Kwame Nkrumah University of Science and Technology, Kumasi.

INSTITUTE OF DISTANCE LEARNING

KNUST

MATHEMATICAL MODEL OF CAR TRAFFIC FLOW

A Thesis submitted to the Department of Mathematics,

in partial fulfilment of the requirements for the award of degree of

Master of Science (Industrial Mathematics)

BY

SOLOMON OKYERE

JUNE, 2011

DECLARATION

I, hereby declare that this thesis entitled "Mathematical Model of Car Traffic Flow" represent my own work towards the award of MSc and that, to the best of my knowledge, except where due reference is made has not been previously submitted to this or other institution for the award of degree, diploma or other qualification.

I also declare that I have wholly undertaken the study reported herein under supervision.

KNUST

Solomon Okyere (PG 3016509)	………………	……………
Student's Name and ID	Signature	Date

Certified by:

Dr. F. T. Oduro	………………	…………….
Supervisor's Name	Signature	Date

Dr. S. K. Amponsah	……………….	……………..
Head of Department's Name	Signature	Date

ACKNOWLEDGEMENT

My sincere thanks goes to the management of Wa Polytechnic and the Ghana Education Trust Fund (GetFund) for their financial support.

The author would like to express his deepest gratitude to his supervisor and mentor Dr. F. T. Oduro for his utmost patience and help throughout this project. He has helped to clarify concepts, suggest alternate approaches to some methods and has posed questions which make the project more interesting.

I also want to acknowledge the contributions of two of my colleagues who gave many thoughtful advices and uncountable technical support: Felix Uba and Fredrick Kuuyine of the Department of Mechanical Engineering, Wa Polytechnic.

I will also like to show my appreciation to my wife Mrs Alberta Boahemaa Okyere for helping out with the typing of the thesis, which ironically turns out to be a more frustrating endeavour than the project itself.

My final acknowledgment is owed to God who is my creator and responsible for all my accomplishments.

DEDICATION

This research work is dedicated to the Almighty God. To my mother, Madam Abenaa Serwaa who sacrificed and encouraged my education with love and affection. To my dear sister, Monica Amoah for her continues support and encouragement. To my lovely wife, Alberta for her endless support, understanding and sacrifice. Most of all to my lovely daughter, Shirley Adwoa Serwaa Okyere.

KNUST

ABSTRACT

The study investigates the macroscopic model of traffic flow characteristics and its accompanied continuity equation of vehicles (Lighthill-Whitham-Richards (LWR) model) on a basic freeway segment. The research presents a mathematical model of car traffic flow using the analogy between vehicles in traffic flows and particles in fluid flows which is based on the conservation laws.

Using regression analysis the Flow-Density curve was found to be quadratic of the form $q = 113.56\rho - 2.53\rho^2$

thus verifying the Lighthill-Whitham-Richards (LWR) model.

The method of characteristics was used to solve the Partial Differential Equation (PDE)

$$\frac{\partial \rho}{\partial t} + \left(113.56 - 5.06\rho\right)\frac{\partial \rho}{\partial x} = 0$$

and the model yielded characteristics curve of the form

$$x = \left(113.56 - 189.24x_0\right)t + x_0$$

Consequently the solution of the Partial Differential Equation was obtained as

$$\rho\left(x,t\right) = \frac{37.4x - 4247.14t}{1 - 189.24t}$$

With the velocity field as

$$u\left(x,t\right) = \frac{113.56 - 189.24x}{1 - 189.24t}$$

TABLE OF CONTENTS

CHAPTER FOUR

DATA ANALYSIS AND MODELLING

CHAPTER FIVE

DISCUSSION, CONCLUSION AND RECOMMENDATION

LIST OF TABLES

KNUST

LIST OF FIGURES

LIST OF ACRONYMS/ABBREVIATIONS

TRB	Transportation Research Board
GDP	Gross Domestic Product
PDE	Partial Differential Equation
MOC	Method of Characteristics
MCO	Moving Car Observer Method
ODE	Ordinary Differential Equation
HCM	Highway Capacity Manual
BVP	Boundary Valued Problem
IVP	Initial Valued Problem
LWR	Lighthill, Whitham and Richards
CBD	Central Business District
Veh	Vehicles
Veh/hr	Vehicles per hour
Veh/km	Vehicles per kilometre
Km/hr	Kilometres per hour

KNUST

CHAPTER ONE

INTRODUCTION

1.1 BACKGROUND OF THE STUDY

Traffic Flow is the study of the movement of individual drivers and vehicles between two points and the interactions they make with one another and they play a vital role in the progress of overall social productivity.

In the 1950s James Lighthill and Gerard Whitham, two experts in fluid dynamics, and independently P. Richards, modelled the flow of car traffic along a single road using the same equations describing the flow of water (Lighthill *et al*.,1955; Richards,1956). The basic idea is to look at traffic on a large scale so as to consider cars as small particles and to assume the conservation of the cars number. The LWR model is described by a single conservation law, a special partial differential equation where the variable, the car density, is a conserved quantity, i.e. a quantity which can neither be created nor destroyed.

Traffic engineering usually deals with the analysis of the behaviour of traffic and to design the roadway facilities for a smooth and safe operation of traffic. Traffic flow like fluid flow has several parameters associated with it; these parameters provide information regarding the nature of traffic flow, which helps the analyst in detecting any variation in flow characteristics. Understanding traffic behaviour requires a thorough knowledge of traffic stream parameters and their mutual relationship.

Traffic Flow Theory is a mathematical tool that helps transportation engineers understand and express the properties of traffic flow. (Transportation Research Board special Report 165, Traffic Flow Theory, 1975).

1.1.1 Why Modelling Traffic

Given the continual and dramatic increase in vehicle ownerships and many more people travelling day in and day out, at any given time, there are millions of vehicles on our roadways; these vehicles interact with each other and have great impact on the overall movement of traffic, or traffic flow. Every driver is therefore faced with the conflicting objective of reducing travel time and avoiding accidents.

Engineers are given the task of designing, controlling and managing the road system to ensure that drivers travel as efficient as possible on our roadway. For this study we seek to obtain a Mathematical model that can provide the understanding necessary for the engineer to undertake his task and to assist in reducing the negative impacts that traffic congestion can have on our lives.

A mathematical model is an inevitable component of scientific and technical progress. The very formulation of the problem of mathematical modelling of traffic flow leads to a better understanding of the traffic phenomena we see around us and a precise plan of action for the engineer.

To better represent traffic flow, relationships have been established between the three main characteristics of traffic flow namely flow rate, density, and velocity. These relationships help in planning, designing, controlling and managing the operations of roadway facilities.

Since 1930s scientist in various fields have focused on traffic problem and great progress has been made. The most widely used model is the Greenshields model

(Greenshields, 1935) which states that the relationship between speed and density is linear. Here we focus our attention on a segment of the roadway where we will analyse traffic phenomena resulting from the complex interaction of many vehicles, instead of studying the phenomena associated with the motion of individual cars.

1.2 STATEMENT OF THE PROBLEM

In recent years, the problem of traffic congestion is becoming endemic due to large and ever-increase number of peoples owing cars and many more travelling. Usually drivers and passengers alike expect to get to their destinations on time without delays, traffic congestion can have great impact on their time of travel.

Sometimes a lot of time is spent in traffic which may result in late arrival for work, meetings, and schools, resulting in lost business, disciplinary action or other personal losses, and even besides the time spent in traffic, which is directly related with congestion, there are other cost such as environmental costs (e.g. air pollution), social costs (e.g. stress) and economic costs (e.g. delayed deliveries, fuel consumption). This delay in traffic can end up reducing man hours which at the end negatively affect the Gross Domestic Product (GDP) and consequently reduce per capital income.

1.3 OBJECTIVES

1.3.1 General Objective

The study investigates the macroscopic model of traffic flow characteristics and its accompanied continuity equation of vehicles (Lighthill-Whitham-Richards (LWR) model) on a basic freeway segment.

It also aims to formulate a model (Partial Differential Equation) that mathematically models car traffic flow similar to the equation of fluid flow.

1.3.2 Specific Objectives

This research seeks:

i. To study traffic flow behaviour of a segment of the roadway
ii. To derive the theoretical relationships between the various traffic variable, i.e., speed, flow and density relationships.
iii. To determine a Partial Differential Equation (PDE) that describes traffic flow using the conservation of cars
iv. To determine the solution of the resulting first-order PDE's using the method of characteristics (characteristic curve)

Although the study is related to particular road section and traffic conditions, it is expected that the findings, conclusions and recommendations may assist the engineers to understand traffic phenomena we see around us in order to eventually make decisions which may reduce travel time, alleviate congestion and avoid accident, to ensure that motorists travel as efficiently as possible on our roads.

1.4 METHODS EMPLOYED IN THE STUDY

We will use the idea of conservation of cars and experimental relationship between car velocity and traffic density to formulation a traffic problems in terms of a nonlinear partial differential equation. Thus we will study some methods of solving partial differential equations particularly the method of characteristics. Mathew and Rao (2007).

Data was collected manually by the Moving Car Observer Method (MCO). It is the most commonly used method to get the relationship between speed, density and traffic flow data by a single experiment. In this method, to measure the traffic flow in a road segment is to drive a car in that road segment yourself. First you drive in the section in the same direction as the observed flow, and then the section is driven on the other side of the road, against the flow. An observer(s) in the car measures the two traveling times, the number of opposing vehicles met, the number of vehicles the test vehicle overtook, and the number of vehicles overtaking the test vehicle. The data are then transferred onto various charts and tables to illustrate the operation of the roadway. The traffic flow pertaining to one direction was considered for this study.

1.5 JUSTIFICATION

Most transportation engineering projects begin with an evaluation of the traffic flow. Therefore, the transportation engineer needs to have a better understanding of the theories behind Traffic Flow Analysis and express their properties. Mathematical models of traffic flow can provide the understanding necessary for their purposes.

In addition to maintaining flow on our existing roadways, we are faced daily with issues of allocating funds to maintenance activities that will ensure the roadways continue to serve the needs into the future. This includes identifying needs for expansion and/or changes in operational strategies, to facilitate efficient traffic flow. Mathematical models of traffic flow can help identify these needs.

1.6 SCOPE OF THE STUDY

For analysis purposes, roadway facilities are separated into categories that are specific to traffic flow type: Uninterrupted and Interrupted traffic flow. (Analysis Procedure Manual 2006).

This project examines commonly used segment (uninterrupted flow) analysis procedures. We will consider the macroscopic view of traffic flow which looks at modelling the number of vehicles passing a specified point on a roadway in some time interval.

We will consider a basic freeway segment (outside the influence area of ramps or weaving areas) which provides uninterrupted flow and to analyse the operating condition primarily resulting from interactions among large number of vehicles in the traffic stream. Here Lighthill-Whitham-Richards (LWR) model concept is introduced.

1.7 LIMITATIONS OF THE STUDY

The study was limited in scope by its budget and time frame. The data collected did not cover the full range that the model is intended to cover, especially the influence of ramp (exit and entrance) segment and weaving sections.

Apart from Lighthill-Whitham-Richards model (1955), there are many ways at looking into car traffic flow (e.g., Payne-Whitham model), and many other ways to solve these situations (e.g., numerical method); I did not however have the time to research them more.

1.8 ORGANIZATION OF THE THESIS

The thesis will be presented in five major chapters.

Chapter one is the introduction which presents the background to the study; why study traffic flow, statement of the problem, objective of the study, methods employed in the study, justification of the study, scope of the study and the limitations of the study.

Chapter two will present review of relevant literature and comprises different perspective of researchers of the problem related to traffic flow.

The methodology to achieve the objectives is outlined in chapter three and here the relationships between the three fundamental traffic variables are developed.

Chapter four presents the data analysis, modelling results and the accompanying discussions.

The summary of finding conclusions and recommendations are presented in chapter five, then the References and Appendix.

CHAP TER TWO

LITERATURE REVIEW

2.1 INTRODUCTION

The scientific study of traffic flow had its beginnings in the 1930's with the application of probability theory to the description of road traffic, (Adams, 1936) and the pioneering studies conducted by Bruce D. Greenshields at the Yale Bureau of Highway Traffic when he measured the actual flows and speeds of one lane of traffic and identified a linear relationship between speed and density. (Greenshields, 1935).

After World War II, with the rapid development of the transportation and the tremendous increase in the use of cars, traffic research got more and more attention and there was a surge in the study of traffic characteristics and the development of traffic flow theories. The 1950's saw theoretical developments based on a variety of approaches. (Dhingra and Gull, 2008).

Some of the early contributions to traffic modelling were those of Reuschel (1950), Pipes (1953) and Lighthill and Whitham (1955). Reuschel and Pipes proposed a traffic model that describes the detailed movement of cars proceeding close together in a single lane, a "microscopic" model of traffic. Lighthill, a world-renowned fluid mechanics theorist, together with Whitham, proposed a "macroscopic" model of traffic, modelling traffic as a continuum akin to a fluid.

2.1.1 Approaches to Modelling Traffic

Modeling the process of traffic flow was previously studied from different points of view and different mathematical methods were used to describe the same process.

It also encounters difficulties in choosing an appropriate method of deriving physical appearance we can notice on our streets and roads. Different authors have different views to the same phenomena and are focusing on different aspects of the same problem (Junevicius and Bogdevicius, 2007; Berezhnoy *et al*., 2007; Akgungor, 2008a and 2008b; Daunoras, *et al*., 2008; Yousefi and Fathy, 2008; Gowri and Sivanandan, 2008; Jakimavicius and Burinskiene, 2007 and 2009; Antov, *et al*., 2009; Knowles, 2008; Gasser, 2003). All authors have an agreement on basic traffic flow parameters like, traffic flow density, traffic flow rate or the average speed of traffic flow. Besides, a lot of different investigations into the use of traffic flow models to deal with various problems of engineering are carried out, for example in Sivilevicius and Sukevicius, 2007 paper.

A comparison of different continuum models has drawn that a number of scientific works were based on fluid dynamic theory and gas-kinetic traffic flow theory. The kinetic traffic flow theory is used for microscopic or macroscopic traffic flow models. The kinetic traffic flow theory is used in Flotterod and Nagel (2007), Gning *et al.* (2008), Li and Xu (2008), Prigogine and Herman (1971), where various approaches to the similar method are discussed. The equations of these models take different values to derive the same process. The kinetic theory was first used by Prigogine and Herman (1971) and co-workers. They suggested an equation analogous to Boltzmann equation. This theory was later criticized by many authors like Tampere, 2004. The papers of whose show the experience of Pavery-Fontna who noticed that Prigogine model had inaccuracies comparing the results of modeling and physical experiments. He suggested vehicle desired velocity towards which its actual velocity tends. Later, many authors mainly focused on a better statistical description of the traffic process.

