KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY (KNUST)

KUMASI

INSTITUTE OF DISTANCE LEARNING (IDL)

ANALYSIS OF LONGITUDINAL DATA ON STUDENT ACADEMIC PERFORMANCE

(Case study, actuarial science (2007-2011))

This Thesis is presented to Kwame Nkrumah University of Science and Technology in partial fulfillment of the requirement for the award of Masters of Science degree in Industrial Mathematics.

BY

SERWAA FELICIA

JUNE 2011

DECLARATION

I hereby declare that this submission is my own work towards the Master of Science and that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the University, except where due acknowledgement has been made in the text.

Serwaa Felicia (PG3017009)	……………………	20 /06/ 2011
Student Name &ID	Signature	Date
Tel.: 0244827949		

Certified by:

Mr. N.K. Frempong	……………………	……………………
Supervisor(s) Name	Signature	Date

Certified by:

Dr. S.K. Amponsah	……………………	……………………
Head of Dept. Name	Signature	Date

Certified by:

Professor I.K. Dontwi	……………………	……………………
Dean of Faculty Name	Signature	Date

DEDICATION

This piece of work is dedicated to my son Nana Yaw, Mother and siblings.

ACKNOWLEDGEMENT

The greatest reverence is ascribed to the Most High God. He is my ‘Insight Source’ and ‘Divine Consultant’. Thank you, Jehovah, for seeing me through another level of educational ladder once again.

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‘Not that we are sufficient of ourselves to think anything as of ourselves, but our sufficiency is of God’. II Corinthians 3:5

‘Without counsel purpose are disappointed but in a multitude of counselors they are established’ Proverbs 15:22

Abstract

One of the most important things in an academic institution that projects its image is the quality of the output or intelligence level of its products (i.e. students from that particular institution). As the saying goes “garbage in garbage out”. The quality of student’s intake of an institution will affect the graduate produced. Therefore, there is a need to take a proper look at how to get the best strategy in selecting fresh student’s base on effective admission criteria.

The data of 62 actuarial students of (2007 – 2010), administrative records of their semester weighted average (SWA) were used for the study. The objective is to determined if academic performance depends on age, is there a difference between the academic performance of regular and fee-paying students, can admission aggregate affect student SWA and also does the type of school attended have effect on academic performance. The data was analyzed using random effect model.

From the chosen model all model, fitted to the data set and the analysis was done, it can then be concluded based on the results from the random effects model that the SWA depends on the age; (older students score low SWA and younger students score high SWA’s), admission aggregate (student with higher aggregate tend to low SWA), type of school attended and the admission mode (fee-paying or regular). The performances of type B and C schools students are better as compare to that of type A. Meanwhile, in all the models it appeared, students decrease from semester one and increase after semester four.

TABLE OF CONTENTS

TABLE **PAGE**

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIAT

ACT	Actuarial
admagg	Admission Aggregate
CWA	Cumulative Weighted Average
GES	Ghana Education Service
ICC	Intraclass Correlation
KNUST	Kwame Nkrumah University of Science and Technology
MLE	Maximum Likelihood Estimation
REM	Random Effect Model
RIM	Random Intercept Model
RMLE	Restricted Maximum Likelihood Estimation
SAS	Statistical Software
SHS	Senior High School
SWA	Semester Weighted Average

CHAPTER ONE

INTRODUCTION

BACKGROUND OF THE STUDIES

In the last few years universities in Ghana have undergone considerable changes not only in terms of numerical expansion but also in the quality of academics work. It is evident that some rating agencies or organization ranks universities based on some other criteria as well as the quality of graduates they produced. This influenced the education policy makers of the academic institutions including Kwame Nkrumah University of Science and Technology (KNUST) to respond to developmental race of education. Ghana had to modify her tertiary education to meet prevailing developmental needs. The devious impact of such a study is that KNUST and other universities in Ghana would be in position to offer admission to only the students who have the greatest probability of succeeding academically.

To compete favorably in terms of high rate of global development, Ghana must use education as a key of resurgence. At present, Ghana needs both technological and educational advancement.

The root of national wealth is base on excellent technological knowledge and education. There is a strong correlation between a country's development and the quality education provided within that country (Borahan and Ziarati, 2002)

Quality education can therefore be achieved by proper monitoring of what students are doing and what is affecting their progress in terms of performance. If there is no quality policy, it would be difficult for institutions to assessed good performance at all levels. The public universities in Ghana including Kwame Nkrumah University of Science and Technology (KNUST) want to produce good and quality technologist and high level standard students. In other to achieve this, frequent and proper measure of academic performance should be put in place; the relevant questions that must be answered are: What are some of the factors that will contribute to quality product? And how are we doing it? How would the university make sure that the planning and quality assurance department monitor and ensure quality production? These are some of the questions that come to mind when we talk about measurement of academic performance.

The basic purpose of any measurement system is to provide feedback relative to the goal that increases the chances of achieving these goals efficiently and effectively. Measurement gains true value when used as the basis for timely decisions. The purpose of measuring is not to know how we are performing but to enable us to perform better. The ultimate aim of implementing a performance measurement system is to improve the academic performance of institution(s). If the performance measurement is right, the corresponding data generate will direct where one is, how one is doing, and where one is going.

