CONSTRUCTIVE HEURISTIC FOR THE INSPECTION OF ELECTRICITY METERS – A CASE STUDY OF ASYLUM DOWN, ACCRA - GHANA

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CHAPTER ONE

1.0 INTRODUCTION

Referring to the 2009 Annual Report and Financial Statements of Electricity Company of Ghana (ECG) under a very important subheading *Outlook for the Year 2010 (page 21 of 57)*, the first two points were:

(i) reduce commercial losses through aggressive monitoring of customer metering facilities to verify the integrity of meters and to check energy theft and

(ii) accelerate the deployment of prepayment metering and general improvement of metering technologies across operational areas;

From above it is undeniably evident that the realization of these sensitive objectives will require movement of some officials to physically inspect, install, repair, report and in some cases arrest defaulting tenants. Hence the need for proper planning of the routing of streets to achieve this at a minimum cost to the ECG or any subcontracting firm.

1.1 BACKGROUND

The Electricity Company of Ghana was incorporated under the companies code, 1963 (Act 179) limited by shares on the 21st day of February, 1997.

All assets and liabilities of the former Electricity Corporation of Ghana and its predecessor the Electricity Division were vested in the company.

The power company installs an electric meter near where its power lines enter a customer's building. It reads the meter periodically and charges the customer for the amount of electricity used.

An electricity meter or energy meter is a device that measures the amount of electric energy consumed by a residence, business, or an electrically powered device.

Electricity meters are typically calibrated in billing units, the most common one being the kilowatt hour. One kilowatt-hour is the amount of electric energy required to provide 1,000 watts of power for a period of one hour. (Ten 100-watt light bulbs left on for one hour consume one kilowatt-hour of electric energy.)

Periodic readings of electric meters establish billing cycles and energy used during a cycle.

In settings when energy savings during certain periods are desired, meters may measure demand, the maximum use of power in some interval. In some areas the electric rates are higher during certain times of day, reflecting the higher cost of power resources during peak demand time periods. Also, in some areas meters have relays to turn off nonessential equipment.

1.1.1 HISTORY

1.1.1.a DIRECT CURRENT (DC)

As commercial use of electric energy spread in the 1880s, it became increasingly important that an electric energy meter, similar to the then existing gas meters, was required to properly bill customers for the cost of energy, instead of billing for a fixed number of lamps per month. Many experimental types of meter were developed. Edison at first worked on a DC electromechanical meter with a direct reading register, but instead developed an electrochemical metering system, which used an electrolytic cell to totalize current consumption. At periodic intervals the plates were removed, weighed, and the customer billed. The electrochemical meter was labor-intensive to read and not well received by customers. In 1885 Ferranti offered a mercury motor meter with a register similar to gas meters; this had the advantage that the consumer could easily read the meter and verify consumption. The first accurate, recording electricity consumption meter was a DC meter by (Aron, 1883). (Hirst, 1888) of the British General Electric Company introduced it commercially into Great Britain. Meters had been used prior to this, but they measured the rate of energy consumption at that particular moment, i.e. the electric power. Aron's meter recorded the total energy used over time, and showed it on a series of clock dials.

1.1.1.b ALTERNATING CURRENT (AC)

The first specimen of the AC kilowatt-hour meter produced on the basis of Hungarian (Blathy, 1889) patent and named after him was presented by the Ganz Works at the Frankfurt Fair, and the first induction kilowatt-hour meter was already marketed by the factory at the end of the same year. These were the first alternating-current watt meters, known by the name of Bláthy-meters. The AC kilowatt hour meters used at present operate on the same principle as Bláthy's original invention. Also (Thomson, 1889) of the American General Electric company developed a recording watt meter (watt-hour meter) based on an ironless commutator motor. This meter overcame the disadvantages of the electrochemical type and could operate on either alternating or direct current.

(Shallenberger, 1894) of the Westinghouse Electric Corporation applied the induction principle previously used only in AC ampere-hour meters to produce a watt-hour meter of the modern electromechanical form, using an induction disk whose rotational speed was made proportional to the power in the circuit. The Bláthy meter was similar to Shallenberger and Thomson meter in that they are two-phase motor meter. Although the induction meter would only work on alternating current, it eliminated the delicate and troublesome commutator of the Thomson design. Shallenberger fell ill and was unable to refine his initial large and heavy design, although he did also develop a polyphase version.

1.1.2 UNIT OF MEASUREMENT

Panel-mounted solid state electricity meter, connected to a 2 MVA electricity substation. Remote current and voltage sensors can be read and programmed remotely by modem and locally by infra-red. The circle with two dots is the infra-red port. Tamper-evident seals can be seen.

The most common unit of measurement on the electricity meter is the kilowatt hour, which is equal to the amount of energy used by a load of one kilowatt over a period of one hour, or 3,600,000 joules. Some electricity companies use the SI megajoule instead.

Demand is normally measured in watts, but averaged over a period, most often a quarter or half hour.

Reactive power is measured in "thousands of volt-ampere reactive-hours", (kvarh). By convention, a "lagging" or inductive load, such as a motor, will have positive reactive power. A "leading", or capacitive load, will have negative reactive power.

Volt-amperes measures all power passed through a distribution network, including reactive and actual. This is equal to the product of root-mean-square volts and amperes.

Distortion of the electric current by loads is measured in several ways. Power factor is the ratio of resistive (or real power) to volt-amperes. A capacitive load has a leading power factor, and an inductive load has a lagging power factor. A purely resistive load (such as a filament lamp, heater or kettle) exhibits a power factor of 1.

Current harmonics are a measure of distortion of the wave form. For example, electronic loads such as computer power supplies draw their current at the voltage peak to fill their internal storage elements. This can lead to a significant voltage drop near the supply voltage peak which shows as a flattening of the voltage waveform. This flattening causes odd harmonics which are not permissible if they exceed specific limits, as they are not only wasteful, but may interfere with the operation of other equipment. Harmonic emissions are mandated by law in EU and other countries to fall within specified limits.

1.1.3 TYPES OF METERS

Electricity meters operate by continuously measuring the instantaneous voltage (volts) and current (amperes) and finding the product of these to give instantaneous electrical power (watts) which is then integrated against time to give energy used (joules, kilowatt-hours etc.). Meters for smaller services (such as small residential customers) can be connected directly in-line between source and customer. For larger loads, more than about 200 ampere of load, current transformers are used, so that the meter can be located other than in line with the service conductors. The meters fall into two basic categories, electromechanical and electronic.

1.1.3.a ELECTROMECHANICAL METERS

The most common type of electricity meter is the electromechanical induction watt-hour meter. The electromechanical induction meter operates by counting the revolutions of an aluminium disc which is made to rotate at a speed proportional to the power. The number of revolutions is thus proportional to the energy usage. It consumes a small amount of power, typically around 2 watts.

The metallic disc is acted upon by two coils. One coil is connected in such a way that it produces a magnetic flux in proportion to the voltage and the other produces a magnetic flux in proportion to the current. The field of the voltage coil is delayed by 90 degrees using a lag coil. This produces eddy currents in the disc and the effect is such that a force is exerted on the disc in proportion to the product of the instantaneous current and voltage. A permanent magnet exerts an opposing force proportional to the speed of rotation of the disc. The equilibrium between these two opposing forces results in the disc rotating at a speed proportional to the power being used. The disc drives a register mechanism which integrates the speed of the disc over time by counting revolutions, much like the odometer in a car, in order to render a measurement of the total energy used over a period of time.

The type of meter described above is used on a single-phase AC supply. Different phase configurations use additional voltage and current coils.

Three-phase electromechanical induction meter, metering 100 A 230/400 V supply. Horizontal aluminum rotor disc is visible in center of meter.

The aluminum disc is supported by a spindle which has a worm gear which drives the register. The register is a series of dials which record the amount of energy used. The dials may be of the *cyclometer* type, an odometer-like display that is easy to read where for each dial a single digit is shown through a window in the face of the meter, or of the pointer type where a pointer indicates each digit. With the dial pointer type, adjacent pointers generally rotate in opposite directions due to the gearing mechanism.

The amount of energy represented by one revolution of the disc is denoted by the symbol *Kh* which is given in units of watt-hours per revolution. The value 7.2 is commonly seen. Using the value of *Kh*, one can determine their power consumption at any given time by timing the disc with a stopwatch. If the time in seconds taken by the disc to complete one revolution is *t*, then the power in watts is $P = \frac{3600 \cdot Kh}{t}$, For example, if Kh = 7.2, as above, and one revolution took place in 14.4 seconds, the power is 1800 watts. This method can be used to determine the power consumption of household devices by switching them on one by one.

Most domestic electricity meters must be read manually, whether by a representative of the power company or by the customer. Where the customer reads the meter, the reading may be supplied to the power company by telephone, post or over the internet. The electricity company will normally require a visit by a company representative at least annually in order to verify customer-supplied readings and to make a basic safety check of the meter.

In an induction type meter, creep is a phenomenon that can adversely affect accuracy, that occurs when the meter disc rotates continuously with potential applied and the load terminals open circuited. A test for error due to creep is called a creep test.

Two standards govern meter accuracy, ANSI C12.20 for North America and IEC 62053.

1.1.3.b ELECTRONIC METERS

Electronic meters display the energy used on an LCD or LED display, and can also transmit readings to remote places. In addition to measuring energy used, electronic meters can also record other parameters of the load and supply such as maximum demand, power factor and reactive power used etc. They can also support time-of-day billing, for example, recording the amount of energy used during on-peak and off-peak hours.

Multiple Tariff (Variable Rate) Meters

Electricity retailers may wish to charge customers different tariffs at different times of the day to better reflect the costs of generation and transmission. Since it is typically not cost effective to store significant amounts of electricity during a period of low demand for use during a period of high demand, costs will vary significantly depending on the time of day. Low cost generation capacity (baseload) such as nuclear can take many hours to start, meaning a surplus in times of low demand, whereas high cost but flexible generating capacity (such as gas turbines) must be kept available to respond at a moment's notice (spinning reserve) to peak demand, perhaps being used for a few minutes per day, which is very expensive.

Some multiple tariff meters use different tariffs for different amounts of demand. These are usually industrial meters.

Domestic Usage

Domestic variable-rate meters generally permit two to three tariffs ("peak", "off-peak" and "shoulder") and in such installations a simple electromechanical time switch may be used. Historically, these have often been used in conjunction with electrical storage heaters or hot water storage systems.

Multiple tariffs are made easier by time of use (TOU) meters which incorporate or are connected to a time switch and which have multiple registers.

Switching between the tariffs may happen via a radio-activated switch rather than a time switch to prevent tampering with a sealed time switch to obtain cheaper electricity.

Appliance Energy Meters

Plug in electricity meters (or "Plug load" meters) measure energy used by individual appliances. There are a variety of models available on the market today but they all work on the same basic principle. The meter is plugged into an outlet, and the appliance to be measured is plugged into the meter. Such meters can help in energy conservation by identifying major energy users, or devices that consume excessive standby power. A power meter can often be borrowed from the local power authorities or a local public library.

Smart Meters

Smart meters go a step further than simple AMR (automatic meter reading). They offer additional functionality including a real-time or near real-time reads, power outage notification, and power quality monitoring. They allow price setting agencies to introduce different prices for consumption based on the time of day and the season.

These price differences can be used to reduce peaks in demand (load shifting or peak lopping), reducing the need for additional power plants and in particular the higher polluting and costly to operate natural gas powered peaker plants. The feedback they provide to consumers has also been shown to cut overall energy consumption.]

Another type of smart meter uses nonintrusive load monitoring to automatically determine the number and type of appliances in a residence, how much energy each uses and when. This meter is used by electric utilities to do surveys of energy use. It eliminates the need to put timers on all of the appliances in a house to determine how much energy each uses.