The macroscopic theory of traffic flows also can be developed as the hydrodynamic theory of fluids that was first introduced by Lighthill-Whitham and Richards model (Chalons and Goatin, 2008; Kim and Keller, 2002; Liu *et al.*, 2008; Bonzani, 2007; Nikolov, 2008). They presented one dimensional model analogous to the fluid stream model. This theory was also criticized by such authors as Tampere (2004), Daganzo *et al.*,(2008) and Liu *et al.*, (2008) who proposed the lattice method. Nagatani and Nakanishi model took into account that all vehicles were moving at the same time-independent speed and in the same gap between vehicles. This method was improved later by considering the next-nearest neighbour interaction Liu *et al.*,(2008).

A lot of traffic flow models are based on car-following theories supported by the analogues to Newton's equation for each individual vehicle.

First International Symposium on The Theory of Traffic Flow was held at the General Motors Research Laboratories in Warren, Michigan in December 1959 Herman (1961). This was the first of what has become a series of triennial symposia on The Theory of Traffic flow and Transportation.

2.2 THE GREENSHIELDS RELATIONSHIP

Greenshields was one of the first traffic engineers to study the relationship between flow and concentration. Using a camera he took sub sequential photos of highways at a constant frame rate, from which he calculated the density and flow rate of the particular stretch of road. Greenshields postulated a linear relationship between the speed of the vehicles and the density of the road in which they were travelling. This relationship has been generally well regarded

2.3 TRAFFIC MODEL CLASSIFICATION

A wide variety of traffic models exists. These models can be classified based on their properties. (Bellemans, 2002).

Some of the way to classify traffic models from the point of view of a traffic control engineer includes:

i. Physical interpretation
ii. Level of detail
iii. Discrete versus continuous
iv. Deterministic versus stochastic

2.3.1 Physical Interpretation

Various mathematical models are presented to understand the rich variety of physical phenomena exhibited by traffic. According to system theory (Ljung, 1987), there are three major approaches towards modelling.

2.3.1.1 The Deductive Approach

In the deductive approach, physical equations describe the relationships between different states of the traffic system are obtained. This approach is used for instance to describe a mechanical system by Newton’s laws and is called first principles modelling. The relationship between the states of the system is described by properties that can be measured (e.g. mass in a mechanical system).

2.3.1.2 Inductive Approach

An alternative approach to the deductive approach is the inductive modelling where input and output data of the system are recorded and a generic parameterized model is fitted to the data. For a traffic system, the input/output data typically consist of the evolution of traffic flows, traffic densities and measured speeds over time. With the inductive modelling method there is in general no physical relationship between the modelled traffic situation and the model structure. An example of inductive traffic modelling is the application of a neural network that is trained to mimic the behaviour of the traffic system e.g. modelling the relationship between the future density as a function of the current density and speed, (Ho *et al.*, 1996).

Finally, we can distinguish an intermediate method between the deductive and inductive modelling. This modelling is a combination of the inductive and the deductive approach. In the deductive phase, parameterized equations between the states of the motorway system are written down. During the inductive phase, the parameters in the model are tuned by fitting the input-output relation of the traffic model to input-output measurements of the traffic system. As an example of this intermediate approach, we mention the traffic models of Lighthill, Whitham (1955) and Richards (1956) and Payne (1971). The equations in these models have a physical interpretation but contain parameters, which need to be fitted using input/output data from the traffic system.

2.3.2 Level of Detail

Modelling traffic flow can be done at various levels of detail ranging from microscopic, to kinetic and macroscopic models, Bellemans *et al.* (2002) as sited in Hoogendoorn and Bovy (2000).

2.3.2.1 Microscopic Model

Microscopic traffic model was first introduced in 1955 when D. L. Gerlough published his dissertation. In this model every vehicle or 'particle' in the system is considered as an individual, and therefore an equation is written for each, usually an Ordinary Differential Equation (ODE). These vehicle models include e.g. the interaction between the vehicles or between the vehicles and the motorway.

The two most important microscopic models are:

i. The car following model, which describes how a vehicle follows preceding vehicles. (Herman *et. al*., 1959). In this theory, the behaviour of each car is entirely dependent on the car in front of it. That is, it describes the headway a driver preserves between himself and the preceding vehicle, how the driver reacts on acceleration or deceleration of the vehicle in front of him. Here the distance to the next car and the reaction time of the driver are the main variables.

ii. The overtaking model describes how a driver decides whether or not to overtake its predecessor. Vehicle and driver properties that are important in the overtaking model are e.g. the desired speed of the driver and acceleration abilities of the vehicle.

The number of lanes is a motorway property that influences overtaking behaviour. Vehicle interaction is also important in overtaken model. A driver will decide to overtake based on the speed difference between his lane and the adjacent lane and on the available gap on the other lane.

2.3.2.2 Macroscopic Model

Macroscopic models describe the traffic flow by continuous aggregate functions like average density, velocity and flow in the space-time domain. The dynamics of traffic flow is modelled by a nonlinear system of Partial Differential Equations. Typically, a macroscopic model defines a relation between the traffic density, the average velocity and the traffic flow. (Gerlough and Huber, 1975; Pensaud and Hurdle, 1991; Ross, 1991; Hall, Hurdle and Banks, 1992; Gilchrist and Hall, 1992; Disbro and Frame, 1992).

Within the class of macroscopic models, a classification based on the order of the models can be made. The oldest model was proposed by Lighthill and Whitham in 1955 and independently by Richards in 1956 and is of first order.

Lighthill-Whitham-Richards model is given below:

$$\rho_t + \left(\rho u(\rho)\right)_x = 0, \qquad u(\rho) = u_{\max}\left(1 - \frac{\rho}{\rho_{\max}}\right),\ 0 \le \rho \le \rho_{\max}. \tag{2.1}$$

The only state variable of this model is the traffic density. The Lighthill, Whitham, Richards model was extended later on in order to be able to cope with shock waves and stop-and-go traffic in congested traffic situations (Newell, 1993). The model described by Payne (1971) is of second order since it has two state variables: traffic density and average velocity.

Payne-Whitham model:

$$\rho_t + (\rho u)_x = 0, \quad (\rho u)_t + (\rho u^2 + p(\rho))_x = 0, \tag{2.2}$$

This model mimics the flow of gas particles. In fact, the above equations are known as the Euler equations of gas dynamics with pressure

$$p(\rho) = a\rho^{\gamma}, a > 0, \gamma \geq 1. \tag{2.3}$$

The disadvantage of this model is that there may be solutions for which the velocity u is negative. Helbing (1996) proposed a third order macroscopic traffic model with as state variables the traffic density, the average velocity and the variance on the velocity.

Since macroscopic traffic models only work with aggregate variables and do not describe the traffic situation on the level of independent vehicles, they are less computationally intensive than microscopic models. Due to the fact that a macroscopic traffic model has fewer parameters to estimate than a microscopic model, it is easier to identify and to tune a macroscopic model.

2.3.3 Deterministic versus Stochastic

In a deterministic traffic model there is a deterministic relation between the input, the states and the output of the model. If we simulate a traffic situation twice, starting from the same initial conditions and with the same inputs and boundary conditions, the outputs of the model will be the same. Payne's macroscopic traffic simulation model is an example of a deterministic traffic model.

A stochastic traffic model contains at least one stochastic variable. It captures variation in e.g. reaction time, arrival processes, route choice. But every simulation run results in different outcome, so you need to replicate simulation runs**.** This implies that two simulations of the same model starting from the same initial conditions, the same boundary conditions and the same inputs may give different results, depending on the value of the stochastic variable during each simulation. The stochastic variable is characterized by a distribution function or a histogram.

2.3.4 Discrete Versus Continuous

A motorway traffic model describes the evolution of the state variables of the traffic network over time. This means that there will be two independent variables, namely space and time. These independent variables can be considered to be either continuous or discretized. Since the continuous traffic models are generally too complicated to solve analytically, especially if the size of the considered traffic network is large, they are discretized in time and space in order to simulate their behaviour using a computer (Lebacque, 1996). Payne's traffic model is from origin a continuous model which is discretized in space (with motorway stretches of typically 500 meter) and time (with time intervals of typically 15 seconds) for implementation and simulation on a computer (Papageorgiou, 1990).

In this thesis, we will consider the Lighthill-Whitham-Richards model.

2.4 THE LIGHTHILL, WHITHAM AND RICHARDS (LWR) MODEL

In the mid 1950's, two mathematicians from Manchester University; James Lighthill and Gerard Whitham, experts in the field of fluid dynamics provided one of the first published theories of the macroscopic modelling of highway traffic flow. Their theory was based on two relationships. One was a continuity equation, and the other was the fundamental relationship between the flow and the density of a traffic stream. Lighthill and Whitham (1955). This paper focuses on the traffic application.

KNUST

Richards (1956) independently proposed the same continuum approach, even though in a slightly different form. The key difference is that Richards focused on the derivation of shock waves with respect to density, whereas Lighthill and Whitham considered the same from the perspective of disruptions in traffic flow. Another difference between the two methods is that Richards fixed the equilibrium relationship, whereas Lighthill and Whitham did not restrict themselves to an *a priori* definition. Because of the nearly simultaneous and independent development of the theory Modern literature often refers to this model as the Lighthill, Whitham and Richards or the LWR Model, Schoenhof and Helbing (2004) as cited by Chan Wenqin (2007).

Unlike microscopic model (Cameron and Duncan, 1996; Fellendorf, 1996; Owen *et al.,* 2000), the macroscopic model use aggregated variables to describe the behaviour of traffic. Macroscopic traffic models are often derived using the analogy between traffic flows and the fluid flows. We can write down a law of conservation of vehicles in the traffic context:

$$\frac{\partial \rho(x,t)}{\partial t} + \frac{\partial q(x,t)}{\partial x} = 0 \tag{2.4}$$

$$\rho_t + \left(\rho u(\rho)\right)_x = 0, \quad u(\rho) = u_{\max}\left(1 - \frac{\rho}{\rho_{\max}}\right), \quad 0 \le \rho \le \rho_{\max} \tag{2.5}$$

where

$\rho(x,t)$, denotes the traffic density in vehicles per lane per kilometre at location x and at time, t $q(x,t)$ the traffic flow in vehicles per hour at location x at time, t. The aggregated variables $\rho(x,t)$ and $q(x,t)$ are continuous functions of time and space; although the system they describe is intrinsically discrete (the cars on the motorway are discrete entities). Equation (2.5) is a 'physical law' in traffic.

The traffic flow $q(x,t)$ from equation (2.5) can be expressed in terms of the traffic density $\rho(x,t)$ and the traffic speed $u(x,t)$:

$$q(x,t) = \rho(x,t)u(x,t) \tag{2.6}$$

Lighthill and Whitham (1955) and also Richards (1956) observed that the average equilibrium speed of the vehicles is a function of the traffic density:

$$u(x,t) = F\left(\rho(x,t)\right) \tag{2.7}$$

The Lighthill-Whitham-Richards (LWR) model consists of the Eqn. (2.5) to Eqn. (2.7). This model is a continuum model in both time and space.

Although advances have been made in many directions, the LWR model is still widely used for the modelling of traffic flow, because of its simplicity and good explanatory power to understand the qualitative behaviour of road traffic. The results that are obtained from the LWR model are generally adequate for many applications such as traffic management and control problems.

This thesis focuses on investigating the LWR model and its usefulness to the traffic engineer and the development of an efficient solution method for this model.

The first-order LWR model can be solved either analytically or numerically. For the analytical aspects, the governing equation of the LWR model is solved by using the method of characteristics.

2.5 FIRST ORDER PARTIAL DIFFERENTIAL EQUATIONS

In mathematics, a first-order partial differential equation is a partial differential equation that involves only first derivatives of the unknown function of *n* variables. The equation takes the form (Evans, 1998).

$$F\left(x_1,\ldots,x_n,u,u_{x_1},\ldots,u_{x_n}\right)=0 \tag{2.8}$$

So the mathematical problem is to find the unknown from the equation and study its properties.

Partial Differential equations are of interest in mathematics and in modelling phenomena in the sciences, engineering, economics, ecology, and other areas

2.5.1 The Order of PDE

The order of a PDE is the order of the highest derivative present in the equation.

2.5.2 Linearity of PDE

The PDE is said to be linear if is linear in all the components (i.e., linear in the unknown functions and its partial derivatives) and is quasi-linear if is linear in the partial derivative of highest order.

Thus a general first and second order linear PDE, respectively will have the form (in two variables)

$$a(x, y)u_x + b(x, y)u_y + c(x, y)u + g(x, y) = 0 \tag{2.9a}$$

$$a(x, y)u_{xx} + b(x, y)u_{xy} + c(x, y)u_{yy} + d(x, y)u_x + e(x, y)u_y + f(x, y)u + g(x, y) = 0 \tag{2.9b}$$

where the coefficients of '*u*', partial derivatives and '*g*' are known functions.

2.6 TYPES OF PARTIAL DIFFERENTIAL EQUATIONS

Three of the most basic types of partial differential equations (PDEs) are hyperbolic, elliptic, and parabolic equations. For linear models, their representatives are the transport equation and the wave equation for the hyperbolic case, the Laplace equation for the elliptic case, and the heat equation for the parabolic case.

2.7 APPROACH TO STUDYING NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS

The most common basic approach to studying nonlinear partial differential equations is to change the variables (or otherwise transform the problem) so that the resulting problem is simpler (possibly even linear). Sometimes, the equation may be transformed into one or more ordinary differential equations.

Another common (though less mathematics) tactic, often seen in fluid and heat mechanics, is to use scale analysis to simplify a general, natural equation in a certain specific boundary value problem. For example, the nonlinear Navier-Stokes equations can be simplified into one linear partial differential equation in the case of transient, laminar, one dimensional flow in a circular pipe; the scale analysis provides conditions under which the flow is laminar and one dimensional and also yields the simplified equation. Other methods include examining the characteristics and using the methods outlined above for ordinary differential equations.

2.8 SOLUTION OF A PARTIAL DIFFERENTIAL EQUATION

A solution of a PDE, by definition, is a function u, which is continuously c^k differentiable satisfying equation. In all problems where a PDE appears, there are associated conditions, like boundary and/or initial values which are known already. In such cases the solution should satisfy these conditions as well.

In this work we will study the first order equations in detail. The interesting fact in this case is that it can be reduced to a system of ODE's, via the method of characteristics.

2.9 TRAFFIC MEASUREMENTS

Traffic flow parameters estimation is very important in the planning and design process for all aspects of the road network. This can be done in various ways. In the beginning of traffic research the only instruments used were a stopwatch and manual counts. Later, these restricted means were replaced with vehicle detectors, connected to registration equipment, which enables us to perform measurements over long periods of time. The processing of the data, in the early days done by hand, was automated when computers entered the measurement campaigns. These measurements revealed many parameters of the traffic stream. The characteristic property of most vehicle detectors is that the measurements are done at one point. The measurements are so-called “spot measurements”.

