0.9295	-0.9752	44.4888
2003	38	21	0.9295	-0.9752	41.0947
2009	33	22	0.9747	-1.3019	35.7324

Appendix 47 Observed and Expected frequencies repartition(12 hr)

Range(mm)	O freq	E freq
30-35	1	0
35-40	2	1
40-45	0	2
45-50	3	2
50-55	0	2
55-60	2	3
60-65	3	2
65-70	1	2
70-75	3	2
75-80	3	1
80-85	1	1
85-90	0	1
90-95	2	1
95-100	0	0
100-105	0	1
105-110	0	0
110-115	0	0
115-120	1	1
	SUM=22	SUM=22

O: Observed Frequency; E: Expected Frequency

Appendix 48 Distribution of Observed and Expected Frequencies(12 hr)



Appendix 49 Chi-square test (12 hrs analysis)

Interval	O	E	$O-E$	$(O-E)^2$	$(O-E)^2/E$
30-35	3	3	0	0	0.00
45-55	3	4	-1	1	0.25
55-65	5	5	0	0	0.00
65-80	7	5	2	4	0.80
80-95	3	3	0	0	0.00
95-120	1	2	-1	1	0.50
				Σ	**1.55**

$\mathbf{1.55} < 7.815$ ***null hypothesis is accepted at 0.05 level of significance***

Appendix 50 Kolmogorov-Smirnov test 12 Hour AMS

Interval	X_o	O freq	Cum freq	$F_O(x)$	u	$F_t(x)$	$\lvert F_t(x) - F_o(x) \rvert$
30-35	35	1	1	0.05	-1.35	0.0211	0.0289
35-40	40	2	3	0.14	-1.04	0.0591	0.0809
45-50	50	3	6	0.27	-0.43	0.2150	0.0550
55-60	60	2	8	0.36	0.18	0.4338	0.0738
60-65	65	3	11	0.50	0.48	0.5386	0.0386
65-70	70	1	12	0.55	0.79	0.6352	**0.0852**
70-75	75	3	15	0.68	1.09	0.7145	0.0345
75-80	80	3	18	0.82	1.40	0.7814	0.0386
80-85	85	1	19	0.86	1.70	0.8330	0.0270
90-95	95	2	21	0.95	2.31	0.9055	0.0445

$\mathbf{0.0852} < 0.284$ ***null hypothesis is accepted at 0.05 level of significance***

Appendix 51 12 Hour estimates

T Years	K(T)	X(T) mm
1	-0.970	46.16
2	-0.164	63.13
5	0.719	81.72
10	1.305	94.05
15	1.635	101.00
20	1.866	105.86
30	2.189	112.66
40	2.416	117.44
50	2.592	121.15
22	**1.942**	**107.46**
25	2.044	109.61

Sample Mean (μ_s)	59.273mm

Sample St. Dev.(σ_s)	18.507mm
Position Parameter(x_o)	50.826mm
Scale Parameter(s)	15.121mm
Gumbel Mean(μ_G)	59.554mm
Gumbel St.Dev.(σ_G)	19.393mm

Appendix 52 24 hr Annual Maxima Series(AMS) Sample and distribution parameters

Appendix 53 24 Hour AMS analysis

Year	24hrs/40mm(X_O)	Rank(m)	P	u	X_G
1976	90	1	0.0253	3.664	106.2219
1972	82	2	0.0705	2.6041	90.3744
2002	82	3	0.1157	2.0956	82.5139
1975	76	4	0.1609	1.7403	77.1408
1982	74	5	0.2061	1.4659	72.9929
1978	71	6	0.2514	1.2396	69.5707
1970	69	7	0.2966	1.0447	66.6240
1973	69	8	0.3418	0.8718	64.0086
2008	65	9	0.3870	0.7147	61.6328
1977	64	10	0.4322	0.5692	59.4336
1974	61	11	0.4774	0.4324	57.3651
1981	60	12	0.5226	0.3019	55.3915
1999	60	13	0.5678	0.1757	53.4827
2004	59	14	0.6130	0.0519	51.6119
1980	56	15	0.6582	-0.0710	49.7524
1971	55	16	0.7034	-0.1951	47.8756
2005	49	17	0.7486	-0.3227	45.9466
1979	46	18	0.7939	-0.4569	43.9178
1983	40	19	0.8391	-0.6025	41.7158
2009	32	20	0.9295	-0.9752	39.2064
2003	30	21	0.9295	-0.9752	36.0800

