KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY – KUMASI

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INSTITUTE OF DISTANCE LEARNING

MANAGING MINING RISK USING ORE RESERVE ESTIMATES

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BY

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DECLARATION

The work submitted in this declaration is the result of my own investigation, except where due reference is made in the text of the thesis.

It has not already been accepted for any degree, and is also not being concurrently submitted for any other degree.

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DEDICATION

I wish to dedicate this project to the

Almighty God,

My family, especially, Mr. Padi N. Joseph and Mr. Padi K. Abraham,

And all who are dear to me.

ABSTRACT

The decision to invest in the mining industry comes with all forms of challenges that will have to be managed with care. Especially because, it involves huge capital outlay and the associated risks are very high. The assessment process of the commercial viability of the ore deposit consists of two major components; the estimation of the quantity (including quality) of the commodity and the decision as to whether or not to mine (or invest). This thesis tries to explore these risk areas by using the geostatistical model and its analysis (the semi-variogram and kriging analysis) to estimate the quantity and quality of the commodity. The decision to invest or not to invest is analyzed with the help of the black-scholes model, taking into consideration the prevailing market price (including price volatility) and the risk derived from the geostatistical estimates. The results indicate that, if the concession is contracted for a period of five years then the mine would be viable for investment during the entire duration when the price, the base case, of the commodity (gold) is about one thousand six hundred dollars. The sensitivity analysis, however, reveals that the mine would not be worth investing within some periods in the duration of the contract especially when the gold price falls to eight hundred dollars per ounce and below.

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As stated in the old good book; in all things, give thanks to the Almighty God. More so because, whenever the other side of life of despair and impossibility stare at the face, He is more than capable to turn them into hope and possibility. For His divine guidance and direction in the preparation of this thesis, I can only say thank you to Him.

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INTRODUCTION

1.0 Introduction

This chapter elaborates on the study background, problem at hand, objective, methods employed in the analysis of data to attain the aim of the study, justification and how the thesis is structured – or organized. It therefore, presents the intention and focus of the writer.

1.1 Background of the Study

Mostly, investors would channel their capital into mining operations, like any other business venture, because they hoped to make some good profit with risk resilient (at least if it cannot be avoided completely). The information about the grade and tonnage as well as the fluctuating price, however, of the commodity of interest is quite complicated and uncertain thereby increasing the risk component of the venture.

The complication and uncertainty of the price, grade and tonnage; and consequently the entire mining investment portfolio may be appreciated if a more concise definition of mining is highlighted. In simple terms, mining is a process by which a valuable, naturally occurring, mineral is taken out of the ground economically with the existing technology. In other words, if the mineral of interest is naturally occurring then the parameters (grade and tonnage) would be undoubtedly complex and uncertain. Interestingly, the economic aspect of the venture is what concerns investors. The question then may be; what information should be available to inform decision makers put resources into the venture with a controlled risk?

Unfortunately the decision to mine is predominantly influenced by the information gathered on the grade and tonnage of the metal in the ground since it helps determine the commercial viability of the mineral deposit. In the decision making process variables like; current price, interest rate and the existing technology are equally vital. As a first step to fairly achieve this, samples are taken at some designated intervals and sent for assaying. The assayed sampled data is treated and the values obtained inform investors the next line of action. It may be worth noting that the issue of "chicken and egg" arises when it comes to commercial viability of ore deposit. Thus, a mineral deposit may exist in quantity and quality but could not be mined because the price of the commodity may not allow. Conversely, the price of the commodity could be very attractive but the deposit may not exist in quantity and quality to be mined. These are the fundamental areas of risk that the thesis tries to explore.

Although it may be impossible for mining engineers, and other expertise such as geologists, to establish the actual price, grade and tonnage, it is still believed that the application of some mathematical models may considerably improve the results and its overall reliability. Geostatistical models are generally used to estimate the grade and tonnage at unsampled points, including the associated variance (that is, the estimated error), probabilities and confidence intervals. Broadly, geostatistical models try to look at reserve estimation in two cases: the isotropy and anisotropy. The isotropy case assumes grade distribution to be independent of direction but that of the anisotropy considers the distribution of grade to be greatly influenced by direction. This thesis, however, shall consider the isotropy case. Generally, geostatistical models help control the number of exploratory drilling. The Black-Scholes model shall be adapted, using these geostatistical estimates, to establish the decision on whether or not to mine.

Risk management is a technique of determining, controlling or minimizing and preventing the possibility of accidental loss in an investment or speculation successfully by taking safety measures. Grant (2004) quoted that risk refers to a situation in which there is more than one possible outcome to a decision, and the probability of each specific outcome is known or can be estimated. In general, the greater the number and range of possible outcomes, the greater is the risk associated with the action or decision.

Ore reserve estimates are assessment of the quantity and tenor of a mineral that may be profitably and legally extracted from a mineral deposit through mining and/or mineral beneficiation.

The definitions above clearly spelled out that, estimation of ore reserves involves not only evaluation of the tonnage and grade of a deposit but also consideration of the technical and legal aspects of mining the deposit and of selling the product. Mining is one of the major areas of investments where the possibility of accidental loss is most likely and that activities prior to production operations should be quite extensive without compromising financial constrains. In short, ore reserve estimation, particularly grade and tonnage is viewed as one of the main safety measures that are employed when techniques are considered to managing mining risk. Most especially because, if both quantitative and qualitative measures point to it that the deposit is not of commercial quantity then decision makers may have either of the following two options: (i) avoid the investment as a way of managing the risk or (ii) invest only risk capital.

The decision to invest only risk capital may arise from the fact that geostatistical measures (both quantitative and qualitative) could be completely misleading. But interestingly computations would mostly help optimize some risk.

1.2 Problem Statement

Most often, the worth of a mining investment portfolio is computed using the net present value (NPV) approach in relation to the prevailing market factors on the commodity such as; the current price, interest rate, and so on. The risk analysis in most feasibility studies are based on stochastic modeling of project NPV which, in most cases, fail to provide decision makers with truly comprehensive analysis of risks associated to technical and management uncertainty and, as the result, are of little use for risk management and project optimization (Botin, 2011). But, in particular, this project attempts to compute the worth of the mine using the Black-Scholes model and the reserve estimates since any projection of returns on the investment may be meaningless without considering the existence of the precious mineral in question. In other words, the value of this important but highly risky venture is directly linked to whether or not the commodity is in a commercial quantity.

The mining industry is most often faced with two main problems. Namely: (a) the uncertain existence of the prospected mineral of interest in commercial quantity. This could be managed through the estimation of grade and tonnage using geostatistical models. (b) The decision to mine or not to mine based on market factors. This could be achieved when the worth of the mine is estimated by the use of Black-Scholes model.

1.3 Objectives

The objectives of this project are to:

i. Use geostatistical modeling to estimate the grade, tonnage and error of the reserve of a gold mine in the western region of Ghana.

ii. Perform an investment decision analysis using the Black-Scholes (or option pricing) model based on the risk derived from the geostatistical modeling.

1.4 Methodology

The data (secondary) was collected from Goldfields Ghana Limited,Tarkwa mine (at site,Teberebie Cut 3). The geostatistical data collected is on grade of gold and their location coordinates. Geostatistical modeling is used to analyse the data to produce a variogram and kriging. From the variogram, together with a geostatstical model, a kriging analysis will be made. Some of such models include: spherical, exponential, nugget effect and Gaussian. The spherical model is, however, selected to analyse the variogram for the kriging analysis. Parameter values on cost of licensing the mining area of study, cost of developing the technology for production and recovery, unit cost of producing gram per ton of gold and cost of land reclamation will be collected for the evaluation of the geostatistical estimates. The Black-Scholes approach will be used for decision analysis. MATLAB is employed to produce variogram, kriging and decision analysis.

1.5 Justification

What may make this area of study a very special one could be based on the report from the World Bank survey. The survey shows that 73% of mining projects failed in North America alone due to problems in their ore reserve estimates, leading to losses of billions in capital investment over a decade. It also added that the greatest uncertainty driving the risk and profitability of mining investment is the geological variability of mineral deposits (Dimitrakopoulas, 2012). Clearly, this thesis tries to re-emphasize that using ore reserve estimates to manage mining risk may go a long way to improve the economic position of the investment. Results obtained from assayed values using geostatistical models may largely inform decision making. In brief, even though the topic may seem quite technical, engineering and mathematical, its economics and social importance cannot be underestimated.

From economic point of view, this research may be very relevant because, there are other investment options available to the investors. Additionally, reliable estimation is critical to both the confidence in a feasibility studies, and also to the day to day operations of the mine. Ore reserve estimates could help adventurous businessmen put their capital to good use more so, because resources are limited and the success of investors may contribute to the overall developmental goals of the state within which they operate. For example, if Ghana has been able to produce 2.97 million ounce of in 2010 (MBendi et al, 1995 – 2012), one could imagine the benefits that the state would have derived from it.

1.6 Thesis Organization

This thesis is divided into five (5) chapters. The first chapter contains the following: introduction, background of the thesis, problem statement, and objective of the thesis, methodology, justification and thesis organization. In chapter two the literature review is presented.

Chapter three entails the methodology where mathematical models and methods of solution are presented. Chapter four explains data analysis and results. Finally, observation, conclusion and recommendation are contained in chapter five.

CHAPTER 2

LITERATURE REVIEW

2.0 Introduction

This chapter presents the literature on ore reserve estimation; with geostatistics (and its analysis) being the underlining mathematical modeling in the estimation process. The literature is extended to cover other versions of research on the Black-Scholes model.

2.1 Ore Reserve Estimation

Ore reserve estimation is the main procedure adapted to obtain, to some level of confidence, both the qualitative and quantitative information about the resource in the ground. Ideally the best information about the resource could be obtained during exploration process if the whole area under study is excavated. This, however, would be far from reality and economics principles since exploration of solid mineral deposits is generally an expensive task. Even more expensive and risky is the successive mining investment. It may be of paramount importance to optimize the exploration expenses and minimize the risk of the mining investment. More importantly so because, according to Terrence (2002); the development and operation of a mining project requires the investment of many millions of dollars and that accurate quantification of uncertainty in grade allows the risk of investment loss to be correctly managed to the benefit of the mine owner, investors, and the wider community.