Another method to observe a traffic stream is to measure the behavior in a road section. Unfortunately the measurement of road section parameters is difficult. One of the methods, used in the early years of traffic measurement, is to make aerial photographs, and to determine the behaviour of the traffic stream from these photographs. The processing of these pictures however, is a time consuming procedure. It is not possible to use this method continuously to watch the traffic flow. Traffic Guidance Systems (2006)

In the 1950’s a method utilising a moving observer was suggested. Basically, it involves an observer in a moving vehicle that follows a road section in two directions. The properties of the traffic stream can be determined from a few easy measurements. The properties of the traffic stream can be determined from a few easy measurements. Wardrop and Charlesworth (1954)

CHAPTER THREE

METHODOLOGY

3.1 TRAFFIC STREAM

A traffic stream that operates free from the influence of such traffic control devices as traffic signals and stop signs is classified as uninterrupted flow (Highway Capacity Manual, 2000). This type of traffic flow is influenced primarily by roadway characteristics and the interactions of the vehicles in the traffic stream. Freeways, multilane highways, and two-lane highways often operate under uninterrupted flow conditions.

Traffic streams that operate under the influence of external means such as traffic signals and stop signs are classified as interrupted flow. Here vehicle-vehicle interactions and vehicle-roadway interactions play a secondary role in defining the traffic flow (Mannering *et al.*, 2005).

Understanding what type of flow is occurring in a given situation will lead to different methods for analysing traffic situations. In this paper, only uninterrupted flows are considered. Traditionally, uninterrupted traffic flows are modelled empirically: speed and flow data are collected for a specific road and econometrically fitted into curves Daganzo (1997).

3.2 FUNDAMENTAL TRAFFIC VARIABLES

Virtually all traffic flow models are based on the relationships between flow rate (q), density (ρ), and velocity (space mean speed), u. These three basic parameters describe the state of an uninterrupted traffic stream, primarily found on freeways and they form the underpinnings of traffic analysis.

In a free flowing network, traffic flow theory refers to the traffic stream variables of speed, density and flow (Henry Lieu, 1999).

3.2.1 Speed

The average speed is an important property of the traffic situation because it relates to safety, time, comfort and convenience (Cuthbert, 2006).). In traffic engineering, speed is defined as the distance travelled by a vehicle over a certain period of time. It is expressed as distance per unit of time or kilometre per hour. (Ergotmc @ GTRI Georgia Tech »http://ergotmc.gtri.gatech.edu/). It is one of the three basic characteristics of the traffic stream.

One way of measuring the speed of a car moving along the highway, is to record the velocity;

$$u_i = dx_i/dt \tag{3.1}$$

of each car. With N cars, there are different velocities, $u_i(t), i = 1, 2, 3, \cdots, N$, each depending on time, t. If the number of cars N are large, the speed of every individual vehicle is almost impossible to track on a roadway. So instead of measuring the velocity of each individual car, we associate to each point in space at each time a velocity field, $u(x,t)$. This would be the velocity measured by an observer fixed at position x at time, t. This velocity is the velocity of a car at that place (if a car is there at that time). Thus the velocity field $u(x,t)$ at the cars position $x_i(t)$ must be the cars velocity $u_i(t)$,

$$u(x_i(t),t) = u_i(t) \tag{3.2}$$

Freeway speed occurs when a single vehicle is operating at effectively zero density in the presence of no other vehicle. As said earlier it is quite impossible to measure the speed of every individual car and due to this average speed is taken into account.

Two different kinds of average speed can be distinguished; the time mean speed and the space mean speed.

3.2.1.1 Time mean speed

If speed is measured by keeping time as reference it is called time mean speed.

This is the arithmetic mean of the vehicle speeds observed at some particular point along the roadway. Time mean speed is an important factor used in the analysis of flow along a roadway segment which has homogeneous geometric and traffic characteristic with no interruptions.

Time-mean speed is expressed as

$$\bar{u}_t = \bar{u}_{spot} = (1/m)\sum_{i=1}^{m} u_i \tag{3.3}$$

where

$\bar{u}_t$ = time-mean speed in unit distance per unit time

u_i = the speed of the ith vehicle

m = represents the number of vehicles passing the fixed point

3.2.1.2 Space Mean Speed

If speed is measured by space reference it is called space mean speed.

The space mean speed is the average speed of the vehicles in a given road section Wardrop and Charlesworth (1954).

Space mean speed is determined on the basis of the time necessary for a vehicle to travel some known length of roadway. It account for both the segment length and travel time of that road section, and is computed by dividing the segment length by the mean or average travel time of a group of vehicles (HCM, 2000).

$$\bar{u}_s = \frac{l}{\bar{t}} \tag{3.4a}$$

where

$\bar{u}_s$ = space-mean space in unit distance per unit time

l = length of roadway used for travel time measurement of vehicles, and

$\bar{t}$ = average vehicle travel time, defined as

$$\bar{t} = \frac{1}{n}\sum_{i=1}^{n} t_i$$

where

t_i = time necessary for vehicle i to travel a roadway section of length l, and

n = number of measured vehicle travel time.

This implies

$$\bar{u}_s = \frac{l}{\frac{1}{n}\sum_{i=1}^{n} t_i} \tag{3.4b}$$

Space mean speed is an important factor in speed studies used for the analysis of flow along a roadway segment.

3.2.3 Volume

An observer fixed at a certain position along the roadway could measure the number of cars that passed in a given length of time.

Volume is simply the number of vehicles that pass a given point on the roadway in a specified period of time (Highway Capacity Manual, 2000). Volume is commonly converted directly to flow (q), which is a more useful parameter in traffic analysis.

3.2.4 Traffic Flow

Flow is traffic volume standardise to an hour, or vehicles per hour. The HCM, Transportation Research Board 2000 defines Flow rate (q) as the equivalent hourly rate at which vehicles pass over a given point of a roadway during a given time interval in less than one hour usually 15-minutes**.** Flow rate is given in terms of vehicles per hour.

3.2.5 Traffic Density

Density is defined as the number of cars (at a fixed time) occupying a given length of a roadway. (Mannering *et al*., 2005). The unit used is vehicles per unit distance or vehicles per kilometre

$$\rho = \frac{n}{l} \tag{3.5}$$

where

ρ = traffic density in vehicles per unit road length.

n = number of vehicles occupying some length of roadway at some specified time, and

l = length of the roadway

Density is a critical parameter for uninterrupted flow facilities because it characterizes the quality of traffic operations. It describes the proximity of vehicles to one another and reflects the freedom to manoeuvre within the traffic stream. Density measurement is a difficult task as it requires a photographing or videotaping a long segment and manually counting individual vehicles. Density is more commonly determined by dividing flow rate by speed, rather than performing a direct field measurement.

High densities indicate that individual vehicles are very close together, while low densities imply greater distances between vehicles.

3.2.6 Occupancy

The density is a fundamental traffic stream property, but can only be approximated by spot measurements, or it must be measured with complicated methods. Therefore several other approximations for the density are investigated. One of the commonly used properties is the occupancy.

Suppose we are able to measure the lengths of the vehicles in a road section. Then we can calculate the following ratio:

$$R_1 = \frac{\sum \text{vehicle lengths}}{\text{section length}} \tag{3.6}$$

When we divide this value by the average vehicle length, we have an approximation for the density. Although it is not possible to measure the vehicle lengths in a road section, it is possible to find an estimate for this value, using time measurements at a point. (Traffic Guidance Systems, 2006)

KNUST

3.3 FUNDAMENTAL RELATIONS OF TRAFFIC FLOW

There is a close relationship between the fundamental variables of traffic flow, namely Speed, flow, and density. The relationship between them is called the fundamental relations of traffic flow. The relationships between speed and density are not difficult to observe in the real world, while the effects of speed and density on flow are not quite as apparent.

Let's consider one of the simplest possible traffic situations. Suppose that on some road, cars are at a constant speed u with a constant density ρ. Since each car moves at the same speed, the distance between cars remains constant. Hence the traffic density does not change.

Consider an observer measuring the traffic flow (the number of cars per hour that pass him), then after a time τ the marked car initially at A, see **fig. 3.1** will cover a distance $\text{AB} = u\tau$. Of course during the same time interval the $\rho \cdot u\tau$ cars initially occupying AB will have moved on, passing B in the process. Thus by definition the number of cars per hour which we have called the flux past B is $q = \rho u\tau / \tau = \rho u$

If the traffic variables depends on x and t, i.e., $q(x,t), \rho(x,t), u(x,t)$, then under uninterrupted flow conditions, the three fundamental traffic variables speed, density, and flow are all related by the following equation:

$$q(x,t) = \rho(x,t)u(x,t) \tag{3.7}$$

This relationship represents the fundamental equation of traffic flow.

Because flow is the product of speed and density, the flow is equal to zero when one or both of these terms is zero. It is also possible to deduce that the flow is maximized at some critical combination of speed and density.

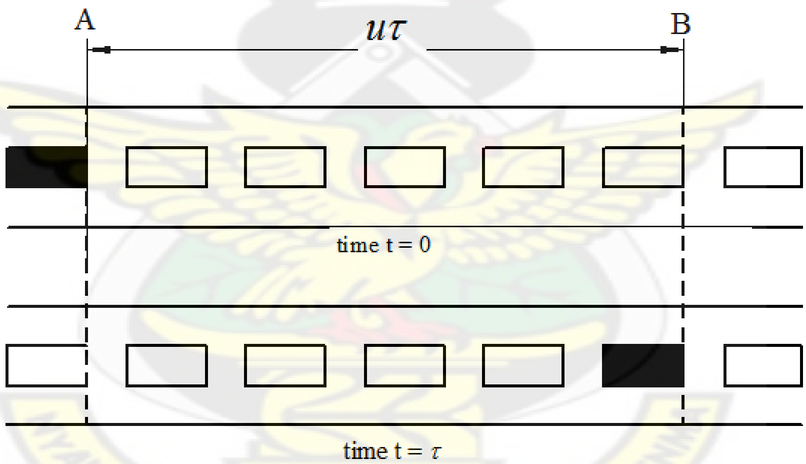


Figure 3. 1: The Flux, Density, Speed Relation

Source: Neville, 1994

3.4 FREEWAYS

The analysis of freeways is generally broken down into the major components of the freeway system including basic freeway segments, ramps and ramp junctions and weaving segments (HCM, 2000). In this work the analysis procedures used for the basic freeway segment is adopted.

3.5 BASIC FREEWAY SEGMENTS

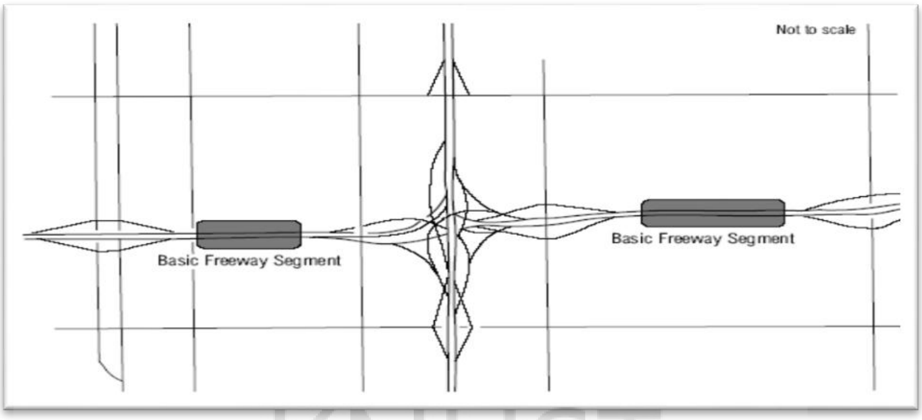


Figure 3.2: Basic Freeway Segments

Source: HCM, 2000

Basic freeway segments include the portions of freeway where traffic flow is not influenced by the diverging or merging movements near ramps/freeway connections. Partial Differential Equation (PDE) would be used for analysing basic freeway segment operations.

Within basic freeway sections, density is used to define the level of service. Density was selected as the parameter because it is sensitive to changes in flow throughout the range from zero to capacity.

3.6 METHODS OF ANALYSIS

When studying traffic patterns, there are many ways to model the flow of cars on the road. Based on the level of detail, traffic flow models have been categorized as macroscopic and microscopic models. But in relevance to the research at hand, we focus our attention on macroscopic model where we will analyse traffic situation resulting from the complex interaction of many vehicles, instead of studying individual cars.

3.7 CONTINUUM FLOW MODEL

Looking into traffic flow from a very long distance, the flow of fairly heavy traffic appears like a stream of a fluid or continuum fluid and also because traffic involves flows, density, and speeds, there is a natural tendency to attempt to describe traffic in terms of fluid behaviour. It seems therefore quite natural to associate traffic with fluid flow and treat it similarly.

In the fluid flow analogy, the traffic stream is treated as a one dimensional compressible fluid. This leads to two basic assumptions:

i. Traffic flow is conserved, which leads to the conservation or continuity equation, and
ii. There is a one-to-one relationship between speed and density or between flow and density.

3.8 THE GLOBAL CONSERVATION LAW

The development of continuum models of traffic flow began from the LWR theory presented by Lighthill and Whitham (1955) and independently by Richards (1956).

Considering a traffic flow on section of a road, if there is no on-ramp and off-ramp (i.e. no sinks or sources) within the interval, then the number of cars coming in equals the number of cars going out of the segment (conservation law).

We choose an interval on any particular roadway between say $x = x_1$ and $x = x_2$ as shown in **Fig 3.3.**

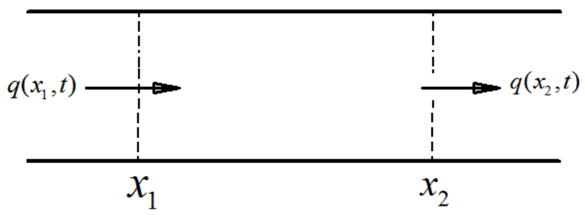


Figure 3.3: Cars entering and leaving a segment of roadway

Source: Richard Habeman, 1977

Our main aim is to take the statement that cars are conserved and turns it into a Partial Differential Equation (PDE). As said earlier we adopt a continuum model of traffic flow rather than modelling individual cars and their flow. We will assume the vehicles to be sufficiently numerous that they can be considered to be distributed continuously from x_1 to $x_2 \cdot$ Accordingly, we defined the continuous and differentiable function $\rho(x,t)$ to be the number of cars per unit length of the road at time t and position x and it is called the vehicle density. We also define the Flux of the vehicle, $q(x,t)$ as the number of cars passing a position x per unit time at time, $t \cdot$ The number of cars which are in the interval, (x_1, x_2) denoted by N can be computed from the sum of vehicles in the segment and is equal to.:

$$N(t) = \int_{x=x_1}^{x=x_2} \rho\left(x,t\right) dx \tag{3.8}$$

If more cars flow into the segment (x_1, x_2) than flow out of it, the number of cars within the segment will increase, and similarly if more cars flow out than in, the number of cars will decrease. Assuming no vehicles are created or destroyed within the segment then Mathematically the rate of change of the number of cars in the given segment of the road with respect to time, dN/dt, is equal to the difference between the cars entering and those leaving the section through its two ends, as illustrated in the equation below since the rate of change of the number of cars per unit time is the traffic flow at position x_1 minus the traffic flow at position x_2 both at time t:

$$\frac{dN}{dt} = q(x_1, t) - q(x_2, t) \tag{3.9}$$

Taking the derivative of both sides of equation (3.8) with respect to time gives the following:

$$\frac{dN}{dt} = \frac{d}{dt}\int_{x_1}^{x_2} \rho(x,t)\,dx \tag{3.10}$$

By equating equation (3.9) and equation (3.10), you get the result:

$$\frac{d}{dt}\int_{x_1}^{x_2} \rho(x,t)\,dx = q(x_1, t) - q(x_2, t) \tag{3.11}$$

This last result is the traffic conservation law in integral form.