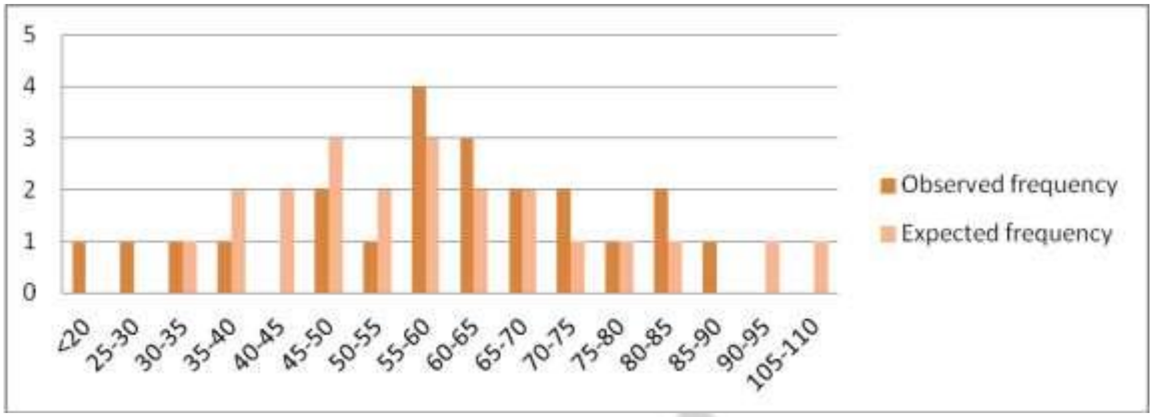
2006	14	22	0.9747	-1.3019	31.1404

Appendix 54 Observed and Expected frequencies repartition

Range(mm)	O freq	E freq
<20	1	0
20-25	0	0
25-30	1	0
30-35	1	1
35-40	1	2
40-45	0	2
45-50	2	3
50-55	1	2
55-60	4	3
60-65	3	2
65-70	2	2
70-75	2	1
75-80	1	1
80-85	2	1
85-90	1	0
90-95	0	1
95-100	0	0
100-105	0	0
105-110	0	1
	SUM=22	SUM=22

O: Observed Frequency; E: Expected Frequency

Appendix 55 Distribution of Observed and Expected Frequencies(24hr)



Appendix 56 Chi-square test (24 hrs analysis)

Appendix 56 Chi-square test (24 hrs analysis)

Interval	O	E	$O-E$	$(O-E)^2$	$(O-E)^2/E$
10-45	4	5	-1	1	0.20
45-60	7	8	-1	1	0.13
60-65	3	2	1	1	0.50
65-70	2	2	0	0	0.00
70-75	2	1	1	1	1.00
75-110	4	4	0	0	0.00
				Σ	**1.83**

$\mathbf{1.83} < 7.815$ ***null hypothesis is accepted at 0.05 level of significance***

Appendix 57 Kolmogorov-Smirnov test 24 Hour AMS

Interval	X_o	O freq	Cum freq	Fo(x)	u	$F_t(x)$	$\lvert F_t(x) - F_o(x)\rvert$
25-30	30	1	1	0.05	-1.38	0.0188	0.0312
30-35	35	1	2	0.09	-1.05	0.0574	0.0326
35-40	40	1	3	0.14	-0.72	0.1282	0.0118
45-50	50	2	5	0.23	-0.05	0.3495	0.1195
50-55	55	1	6	0.27	0.28	0.4697	**0.1997**
55-60	60	4	10	0.45	0.61	0.5808	0.1308
60-65	65	3	13	0.59	0.94	0.6766	0.0866
65-70	70	2	15	0.68	1.27	0.7551	0.0751
70-75	75	2	17	0.77	1.60	0.8172	0.0472
75-80	80	1	18	0.82	1.93	0.8649	0.0449
80-85	85	2	20	0.91	2.26	0.9009	0.0091
85-90	90	1	21	0.95	2.59	0.9277	0.0223

$\mathbf{0.1997} < 0.284$ ***null hypothesis is accepted at 0.05 level of significance***

Appendix 58 24 Hour estimates

30	2.189	102.01
40	2.416	106.41
50	2.592	109.82
22	**1.942**	**97.22**
25	2.044	99.19
T Years	K(T)	X(T) mm
1	-0.970	40.74
2	-0.164	56.37
5	0.719	73.50
10	1.305	84.86
15	1.635	91.26
20	1.866	95.74

Heavy Falls of Rain

Appendix 59 Sample of Rainfall Data Gathered

Station: ……. Kumasi

Duration of Period	0.2 hours	0.4hours	0.7 hours	1 hours	2 hours	3hours	6 hours	12 hours	24 hours
Lower limits of falls entered	12mm	15mm	20mm	25mm	25mm	30mm	30mm	30mm	40mm

Year	Month	0.2 hours	0.4hours	0.7 hours	1 hours	2 hours	3hours	6 hours	12 hours	24 hours
1970	3		38	42	46	46	46	46	46	46
1970	4		16		28	37	38	38	38	
1970	5	20	30	33	34	38	40	43	43	43
1970	7	12	33	43	43	44	73	73	73	73
1970	9	20	25	26						
1970	10		19	22						

1970 11 14 16

1971 3 24 32 34 35

1971 4 16

1971 5 14 15 26 30 33 1971 9 25

45 88

1971 11 16 .

