

OPTIMAL HARVESTING POLICY FOR AN AGE-STRUCTURED TILAPIA POPULATION

By

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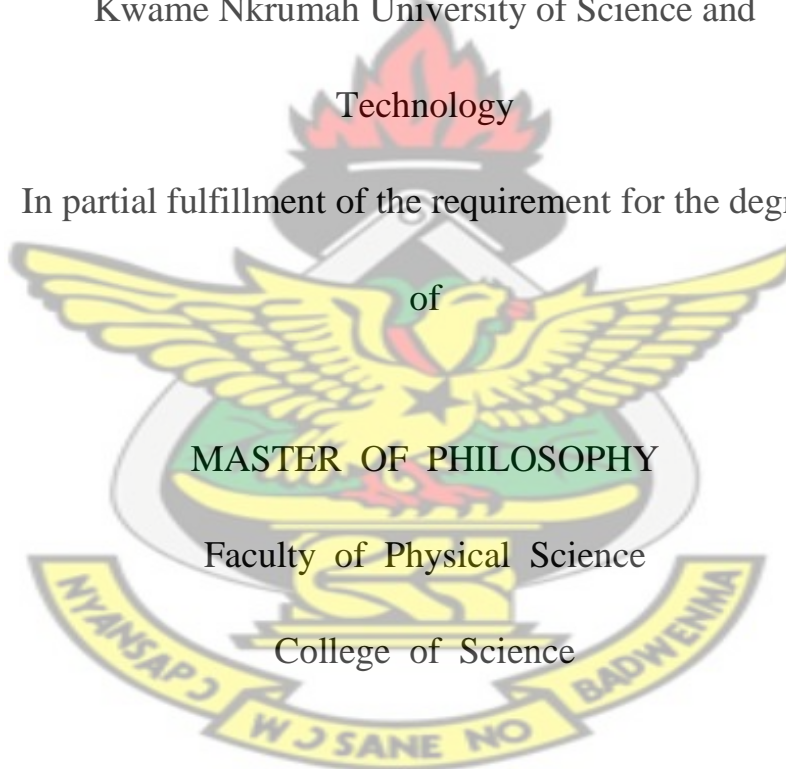
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DEDICATION

This piece of work is dedicated to the Almighty God for His protection throughout my academic years in this university.

I also dedicate it to my mum, Madam Janet Amankwah, who single handedly saw me through my education.

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DECLARATION

I hereby declare that this submission is my own work towards the M. Phil. And that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the University, except where due acknowledgement has been made in the text.

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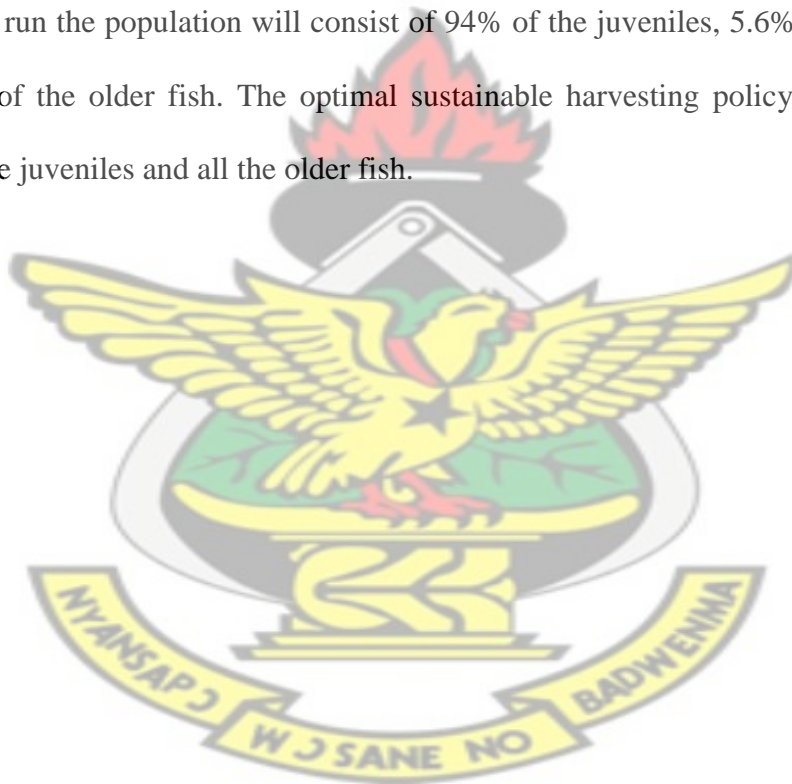
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ABSTRACT

This research was carried out to determine an efficient optimal sustainable harvesting policy for an age-structured tilapia population using the Leslie model. The population was grouped into three age classes of six months intervals: juvenile, matured and older classes. The dynamics of the population was investigated by finding the eigenvalues and eigenvectors of the Leslie matrix of the tilapia population and develop an optimal sustainable harvesting policy for the population.

The dominant eigenvalue was $\lambda = 12.652$ and the corresponding eigenvector indicated that in long run the population will consist of 94% of the juveniles, 5.6% of matured and only 0.4% of the older fish. The optimal sustainable harvesting policy was harvesting 99.3% of the juveniles and all the older fish.



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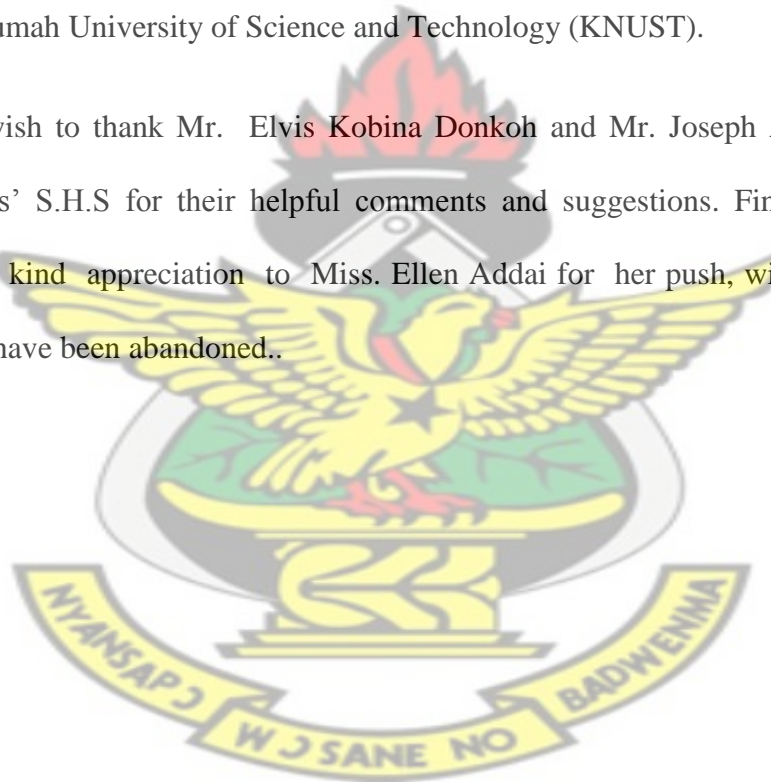


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CHAPTER 1

INTRODUCTION

1.1 BACKGROUND

Fifty years ago, scientists were beginning to recognize that many renewable resources, once plentiful and seemingly limitless, were in decline; stocks were diminishing and increasing amounts of effort were required to maintain harvest level.

At the time, biologists played the leading role in policy designed and analysis; primarily focused on fisheries. Only later would economists engage in this discussion and convincingly articulate the role economic behavior played in the problem and potential role economic institutions could play in the solution (Gordon, 1954; Scott, 1955).

Open access resources tend to be overexploited (Clark, 1990), and management actions are usually necessary in order to achieve sustainable use of the renewable resources. Optimal harvesting strategies are management tools which help to decide how the annual harvesting from a fish stock should be adjusted to in response to stock population, to obtain sustainable maximum yield (SMY). In population ecology and economics, SMY is, theoretically, the largest yield (or catch) that can be taken from a species' stock over an indefinite period.

The world's fishing fleet are losing 50 billion USD each year through depleted stock and poor fisheries management (UN report, 2008). The report, produced jointly by the World Bank and UN food and Agriculture (FAO), assent that half of the world's fishing fleet could be scrapped off with no change in catch.

Fish and other marine products form an import sources of food, in particular protein, for mankind. Thus role of marine products is still rising, because land for agricultural has

been over exploited to a large extent. But no new areas for agriculture can be found, because of a limited of water. Moreover the recent spate of diseases among livestock and poultry causing mankind to seek for other alternates sources of proteins hence, the need to control the stock in order to optimize harvest.

The control and optimal yield will require a series of political and technological steps like binding international agreements on size and structure of catch, verification of size and structure of fish stocks or limits to equipment and modification of equipment. Based on reliable data, it should be possible to give detailed recommendation for fishing industry so that at the long round, the catch can be optimized.

Between 1960 and 1996, global fish production for human consumption grew from 27 million to 91 million metric tons. The demand for fish will continue to rise with growing population, increasing incomes and improved diet in developing countries. As the population of countries rise, so will demand for fish. Unfortunately, about 70 percent of the world's major fish species and all of the 15 major fishing are in decline and need urgent management. The decline in the world's fish stock resulted from; overfishing, indiscriminate harvesting methods, and degradation of coastal and inland ecosystem.

One hope for meeting the rising demand for fish is aquaculture. Since 1984, the output of aquaculture has increased annually by 10 percent, where as captured fish output has only increased 1.6 percent each year. Today, about one-fourth of the fish eaten by humans comes from aquaculture systems. Aquaculture offers developing countries a means to earn foreign exchange through high-value species, such as pear, prawns and salmon and a way for poor communities to maintain a healthy diet and earn an income. But if aquaculture is not carried out in an environmentally responsible way, they can cause water pollution, wetland loss and mangrove swamp destruction. Research has been one

key to the successful growth of aquaculture. Scientists also are seeking to improve the production and management of all fisheries resources in ways that benefit users and promote a healthy environment. Some examples include:

- Breeding new fish species. Researchers have produced and improved strain of Nile tilapia, a hardy freshwater fish that grows 60 percent faster than other farmed strains and yields three fish crops per year, instead of the usual two. Fish farmers can expect higher productivity, higher profit and higher yield.
- Integrating crop and livestock farming with aquaculture. Through assistance from research organization, farmers in Malawi and other African countries are introducing small ponds into their homes gardens for irrigation and to grow fish. They are also using the mud from the bottom of the fish ponds as rich organism fertilizer for their farming.
- Improving the livelihood of the rural poor. In Bangladesh scientist are helping to turn numerous unused ponds and other water bodies into viable fish farms.
- Fish farming provides a new source of income for the rural poor particularly using new systems developed through research, fish production in existing ponds has increased eightfold (Future Harvest, 2010).

1.2 STATEMENT OF THE PROBLEM

The demand for marine products in Ghana is increasing annually with improved income and lifestyle (Kwadjosse, 2009). Also a projection of demand and supply of fish in Ghana, computed from a baseline demand of 80000 tons in 2002 shows an increase deficit in supply (Ghanaweb, 2010). Thus the need to augment aquaculture in, the best way of increasing the supply of marine product. Tilapia with its viable commercial aquaculture potential makes it a good choice.

1.3 PURPOSE OF THE STUDY

Aquaculture of tilapia has been booming in recent times (Helga, 2007). However, one of the main drawbacks associated with tilapia farming is overpopulation due to their high fecundity. To solve this problem, many methods including monosex culture and periodic harvesting of fries, among many others, have been employed by farmers (Fortes, 2005). These methods of getting all males for the monosex culture are either too tedious or are not readily available to common farmer and even if it is available, have health implications on the consumer because of the chemicals used.

In this thesis, an attempt is made at finding a solution that can be made readily available to famers by finding an optimal harvesting policy for an age-structured tilapia population.

The specific objectives are:

1. To formulate a Leslie matrix for the Nile tilapia (*Oreochromis niloticus*)
- 2 To study the dynamics of the *Oreochromis niloticus* (*O. niloticus*)
- 3 To determine an optimal harvesting policy for the Nile tilapia population.

1.4 COMMERCIAL IMPORTANCE OF TILAPIA

Tilapias are among the most important warm water fishes used for aquaculture production. They originated from the tropical and subtropical parts of Africa (Fryer and Iles, 1972) but are now farmed throughout the world. In temperate and some sub-tropical regions, their culture is highly affected by sensitivity to low ambient temperatures leading to poor growth and mass mortality during over-wintering (Chervinski and Lahav, 1976; Tave *et al.*, 1990). This restricts the grow-out period in these regions to between three and seven months (Hofer and Watts, 2002). To maximize the grow-out season, fingerlings are usually produced indoor during colder months and stocked during warmer summer periods. The optimal temperature for growth of most tilapia species is between 25- 28 °C. Reproduction stops at 22 °C and feeding below 20 °C (Wohlfarth and Hulata, 1983).

Tilapia cannot survive temperatures less than 10-12 °C for more than a few days (Chervinski, 1982).

A few studies have been carried out on the genetic basis of cold tolerance in tilapia. Wohlfarth *et al.* (1983) and Cnaani *et al.* (2000) studying some tilapia species and their hybrids found that a large component of the trait's variance was a result of dominance effects. However, in Nile tilapia, *Oreochromis niloticus*, Tave *et al.* (1990) and Behrends *et al.* (1990) suggested that cold tolerance shows mainly additive genetic variance. Knowledge of the genetic nature of cold tolerance could aid in improving the tolerance of Nile tilapia to low temperature and extend its grow-out period.

A collaborative project aimed at selecting for fast growth of *Oreochromis niloticus* (*O. niloticus*) in ponds is currently being carried out by Wageningen University, the Netherlands, and the World Fish Centre (formerly ICLARM) Regional Centre for Africa and West Asia, Abbassa, Egypt. Abbassa is located east of the Nile Delta, which experiences cold spells during winter. Knowledge and improvement of the temperature tolerance of fingerlings could help extend the grow-out period and reduce economic losses in this region. The main objective of the study was to estimate the heritability of cold tolerance in juveniles of *O. niloticus*.

The total world tilapia landing from capture and culture has been estimated at 1.16 million tons (FAO, 1997), with cultured tilapia accounting for 57 percent of the total (659,000 tons). The most important tilapias in aquaculture are the maternal mouth brooders (Schoenen 1982; Pullin 1985); the Nile tilapia (*O. nilotical*) Mozambique tilapia (*O. mossambica*); and the blue tilapia (*O. aurea*); plus a number of a mouth brooding tilapia hybrids used in aquaculture (especially red *O. Mossambica* hybrids) with *O. aurea*,

O. nilotica, and *O. urolepis hornorum*. These species account for 99.5% of global tilapia production (FAO, 1997).

Of secondary importance are *T. galilea* and *T. melanotheron* (principally in West African lagoons). Since the mid-1980s, there has been a shift in producer preferences away from the Mozambique tilapia towards growing Nile tilapia. Nile tilapia now dominates global tilapia aquaculture, accounting for 72% or 474,000 tons in 1995 (FAO, 1997). Cuba is the world's largest producer of blue tilapias, which are grown in an enhanced reservoir fishery supplemented by hatcheries (Fonticiella and Sonesten, 2000). The largest tilapia producing nations are in Asia. China is the world's largest tilapia producer (315,000 tons), accounting for 48% of global production, followed by the Philippines, Thailand, Indonesia and Egypt (FAO, 1997).