Prepayment Meters

Prepayment meter and magnetic stripe tokens, from a rented accommodation in the UK. The button labeled A displays information and statistics such as current tariff and remaining credit. The button labeled B activates a small amount of emergency credit should the customer run out

A Prepayment Key

The standard business model of electricity retailing involves the electricity company billing the customer for the amount of energy used in the previous month or quarter. In some countries, if the retailer believes that the customer may not pay the bill, a prepayment meter may be installed. This requires the customer to make advance payment before electricity can be used. If the available credit is exhausted then the supply of electricity is cut off by a relay.

In the UK, mechanical prepayment meters used to be common in rented accommodation. Disadvantages of these included the need for regular visits to remove cash, and risk of theft of the cash in the meter.

Modern solid-state electricity meters, in conjunction with smart cards, have removed these disadvantages and such meters are commonly used for customers considered to be a poor credit risk. In the UK, one system is the PayPoint network, where rechargeable tokens (Quantum cards for natural gas, or plastic "keys" for electricity) can be loaded with whatever money the customer has available.

Recently smartcards are introduced as much reliable tokens that allow two way data exchange between meter and the utility.

In South Africa, Sudan and Northern Ireland prepaid meters are recharged by entering a unique, encoded twenty digit number using a keypad. This makes the tokens, essentially a slip of paper, very cheap to produce.

Around the world, experiments are going on, especially in developing countries, to test pre-payment systems. In some cases, prepayment meters have not been accepted by customers. There are various groups, such as the Standard Transfer Specification (STS) association, which promote common standards for prepayment metering systems across manufacturers. Prepaid meters using the STS standard are used in many countries.

1.1.4 ENTITIES IN GHANA RESPONSIBLE FOR ELECRTICITY

There are several key entities in the Ghanaian electric power industry. They will be discussed and referred to in later chapters in more detail. In this section, we provide a short introduction to each of these entities.

The Ministry of Energy

Ultimate body responsible for development of electricity policy for Ghana.

The Volta River Authority (VRA)

State-owned entity that is responsible for generation and transmission of electricity in Ghana. VRA operates the largest generation facility in Ghana, the Akosombo hydroelectric plant.

The Electricity Company of Ghana

(ECG): State-owned entity that is responsible for distribution of electricity

to consumers in southern Ghana, namely Ashanti, Central, Greater Accra, Eastern and Volta Regions of Ghana. ECG is the entity that consumers interact with when they receive and pay their bills or when they have service questions (billing, metering, line connection etc.).

The Northern Electrification Department (NED)

A subsidiary of VRA and responsible for power distribution in northern Ghana namely, Brong-Ahafo, Northern, Upper East and Upper West Regions.

The Public Utilities Regulatory Commission (PURC)

Independent agency that calculates and sets electricity tariffs, educates customers about electricity services as well as energy efficiency and conservation and ensures the effectiveness of investments.

The Energy Commission

Independent agency that licenses private and public entities that will operate in the electricity sector. EC also collects and analyses energy data and contributes to the development of energy policy for Ghana.

The Private Generators

Domestic or international entities that build power generation facilities in Ghana. They sell their electricity to VRA or ECG. The Energy Foundation: a Ministry of Energy – Private Enterprises Foundation (PEF) initiative, which was set up in 1997 to promote energy efficiency and conservation programmes. Initial activities focused primarily on provision of technical support to industries, introduction of compact fluorescent lamps (CFLs) countrywide and public education.

1.2 PROBLEM STATEMENT

A meter reader is seeking the shortest (time, distance) way to complete his assigned route of streets. Each street must be traversed at least once and the meter reader must end up at the same place where he began his route.

1.3 OBJECTIVE

To obtain the optimal solution for a meter reader to traverse his assigned route of streets within the shortest (time, distance) way.

1.4 METHODOLGY

1.4.a PROBLEM AT HAND

A meter reader is seeking the shortest (time, distance) way to complete his assigned route of streets. Each street must be traversed at least once and the meter reader must end up at the same place where he began his route.

1.4.b MODEL FOR PROBLEM

A meter reader is seeking the shortest (time, distance) way to complete his assigned route of streets at Asylum Down. Each street must be traversed at least once and the meter reader must end up at the same place where he began his route.

1.4.c DATA

Google Map

1.4.d METHOD/ALGORITHM

Chinese Postman Problem and Fleury Algorithms

1.5 JUSTIFICATION

This study will have positive impact in the development of Ghana as spelt below:

(i) Reduce commercial losses through aggressive monitoring of customer metering facilities to verify the integrity of meters and to check energy theft;

(ii) Accelerate the deployment of prepayment metering and general improvement of metering technologies across operational areas;

(iii) Reduce commercial losses through efficient routing of streets resulting in less fuel usage, less time translating into increase revenue for the nation.

1.6 LIMITATIONS OF THE STUDY

This study is constrained by the following points:

(i) The actual amount of time spent by a meter reader in a residence or a building

(ii) The occupancy of a structure by a tenant at a particular point in time will determine whether a meter will be inspected or not.

1.7 THESIS ORGANIZATION

The first chapter deals with the general introduction of the study. Chapter two dwells on the literature review touching on the applications of the Chinese Postman Problem algorithm. Chapter three states the application of the Chinese Postman Problem and a solved example for better understanding. Chapter four is about the data collection, analysis and results. Finally chapter five talks on conclusion and recommendation of the study.

1.8 SUMMARY

In the next chapter, we shall put forward pertinent and adequate literature on Chinese Postman Problem.

CHAPTER TWO

LITERATURE REVIEW

2.0 INTRODUCTION

This chapter talks more on the application of the Chinese Postman Problem and its variants.

2.1 APPLICATIONS AND VARIANTS OF THE CHINESE POSTMAN PROBLEM

A postman delivering letters in a village may wish to know a circuit that traverses each street (in the appropriate direction if one-way streets), starting and returning to their office. This is a graph theoretic problem: roads are directed edges (arcs), and road junctions are vertices.

The postman requires a Chinese Postman Tour (CPT). The postman probably wants a shortest tour, with few repeated street visits. The *cost* of a CPT is defined as the total arc weight, summed along the circuit (e.g., the total distance walked). An *optimal CPT* is a CPT of minimal cost. If some weights are negative an optimal CPT may not be defined: if there is any circuit with an overall negative weight, the postman could arbitrarily repeat it and get a total cost lower and lower without bound.

Conventional applications of the CPP are concerned with routing more generally than postmen, as in routing snow ploughs or planning street maintenance. For example for a simple, isolated village, with four intersections and six one-way streets. The least cost CPT in this example requires travelling down streets ten times; some streets are used more than once, but this is not always the case, and the optimal solution obviously depends on the relative costs of using each arc.

2.1.1 CHINESE POSTMAN PROBLEM

Imagine trying to understand your mobile phone. Pressing buttons takes the phone to new states, and corresponds to travelling down one-way streets. After some considerable work, one might obtain a map of how the phone works. The question then is, is this map correct?

Unfortunately, the map may be complex and difficult to test systematically. Given the map, a CPT will provide a systematic test sequence that will exercise each transition in every state.

The optimal CPT will give the shortest test sequence possible; you could then step through the CPT 'instructions' and note any unexpected behaviour of the mobile. The length of the optimal CPT is therefore a measure of a machine's complexity.

As a concrete example, the Nokia 2110 mobile phone has a menu subsystem of 88 menu-items and 273 actions; the optimal CPT of this takes 515 button presses, plus 79 presses (done at appropriate points) to check presses that do nothing. (Some states have fewer options than there are buttons; each unused button in a state corresponds to a self-loop in the graph.) In comparison, the shortest trip that visits each vertex at least once (a solution to the TSP), for instance to check that each menu-item function corresponds correctly to its name, is only 98 button presses. Thus it is much easier to check the functionality than check the user interface.

It is clear that usability tests involving real users — even for such a "simple" device — are unlikely to be exhaustive, even if users make no mistakes. Indeed the Nokia 2110's user interface is quirky, perhaps a corroboration of the difficulty of performing effective user tests.

Web sites notoriously have broken links; but possibly worse is a link that takes the user to the wrong page. A web author should check every link of a site for correctness, certainly if it provides medical or legal information. Since links are often descriptive, and require an understanding of their purpose, they must be checked manually to see whether they link to appropriate pages. However, on following a link, the human checker is now on another page.

An optimal CPT gives a route around a web site (or other multimedia resource) that exercises every link, with minimal effort.

For well designed web sites, it is not necessary to check every link explicitly. The web site for Benjamin Franklin's House had 66 pages and 1191 links at the time of writing. Its optimal CPT of 2248 steps is excessive for a human to follow unaided; without following a CPT users would do even more work and be unable to guarantee thorough checking. Ideally authoring systems should provide mechanisms to help check sites (for instance, so that the human checker can take a break and not lose track of where they were), in conjunction with using an optimal CPT algorithm to minimise workload. In fact, the Benjamin Franklin site was generated by compiling 78 pages (12 being design templates) needing only 201 explicit links (most being checked by the compiler itself). The CPT of this specification site had 241 steps, including the links the compiler can check itself. The tenfold improvement makes checking the site manageable for a human.

To find the map of a mobile phone, video recorder or a web site in the first place is not easy if it has to be done by reverse engineering. This problem is equivalent to the *mobile robot exploration problem*, where a robot has to explore (by physically moving around in) a network when it does not know, to start with, what the network is. The robot has to explore every arc and vertex of the network (obeying any one-way restrictions), and it has to do so travelling a minimum distance. For a network of m arcs, an algorithm has been found that takes at most $m\varphi O(\log \varphi)$ steps, where φ is the *deficiency* of the graph, a term we define below.

We will see 4 H. THIMBLEBY that the algorithm for the CPT also determines the deficiency. Deficiency is a measure of the ease of use of a system: it relates to how hard a device is to fully understand without benefit of a map, or how hard a network (e.g., a building or interactive device) is to learn.

There are important variations to the CPP. The problem may be closed, with the postman starting *and* returning to their office; in the open problem, the postman need not return. In the real world, where postmen or vehicles traverse a network of roads, this may not seem very useful because they will eventually need to return home; however, in testing once every transition has been tested it may not be necessary to return the device to its initial conditions.

This is certainly true for testing web sites: once a user has tested a site, they can simply walk away and they do not have to return the browser to the home page. The optimal test sequence for a web site is therefore an open CPT.

If the open tour of an interactive device is much shorter than a closed tour, this suggests a possible design fault because it indicates that a tester is (and users in general are) better off walking away from a device than switching it off (or otherwise returning it to its initial state).

Orloff originally introduced the idea of the *General Routing Problem*: to visit some edges and some vertices in a graph — if all edges are to be visited, the problem is the CPP; if some edges are to be visited, the problem is the rural CPP; if all vertices are to be visited the problem is the TSP. Further, there may be one or more postmen, with one or more offices, and each postman may be capacitated (e.g., in the total edge cost they can bear).

The *selecting CPP* (also known as the *rural CPP*) is a variation where the postman must visit certain roads but not necessarily all of them [15]. In machine test terms, the selecting CPP arises, for instance, if we wish to find the shortest test sequence that tests each button on a machine at least once but not necessarily to test all state transitions. The selecting CPP can be used to solve the *incremental CPP*, where an impatient postman sets off before finding the optimal CPT solution. The selecting CPP can also be used for solving the problem of checking a web site where some links have been mechanically constructed (e.g., from templates and are therefore known to be correct) but where others, the ones to be checked, were created by hand.

The graph may be undirected, directed or mixed. Computationally, the undirected and directed cases are polynomial, whereas the mixed is NP-hard. This paper considers the directed case on multidigraphs (graphs with parallel arcs): this case arises naturally in analysing the World Wide Web and finite state machines.