Admission to KNUST based on results obtained in Senior High School (SHS) or 2nd cycle institution is a pre-tool used in the admission process. The main objective of admission system is to determine candidate who have the potential to excel in the field of interest in the university. The quality of students admitted to the institution affects the prestige of the institution as well as development of the country, as this potential student's eventually become key to development. Kenneth Mellamby (1956) observed that universities worldwide are not completely satisfied by the methods used for selecting undergraduates.

Institution care a lot about producing quality graduates, therefore the initial selection and admission of students from among all applicants is of utmost importance. The admission selection process should choose students who are likely to be successful and likely to provide the most glorious name to the institution. During this process, both objective and subjective criteria are used to determine eligibility and make selections. It is important that, when possible, criteria are genuine factors of the outcome of the institutions. One might assume that as long as there has been graduate level education, the institution offering it must have used a process to select students for their programs. An appropriate selection process provides benefit to the institution and student. Institutions want to prevent the admission of less-than-qualified individuals because that could diminish both the quality of education provided and academic reputation of the institution. Student prefer to attend the schools with the best reputation if possible, because in addition to receiving a quality education, earning a degree from a highly respected graduate degree program can provide a competitive edge when seeking for employment. Performance measurements involve determining what to measure, identifying data collection methods, and collecting the data.

A Semester Weighted Average (SWA) as a measurement of academic performance would be used as a dependent variable in this thesis. The SWA is basically a single score representing a students' performance in all the courses taken in a semester and is calculated to capture numerically a student's quality of academic performance. It is calculated by multiplying the marks obtained for each course by the credits of the particular course adding up the products and dividing by the total number of units of credit for the courses registered.

An up to date assessment from the time the student entered the program of study is obtained by calculating Cumulative weighted average (CWA) which is ultimately used for the award of the degree. The (CWA) therefore depends on the SWA.

In this thesis we study how the identified factors can predict academic performance of students. In applied science many studies are often conducted using longitudinal data, the techniques devised for some of this data are SAS procedure which have been implemented in SAS version 9.1 for general linear models. This study would enable us to model the evolution of SWA/over semesters, while adjusting for differences in some factors affecting some of the students.

1.1 Statement of the Problem

The KNUST like any other university use various criteria in selecting student for admission. However to our knowledge no scientific studies have been conducted to evaluate the effect of admission variables on students' performance academically in relation to factors such as SHS attended, entry age of student, entry aggregate and the status of the student whether a student is on government scholarship or fee paying.

1.2 Objectives

The primary aim of the study is to use random effect model to assess the academic performance based on student's SWA's. Other secondary objectives are:

- If the type of senior secondary school attended has impact on student's academic performance.
- The impact of entry age on academic performance of the student.
- Can entry aggregate have impact on students' academic performance?

- Whether the status of the student (fee paying/regular) has influence on their academic performance.

1.3 Scope and Limitations

The research would be limited to final year actuarial science students. The four factors mentioned in the objectives of this thesis are not the only factors that could be considered, however to make our analysis tractable and more simplified in this initial study, the other factors can be considered in the subsequent study. Time and financial obligations were the other limitation.

1.4 Organization of the Thesis

In subsequent sections of this thesis, admission criteria of a number of graduate programs will be discussed. It is evident that institutions use different criteria and different combinations of factors when making admission decisions. Previously reported research studies which have employed a variety of statistical models to predict graduate school success have not always arrived at consistent results. This study seeks to contribute to the body of research that explores the appropriate admission criteria for perhaps a less common but more specialized graduate program in hope that other schools may also benefit from these findings. However, as Braunstein (2002) points out, prior authors of studies suggest that graduate programs conduct their own investigations into appropriate admission criteria, in light of differences that exist among institutions.

The chapter two deals with literature review of methods and models that reflect on the academic performance of students in general.

Chapter three discusses how random effect model (REM) is been use to analyze the data and what it entails. Graphical representation of each technique of some stages has been displayed. All this techniques are been implemented through the use of statistical software (SAS).

The results of the model (REM) using SAS is discussed in chapter four.

Chapter five, we represent our recommendations and conclusions.

1.5 Justification

The quality assurance and planning unit of KNUST has some guidelines or policy for assessing performance of students in the institution. The purpose of this research is targeted to find out how some factors like previous school, entry age, aggregate of entering, and fee paying status affect the academic performance of students, this will help the planning and quality control unit to improve on their analysis and also include some of these factors if they are not considering. It would help to determine the standards leads to satisfaction of both the student and the authority's capabilities of producing knowledgeable, effective skilled and talented graduates.

The research shall enable the university and other policy maker, both academics and social groups to determine performance level of some categories of students in their institution.

CHAPTER TWO

LITERATURE REVIEW

A random effect model is also known as variance components model which is hierarchical linear model and it assumes that the data set being analyzed consists of a hierarchy of different factors whose differences relate to that hierarchy. Random effect model is used in the analysis of hierarchical or panel data when one assumes no fixed effects. The fixed effects model is a special case of the random effects model. According (Roberto G. Gutierrez, (2008)) random effects are not directly estimated but summarized by their variance components, which are estimated from the data. In this chapter, we are going to look at what some fame in statistical field has done in this area.