The USA is the world's largest tilapia consumer. US tilapia consumption is estimated at 51,645 tons of live weight equivalent (Engle, 1997). The USA imports over 3 times the amount of tilapia it grows, with the major importers (in order by value): China, Thailand, Costa Rica, Indonesia, and Columbia. Tilapia imports contribute a measurable share of the large US trade deficit in seafood products. It is the third largest imported aquaculture product to the US after shrimp and salmon and has received rapid consumer acceptance in US seafood circles as the 'new white fish' (ATA, 2000).

Tilapias are the most frequently requested fish in the US restaurant trade, and new markets carrying the fish for the first time report rapid acceptance (ATA, 1995). The culinary characteristics of the fish match almost perfectly the desires of the US consumer, for example, a white flesh, boneless, relatively odorless, with a very mild flavor. Tilapias are increasingly being seen as a replacement for cod and hake which are in short supply. Tilapia sales have exceeded those of trout in the US each year since 1995 (ATA, 2000).

As a result, tilapia production in the Americas is expected to exceed 500,000 tons by 2010. (Fitzsimmons, 2000). However, in some countries like Australia (FAO, 1980), tilapia are not seen as an economic resource but an ecological pest, that eradication programs are being undertaken through several means: use of organic toxicants, construction of outfalls and others were undertaken unfortunately, but have not succeeded.

1.5 TILAPIA FARMING

Tilapias have good characteristics for farming, and are now so domesticated that they have earned the title “the aquatic chicken”. They are fast-growing, able to survive in poor water conditions, eat a wide range of food types, and breed easily with no need for special hatchery technology.

Tilapias are one of the best researched species for aquaculture, and there is a wealth of experience in their husbandry. Tilapia are tough and tolerate a wide range of environmental conditions: little environmental modification is needed, so aquaculture systems can be low-tech. Earthen ponds of appropriate design in non-flood-prone areas will be sufficient for tilapia farming. Concrete tanks or raceways can be used, but are more expensive to build and usually cannot be justified in rural areas. Cages in lakes, dams or rivers can also be used. All these systems are in use in Pacific Island countries, depending upon local circumstances.

Tilapias have some bad characteristics. Uncontrolled breeding in ponds lead to overcrowding and stunted growth. Tilapia can be a pest that eats other fish species. Once tilapia are present in a pond, they are difficult to get rid of except by poisoning or by draining the place and leaving it to dry until the bottom has baked hard in the sun. They

need to live in warm water and do not grow well if the water temperature is lower than 22°C.

The trick to efficient, successful and profitable tilapia farming is to stock with large batches of fingerlings of similar size and age. Only then can all fish in a pond be harvested at the same time, after which the pond must be completely dried out to kill any leftover, unwanted fish. A sign of inefficient farming is ponds filled with fish all of different sizes and ages. It is easy to farm tilapia badly, but to farm them well and produce large batches of similar-sized fish; management skills and a good supply of quality fingerlings are needed.

1.6 TYPES OF TILAPIA

Tilapias belong to a group of fish called cichlids and are native to Africa. Tilapia are shaped like snapper but can be identified by an interrupted lateral line, which is a characteristic of the cichlid family of fishes. They are laterally compressed (flat-sided) and deep-bodied with long dorsal fins. The front portion of the dorsal fin is spiny and the rear is soft rayed. Spines are also found in the pelvic and anal fins. The external anatomy of tilapia is given in Fig. 1.

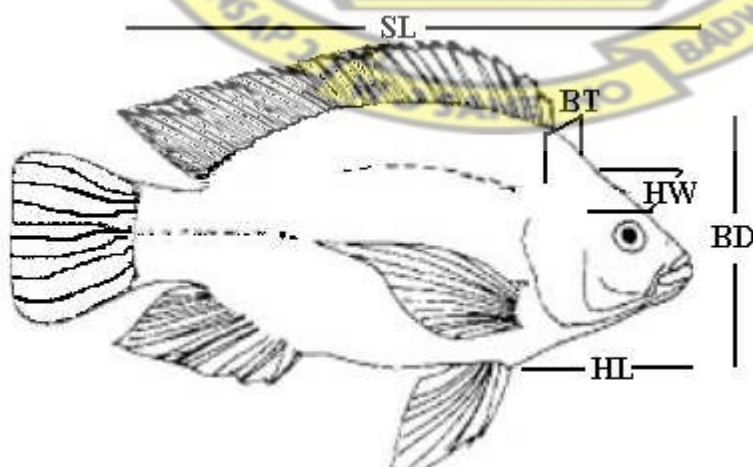


Figure 1.1: External anatomy of tilapia.

Body measurements taken on each fish: total length (TL), standard length (SL), body depth (BD), body thickness (BT), head width (HW) and head length (HL)

The tilapia group consists of three important genera, *Oreochromis*, *Sarotherodon* and *Tilapia*. Several characteristics distinguish these three genera, but the most important one relates to reproductive behavior. Tilapia build nests and the fertilized eggs are guarded in the nest by a brood parent. Species of both *Oreochromis* and *Sarotherodon* are mouth brooders: eggs are fertilized in the nest but parents immediately pick up the eggs in their mouths and hold them during egg incubation. They continue to hold the fry in their mouths for several days after hatching. In *Oreochromis* species, only the females practice mouth brooding, while in *Sarotherodon* species either the male or both male and female are mouth brooders.

All commercially important tilapia outside of Asia and Africa belong to the genus *Oreochromis*, of which there are three main species used for farming. These are Nile tilapia (*Oreochromis niloticus*), Mozambique tilapia (*Oreochromis mossambicus*) and blue tilapia (*Oreochromis aureus*). The Nile tilapia is the most popular farmed species. Its classification is given in Table 1.

“Family tree” of the Nile tilapia	
Phylum	Chordata – Notochord group of animals
Subphylum	Vertebrate – Animals with a backbone
Class	Osteichthyes – Bony fishes
Order	Perciformes – Perch-like fishes
Family	Cichlidae – Cichlid fishes
Genus	<i>Oreochromis</i> – Mountain cichlid group
Species	<i>niloticus</i> – Nile tilapia

Table 1.1: Binomial Classification of Tilapia.

These species can be differentiated from each other by examining their body colouration and fins (Table 1.2). Nile tilapias have strong vertical bands on the caudal fins whereas Mozambique tilapias have weak or no bands on the caudal fins. Mature male Nile tilapias have grey or pink pigmentation in the throat region, while Mozambique tilapias have a more yellow colouration. However, colouration is often an unreliable method of distinguishing *Oreochromis* tilapia species because environment, state of sexual maturity, and food source greatly influence colour intensity.

Species	Body	Fins
<i>O. mossambicus</i>	Dark colour	No bars on caudal fins
<i>O. aureus</i>	Bluish colour	Red margins on dorsal, caudal and anal fins
<i>O. niloticus</i>	Reddish to white	Prominent bars on caudal fins; white colour strips on dorsal and anal fins

Table 1.2: Distinguishing features of the three main farmed species

1.7 BREEDING CHARACTERISTICS

Oreochromis species usually seek out shallow areas and group together for breeding. Males develop bright colouration and set up territories in which they build their nest: they hollow out a small area on the pond bottom approximately 10–15cm in diameter. They display courtship behavior and lure females to the nesting site. The nest is used as temporary site for courtship. If the female is receptive to the courting male, she will be induced to spawn. In the nest, eggs laid are fertilized by the male, who discharges sperm

over the eggs. The female collects the fertilized eggs in her mouth, and mouth-broods the eggs for around 6–10 days. After hatching, the newly hatched fry continue to shelter in her mouth for another 4–7 days. The fry begin to swim freely in schools, but may return to the mouth of the mother when threatened. Females do not feed while they are incubating the eggs or mouth-brooding the newly hatched fry.

Male tilapia can mate with several females (polygamy) and females may mate with several males (polyandry). In ponds, Nile tilapia become sexually mature at three to five months of age (150–200g weight), while Mozambique tilapia mature earlier and at smaller size. The fact that Nile tilapia mature late is why they are a better fish for farming. They have a longer time to feed and grow to a good size before they breed and so come into competition with their own offspring for food, space and oxygen.

As soon as sexual maturity is attained, most female tilapia are able to undergo successive spawning to produce a new brood every 4 to 6 weeks. Temperature plays an important role here. If water temperature remains at 22°C and above, tilapia will spawn throughout the year. Temperature in the range of 25–30°C is considered ideal.

The number of eggs per clutch increases as the female fish grows from four months to two years. On average each clutch of eggs will produce 100–500 fry. However as the female fish get older (more than two years old) they produce less fry compared with when they were younger. The best size of female for breeding is 150–300g. On average a 200g breeder would produce 200–500 fry per month. Fry production will also depend on the condition and health of the breeders. In ponds or any confined conditions, egg size and clutch size may vary.

In general fry production decreases as salinity of the water increases. Adult Nile tilapia can tolerate salinity of up to 30 parts per thousand (ppt), while fry and small juveniles are much less tolerant of saline water (they prefer less than 10 ppt). This suggests a potential for brackish water culture from the point of view of reproductive control, to lessen the problem of adults competing for food with their offspring.

1.8 SPECIES SELECTION

Potential tilapia culturists should first determine which species, if any, can be legally cultured in their state. Assuming there are no restrictions, selection of a species will depend mostly on growth rate and cold tolerance. Rankings for growth rate in ponds are *T. nilotica* > *T. aurea* > *T. rendalli* > *T. mossambica* > *T. hornorum*. Most of the hybrids tested grow as fast as their parent species. Cold tolerance may become an increasingly important criterion for selecting a species in more northerly latitudes. *Tilapia aurea* is generally recognized as being the most cold tolerant.

Pond culture is the most popular method of growing tilapia. One advantage is that the fish are able to utilize natural foods. Management of tilapia ponds ranges from; extensive systems, using only organic or inorganic fertilizers, to intensive systems, using high-protein feed, aeration and water exchange. The major drawback of pond culture is the high level of uncontrolled reproduction that may occur in grow-out ponds. Tilapia recruitment, the production of fry and fingerlings, may be so great that offspring compete for food with the adults. The original stock becomes stunted, yielding only a small percentage of marketable fish weighing 1 pound (454 grams) or more. In mixed-sex populations, the weight of recruits may constitute up to 70 percent of the total harvest weight. Two major strategies for producing tilapia in ponds, mixed-sex culture and male mono-sex culture, revolve around controlling spawning and recruitment.

1.9 POND CONSTRUCTION

There is no restriction on pond size, but for ease of management and economical operation, shallow (3 to 6 feet), small (1 to 10 acres) ponds with drains are recommended. Draining is necessary to harvest all of the fish. A harvesting sump is needed to concentrate the fish in the final stage of drainage. The pond bottom should be dried to eradicate any fry or fingerlings that may interfere with the next production cycle.

1.10 THE PROBLEM OF OVERPOPULATION IN PONDS

Natural reproduction of cultured tilapia species occurs in one of two ways. The species *Oreochromis aureus*, *O. mossambicus* and *O. niloticus* are called mouth-brooders. The female incubates and hatches her eggs in her mouth after they are laid and the male fertilizes them. The species *T. rendalli* and *T. zillii* are called substrate spawners because eggs are laid and hatched on bottom substrates in a nest dug by the male and female. Parents guard their eggs and fry, but do not protect them in their mouths. The ease with which tilapia spawn and produce offspring makes them a good fish to culture. However, this trait also creates problems. Survival of young ones is very high and this results in overcrowding in grow-out ponds. Fish become stunted as the supply of natural food organisms in the pond is insufficient. Nearly 75% or more of the stock may be less than 100 grams in such cases. This may not present a serious problem in the Orient where even tiny fish are eaten. However, if fish larger than 150 g are preferred by the market, special culture techniques may be required to grow them. These technologies require different levels of skill and management to yield varying degrees of success in producing large tilapia. Some may be combined for efficiency in resource use.

CHAPTER 2

LITERATURE REVIEW

INTRODUCTION

The review of the literature summarizes a number of works in the field of the modeling which have application to age structured population. The chapter begins by reviewing publications related specifically to harvesting age-structured populations, optimal harvesting strategies and Leslie matrix model applications.

2.1 BIOLOGY OF TILAPIA

2.1.1 HABITAT

Fishes have been living on this earth for over six thousand years. Thus they occupy all possible niches of the sea and vary greatly in their structure and habitat. All life in the sea eventually is based on small plants in the top 50metre photic layer of the sea (Mustafa, 2005). These plant algae mostly, are eaten by small copepods. These are in turn eaten by smaller fish and those again by bigger fish. All tilapia are more or less herbivores, but prefer higher plants where as others are adapted to feed on plankton. Some are relatively omnivores and will benefit from artificial feed. Tilapia is essentially tropical lowland fish, but some species and some stocks withstand cool temperature much better than others. Tilapia is eurythermal and can tolerate a wide range of temperatures (8 – 42°C). They can tolerate temperatures of 8°C at night for several hours .They however, prefers temperatures between 28 –30°C depending on climate temperature .They can tolerate low dissolved oxygen (0.1part per million(ppm)), withstand salinity range of 11 to 29% . Reduced salinity tolerance when compared to many other introduced tilapia.

They are also less aggressive than most carnivorous cichlids, but they may attack and nip fins of other species, an undesirable habit for polyculture. Most tilapia is tolerant of

brackish water, but some are better adapted to it than others and may thrive and even breed in sea water (Bardach *et al.*, 1972).

2.1.2 SPAWNING AND GROWTH OF YOUNG

In many form of fish culture, obtaining spawn is one of the most difficult tasks. Tilapia presents no such problem; indeed it is difficult to prevent them from spawning. Spawning habitat is variable, from firm sand to muddy lagoons to steep lava shores. A female can deposit 75 to 250 eggs in a simple crater-shaped nest constructed in sand or mud by – the male. Eggs are laid in approximately 20 batches over a 45-min to 2 hour period. The nest is two times the male length or up to 1 m in diameter. The female then pick them up in her mouth. A female may brood up to 2000 eggs at once. Next the male discharges sperms into the depression and this too is picked up by the female where hatching occur within 3 to 5 days when young are 4.5 mm in length. The larvae are retain in the mouth until the yolk sac is absorbed, after which they may venture forth, but for 10 to 50 days they will return to the female's mouth when threatened. The female seldom eat during this time and develop a characteristic coloration pattern while brooding young (dark stripes across forehead and a dark operculum, chin, and eye). The young tilapia matures at an age of 2 to 3 months, at which time they are 6cm to 10cm long. From then on they breed every 3 to 6 weeks as long as the water is warm.

Some tilapia species like *Tilapia zillii* and *T. sparamanni* which do not mouth-brood produce more eggs- up to 5000 in large *T. zillii* .Thus *T. sparamanni* and *T. zillii* are even more prone to overpopulate a pond and produce a stunted population than are other tilapia (Bardach *et al.*, 1972). Growth of tilapia varies greatly with stocking density, frequency of spawning and food supply. Under favorable condition, individual lava tilapia may reach a weight of 850g in 1 year; in brackish water they may reach 450g in 8 months. But

in most ponds 85 to 140g is a more realistic weight to expect after a year if sexes are raised together. Male grow two to three times faster than female. (Bardech *et al.*, 1972).

2.1.3 FECUNDITY

Fecundity is the total amount of eggs produced by a female during a single reproductive cycle. Fish are very fecund and can lay as many as 10^4 to 10^7 eggs in each spawning per female. In fish population an estimate of the number of eggs laid by an age group is obtained by multiplying the number of breeding females by their average fecundity (Pitcher and MacDonald, 1973). Fish population is more fecund in lower latitudes due to the pressure of predators. Fecundity is usually proportional to the weight i.e. if the eggs are of the same size, bigger fish, are more fecund than smaller ones. It is sometimes related to the biomass. Fecundity of Nile tilapia ranges from 149 to 2797 ova for fish between 36 and 975g.