The Chinese Postman Problem (CPP) is a close cousin to TSP. In this routing problem the traveller must traverse every arc (i.e. road link) in the network. The name comes from the fact that a Chinese mathematician, Mei-Ko Kwan (1962), developed the first algorithm to solve this problem for a rural postman. It is an extension to one of the earliest graph theory questions, the Königsberg Bridge Problem, which was studied by Euler (1736). The Pregel River runs through the city of Königsberg in Germany. In a city park seven bridges cross branches of the river and connect two islands with each other and with opposite banks of the river. For many years the citizens of Königsberg tried to take walks that took them over each bridge once without retracing any part of their path. Euler was able to prove that such a walk is impossible. In general, graph theorists are interested in understanding whether or not a circuit exists that does not require traversing the same arc twice. Operations researchers are interested in finding the shortest route in any type of network.

The Chinese Postman class of problems is relevant to a number of other services. Garbage collection, street sweeping, salting or gritting of icy roads, and snow plowing are some of the other services for which vehicle routing algorithms have been applied. Meter readers also must travel up and down every street. Checking roads for potholes or serious deterioration or checking pipelines for weak spots also fall into this class of problems.

In the ever complex real-world additional constraints can arise that complicate the search for efficient routes. Labour contracts may require that the routes of different drivers must be approximately of equal length. There may be significant time restrictions or time windows on when a vehicle must visit a specific location to make a delivery or pick-up. The vehicle making pick-ups may also have capacity limitations such as a garbage truck which would restrict the maximum length of a route. Uncertainty can also complicate route planning. Trucks that deliver gasoline or oil, can't be sure when they set out as to how much they will have to pump into each of the tanks on their route.

2.1.2 THE CHINESE POSTMAN PROBLEM REVISITED

In the Chinese postman problem, we needed to find the minimum number of multiple edges to add to a graph, so that the multigraph obtained would have a closed eulerian chain, that is, so that every vertex would have even degree. We can now describe a procedure to do this.

First we construct, from the given graph G, a graph G' consisting of the vertices of odd degree in G. We join the vertices in G' with edges if there is a chain in G between them, and we label each edge by the length of the shortest path between the two endpoints. This information is obtained by using Dijsktra's algorithm in G. When all the edges of G' have been labelled, then we seek a minimum weight perfect matching in G'. The matching will tell us which vertices in G will be joined by chains of multiple edges to obtain the minimum number of edges to add to G.

2.1.3 SOLVING THE CHINESE POSTMAN'S PROBLEM

We now return to the Chinese postman's problem and solve the problem of finding the minimum length, edge-covering tour of a graph G(N, A) with no restrictions placed on G other than that it be connected and undirected.

The solution procedure consists of adding, in a judicious manner, artificial edges, parallel to the existing ones, to the original graph G to obtain a new graph $G^1(N, A^1)$ on which an Euler tour can be drawn. This means that the addition of the artificial edges should be such as to turn all odd-degree nodes on G into even-degree nodes on G^1. Moreover, this should be accomplished by selecting that combination of artificial edges which results in the minimum length tour at the end. The addition of an artificial edge in parallel to an existing one in the original network simply means that the existing edge will have to be traversed twice in the final Chinese postman's tour.

An important observation is helpful at this point.

Lemma: The number of nodes of odd degree in an undirected graph G is always even.

Hint: The sum of the degrees of all the nodes in G is an even number since each edge is incident on two nodes.

We now describe the solution approach in general terms:

Chinese Postman Algorithm--Undirected Graph (Algorithm 6.5)

Identify all nodes of odd degree in G(N, A). Say there are m of them, where m

STEP 1	Identify all nodes of odd degree in G(N, A). Say there are m of them, where m is an even number according to the lemma.
STEP 2	Find a minimum-length pairwise matching (see below) of the m odd-degree nodes and identify the m/2 shortest paths between the two nodes composing each of the m/2 pairs.
STEP 3	For each of the pairs of odd-degree nodes in the minimum-length pairwise matching found in Step 2, add to the graph G(N, A) the edges of the shortest path between the two nodes in the pair. The graph $G^1(N, A^1)$ thus obtained contains no nodes of odd degree.
STEP 4	Find an Euler tour on $G^1(N, A^1)$. This Euler tour is an optimal solution to the Chinese postman's problem on the original graph G(N, A). The length of the optimal tour is equal to the total length of the edges in G(N, A) plus the total length of the edges in the minimum-length matching.

A pairwise matching of a subset $N' \subset N$ of nodes of a graph G is a pairing of all the nodes in N' (assuming that the number of nodes in N' is even). A minimum-length pairwise matching of the nodes in An is then a matching such that the total length of the shortest paths between the paired nodes is minimum.

Example 1: Application of Algorithm 6.5

Consider the graph of Figure 6.15a. The numbers on the edges indicate the lengths of the edges. The graph contains four nodes of odd degree (a, b, d, and e). Thus, there are three possible matchings of the odd-degree nodes: a-b and d-e; a-d and b-e; and a-e and bad. The augmented networks that would result from the addition of the artificial edges corresponding to each of these three matchings are-shown in Figure 6.15b-d, respectively. Of the three matchings, the optimal is obviously the one shown in Figure 6.15c. It adds only 12 units of length to the tour as opposed to 16 for Figure 6.15b and 20 for 6.15c. Thus, the graph shown on Figure 6.15c should be the result of the procedure outlined in Steps 2 and 3 of Algorithm 6.5. Supposing that the required tour must begin at node a, a solution to the Chinese postman's problem for the graph of Figure 6.15a is the tour {a, d, a, c, d, e, c, b, e, b, a}. Its total length is 60 units, of which 48 is the total length of the graph 6.1 5a, while 12 units are due to the artificial edges. In other words, edges (a, d) and (b, e) will be traversed twice.

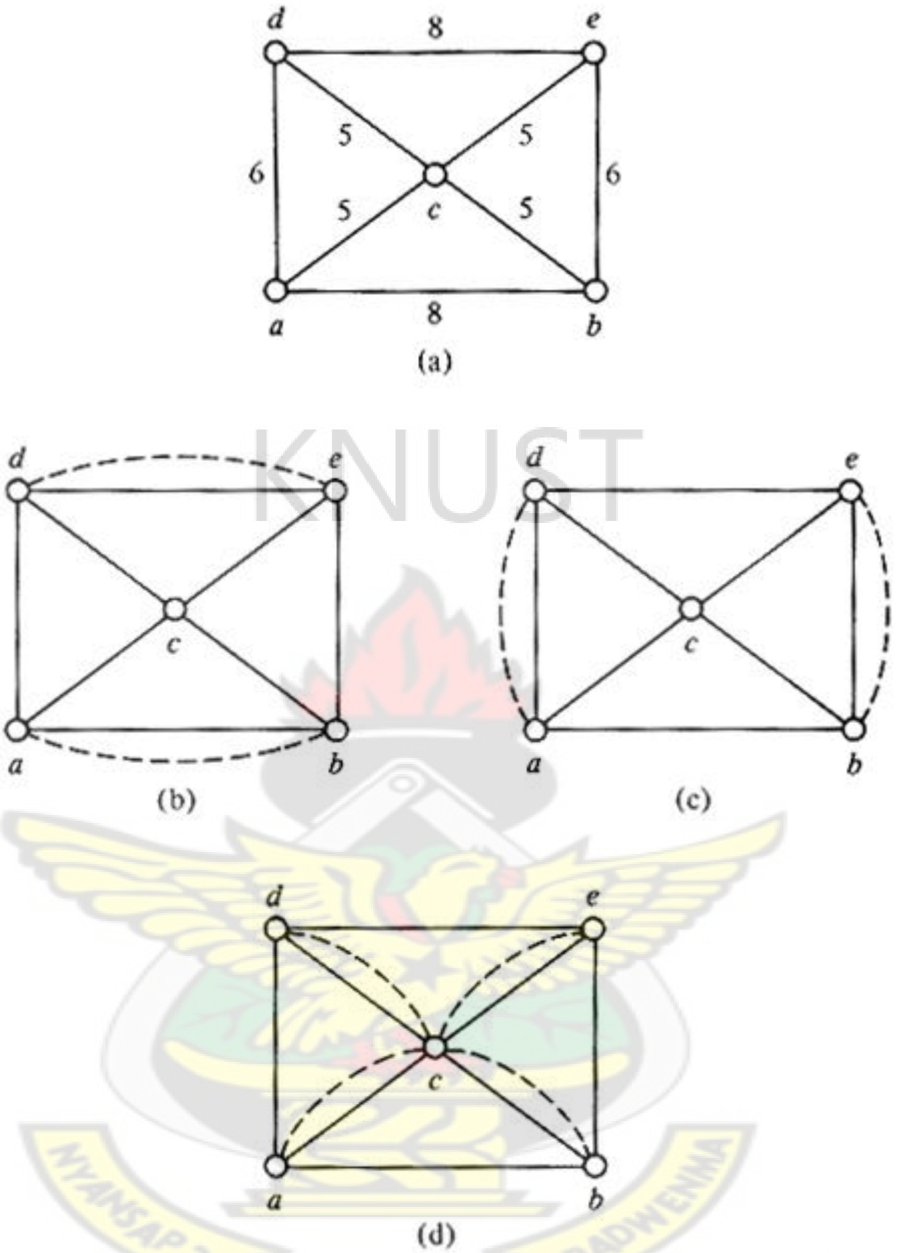


FIGURE 6.15 Illustration of the application of Algorithm 6.5.

In Algorithm 6.5, we have concentrated exclusively on the *shortest paths* between pairs of odd-degree nodes. That this should be so can be seen as follows. Were a supposedly optimal pairwise matching between odd-degree nodes to contain a path, P^1, between any given pair of nodes that is not the shortest path, P. between these two nodes, the solution could be improved by simply substituting P for P^1 in the drawing of the artificial edges, since the length of P is, by definition, smaller than the length of P^1.

In reviewing Algorithm 6.5, we can see that its only difficulty lies in Step 2, since Steps 1, 3, and 4 are all very simple. For Step 2, however, the number of possible pairwise matching combinations increases very quickly with the number of odd-degree nodes. Some thought will show that with m odd degree nodes in G (remember that m is always even), there are

$$S = \prod_{i=1}^{m/2} (2i - 1) \qquad (6.4)$$

possible distinct pairwise matching combinations. Thus, for m = 4, there are I · 3 = 3 possible combinations (as we have already seen in our last example), for m = 10,945 combinations, and for m = 20, about 655 million combinations.

Fortunately, an efficient, but quite complicated algorithm for minimum length matchings on undirected graphs is now available. This algorithm is based on the theory of matching on graphs that has been developed in recent years primarily by J. Edmonds. The interested reader is referred to [EDMO 73] or to [CHRI 753] for its details. Here we only note that the algorithm is an exact one (i.e., finds the optimum matchings) and efficient computer-implemented versions solve the problem in time proportional to n^3, where n is the number of nodes in the graph under consideration. The algorithm not only finds the minimum-length matchings but also identifies the shortest path associated with each pair. It follows that in the worst case Algorithm 6.5 is also $O(n^3)$.

Manual approach to matching. When Edmonds' minimum-length matching algorithm is used in Step 2 of Algorithm 6.5, an optimum solution to the Chinese postman's problem is obtained. Experience has shown, however, that whenever the Chinese postman problem is posed in a geographical context (and this of course is the case in urban service systems applications), then excellent--in the sense of being very close to the optimum--manual solutions can be obtained practically by inspection with the aid of a good map. The map can be used to perform all four steps of Algorithm 6.5, and the only step where one risks deviating from the optimum solution is in finding the minimum-length matching in Step 2.

The manual search for an approximately minimum-length matching of odd-degree nodes in Step 2 is greatly aided by the following key observation:

In a minimum-length matching of the odd-degree nodes, no two shortest paths in the matching can have any edges in common.