In order to carry out thorough analysis of traffic, a conservation law in partial differential form is required.

Now, taking the partial derivative of the right hand side of equation (3.11) with respect to, x and then taking the integral from $x = x_2$ to $x = x_1$, gives the following equation:

$$\frac{d}{dt}\int_{x_1}^{x_2} \rho\left(x,t\right)dx = \int_{x_2}^{x_1} \frac{\partial q(x,t)}{\partial x}dx \tag{3.12}$$

To have the integral with the same interval, we need to use an integral property, which is to take the negative of the right hand side of equation (3.12):

$$\frac{d}{dt}\int_{x_1}^{x_2} \rho\left(x,t\right)dx = -\int_{x_1}^{x_2} \frac{\partial q(x,t)}{\partial x}dx \tag{3.13}$$

Moving the negative sign inside of the integral gives:

$$\frac{d}{dt}\int_{x_1}^{x_2} \rho\left(x,t\right)dx = \int_{x_1}^{x_2} -\frac{\partial q(x,t)}{\partial x}dx \tag{3.14}$$

We can now move the d/dt inside of the integral to get the following equation; we can do this because derivatives and integrals are interchangeable. If you move the derivative inside the integral and it has a function of two variables, then the derivative becomes a partial derivative:

$$\int_{x_1}^{x_2} \frac{\partial}{\partial t}\rho\left(x,t\right)dx = \int_{x_1}^{x_2} -\frac{\partial q(x,t)}{\partial x}dx \tag{3.15}$$

Equation (3.15) implies:

$$\int_{x_1}^{x_2} \left(\frac{\partial}{\partial t}\rho(x,t) + \frac{\partial q(x,t)}{\partial x}\right)dx = 0 \tag{3.16}$$

This equation states that the definite integral of some quantity is always zero for all values of the independent varying limits of the integral. The only function with this feature is the zero function. Therefore, assuming $\rho(x, t)$, and $q(x, t)$ are both smooth, the **1-D conservation law** is found to be

$$\frac{\partial}{\partial t}\rho(x,t)+\frac{\partial q(x,t)}{\partial x}=0 \tag{3.17a}$$

And from equation (3.17a) we get

$$\frac{\partial \rho(x,t)}{\partial t}+\frac{\partial q(x,t)}{\partial x}=0 \tag{3.17b}$$

Suppressing dependencies, Equation (3.17b), becomes:

$$\frac{\partial \rho}{\partial t}+\frac{\partial q}{\partial x}=0 \tag{3.18}$$

This PDE describes the conservation of vehicle on a simple road. Its major underlying assumption is that no vehicles are created or destroyed (or join/leave the road if junctions are present) between two elementary points on the road.

If sinks or source exist within the section of the roadway, then the conservation equation takes the more general form

$$\frac{\partial \rho}{\partial t}+\frac{\partial q}{\partial x}=g(x,t) \tag{3.19}$$

where $g\left(x,t\right)$ is the generation (or dissipation) rate in vehicle per unit time per length.

We know from above that, for vehicle traffic flow, the flow is given by equation (3.7), and so therefore we can rewrite the $\partial q/\partial x$ as the following:

$$\frac{\partial q}{\partial x} = \frac{\partial}{\partial x}(\rho u) \quad . \tag{3.20}$$

Equation (3.18) then becomes

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} = 0 \tag{3.21}$$

This is another form of the conservation equation.

3.9 A VELOCITY-DENSITY RELATIONSHIP

Again let's consider a section of the highway with only a single vehicle on it. Under this condition the density will be very low and, the vehicles can move freely without hindrance from the other vehicles, (Wardrop and Charlesworth, 1954). As more and more vehicles begin to use the segment of highway, the traffic density will increase and the average operating speed of vehicle will decline from the free flow value due to the presence of other cars. With all of these types of observations, we can make a simplifying assumption that at any point along the road the velocity of a car only depends on the density of cars (Lighthill *et al.*, 1955).

$$u = u(\rho) \tag{3.22}$$

3.10 FUNDAMENTAL DIAGRAMS OF TRAFFIC FLOW

The fundamental diagrams of traffic flow are vital tools which enables analysis of fundamental relationships. The relation between flow and density, density and speed, speed and flow, can be represented with the help of some curves. They are referred to as the fundamental diagrams of traffic flow. There are three diagrams; speed-density, speed-flow and flow-density.

All the graphs are related by the equation “flow = speed * density”; this equation is the essential equation in traffic flow. The fundamental diagrams were derived by the plotting of field data points and giving these data points a best fit curve. With the fundamental diagrams road engineers can explore the relationship between speed, flow, and density of traffic.

3.11 ELEMENTARY TRAFFIC FLOW MODEL

As shown above, in general the car velocity is a decreasing function of density Thomson (1967) as cited in Lighthill *et al.* (1955). The graph of this function estimated in the literature approximate the downward sloping of this relationship. One possible representation of the process is the linear relationship shown in **Fig. 3.4**.

The simplest macroscopic stream model developed by Greenshield (Greenshied, 1935) in which density and speeds are negatively linearly related is given by

$$u(\rho) = u_{\max}\left(1 - \frac{\rho}{\rho_{\max}}\right) \tag{3.23}$$

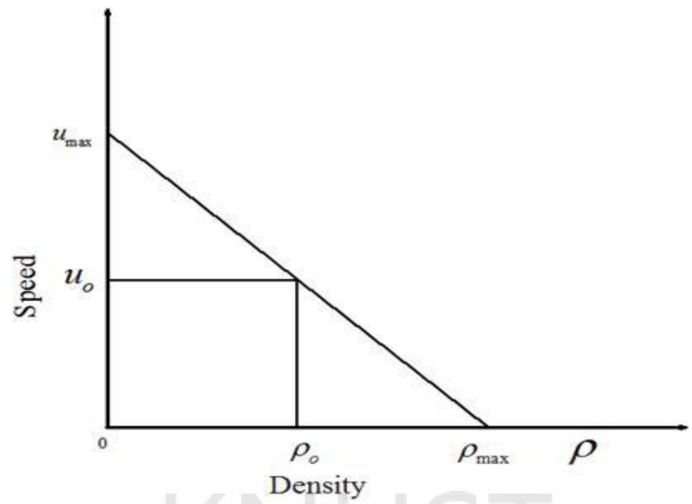


Figure 3. 4: Illustration of a typical linear speed-density relationship

Source: Mannering, 1998

As mentioned above, free-flow speed occurs when a single vehicle is operating at effectively zero density in the presence of no other traffic (i.e., when there are very little or no cars at all on the road with them).

So the velocity will be at a maximum when the density is zero or the vehicle flowing with the free flow speed as illustrated with equation (3.23)

$$u(0) = u_{\max} \tag{3.24}$$

As more and more cars join the road way, their presence will slow down the car, and as the density increases further, the velocity of the cars would decrease linearly from free-flow speed to zero speed. Thus the rate of change, which is the derivative of the velocity with respect to density, is defined as below:

$$\frac{du}{d\rho} \equiv u'(\rho) \leq 0, \qquad \rho > 0 \tag{3.25}$$

That is traffic speeds tend to go down with increasing traffic density.

At a certain density highly congested roadways result in minimal traffic movement, and then cars will move at zero velocity, or stand still. This maximum density $\rho_{\max}$ usually corresponds to what is called bumper to bumper traffic.

$$u(\rho_{\max}) = 0 \tag{3.26}$$

Therefore the car velocity vs. the traffic density is steady decreasing. Optimum speed occurs between free-flow speed and zero speed and optimum density occurs at optimum speed. As illustrated in **Fig 3.4.**

3.12 BASIC FLOW RELATIONSHIP

3.12.1 Flow-Density Model

Since the traffic flow (the number of car per hour) equal density times velocity, $q = \rho u$ and $u = u(\rho)$

$$q = \rho u(\rho) \tag{3.27}$$

Thus using the assumption of a linear speed-density relation, a parabolic flow-density model can be obtained by substituting Eqn. (3.23) into the Eqn. (3.27)

This gives

$$q(\rho) = \rho u = \rho u_{\max}\left(1 - \frac{\rho}{\rho_{\max}}\right) \tag{3.28}$$

$$q(\rho) = u_{\max}\left(\rho - \frac{\rho^2}{\rho_{\max}}\right) \tag{3.29}$$

where all terms are as defined before.

Thus, in general, the traffic flow dependence on the density is illustrated in **Fig 3.5.**

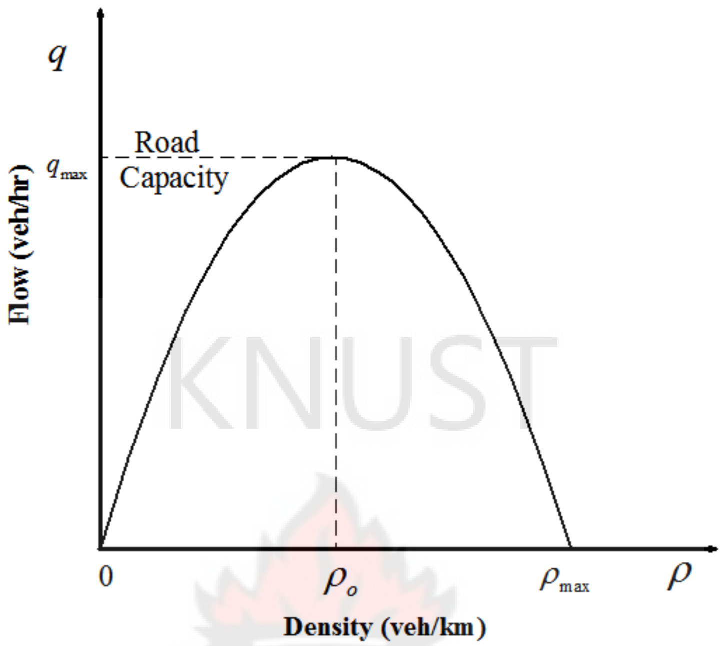


Figure 3.5: illustration of parabolic Flow – Density relationship
Source: Mannering, 1998

This curve has a characteristic shape that is the same for every motorway section and is known in the traffic literature as the "fundamental diagram". (Bellemans, 2002 as cited in Greenshields, 1935).

Note from **Fig. 3.5** that the maximum flow rate q_{max} represents the highest rate of traffic flow that the highway is capable of handling. This is referred to as traffic flow at capacity. The traffic density that corresponds to this capacity flow rate, the optimum density is ρ_0 and the corresponding speed, the optimum speed is u_0. To determine the density, ρ_0 and the speed, u_0 at which the flow or volume is maximum we differentiate Eqn. (3.29) because at maximum flow

$$\frac{dq}{d\rho} = u_{\max}\left(1 - \frac{2\rho}{\rho_{\max}}\right) = 0 \tag{3.30}$$

And because the free-flow speed $u_{\max}$ is not equal to zero

$$\rho = \rho_0 = \frac{\rho_{\max}}{2} \tag{3.31}$$

KNUST

Substituting Eqn. (3.31) into Eqn. (3.23)

$$u_0 = u_{\max}\left(1 - \frac{\rho_{\max}/2}{\rho_{\max}}\right) \tag{3.32}$$

$$u_0 = \frac{u_{\max}}{2} \tag{3.33}$$

And putting Eqn. (3.31) and (3.33) into $q = \rho u$ gives the maximum flow

$$q_{\max} = \frac{u_{\max}\rho_{\max}}{4} \tag{3.34}$$

3.12.2 Speed-Flow Model

Again returning to linear speed-density model Eqn. (3.23), a corresponding speed-flow model can be developed by rearranging to obtain

$$\rho = \rho_{\max}\left(1 - \frac{u}{u_{\max}}\right), \tag{3.35}$$

and by substituting Eqn. (3.35) into $q = \rho u$ and simplifying we obtain

$$q(u) = \rho_{\max}\left(u - \frac{u^2}{u_{\max}}\right) \tag{3.36}$$

This speed- flow model defined by Eqn. (3.36) again gives a parabolic function, as shown in **Fig.3.6**

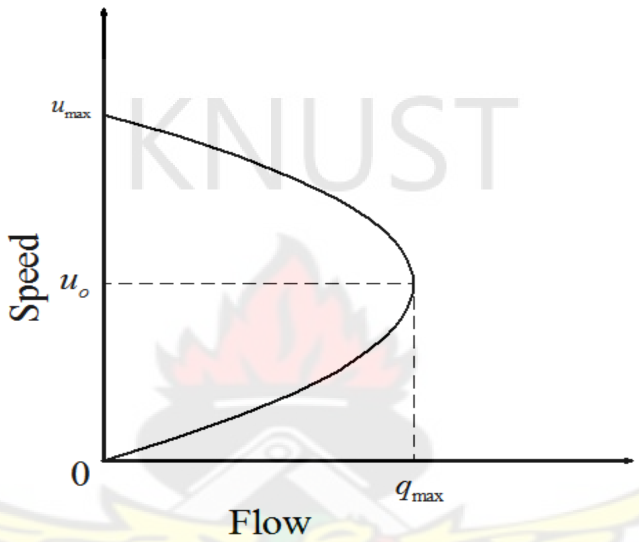


Figure 3.6: illustration of parabolic Speed – Flow relationship

Source: Mannering, 1998

3.13 VELOCITY AS A FUNCTION OF DENSITY

The equation we now have for vehicle conservation, Equation (3.21) is one relation involving two unknowns. Conventionally, we would need another relation to close the system in two unknowns. A major assumption that is often made by traffic modellers is that velocity may be reasonably assumed to be a function of the density alone. That is, we can assume $u = u(\rho)$ and our equation becomes a relation in ρ and its derivatives:

$$\frac{\partial \rho}{\partial t} + \frac{\partial\left(\rho u(\rho)\right)}{\partial x} = 0 \tag{3.37}$$

Such an equation is called a partial differential equation (PDE) of first order.

It is a PDE because of the two variables involved, *x*, *t*, and the partial differentiations with respect to these variables. It is a first-order equation because only first partials are involved. But we know that $q(\rho) = \rho u(\rho)$. Then the Eqn. (3.37) can be written

$$\frac{\partial \rho}{\partial t} + q'(\rho)\frac{\partial \rho}{\partial x} = 0 \qquad (3.38)$$

where the prime denotes the differentiation of q with respect to ρ.

This first order nonlinear partial differential equation for the traffic density contains both the driver behaviour information and the conservation information required to determine how density changes occurs in traffic. (Fowkes and Mohany, 1994).

The first term $\partial\rho/\partial t$ represents the change in the traffic density at fixed position, whiles $q'(\rho)\partial\rho/\partial x$ represents the change due to the fact that the observer moves in to a region of possibly different traffic density.