Arredondo-Figueroa and Guzmán-Arroyo (1986) mentioned that *Oreochromis* spp. is a mouth brooder with small gonads of less than 700 eggs corresponding to fish total length of 10-23cm. Pickering and Nandlal (2004) 200eggs; Gómez-Márquez *et al.* (2005) in their study of 'Reproductive biology of *Oreochromis niloticus* (Perciformes: Cichlidae) at Emiliano Zapata dam, Morelos, Mexico', reported fecundity range of 243 to 847 eggs. This result was lower than that observed by other workers for *O. niloticus*. Fryer and Iles (1972) obtained a fecundity of 3 706 eggs from a gigantic female specimen with total length of 57 cm of *T. nilotica*. The maximum numbers of eggs counted in the ripe ovaries of any mouth-brooding cichlid are 4 300 in a specimen of the *T. aurea* with 25cm of total length. Moyle and Cech (2000) mentioned that in mouth brooding cichlids, the fecundity is considerably low because the parents assure the survival of the offspring, and in

consequence less mortality. In addition, the variation in fecundity may be attributed to differential abundance of food within the members of the population.

2.2 TILAPIA PRODUCTION CONSTRAINTS

Tilapias are useful for aquaculture with a lot of positive properties. However, they can be cultured in especially subtropical regions since they cannot tolerate cold water. Tilapias die in few days, depending on species, when water temperature decrease, generally to 13-14°C but lower in some species (Altun *et al.*, 2006).

In addition to this, even at water temperature being few degrees over the lethal limits, some fungal, viral, bacterial and parasitic infections such as saprolegniasis (Zaijie *et al.*, 2006). Another problem associated with tilapia farming is overcrowding (Charo-Karisa, 200). Different methods have been developed to control overpopulation of tilapia. The most reliable and economical method is to produce all male culture because they grow faster and produce more market sizes fishes than mixed-sex. All male tilapia can be produced by methods such as separating of sex manually via visual examination (or hand sexing), sex reversal with hormone treatment, hybridization or production of super male by applying sex reversal with hormone and hybridization. (Altun *et al.*, 2006)

2.3 IMPORTANCE OF TILAPIA

The importance of tilapia cannot be overemphasized. Among the cultured fish, it is the fifth most important, with the world production reaching 1.5 million metric tons in 2000 (Yonas, 2006). Because of the large size, fast growth, and palatability, tilapia are the focus of major aquaculture efforts. Tilapia contains both Omega-3 and 6 fatty acids which are very important nutrients. Apart from the use of tilapia as food for both man and animals, they serve as a natural biological control of most aquatic weeds. They feed on floating aquatic plants such as duckweed and watermeal, and most forms of algae. They

are becoming plant control method choice in the United States and Thailand (Wikipedia, 2010). In Kenya they have been used to help control mosquitoes which carry malaria parasite by consuming mosquito larvae.

2.3.1 DISEASES OF TILAPIA

Tilapias tolerate adverse water quality and other stressors better than other commercial aquaculture fish. In the presence of pathogens tilapia are the last to break with disease. The clinically significant tilapia pathogens fall into the general categories of virus, bacteria, and protozoa. Mycotic (fungal) diseases are only significant if the tilapias are under constant stress. In certain system, metazoan ectoparasite and endoparasites cause problems, but do not impact the tilapia industry significantly.

2.4 AQUACULTURE IN AFRICA

The majority of global aquaculture production (approximately 90%) is currently in developing countries (Hishamunda and Ridler, 2002) with Africa contributing approximately 1% of the global production (Jamu and Brummet, 2004). Given the under-utilized water resources available and inexpensive labor, high demand for fish, and suitable climate, there is a large potential for improvement of aquaculture in Africa. However, African aquaculture has had a long history of failure (Machena and Moehl, 2001). The failure for aquaculture to take off has been blamed on lack of clear policy and technological knowhow, inadequate infrastructure, poor extension support, lack of fish seed and feed, and poor financing (Hempel, 1993; Machena and Moehl, 2001).

About 95% of aquaculture production in Africa comes from small-scale farmers, with tilapia accounting for about 40% of total production. Small-scale production systems are extensive/semi intensive utility oriented pond systems (100-500m²) operated by households and integrated with other agricultural activities (Machena and Moehl, 2001).

In highly productive agricultural areas where land-subdivision has led to small parcels of land, small scale aquaculture is the only option left. Targeting small-scale systems for improved production is therefore synonymous with increasing total production of the aquaculture sector in Africa. Currently, these systems are characterized by low yields and small fish size at marketing. Because tilapia prices in most cases depend on fish size and quality, farmed fish should be of acceptable standards (Adesulu, 2000; Afolabi *et al.*, 2000). Hence, fish farming must compete with other agricultural activities for labor, water, and farm inputs at the farm level as well as with fish from capture fisheries at the market place.

Success in aquaculture depends on a viable concept, sound management, adequate financing and an organism suitable for the production system (Shultz, 1986). A major setback to aquaculture production in Africa has been the problem of dependence on donor funding, making aquaculture unsustainable (Machena and Moehl, 2001). Kosgey *et al.*, (2006) reviewed several small ruminant breeding programs among resource poor farmers in the tropics and highlighted issues determining their success or failure. One important issue causing failure is that of top-down approaches in setting breeding goals and insufficient involvement of farmers which leads to setting up of wrong breeding goals i.e. not suitable for the target conditions, and lack of continual monitoring of animals at the multiplier level. These issues may apply in selective breeding programs for fish breeding as well. Management practices and breeding goals in resource poor regions may differ from those in favorable environments. The choice of appropriate selection strategy should therefore be informed by farmer needs, the production environments, and climatic conditions.

2.5 STOCK AND RECRUITMENT

Recruitment describes the number of fish reaching a specific size or age. This may be one year, or maturity or the time when they are caught by commercial fishing gear. Recruitment can be expressed in units of number or biomass. A fish stock will consist of a number of different age or size classes of fish. Spawning level may be measured as the number of spawners, biomass of spawners, number of eggs or some other measure spawning potential, that is, the process lies in the mortality from egg to recruit. Since adult fish migrate freely in the open sea (this does exist in ponds) density dependent effects due to competition or predation will be rather small. In fact, so far this has not been observed (Mustafa, 2005). Cushing (1981) argues that density dependent effects for fish population arise during the larval drift.

Density dependence can arise through cannibalism, competition for food and predation. These effects should be more pronounced for fish with pelagic eggs because for such fish the number of eggs tends to be higher thus leading to stronger competition and predation. Cushing (1981) showed that highly fecund fish shows higher density dependence. According to him density dependent control is most likely not active in the egg stage but predominant during larval drift. In this case competition for food seems to be the main cause. Results on density depend effects are difficult to obtain, because the high variability of external conditions and influence of other external feature. The main reasons that competition for food is the dominant factors are:

- The spawning of fish is timed closely to the plankton production cycle (Cushing, 1981)
- Highly fecund fishes exhibit the most pronounced density dependent .The total process of recruitment involves the stages:

Eggs → Larvae → Juveniles → Recruits → Spawners → Eggs

As recruitment here is endogenous, this result generalizes the above mentioned pulse harvesting found in Clark (1990).

Uraivan (1988) confirmed that a genetic relationship exists between growth rate, age and size at maturity in tilapia. This observation led to a suggestion that selection can be made more efficient by combining selection for body weight at a particular age with selection for increasing fish growth. It was further suggested that environment and genotype interactions were important factors that must be considered in selection for growth and maturation. Similar to growth variation, most of the variation in age and size at maturation was found to be due to environment and genotype interaction. High environmental effects were found to be responsible for low estimated heritability for these traits and inconsistent genetic gains. Falconer (1990) and Lynch and Walsh (1988) indicated that most phenotypic characters are correlated and that an improvement in one may result in positive or negative impact on other traits.

Alverson and Paulik (1973) noted that traditional management models, such as the Beverton-Holt yield per-recruit model Ricker (1975), were of limited value for management of developing fisheries because these fisheries typically have not reached steady-state conditions. The same limitation also applies to management of overfished stocks. The latter case is of considerable interest because many stocks are fully exploited or have been overharvested (Cushing, 1981). For example, Atlantic menhaden (*Brevoortia tyrannus*) and Pacific Ocean perch (*Sebastes ulufus*) stocks have been overfished for several years and reductions in fishing effort have been recommended (Archibald, 1983). To facilitate rehabilitation of overexploited stocks, management models are needed to provide quantitative recommendations for the transitional period to lower fishing effort. Age-structured population models have been used to develop rehabilitation policies for several overexploited fish stocks.

Huang and Walters (1983) also used an age-structured model to compare short-term approaches for rehabilitation of a large yellow croaker (*Pseudosciaena crocea*) stock. In their study alternative strategies were selected arbitrarily and compared through simulation studies. Stock rehabilitation required about 15 years under a constant effort policy, but occurred much more rapidly if the fishery was closed for 3 years.

Ruppert *et al.* (1985) used an age-structured model to develop optimal rehabilitation policies for the Atlantic menhaden stock. They found that the stock was restored to the optimal steady-state level in about 9 years using either a constant effort policy or one of four “egg escapement” policies. The steady-state performance of the five policies was somewhat similar, although there were notable differences in harvest during the rehabilitation period.

Nile tilapia, *Oreochromis niloticus* is an important species in aquaculture because of its fast growth and its resistance against harsh conditions. Fingerlings of *O. niloticus* are easy to produce and reproductive potential is influenced by the environment (Lowe-McConnell, 1982), the year class (Siraj *et al.*, 1983; Rana and Macintosh, 1988) the protein level of the feed (de Silva and Radampola, 1990) or by strain differences (Hulata *et al.*, 1988).

Tsadik, (2008) studied “the effect of maternal age on fecundity spawning interval, and egg quality of Nile tilapia, *Oreochromis niloticus* ” he examined four age groups of 4, 9, 16, and 24 months in a circular concrete tanks in a recirculation systems. He monitored fecundity (eggs per spawn), weight (g), and egg quality every 4-days for the 168-day experimental period. According to him, egg per spawn correlated with maternal age, while it did not correlate with body weight. Eggs per female per day were 1.8 and 2.4 times higher in 9-months old female than 16 and 24 months female respectively. Eggs per

female per day showed a decline concurrent with per spawn per female trend after 18-month old. Four month old females produced eggs that were more uniform in size and weight than others. While egg size (mm), weight (mg) and their variations with eggs per spawn increased. The percentage fertilization and hatchability decrease with increase in age of females. Biological optimum age range for reproducible reproduction of good quality eggs was 6-18 months. This according to him could be adopted in selecting breeders for better seed production.

2.6 CONSERVATION BIOLOGY

The judicious use of matrix population models has become an indispensable tool for conservation biology (Beissinger and Westphal, 1998; Benton and Grant, 1999). The rate of increase of a population is a critical parameter of interest in conservation biology, since a robust growth rate – and low variance in this growth rate – is the best insurance against extinction. The asymptotic growth rate of an age-structured population is the dominant eigenvalue of the projection matrix. This dominant eigenvalue, one is guaranteed to be unique, strictly greater than all other eigenvalues, and real if standard conditions for irreducibility and aperiodicity are met (Caswell, 2001). Sensitivity and elasticity analyses of dominant eigenvalue with respect to perturbations of vital events provide important insights into the relative importance of different vital rates for population viability and the potential impact of measurement error on the estimation of a population's rate of increase (Caswell, 2001). The assumptions of the perturbation analysis that lie behind demographic elasticity analysis limit the uncritical use of elasticity in conservation biology. For example, real perturbations in a conservation context can be large and affect multiple vital rates simultaneously (Mills *et al.*, 1999).

2.7 THE LIFE TABLE AND THE LESLIE MODEL

Most managed populations are age-structured, birth-pulse populations (Caughley, 1977). Two approaches to modeling the population dynamics of species that reproduce in a discrete rather than continuous fashion have been developed: the Leslie (1945) matrix and the life table (Caughley, 1977). The main difference between the two is the recruitment term (Taylor and Carley, 1988). The difference is noteworthy because the Leslie matrix recruitment term includes both recruitment and survival information, and cannot be estimated directly from the standing age distribution, while the classical life table recruitment term is simply the number of female offspring produced by females of age n divided by the number of females of age n .

Individuals simultaneously survive and reproduce with the Leslie matrix approach, whereas the life table approach has the females survive first, then they reproduce. The mathematical relationship between the Leslie matrix recruitment rate and the life table recruitment rate also depends on the time chosen for the census (Taylor and Carley, 1988). Both approaches identify a single time of census that defines both survival and recruitment values (Taylor and Carley, 1988).

The management of a fishery is a decision with multiple objectives. One of the desirable objectives in the management of fish resources is the conservation of the fish population. The formulation of good harvesting policies which take into account this objective is complex and difficult. For this reason, models of fish population dynamics are essential to provide assessment of fish biomass and fishing pressure. Their use forms the basis of scientific advice for fisheries management. Their nonlinearity and their complexity that are associated with biological phenomena (birth, death, growth, cannibalism, intra-stage competition for food and space, etc.) offer many challenges for scientists and engineers,

in order to manage fish population resources. The development of fishing management modeling was motivated by the need to understand mechanisms governing production flows of marine reserves. Several models have been developed and their analyses help identify effective management measures to be adapted to specific objectives. The control theory can be used to address the problem of defining a good harvesting policy, by stabilizing the stock states around the references equilibrium, which means biologically the sustainability of the fish stock. When solving this control engineering problem, it is often necessary to know the state of a dynamical system.

2.9 CONSIDERATION OF SUSTAINABILITY AND YIELD

We know of only one general analysis that included aspects of both sustainability and yield. A comparison of the spatial configuration likely to be best for conservation with that best for yield indicated that for conservation one could use a small number of large reserves of a size that would allow persistence of the longest-distance disperser one desired to protect (Hastings and Botsford, 2003). This was contrasted with the best configuration for yield, which was that configuration that sustained the population but also supplied the greatest export of larvae from reserves, a system of many reserves as small as practicable. For sustainability, this system needed to cover a certain fraction of the coastline, the minimum fraction Lifetime eggs production (FLEP) required for sustainability of the fished species, as noted above (Botsford *et al.*, 2001). This analysis assumed the cost of a reserve for conservation was proportional to the shoreline placed in reserve and did not include the cost of fishery yield lost due to displaced effort. While that analysis provides valuable insight into the benefits supplied by different spatial considerations, it is now clear that cost of foregone yield is a significant issue in real policy decisions.

2.10 POPULATION HARVESTING

Braverman and Mamadani (2008) considered both autonomous and no autonomous population models and found that constant harvesting is always superior to impulsive harvesting even though impulsive harvesting can sometimes do as good as constant harvesting. Their results contrast with the results of Ludwig (1980) and Xu et al (2005). For single species, Ludwig (1980) studied models with random fluctuations and found that constant effort harvesting does worse than other harvesting strategies.

Xu *et al.*(2005) investigated harvesting in seasonal environments of a population with logistic growth and found that pulse harvesting is usually the dominant strategy and that the yield depends dramatically on the intrinsic growth rate of population and the magnitude of seasonality. Furthermore, for large intrinsic growth rate and small environmental variability, several strategies such as constant exploitation rate, pulse harvest, linear exploitation rate, and time-dependent harvest are quite effective and have comparable maximum sustainable yields. However, for populations with small intrinsic growth rate but subject to large seasonality, none of these strategies is particularly effective, but still pulse harvesting provides the best maximum sustainable yield.

AlSharawi and Rhouma (2009) examined the effect of harvesting and stocking on competing species governed by a Leslie-Gower model and found that careful harvesting of the dominant species in an exclusive competitive environment can sometimes lead to the survival of the weaker species. More recently, the authors have also studied the Beverton-Holt equation under periodic and conditional harvesting and have found that in a constant capacity environment, constant rate harvesting is the optimal strategy.