To see this, consider Figure 6.16, assuming that nodes a, b, c, and d are all of odd degree and that the shortest paths between, say, the pairs of nodes a-c and Kd have edge (p, q) in common. Then it is impossible that the minimum-length pairwise matching contain the pairs a-c and Fd, since it would then be possible to obtain a better (shorter) matching by coupling together nodes a and b into one pair and nodes

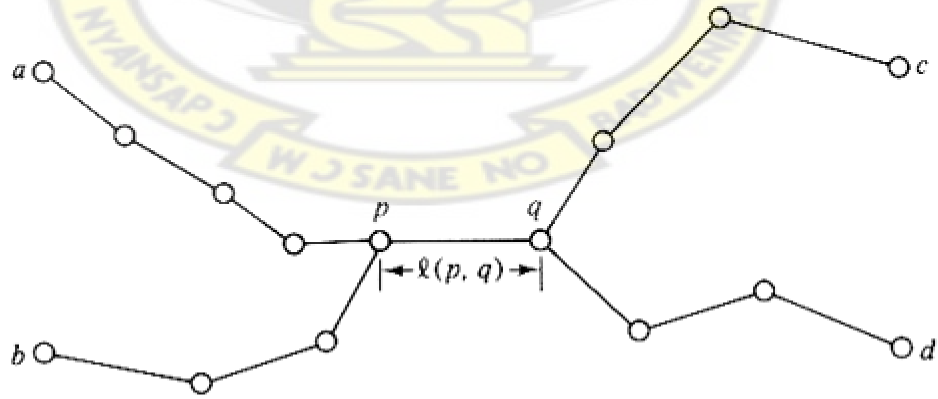


FIGURE 6.16 In an optimum pairwise matching, node *a* can only be matched with node *b* and node *c* with node *d*.

c and d into another. This latter matching will reduce the sum of the shortest path lengths by at least 2 ℓ(p, q) units.

From the practical point of view, this observation has two effects: 1. It eliminates from consideration a very large percentage of the possible sets of matchings that might be considered. (This percentage might be expected to increase as the number of odd-degree nodes increases.) 2. It indicates that in a minimum-length matching odd-degree nodes will be matched to other odd-degree nodes in their immediate neighbourhood.

Other observations (of a somewhat more complicated nature) have also been made (see [STRI 70]) with regard to finding good approximate solutions to minimum-length matching problems without using a matching algorithm. It has been the authors' experience that, whenever good maps of urban areas are available, it takes only a limited amount of practice with some examples before one develops the ability to find *virtually by inspection* excellent solutions to matching problems (by taking advantage of the foregoing observations).

Example 2

Consider again our initial Chinese Postman problem shown in Figure 6.12. The odd-degree nodes on Figure 6.12 are C, D, F, G, I, J, K, and L. They are shown on Figure 6.17, with that part of the network model of the district that contains all the shortest paths between the odd-degree nodes.

Inspection of Figure 6.17 leads to the conclusion that (although theoretically there are $7 \cdot 5 \cdot 3 \cdot 1 = 105$ possible sets of matchings among the eight odd-degree nodes) very few sets of matchings would even qualify for consideration. In fact, a couple of trials should be sufficient to convince the reader that the minimum-cost matching can only be the matching I-J, K-L, C-D, and F-G for a total cost (i.e., duplicated street length) of 490 units of length. The final result for the Chinese postman,s problem in this case is then shown on Figure 6.18, where edges to be traversed twice have been substituted by two edges (or pseudo-edges) each of equal length to the original one. The graph of Figure 6.18 now contains no nodes of odd degree, and thus an Euler tour can be drawn on it, beginning at any desired node and ending at the same node. The length of all Euler tours on this graph will be equal to the total length of the original graph G (3,340 units) plus the distance to be traversed twice (490 units), for a total length of 3,830 units, or about 15 percent more than the total street length of the district that the mailman is responsible for. (This turns out to be the optimum solution.)

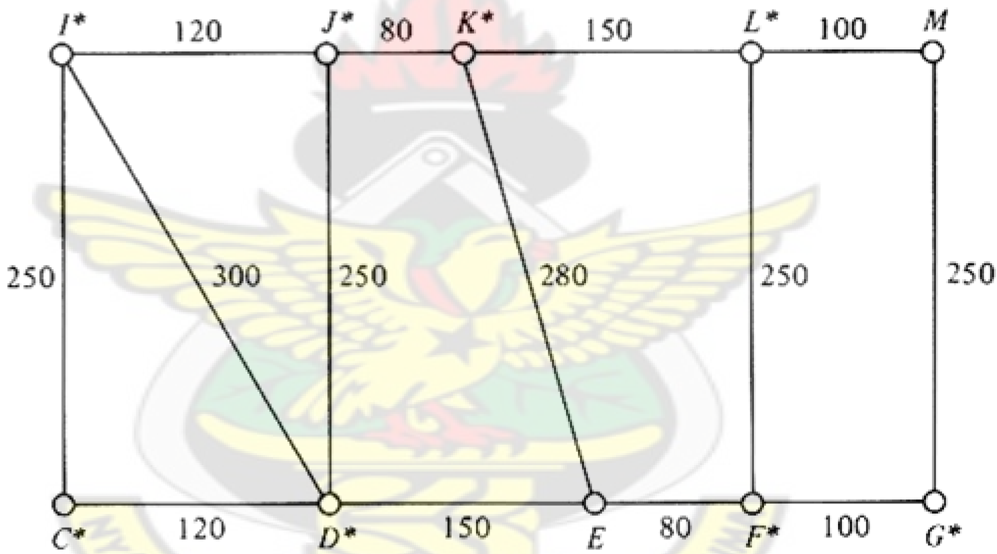


FIGURE 6.17 Odd-degree nodes from the graph of Figure 6.12 are indicated with a *.

2.1.4 MULTIROUTE CHINESE POSTMAN PROBLEM

Just as in the case of node covering, multiroute edge-covering problems are very meaningful and applicable in the urban environment. Urban areas are obviously subdivided on a routine basis into smaller districts that can be covered by a single mailman or refuse-collection truck or parking-meter reader or snowplowing truck. This districting aspect is an integral part of the multiroute Chinese postman problem.

This problem is usually referred to as the *constrained Chinese postman problem* (CCPP), since the need to subdivide an area into many routes arises due to some constraint(s), such as the maximum distance that a mailman can cover walking during a normal day or the maximum weight or volume of solid waste that a refuse collection truck can carry or, very often, other limits on some measures of workload that have been agreed on in a labour contract.

The CCPP has not been investigated extensively to date, but practical approaches to it--in the context of the delivery of urban services--have been suggested for both undirected [STRI 70] and directed [BELT 74] networks. Because of the relative ease with which the single-tour Chinese postman problem can be solved, the "route first, cluster second" strategy seems to be the favoured one in this case: a giant tour is first found and then divided into m subtours, where m is the number of available vehicles. However, with no "best way" available for breaking up the giant tour into shorter subtours, this approach depends to a large extent on the ability and experience of the analyst. In fact, the approach described below for an undirected network [STRI 70] is most effective when carried out manually with the assistance of a good map.

The key to the success of the approach is to subdivide the graph G', on which the large, single tour is drawn in such a way as not to create odd-degree nodes on the boundaries between subtours. Since G' has been derived by applying a CP algorithm to the original graph G. G' has no odd-degree nodes. Therefore, all the nodes in the *interior* of subtours will be even-degree nodes and the partitioning process can create odd-degree nodes only on the boundaries between subtours. To avoid this, it is important to draw continuous boundaries for each subtour, so that an even number of edges is incident on each node. The following describes informally a possible heuristic approach:

Constrained Chinese Postman "Algorithm" (Algorithm G.9)

STEP 1: Using a CP algorithm, create an Eulerian graph from the given network whose edges are to be covered.

STEP 2: Sketch out roughly the boundaries of the m subtours in accordance with the given constraints on tour lengths, vehicle capacities, and so on.

STEP 3: Carefully draw a continuous boundary for each subtour so that an even number of edges is incident to every node.

It is clear that the procedure above does not really describe an algorithm in the strict sense but rather outlines a procedure for obtaining a good solution to the CCPP.

Example 3: Tour of the Mailman, Revisited

Consider again the mailman’s problem that we used in an earlier example (Figure 6.12). The modified graph G' which was found by applying our CP algorithm for the solution of the single tour was shown in Figure 6.18 and is copied in Figure 6.34. It is a network with no nodes of odd degree and with total edge length equal to 3,830 distance units.

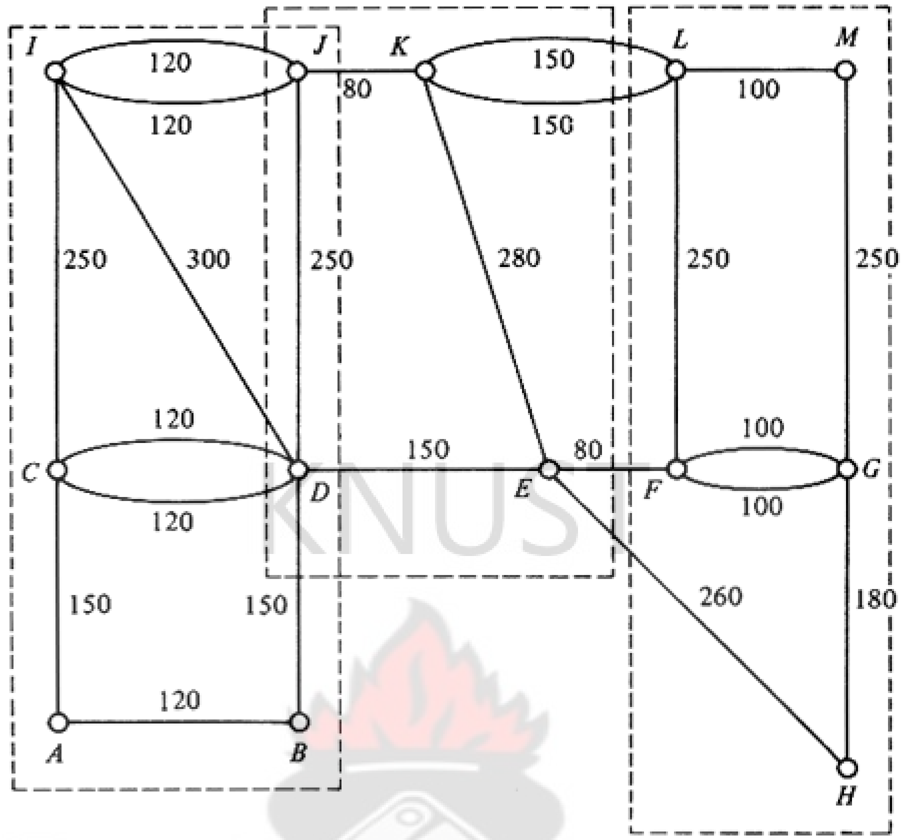


FIGURE 6.34 Tentative partitioning of the single-CP tour into three approximately equal subtours.

Suppose now that an upper limit of 1,500 distance units is placed on the length of a mailman's tour. We then attempt to subdivide the single 3,830-unit tour into three approximately equal tours, each of which satisfies the 1,500-unit limit. (Alternatively, it might have been specified that the district in question must be covered by three mailmen.)

On Figure 6.34 the rough outlines of three approximately equal-length tours are sketched in accordance with Step 2 of the CCPP algorithm. Note that these outlines may overlap since they serve only as an aid in defining the approximate physical boundaries of the subtours. In Step 3 the three subtours are designed in detail with continuous boundaries to ensure both the existence of an Eulerian tour and an increase in the total distance covered, which is as small as possible. The three subtours shown in Figure 6.35 are 1,210, 1,300, and 1,320 units long. Their total length in this particular case turns out to be exactly equal to the length of the single tour from which they were derived.