As stated by Xiping (1992), reserve estimation involves the modeling of spatial variation and distribution of ore grade in the region of exploration. Current approaches are based essentially on either geometrical reasoning or statistical techniques, and generally assume that the spatial distribution of ore grade is a function of distance. Ore reserve estimates and their interpolations do really have effect on the mining operations. Many mining operations developed in the last 15years have not performed to expectations and most of the problems can be directly attributed to decisions made on the basis of geosciences interpolations and estimates (Berry et al. 2006).

As with all reserve estimation, a thorough understanding of the geology is essential, including a practical model of the mineralization structures, and the definition of the various ore types present. This will ensure that the geostatistical analysis and valuations are carried out within defined geologically homogeneous zones so as to avoid obvious non-stationarity, and that significant mineralization structures are modeled geostatistically and used in the valuation process (Krige, 1994).

2.2 Managing Mining Investment Risk

The holistic management of risk associated with mining investment is quite a comprehensive task; ranging from acquisition of the concession, legal requirements, political, exploration, production/procession through to the sale of the product. Each of these stages, and beyond, defines a cost center that, if not treated with caution, may threaten the entire venture.

The decision on how to allocate capital in mining has always been complex, but the unpredictability of the markets and the unprecedented level of volatility they have displayed recently makes mining investment decision-making even more difficult and increasingly multifaceted. NPV does not tell the whole story, and robust risk analysis provides more valuable insight about how to quantify and manage the downside risks and upside opportunities of major project like mining. In all business and finance, uncertainties are associated with every aspects of the economy. Beside the few examples of global factors that are characterized by uncertainties such as the political environment for doing business, commodity prices and exchange rate, there are equally important project-level factors associated with large uncertainties like; technical success of a mining method or understanding geology of new resource (Jacet, 2010).

According to Frimpong (1998), successful management in competitive markets requires evaluation methods that respond to global market dynamics and provide investors with relevant information to make strategic investment decisions. These strategic decisions include investment timing, feasibility study and risk management and mine operating options. Conventional methods do not have the built-in capabilities to help investors handle these strategic issues. Advances in modern finance have had profound impacts on financial markets for options, future and collateralized securities and offers appropriate tools in solving these problems. In brief, the results obtained from the derivative mine evaluation method is one of the best ways that allow investors to maximize the venture's market value.

As stated by Rui (2008), an analytically tractable dynamic stochastic general equilibrium model to study asset pricing and welfare implications of imperfect investor protection is developed. Consistent with empirical evidence, the model predict that countries with weaker investor protection have more incentives to overinvest, higher return volatility, larger risk premia, and higher interest rate. Calibrating the model to the Korean economy, reveals that perfecting investor protection increases the stock market value by 22%, a gain for which outside shareholders are willing to pay 11% of their capital stock. This is to say that dynamic stochastic model (like the Black-Scholes) could help take a decision on the value of stock for which investment should take place.

According to Mitchell (1997), as gold prices rise, its extraction cost also rise as they switch to mining lower grade ores, which are more costly to extract. A rise of 10% in the price of gold during the year is associated with a 2.4% increase in its extraction cost. A run up in price in the previous years may have prompted a decision to increase production since the investment may take time to adjust to a new environment (different level of gold prices).

For a mining company to be profitable, it needs to locate an accessible area that contains gold in 2000 to 3000 parts per billion (ppb) of surrounding soil and rock. Thus, geological, geophysical, and geochemical data is collected to determine the probability of the deposit. If sufficient data is acceptable, then test drill holes are sunk in predefined area to take sub-surface samples, which are sent to a laboratory to determine the evidence of any gold ore (CME Group, 2011).

A hypothetical financial model based on a gold operation has been used to estimate the potential effect of resource/reserve on revenue. The results show that realistic uncertainty ranges can generate changes in the estimate of potential revenue of plus or minus 30%. Therefore it is important to allow for errors in these processes in any financial analysis or feasibility study (Snowden et al. 1993; 1998).

2.3 Geostatistics

Geostatistics is a set of statistical estimation tools involving quantities which vary in space. It is originally very popular in the mining and oil exploration. Now used extensively in almost all branches of hydro science (TTU Pride, 2006).

2.3.1 *Historic Background of Geostatistics*

According to Agterberg (2005), the founder of spatial statistics, Professor Georges Matheron (1930 – 2000) made fundamental contributions to science by establishing new theoretical frameworks in spatial statistics, random set, mathematical morphology, and the physics of random media. Matheron, inspired by De Wijs and Krige, commenced work in 1954 on regionalized random variables. One of his first publications (Matheron, 1955a) concerns the Gara Djebilet oolithic iron deposit that has been discovered in 1952. This is a standard geological publication with folded geological map and description of the stratigraphy, structure and genesis of this deposit of early Devonian age. The final section on ore grades and tonnages provides estimates of the ore reserve. These estimates exceed 2 billion metric tons in total.

The basis of geostatistics is the study of the spatial correlation of data measured at various points in three-dimensional space. The fundamental technique is called kriging, in recognition of the work carried out by Dr. Krige on gold deposits in South Africa. Geostatistics is used to interpolate isolated data to map it into a continuous space. Given that the estimation is subject to error, geostatistics can also be used to assess how much confidence can be placed in a map produced in this way (TTU Pride, 2006). This quality control is not part of standard interpolation methods (such as Inverse Distance). Furthermore, standard interpolation methods do not take into account the intrinsic properties of the interpolated phenomena (for example rapid changes in the data with distance, or more gradual changes), as they only take account of the position of the measurement points. Geostatistics uses a probabilistic model to overcome these problems. Geostatistics was originally used in prospecting where it was necessary to estimate the potential of a deposit as accurately as possible using spatially dispersed sampling. The petroleum industry then became interested in geostatistics to improve the modeling of the geometry of oil reservoirs using data from a small number of wells combined with more detailed data from seismic surveys. It's now used for all types of environmental study - air, water, and ground - for which there is an increasing demand for justifiable maps where the uncertainty is quantified.

In geostatistics (and kriging), variogram and covariance function are the 'heart' of the analysis – it mathematically formalizes the spatial variability using some model which is then used to estimate the entire field.

The kriging started when the founder, Dr. Krige in South Africa (about 80 years ago), was required to prospect a very large area for gold. He had all the necessary tools for drilling to mine a spot for gold. However, due to costs and technical difficulty he did not have the luxury to mine physically the whole area (with extensive drilling) in order to find out the locations where gold is deposited in high amounts. Another problem that complicates his objective is that there was no precedence of gold mining in his area (i.e., nobody really knows the geology or any historical fact to guide him to choosing drilling locations that may have a high probability of having gold deposits (TTU Pride, 2006).

2.3.2 *Importance of Geostatistics*

Geostatistics is important for modeling/understanding the spatial variability of a quantity. There is virtually few 'quantity' (at least in natural sciences) that does not vary in space. Gravity, molecular weight of chlorine, relative abundance of oxygen and nitrogen in air are some few examples.

Examples of quantities that show clear spatial variation are: rainfall, vegetation, soil texture, population density, economic wealth etc (TTU Pride, 2006).

Noppe' (1994), states that geostatistical analysis provides a powerful tool for enhancing the prediction and decision-making capabilities of mine planners and geologists. It provides the only method for quantifying differences between alternative sampling philosophies as well as providing the errors associated with various reserve tonnage and quality estimates. It provides the best possible weighting for samples used in reserve estimation so as to produce the lowest possible error of estimation. Geostatistics provides the person involved with routine tonnage and quality estimates the best estimation technique that quantifies the error associated with making such estimates, and therefore allows confidence limits to be placed on these estimates.

Glacken (2001), writes that as a precursor to any of the kriging or conditional simulation techniques, spatial (geostaistical) analysis of the domained data – that is, the calculation and modeling of semi-virograms – is an obvious and necessary step. In brief, geostatistics deals with spatially autocorrelated data where the correlation between elements of a series and others from the same series separated from them by a given interval is analysed.

According to Myers's (2011), geostatistics is only good science brought up to date by the recognition that natural phenomena are subject to spatial variation. That is, each data value is associated with a location in space and there is at least an implied connection between the location and the data value. Location has at least two meanings; one is simply a point in space (which only exists in an abstract mathematical sense) and secondly with an area or volume in space. For example, a data value associated with an area might be the average value of an observed variable, averaged over that volume. In the latter case the area or the volume is often called the support of the data. This is closely related to the idea of the support of a measure. Let x, y, …, w be points (not just coordinates) in 1, 2, or 3 dimensional space and Z(x), Z(y), … denotes observed values at these locations. For example, this might be the grade of gold, copper, temperature, hydraulic conductivity, concentration of a pollutant. Now suppose t is a location that is not sampled. The objective the is to estimate/predict the value Z(t) (and the data locations as well as the location t). If only this information is given then the problem is ill-posed, that is, it does not have a unique solution and one way to obtain a unique solution is to introduce a model into the problem. The two ways of doing this is (i) deterministic and (ii) stochastic or statistical. Both approaches however, must incorporate the idea that there is uncertainty associated with the estimation/ prediction step. One approach is to treat Z(x), Z(y), … and Z(t) as being the values of random variables.

Geoff (2007), added that the basic components of geostatistics are (i) (semi)varogram, where the spatial correlation is characterized (ii) kriging – which is used for optimal interpolation that generate best linear unbiased estimates at each location by employing semivarogram model and (iii) stochastic simulation – in which multiple equiprobable images of the variable is generated by employing semivarogram model. It all begins with the fundamental knowledge in covariance and correlation both of which measure the similarities between two different variables. The magnitude of the covariance increases with increasing similarity in the patterns of variation of the two variables about their respective means. The correlation coefficient ranges from 1 for perfect positive correlation to -1 for perfect negative correlation and is in the vicinity of 0 for uncorrelated variables.