3.14 METHOD OF CHARACTERISTICS

In mathematics, the method of characteristics is a common method for solving initial value problem (IVP). Notable among them is traffic flow problem. (Metcalf, 2006). The method is to reduce a partial differential equation to a family of ordinary differential equations along which the solution can be integrated from some initial data given. It is a technique for solving partial differential equations. Typically, it applies to first-order equations, although more generally the method of characteristics is valid for any hyperbolic partial differential equation (John, 1991).

3.15 CHARACTERISTICS OF FIRST-ORDER PARTIAL DIFFERENTIAL EQUATIONS

The general theory of how to deal with partial differential equations of first order was developed by Cauchy as cited by Carrier and Pearson (1976).

For a first-order PDE, the method of characteristics discovers curves (called characteristic curves) along which the PDE becomes an ordinary differential equation (ODE). Once the ODE is found, it can be solved along the characteristic curves and transformed into a solution for the original PDE (Hood, 2000).

The general characteristics can be obtained by analyzing the conservation of traffic partial differential equation in Eqn. (3.38), as done by Coleman (2008).

Consider a function $\rho(x,t)$ satisfying a first order linear PDE of the form

$$\frac{\partial \rho}{\partial t} + q'(\rho)\frac{\partial \rho}{\partial x} = 0 \tag{3.39}$$

We want to transform this linear first order PDE into ODE along the appropriate curve.

Assume the curve is identified with the graph of a function $x = x(t)$ and let $\rho(x(t),t)$ be the value of the solution along it. We compute the rate of change in the solution along the curve by differentiating $\rho(x(t),t)$ with respect to t. Invoking the multi-variable chain rule.

$$\frac{d}{dt}\rho(x(t),t) = \frac{\partial \rho}{\partial t} + \frac{dx}{dt}\frac{\partial \rho}{\partial x} \tag{3.40}$$

In particular, if $x(t)$ satisfies

$$\frac{dx}{dt} = q'(\rho) \tag{3.41a}$$

Then

$$\frac{d}{dt}\rho(x(t),t) = \frac{\partial \rho}{\partial t} + q'(\rho)\frac{\partial \rho}{\partial x} = 0 \tag{3.41b}$$

So, along the characteristic line $(x(t),t)$, the original PDE become the ODE

$$\frac{d}{dt}\rho(x(t),t) = F(\rho, x(t), t) = 0 \tag{3.42}$$

Since the derivative is zero, the solution $\rho(x(t),t)$ must be a constant. That is to say that along the characteristics, the solution is constant.

Thus $\rho(x_t, t) = \rho(x_0, 0)$

Where $\rho(x_t, t)$ and $\rho(x_0, 0)$ lie on the same characteristic.

So to determine the general solution, it is enough to find the characteristics by solving the characteristic system of ODEs

$$\frac{dx}{dt} = q'(\rho), \tag{3.43}$$

In the (x,t) plane.

We can now determine $\rho(x,t)$, given the traffic conditions at initial time.

One can even graphically construct the answer in the following way as shown in **Fig.3.7**

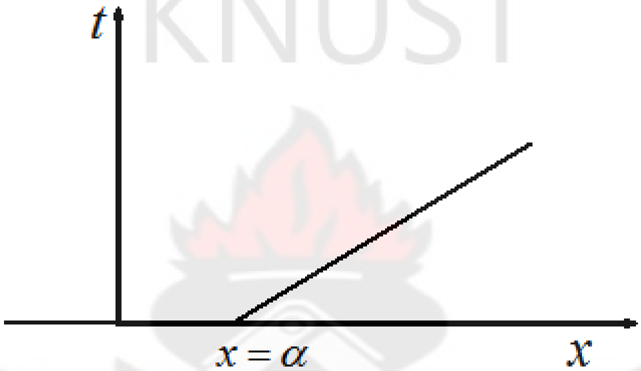


Figure 3.7 : Characteristic initially at $x = \alpha$

At each point $x = \alpha$ along the $t = 0$ axis draw a straight line with the required characteristic slope $q'(\rho(\alpha,0))$. Along this line, ρ retains the value $\rho(\alpha,0)$. A corresponding analytic description is given by

$$\rho(x,t) = \rho(\alpha,0) \text{ along } x = q'(\rho(\alpha,0))t + \alpha \tag{3.44}$$

This characteristic solution is ideal for graphical representation.

3.16 INITIAL AND BOUNDARY CONDITIONS

Formulation of boundary conditions is very crucial to correctly solving the governing equation. Solution of the governing partial differential equation (Eqn. (3.39) starts with calculations at a specified time. At this time, the flow conditions should be known. These flow conditions at the initial time are termed as the "initial conditions"

Any physical system has finite boundaries. For a one-dimensional flow, there will be a boundary at $x = 0$ and another boundary at some distance $x = x_c$.

In the solution of governing equations, one has to specify one or more conditions at the boundaries of the system. These are known as the "boundary conditions".

3.17 TRAFFIC DATA COLLECTION

Unlike many other disciplines of the engineering, the situations that are interesting to a traffic engineer cannot be reproduced in a laboratory. Even if road and vehicles could be set up in large laboratories, it is impossible to simulate the behavior of drivers in the laboratory. Therefore, traffic stream characteristics need to be collected only from the field. There are several methods of data collection depending on the need of the study (Mathew, 2007; Traffic Guidance Systems, 2006).

3.18 MEASUREMENTS AT ONE POINT

Traffic theory started in the early days of traffic research with data, measured at a fixed point besides the road, because the existing measurement equipment offered only those data. Up to now this method provides the major part of the traffic measurements. The simplest measurements are counts. Normally, data will be collected for short interval of 5 minutes, 15 minutes or 1 hour etc., although the total measurement duration covers a larger period. The properties of the traffic stream are often presented as average values. Therefore averaging techniques was employed.

Density, which is defined as vehicles per unit length, does not make sense for a point measurement, because no length is involved. (Traffic Guidance Systems, 2006; May *et al*., 1963; Athol, 1965).

3.19 MEASUREMENTS ON A ROAD SECTION

According to Mathew (2007) measurement on a road section is normally used to obtain variations in speed over a stretch of road. We can also get density.

Vehicles on a road section can be observed by using aerial photographs or by using video-image processing. When distance markers are available on the road, the number of vehicles in a section can be counted, giving the density. From a single frame on the road the number of vehicles in a section can be counted, given the density, but not speed or volumes. By taking two images with a short interval, the speed of the vehicles can be determined by the distance covered between the two frames and the time interval between them. These properties are sufficient to compute the flow rate. (Payne and Tignor, 1978; Collins, 1983).

If N is the number of vehicles in the observed road section, and the length of the section is L, the density follows from:

$$\rho = \frac{N}{L} \tag{3.56}$$

When two images are taken with a time difference of Δt, and the displacement of vehicle i is s_i, the space mean speed can be computed with:

$$u_i = \frac{s_i}{\Delta t} \quad \Rightarrow \bar{u}_s = \frac{1}{N}\sum_{i=1}^{N} u_i = \frac{1}{N}\sum_{i=1}^{N}\frac{s_i}{\Delta t} = \frac{1}{N \cdot \Delta t}\sum_{i=1}^{N} s_i \tag{3.57}$$

The flow rate can be found with the well-known formula: $q = \rho \cdot \bar{u}_s$

$$q = \frac{\sum_{i=1}^{N} s_i}{L \cdot \Delta t} \tag{3.58}$$

NB: Theoretically these formulas are only valid when the length of the road section approaches infinity.

3.20 MOVING OBSERVER MEASUREMENT

The moving observer method was developed in the UK by the Road Research Laboratory (Traffic and Safety Division) and was first described in a paper by Wardrop and Charlesworth (1954). Their method involved a series of runs in a test vehicle made travelling 'with' and 'against' a one-way traffic stream. O'Flaherty and Simons (1970).

The observer(s) in the test vehicles record the following information for each run:

The number of opposing vehicles met;

The number of vehicles overtaking the test vehicle while it was travelling;

The number of vehicles the test vehicle overtook;

The journey times of the observer, with and against the stream.

These observations form the basis for the estimate of the traffic flow rate. Equations were derived enabling the traffic flow rate to be calculated from the collected information.

Moving observer method of measurement is the most commonly used method to get the relationship between speed, density and traffic flow data by a single experiment Mathew (2007).

KNUST

3.21 THEORY

Consider an observer watching a stream of vehicles. Two different cases of motion can be considered, Wardrop and Charlesworth (1954). The first case considers the traffic stream to be moving and the observer to be stationary. If m_0 is the number of vehicles overtaking the observer during a time period, t, then the flow q is $\frac{m_0}{t}$, or

$$m_0 = q \cdot t \tag{3.59}$$

The second case assumes that the stream is stationary and the observer moves with speed u_0. If m_p is the number of vehicles overtaken by observer over a length l travelled by the observer then by definition, density ρ is $\frac{m_p}{l}$, or

$$m_p = \rho \cdot l \tag{3.60a}$$

or

$$m_p = \rho \cdot u_0 \cdot t \tag{3.60b}$$

Where u_0 is the speed of the observer and t is the time taken for the observer to cover the road stretch.

These two cases are now merged into a situation where both the stream and the observer are moving in the same direction, but at different speeds. Obviously some cars will overtake the observer and he will in turn over take some of the cars.

On the trip in the same direction as the observed flow, the observer adds up the cars that pass the observer, and subtracts the cars that are passed by the observer. On the trip against the observed flow, the observer counts the cars that pass the observer in the opposite direction. Wardrop and Charlesworth (1954). If the length of the road section is L. This can be represented mathematically.

Assuming m_0 vehicles overtake the observer, and assuming m_p is the number of vehicles the observer passes, then, using Eqn. (3.59) and Eqn. (3.60b) we can describe the difference between m_0 and m_p called the tally counts as:

$$m = m_0 - m_p \tag{3.61a}$$

From above

$$m = qt - \rho u_0 t \tag{3.61b}$$

This equation is the basic equation of moving observer method, which relates q and ρ to the counts, m_w, t and u_0 that can be obtained from the test.

Assume that the trip against the observed flow yields a traveling time t_a and a vehicle count is m_a,

$$m_a = qt_a + \rho u_a t_a \tag{3.62}$$

and the trip in the same direction of the observed stream has a traveling time t_w and a vehicle count m_w (i.e. those passing minus those overtaken-Tally counts).

$$m_w = qt_w - \rho u_w t_w \tag{3.63}$$

Manipulating Eqn. (3.62) and Eqn. (3.63), the following formulas can be found (refer to Appendix A3.1 for derivation):

$$q = \frac{m_a + m_w}{t_a + t_w} \tag{3.64}$$

$$t_{av} = t_w - \left(\frac{m_w}{q}\right) \tag{3.65}$$

$$u_{av} = \frac{l}{t_{av}} \tag{3.66}$$

$$\rho = \frac{q}{v_{av}} \tag{3.67}$$

These formulae allow one to estimate both speeds and flows for one direction of travel.

where,

q is the estimated flow on the road in the direction of the stream.

m_a is the number of vehicles met in the opposite direction while traveling against the direction of interest,

m_w is the net number of vehicles that overtake the survey vehicle while traveling in the direction of interest (i.e. those passing minus those overtaken),

t_a is the travel time taken for the trip against the stream,

t_w is the estimate of mean travel time for the trip in the direction of the stream.

3.22 REGRESSION ANALYSIS

3.22.1 Linear Regression

The goal of regression analysis is to model the expected value of a dependent variable *y* in terms of the value of an independent variable (or vector of independent variables) *x*. In simple linear regression, the model

$$y = \beta_0 + \beta_1 x + \varepsilon , \tag{3.68}$$

is used, where ε is an unobserved random error with mean zero conditioned on a scalar variable *x*. In this model, for each unit increase in the value of *x*, the conditional expectation of *y* increases by a_1 units.

In many settings, such a linear relationship may not hold. In that case, we might propose a quadratic model of the form

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \varepsilon \qquad (3.69)$$

In this model, when there is an increased from x to x + 1 unit, the expected yield changes by $\beta_1 + 2\beta_2 x$. The fact that the change in yield depends on x is what makes the relationship nonlinear (this must not be confused with saying that this is nonlinear regression; on the contrary, this is still a case of linear regression).

In general, we can model the expected value of y as an nth order polynomial, yielding the general polynomial regression model

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \ldots + \beta_m x^m + \varepsilon \qquad (3.70)$$

Conveniently, these models are all linear from the point of view of estimation since the regression function is linear in terms of the unknown parameters $\beta_0, \beta_1, \beta_2, \ldots$ Therefore, for least squares analysis, the computational and inferential problems of polynomial regression can be completely addressed using the techniques of multiple regressions. This is done by treating $x, x^2 \ldots$ as being distinct independent variables in a multiple regression model.

3.22.2 Polynomial Regression

In statistics, polynomial regression is a form of linear regression in which the relationship between the independent variable x and the dependent variable y is modelled as an nth order polynomial. Polynomial regression fits a nonlinear relationship between the value of x and the corresponding conditional mean of y, denoted E(y|x),. Although polynomial regression fits a nonlinear model to the data, as a statistical estimation problem it is linear, in the sense that the regression function E(y|x) is linear in the unknown parameters that are estimated from the data. For this reason, polynomial regression is considered to be a special case of multiple linear regressions.

Polynomial regression models are usually fit using the method of least squares. The least-squares method minimizes the variance of the unbiased estimators of the coefficients, under the conditions of the Gauss–Markov theorem. The least-squares method was published in 1805 by Legendre and in 1809 by Gauss. The first design of an experiment for polynomial regression appeared in an 1815 paper of Gergonne. In the twentieth century, polynomial regression played an important role in the development of regression analysis, with a greater emphasis on issues of design and inference. More recently, the use of polynomial models has been complemented by other methods, with non-polynomial models having advantages for some classes of problems.

3.23 MATRIX FORM AND CALCULATION OF ESTIMATES

The polynomial regression model

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \ldots + \beta_m x^m + \varepsilon \quad (i = 1, 2, \ldots, n) \tag{3.71}$$

can be expressed in matrix form in terms of a design matrix X, a response vector $\vec{y}$, a parameter vector $\vec{\beta}$, and a vector ε of random errors. The ith row of X and $\vec{y}$ will contain the x and y value for the ith data sample. Then the model can be written as a system of linear equations:

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} 1 & x_1 & x_1^{\ 2} \dots & x_1^{\ m} \\ 1 & x_2 & x_2^{\ 2} \dots & x_2^{\ m} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_n & x_n^{\ 2} \dots & x_n^{\ m} \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \vdots \\ \beta_m \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{pmatrix} \tag{3.72}$$

which when using pure matrix notation is written as

$$Y = X\vec{\beta} + \varepsilon \tag{3.73}$$

The vector of estimated polynomial regression coefficients (using ordinary least square estimation is

$$\hat{\vec{\beta}} = \left(X^T X\right)^{-1}\left(X^T Y\right) \tag{3.74}$$

This is the unique least squares solution as long as X has linearly independent columns.