Two disadvantages of the approach that we just illustrated are readily apparent. First, some trial-and-error work may be required before a set of feasible tours is obtained. This is due to the fact that the subdivision of the tour is initially made by inspection alone. Second, the algorithm above does not take into consideration the distances

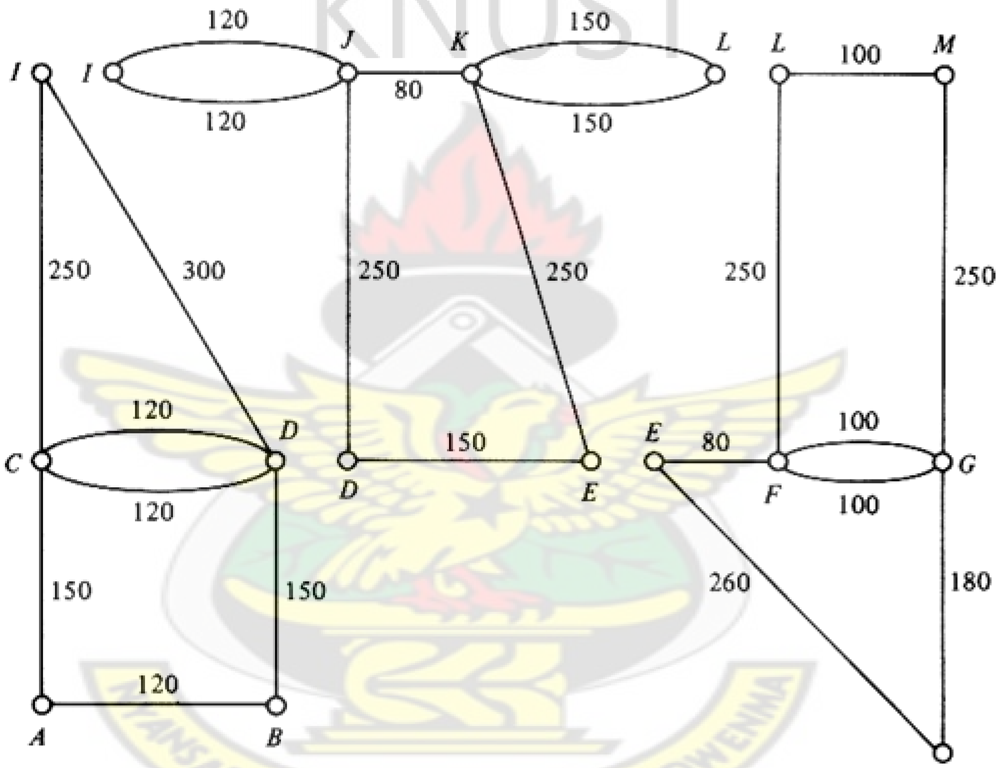


FIGURE 6.35 Final partitioning of the single CP tour into three approximately equal subtours.

involved in getting to each district from the central station (post office, depot, etc.) and back. These distances--or, better, the time required in practice to cover them--are considered to be second-order-effect quantities.

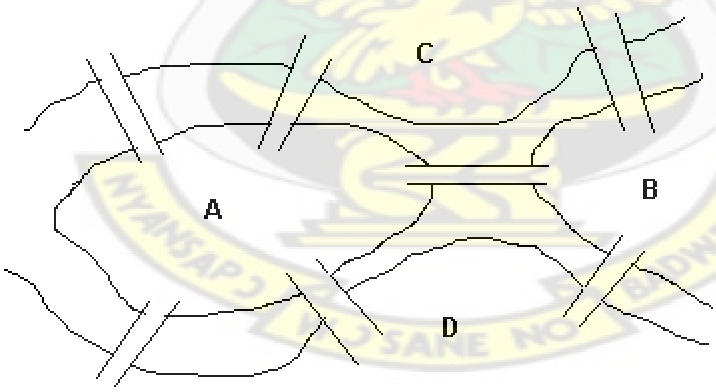
CHAPTER THREE

3.0 INTRODUCTION

A postman is seeking the shortest (time, distance) way to complete his assigned route of streets. Each street must be traversed at least once and the postman must end up at the same place where he began his route.

3.1 BACKGROUND

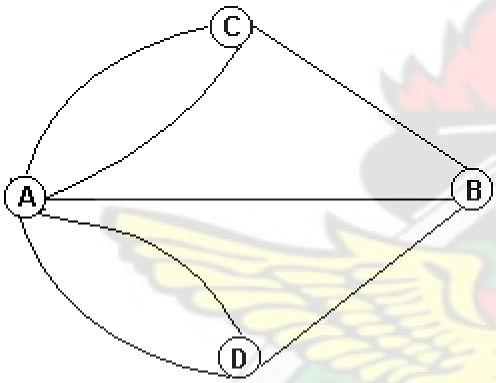
In the 1730s, the Swiss mathematician Leonhard Euler decided to study the city of Köningsberg in Prussia. Köningsberg was built on the banks of a river and had seven bridges, which connected the two islands to the river banks. What Euler wanted to discover was whether it would be possible to cross all the bridges in a type of "tour" without crossing the same bridge twice.



There are some differences between the Köningsberg Bridge Problem (KBP) and the Chinese Postman Problem (CPP). First, Euler did not require the starting and ending points for the KBP to be the same. Second, Euler was not trying to minimize the distance travelled. The CPP gained its name because it was proposed by a Chinese mathematician (Mei-ko Kwan) in 1962. It is also known under the alternative name of the *Route Inspection Problem*. The CPP has many practical applications including bus routing, trash collection, road sweeping, snow-plowing, and transmission line inspection.

3.2 MATHEMATICAL FORMULATION

We can turn both of these problems into network models. For example, in the KBP, let the four land masses be represented by nodes and the seven bridges by edges. Then we have the *multigraph* shown below.



Definition: A multigraph is a graph where multiple edges (between two distinct nodes) and self loops (an edge that joins a node to itself) are allowed. A graph that does not allow multiple edges or self loops is called a simple graph.

Definition: Let G be a graph. A walk in G that traverses every edge in G exactly once is called an Eulerian trail. In addition, if the trail begins and ends at the same vertex, it is called an Eulerian circuit (or Eulerian tour). If G has an Eulerian tour, we say G is an Eulerian graph.

We see that the KBP is the problem of finding an Eulerian trail in a graph. The CPP can be converted into the problem of finding an Eulerian tour in a graph that has minimum cost.

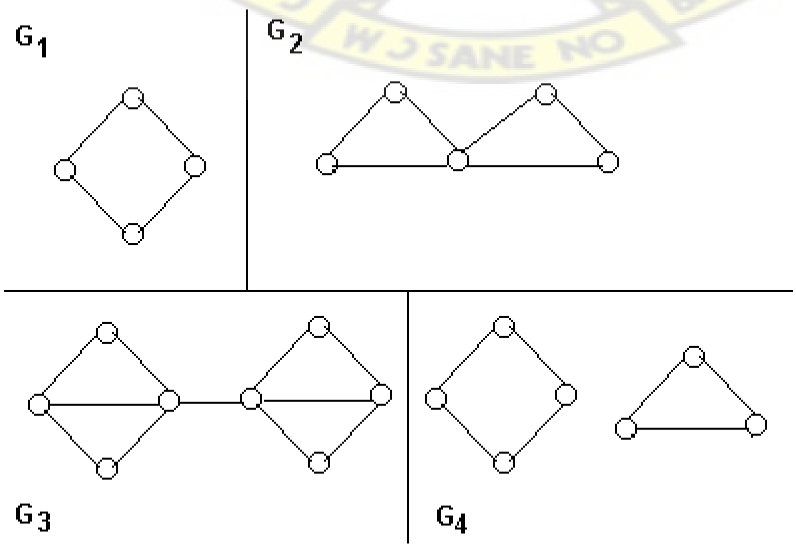
Definition: Let *v* be a vertex in G. The degree of *v*, degree (*v*) is the number of edges that are attached to *v*. If degree (*v*) is odd, we say v is an odd vertex; if degree (*v*) is even, we say v is an even vertex.

Theorem: There is always an even number of odd vertices in a graph.

Definition: Let G be a graph. A nontrivial component of G is a subgraph containing at least two distinct vertices and at least one path between any pair of vertices in the subgraph.

Theorem: Let G be a graph. G is an Eulerian graph if all the vertices are even and all the edges belong to a single component.

Examples



G_1 and G_2 are Eulerian graphs but G_3 and G_4 are not. G_3 is not Eulerian because it contains odd vertices. G_4 is not Eulerian because it is composed of two nontrivial components.

Theorem: Let G be a graph. G has an Eulerian trail if all the edges belong to a single component and there are at most two odd vertices.

Observation 1: The Eulerian trail would necessarily begin and end at the odd nodes.

Observation 2: The KBP does not have an Eulerian trail since all four vertices have odd degree. Hence, there is no way to traverse all seven bridges exactly once.

Observation 3: If G is a graph with only even vertices and one nontrivial component then the solution to the CPP uses every edge in G EXACTLY once and the total cost of the tour is the sum of all of the edge weights.

3.2.1 FLEURY'S ALGORITHM

If G is a graph with only even vertices and one nontrivial component then we may identify an *Eulerian tour* as follows:

Start with any vertex.

From the current vertex traverse any unselected edge whose deletion would not result in a graph with two nontrivial components.

Delete the selected edge from the graph. If there are no edges remaining STOP; otherwise, go back to STEP 2.

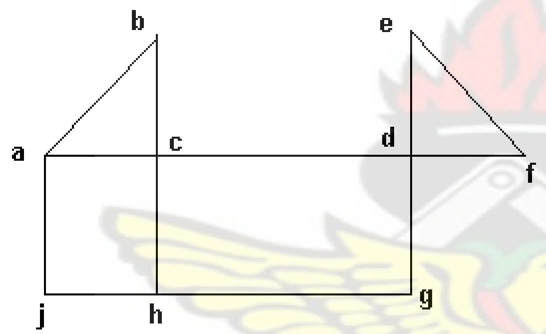
If G is a graph with at most two odd vertices and one nontrivial component then we may identify an *Eulerian trail* as follows:

Start with any odd vertex. (If all vertices are even, start anywhere)

From the current vertex traverse any unselected edge whose deletion would not result in a graph with two nontrivial components.

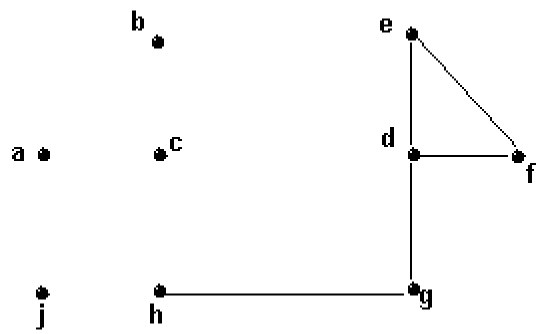
Delete the selected edge from the graph. If there are no edges remaining STOP; otherwise, go back to STEP 2.

Example

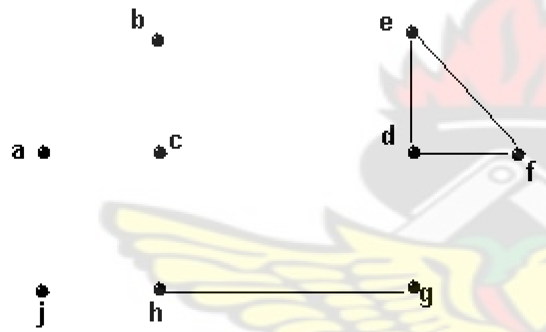


In the graph above, degree(a) = 3 and degree(h) = 3. All other vertices are even. Hence, the graph does not have an Eulerian tour but it does have an Eulerian trail beginning at a and ending at h.

Starting at a, we have a→b→c→a→j→h→c→d. At this point we have the graph shown below (with the edges that have been already traversed deleted).



We cannot visit *g* next because deleting *dg* would leave a graph with two nontrivial components:



So we visit *e* next and complete the trail with e→f→d→g→h.