2.3.3 *The Variogram*

In a statistical analysis, a variogram shows how well a data set performs in space – it is a measure of spatial correlation. A variogram contains two elements: the experimental variogram and the variogram model. Variograms are used to show spatial continuity in a set of a data and can show textural differences that cannot be represented on a regular graph (Stephanie, 1999-2011). In short, variogram characterizes the spatial continuity or roughness of a data set.

Barnes (2003), states that the experimental variogram is calculated from the data while the variogram model is fitted to the data. The experimental variogram is calculated by averaging one-half the difference squared of the z-values over all pairs of observations with the specified separation distance and direction. It is plotted as a two-dimensional gragh. The variogram model is, however, chosen from a set of mathematical functions that describe spatial relationship. The appropriate model is chosen by matching the shape of the curve of the experimental variogram to the shape of the curve of the mathematical function. In order to clearly see what variogram represent, let us consider two synthetic data sets; A and B with some common descriptive statistics are given in table 2.1

Table 2.1 Descriptive Statistics of Two Data Sets.

Source: (www.goldensoftware.com)

	A	B
Count	15251	15251
Average	100.00	100.00
Standard deviation	20.00	20.00
Median	100.35	100.95
10^{th} percentile	73.89	73.95
90^{th} percentile	125.61	124.72

Even though, the respective histograms may also be very similar, the two data sets may be significantly different in ways that are not captured by the common descriptive statistics and histograms.

Pincock et al.(2007), added that a variogram represents both structural and random aspects of the data under consideration. The range of a variogram represents the structural part of the variogram model. The variogram values increase with increase in the distance of separation until it reaches a maximum (C) at a distance known as the range (a). if at a distance nearly equal to zero, that is, $h \rightarrow 0$, the variogram value is greater than zero, this value is known as the nugget-effect, (C_0).

Low range values exhibit randomness in spatial distribution of a quantity being measured. On the other hand, high values of range reflect a distinct spatial structure of clusters of similar values.

According to Schlumberger (2011), the term variogram is sometimes used incorrectly in place of semi-variogram. The two differ only in that the semi-variogram uses each pair of data element only once, where as the variogram uses all possible data pairs. Semi- variograms are usually used instead of variograms, but opposite vector directions (for example, north and south) are recognized as representing the same thing and having identical ranges, sills, nugget points and the like. Variogram is therefore viewed as a two point statistical function that describes the increasing differences or decreasing correlation, or continuity, between sample values as separation between them increases.

The fundamental principle is that, things that are closer are more alike than things farther apart. Thus pairs of locations that are closer (far left on the x-axis) should have more similar values (low on the y-axis). As pairs of locations become farther apart (moving to the right on the x-axis), they should become more dissimilar and have a higher squared difference (move up on the y-axis) as shown in fig 2.1.

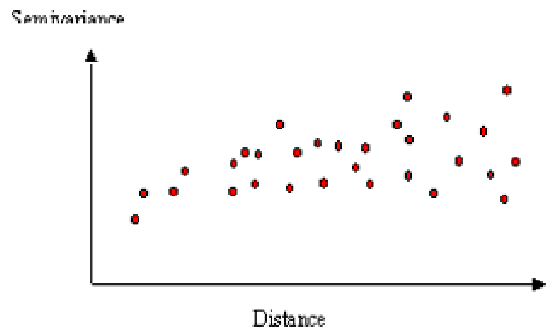


Fig 2.1: Things closer are more alike than things farther apart.

Source (resources.esri.com)

Data analysis exercise should have some geological backing and simple variogram analysis is not a magic tool to get the best results. The right approach is that the variogram model must conform the geology. The effect of a variogram should be related partly on the data set but mostly on the knowledge of the geological setting and other external facts. The geological understanding should also determine the range, direction, etc. variogram analysis can also be a method for identifying environments like fluvial systems and lower energy levels. Other areas include: finding possible outliers, detecting anomalies and level of heterogeneity, determining anisotropies (that is, different variograms in different directions), detecting changes in functions due to scale (that is, specially geological trend) and use it to quantify geology (the link between geology and maths/statistics) (Janine, 2011).

Spatial prediction involves two stages. First, the covariance or semi-variogram of the spatial process is modeled. This involves choosing both a mathematical form and the values of the associated parameters. Second, this dependence model is used in solving the kriging system at a specified set of spatial point, resulting in predicted values and associated standard errors (SAS, 1999).

2.3.4 *Kriging*

Kriging is estimating values at those locations which have not been sampled. The basic technique is "ordinary kriging" uses a weighted average of neighbouring samples to estimate the unknown value at a given location. Weights are optimized using the semi-variogram model, the location of the samples and all the relevant inter-relationship between known and unknown values. The technique also provides a standard error which may be used to quantify confidence levels (Royal School of Mines, 1978).

According to Geoff (2007), kriging is optimal interpolation based on regression against observed z values of surrounding data points, weighted according to spatial covariance values. Interpolation is, however, an estimation of a variable at an unmeasured location from observed values at surrounding locations. Kriging assigns weights according to a (moderately) data-driven weighting function, rather than an arbitrary function. The goal of kriging is to determine weights that minimize the variance of the estimator. Some advantages of kriging are that, it helps to compensate for the effect of data clustering, assigning individual points within a cluster less weight than isolated data points (or treating clusters more like single point), it gives the kriging variance along with the estimate of the variable, z itself, availability of estimation error provides basic for stochastic simulation of possible realization of z(u).

Kumar (2006), describes kriging as an application of spatial statistical technique for spatial analysis. Kriging is a technique of making optimal, unbiased estimates of regionalized variables at unsampled locations using the structural properties of the variogram and the initial set of the data values. Kriging takes into consideration the spatial structure of the parameter and hence score over other methods like arithmetic mean method, nearest neighbor method, distance weighted method, and polynomial interpolation. Also, kriging provides the estimation variance at every estimated point, which is an indicator of the accuracy of the estimated value.

The kriging weights are obtained by substituting the respective distances into the covariance function. The problem of finding the weights subject to the constraints that they sum to unity is recast in its Lagrangian equivalent, and solution of the resulting set of Lagrangian equations then simultaneously yields both a set of weights and the Lagrange parameter. Once the weights are known the kriging prediction $z(u_k)$ and its expected variance, $\sigma^2(u_k)$, are obtained (Kumar, 2006).

2.4 The Black-Scholes Model

By way of history, the Black-Scholes model was first discovered in 1973 by Fischer Black and Myron Scholes, and then further developed by Robert Merton. Fischer Black started out working to create a valuation model for stock warrants. Soon after this discovery, Myron Scholes joined Black and the result of their work is a pricing model we use today which is surprisingly accurate. The work of Black and Scholes is actually an improved version of a previous model developed by A. James Boness in his Ph.D. dissertation at the University of Chicago. The Black and Scholes' improvement on the Boness model comes in the form of a proof that the risk-free interest rate is the correct discount factor, and with the absence of the assumptions regarding investor's preferences (SOME, 2005).

According to Maxi-Pedia (2012), since Robert Merton participated in the model's creation, it is sometimes referred to as Black-Scholes-Merton model. Just as every model is only an approximation of the real world and every model has some limitations, the Black-Scholes model has its own shortcomings. For this reason, throughout the years, many other models emerged trying to provide more accurate approach to option valuation. However, all of them are based on the same valuation principle. The difference between the models is mostly how they address the assumptions of the Black-Scholes model. For example, Generalized Auto Regressive Conditional Heteroskedasticity (GARCH) substitutes constant volatility with a stochastic or random one. Other models address other assumption, for example the assumption of constant interest rate. Example of some few models related to valuations are: Garman-Kolhagen Option Pricing Model which is used for currency pricing, Hull-White, Cox-Ingersoll-Ross, Vasicek, Cox-Ross-Rubenstein model etc.

2.4.1 *Importance of Black-Scholes Model*

The Black-Scholes model is a mathematical formula designed to price an option as a function of certain variables – generally stock price, striking price, volatility, time to expiration,, dividends to be paid, and the current risk-free interest rate. It is a tool for equity option pricing. Options traders compare the prevailing option price in the exchange against the theoretical value derived by the Black-Scholes model in order to determine if a particular option contract is over or under valued, hence assisting in the option trading decision (Wikinvest, 2007).

ICMBA (1999-2010), states that the Black-Scholes model became the standard method of pricing options with the following assumptions that: the option can be exercised only at expiration, it requires that both the risk-free rate and the volatility of the underlying stock price remain constant over the period of analysis, the underlying stock does not pay dividend; adjustments can be made to correct for such distributions.

According to Coelen (2002), common stock represents an ownership in a corporation. Stocks provide a claim to the corporation's income and assets. A person who buys a financial asset in hopes that it will increase in value has taken a long position. A person who sells a stock before he/she own it hoping that it decreases in value is said to be short in asset. People who take short positions borrow the asset from large financial institutions, sell the asset, and buy the asset back at a later time. In recent years, derivatives have become increasingly complex and important in the world of finance. A derivative is a financial instrument whose value depends on the value of other basic assets, such as common stock. Many individuals and corporations use derivative to hedge against risk. A call option gives the owner the right to buy the underlying asset on a certain date for a certain price. On the other hand, a put option gives the owner the right to sell the underlying asset on a certain date for a certain price.

Bahuguna (2000), states that there are industries in which Black-Scholes has important application. Theorists would argue that asset ownership decisions are like evaluations of investment opportunities and, as such, can be treated like financial options. An option with a strike price of $100 is essentially a loan of $100 from the asset owner which is payable only when one exercises the right to own the asset. Oil companies, and for that matter solid mineral companies, are likewise partial to the technique since the volatility of outcomes can be readily modeled. The volatility of finding oil (or solid mineral) can be reliably modeled using seismic survey (or drilling) information.

As stated by FAS123 solutions (2008), the Black-Scholes model with the traditional Black-Scholes closed-form, but modified for employee stock options by inputting an expected term in place of the contractual option term- say ten years, is among the most widely accepted fair option value models. The Black-Scholes, when modified for employee stock option, summarizes all behavioral information with a single number – the expected term. All options are treated as if they disappear at the same time independent of how the stock price evolves over the option term. The Black-Scholes is easy to implement once the expected term is estimated.