CHAPTER FOUR

DATA ANALYSIS AND MODELLING

4.1 DATA COLLECTION

The data were collected on March 22, 2011 during good weather. The field data were collected during evening peak period. Peak hour was taken generally from 5.30 p.m. to 6.30 p.m., where the civil servants and traders from the Central Business District (CBD) areas are going back home. The traffic count was done manually using the Moving Car Observer method (MCO) along 1.2 Km way of Kumasi-Accra road (Ghana) between (KNUST Police Station and Boadi Junction). The site is a level grade straight segment. In all a one hour data were collected for the study using the Moving Car Observer method.

Basically in the moving car observer method, observer (s) in a moving vehicle travelling along a known section of road in both direction records the number of vehicles met in the opposite direction (m_a), number of vehicles passing the test car (m_o), number of vehicles overtaken by test car (m_p) and the journey time of the test vehicle in both directions is also noted (t_w, t_a). Therefore, based on the Wardrop and Charlesworth formula the number of vehicles per hour (q), number of vehicles per length (l) of roadway (ρ), mean space speed (u), and average journey time (t_{av}) is equal:

$$q = \frac{m_a + m_o - m_p}{t_w + t_a} \qquad t_{av} = t_w - \frac{m_o - m_p}{q} \qquad u = \frac{l}{t_{av}} \qquad \rho = \frac{q}{u}$$

In all seven (7) runs by the observer's vehicle was done to collect the data and the results are presented in Tables 4.1 and Tables 4.2.

TABLE 4. 1: Vehicle Stream Data

Sample No	Start time (pm)	Journey Time (min)	No of Vehicles Met m_a	No of Vehicles that Overtakes the test car m_o	No of Vehicles Overtaken by the test car m_p
1	5:30	1.11		0	0
	5:32	1.28	34		
2	5:34	1.13		3	3
	4:36	1.30	49		
3	5:38	1.16		1	2
	5:40	1.23	54		
4	5:42	1.19		0	2
	5:44	1.27	60		
5	5:45	1.25		0	4
	5:47	1.30	62		
6	5:50	1.46		1	5
	5:53	1.34	63		
7	5:55	2.00		0	10
	5:58	1.3	55		

TABLE 4. 2: Typical Analysed Manual Data Count

No	No of Vehicles overtaking the test Car m_o	No of Vehicles overtaken by the test Car m_p	Relative Flow against test car m_a	Relative Flow with test car $m_w = m_0 - m_p$	Travelling time against The flow t_a	Travelling time With the flow t_w	Flow rate (veh/hr) $q = 60\left(\frac{m_a + m_w}{t_a + t_w}\right)$	Average time (min) $t_{av} = t_w - 60\left(\frac{m_w}{q}\right)$	Speed (km/hr) $u = \frac{l}{t_{av}}$	Density (veh/km) $\rho = \frac{q}{u}$
1	0	0	34	0	1.28	1.11	853.56	1.11	64.86	13.16
2	3	3	49	0	1.3	1.13	1209.88	1.13	63.72	18.99
3	1	2	54	-1	1.23	1.16	1330.54	1.21	59.75	22.27
4	0	2	60	-2	1.27	1.19	1414.63	1.27	56.48	25.05
5	0	4	62	-4	1.31	1.25	1359.38	1.43	50.47	26.93
6	1	5	63	-4	1.34	1.46	1264.29	1.65	43.64	28.97
7	0	10	55	-10	1.3	2	818.18	2.73	26.34	31.06

From the data shown in the Table 4.2 above, the results of stream flow for KNUST Police Station – Boadi junction are summarized in Table 4.3

Table 4. 3: Stream flow of road section on Kumasi-Accra road

No	Relative flow With test car	Relative flow Against test car	Travelling time Against Flow	Travelling time with Flow	Flow rate	Average time	Speed	Density
1	0	34	1.28	1.11	853.56	1.11	64.86	13.16
2	0	49	1.3	1.13	1209.88	1.13	63.72	18.99
3	-1	54	1.23	1.16	1330.54	1.21	59.75	22.27
4	-2	60	1.27	1.19	1414.63	1.27	56.48	25.05
5	-4	62	1.31	1.25	1359.38	1.43	50.47	26.93
6	-4	63	1.34	1.46	1264.29	1.65	43.64	28.97
7	-10	55	1.3	2	818.18	2.73	26.34	31.06

4.2 FUNDAMENTAL RELATIONSHIPS ANALYSIS

In this section, fundamentals of the data were studied. A detailed description of the data is presented. In addition, procedures for computing traffic flow measures are explained. Finally, traffic conditions of the data and the relationships among the traffic condition parameters are studied.

The average rate of flow ranged from 818.18 to 1414.63 vehicles per hour; the average traffic density ranged from 13.16 to 31.06 vehicles per kilometre; and the average travel speed ranged from 26.34 to 64.86 kilometres per hour. The rate of flow and density increased with time during the study period whiles the speed decreased.

The pair-wise relationship curves among the rate of flow, density, and speed closely matches the basic relationship curves among these three parameters within the range of the data. The result supports the adequacy of the data.

Before a regression model was built and developed for analyzing the basic freeway segment performance, the empirical relationships between traffic speed, rate of flow and density were examined to gain a better understanding of the operational characteristics of the basic freeway section.

KNUST

4.2.1 Speed-Density Relationships

Relationships between speed and density were examined. Speeds and densities were calculated from the field data over the length of the freeway section.

TABLE 4.4: Model Summary

R	R Square	Adjusted R Square	Std. Error of the Estimate
.858	.736	.684	7.659

The independent variable is Density.

TABLE 4. 5: ANOVA of Speed-Density Function

	Sum of Squares	df	Mean Square	F	Sig.
Regression	819.864	1	819.864	13.975	.013
Residual	293.331	5	58.666		
Total	1113.195	6			

The independent variable is Density.

For this model F-ratio is 13.975, which is very unlikely to have happened by chance (p<.001). therefore the model significant at 5%

TABLE 4. 6: Coefficients of Speed-Density Function

	Unstandardized Coefficients		Standardized Coefficients	T	Sig.
	B	Std. Error	Beta		
Density	-1.890	.506	-.858	-3.738	.013
(Constant)	97.113		12.363	7.855	.001

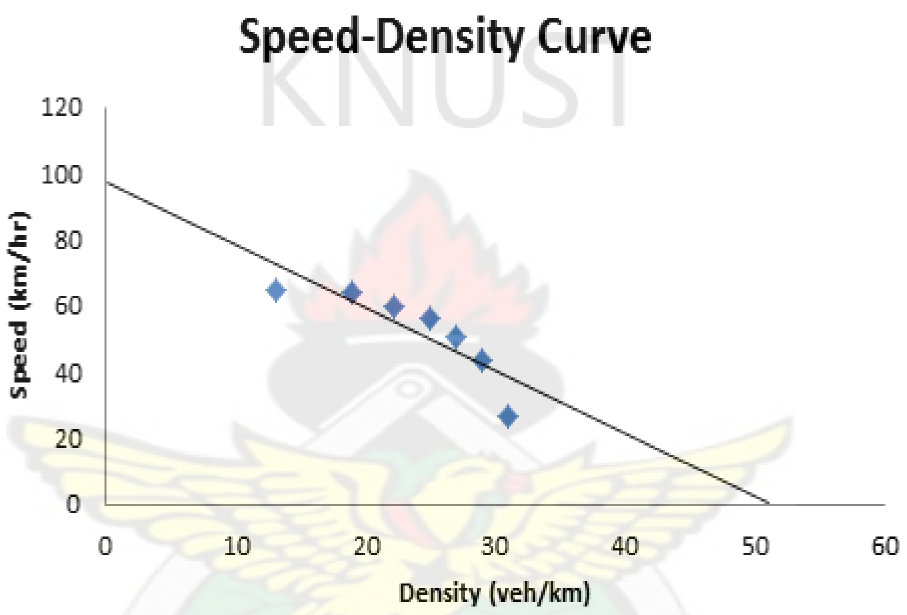


Figure 4. 1: Result of Regression for Average Speed versus Average Density.

Figure 4.1 presents the relationships between the average speed and average density over the observed periods. This it significantly from most speed-density models in which the average speed generally decreases linearly with the increase of density.

4.2.2 Rate of flow - Density Relationships

Relationships between density and rate of flow were also examined. Densities were obtained from the flow rate and the velocities. The following results were obtained.

TABLE 4.7: Model Summary[a]

R	R Square	Adjusted R Square	Std. Error of the Estimate
.992	.983	.976	184.531

The independent variable is density.

a. The equation was estimated without the constant term.

TABLE 4. 8: ANOVA[a] of Flow-Density Function

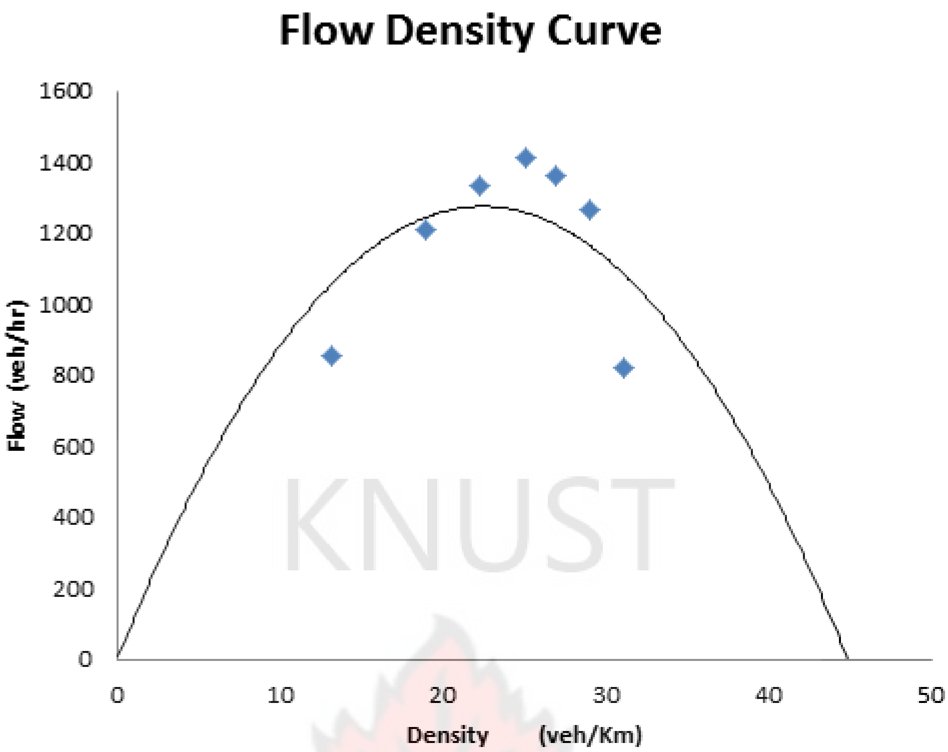
	Sum of Squares	Df	Mean Square	F	Sig.
Regression	9909379.543	2	4954689.771	145.504	.000
Residual	170259.139	5	34051.828		
Total	1.008E7	7			

The independent variable is density.
a. The equation was estimated without the constant term.

For this model F-ratio is 145.504, which is very unlikely to have happened by chance ($p<.001$). Therefore the model is significant.

TABLE 4. 9: Coefficients of Flow-Density Function

	Unstandardized Coefficients		Standardized Coefficients		
	B	Std. Error	Beta	t	Sig.
Density	113.561	16.703	2.314	6.799	.001
density ** 2	-2.530	.628	-1.371	-4.027	.010



KNUST

Figure 4. 2: Result of regression for average rate of flow versus average density

The figure shows a fitted second order polynomial to the data (which is expected to explain the nature of any flow-density diagram). The maximum of the fitted curve, i.e., the highest flow value indicates the capacity of the road stretch.

It was determined that a model for predicting densities on the basis of flow would be the most effective procedure for predicting traffic operations in the basic freeway section of the roadways.

4.2.3 Speed-Rate of flow Relationships

The relationships between speed and rate of flow were studied and analyzed using SPSS 17.00 and the following results were obtained.

TABLE 4. 10: Model Summary[a]

R	R Square	Adjusted R Square	Std. Error of the Estimate
.992	.983	.977	183.765

The independent variable is speed.

a. The equation was estimated without the constant term.

TABLE 4. 11: ANOVA[a] of Flow-Speed Function

	Sum of Squares	Df	Mean Square	F	Sig.
Regression	9910790.738	2	4955395.369	146.741	.000
Residual	168847.943	5	33769.589		
Total	1.008E7	7			

The independent variable is speed.

a. The equation was estimated without the constant term.

For this model F-ratio is 146.741, which is very unlikely to have happened by chance ($p<.001$). Therefore the model significantly improves our ability to predict the outcome.

TABLE 4. 12: Coefficients Flow-Speed Function

	Unstandardized Coefficients		Standardized Coefficients		
	B	Std. Error	Beta	t	Sig.
Speed	51.841	8.495	2.319	6.103	.002
speed ** 2	-.527	.147	-1.366	-3.593	.016

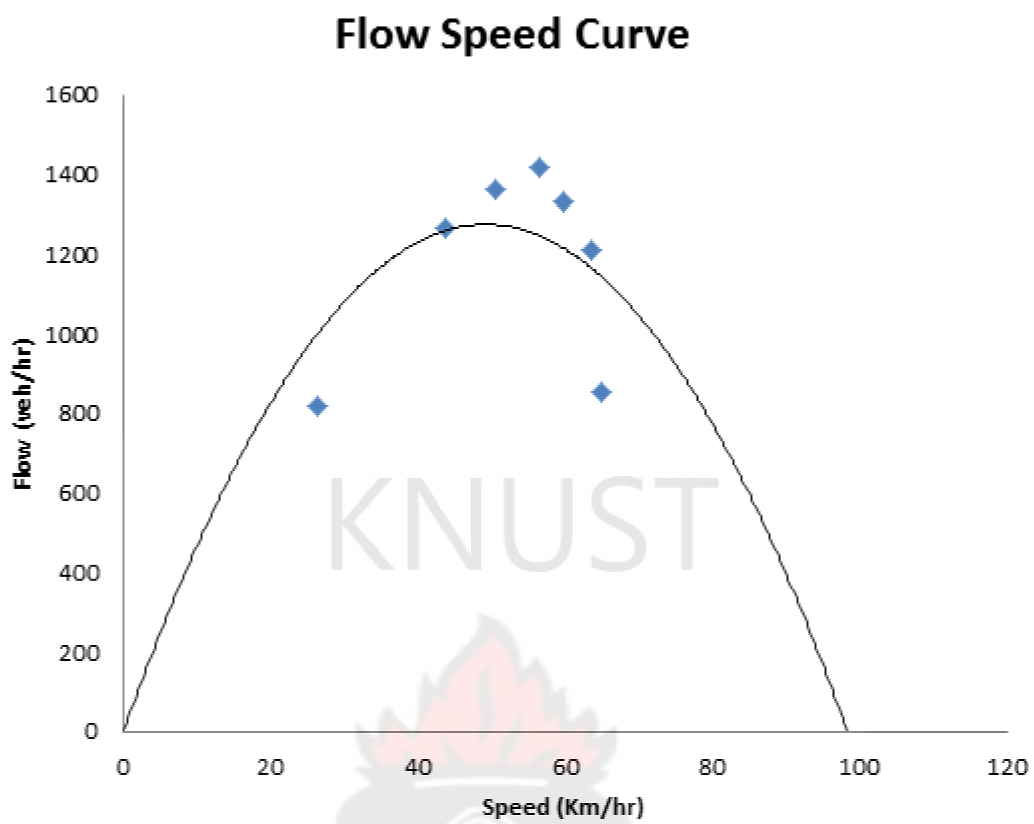


FIGURE 4. 3: Result of Regression for Average Rate of Flow Versus average Speed

From **Fig. 4.3** Speed appears to be sensitive to flow for the flow rates measured. From the data collected, an obvious relationship between speed and flow was found.