2.11 SUSTAINABILITY OF THE TILAPIA POPULATION

Sustainability of fisheries is an issue that has received increasing attention since the late 1980s, resulting in explicit focus on the persistence of fished populations. Persistence has been described in terms of equilibrium conditions for age-structured populations (Mace and Sissenwine (1993). These conditions specify that a population with density-dependent recruitment will have a nonzero equilibrium as long as the number of eggs produced in the lifetime of an individual exceeds the inverse of the slope of the relationship describing the number of recruits produced by a specified number of eggs spawned (i.e., the stock– recruitment relationship with stock depicted in terms of total egg production). The lifetime egg production (LEP) used in ecology is equivalent to eggs per recruit or spawning biomass per recruit in fisheries (Goodyear, 1993). To compare that condition across species in general terms, fishery biologists express it as the fraction of the natural, unfished LEP, which is denoted as FLEP (Botsford *et al.*, 2004). This quantity is referred to as spawning potential ratio in fisheries. Considerable effort has gone into determining the value of that quantity required for persistence of marine fish, (Clark ,1990), Mace and Sissenwine (1993) with more recent efforts on meta-analysis to determine generic consistencies based on taxonomy (Myers *et al.*, 1999) and upward adjustment of previously low values (Clark, 2002; Ralston, 2002). The reason that this persistence condition is not useful in the assessment of population persistence in marine reserves is that a population distributed across a system of marine reserves is not the single, well mixed population assumed above, but rather a meta-population of such populations distributed over space, connected by dispersing larvae (Botsford *et al.*, 1994). The recruits produced at each location end up distributed along the coast through the process of larval dispersal, and much of the recruitment at each location is produced

elsewhere. Because of this additional complexity, a general expression for stability of these marine meta-populations has been difficult to obtain (Armsworth, 2002).

Researchers have, however, developed useful ways of describing population dynamics in terms of source–sink dynamics (Lipcius *et al.*, 2001). Some results regarding persistence of simple meta-populations have been developed in the context of marine reserves. Initial results assumed a population with sedentary adults, post-settlement density dependent recruitment of the Beverton–Holt type (Beverton and Holt, 1957) and larvae with a Laplacian dispersal pattern (i.e., exponentially decaying with distance in both directions) in a system of uniformly spaced reserves of width w and spacing s , along a coastline with uniform habitat, with fishing removing all fish between reserves (Botsford *et al.*, 2001). The result was that for a single reserve of a certain width, species with mean dispersal distances less than or roughly the same as that width would persist. However, a system of uniformly spaced reserves of any width that covered a specific fraction of the coastline would allow persistence of all species, regardless of dispersal distance. The latter result indicated that a network of reserves could function in a way that was greater than the sum of the workings of the individual reserves (i.e., it was a “network result”). The specific fraction of coastline that needed to be covered was the value of FLEP required for persistence of the single, well-mixed population, as discussed in the previous paragraph. In this simple case, one can gain some intuitive understanding of why FLEP sets the minimum area in reserves by observing that the role of fishing in reducing LEP in the single, well-mixed population can be thought of as being replaced in the

2.12 FISHERY YIELD

In its simplest form, the question in the minds of policy makers of how implementation of a marine reserve will affect fishery yield is essentially whether the loss of fishable area will be compensated for by changes in the population brought about by implementation of the reserve. Assuming that reserves are not going to affect individual growth and mortality rates outside the reserve, they will not affect yield per recruit, and we need consider only the effect on recruitment. For yield to increase with the implementation of reserves, the increase in recruitment due to the increased egg production from the reserves must be large enough to compensate for the fraction of the area placed in reserves.

This requires not just an increase in egg production, but that the larvae produced are able to reach the fished areas to increase recruitment by the required amount. Thus, if recruitment has not been substantially reduced by the current fishery, there is little scope for recruitment to be increased by the additional egg production supplied by a system of marine reserves. In turn, the amount by which recruitment will have been reduced by fishing a certain amount will depend to some degree on the slope of the egg–recruitment relationship at the origin.

This observation regarding the potential for reserves to increase yield raises the policy question of whether yield can be increased as much by changing management in a conventional way (i.e., by decreasing fishing effort). That question was addressed by analysis of a simple model with no adult movement, larval settlement equally distributed across the population, post-dispersal density-dependent recruitment only (Hastings and Botsford, 1999). The answer, that the maximum yield problem for conventional management was mathematically the same as maximizing yield using reserves, indicates that there is a rough equivalence between reducing effort in conventional management and implementing marine reserves. This conclusion is also indicated by the invariant

noted by Mangel (1998). This rough equivalence is consistent with results from a number of simulation studies of more complex models. The typical result is that marine reserves produce greater yield only for fishing mortality rates greater than a certain minimum value Holland and Brazee (1996), or conversely, marine reserves are a means of guaranteeing sustainability even if the fishing mortality increases to very high values, (Quinn *et al.*, 1993). The rough equivalence between conventional management and management by reserves provides policy makers with an easily computed benchmark estimate of the yield possible with marine reserves.

Whether reserves or conventional management are actually superior depends on further detail, usually in an obvious way. For example, if compensatory density-dependence occurs prior to dispersal (e.g., density-dependent fecundity or indirect effects on fecundity such as density-dependent growth), reserves will have less advantage because reserves will increase density. On the other hand, if pre-dispersal density dependence is dispensatory, such as in broadcast spawning, reserves will have greater advantage. Another example, fishermen shifting effort from inside reserves to outside, rather than simply leaving as assumed implicitly or explicitly in virtually all models of marine reserves, is treated in several recent publications (Smith and Wilen 2003). In that case also, reserves have less advantage.

Situations involving substantial movement and heterogeneity in productivity can lead to exceptions to this rough equivalence. One example is populations with ontogenetic movement, in which specific life history stages can be protected (e.g., reserves can be placed to protect spawning and rearing areas (Apostolaki *et al.*, 2002). Other examples involve differences in larval productivity and substantial alongshore advection. When there are areas with excess larval production, and other areas with populations at less than

the benthic carrying capacity, then reserves can provide greater yield than can any scheme with the same fishing effort at all locations.

Morgan and Botsford (2001) showed that increasing protection of a single source population coupled to three sink populations could increase yield by an amount greater than that possible through conventional fishery management of all four populations. However, that advantage was not possible unless the source was known. Gaines *et al.* (2003) showed another example in which populations in the middle of a bounded area along a coastline with reversing currents could produce greater yield with reserves than with conventional fishing throughout. In the cases of these exceptions to the rough equivalence between conventional management and management by reserves, yield could increase even when the fishery has not diminished recruitment.

Much of the literatures focusing on fisheries bio-economic models are analytical in which equilibrium solutions are determined and discussed. Many solutions of dynamic bioeconomic models have been presented in the literature (Clark, 1990). These studies present dynamic optimization problems constrained by the biological model of the resource stock. The results of these studies confirm what has become known as the 'modified Golden Rule' of resource or capital accumulation, as it provides a rule for determining the extent to which society should invest in a resource. Usually the optimal biomass lies between the biomass at bionomic equilibrium (no rent generated) and the biomass at maximum sustainable yield. The path taken to the optimal solution is complex, (Clark *et al.*, 1979). The policy implications of the modified golden rule are such that it is only rational for a participant in a fishery to invest in the future if the benefits of the investment accrue to them. Also the gains they make should at least equal the gains from

alternative investments. It is difficult to include age of fish in these analytical models thus the approach taken here is to use a simulation model, which is forward projecting.

Harvesting of multispecies fisheries is an important area of study in fishery modeling.

The basic ideas related to this field of study were first provided by Clark (1976). He also considered the harvesting of one species in a fishery consisting of two competing species. Chaudhuri (1987) has studied the combined harvesting of two competing species from the standpoint of bioeconomic harvesting and has discussed dynamic optimization of the harvest policy. Chaudhuri and SahaRay (1996) have studied combined harvesting of a prey-predator community with some prey hiding in refuges. The problem of nonselective (that is, combined) harvesting of a prey-predator fishery with infected prey has been studied by Chattopadhyay, *et al.*, (1996). Pradhan and Chaudhuri (1999) developed a dynamic reaction model for harvesting a two species fishery with taxation as a control instrument. Multispecies harvesting models have also been studied by Silvert and Smith (1977), Ragozin and Brown (1985), Wilen and Brown (1986) and Mesterton-Gibbons (1988) amongst others.

Age-structured models have been popular among fishery scientists and fishery managers for many years (Baranov, 1918; Leslie, 1945) and the Beverton and Holt (1957) model is the most commonly applied such model. Such models have been dominating among biologists for several decades (Hilborn and Walters, 2001), whereas surplus growth models have retained a strong position in economics and in bioeconomic modeling (Scott and Munro, 1985). This has changed recently, and more and more economists agree that age-structured models are necessary in order to cover the complexity of real world fisheries and fish stocks (Townsend, 1986; Wilen, 2000). A large variety of bioeconomic age-structured models have now been developed (Tahvonen, 2010). Such models can be

used both for simulation as well as numerical optimization. Simulation models have traditionally been the most common ones as numerical optimization has been regarded as much more difficult, and analytical optimization as almost impossible (Clark, 1990). The possibility to do numerical optimization with such models has become more realistic along with the occurrence of gradually more sophisticated software. Even the long held belief that analytical optimization is almost impossible has been challenged recently by, among others, Tahvonen (2010) who also presented a useful survey of age-structured optimization models. Notwithstanding the large variety of age-structured bioeconomic models, the basic structure has more or less remained the same. Consequently these models have up to now primarily been representative of so-called search fisheries (e.g. bottom-trawl) where the fish is uniformly distributed in the water. Many of the most important fisheries in the world are, however, based on pelagic schooling species, for which existing age-structured models are not adequate.

2.13 HARVESTING IN AN AGE-STRUCTURED POPULATION

The sustained yield from a harvested population depends partly on the sex and age distribution of the kill. The yield of age zero is higher than mature. Additionally most harvests are selective for particular sex and age strata, and for most species particular sex and age strata are more vulnerable than others. The sex and age structure of the harvest are partly determined by hunter selectivity, but also depends on relative availability of the various sex, age, and family status classes. For instance, if the males in a population became depleted, even a harvest that was selective for males might result in mainly females being taken. When selectivity/vulnerability is constant, relative availability changes in a dynamic fashion. The sex and age distribution changes through time as a result of changes in relative availability caused by selective harvest. If the harvest is exactly sustainable, the population standing sex/age distribution will eventually become

constant. However, if selectivity, vulnerability, or numbers harvested change; the sex and age distribution will also change. The sex/age distribution of a selective harvest affects the sex/age distribution of the population each year. Additionally, the sex/age distribution of the population affects the sex and age of the harvest in a dynamic fashion. The dynamics of both interactions are partly determined by the overall population numbers, which are affected directly by harvest, and indirectly (i.e. recruitment) by the standing age distribution.

Current methods for determination of optimal exploitation rates use simple analytic analysis of very simple stock recruitment models to determine optimal exploitation rates at equilibrium. It is possible to use more complex models to test very simple control laws; for instance, constant harvest or constant escapement. You simply have the same harvest taken every year and then calculate the average catch by simulating a large number of years. This method has been used to look at the role of stochastic variation on simple stock recruitment models Larkin and Hourston (1964).

Kotani *et al.* (2008) considered the optimal control of exotic species and numerically obtained an optimal harvesting policy when the utility of standing population is negative. Even for the optimal control of exotic species, the constant escapement is optimal.

2.14 OPTIMAL SUSTAINABLE HARVEST

Watt (1968) stated the first of the two basic approaches used by several authors who examined, usually under some poorly defined criterion such as “optimal sustained yield”, the problem of an optimal age-specific harvest. Watt states without prove, that the largest possible yield that can be harvested over a long horizon, without endangering the population, is to harvest an amount just equal to the total productivity over the whole planning `period. Productivity here is the net additional production by the original

populations. He defines his problem as harvesting “the total productivity in a sustainable way”. Inherent in this incremental policy that Watt (1960) seeks, is the assumption that harvesting as much as possible at each time period will produce the largest total harvest that will satisfy the constraints to which the harvest is subjected to. While this is true for certain classes of mathematical problem Sobel (1976), it does not necessary describe an optimal harvesting strategy.

The optimal management of the environment is becoming an increasingly important concern in our current developed societies, as awareness of the problem regarding the loss of biodiversity and overexploitation of natural resources is today considered the first issue in whatsoever political agenda. The main problem that current governments are basically dealing with is that renewable sources are usually characterized by open access, with very poorly defined property right (Brown, 2000). Therefore, an indiscriminate and uncontrolled harvest is assumed to take place, as a common threat on the survivability of the ecosystem for those generations coming after us. The problem is that when a natural resource is depleted an irreversible process created; for the same natural capital cannot be created again (Xepapadeas, 2005). In this light, inside the large amount of researches trying to solve the problem of the efficient management of natural resource available, numerous political solutions have been consequently suggested, either focusing on some particular endangered resources, or formalizing a more general model theory of resources harvesting. (Brender and Taylor, 1998).

For instance, the economic literature on forest, or rather the matter of finding some kind of regulation to guarantee the safety of the world fisheries has had a very long tradition in exploring the conditions under which an optimal resource management can be achieved, (Amacher *et al.*, 2002).

Another early contribution is Reed (1980) who analytically studies the maximum sustainable yield problem. He finds that optimal harvesting includes at most two age classes (Getz, 1985). Getz and Haight (1988) reviewed various stage-structured models and the maximum sustainable yield problems, while Caswell (2001) gives a broad overview of various types of stage and age-structured models (linear as well as nonlinear). Tahvonen (2009) has recently derived both analytical and numerical results on optimal harvesting in a dynamic setting under various simplifying assumptions. When assuming non selective technology, he finds the optimal solution to be impulse control.



CHAPTER 3

METHODOLOGY

3.1 MATRIX POPULATION MODELS

Matrix population models have become increasingly important and useful in predicting population growth. Projections of population growth on the basis of survival and fertility assumptions date back to 1895 by Cannan. In the early 40's, Bernardelli (1941), Lewis (1942), and Leslie (1945) successfully formalized the matrix methods introduced by Whelp ton in 1936 to project population's. P.H. Leslie was born in 1900 and died in 1974. At the age of 21 he studied physiology at Oxford and due to a health problem did not continue with his medical career, instead he became an ecologist. By 1935 he was working at the Bureau of Animal Population (BAP) in Oxford, and did so until his retirement in 1968. In 1948 he wrote a paper that started by using the basic age-specific projection equations in a matrix form. He developed this tool after Elton; his useful director at BAP suggested to him that it would be useful if the mortality and fertility function could be combined into a single expression Caswell (2001) and Kofi (2001). This tool was named after him as, "The Leslie Matrix Population Model".

In 1959 Leslie proposed a modified form of projection matrix to allow for the effect of the presence of other population members on population growth. Due to the importance of this deterministic model, J.H Pollard developed a stochastic version of the basic model in 1966 by giving each complete point of time the mean and variance of the number of animals in each age group (Usher, 1971).

Matrix population models are commonly used by researchers in studying the demography of a population. They provide a versatile method that can be used in a wide range of situation. This model has been used to describe the population dynamics of a

wide variety of organism including brook trout, rabbits, lice, beetles, pine trees, buttercups, killer whales and humans. This thesis focuses on the use of the Leslie model to determine the growth of a tilapia population as well as the age distribution of the population overtime. We will apply the Leslie model to a population of tilapia.