The write up by Bowman (2001), states that the investment decision represents a stylized description of the critical process by which organizations commit resources to future growth. Real options analysis appears to present formidable for implementation. In valuing strategic options, perhaps the most important assumption concerns the distribution of the underlying stock price. In the Black-Scholes formula, the stock price is assumed to follow a lognormal distribution, with a constant level of volatility.

Thinking options is very useful for forming strategy. But options can be very costly to acquire and to maintain or keep alive. Investment should be made in options with value greater than their cost. In markets with uncertainty both strategy and finance are focus on valuing risky assets. This means that the risk/return tradeoff of traded assets should be important benchmarks for corporate assets as well. Using the financial options tools for real options brings this type of discipline to corporate strategy (Amram, 2004).

According to Breman et al. (1985), to illustrate the use of option pricing to value natural reserves, consider an offshore oil property with an estimated reserve of 50 million barrels of oil; the cost of developing the reserve is expected to be $600 million, and the development lag is two years. The firm has the rights to exploit this reserve for the next 20 years, and the marginal value per barrel of oil is $12 currently (that is, price per barrel – marginal cost per barrel). Once developed, the net production revenue each year will be 5% of the value of reserve. The riskless rate is 8%, and the variance in

The basic mathematics behind the Black-Scholes model is also presented; the main model that would be used for decision analysis. The analytical approach to the models is highlighted with some hypothetical data.

3.1 Geostatistics

Geostatistics is a class of statistics used to model, analyze and predict the values associated with spatial or spatiotemporal phenomena. Geostatistics is widely used in many areas of science and engineering, for example:

i. The mining industry uses geostatistics for several aspects of a project: initially to quantify mineral resources and evaluate the project economic feasibility.
ii. In the environmental sciences, geostatistics is used to estimate pollutant levels in order to decide if they pose a threat to environmental or human health and warrant remediation.
iii. Relatively new applications in the field of soil science focus on mapping soil nutrient levels (nitrogen, phosphorus, potassium, and so on) and other indicators (such as electrical conductivity) in order to study the relationships to crop yield and prescribe precise amount of fertilizer for each location in the field.
iv. Meteorological applications include prediction of temperatures, rainfall, and associated variables (such as acid rain).
v. Most recently, there have been several applications of geostatistics in the area of public health, for example, the prediction of environmental contaminant levels and their relation to the incidence rates of cancer.

In all these examples, the general context is that there is some phenomenon of interest occurring in the landscape (Esri, 2011).

According to Geoff (2007), the basic components of geostatistics are: (semi)variogram analysis, kriging and stochastic simulation. The basic statistics which define geostatistics are:

(i) Covariance,

With the residual component treated as a random field with a stationary mean of 0 and a stationary covariance (a function of lag, h, but not of position, u); $E(R(u)) = 0$ and

$$Cov\{R(u), R(u+h)\} = E\{R(u).R(u+h)\} = C_R(h) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(3.16)$$

The residual covariance function is generally derived from the input semivariogram model:

$$C_R(h) = C_R(0) - \gamma(h) = Sill - \gamma(h) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.17)$$

For simple kriging, we assume that the trend component is a constant and known mean, $m(u)=m$, so that

To minimize the error variance, we take the derivative of the above expression with respect to each of the kriging weights and set each derivative to zero. This leads to the following system of equations:

3.1.1 *Geostatistical Analysis Workflow*

The figure 3.1 below presents the summary of geostatistical analysis workflow.

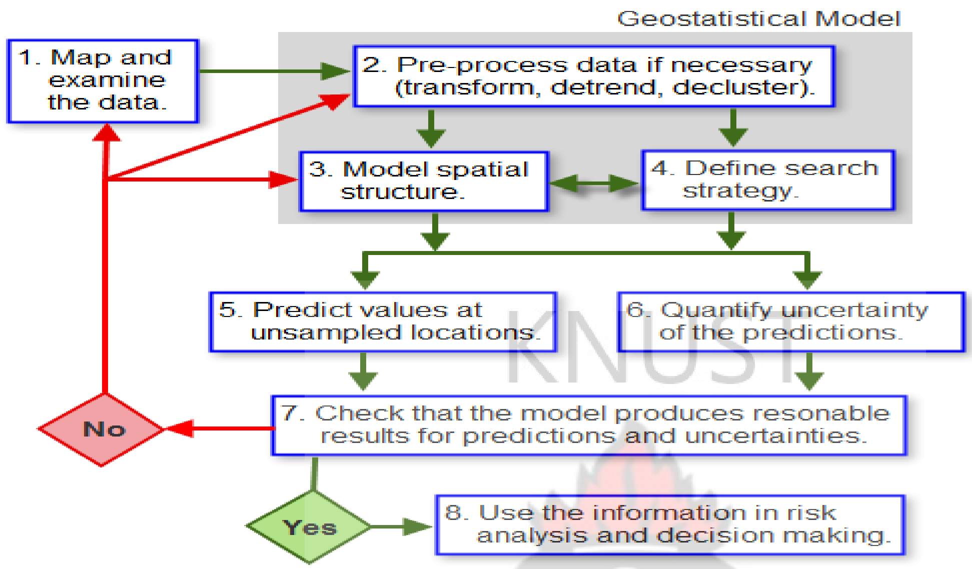


Fig 3.1: The geostatistical analysis workflow

(source: Esri.com)

3.1.2 *Geostatistical Modeling and Semi-variogram Analysis*

The semi-variogram is a graph of semi-variance, γ(h) plotted against the lag distance, h. The simi-variance is, however a statistical formula which is given by;

γ(h) =

representing separation between two spatial locations; N(h) is the number of pairs separated by lag h (plus or minus the lag tolerance).

During the construction of the semi-variogram, the semi-variance(γ(h)) is computed for all variable pairs with separation distance, h and the points (h, γ(h)) are plotted. Consequently, the appropriate geostatistical model is fitted to allow for kriging. Among the various geostatistical models: exponential, nugget effect, spherical, Gaussian, power method, etc, the most commonly used model for gold estimation is the spherical model. For the purpose of this thesis, the spherical model will be fitted to the experimental variogram. From the variogram model, the basic parameters that are noted for kriging include: the range, the sill value and the nugget value. In constructing the semi-variogram, the data is collected on a grid as shown in fig 3.2

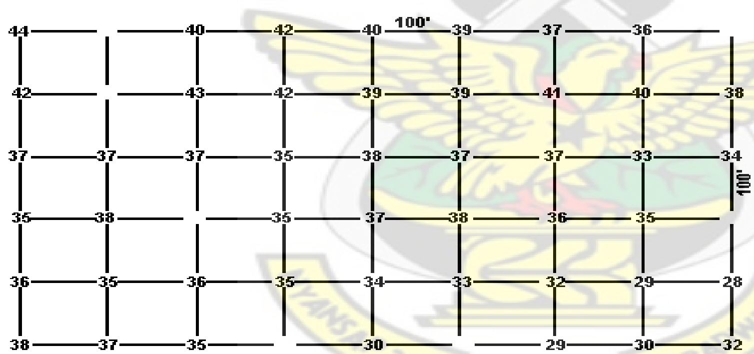


Fig 3.2: Example of data on a grid.

Source (Isobel, 1979).

If the east-west direction is considered to construct an experimental semivariogram for this relative orientation then, the grid on which the holes have been so conventionally placed is 100ft by 100ft, so that the values of the semi-variogram, $\gamma^*(h)$, for distances which are multiples of this. At zero, $\gamma^*(0)$ is equal to zero. At 100ft all pairs of the sample is found at a separation of 100ft in the east-west direction as shown in fig 3.3

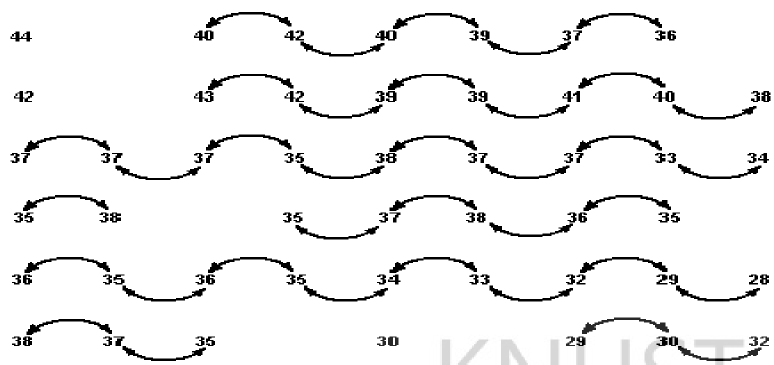
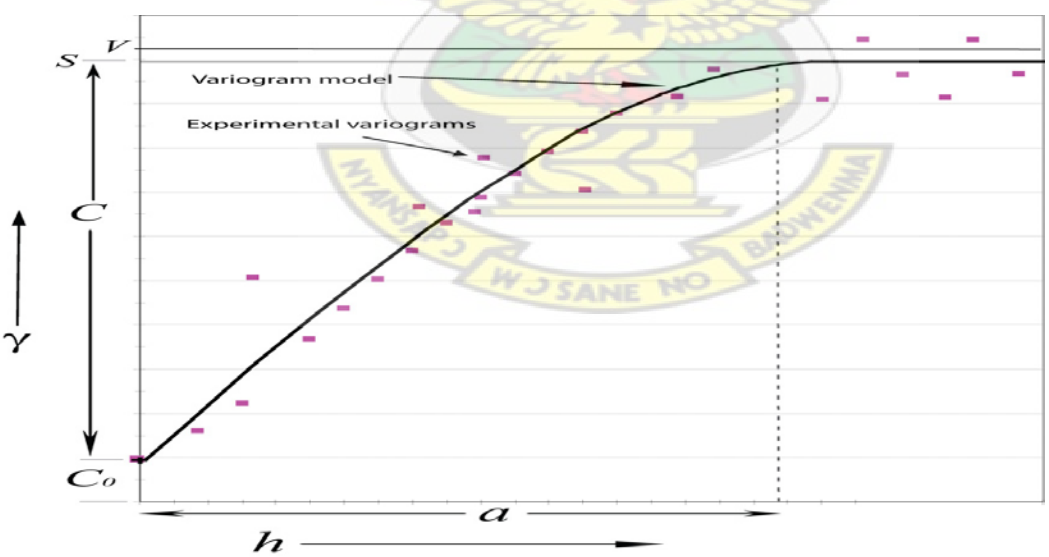


Fig 3.3: Identifying all the pairs at 100ft apart in the east-west direction.