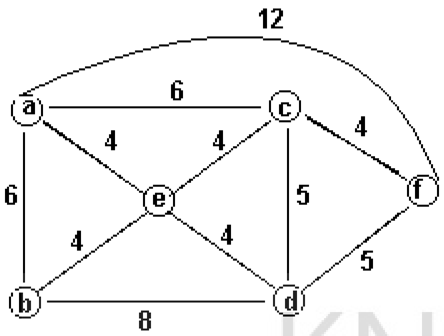
3.2.2 SOLVING THE CPP

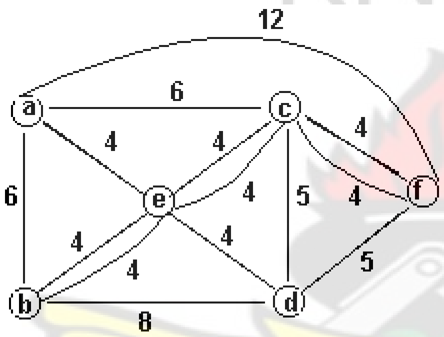
QUESTION: What should our postman do if the underlying graph does not have only even vertices?

ANSWER: He pairs up the odd vertices and tries to find the shortest path between the two vertices in each pair.

Example

In the graph below, there are two odd vertices, *b* and *f*. If we only wanted an Euler trail, we could just use Fleury's Algorithm. But we want to solve the CPP on this graph so we need to find the shortest path between b and f and augment the graph with the edges in the shortest path.





The new graph is Eulerian and now we can use Fleury's Algorithm to find an Euler tour in it.

3.2.3 CPP ALGORITHM

List all the odd vertices in the graph. If there are no odd vertices, go to STEP 5.

Find all possible SETS pairings of the odd vertices.

For each SET of pairings, find the shortest path between the two vertices in each pair. Compute the total cost of the SET of pairings by adding up the costs of the shortest paths.

Select the SET of pairings with minimum weight and repeat these edges in the graph.

Use Fleury's algorithm to find an Euler tour in the resulting graph.

What we have done in STEPS 2 through 4 is to convert a non-Eulerian graph into an Eulerian graph by adding edges to the graph. This is equivalent to having our postman walk up and down the same street.

Example

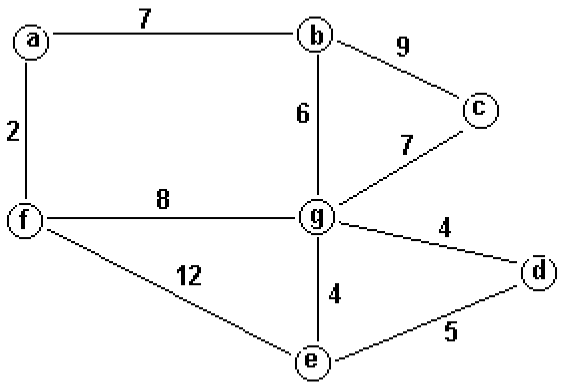
In the graph below, there are four odd vertices, *b*, *e*, *f*, and *g*. There are three SETS of pairings: {*b*&*e*, *f*&*g*}; {*b*&*f*, *e*&*g*}; and {*b*&*g*, *f*&*e*}.

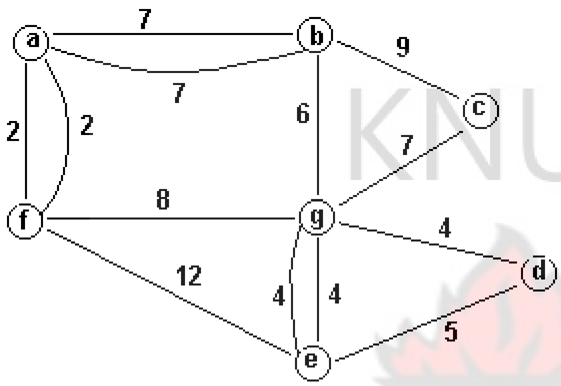
Pair	Shortest Path	Cost
b&e	*b,g,e*	10
f&g	*f,g*	8
b&f	*b,a,f*	9
e&g	*e,g*	4
b&g	*b, g*	6
f&e	*f, e*	12

Set of Pairs	Cost
b&e, f&g	10 + 8 = 18
b&f, e&g	9 + 4 = 13
b&g, f&e	6 + 12 = 18

The cheapest SET of pairs is {*b*&*f*, *e*&*g*} so we repeat the edges *ba*, *af*, and *eg* in the graph.

The new graph is Eulerian and now we can use Fleury's Algorithm to find an Euler tour in it.





3.3 DIJKSTRA'S SHORTEST PATH ALGORITHM

Given a directed network and interpreting the arc weights as distances, a very common practical problem is to find the path of minimum distance (shortest path) between some specified pair of vertices, say x and y. An algorithm for doing this is due to Dijkstra.

We will maintain two lists, W a list of vertices and B a list of arcs. For each vertex that gets put in W we will also record the distance from x to that vertex. Initially, the list B is empty, x is placed in W and d(x,x) = 0.

The algorithm proceeds as follows:

For each vertex u in W and v not in W we calculate the number d(x,u) + wt(u,v). Select the u and v that give the minimum value (in case of ties, chose arbitrarily). If there is no minimum then stop and indicate that there is no path from x to y. If there is a minimum then put v in W, put (u,v) in B and record d(x,v) = d(x,u) + wt(u,v). Repeat the procedure until y is placed in W. The shortest path is the unique path of arcs in B that lead from x to y (most easily obtained by working backwards from y). There is an animated explanation of this algorithm on the web.

3.3.1 SCHEDULING PROBLEMS

Instead of interpreting the edge weights as distances, they may represent costs or times. The algorithm we have used can then be applied to find schedules of minimum cost or minimum time. In these situations the network is often called a PERT (Project Evaluation and Review Technique) network or a CPM (Critical Path Method) network.

CHAPTER FOUR

4.0 INTRODUCTION

This chapter is mainly about the data collection, Analysis and Results of this study. Because of nature of streets and work load, we have two team of one members each to inspect both Zone One and Zone Two respectively of this area.

4.1 DATA COLLECTION

Below is the map (see Fig 4.1) showing the streets of Asylum Down as obtained from Google Map. Please refer to actual map (see Fig 4.2) used for the analysis on Page 45 of this document. Scale = 1.7cm : 200m

4.1.a ASSUMPTIONS

Below are the assumptions for this exercise:

(i) There is at least a tenant in a building to allow for inspection.

(ii) Streets are safe for pedestrian

(iii) Distance is translated into cost

(iv) Streets which are dead ends are self loop (i.e. to be traversed twice)

(v) Diagrams are not drawn to scale

Fig 4.1 Entire map of Asylum Down, Accra - Ghana

4.2 SOLVING THE CHINESE POSTMAN PROBLEM FOR ZONE ONE – TEAM A

4.2.1 TRACING THE PATH FOR ZONE ONE ON THE MAP

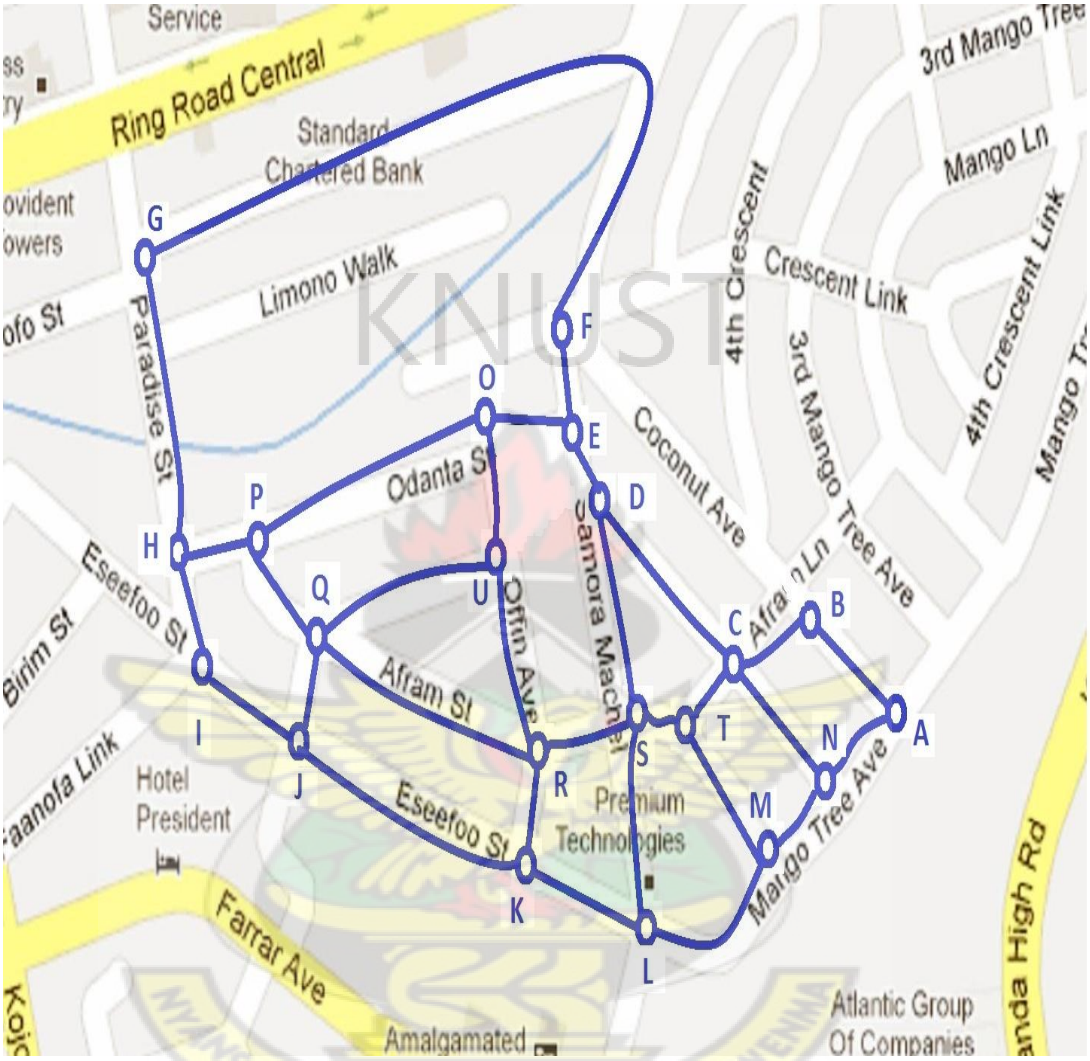


Fig 4.3 Map of Zone One – inspected by Team A

4.2.2 TABULATION OF ACTUAL DISTANCES – ZONE ONE

Table 4.1 Tabulation for Actual Distance for Zone One

Segments	Distance on Map (cm)	Actual Distance (m)	Actual Distance to the nearest whole number (m)
AB	1.35	158.82	159
AN	0.70	82.35	82
BC	0.70	82.35	82
CD	1.85	217.65	218
CN	1.20	141.18	141
CT	0.70	82.35	82
DE	0.35	41.18	41
DS	1.50	176.47	176
EF	0.70	82.35	82
EO	0.80	94.12	94
FG	7.00	823.53	824
GH	1.95	229.41	229
HI	0.60	70.59	71
HP	0.70	82.35	82
IJ	1.00	117.65	118
JK	2.25	264.71	265
JQ	0.70	82.35	82
KL	1.50	176.47	176

KR	0.70	82.35	82
LM	1.50	176.47	176
LS	1.25	147.06	147
MN	0.60	70.59	71
MT	1.40	164.71	165
OP	2.35	276.47	276
OU	0.60	70.59	71
PQ	0.90	105.88	106
QR	2.20	258.82	259
QU	1.75	205.88	206
RS	0.70	82.35	82
RU	1.10	129.41	129
ST	0.60	70.59	71
Total		4847.06	4847