3.2 THE LESLIE MATRIX

The Leslie matrix is a discrete, age-structured model of population growth that is very popular in population ecology. It was invented by and named after Patrick H. Leslie. The Leslie matrix (also called the Leslie Model) is one of the best known ways to describe the growth of populations (and their projected age distribution), in which a population is closed to migration and where only one sex, usually the female, is considered.

The Leslie Matrix is used in ecology to model the changes in a population of organisms over a period of time. In a Leslie Model, the population is divided into groups based on age classes. A similar model which replaces age classes with life stage is called a Lefkovitch matrix, whereby individuals can both remain in the same stage class or move on to the next one. At each time step the population is represented by a vector with an element for each age classes where each element indicates the number of individuals currently in that class.

The Leslie Matrix is a square matrix with the same number of rows and columns as the population vector has elements. The (i,j) th cell in the matrix indicates how many individuals will be in the age class i at the next time step for each individual in stage j . At each time step, the population vector is multiplied by the Leslie Matrix to generate the population vector for the following time step.

The Leslie model uses the following assumption

- a) We consider only the females in the population.
- b) The maximum age attained by any individual is m years.
- c) The population is grouped into m one-year age classes.
- d) An individual's chance of surviving from one year to the next is a function of its age.
- e) The survival rate P_k of each age group is known.
- f) The reproduction (fecundity) rate F_k for each age group is known.
- g) The initial age distribution n_t at time t is defined.

3.3 AGE-STRUCTURED POPULATION MODELS

3.3.1 THE LESLIE MATRIX

In many species, reproduction is highly age-dependent. For instance, periodical cicadas spend 13-17 years in the nymphal stage; they only reproduce once in their lifetime. Many animals, such as humans, elephants, etc., do not reproduce during their first years and then their reproductive success is age-dependent. To model such situations, age-dependent population models are appropriate. Patrick Leslie introduced matrix models that have discrete age classes with synchronous reproduction (Leslie 1945). The models are parameterized by age-specific survival probabilities and average number of female offspring.

To build a matrix, some information must be known from the population:

- n_k the number of individual (n) of each age class k
- P_k the fraction of individuals that survives from age class k to age class $k+1$,
- F_k , fecundity, the per capita average number of female offspring reaching n_0 born from mother of the age class.

An age-structured population with k age classes is described by a population vector of length k

$$n(t) = \begin{bmatrix} n_1(t) \\ n_2(t) \\ \vdots \\ n_k(t) \end{bmatrix}$$

where $n_k(t)$ denotes the number of females in age class k . Individuals within an age class are assumed to have equal birth and death probabilities. Survival from age class k to $k + 1$ is given by probability P_k ; female individuals in age class k give birth on average to F_k female offspring. This can be illustrated using life cycle graphs in Figure 3.1.

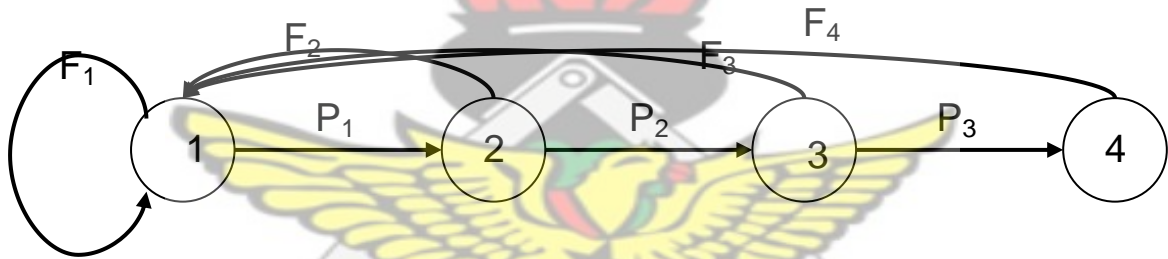


Figure 3.1: A life cycle graph with four age classes

Translating the life cycle graph into equations, we find

$$n_0(t+1) = F_1 n_0(t) + F_1 n_1(t) + \dots + F_k n_k(t)$$

$$n_1(t+1) = P_1 n_1(t)$$

$$n_2(t+1) = P_2 n_2(t)$$

$$\vdots$$

$$n_k(t+1) = P_{k-1} n_{k-1}(t)$$

This can be written in matrix notation

$$\begin{bmatrix} n_0(t+1) \\ n_1(t+1) \\ \vdots \\ n_k(t+1) \end{bmatrix} = \begin{bmatrix} F_1 & F_2 & \dots & F_k \\ P_1 & 0 & \dots & 0 \\ 0 & P_2 & 0 & \vdots \\ 0 & 0 & P_{k-1} & 0 \end{bmatrix} \begin{bmatrix} n_0(t) \\ n_1(t) \\ \vdots \\ n_k(t) \end{bmatrix}$$

The matrix contains both the survival probabilities (just below the diagonal) and the fecundities (in the first row). The matrix is called Leslie matrix. The characteristic polynomial of the Leslie matrix $\det(L - \lambda I)$ is given by the Euler–Lotka equation;

$$1 = \sum_{k=1}^m \lambda^{-k} F_k P_k$$

Where λ is the discrete growth rate P_k is the fraction of individuals surviving to age class k and F_k is the number of individuals born at time step.

The Leslie model is very similar to a discrete-time Markov chain. The main difference is that in a Markov model, one would have $F_k + P_k = 1$ for each k , while the Leslie model may have these sums greater or less than 1.

3.4 DOMINANT EIGENVALUE AND THE STABLE POPULATION VECTOR

Since the Leslie matrix is a $m \times m$ matrix, it can be concluded that there are m possible eigenvalues and eigenvectors which satisfy the equation

$$Lv = \lambda v \tag{3.0}$$

where λ is any eigenvalue and v is an eigenvector corresponding to λ . Eigenvalues and eigenvectors are usually used to study the changes in populations over time in a dynamic system and to give a meaningful biological interpretation. The aim is to determine the long term dynamics of the population, that is, to demonstrate whether the population is increasing or decreasing or staying constant.

After the computation the eigenvalues from the projection matrix using the analytical method (or any other suitable method), the eigenvalues of interest is the one which is more positive in comparison with the others. This eigenvalue is called dominant eigenvalue. The reason why λ is important is that it defines the rate of growth, or gives

valuable information about the state of the population. The significant of the dominant eigenvalue is supported by the Perron-Frobenius theorem for non-negative and irreducible matrices, which has the following properties;

- There exist one eigenvalue that is greater than or equal to any of the others in magnitude called the dominant eigenvalue of L
- There exist an eigenvector such that its elements are non-negative
- λ is greater or equal to the smallest row sum of L and less or equal to the greatest row sum

The eigenvalue is obtained from the equation

$$|L - \lambda I|$$

where I is an identity matrix, the values obtained represent:-

When $\lambda=1$, the population is stationary, $\lambda>1$, overpopulation is experienced.

When this is experienced, the population is harvested to keep it stable When $\lambda<1$, the population is diminishing and the population will be zero (extinct) if care is not taken

The annual rate of increase of the population is given by the logarithm of the dominant eigenvalue

$$r = \log \lambda$$

3.5 ANALYSIS OF THE LESLIE MODEL

To analyze the behavior of this discrete dynamical model, we find the eigenvalues and eigenvectors of L . The eigenvalue of the Leslie matrix L are found by finding the zeros of the characteristic polynomial $p(\lambda) = \det(L - \lambda I)$. There is a single

dominant eigenvalue, λ_1 with corresponding eigenvector (normalized) w , then asymptotically the population growth is

$$n(t) = c_1 \lambda_1^t v$$

It follows that the population grows or decays much like the Malthusian growth

3.5.1 EIGENVALUE ANALYSIS

The most common forms of analysis of the projection matrix are contained in the function `eigen.analysis`, which takes the projection matrix as its sole argument. The return value of eigenvalue analysis is a list containing: (1) λ_1 : the dominant eigenvalue of the projection matrix, (2) rho: the damping ratio $\rho = \frac{|\lambda_i|}{|\lambda_{i+1}|}$, which is a measure of the rate of convergence to the stable age distribution, (3) sensitivities: a matrix of eigenvalue sensitivities, (4) elasticity: a matrix of eigenvalue elasticities, (5) stable age: the stable age distribution u normalized so that $\sum u = 1$, and (6) reproductive value: reproductive value v normalized such that $v_1 = 1$. $ratio = \frac{|\lambda_i|}{|\lambda_{i+1}|}$

3.6 AGE STRUCTURED

Age-structured population models have been used to develop rehabilitation policies for several overexploited fish stocks. For example Huang and Walters (1983) used an age structured model to compare short-term approaches for rehabilitation of a large yellow croaker (*Pseudosciaena crocea*) stock. In their study alternative strategies were selected arbitrarily and compared through simulation studies. Stock rehabilitation required about 15 years under a constant effort policy but occurred much more rapidly if the fishery was closed for 3 years. Ruppert *et al.* (1985) used an age structured model to develop optimal rehabilitation policies for the Atlantic menhaden stock. They found that the stock was restored to the optimal steady-state level in about 9 years using either a constant effort

policy or one of four “egg escapement” policies. The steady-state performance of the five policies was somewhat similar, although there were notable differences in harvest during the rehabilitation period. Archibald *et al.* (1983) compared several constant effort policies for rehabilitation of a Pacific wean perch stock. Results for a 30-year planning horizon were used to evaluate the “short-term” performances of these policies: however, simulation runs of 100yr or more were required to achieve steady-state levels. These results demonstrate that the life history characteristics of the stock and the form of the harvesting policy can have important effects on the management strategies employed during the rehabilitation period. Our objective in this study was to examine the effects of planning horizon length, stock demographic characteristics, and objective function type on the form of optimal rehabilitation policies for three overexploited fish stocks. Our results can be used to develop rehabilitation policies tailored to specific fisheries; for example a policy with gradually decreasing effort levels might be preferable to one that achieves stock rehabilitation quickly.

3.7 STABLE AGE STRUCTURE

This age-structured growth model suggests a steady-state, or stable age-structure and growth rate. Regardless of the initial population size, N_0 , or age distribution, the population tends asymptotically to this age-structure and growth rate. It also returns to this state following perturbation. The Euler-Lotka equation provides a means of identifying the intrinsic growth rate. The stable age-structure is determined both by the growth rate and the survival function (i.e. the Leslie matrix). For example, a population with a large intrinsic growth rate will have a disproportionately “young” age-structure. A population with high mortality rates at all ages (i.e. low survival) will have a similar age-structure. Charlesworth (1980) provides further details on the rate and form of convergence to the stable age-structure.

We compute the fraction of individuals in each of the age classes. If these fractions stabilize over time, they represent the stable age distribution. A stable age distribution does not always exist. Suppose that

$v_1 = v_{11}, v_{12}, \dots, v_{1k}$ is a stable age distribution, then applying the Leslie matrix to this vector, that is computing Lv_1 results in a new vector that can be scaled so that it is equal to v_1 . This means that v_1 is a right eigenvector. The scaling can be expressed as $Lv_1 = \lambda_1 v_1$, implying that λ_1 is the corresponding eigenvalue.

Assume that the eigenvalues of the Leslie matrix are real and distinct. Because of the assumption of distinct eigenvalues, the corresponding right eigenvectors are also distinct, and we can write any initial population vector $n(0)$ as a linear combination of the right eigenvectors of the Leslie matrix.

$$n(0) = \sum_{i=1}^k c_i v_i$$

where the coefficients c_i are determined by the initial vector $n(0)$. Applying the Leslie matrix to this, we see what the advantage of writing the initial vector in this way is

$$n(1) = Ln(0) = L \sum_{i=1}^k c_i v_i = \sum_{i=1}^k c_i Lv_i = \sum_{i=1}^k c_i \lambda_i v_i$$

If we compute the population size at time 2, we find

$$n(2) = Ln(1) = L \sum_{i=1}^k c_i \lambda_i v_i = \sum_{i=1}^k c_i \lambda_i Lv_i = \sum_{i=1}^k c_i \lambda_i^2 v_i$$

and so on. We thus find

$$n(t) = Ln(t-1) = L \sum_{i=1}^k c_i \lambda_i^{t-1} v_i = \sum_{i=1}^k c_i \lambda_i^{t-1} Lv_i = \sum_{i=1}^k c_i \lambda_i^t v_i$$

We find that the population vector at time t is a weighted average of exponentials λ_i^t where the weights are determined by the initial population vector. If all the eigenvalues

are distinct, we can order them so that $\lambda_1 > \lambda_2 > \lambda_3 > \dots$. We then find that for t large, the largest eigenvalue dominates in the sense that

$$n(t) = \lambda_1^t \sum_{i=1}^k c_i \left(\frac{\lambda_i}{\lambda_1} \right)^t v_i = \lambda_1^t \underbrace{\left(c_1 v_1 + c_2 \left(\frac{\lambda_2}{\lambda_1} \right)^t v_2 + \dots + c_k \left(\frac{\lambda_k}{\lambda_1} \right)^t v_k \right)}_{\rightarrow 0 \text{ as } t \rightarrow \infty}$$

which we can also express as

$$\lim_{t \rightarrow \infty} \frac{n(t)}{\lambda_1^t} = c_1 v_1$$

The eigenvector v_1 corresponding to the largest eigenvalue λ_1 is also the stable age distribution. The largest eigenvalue λ_1 tells us how quickly the population grows for large times. When $\lambda_1 > 1$, the population will grow exponentially without bound, and when $\lambda_1 < 1$, it will decline exponentially fast. It is assumed that all the eigenvectors are distinct. We wish to relate this property to biological properties of the population. We begin with a number of definitions. A matrix is nonnegative if all its entries are nonnegative. A matrix is positive if all its entries are positive. The Leslie matrices are nonnegative. A nonnegative matrix or a life cycle graph is irreducible if and only if every stage can be reached from every other stage. In terms of the life cycle graph this means that there is a path from every node to every other node. Life cycle graphs with post reproductive age classes do not result in an irreducible matrix.

A loop is a path that has the same start and end point and passes through each node at most once. A life cycle graph is primitive if it is irreducible and the greatest common divisor of the lengths of its loops is one.

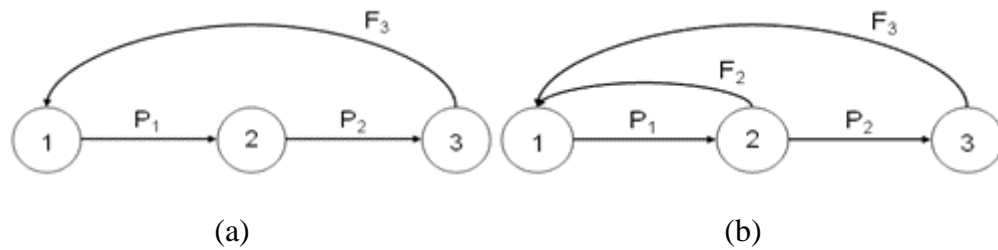


Figure 3.2: A life cycle graphs with three age classes

The life cycle graph (a) is not primitive since there is only one loop and it is of length three. The life cycle graph (b) is primitive since there are two loops, one of length two, the other of length three; consequently, the greatest common divisor is one.

3.8 OPTIMAL YIELD PROBLEMS

The optimum yield problem is to determine the maximum number of individuals that can be removed from a population without impairing the ability of the remainder to produce the maximum on a sustainable basis. The problem of optimal yield arises whenever a population of living organism is subjected to systematic exploitation by man, whether this is in the form of fishing, hunting, harvesting or lumbering (Watt ,1955). In the process of harvesting a population of organism, man assumes the role of predator in his ability to attempt to calculate his strategy of predation to maximize both the harvest and the likelihood that the exploited population will persist on a sustainable basis Slobodkin and Richman (1956).