Source (Isobel, 1979).

A spherical variogram model fitted to the experimental semi-variogram points is shown in fig 3.4



$$\gamma = C_0 + C*(1.5*\left(\frac{h}{a}\right) - 0.5*\left(\frac{h}{a}\right)^3)$$

Fig 3.4: Example of spherical variogram model fitted.

Source (Pincock, 2007)

3.1.3 *MATLAB Spatial Statistics Toolbox*

The MATLAB spatial statistics toolbox includes simultaneous spatial autoregression (SAR), conditional spatial autoregression (CAR), mixed regressive spatially autoregressive (MRSA) models. In addition, it contains codes for creating sparse spatial weight matrices and finding the log determinants (needed for maximum likelihood). Hence, the MATLAB spatial statistics toolbox includes the most common estimators employed in spatial econometrics. These products use sparse matrices and other computational techniques to greatly accelerate computations and to expand the size of potential data sets analyzed (Kelley, 1997).

A simple implementation of variogram function using matlab code is found in APPENDIX G–J.

In the MATLAB command window, each pair of value separated by a lag distance, h will be looked at in the following manner to produce the experimental semi-variogram as well as fit a spherical variogram model. From the fitted spherical variogram model, the range, sill, nugget-effect will be deduced for kriging.

During the programming process in MATLAB, the sets (a1, a3, a5, a7, a9, …) and (a2, a4, a6, a8, a10, …) represent the head and tail values of gold grade separated by lag distance, h, respectively such that: for h= (say 10m), we have (a1,a2) as the two row vectors that contain the pairs (that is, head values in a1 and tail values in a2) that will be computed to obtain the semi-variance (v1). Similarly, for h=20m, (a3,a4) represent the two row vectors that contain the pairs that will be computed to obtain the semi-variance (v2). From the foregoing analysis, the points (h=10m, v1) and (h=20m, v2) will be plotted on the variogram. Consider the following formulation:

h=10m;

But as indicated earlier, the spherical model is considered for this thesis. For each of the models above, the covariance, C(h), is given by:

$C(h) = C(0) - \gamma(h)$, ………………………………………………………………..(3.37)

3.1.4 *Kriging Process*

As already stated in the previous chapters, kriging is the process of estimating values at those locations which have not been sampled. Among the various kriging techniques which include simple, ordinary, cokriging, indicator kriging, etc, the ordinary kriging technique will be used for the estimates. During the process of kriging, some few data points (say 8) surrounding the point to be estimated are used for the estimation as in figure 3.5. The estimated mean grade of the point (PE) gives the average grade of the block, A. In the analysis, weights (w_i) are assigned to each of the sampled location (or value) such that:

$B^1 W^1 = c$ ……………………………………………………………………(3.39)*

If PE is the point to be estimated then P1PE, P2PE, P3PE,…, P8PE are the separation distances between samples and the block to be estimated. Similarly, the set P1P2, P1P3, P1P4,…, P1P8 is an example of separation distances between samples. The illustration is shown on table 3.1.

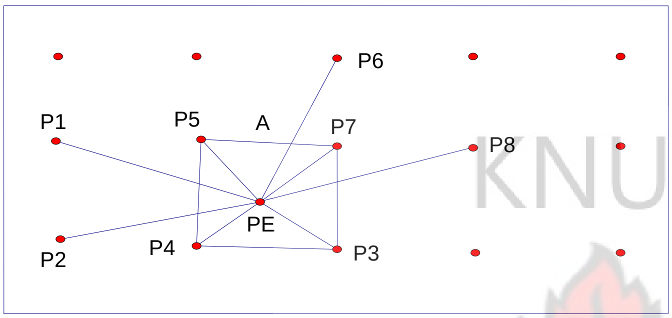


Figure 3.5: Diagram of 8 sampled points used to estimate block A

Table 3.1 Distances between samples and point to be estimated.

	P1	P2	P3	P4	P5	P6	P7	P8	PE
P1	0	25	37	18	23	29	33	19	14
P2	25	0	16	26	28	17	30	22	21
P3	37	16	0	34	38	31	35	24	16
P4	18	26	34	0	37	27	39	31	20
P5	23	28	38	37	0	34	36	20	15
P6	29	17	31	27	34	0	39	38	27
P7	33	30	35	39	36	39	0	21	19
P8	19	22	24	31	20	38	21	0	35
PE	14	21	16	20	15	27	19	35	0

If C(0)= 0.9 and range, a= 14.5 are the hypothetical variogram parameters based on a spherical model fit then,

C(h)=0.9-0.9*(1.5*(h/14.5)-0.5*(h/14.5).^3), ………………………………(3.40) and the following system of equations, B, is generated from Table 3.1.

$$B=\begin{pmatrix}
0.9000 & 0.8788 & 4.9319 & 0.0850 & 0.5546 & 1.8000 & 3.1322 & 0.1435 \\
0.8788 & 0.9000 & 0.0149 & 1.0737 & 1.5334 & 0.0424 & 2.0923 & 0.4234 \\
4.9319 & 0.0149 & 0.9000 & 3.5360 & 5.4616 & 2.4112 & 3.9701 & 0.7060 \\
0.0850 & 1.0737 & 3.5360 & 0.9000 & 4.9319 & 1.2916 & 6.0249 & 2.4112 \\
0.5546 & 1.5334 & 5.4616 & 4.9319 & 0.9000 & 3.5360 & 4.4351 & 0.2188 \\
1.8000 & 0.0424 & 2.4112 & 1.2916 & 3.5360 & 0.9000 & 6.0249 & 5.4616 \\
3.1322 & 2.0923 & 3.9701 & 6.0249 & 4.4351 & 6.0249 & 0.9000 & 0.3118 \\
0.1435 & 0.4234 & 0.7060 & 2.4112 & 0.2188 & 5.4616 & 0.3118 & 0.9000
\end{pmatrix}$$

If the Lagrange multiplier and unity-sum is taking into consideration, the matrix (B^1) emerges:

$$B^1=\begin{pmatrix}
0.9000 & 0.8788 & 4.9319 & 0.0850 & 0.5546 & 1.8000 & 3.1322 & 0.1435 & 1.0000 \\
0.8788 & 0.9000 & 0.0149 & 1.0737 & 1.5334 & 0.0424 & 2.0923 & 0.4234 & 1.0000 \\
4.9319 & 0.0149 & 0.9000 & 3.5360 & 5.4616 & 2.4112 & 3.9701 & 0.7060 & 1.0000 \\
0.0850 & 1.0737 & 3.5360 & 0.9000 & 4.9319 & 1.2916 & 6.0249 & 2.4112 & 1.0000 \\
0.5546 & 1.5334 & 5.4616 & 4.9319 & 0.9000 & 3.5360 & 4.4351 & 0.2188 & 1.0000 \\
1.8000 & 0.0424 & 2.4112 & 1.2916 & 3.5360 & 0.9000 & 6.0249 & 5.4616 & 1.0000 \\
3.1322 & 2.0923 & 3.9701 & 6.0249 & 4.4351 & 6.0249 & 0.9000 & 0.3118 & 1.0000 \\
0.1435 & 0.4234 & 0.7060 & 2.4112 & 0.2188 & 5.4616 & 0.3118 & 0.9000 & 1.0000
\end{pmatrix}$$

1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 0.0000

$c = (0.0016, 0.3118, 0.0149, 0.2188, 0.0016, 1.2916, 0.1436, 3.9701, 1.0000)^T$

w_i and μ are computed from the relation:

$w^1 = (w_1, w_2, w_3, w_4, \ldots w_8, \mu)^T = inv(B^1).*c$, ……………………………………(3.41)

$w^1 = (w_1\ w_2\ w_3\ w_4\ w_5\ w_6\ w_7\ w_8\ \mu)^T = (0.1606, 0.7380, -0.7321, -0.4948, -0.5047, 1.0207, 0.5698, 0.2424, -0.5157)^T$.

Knowing the respective weights assigned to each sample value and the Lagrange multiplier, the estimated mean grade and variance of block, A (at point PE) are easily calculated as follows: $Z_{PE} = sum(Z_P.*w) = Z_{P1}w_1 + Z_{P2}w_2 + Z_{P3}\ w_3 + \ldots + Z_{P8}w_8$ ………………………(3.42)

Supposed that $Z_{P1}=3.50$, $Z_{P2}=1.90$, $Z_{P3}=4.80$, $Z_{P4}=2.00$, $Z_{P5}=2.70$, $Z_{P6}=3.58$, $Z_{P7}=4.78$ and $Z_{P8}=4.50$ are the realization of gold grades (g/t) at the locations P1, P2, …, P8 respectively then, $Z_{PE} = 0.0241$g/t. Again, if the mean grade of the 50 sample points is realized to be 3.98g/t then the residuals at the 8 (selected) locations are given by:

$R_1 = 3.50 - 3.98 = -0.48$, $R_2 = 1.90 - 3.98 = -2.08$, $R_3 = 0.82$, $R_4 = -1.98$, $R_5 = -1.28$, $R_6 = -0.40$, $R_7 = 0.80$, $R_8 = 0.52$.

The estimated residual (R_{PE}) from the mean of the block PE is given by: $R_{PE} = sum(w.*R) = w_1R_1 + w_2R_2 + ... + w_8R_8 = -0.4131$ ……………………………(3.43)

The estimated grade of block, A (at point PE) is evaluated to be:

$Z^{\circ}_{PE} = 3.98 + R_{PE} = 3.98 + (-0.4131) = 3.5669$g/t ……………………………….(3.44)

The estimated variance, otherwise known as the kriging variance,

$\sigma^2_E = \sigma^2_{max}$-

Also,

1oz of gold =28g of gold.