1972 1 12 15

Year	Month	0.2 hours	0.4hours	0.7 hours	1 hours	2 hours	3hours	6 hours	12 hours	24 hours
1972	2	14	19	23						
1972	4	14	19	22						
1972	5	23	51	62	67	74	79	89	91	91
1972	6	15	24	33	34	39	39	39	40	40
1972	7	14	19	24	25	38	41	41	41	41
1972	9	20	45	47	47	49	64	65	65	74

Appendix 60: Relationship between Calculated WMO and Sub-Hourly IDF estimates for 2 Year Return Period.

Duration/min.	WMO	Calculated Value	Actual Value	% error(absolute)
12	144.0	0.88*144.0=126.72	127.2	0.4
24	105.7	0.88*105.7=93.02	94.2	1.3
42	76.6	0.88*76.6=67.41	61.2	10.1
60(1hr)	56.7	56.7	56.7	0

Appendix 61: Relationship between Calculated WMO and Sub-Hourly IDF estimates for 5 Year Return Period.

Duration/min.	WMO	Calculated Value	Actual Value	% error(absolute)
12	180.7	0.88*180.7=159.0	161.6	1.6
24	132.8	0.88*132.8=116.9	120.7	3.1
42	96.2	0.88*96.2=84.7	75.4	12.3

60(1hr)	71.3	71.3	71.3	0

Appendix 62: Relationship between Calculated WMO and Sub-Hourly IDF estimates for 10 Year Return Period.

Duration/min.	WMO	Calculated Value	Actual Value	% error(absolute)
12	205.3	0.88*205.3=180.7	184.4	2.0
24	150.8	0.88*150.8=132.7	138.3	4.0
42	109.3	0.88*109.3=96.2	84.8	13.4
60(1hr)	81.0	81.0	81.0	0

Appendix 63: Relationship between Calculated WMO and Sub-Hourly IDF estimates for 20 Year Return Period.

Duration/min.	WMO	Calculated Value	Actual Value	% error(absolute)
12	228.9	0.88*228.9=201.4	206.3	2.4
24	168.1	0.88*168.1=147.9	155.2	4.7
42	121.8	0.88*121.8=107.2	93.9	14.2

60(1hr)	90.3	90.3	90.3	0

Appendix 64: Goodness-of-Fit for Gumbel Distribution(0.4 hrs)

Kolmogorov-Smirnov					
Sample Size Statistic P-Value Rank	22 0.08529 0.99282 1				
□	0.2	0.1	0.05	0.02	0.01
Critical Value	0.22115	0.25283	0.28087	0.31394	0.33666
Reject?	No	No	No	No	No
Chi-Squared					
Deg. of freedom Statistic P-Value Rank	2 0.02854 0.98583 1				
□	0.2	0.1	0.05	0.02	0.01
Critical Value	3.2189	4.6052	5.9915	7.824	9.2103
Reject?	No	No	No	No	No

Appendix 66: Goodness of Fit(0.4 hrs) Summary

#	Distribution	Kolmogorov Smirnov		Anderson Darling		Chi-Squared	
		Statistic	Rank	Statistic	Rank	Statistic	Rank

1	Gen. Extreme Value	0.08529	1	0.12336	1	0.02854	1
2	Log-Pearson 3	0.08988	2	0.13133	2	0.05457	2

Appendix 65: Goodness-of-Fit for Log Pearson Type III Distribution(0.4 hrs)

Kolmogorov-Smirnov					
Sample Size Statistic P-Value Rank	22 0.08988 0.98711 2				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	0.22115	0.25283	0.28087	0.31394	0.33666
Reject?	No	No	No	No	No
Anderson-Darling					
Sample Size Statistic Rank	22 0.13133 2				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	1.3749	1.9286	2.5018	3.2892	3.9074
Reject?	No	No	No	No	No
Chi-Squared					

Deg. of freedom Statistic P-Value Rank	2 0.05457 0.97308 2				
□	0.2	0.1	0.05	0.02	0.01
Critical Value	3.2189	4.6052	5.9915	7.824	9.2103
Reject?	No	No	No	No	No