Fish is a major food source for people throughout the world and the main source of protein for 1 billion people. For at least 150 million people, fish provide not only vital nutrition but also a source of employment and income. Most of these who rely on fish for their livelihood are developing countries. Aquaculture, the farming of fish shellfish, seaweeds and other products represent the fastest growing sector of the global food production.

CHAPTER 4

DATA ANALYSIS AND MODELING

INTRODUCTION

In this chapter a Leslie population matrix for an age-structure tilapia population is set up and the dynamics of the populations is analyzed using the Leslie population model described in the previous chapter. A harvesting functions is then introduced in the model and the optimal sustainable harvesting determined for this particular population of tilapia

4.1 TILAPIA POPULATION MODEL

The population of tilapia at time t (months) is divided into three age groups, n_1 (age < 6 months), n_2 (6 months \leq age < 12 months), and n_3 (12 months $<$ age < 18 months). All stocks are measured in number of fish and every age class is harvestable. Age class n_1 consist of immature fish (i.e. eggs, fingerlings and juveniles), the other age classes consist of sexually matured female tilapia. This age interval is chosen because in grow-out ponds, *O. niloticus* matures and starts reproducing at 4-6months at most (Satya and Timothy, 2004).

The survival rate for age class n_1 is 70 - 80% (mean 75%), (de Graaf , 2004 ;Leonard and Pompma, 1995) , that of age class n_2 is 75-95% (mean 85%), (Engle, 2004) , 70-75% (mean 72.5%) and 62-76% (mean 69%), (Charo-Karisa, 2006) and 70% expected survival of McGinty and Rackecy (1989). The longevity of *O. niloticus* is more than 18 months but it is expected that no fish will be allowed to stay in the pond beyond 2years.

Suppose each spawning mother can reproduce, in 6months, an average 200 fry (Pickering and Nandlal, 2004). Then $P_1 = 0.75, P_2 = 0.85$ and $F_2 = F_3 = 200$. The corresponding Leslie matrix for this fish population is

$$L = \begin{pmatrix} 0 & 200 & 200 \\ 0.75 & 0 & 0 \\ 0 & 0.85 & 0 \end{pmatrix}$$

4.2 DYNAMICS OF THE TILAPIA POPULATION

Suppose we introduce 3000 of the age first age class of the *O. niloticus* into a 48m³ pond (i.e. is 42fish/ m³),so

$$n_0 = \begin{pmatrix} 3000 \\ 0 \\ 0 \end{pmatrix}$$

and $N_0 = 3000$

Now we want to observe what happens to the population in the next few years, therefore

$$n_1 = Ln(0)$$

$$n_1 = \begin{pmatrix} 0 \\ 2250 \\ 0 \end{pmatrix}$$

and $N_1 = 2250$

$$n_2 = \begin{pmatrix} 450000 \\ 0 \\ 1912.5 \end{pmatrix}$$

And $N_2 = 451910$

The age structure of the tilapia after 18months is $n_3 = \begin{pmatrix} 382500 \\ 337500 \\ 0 \end{pmatrix}$, $N_3 = 720000$

We can continue until 5 time steps, i.e. the population of the tilapia after $2\frac{1}{2}$ years which is given by

$$n_5 = \begin{pmatrix} 1.1475e08 \\ 5.0625e07 \\ 2.4384e05 \end{pmatrix}$$

And $N_5 = 1.6562e08$. These results indicate that the population is increasing at a very fast rate. We use the Matlab code below to graph this growth over $5\frac{1}{2}$ years.

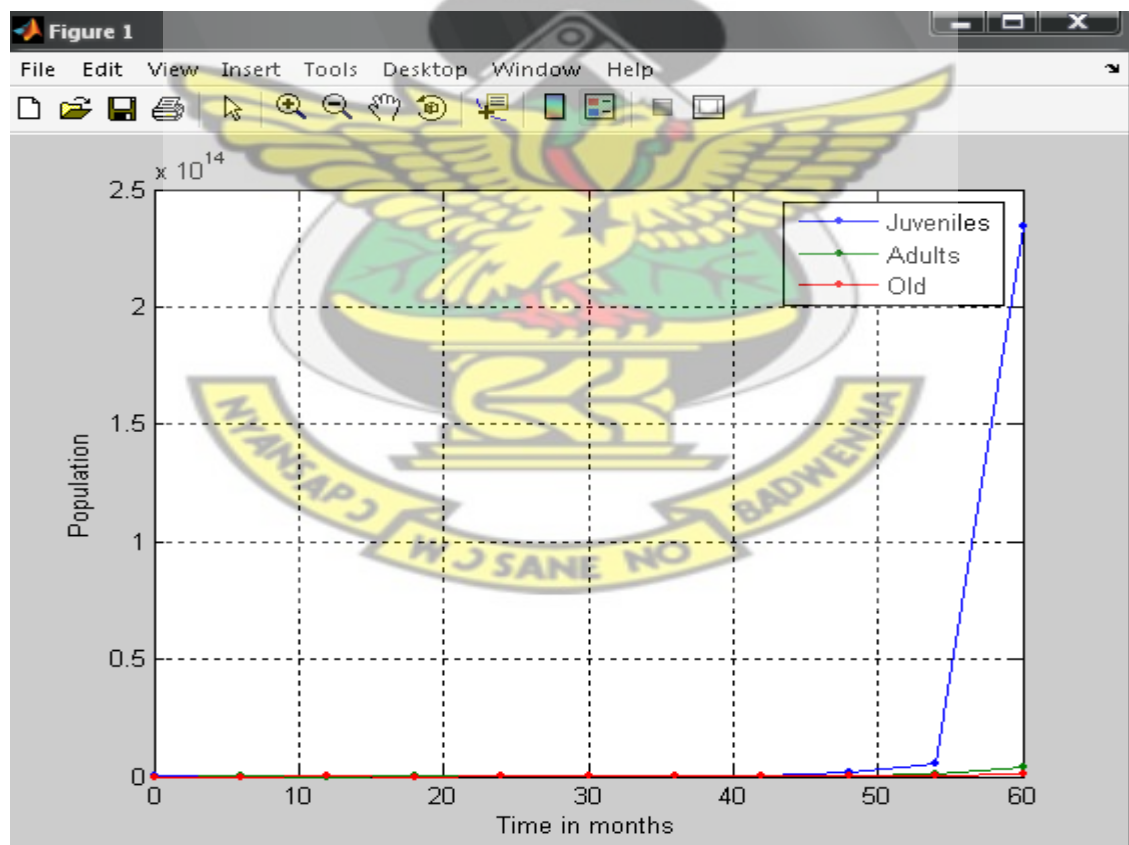


Figure 4.1: The trajectory of the tilapia population over 60 months.

```
>>L=[0 200 200 ;0.75 0 0 0;0 0.85 0 ];
```

```
>>n0=[3000;0;0];
```

```
>>N=zeros(3,11);
```

```
>>N(:,1)=n0;
```

```
>>for t=1:11
```

```
>>nt=L^(t-1)*n0;
```

```
>>%do for all k=2:30,
```

```
>>for k=2:11,
```

```
>>format short g
```

```
>>N(:,k)=L*N(:,k-1);
```

```
>>end
```

```
>>end
```

```
>>T=0:6:60;
```

```
>>plot(T,N')
```

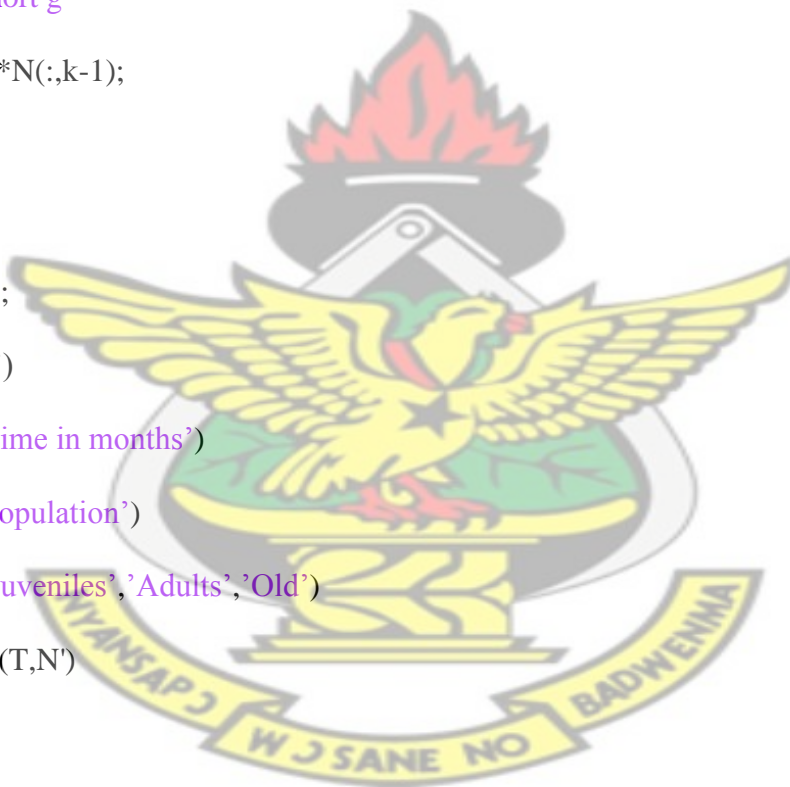
```
>>xlabel('Time in months')
```

```
>>ylabel('Population')
```

```
>>legend('Juveniles','Adults','Old')
```

```
>>semilogy(T,N')
```

KNUST



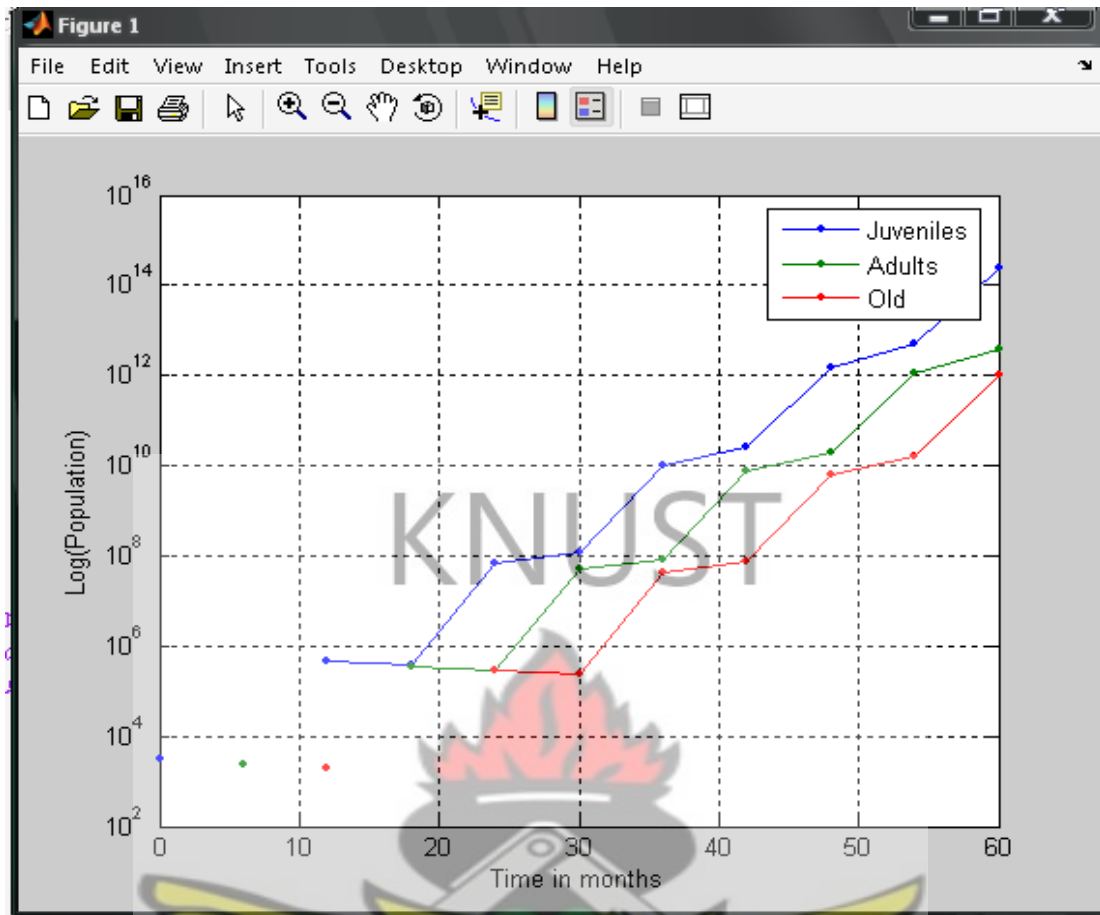


Figure. 4.2: Logarithm of the Tilapia population over 5years.

Each line on the graph represent an age class, the blue line represent the first age class, the deep green represents the second age class, and the red line represent the third age class. A matlab code for the total population is given below:

```
>>L=[0 200 200 200;0.75 0 0 0;0 0.725 0 0;0 0 0.85 0];
>>n0=[500;500;500;500];
>>N=zeros (4,10);
>>for t=2:10;
>>n1=L*n0;
>>nt=L^t*n0 ;
>>N(:,1)=n0;
```

```

>>N(:,t)=L*N(:,t-1);

>>N(:,2)=n1;

>>Nt=zeros(1,10);

>>for t=1:10, Nt(:,t)=sum(N(:,t)); end

>>format short g

>>display(Nt);end

>>t=0:6:54;

>>semilogy(t,Nt')

>>xlabel('time in months')

>>ylabel('Log(Total Population)')