From this information, the total ounce of gold of the estimated block will be evaluated.

From the foregoing analysis, the following deductions are made: Area (A) of the estimated block= length (l)*breadth (b); Volume (V) of gold of estimated block= thickness (h)*area (A); Quantity (m) of gold of estimated block= Z°_{PE} *volume (V)*specific gravity (ρ) of gold, where Z°_{PE} (g/t) represents the estimated grade of block under study. For example, Z°_{PE} =3.5669g/t for the hypothetical data considered above. The resulting quantity (m) of gold will be in grams.

If 28grams of gold is equivalent to 1ounce (oz) of gold then the estimated quantity of gold, Z°_{PE}* V* ρ, will be equivalent to

$\{(Z^{\circ}_{PE}* V* \rho)/28\}$ ounce of gold …………………………………………..(3.49)

Given the current world market price of $1,600.00 per ounce, the approximate current value of the estimated block will be:

$\$(1600*Z^{\circ}_{PE}* V* \rho)/28$ …………………………………………………….(3.50)

Consider the hypothetical data in figure 3.1 above, if the area of the estimated block=10*10 = 100m^2, thickness= pit limit (depth) =10m, specific gravity of gold= 19.3t/m^3, then volume of gold= 10*10*10= 1000m^3, quantity of gold of estimated block= 3.5669*1000*19.3= 68841.17g. The ounce of gold of the estimated block= 68841.17/28= 2458.61(oz). With the current world market price of $1,600.00 per ounce, the approximate current value (or price) of the estimated block will be 1600*3.5669*1000*19.3/28= $3,933,781.143.

3.2 Derivation of the Black-Scholes Differential Equation

According to Coelen (2002), the Black-Scholes model is derived as follows: consider the price of a stock S at time t. Consider also a small time interval dt during which the price of the underlying asset S changes by an amount dS. The most common model separates the return on the asset, dS/S into two parts. The first part is completely deterministic and this yields a contribution of μdt to dS/S. Here μ is a measure of the average rate of growth of the stock, also known as the drift. In this model, μ is assumed to be the risk-free interest rate on a bond. The second part of the model accounts for the random changes in the stock price due to external effects, such as unanticipated news. It is best modeled by a random sample drawn from a normal distribution with mean zero and contributes σdB to dS/S, where σ is defined as the volatility and B denotes Brownian motion. σ and μ are functions of S and t. putting this information together, the stochastic differential equation:

$$dS/S = \mu dt + \sigma dB \qquad (3.51)$$

The Brownian motion is a stochastic process $B(t)$ characterize by the following three properties:

1. Normal increments: $B(t)-B(s)$ has a normal distribution with mean zero(0) and variance , $t-s$ if $s=0$ then $B(t)-B(0)$ has normal distribution with mean 0 and variance t.

2. Independence of increments: $B(t)-B(s)$ is independent of the past.

3. Continuity of paths: $B(t)$, $t>0$ are continuous functions of t.

The stochastic integrals with respect to Brownian motion which are commonly called Itô integrals will be introduced. In order to derive the Black-Sholes formula, the stochastic integral is defined. If X(t) is a constant, c, then:

Substituting dS for $\mu Sdt+\sigma SdB$,the result is ,

$dV(S,t)=$

where r is the risk-free interest rate. Substituting the expressions for $d\pi$ and π gives:

and

The Black-Scholes, the call option, model is given by:

$C(S,t) = SN(d_1) – Ke^{-rt}N(d_2)$

where $C(S,t)$ – is the theoretical call premium, S – the current stock price, t – time until option expiration, K – option striking (or exercise) price, r – risk-free interest rate (or the drift), N – cumulative standard normal distribution, e – exponential term (2.7183),

$d_1 =$

Since the investment decision is based on cost of mining the estimated block (C_p), that is, if C $>C_P$, the decision will be to mine, if C $<C_p$, the decision will be not to mine. With the critical look at the results of C, the decision is that mining will take place over the period of the first five (5) years if the base price of the estimated gold is $3933781.143. However the maximum revenue is more likely at the end of the expiration period (that is, in the fifth year).

CHAPTER 4

DATA ANALYSIS AND RESULTS

4.0 Introduction

In this chapter, the exploratory data - secondary (including some basic information on parameter values) which is available to Goldfields Ghana Limited (from site, Teberebie Cut3 of the mine) was collected for analysis. The detail of the data available for geostatistical analysis include: average grade of gold (assayed values, in g/t) per each drill hole of 50, spacing of drill holes (in metres), coordinates (x, y) of each drill hole location (otherwise known as easting and northing). For evaluation of the geostatistical estimates and decision making analysis, the cost per ounce of gold was collected.

The other vital information that was collected includes; specific gravity (ρ) of the commodity (this is standard for gold, 19.3t/m^3), pit limit (depth) and the current world market price of gold. The area of site over which the data was taken, volume of commodity recoverable, quantity (both in grams and ounce) of gold recoverable, were computed from the information available.

In order to make a decision, the time until the expiration of the contract was noted, the current price of the estimated gold was used as the base price. With this base price, other five sensitivity cases were analyzed. The cost of purchasing the site (mine), projected cost of land reclamation as well as initial outlay to initialize the investment was collected (this information will form the basis for the strike price when using the Black-Scholes, call option, model).

4.1 The Area of Study

Goldfields Ghana Limited is currently undertaking surface mining with its concession spreading over 176km^2 and which is located to the north and northwest of the town of Tarkwa in the Western region of Ghana. In August 2000, the Tebrebie concession was acquired as part of the mine. This statistics on the site was necessary, especially to management, because management was considering the expansion of the mine. Fig. 4.1 is the map of Southern Ghana showing the location of Tarkwa and the concession of Goldfields Ghana Limited (GFGL).

Fig 4.1: The map of Southern Ghana showing the Tarkwa township and the concession of GFGL.

4.2 Geostatistical Data Analysis

The data (secondary) for geostatistical analysis which was collected from the site, on 13th Dec, 2011 by Mr Agbati Sebastian (senior staff in charge of drill and blast operations) is found in table 4.1a of APPENDIX K. It contains the easting and northing co-ordinates of the locations with their respective average grade of gold realized from the assay. Table 4.1b below, however, shows the parameter values for evaluation of geostatistical estimates and decision analysis.

The analysis of the data was done using MATLAB software to estimate averages, construct semi-variogram, compute kriging values, and obtain results for decision analysis using the Black-Scholes (call option) model. Figure 4.2 depicts the data points on the field indicating the ten points that are used to estimate block 1 (25x50). In the geostatistical data analysis, the following assumptions are made:

I. Isotropy – the grade distribution is independent of direction.

II. Drilled holes are equally spaced.

Table 4.1b Parameter values for evaluation of geostatistical estimates and decision analysis

specific gravity of gold	$19.3t/m^3$
pit limit (thickness or depth)	30m
area of site	$5000m^2$(50X100)
Block size	$50x25=1250m^2$
Cost of initial outlay per block	$42000000
Cost of purchasing site (includes cost of land reclamation)- per block	$106123856.1
cost per ounce of gold	$540
current price per ounce of gold (3rd Jan. 2012)	$1600.20

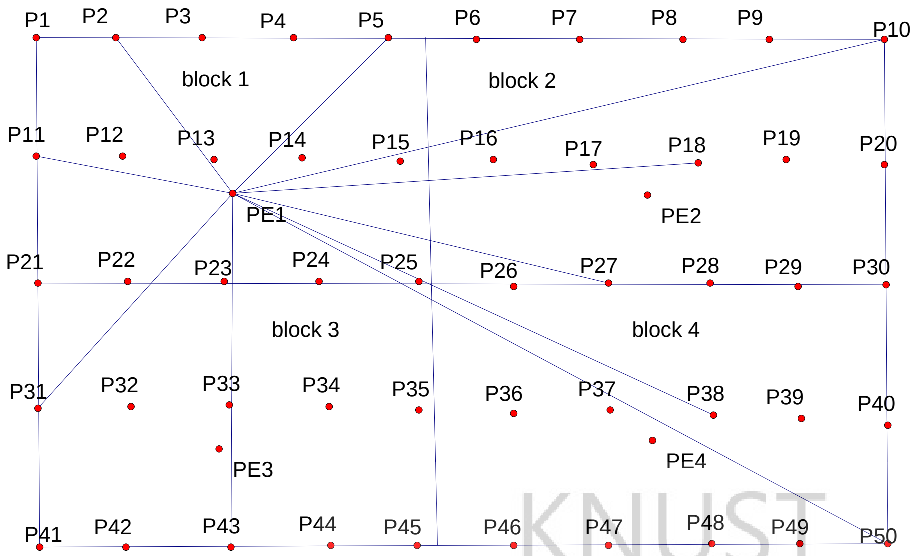


Fig 4.2: Data points of the field at Teberebe Cut3 of Goldfields Ghana Limited

4.2.1 *The Semi-variogram of Field Data*

Here, the omnidirectional semi-variogram is considered since that reveals the average behavior of the distribution. This is shown in figure 4.3.

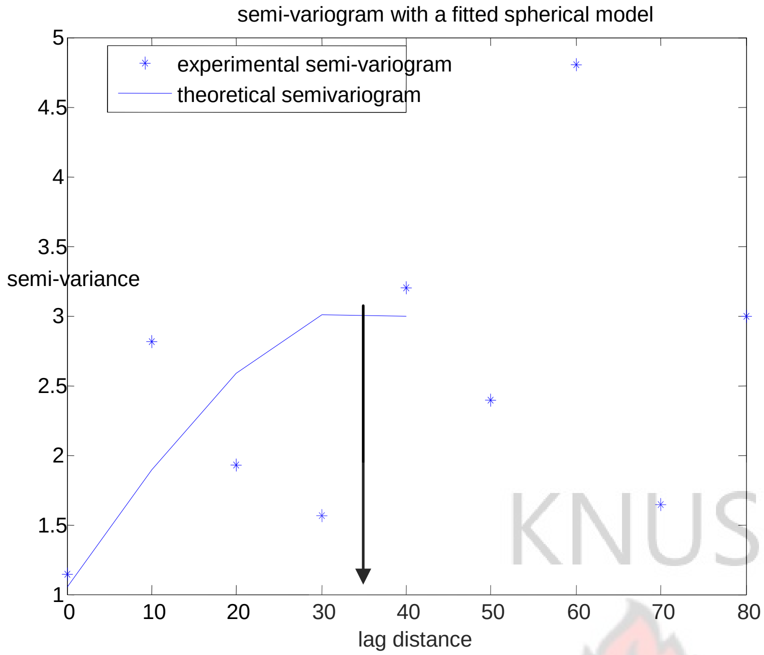


Figure 4.3: The semi- variogram of grade distribution

4.2.2 *Application of Ordinary Kriging to Field Data*

In the kriging process ten sample points are used to estimate the grade at point, PE1 (which represents the average grade of block 1), with co-ordinates (251,139) as indicated in fig 4.2. The distances between the selected sample points (including point, PE1, to be estimated) are shown in table 4.2.