4.2.3 CONVERSION OF ZONE ONE MAP TO GRAPH

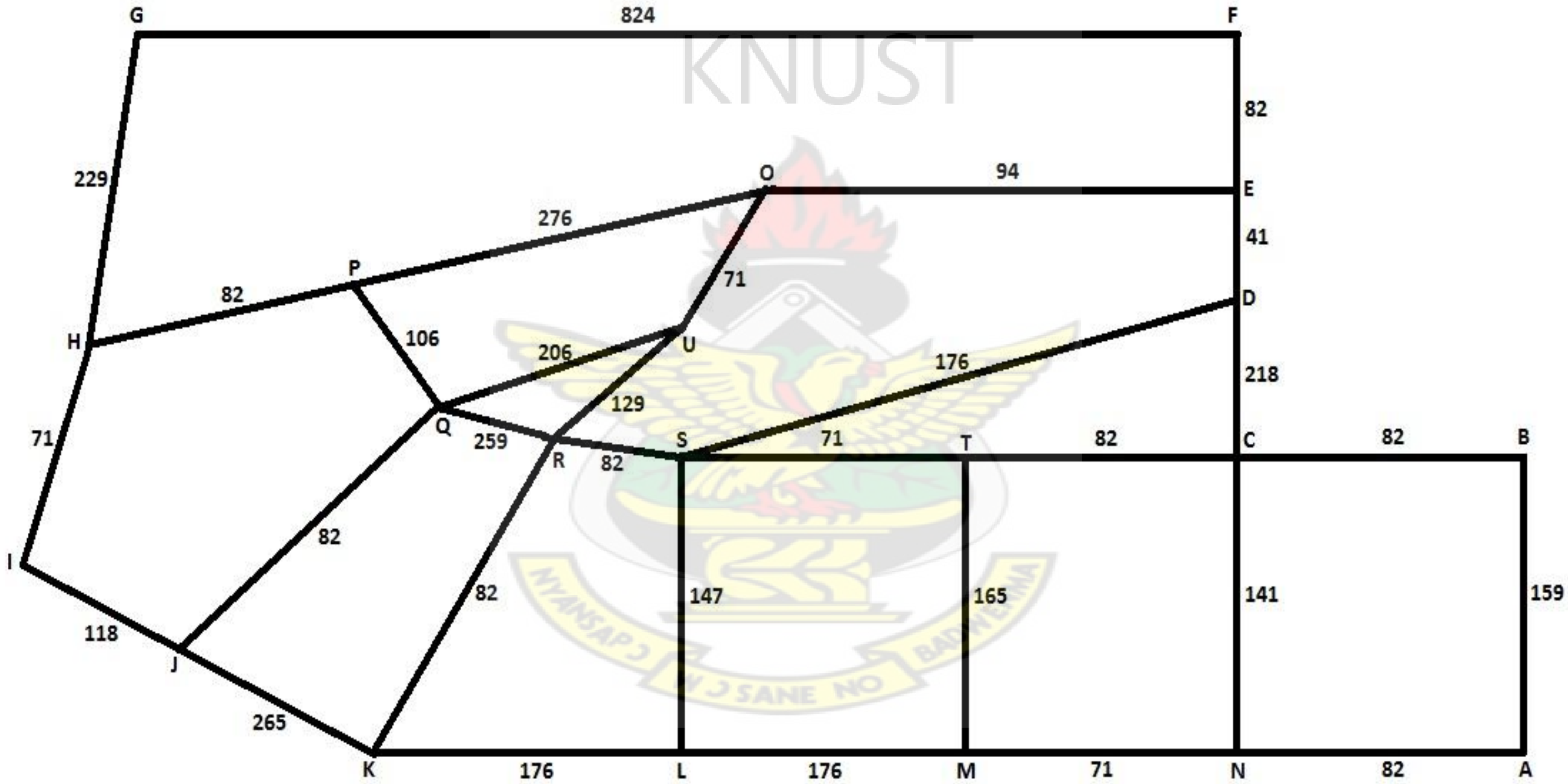


Fig 4.4 Graph for Zone One (not drawn to scale)

4.2.4 FINDING THE OPTIMAL SOLUTION – ZONE ONE

The above graph is non-Eulerian. We need to convert graph into an Eularian graph by adding edges after which an Euler tour will be found with Fleury's algorithm.

Step 1 : Identify all the odd vertices in the graph

Odd vertices: D, E, H, J, K, L, M, N, O, P, T, U

Step 2: Find all possible sets pairings of the odd vertices.

Referring to Equation 6.4 on Page 25

S = 10395, where S is all the possible sets pairing of the vertices.

Step 3

For each set of pairings, we find the shortest path between the two vertices in each pair. We then compute the total distance of the set of pairings by adding up the distances of the shortest paths.

Although theoretically there are 11 * 9 * 7 * 5 * 3 * 1 = 10395 possible sets of matchings among the twelve odd-degree nodes very few sets of matchings would even qualify for consideration.

Step 4

We select the set of pairings with minimum weight and repeat these edges in the graph.

In fact, a couple of trials was sufficient to convince me that the minimum-cost matching can only be the matching D-E, H-J, K-L, M-N, O-P, T-U for a total cost (i.e., duplicated street length) of 1035m of length. Please refer to Example 2 on pages 27 and 28.

Table 4.2 Minimum cost matching – Zone One

Pair	Shortest Path	Cost
ED	E-D	41
HJ	H-I-J	189
KL	K-L	176
MN	M-N	71
OP	O-P	276
TU	T-S-R-U	282
TOTAL		1035

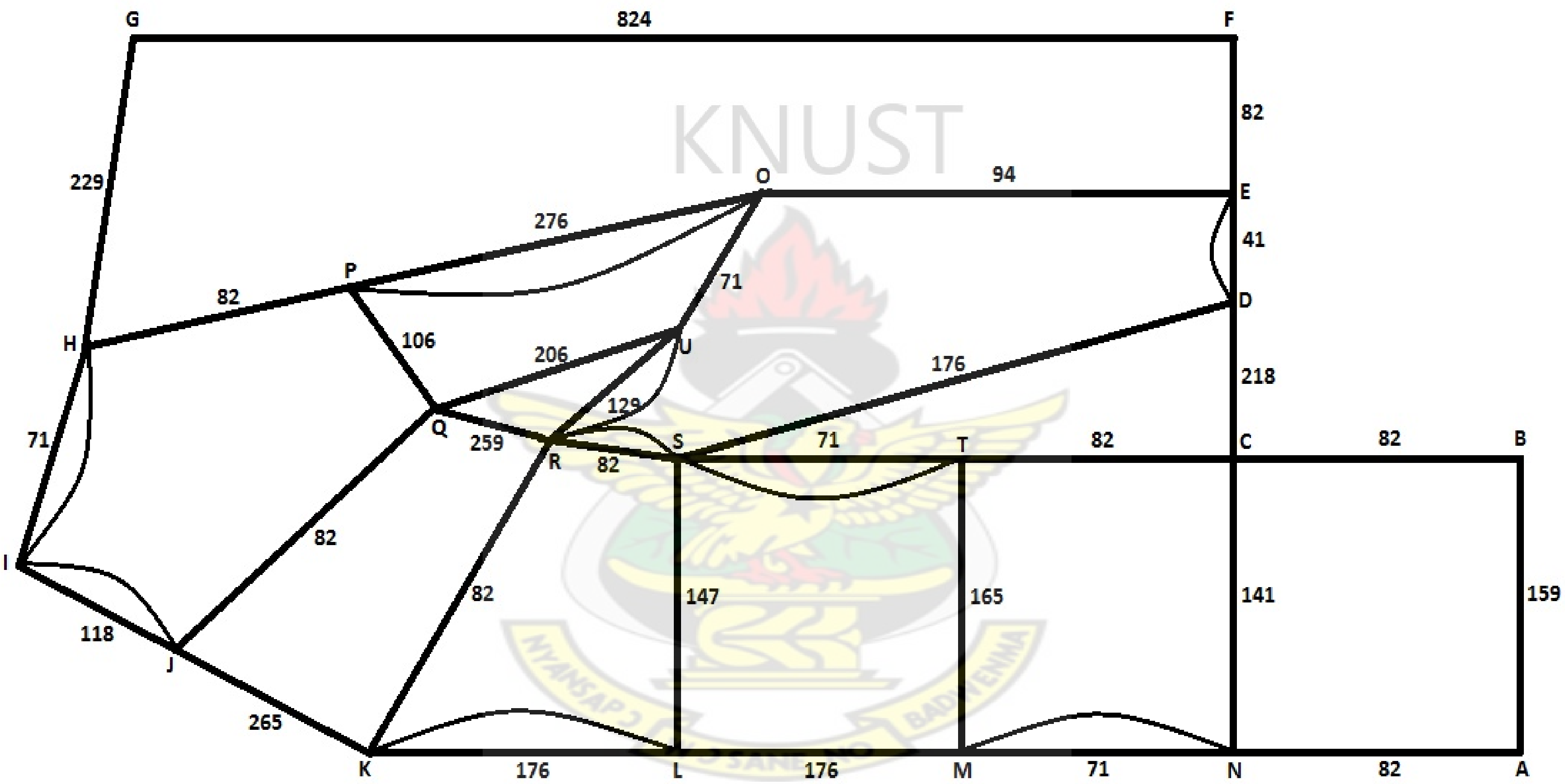


Fig 4.5 The Eulerian graph (with the repeated edges) for Zone One

Step 5

We finally use Fleury's algorithm to find an Euler tour in the resulting graph.

The final result for the Chinese postman,s problem in this case is then shown on Figure 4.5, where edges to be traversed twice have been substituted by two edges (or pseudo-edges) each of equal length to the original one. The graph of Figure 4.5 now contains no nodes of odd degree, and thus an Euler tour can be drawn on it, beginning at any desired node and ending at the same node.

In this case we start at A and end at A. Below is the path:

A-B-C-D-E-F-G-H-I-H-P-O-P-Q-U-O-E-D-S-R-U-R-Q-J-I-J-K-R-S-L-K-L-M-T-S-T-C-N-M-N-A

The length of all Euler tours on this graph is equal to the total length of the original graph (4847m) plus the distance to be traversed twice (1035m), for a total length of 5,882m.

Considering two streets which lead to dead ends of total distance of approximately 129m, the total length of street to be transversed by Team A is 5882 + (129*2) is equal to 6,140m. This represents 27 percent more than the total street length of the district that the mailman is responsible for. This turns out to be the optimum solution.