```

The graph of the total population is shown in the figure below

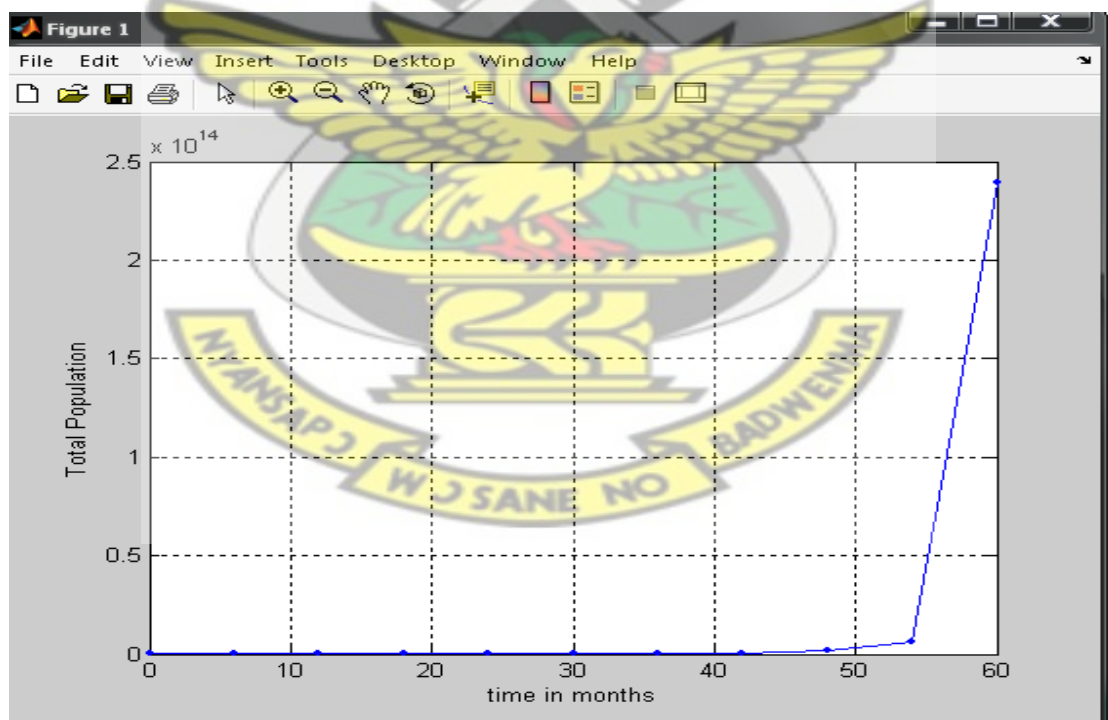


Figure 4.3: Graph of total population

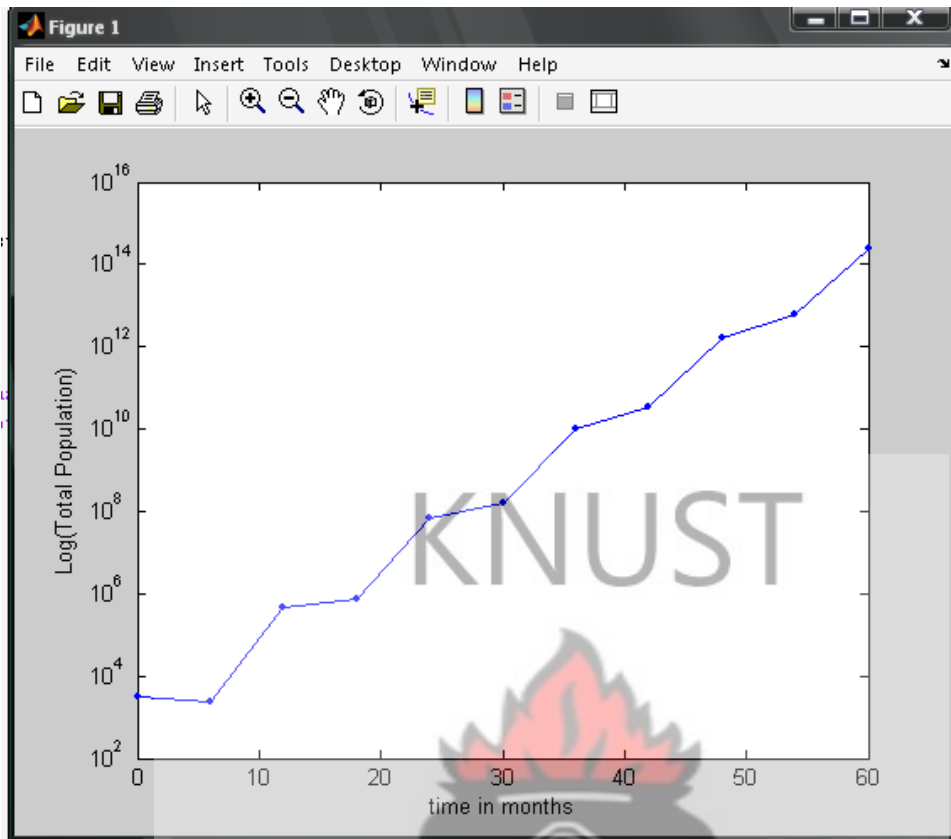


Figure 4.4: The log scale graph of the total population of Tilapia.

The graph above shows that the population is growing exponentially and there is a bit of fluctuation being experience due to the seasonal changes.

We use MatLab to find the eigenvalues of the Leslie matrix as $\lambda_1 = 12.652, \lambda_2 = -11.798$ and $\lambda_3 = -0.85415$. The dominant eigenvalue is $\lambda_1 = 12.652$ and the corresponding eigenvector is $v_1 = [0.9982 \ 0.0592 \ 0.0040]^T$. It follows that, in the long run, the population grows by 1165% every six months, the age distribution among the three age classes is 94% of the fish will be in the youngest age class, 5.6% in the second age class and only 0.4% in the oldest class.

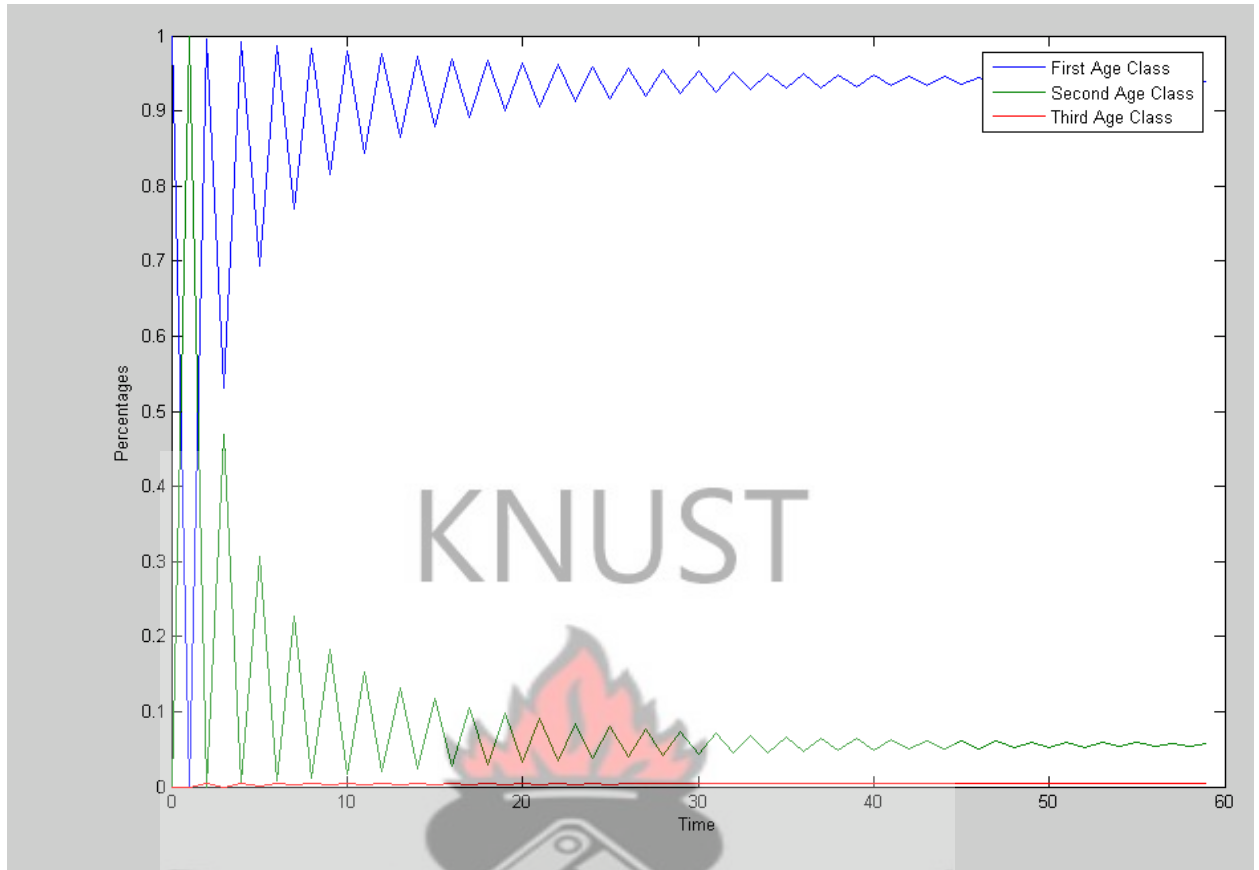


Figure 4.5: Percentage age distribution of stable population.

From equation (4.0) the state vector after six months is Ln_0 and the growth in that period (distribution in age classes) is $Ln_0 - n_0$.

4.3 HARVESTING POLICIES

A desirable goal for management of any renewable sources is to find a stable configuration from which one can harvest the growth of regular intervals thereby producing income and returning the population to its previous configuration.

In harvesting a population, the age structure is usually ignored but there may be occasions when specific age classes can be harvested. In the later case, a number of investigators have shown that the greatest harvest would require cropping of no more than two ages (Beddington and Taylor 1976;Rorres and Fair 1975;Doubleday 1975;Rorres 1976). Algorithms for finding the optimal harvesting strategy with regard to age structured

populations have been published (Beddington and Taylor, 1973; Rorres, 1976; Law, 1979). Methods such as linear programming (Doubleday, 1975; Cullen, 1986) and dynamic programming (Mendelssohn, 1976) have been used to solve the optimization problem.

Linear programming is the most commonly used method for solving the optimal harvesting problem in Leslie-type models. For example, Doubleday (1975) maximized yield subject to the constraints that population size and age structured are restored after each time period.

A sustainable harvest policy seeks to harvest on a regular schedule in such a way that the harvest is always the same and the state of the population after harvesting is always the same. Thus we define

$$0 \leq h_i \leq 1, i = 1, 2, \dots, n \quad (4.2)$$

to be the proportion of individual removed by harvesting from the $i - th$ age class at the end of each growth period, and we let H be the diagonal matrix whose entries are the h_i 's. If we start a growth period with age-distribution state n , then the state vector after growth will be Ln_0 . The harvest after growth will be

$$HLn_0,$$

and that will reduce the population to

$$Ln_0 - HLn_0 \text{ or } (I - H) Ln_0 \quad (4.3)$$

where I is identity matrix of appropriate dimension. To be sustainable, the population state after harvest must much the starting state, i.e.,

$$(I - H)Ln_0 = n_0 \quad (4.4)$$

This is an eigenvalue equation and it means that n_0 must be an eigenvector for the matrix $(I - H)L$. We wish to find harvest H such that $(I - H)L$ has dominant eigenvalue of 1. That is, we set the long-term growth rate of the population equals to 1, replacement only.

$$(I - H)L \quad (4.5)$$

Suppose the Leslie matrix of the population is

$$L = \begin{bmatrix} F_1 & F_2 & F_3 & \dots & F_{k-1} & F_k \\ P_1 & 0 & 0 & \dots & 0 & 0 \\ \cdot & P_2 & 0 & \dots & 0 & 0 \\ \cdot & \cdot & P_3 & \dots & 0 & 0 \\ 0 & \cdot & \cdot & P_{k-2} & \cdot & 0 \\ 0 & 0 & 0 & 0 & P_{k-1} & 0 \end{bmatrix}$$

Then the matrix (4.5) is easily computed:

$$\begin{bmatrix} (1 - h_1)F_1 & (1 - h_1)F_2 & (1 - h_1)F_3 & \dots & (1 - h_1)F_{k-1} & (1 - h_1)F_k \\ (1 - h_2)P_1 & 0 & 0 & \dots & 0 & 0 \\ \cdot & (1 - h_3)P_2 & 0 & \dots & 0 & 0 \\ \cdot & \cdot & \cdot & \dots & 0 & 0 \\ 0 & \cdot & \cdot & (1 - h_{k-1})P_{k-2} & \cdot & 0 \\ 0 & 0 & 0 & 0 & (1 - h_k)P_{k-1} & 0 \end{bmatrix} \quad (4.6)$$

We thus see that (4.5) is a matrix which has the same mathematical form as the Leslie matrix. The necessary and sufficient condition for a Leslie matrix, to have one as an eigenvalue, is that its net reproduction rate should also be one. Calculating the net reproduction rate of equation (4.5) and setting it equal to one, we obtain

$$(1 - h_1) [F_0 + F_1 P_0 (1 - h_2) + F_3 P_0 P_1 (1 - h_2)(1 - h_3) + \dots + F_k P_0 P_1 \dots P_{k-1} (1 - h_2)(1 - h_3) \dots (1 - h_k)] = 1 \quad (4.7)$$

This equation places a restriction on the allowable harvesting fractions. Only those values of h_1, h_2, \dots, h_k which satisfy equation (4.7) and which lies in the interval $[0,1]$ can produce a sustainable yield. If h_1, h_2, \dots, h_k do satisfy (4.7), then the matrix $(I - H)L$ has

the defined eigenvalue λ_1 and, furthermore this eigenvalue has multiplicity one since the positive eigenvalue of a Leslie matrix always has multiplicity one. This means that there is only one linearly independent eigenvector v satisfying equation (4.4).

We pick the following normalized eigenvector:

$$v = \begin{bmatrix} 1 \\ P_1(1 - h_2) \\ P_1P_2(1 - h_2)(1 - h_3) \\ P_1P_2P_3(1 - h_2)(1 - h_3)(1 - h_4) \\ \vdots \\ P_1P_2P_3 \dots P_{k-1}(1 - h_2)(1 - h_3) \dots (1 - h_k) \end{bmatrix} \quad (4.8)$$

Any other solution of equation (4.4) is a multiple of v , the vector that determines the portion of females within each group of the n classes after a harvest under a sustainable harvesting policy. But there is an ambiguity in the total number of females in the population after each harvest. This can be determined by some auxiliary condition such as an ecological or economic constraint. For example, for a population economically supported by the harvester, the largest population the harvester can afford to raised between harvest would determine the population; the particular constant v in (4.8) must be multiplied by to produce the appropriate vector v in (4.4)

We see that there is wide choice in the values of h_1, h_2, \dots, h_k which will produce sustainable yield. But once these values are selected, the proportional age distribution of the population after each harvest is uniquely determining the normalized eigenvector v defined by (4.8).

There are infinitely many ways to construct a sustainable harvesting policy. In this thesis, we consider three of those; uniform harvesting, harvesting only the young age class and proportional harvesting.

4.3.1 HARVESTING THE YOUNG AGE CLASS

In some population, only the youngest female are of any economics value, and so harvester seeks to harvest only the female from the young age .In our case the farmer can decide to harvest the young age class and sell to other farmers to rear. Accordingly, we set

$$h_1 = h$$

$$h_2 = h_3 = \dots = h_k = 0$$

The equation (4.7) then reduces to

$$(1 - h_1)[F_1 + F_2P_1 + F_3P_1P_2 + \dots + F_kP_1P_2 \dots P_{k-1}] = 1$$

$$\text{or} \quad (1 - h)R_0 = 1 \quad (4.9)$$

where R_0 is the net reproduction rate of the population. Solving for h , we obtain

$$h = 1 - \frac{1}{R_0}$$

we notice from this equation that only if $R > 1$ is a sustainable harvesting policy possible. This is reasonable since only if $R > 1$ is the population growing. From equation (4.8) the age distribution vector

$$v = \begin{bmatrix} 1 \\ P_1 \\ P_1P_2 \\ P_1P_2P_3 \\ \vdots \\ P_1P_2P_3 \dots P_{k-1} \end{bmatrix} \quad (4.10)$$

We apply this type of sustainable harvesting policy to the fish population. For the net reproduction rate of the population, we find

$$R_0 = F_1 + F_2P_1 + F_3P_1P_2 + \dots + F_kP_1P_2 \dots P_{k-1}$$

From equation (4.9) the fraction of the age class harvested is

$$h = 1 - \frac{1}{R_0} = 1 - \frac{1}{277.5} = 0.9964$$

from equation (4.10), the age distribution of the fish population after the harvest is proportional to the vector

$$v = \begin{bmatrix} 1.000 \\ 0.7500 \\ 0.6375 \end{bmatrix}$$

A direct calculation gives us the following

$$Lv = \begin{bmatrix} 277.5 \\ 0.7500 \\ 0.6375 \end{bmatrix}$$

The vector Lv is the age distribution vector immediately before the harvest. The total of all entries is 278.8875, so that the first entry, 277.5 is 99.502% of the total. This means that immediately before each harvest, 99.5% of the population is the youngest age class. Since 99.64% of this class is harvested, it follows that 99.14 % (=99.64% of 99.50%) of the entire fish population is harvested each time step.