Table 4.2 Distances between selected sample points

	P2	P5	P10	P11	P18	P27	P31	P38	P43	P50	PE1
P2	0.00	88.39	32.45	92.66	25.08	21.02	12.37	24.04	163.53	107.47	133.45
P5	88.39	0.00	100.18	76.38	111.40	183.86	98.67	71.28	86.31	118.07	122.64
P10	32.45	100.18	0.00	77.01	25.96	18.03	24.74	53.26	161.90	80.40	109.90
P11	92.66	76.38	77.01	0.00	101.55	73.41	94.37	95.19	94.59	43.38	48.47
P18	25.08	111.40	25.96	101.55	0.00	30.61	13.04	48.84	181.60	106.07	135.86
P27	21.02	183.86	18.03	73.41	30.61	0.00	21.10	36.88	151.01	86.47	112.64
P31	12.37	98.67	24.74	94.37	13.04	21.10	0.00	36.40	170.70	104.02	132.02
P38	24.04	71.28	53.26	95.19	48.84	36.88	36.40	0.00	152.62	118.68	140.51
P43	163.53	86.31	161.90	94.59	181.60	151.01	170.70	152.62	0.00	131.03	112.38
P50	107.47	118.07	80.40	43.38	106.07	86.47	104.02	118.68	131.03	0.00	33.42
PE1	133.45	122.64	109.90	48.47	135.86	112.64	132.02	140.51	112.38	33.42	0.00

From the semi-variogram in fig 4.3, the range, a, = 30m, the nugget variance, C(0), =3.0 and the covariance is given by:

C(h) =3.0-3.0*(1.5*(h/30)-0.5*(h/30).^3) ………….. (4.01)

Thus, using the actual distances (h) between the sample points computed from their co-ordinates in Table 4.2, the following system of data covariance matrix is generated.

$c = (115.02,87.08,60.26,2.06,121.94,65.50,111.03,136.04,64.99,0.06,1.00)^T$

$$
B=\begin{pmatrix}
3.00 & 28.11 & 0.03 & 33.30 & 0.11 & 0.36 & 1.25 & 0.17 & 221.42 & 55.84 \\
28.11 & 3.00 & 43.83 & 16.30 & 63.09 & 320.72 & 41.57 & 12.43 & 25.77 & 76.73 \\
0.03 & 43.83 & 3.00 & 16.82 & 0.08 & 0.62 & 0.13 & 3.40 & 214.47 & 19.81 \\
33.30 & 16.30 & 16.82 & 3.00 & 45.95 & 13.97 & 35.54 & 36.64 & 35.83 & 1.03 \\
0.11 & 63.09 & 0.08 & 45.95 & 3.00 & 0.002 & 1.17 & 2.15 & 308.48 & 53.39 \\
0.36 & 320.72 & 0.62 & 13.97 & 0.002 & 3.00 & 0.36 & 0.25 & 171.66 & 25.95 \\
1.25 & 41.57 & 0.13 & 35.56 & 1.17 & 0.36 & 3.00 & 0.22 & 253.73 & 49.93 \\
0.17 & 12.43 & 3.40 & 36.64 & 2.15 & 0.25 & 0.22 & 3.00 & 177.60 & 78.06 \\
221.42 & 25.77 & 214.47 & 35.83 & 308.48 & 171.66 & 253.73 & 177.60 & 3.00 & 108.33 \\
55.84 & 76.73 & 19.81 & 1.03 & 53.39 & 25.95 & 49.93 & 78.06 & 108.33 & 3.00
\end{pmatrix}
$$

$$
\mathbf{B^1}=\begin{pmatrix}
3.00 & 28.11 & 0.03 & 33.30 & 0.11 & 0.36 & 1.25 & 0.17 & 221.42 & 55.84 & 1.00 \\
28.11 & 3.00 & 43.83 & 16.30 & 63.09 & 320.72 & 41.57 & 12.43 & 25.77 & 76.73 & 1.00 \\
0.03 & 43.83 & 3.00 & 16.82 & 0.08 & 0.62 & 0.13 & 3.40 & 214.47 & 19.81 & 1.00 \\
33.30 & 16.30 & 16.82 & 3.00 & 45.95 & 13.97 & 35.54 & 36.64 & 35.83 & 1.03 & 1.00 \\
0.11 & 63.09 & 0.08 & 45.95 & 3.00 & 0.002 & 1.17 & 2.15 & 308.48 & 53.39 & 1.00 \\
0.36 & 320.72 & 0.62 & 13.97 & 0.002 & 3.00 & 0.36 & 0.25 & 171.66 & 25.95 & 1.00 \\
1.25 & 41.57 & 0.13 & 35.56 & 1.17 & 0.36 & 3.00 & 0.22 & 253.73 & 49.93 & 1.00 \\
0.17 & 12.43 & 3.40 & 36.64 & 2.15 & 0.25 & 0.22 & 3.00 & 177.60 & 78.06 & 1.00 \\
221.42 & 25.77 & 214.47 & 35.83 & 308.48 & 171.66 & 253.73 & 177.60 & 3.00 & 108.33 & 1.00 \\
55.84 & 76.73 & 19.81 & 1.03 & 53.39 & 25.95 & 49.93 & 78.06 & 108.33 & 3.00 & 1.00 \\
1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 0.00
\end{pmatrix}
$$

The Lagrange multiplier (μ) and weights (w_i) assigned to each of the sample point are computed as, $W^1 = (w_2\ w_5\ w_{10}\ w_{11}\ w_{18}\ w_{27}\ w_{31}\ w_{38}\ w_{43}\ w_{50}\ \mu)^T$ = inv (B^1)*c.

W^1= (0.27, 0.0002, -0.43, 0.07, -0.02, -0.05, 0.16, -0.48, 0.13, 1.36, 7.74)^T

The estimate at PE1 is given by: PE1=sum (W_i .*g_i) = 3.86g/t. Average (mean) grade of the 50 sample locations, AVG,= sum (g_i)/50, where g_i represents the grade at the ith sample location. Using MATLAB software, AVG =2.43g/t. The residual at each of the ten (selected) points is given by: $R_i = g_i$ – AVG. Table 4.3 shows the summary of residuals at the selected sample points.

Table 4.3 Summary of residuals at the selected sample points

selected point (P_i)	grade realized (g_i)	Residual (R_i)
p2	1.25	-1.18
p5	2.33	-0.1
p10	1.84	-0.59
p11	0.81	-1.62
p18	3.43	1
p27	2.2	-0.23
p31	3.22	0.79
p38	1.1	-1.33
p43	2.89	1.46
p50	3	0.57

The estimated residual (R_E) from the mean of the area under study is given by:

R_E=sum(W_i*R_i)=$w_2R_2+w_5R_5+ w_{10}R_{10} +...+w_{50}R_{50}$ = 1.56g/t.

The estimated grade of the block under study,

Z°_E = AVG +R_E = 2.43 +1.56 = 3.99g/t.

The kriging variance,

$\sigma^2_E = \sigma^2_{max}$-

4.3 Decision Analysis of Data

As stated in chapter three, the Black-Sholes (call option model) shall be used to compute the common parameters available for decision analysis on the mining project.

The parameters include, stock price (S) - the base case and sensitivity cases, exercise price (K), interest rate (r), volatility (σ) of gold price (see figure 4.4) and time (t) until expiration.

The Black-Sholes (call option model) is revisited:

$C(S,t)= SN(d_1) – Ke^{-rt}N(d_2)$

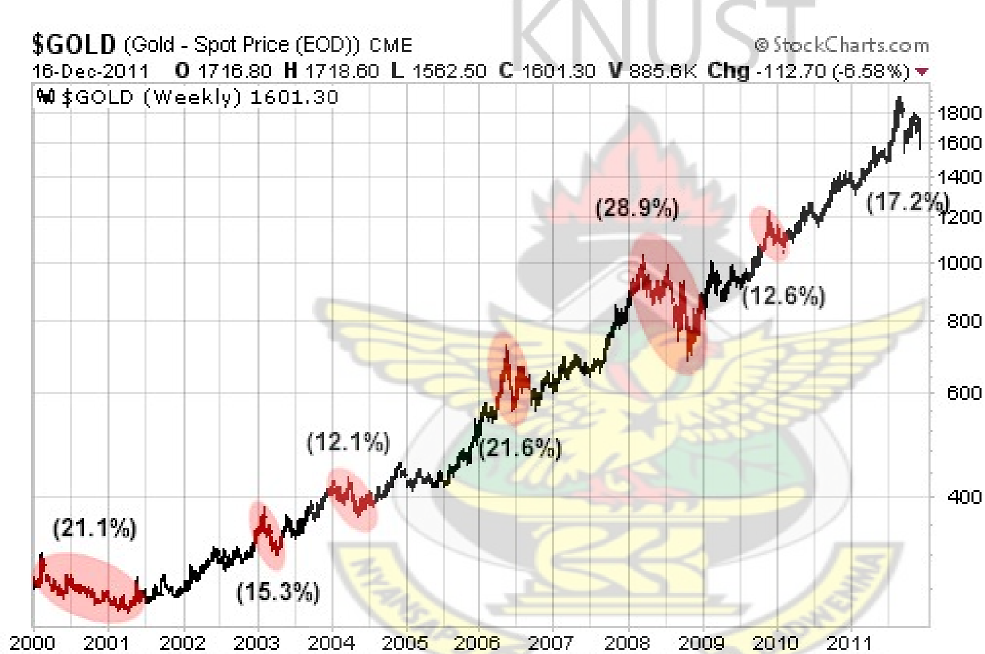
However, d_1=

Computation for the base case over a period of five years

(When price of gold is $1600.20 per ounce):