CHAPTER FIVE

DISCUSSION, CONCLUSION AND RECOMMENDATION

5.1 ANALYSIS OF RESULTS AND DISCUSSION

The purpose of this study and analysis for basic freeway is to build up the regression model of the correlation of traffic speed, rate of flow, and density, and predict the trend of traffic flow characteristics.

KNUST

5.2 RESULTS OF ANALYSIS

5.2.1 Density Versus Speed

The model of the result of regression for density versus speed is indicated as follows:

$$u = 97.113 - 1.890\,\rho \qquad (5.1)$$

where: ρ = average traffic density, vehicle per kilometre, and

u = average traffic speed, kilometre per hour.

The coefficient of correlation $\left(R^2 = 0.736\right)$ and the t value of constant is 7.855. The density variable is the significance of the speed. In this case, it is explained by the variability of the dependent variable, the average speed in this case. The adjusted R^2 value for Eqn. (5.1) is 0.684. The result of regression for average traffic density versus average traffic speed equation is shown as **Fig. 4.1.**

Notice that Eqn. (5.1) is linear with respect to space mean speed and density and is of the form of Greenshield's equation.

Greenshield's equation: $u_s = u_{\max} - \left(\dfrac{u_{\max}}{\rho_{\max}} \right) \rho$ (5.2)

Comparing Eqn. (5.1) and Eqn. (5.2)

Free flow speed $u_{\max} = 97.113$

To calculate jam density:

$\dfrac{u_{\max}}{\rho_{\max}} = 1.890$ Implying $\rho_{\max} = 51.383$

This model indicates when the critical (optimum) density at maximum flow (capacity) equals to 22.44 vehicles per kilometre. This coincides with the speed 54.701 kilometres per hour at that point of capacity, and the rate of flow is 1274.318 vehicles per hour per. The jam density equals to 51.383 vehicles per kilometre and then the traffic rate of flow and speed equals to zero. The results are shown as **Fig. 4.2** and **Fig. 4.3.**

5.2.2 Rate of Flow versus Density

The model of the result of regression for the density versus rate of flow is indicated as follows:

$$q(\rho) = 113.5615\rho - 2.530\rho^2 \tag{5.3}$$

$\rho_{\max} = 44.88 \quad (\text{veh/km})$

where: q = average rate of flow, vehicle per hour, and

ρ = average traffic density, vehicle per kilometre.

The coefficient of correlation $\left(R^2 = 0.983\right)$. The density variable is significant to the rate of flow. In this case, it is explained by the variability of the dependent variable, the average rate of flow in this case.

The adjusted R^2 value for Equation (5.3) is 0.976. The result of regression for the average traffic density versus the average rate of flow equation is shown as Figure 4. 2.

KNUST

The flow and speed are ***1274.318 veh/hr*** and ***56.788 km/hr*** respectively from appendix A3.1.

The model indicated that the critical density is 22.44 vehicles per kilometre, which coincides with the rate of flow at 1274.318 vehicles per hour (capacity).

5.2.3 Speed versus Rate of Flow

From the model of the result of regression for the speed versus rate of flow is indicated as follows:

$$q = 51.841u - 0.527u^2 \tag{5.4}$$

where: q = average rate of flow, vehicle per hour per lane, and

u = average traffic speed, vehicle per hour per lane.

The coefficient of correlation $\left(R^2 = 0.983\right)$. The adjusted R^2 value for Eqn. (3) is 0.977. The result of regression for the average traffic speed versus the average rate of flow equation is shown as Fig. 4.3 . The model indicated the critical speed is

54.701 kilometres per hour, which coincides with the rate of flow at 1274.318 vehicles per hour per.

The speed variable is significant to the rate of flow. In this case, it is explained by the variability of the dependent variable, the average rate of flow in this case.

5.3 CHARACTERISTIC CURVES AND THE SOLUTION OF THE TRAFFIC FLOW EQUATION

For the (LWR) model, It was determined that a model for predicting densities on the basis of flow would be the most effective procedure for predicting traffic operations in the basic freeway section of the roadways.

The model of the result of regression for the flow rate versus density was indicated in Eqn. (5.3). Differentiating Eqn. (5.3) and substituting into Eqn. (3.38) we have

$$\frac{\partial \rho}{\partial t} + (113.561 - 5.06\rho)\frac{\partial \rho}{\partial x} = 0 \tag{5.5}$$

Equation (5.5) is a first order partial differential equation.

The method of characteristics can be used to find a solution for the initial boundary value problem. An initial boundary value problem assumes (beside the differential equation) two extra equations:

• Initial values: the density values at time $t = 0$

$$\rho(x,0) = f(x) = \alpha + \beta x \tag{5.6}$$

• Boundary values: the density values at distance

$$x = 0 \qquad \rho(0,t) = g(t) \tag{5.7a}$$

And $x = x_c$ where $\rho(x_c,t) = h(t)$ (5.7b)

We assume that the initial density distribution $f(x)$ is given by a linear function

$$\rho(x,0) = kx \tag{5.8}$$

Consider the boundary conditions

$$\rho(0,0) = 0 \text{ and } \quad \rho(x_c,0) = \frac{\rho_{\max}}{2} = 22.44$$

This implies that

$$k = \frac{\rho(x_c,0)}{x_c} = \frac{22.44}{0.6} = 37.4$$

Hence the initial condition

$$\rho(x,0) = 37.4x \tag{5.9}$$

Now applying the method of characteristics to Eqn. (5.5)

along with the initial condition $\rho(x,0) = 37.4x$

where $\rho = \rho(x,t)$ is the unknown to be determined

The traffic density measured by the moving observer depends on time and the position, $\rho(x(t),t)$. The rate of change of this density depends both on the variation traffic and on the motion of the observer, since the chain rule of partial derivatives implies equation (3.40).

The characteristic ordinary differential equations is given as

$$\frac{dx}{dt} = \frac{dq}{d\rho} = 113.561 - 5.06\rho \tag{6.0}$$

Therefore $x = (113.56 - 5.06\rho)t + k$ (6.1)

Thus we have

$$x = (113.56 - 5.06\rho)t + x_0 \tag{6.2a}$$

as the characteristic curve which starts at $x = x_0$ when $t = 0$.

Substituting $\rho(x_0, 0) = 37.4x_0$ into eqn. (6.2a)

we have $x = (113.56 - 189.24x_0)t + x_0$ (6.2b)

For $x = 0$, $\rho = 0$ and $x = 113.56t$ (6.3a)

For $x = 0.24$, $\rho = 8.976$ and $x = 68.14t + 0.24$ (6.3b)

For $x = 0.48$, $\rho = 17.952$ and $x = 22.725t + 0.48$ (6.3c)

For $x = 0.6$, $\rho = 22.44$ and $x = 0.016t + 0.6$ (6.3d)

For $x = 0.84$, $\rho = 31.416$ and $x = -45.40t + 0.84$ (6.3e)

For $x = 1.08$, $\rho = 40.392$ and $x = -90.82t + 1.08$ (6.3f)

The graphical construction of these characteristics curves (6.3) are shown in Fig. 5.1

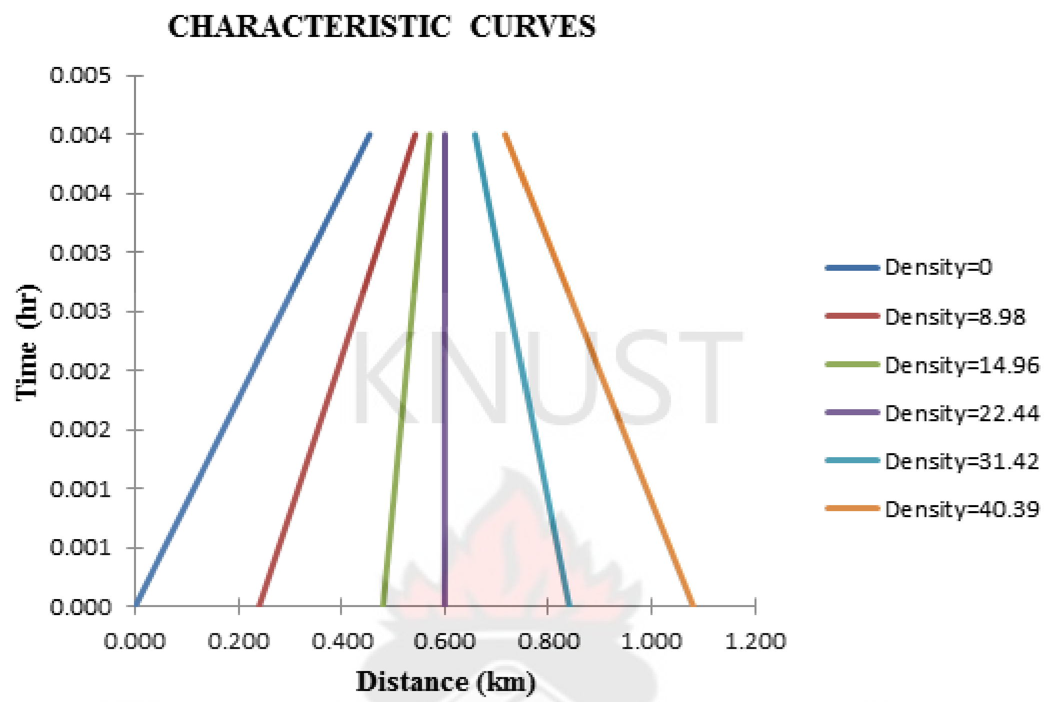


Figure 5.1: Solution of the traffic flow equation by method characteristics

This $x-t$ diagram shows what happens to a gradually increasing traffic density.

Notice that, since ρ is constant on each characteristic, we can compute the traffic density as a function of x for any time.

Recall

$$x = (113.57 - 189.24x_0)t + x_0$$

Solving for $x_0(x,t)$ we have

$$x_0 = \frac{x - 113.56t}{1 - 189.24t} \tag{6.4}$$

But we know $\rho = 37.4x_0$ and substituting from (6.4) we obtain

$$\rho = 37.4\left(\frac{x-113.56t}{1-189.24t}\right) \tag{6.5a}$$

$$\rho(x,t) = \frac{37.4x - 4247.14t}{1-189.22t} \tag{6.5b}$$

Also $$\frac{dx}{dt} = u = 113.56 - 189.24x_0$$

Hence $$u(x,t) = \frac{113.56-189.24x}{1-189.24t} \tag{6.6}$$

5.4 CONCLUSION

The presented traffic flow model gave the theoretically expected results.

The Lighthill-Whitham-Richards (LWR) model has been verified for the basic freeway segment between KNUST Police station and Boadi junction on Kumasi-Accra road

Using regression analysis, the Flow Density curve was found to be quadratic of the form $q = 113.5615\rho - 2.530\rho^2$

The method of characteristics was used to solve the PDE and the model yielded characteristics curve of the form $x = (113.57 - 189.24x_0)t + x_0$

Consequently the solution of the Partial Differential Equation was obtained as

$$\rho(x,t) = \frac{37.4x - 4247.14t}{1-189.24t}$$

5.5 RECOMMENDATION

It is recommended to conduct the same study on working days, during morning peak hour and on various segments of the road on the test site.

To prevent congestion and to improve efficiency, traffic should somehow be forced to move at a density (and speed) corresponding to maximum traffic flow. A signal which literally stops traffic and then permits it to go (in intervals yielding the density corresponding maximum flow) would result in an increased flow of cars on the road. Thus momentarily stopping traffic would actually result in an increase flow.

KNUST

REFERENCES

Banks, J. H. (1991). *Two Capacity Phenomenon at Freeway Bottlenecks: A Basis for Ramp Metering?* Transportation Research Record 1320, pp. 83-90.

Banks, James H. (1992). *"Freeway Speed-Flow-Concentration Relationships: More Evidence and Interpretations."* Transportation Research Record 1225, pp. 53-60.

Bellemans T., B. De Schutter, and B. De Moor (2002). *"Models for traffic control,"* Journal A, vol. 43, no. 3–4, pp. 13–22, 2002.

Bellomo. N., V. Coscia, M. Delitala (2002). *On the Mathematical Theory of Vehicular Traffic Flow I*. Fluid Dynamic and Kinetic Modelling, Math. Mod. Meth. App. Sc., Vol. 12, No. 12 (2002) 1801–1843

Ben-Akiva, M. Cyna, and A. de Palma **(1984)**. *Dynamic models of peak period traffic congestion.* Transportation Research, 18(4):-355, 1984.

Blensly R. C. (1956). *Moving vehicle method of estimating traffic volumes.* Traffic Eng. 27, 127-9, 147.

Cassidy, M.J. and R.L. Bertini (1999). *Some Traffic Features at Freeway Bottlenecks.* Transportation Research Part B Vol. 33, pp. 25-42

Castillo, J. M. D. and F. G. Benitez, (1995). *"On the Functional Form of the Speed–Density Relationship-I: General Theory,"* Transportation Research Part B: Methodological, 29, pp. 373–389.

Coleman J.P (2009). *Partial differential equation iv math 4041*, michaelmas term lecture notes. Mathematics Department, University of Durham, 2008/09.

Daganzo C.F (1997). *Fundamentals of transportation and traffic operations.* Elsevier Science, Oxford, United Kingdom

Dhingra S. L , Gundaliya P. J. And Tom V. Mathew (2005). *"Cellular Automata: An approach for Traffic Flow Modelling"* Accepted in START 2005, at IIT Kharagpur.

Disbro, John E. And Frame, Michael. (1992). "Traffic Flow Theory and Chaotic Behavior." Transportation Research Record. 1225: 109-115.

Drew, D. R., (1968). *Traffic Flow Theory and Control*, Chapter 12, McGraw-Hill Book

Edie L.C. (1960). *"Car Following and Steady-State Theory for non-congested Traffic,"* Tunnel Traffic Capacity Study Report VI, Port of New York Authority, New York, May, 1960.

Evans L.C.. *Partial Differential Equations*, American Mathematical Society, Providence, 1998. ISBN 0-8218-0772-2

Faber, T. E. (1995) *Fluid Dynamics for Physicists*. New York: Cambridge University Press, 1995.