4.3.2 UNIFORM HARVESTING

With many populations it is difficult to distinguish or catch animals of specific age. If animals are caught at random, we may reasonably assume the same fraction of each age class is harvested. Let us therefore set

$$h = h_1 = h_2 = \dots = h_k$$

equation (3.4) reduces to

$$Lv = \frac{1}{1 - h}$$

Hence, $\frac{1}{1-h}$ must be the unique positive eigenvalue λ_1 of the Leslie growth matrix L . That is,

$$\lambda_1 = \frac{1}{h}$$

Solving for the harvesting fraction, h we obtain

$$h = 1 - \frac{1}{\lambda_1} \quad (4.11)$$

The vector x_1 , in this case is the same as the eigenvector of L corresponding to the eigenvalue λ_1 is

$$v = \begin{bmatrix} 1 \\ P_0/\lambda_1 \\ P_0P_1/\lambda_1^2 \\ P_0P_1P_2/\lambda_1^3 \\ \vdots \\ P_0P_1 \dots P_{k-1}/\lambda_1^{n-1} \end{bmatrix} \quad (4.12)$$

From equation (4.11), we can see that the larger λ_1 is, the larger is the fraction of animal we can harvest without depleting the population. We also notice that we need $\lambda_1 > 1$ in order that the harvesting fraction h lies in the interval $(0, 1]$. This is to be expected since λ_1 is the condition that the population is increasing.

For the tilapia species the following Leslie matrix was formulated. The growth period has been set as six month since it takes 4–6 months for tilapia to reach breeding size of 150–300g (Pickering and Nandlal, 2004)

$$L = \begin{bmatrix} 0 & 200 & 200 \\ 0.75 & 0 & 0 \\ 0 & 0.85 & 0 \end{bmatrix}$$

We have assumed growth-out period of 2 years so that no tilapia live beyond 2 years since fish kept for brood stock should not be used for more than 1.5–2 years (Pickering and

Nandlal, 2004). By the use of Mat lab we found the unique eigenvalue of the L to be $\lambda_1=12.652$. From equation (4.11), the harvesting fraction h is

$$h = 1 - \frac{1}{\lambda_1} = 1 - \frac{1}{12.652} = 0.921$$

Thus, the uniform harvesting policy is one in which 92.1% of the fish from each of the three classes is harvested every six month. From equation (4.12) the age distribution vector of the fish after each harvest is proportion to

$$v = \begin{bmatrix} 1 \\ 0.059 \\ 0.004 \end{bmatrix} \quad (4.13)$$

From equation (4.13) we see that for every 1000 fishes between 0-6 months of age which are not harvested, there are 59 fish between 6 months and 12 months of age, 4 fish above 12 months. Suppose a fish farmer wishes that the total population after harvesting is to be 10000. How many females does he harvest in each age class, and how many remain in each age class after the harvest?

We have $\lambda_1 = 12.652$ and we calculated the $h = 92.1\%$ for the uniform harvesting; thus 92.1% of the fish in each age class is harvested. The eigenvector for λ_1 is found to be $v = (1, 0.059, 0.004)$. Let $n = 9407v = (9407, 555, 38)$; we chose the 9407 so that the population is 10000. Then n gives the population, by age class, after each harvest; 9407 in the youngest age class, 555 in the second age class and 38 in the oldest age class. This is also the population at the beginning of each growth period. The population at the end of each growth period, just before the harvest, is Ln which, as equation (3.0) indicates, works out to $12.652n = (119017, 7022, 481)$. Then we harvest 92.1% of the fish in each age class; that's 109615 of the fingerlings 6467 of the second age class and 443 of the oldest age class. What remains is $(119017, 7022, 481) - (109615, 6467, 443) = (9402, 555, 38) \approx n$, as it should be.

4.4 OPTIMAL SUSTAINABLE YIELD

The sustainable harvest policy in which the same fraction of each age class is harvested produces a yield of 92.1% of the fish population. Also if only the young age class is harvested the resulting yield is 99.14% of the total population. There are many other possible sustainable harvesting policies, and each will provide a generally different yield. The interest of this thesis is to find a sustainable harvesting policy which produces the largest possible yield called the Optimal Sustainable harvesting policy. The resulting yield is the optimal sustainable yield. We use the following linear programming theory.

4.4.1 OPTIMAL HARVESTING THEOREM

An optimal sustainable harvesting policy is one in which either one or two age classes are harvested. If two age classes are harvested, then the older age class is completely harvested. (Beddington & Taylor, 1973)

The results of theorem 4.0 suggest the following algorithm for determining the optimal sustainable yield;

1. For each value of $i = 1, 2, \dots, n$ we set $h_i = h$ and $h_k = 0$ for $k \neq i$ and calculate the respective yields. These n calculations give the one-age-class result. Any calculations leading a value of h which does not satisfy equation (4.5) is rejected
2. For each value of $i = 1, 2, \dots, n-1$ and $j = i+1, i+2, \dots, n$ we set $h_i = h, h_j = 1$ and $h_k = 0$ for $k \neq i, j$ and calculate the respective yields. These $n(n-1)/2$ calculations give the two way-age-class results. Of course any calculation leading to a value of h not between 0 and 1 is rejected
3. Of the yield calculated in part (1) and (2), the largest is the optimal sustainable yield.

We will have at most $n + n(n-1)/2 = n(n+1)/2$ calculations in all. The table below shows a summary of the calculations.

h_1	h_2	h_3	Percentage yield of the total population
0.996	0	0	93.62%
0	Infeasible	0	-
0	0	Infeasible	-
0.993	0	1	93.74%
0	0.993	1	5.96%
0.993	1	0	98.94%

Table 4.0: Summary of values of h 's and the percentage yield.

4.5 SENSITIVITY ANALYSIS

Substituting this optimal harvest strategy into equation (4.6), gives a new Leslie matrix

$$L = \begin{bmatrix} 0 & 1.4 & 1.4 \\ 0.75 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (4.14)$$

The characteristic equation of this matrix is $-\lambda(\lambda^2 - 1.05)$. The 1.05 is the product of a_{12} and a_{21} . Whenever this product is less than 1, the dominant eigenvalue of the matrix is also less than 1. Thus, the population is diminishing to extinction. If a tilapia fish farmer harvest 0.994 instead of the recommended 0.993 of the fingerlings, the whole fish population will collapse. The farmer has an error margin of 0.03% above which the population declines.

CHAPTER 5

SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.0 SUMMARY

This thesis established the harvesting policy for an aged-structured tilapia population with sustainable optimal yield. A Leslie matrix for the Tilapia is formulated from the available literature on the fecundity and survivability of the Tilapia. The population was divided into three age classes namely, juveniles, matured and old fish. Different harvesting policies for this Leslie-type population are presented, uniform harvest, where equal fraction of each age class is harvested; harvesting only the youngest age class and harvesting unequal fractions of each age class.

Harvesting management is never a simple task. The challenges of harvest management are compounded when structured population are considered.

The most intrinsic finding in this piece of work was first the numerous published works on harvesting of renewable resources in general; the usefulness of Tilapia and its potential to solving Ghana's annual fish-demand deficit problem of 460000 tons, Kwadjosse (2007), due to its high fecundity and fast growth rate. However, not much has been done on Tilapia harvesting particularly in Africa and Ghana. Optimal harvesting has been studied under different approaches and using different tools.

The recent outbreak of diseases in livestock and poultry led to the decision to find alternative source of protein which has resulted in research into fish farming. And the high fecundity of Tilapia, its growth rate couple with its taste and nutritional value led the researcher to decide on finding a method of controlling the fish without impeding their reproduction.

The dominant eigenvalue of the Leslie matrix of the Tilapia population of 12.65 indicated that the population is growing at an astronomical rate of 1165% semi-annually.

Many methods of controlling the population of Tilapia in ponds have been developed; mono sex culture of male (sex reversal) to using predators like catfish (Popma and Lovshin1995).

The results of the different harvesting strategies are given in table 3. The constraints (4.2) were not met for the harvesting strategy which harvests only the matured (age class two) and old (age class three) fishes. Harvesting all of the matured fishes and a portion of the old fishes was not sustainable since this results in the extinction of the entire fish population. The optimal harvesting policy for this Tilapia population is when all the older fishes are harvested and 0.993 of juveniles are harvested. This results in a yield which is 93.74% of the total Tilapia population.

5.1 CONCLUSION

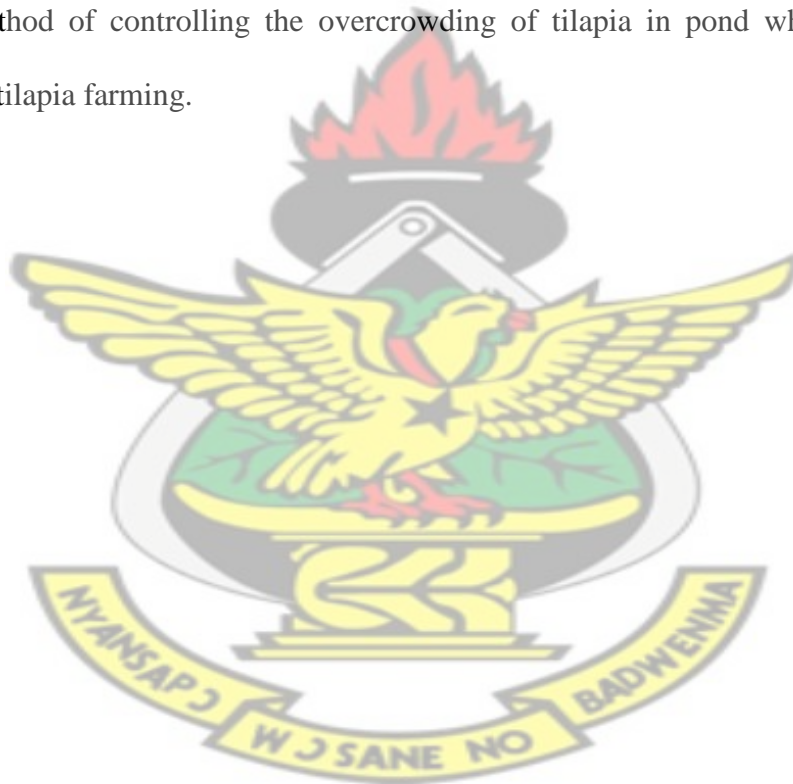
Generally, it is evident that more research needs to be done to investigate the profitability of Tilapia farming and how it can be used to create employment and boost the GDP of developing nations like Ghana. Besides, future researchers can seek to optimize profit of Tilapia farming using this optimal harvesting policy and compare it with the profit of other tilapia farming methods such as monosex culture. Further studies can also be done on tilapia consumption pattern in the nations.

5.2 RECOMMENDATIONS

Over-exploitation and extinction of fisheries is a serious global problem in recent times, which current management policies struggle to solve. Traditionally, the management of

fisheries is based on adjusted quotas or effort control. Unfortunately, such management strategies are expensive to implement due to its enormous amount of data collection needed for exact stock assessments. This paper has described another method of sustainably exploring the fish populations to optimize harvest and still maintain the population.

It is recommendation that the aquaculture of Nile tilapia be expanded throughout Ghana to create jobs. This can be done if the government construct pond for the farmer for them to pay over a period. Also the policy described in this thesis is recommended to be use as another method of controlling the overcrowding of tilapia in pond which has been a problem in tilapia farming.



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APPENDIX

CALCULATIONS OF YIELD FOR DIFFERENT HARVESTING STRATEGIES

$$(1 - h_1)[F_1 + F_2 P_1(1 - h_1) + F_3 P_1 P_2(1 - h_2)(1 - h_3) + \dots + F_n P_1 P_2 \dots P_{n-1}(1 - h_n)] = 1$$

We set $h_2 = h_3 = 0$, and find $h_1 = h$

$$(1 - h)[0 + 200 \times 0.75 + 200 \times 0.75 \times 0.85] = 1$$

$$(1 - h)(150 + 127.5) = 1$$
$$1 - h = \frac{1}{277.5}$$

$$1 - h \approx 0.004, h \approx 0.996 \text{ i.e. } H = \text{diag}(0.996, 0, 0)$$

When $h_1 = h_3 = 0, h_2 = h$

$$\Rightarrow [0 + 200 \times 0.75 + 200 \times 0.75 \times 0.85(1 - h)] = 1$$

$$\Rightarrow 150 + 127.5 - 127.5h = 1$$

$$\Rightarrow h = h_2 = -5.622$$

For $h_1 = h_2 = 0, h_3 = h$

$$\Rightarrow [0 + 200 \times 0.75 + 200 \times 0.75 \times 0.85(1 - h)] = 1$$

$$\Rightarrow 150 + 127.5 - 127.5h = 1$$

$$\Rightarrow 127.5h = 276.5$$

$$\Rightarrow h = 2.169, h_3 = 2.169,$$

When $h_1 = h, h_2 = 1, h_3 = 0$

$$\Rightarrow (1 - h)[0 + 200 \times 0.75(1 - 1)] = 1$$

$$\Rightarrow h_1 = 0$$

$$\text{When } h_1 = h, h_2 = 0, h_3 = 1$$

$$\Rightarrow (1 - h)[(0 + 200 \times 0.75 + 200 \times 0.75 \times 0.85 \times (1 - 1))] = 1$$

$$\Rightarrow (1 - h)150 = 1$$

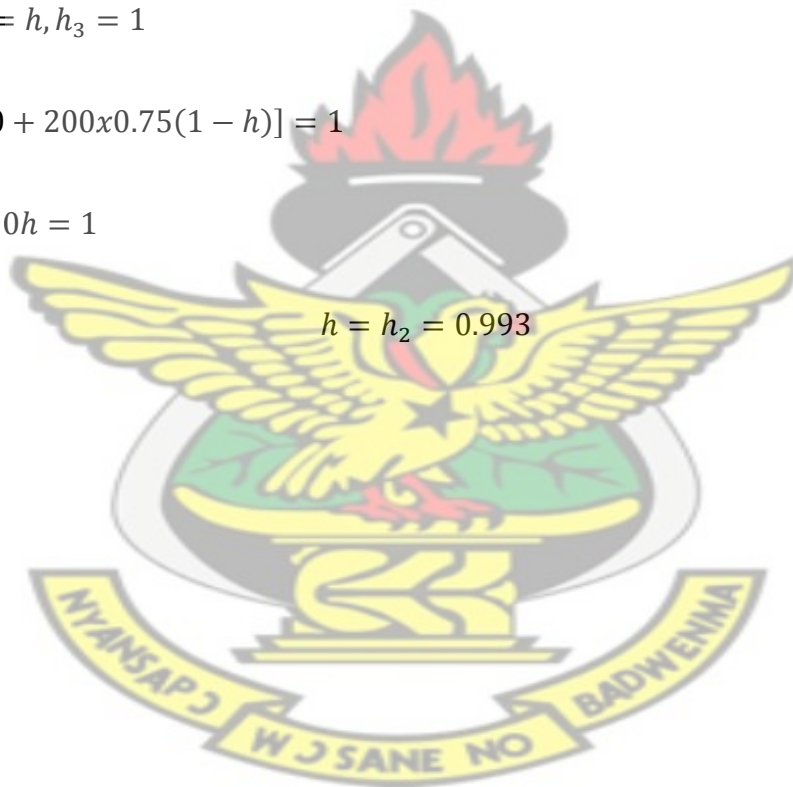
$$\Rightarrow h_1 \approx 0.993$$

$$h_1 = 0, h_2 = h, h_3 = 1$$

$$\Rightarrow (1 - 0)[0 + 200 \times 0.75(1 - h)] = 1$$

$$\Rightarrow 150 - 150h = 1$$

$$h = h_2 = 0.993$$



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