End of year 1 (time, t = 1)

$d_1 =$



$d_1 =$

$N(d_1) = 0.97982$

$d_2 = d_1 -$

End of year 3 (time, t=3)

$d_1 =$

Sensitivity case 2: (When the price of gold is around $1000 per ounce)

End of year 1 (time, t=1)

$d_1 =$

End of year 4 (time, t=4)

$d_1 =$

End of year 2 (time, t=2)

$d_1 =$

End of year 5 (time, t=5)

$d_1 =$

End of year 3 (time, t=3)

$d_1 =$

<u>Sensitivity case 5</u>: (When the price of gold is around \$700 per ounce)

End of year 1 (time, t=1)

$d_1 =$

End of year 4 (time, t=4)

$d_1 =$

Table 4.4: Summary of Project Call Option valuation Analysis

		Sensitivity Cases				
	Base Case	#1	#2	#3	#4	#5
common parameters:						
stock price ($)	165,035,634.9	123,761,256	103,134,380	92,820,942	82,507,504	72,194,066
exercise (strike) price ($)	148123856.1	148123856.1	148123856.1	148123856.1	148123856.1	148123856.1
interest rate (%)	28%	28%	28%	28%	28%	28%
price volatility (%)	18.4%	18.4%	18.4%	18.4%	18.4%	18.4%
first year time horizon cases						
option value ($)	58,839,000	26,495,000	14,229,000	9,465,000	5,728,000	3,054,000
Decision	invest	Not invest	Not invest	Not invest	Not invest	Not invest
second year time horizon case						
option value ($)	85,272,000	49,031,000	32,894,000	25,596,000	18,676,000	13,198,000
Decision	invest	Not invest	Not invest	Not invest	Not invest	Not invest
third year time horizon cases						
option value ($)	104,520,000	66,248,000	48,248,000	39,701,000	31,591,000	24,020,000
Decision	invest	invest	Not invest	Not invest	Not invest	Not invest
fourth year time horizon cases						
option value ($)	119,190,000	79,629,000	60,771,000	51,547,000	42,583,000	33,965,000
Decision	invest	invest	invest	Not invest	Not invest	Not invest
fifth year time horizon cases						
option value ($)	130,070,000	90,122,000	70,503,000	60,869,000	51,402,000	42,160,000
Decision	invest	invest	invest	invest	Not invest	Not invest

Note: The decision to mine (invest) or not to mine (not invest) is in respect with the cost of mining the estimated block of the mineral bearing rock (that is, $55,692,565.20).

CHAPTER 5

CONCLUSION AND RECOMMENDATIONS

5.0 Introduction

This chapter seeks to present the findings from the rigorous analysis of the geostatistical data as well as the decision analysis process; all pointing to managing the risk and establish the commercial viability of a discovered gold deposit of a mine in the western region of Ghana. The methods and models employed in the analysis assist in quantifying and presenting to management (decision makers) the various investment options.

The findings from the geostatistical analysis (which employed semi-variogram plot and kriging) reveal that the area under exploration would yield 3.99g/t and 2.89tons (or 103134.38 oz) of gold being the quality and quantity respectively, with volatility of 35.16.

Monitoring the world market price (of gold) and its (price) volatility closely, and with the careful application of the Black-Scholes model, the appropriate decision is taken.

5.1 Conclusion

The carefully application of the Black-Scholes model to the geostatistical estimates yielded the results in Table 4.4 from which the following conclusion is drawn:

i. The mining venture is lucrative from year1 up to the time until expiration under the base case (that is, when gold price is $1600.20 per ounce). Investment is recommended.
ii. For the sensitivity case 1 (that is, when gold price is around $1200 per ounce), investment may take place in the third, fourth and fifth years. Investment should not be made in first and second years.

iii. For the sensitivity case 2 (if gold price per ounce is around $1000), investment may take place in the fourth and fifth years. Investment should not be made in the first three years of the contract.

iv. For the sensitivity case 3 (if gold price per ounce is around $900), investment should not be made in the first four years of the contract. Investment could take place in the fifth year.

v. Sensitivity cases, 4 and 5, indicate clearly that mining will not be profitable if the price of gold falls to a value around $800 per ounce and below.

vi. At a price of $800 per ounce of gold, the cut-off grade should be higher than 3.99g/t.

vii. Investing into the mining of gold involves a huge capital outlay. Especially when one refers to the monetary values quoted (calculated) in dollars.

viii. The investment opportunity brightens with increasing expiration time of the contract.

5.2 Recommendation

After a thorough analysis of the data and the findings, this thesis suggests that:

i. The mining company should intensify the use of geostatistical model in its mining operations in order to produce more reliable estimates.

ii. The management should consider the use of Black-Scholes model in its decision making analysis. This would help decision makers to ascertain exactly when within the contract period to invest in order to make an optimal profit.

iii. Further research should be made into the topic, especially in the area of anisotropy and the use of modified forms of Black-Scholes model. This is because grade distribution may generally be influenced by direction, and the modified forms of Black-Scholes could also help overcome the limitations associated with the Black-Scholes model itself. This could help produce quite authentic results for more reliable decision to be taken. Some few examples of the modified forms of the Black-Scholes that could be looked at include: continuous dividend yield model, discrete yield model, time dependent parameters and transaction cost.

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APPENDIX A

S=E

APPENDIX B

APPENDIX C

The terms in *u* and

APPENDIX D

APPENDIX E

where

APPENDIX F

According to Ömür (2008), let us denote by $V_C(S, t)$ and $V_P(S, t)$ the values of the European call and put options respectively. Then the payoff functions are:

$V_C(S, t) = \max\{S-K, 0\}$ …………………………………………… (102)

$V_P(S, t) = \max\{K-S, 0\}$ ……………………………………………. (103)

Similar calculations for equation (3.101) shows that the value of a European put option $V_P(S, t)$ can be written as:

$V_P(S, t) =$

APPENDIX G

```
% computes 2D variogram {directional or omnidirectional}

% copyright alghalandis.com | Younes Fadakar Alghalandis

Function[h,g,c]=variogram(x,y,v,nL,dir,td)

n=numel(x);

t=zero (n*(n-1)/2, 1);

r=t;

k=1;

for 0=1:n-1                         % computes distances and angles

for p=0+1:n

  dx=x(p)-x(0);

dy=y(p)-y(0);
t(k)=atan2(dy,dx);

r(k)=sqrt(dx^2+dy^2);

k=k+1;

end

end
```

APPENDIX H

```
L=max(r)/nL                % initialize lag length

g=zeros(1,nL)              % initialize gammas with zeros

c=g;                       % and count of pairs

b=1;

ta=deg2rad(td);            % degree to radian

for a=deg2rad(dir)

for s=1:nL

h=s*L;

th=L/2;

k=1;

q=0;

g(b,s)=0;

c(b,s)=0;

for 0=1:n-1                % computes variogram for different h's

for p=0+1:n

if (abs(r(k)-h) & 1t;=th) && (abs(t(k)-a) & 1t;=ta)
```

APPENDIX I

```
g(b,s)=g(b,s)+(v(p)-v(0))^2;

q=q+1;

end

k=k+1;

end

end

g(b,s)=g(b,s)/(2*g);

c(b,s)=q;

end

b=b+1;

end

h=(1:Nl)*L;              % returns lags as h vector
```

Note: the above function has to be saved as separate file variogram.m according to matlab rule which requires the name of defined function and the container file to be exactly the same.

It can now be used in the main code as presented in the following example;

```
[X,Y,V]=textread('xyv.txt');   % reads data into variable vectors
```

APPENDIX J

```
                              % xyv.txt is a tab-delimited text format data file without headers

Vr=var(v);                    variance of the variable of interest

[h,g,c]=variogram(X,Y,V,30,0,90);   % computes variogram

Plot(h,g,’0-‘,[0,max(h)],[vr,vr],…  % draws the variogram and a line of variance

‘r--‘,’Linesmooth’,’on’);

x label(‘h(distance in m)’);        % label for x axis

y label(‘\gamma’);                  % label for y axis

title (‘variogram’);                % title of the chart

grid on;                            % show grids
```

(Source: alghalandis.com).

APPENDIX K

Table 4.1a Data for geostatistical analysis.

drill hole no.	easting (x-cordinate)	northing (y-cordinate)	grade (g/t)
1	213	126	2.91
2	118	128	1.25
3	131	147	1.22
4	204	129	1.79
5	151	210	2.33
6	106	132	1.1
7	149	216	1.46
8	259	146	1
9	214	212	3.9
10	145	110	1.84
11	206	157	0.81
12	233	257	2.66
13	254	247	2.3
14	154	137	1.47
15	223	305	0.05
16	217	236	0.1
17	201	244	3.3
18	120	103	3.43
19	119	113	2.12
20	100	114	3.6
21	124	120	4.2
22	128	123	2.8
23	144	135	3.92
24	132	143	1.75
25	148	140	5.4
26	138	230	4.93
27	139	127	2.2
28	246	222	4.5
29	234	112	3.89
30	220	109	1.86
31	121	116	3.22
32	312	216	0.09
33	215	310	1.72
34	304	200	3.79
35	195	122	1.5

36	220	220	1.98
37	238	341	1.2
38	111	151	1.1
39	316	208	2
40	242	98	2.8
41	246	277	3.55
42	224	317	4.4
43	228	249	2.89
44	292	154	1.71
45	317	212	0.87
46	339	236	1.51
47	196	226	4.2
48	119	155	3.9
49	241	323	1.85
50	225	118	3

Spacing of drill hole: fairly regular (10m) apart