Fan, Jianqing (1996). *"From linear regression to nonlinear regression". Local Polynomial Modelling and Its Applications*. Monographs on Statistics and Applied Probability. Chapman & Hall/CRC. ISBN 0-412-98321-4.

Fred, L. M., And Walter, P. K.(1998). *Principles of Highway Engineering and Traffic Analysis*. New York, 1998.

Garavello M. and B. Piccoli *Traffic Flow on Networks*, American Institute of Mathematical Sciences (AIMS), Springfield, MO, 2006. pp xvi+243 ISBN 978-1-60133-000-0

Gergonne, J. D. (November 1974 [1815]). *"The application of the method of least squares to the interpolation of sequences"*.

Gerlough, Daniel L. And Huber, Matthew J. (1975). *Traffic Flow Theory:* A Monograph TRB Special Report 165. Transportation Research Board, Washington DC.

Gilchrist, Robert S. And Fred L. Hall. (1992). *"Three-Dimensional Relationships Among Traffic Flow Theory Variables."* Transportation Research Record. 1225:99-108.

Greenberg H. *"A Mathematical Analysis of Traffic Flow",* Tunnel traffic Capacity Study, the Port of New York Authority, New York, 1958.

Greenberg, H., (1959), *"An Analysis of Traffic Flow,"* Operations Research, 7, pp. 78–85.

Greenshield, B. D., (1935). *"A Study of Traffic Capacity,"* Highway Research Board Proceedings, 14, pp. 448–477.

Greenshields B.D and F.M. Weids. (1952). *Statistics with Applications to Highway Traffic Analyse,* Eno Foundation for Highway Traffic Control,Saugatuck,Conn.,1952

Guell, D. L. and Virkler, M. R. (1988). *"Capacity Analysis of Two-Lane Highway",*

Haberman, Richard (2003). *Applied Partial Differential Equations.* 4th Edition . Prentice Hall, 2003. 564-567

Haberman, Richard. (1977). *Mathematical Models in Mechanical Vibrations, Population Dynamics, and Traffic Flow.* Prentice-Hall, 1977. 259-394.

Haight, F.A. (1963). *Mathematical theories of Traffic Flow.* New York: Academic Press Inc,1963

KNUST

Hall, F. L., Hurdle, V. F. and Banks, J. H. (1992). *"Synthesis of Recent Work on the Nature of Speed-Flow and Flow-Occupancy (or Density) Relationships on Freeways",* Transportation Research Record 1365, TRB, National Research Council, Washington, D. C., pp. 12-18.

Hall, F.L. and K. Agyemang-Duah (1991). *Freeway Capacity Drop and the Definition of Capacity.* Transportation Research Record 1320, pp. 91-98

Henry Lieu (1999·). *"Traffic-Flow Theory".* Public Roads (US Dept of Transportation) (Vol. 62· No. 4) .   http://www.tfhrc.gov/pubrds/janfeb99/traffic.htm.
Ergotmc @ GTRI Georgia Tech http://ergotmc.gtri.gatech.edu/
http://en.wikibooks.org/wiki/Fundamentals_of_Transportation/Traffic_Flow (last accessed on 07/05/2011)

Herman R., E. Montroll, R. Potts, and R.Rothery (1959). *"Traffic Dynamics: Analysis of Stability in Car Following,"* Operations Res., vol. 7, 1959.

Highway Capacity Manual 2000 (HCM) Retrieved from "http://en.wikipedia.org/wiki/Traffic_flow"

Highway Capacity Manual, Bureau of Public Roads, U.S. Dept. of Commerce, (1950)
Hood, Alan (2000). *"Method of Characteristics."* <http://www-solar.mcs.st-and.ac.uk/~alan/MT2003/PDE/node8.html>.

John, Fritz (1991), Partial differential equations (4th ed.), Springer, ISBN 978-0387906096

Khisty, C. J. and Lall, B. K. (1998), *Transportation Engineering, An Introduction*, 2nd Edition, Prentice Hall, Upper Saddle River, New Jersey.

Knobel, Roger (2000). *An Introduction to the Mathematical Theory of Waves.*Vol.3. American Mathematical Society, 2000, pp. 153-154.

Koshi, M., M. Iwasaki, and I. Okhura. (1983). *"Some Findings and an Overview on Vehicular Flow Characteristics."* Proceedings of the Eighth International Symposium on Transportation and Traffic Theory: 403-426.

Lighthill, M. J. and Whitham, G. B. (1955), *"On Kinematic Waves II. A Theory of Traffic Flow on Long Crowded Roads",* Proceedings of Royal Society of London, Vol. 229, No. 1178, May, pp. 317-345.

Maerivoet S. *Modelling Traffic on Motorways: State-of-the-Art, Numerical Data Analysis, and Dynamic Traffic Assignment*, Katholieke Universiteit Leuven, 2006

Mannering, F.L.; Kilareski, W.P. and Washburn, S.S. (2005). *Principles of Highway Engineering and Traffic Analysis,* Third Edition. Chapter 6

May A. D. (1990). *Traffic Flow Fundamentals*, Prentice Hall, Englewood Cliffs, New Jersey 07632.

Michalopoulos, P. G., Yi, P. and Lyrintzis, A. S. (1993). *"Continuum Modelling of Traffic Dynamics for Congested Freeways",* Transportation Research *B*, Vol. 27B, No. 4, pp. 315-332.

Morales, J. M. (1987), *"Analytical Procedures for Estimating Freeway Traffic Congestion",* ITE Journal, Jan. pp. 45-49.

Mortimer W. J. (1956). *Moving vehicle method of estimating traffic volumes and speeds.* Traffic Eng. 28,539-44.

Moskowitz, K. and Newman, L. (1963), *"Notes on Freeway Capacity",* Highway Research Record 27, pp. 44-68.

Nam, D. H. (1995), *Methodologies for Integrating Traffic Flow Theory, Its and Evolving Surveillance Technologies*, Unpublished Ph.D. dissertation, Virginia Polytechnic Institute and State University, Blacksburg, VA.

Nam, D. H. (1998), *"Analyzing Freeway Traffic under Congestion: A Traffic Dynamics Approach*", 77th Annual Meeting of the Transportation Research Board, January, Washington, D.C.

Newell, G. F. (1988*), "Traffic Flow in the Morning Commute",* Transportation Science, Vol. 22, No. 1, pp. 47-58.

O'Flaherty, C.A and Simons F. (1970*). An Evaluation of the Moving Observer Method of Measuring Traffic Speeds and Flows.* Australian Road Research Board Proceedings, 5, Part 3, 40-54.

Papageorgiou M. (1980). *A new approach to time-of-day control based on a dynamic freeway traffic model.* Transportation Research, Vol. 14B:-360, 1980.

Payne H. J (1971). *Models of freeway traffic and control, Simulation*. Councils Proc. Ser.: Mathematical Models of Public Systems, Vol. 1, No. 1, Editor G.A. Bekey, 1971.

Payne H. J. (1973). *Freeway traffic control and surveillance model*. Trasportation Engineering Journal, 99 (TE4):-783, 1973.

Payne H. J. (1978). FREFLO. *A macroscopic simulation model of freeway traffic:* Version 1 - User's guide. Technical report, ESSOR Report, 1978.

Pensaud, B.N. and Hurdle, V. F. (1991). *"Some New Data That Challenge Some Old Ideas About Speed-Flow Relationships."* Transportation Research Record. 1194: 191-8.

Pline J. L (1992). *Traffic Engineering Handbook*. Prentice Hall, Englewood Cliffs, New Jersey, 4th edition, 1992.

Polyanin A. D (2000). *Handbook of Linear Partial Differential Equations for Engineers and Scientists*, Chapman & Hall/CRC Press, Boca Raton, 2002. ISBN 1-58488-299-9

Polyanin A. D., V. F. Zaitsev And A. Moussiaux, *Handbook of First Order Partial Differential Equations*, Taylor & Francis, London, 2002. ISBN 0-415-27267-X

Ross, Paul. (1991). *"Some Properties of Macroscopic Traffic Models."* Transportation Research Record. 1194: 129-34.

Salter, R. j., and Hounsell, N. B (1996). *Highway Traffic Analysis and Design.* New York,

Sarra, Scott (2003). *The Method of Characteristics with applications to Conservation Laws*, Journal of Online Mathematics and its Applications.

SATURN ITS Transport Software Site. *A survey about the state of art in traffic flow modelling*

Smith S. A. (1985). *Freeway data collection for studying vehicle interactions.* Technical Report FHWA/RD-85/108, Federal Highway Administration, Office of Research, Washington D. C., 1985.

Subramanian H. (1996). *Estimation of a car-following model for freeway simulation.* Master's thesis, Massachusetts Inst. of Tech., Cambridge, MA, 1996.

Transportation Research Board (1998). *Special Report 209, Highway Capacity Manual 2000*,National Research Council, Washington D.C.

Tritton, D. J.(1988). *Physical Fluid Dynamics,* 2nd ed. Oxford, England: Clarendon Press, pp. 52-53 and 59-60, 1988.

Underwood, R. T., (1961). *"Speed, Volume, and Density Relationships: Quality and Theory of Traffic Flow,"* Yale Bureau of Highway Traffic, pp. 141–188.

Wang, H., Li, J., Chen, Q. and D. Ni, (2009), *"Speed–Density Relationship: From Deterministic to Stochastic,"* Transportation Research Board Annual Meeting, Paper #09–1527

Wardrop, B.A. and Charlesworth G. (1954*). A Method of Estimating Speed and Flow of Traffic from a Moving Vehicle.* Proceedings of the Institution of Civil Engineers, 3, Part II, 158-169.

Williamst . E. H. and Emmersonj. (1961). *Traffic volumes journey times and speeds by moving-observer method.* Traffic Eng. and Control j, 159-162, 167.

Wright C. C., Hyde T., Holland P. J. and Jackson B. J. (1973). *Estimating traffic speeds from flows observed at the ends of a road link.* Traffic Eng. and Control

APPENDIX A3.1

<u>Determination of maximum flow, optimum density and optimum speed.</u>

Recall $q = 113.5615\rho - 2.530\rho^2$

at the maximum point

$$\frac{dq}{d\rho} = 113.561 - 2.530(2)\rho = 0$$

$$113.561 - 2.530(2)\rho = 0$$

$$5.06\rho = 113.561$$

$\rho = 22.44 = \rho_o =$ density at maxmum flow

Substituting $\rho = 22.44$ into (5.3), gives

$$q = -2.530\rho^2 + 113.561\rho$$

$$q = -2.530(22.44)^2 + 113.561(22.44)$$

$$q = -1273.991 + 2548.309$$

$q = 1274.318$ veh/hr

Speed at maximum flow is $u_o = \frac{q_{\max}}{\rho_o} = \frac{1274.318}{22.44} = 56.788$ km/hr

APPENDIX A3.2:

Derivation of formulas for the estimation of speed, flow rate and density for one direction of travel.

Consider an observer watching a stream of vehicles. Two different cases of motion can be considered J.G Wardrop and G. Charlesworth (1954). The first case considers the traffic stream to be moving and the observer to be stationary. If m_0 is the number of vehicles overtaking the observer during a time period, t, then the flow q is $\frac{m_0}{t}$,

or

$$m_0 = q \cdot t \qquad (1)$$

The second case assumes that the stream is stationary and the observer moves with speed u_0. If m_p is the number of vehicles overtaken by observer over a length l travelled by the observer then by definition, density ρ is $\frac{m_p}{l}$, or

$$m_p = \rho \cdot l \qquad (2a)$$

or

$$m_p = \rho \cdot u_0 \cdot t \qquad (2b)$$

Where u_0 is the speed of the observer and t is the time taken for the observer to cover the road stretch

These two cases are now merged into a situation where both the stream and the observer are moving in the same direction, but at different speeds. Obviously some cars will overtake the observer and he will in turn over take some of the cars.

On the trip in the same direction as the observed flow, the observer adds up the cars that pass the observer, and subtracts the cars that are passed by the observer. On the trip against the observed flow, the observer counts the cars that pass the observer in the opposite direction. Wardrop, J. G., and Charlesworth (1954). If the length of the road section is L. This can be represented mathematically.

Assuming m_0 vehicles overtake the observer, and assuming m_p is the number of vehicles the observer passes, then, using Eqn. (1) and Eqn. (2b) we can describe the difference between m_0 and m_p called the tally counts as:

$$m = m_0 - m_p \tag{3a}$$

From above

$$m = qt - \rho u_0 t \tag{3b}$$

This equation is the basic equation of moving observer method, which relates q and ρ to the counts, m, t and u_0 that can be obtained from the test.

Assume that the trip against the observed flow yields a traveling time t_a and a vehicle count is m_a,

$$m_a = qt_a + \rho u_a t_a \tag{4}$$

and the trip in the same direction of the observed stream has a traveling time t_w and a vehicle count m_w (i.e. those passing minus those overtaken-Tally counts).

$$m_w = qt_w - \rho u_w t_w \tag{5}$$

Adding Eqn. (4) and Eqn. (5), gives the flow rate in both directions (in minutes)

$$q = \frac{m_a + m_w}{t_a + t_w} \tag{6}$$

Now from $m_w = qt_w - \rho u_w t_w$

$$\frac{m_w}{t_w} = q - \rho u_w$$

But $\rho = \frac{q}{u}$ and $u_w = \frac{l}{t_w}$

$$\frac{m_w}{t_w} = q - \left(\frac{q}{u}\right)\left(\frac{l}{t_w}\right)$$

$$\frac{m_w}{t_w} = q\left(1 - \frac{l}{u} \cdot \frac{1}{t_w}\right)$$

$$\frac{m_w}{t_w} = q\left(1 - \frac{t_{av}}{t_w}\right) \qquad \text{since } t = \frac{l}{u}$$

$$\therefore \quad \frac{m_w}{q} = t_w\left(1 - \frac{t_{av}}{t_w}\right)$$

$$\frac{m_w}{q} = t_w - t_{av}$$

$$t_{av} = t_w - \frac{m_w}{q} \tag{7}$$

$$t_{av} = \frac{l}{u_{av}} = t_w - \frac{m_w}{q}$$

$$u_{av} = \frac{l}{t_w - \frac{m_w}{q}} \tag{8}$$

$$u_{av} = \frac{l}{t_{av}}$$

From $q = \rho u$

$$\Rightarrow \rho = \frac{q}{u_{av}} \tag{9}$$

APPENDIX A4.1

<u>The Result of Stream Flow for the First Test Run</u>

Flow rate is given by equation, $q = 60\left(\dfrac{34+(0-0)}{1.28+1.11}\right) = 853.5565$ veh/hr

Average time of the stream $t_{av} = 1.11 - 60\left(\dfrac{0}{853.5565}\right) = 1.11$ hrs

Stream speed v_s can be found out from equation $v_s = 60\left(\dfrac{1.2}{1.11}\right) = 64{,}86$ km/hr

Density can be found out from equation as $\rho = \dfrac{853.5565}{64.85486} = 13.159$ veh/km

KNUST