4.3 SOLVING THE CHINESE POSTMAN PROBLEM FOR ZONE TWO – TEAM B

4.3.1 TRACING THE PATH FOR ZONE TWO ON THE MAP

Fig 4.6 Map of Zone Two – inspected by Team B

4.3.2 TABULATION OF ACTUAL DISTANCES – ZONE TWO

Table 4.3 Tabulation for Actual Distance for Zone Two

Segments	Distance on Map (cm)	Actual Distance (m)	Actual Distance to the nearest whole number (m)
A'B'	0.60	70.59	71
B'C'	0.70	82.35	82
B'M'	1.20	141.18	141
C'D'	3.40	400.00	400
C'U'	0.70	82.35	82
D'E'	0.90	105.88	106
D'R'	1.10	129.41	129
E'F'	2.90	341.18	341
E'Q'	1.60	188.24	188
F'G'	1.30	152.94	153
F'Q'	0.70	82.35	82
G'H'	0.80	94.12	94
G'P'	0.80	94.12	94
H'I'	2.60	305.88	306
H'O'	2.00	235.29	235
I'J'	0.85	100.00	100
J'K'	2.35	276.47	276
K'L'	0.35	41.18	41

K'O'	1.85	217.65	218
L'M'	0.55	64.71	65
M'N'	1.50	176.47	176
M'V'	0.60	70.59	71
N'O'	0.65	76.47	76
N'P'	1.65	194.12	194
O'I'	0.90	105.88	106
P'Q'	1.30	152.94	153
P'X'	0.70	82.35	82
Q'R'	0.70	82.35	82
R'S'	0.80	94.12	94
R'X'	1.25	147.06	147
S'T'	1.20	141.18	141
S'Y'	1.35	158.82	159
T'U'	0.50	58.82	59
T'Y'	0.80	94.12	94
U'V'	0.60	70.59	71
V'W'	1.10	129.41	129
W'X'	0.90	105.88	106
W'Y'	0.50	58.82	59
Total		5205.88	5206

4.3.3 CONVERSION OF ZONE TWO MAP TO GRAPH

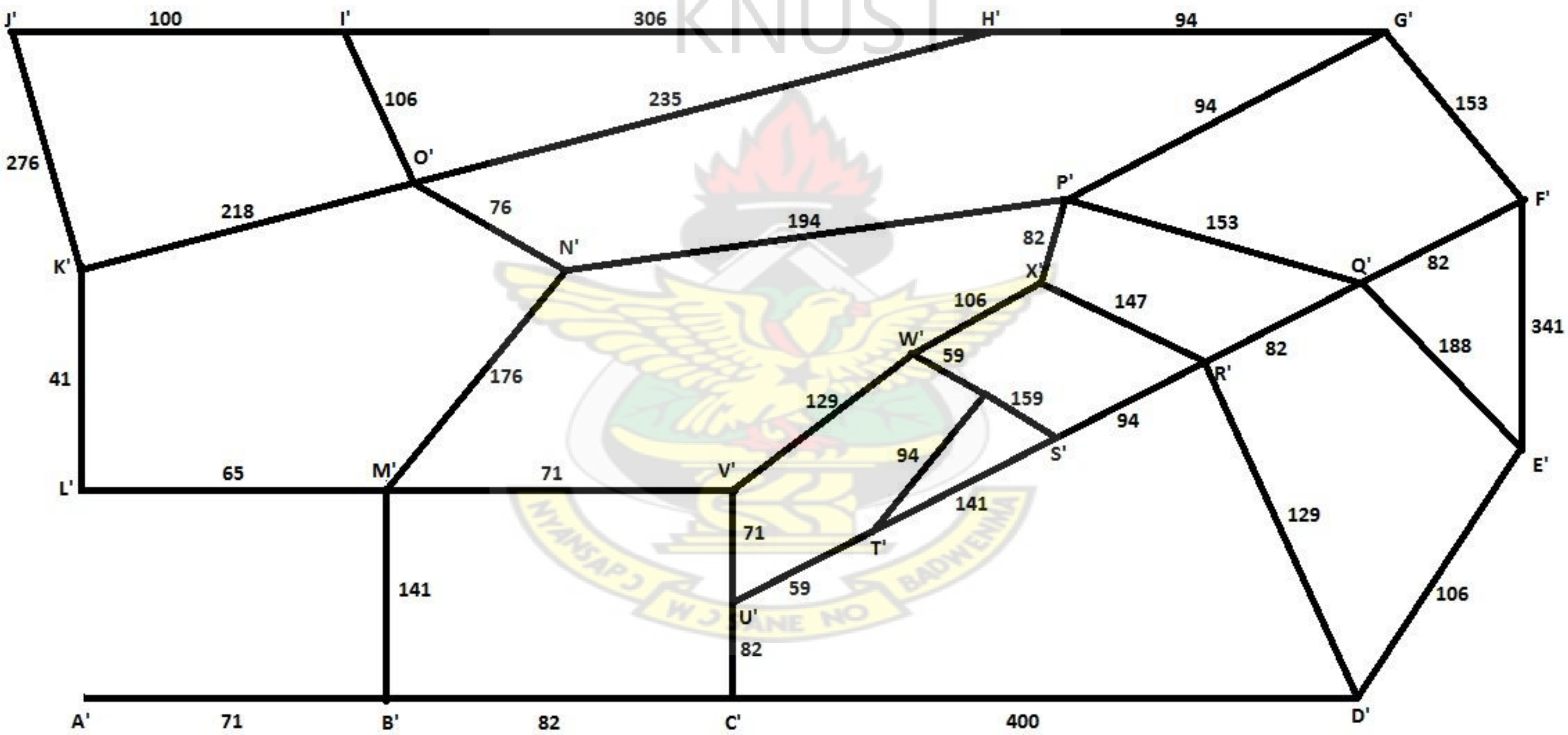


Fig 4.7 Graph for Zone Two (not drawn to scale)

4.3.4 FINDING THE OPTIMAL SOLUTION – ZONE TWO

The above graph is non-Eulerian. We need to convert graph into an Eularian graph by adding edges after which an Euler tour will be found with Fleury's algorithm.

Step 1 : Identify all the odd vertices in the graph

Odd vertices: A', B', C', D', E', F', G', H', I', K', N', S', T', U', V', W', X', Y'

Step 2: Find all possible sets pairings of the odd vertices.

Referring to Equation 6.4

S = 34,459,425, where S is all the possible sets pairing of the vertices.

Step 3

For each set of pairings, we find the shortest path between the two vertices in each pair. We then compute the total distance of the set of pairings by adding up the distances of the shortest paths.

Although theoretically there are 17 * 15 * 13 * 11 * 9 * 7 * 5 * 3 * 1 = 34,459,425 possible sets of matchings among the eighteen odd-degree nodes very few sets of matchings would even qualify for consideration.

Step 4

We select the set of pairings with minimum weight and repeat these edges in the graph.

In fact, a couple of trials was sufficient to convince me that the minimum-cost matching can only be the matching A'-B', C'-D', E'-F', G'-H', 'I-K', N'-S', 'T-U', V'-W', X'-Y' for a total cost (i.e., duplicated street length) of 1035m of length.

Table 4.4 Minimum cost matching – Zone Two

Pair	Shortest Path	Cost
A'B'	A'-B'	71
C'D'	C'-D'	400
E'F'	E'-F'	341
G'H'	G'-H'	94
I'K'	I'-O'-K'	324
N'S'	N'-P'-X'-R'-S'	517
T'U'	T'-U'	59
V'W'	V'-W'	129
X'Y'	X'-W'-Y'	165
TOTAL		2100

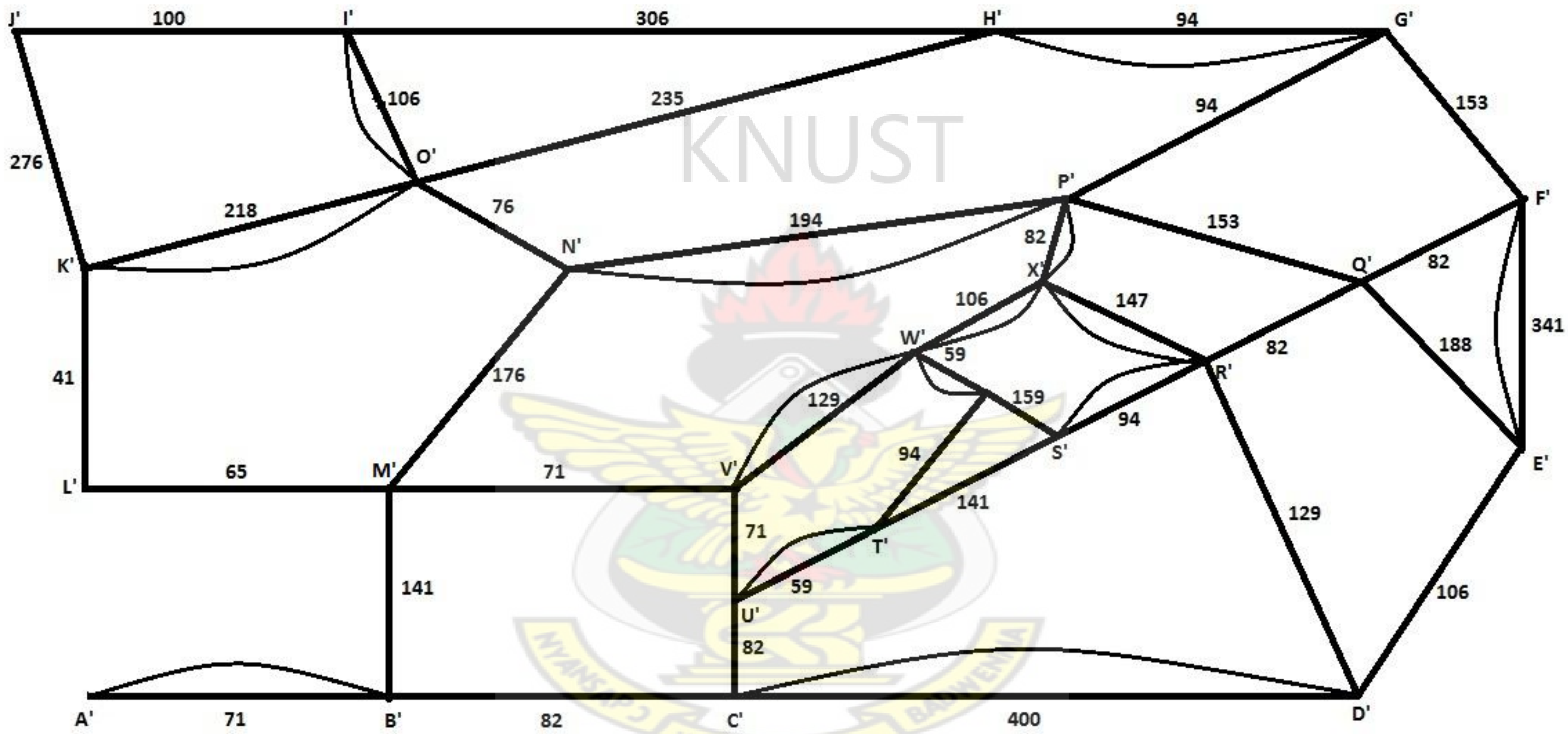


Fig 4.8 The Eulerian graph (with repeated edges) for Zone Two

Step 5

We finally use Fleury's algorithm to find an Euler tour in the resulting graph.

The final result for the Chinese postman,s problem in this case is then shown on Figure 4.8, where edges to be traversed twice have been substituted by two edges (or pseudo-edges) each of equal length to the original one. The graph of Figure 4.8 now contains no nodes of odd degree, and thus an Euler tour can be drawn on it, beginning at any desired node and ending at the same node.

In this case we start at A' and end at A'. Below is the path:

A'-B'-C'-D'-E'-F'-G'-H'-G'-P'-N'-M'-L'-K'-J'-I'-O'-H'-I'-O'-K'-O'-N'-P'-Q'-F'-E'-Q'-R'-X'-P'-X'-R'-D'-C'-U'-V'-W'-X'-W'-Y'-S'-R'-S'-T'-U'-T'-Y'-W'-V'-M'-B'-A'

The length of all Euler tours on this graph is equal to the total length of the original graph (5,206m) plus the distance to be traversed twice (2,100m), for a total length of 7,306m.

This represents 40 percent more than the total street length of the district that the mailman is responsible for. This turns out to be the optimum solution.

CHAPTER FIVE

5.0 INTRODUCTION

This chapter is about the conclusion on the study and some recommendations to be considered for future improvement.

5.1 CONCLUSION

The objective of finding an optimal solution for the inspection of electricity meters in the Asylum Down by two persons was achieved. As the total distance traversed is translated into cost, minimum cost for this exercise was obtained.

5.3 RECOMMENDATIONS

Below are some of the recommendations to be considered:

(i) Step 2 of the Chinese Postman Problem i.e finding all possible sets pairings of the odd vertices especially when the number of odd vertices is large (say greater than 6). Fortunately, an efficient, but quite complicated algorithm for minimum length matchings on undirected graphs is now available. This algorithm is based on the theory of matching on graphs that has been developed in recent years primarily by J. Edmonds. This algorithm should be investigated and used for subsequent similar research study. All other steps are simple to obtain.

(ii) Further, algorithm for sophisticated network like this study should be developed so that once diagram is keyed in, optimal solution for the Chinese Postman Problem will be obtained.

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