The research on Impact of Eyeglasses on the Academic Performance using random effect model by (Paul Glewwe, Albert Park and Meng Zhao, (2006)) shows the extent of which vision problems among students in developing countries and the impact of those problems on student academic performance. The school academic performance data included student's result of each semester's exams which is conducted regularly in each grade since the student enrolled was used for the research. First, to which extent can vision be correlated with other factors that determine academic performance like sex, ethnicity, birth date, and the occupation and education level of the head of the household (usually the father) in which the student lives? The test score data suggest that vision problems have little effect on students' academic performance.

(Henry May and Supovid David, (2004)) Said Education is a cumulative process. According to them while students' knowledge and skills are built up over time, educational researches are not often afforded the opportunity to examine the effects of interventions over multiple years. In their study about America's Choice school reform design which is just an opportunity. Using 11 year longitudinal data of student academic performance from Rochester, N Y, they examined the effects of America's Choice on student learning gains from 1998 to 2003. To analyze these longitudinal data, they used an advanced sophisticated statistical modeling technique called Bayesian hierarchical growth curve analysis with crossed random effects. This technique enabled them to model the annual growth in individual students' academic performance. Most importantly, in statistics this method allowed them to determine the extent to which differences in students' academic performance and growth were due to particular individual factors. They compared the longitudinal gains in test performance of students attending America's Choice schools to those of students attending other Rochester schools. Their analytical method allowed them to examine student test performance over time, which account for the nested structure of students within the schools, and account for the very real problem within-district student mobility. The modern statistical methods used in their analyses were able to help them to compare the individual performance of America's Choice students over multiple years to both the performance of their similar students in other schools. They finally came out with a three-level hierarchical growth curve model for the analysis with time points nested within.

(Ralph Stinebrickner and Todd R. Stinebrickner, 2007) from their statistical point of view very small amount of existing work has provided direct evidence about the relationship between studying and academic performance, they focused on collecting measures of study-effort and obtained estimates of the (conditional) correlation between the number of hours that a person studies and his/her academic performance.

The bias associated with viewing the descriptive relationship in estimation of the causal role that studying plays in the grade production process arises, in part, because students who spend more time in studying may be unable to perform very well. In mathematics not only is it not possible to know the size of the bias that is present if one views the correlations found directly, but it is also not possible to know the direction of the bias. In their paper they examine the effect of studying on college grade performance by using an Instrumental Variable (IV) approach that takes advantage of a real-world situation which they find closely resembles this ideal experiment, in this case the analysis was possible because they designed a sequence of surveys with the specific factors/goal. Finally, because they designed their own longitudinal survey with a well-defined issue in mind, they were able to directly examine the possible theoretical reasons that the instrumental variable might not be valid even in the presence of the random assignment the effect of studying on academic performance was statistically significant.

From the practical standpoint, educational institutions need to find a "tool" that would allow them to measure whether they are meeting the needs of their customers. Customer feedback is an established concept of strategic planning. It is therefore critical that educational institutions monitor their performance on a regular basis. Marketing controls are necessary if the institution is to remain as an attractive proposition for potential students. (Lovelock, 1991)

From (Minerva, 2009) performance analysis involves gathering formal and informal data to help sponsors define and achieve their goals. To her performance analysis uncovers several perspectives on a problems or opportunities, determining any factors and all drivers towards or barriers to successful performance and proposing a solution system based on what is discovered. Empirical research by (Ortinau *et al,* 1987) has found out that students' perceptions of importance with respect to specific course features influence their expectations of the course over time. The change justifies the use of performance analysis for the evaluation of the quality of educational services.

While several criteria for performance measurement are important only a few are most important. These determinant attributes are the ones that will define service quality from the consumer's perspective (Loudon and Della Bitta, 1988)

For (Ennew *et al,* 1993), the issue is to develop a better measurement for quality performance. He state that the qualitative nature of performance quality implies that cardinal scales of measurement are inappropriate, but the process of applying ordinal ranking (performance) to concepts is well established as a research methodology. (Ennew *et al,* 1993) state that a comparison of mean scores on the importance of performance attributes provides a straightforward measure of how well a performance meets its needs.

(Cronbach's Anderson *et al.*, 1994) studied the effect of some factors such as gender, student age, and students' high school scores in Mathematics, English and Economics, on the level of university attainment. According to their study, students who have better scores in high school also performed better in university. Another aspect discovered was that men had better grades than women and choose to drop from school less often.

(Ragothaman, *et al,* 2005) in their studying about master of performance and admission criteria said while there is lot of published research on MPA student performance, there is very little research to predict student success in MPA programs. Their research study examines MPA student performance and its association with junior-senior year grade point average (2UGPA) which is equivalent to our SSS or SHS, verbal GMAT score (Oral English), quantitative GMAT score (mathematics), age of the student (AGE), formal school (COLLEGE), and campus location

(LOCATION) were used to measure student performance. Their study shows that there is a high correlation between the junior-senior year grade (SSS or SHS attended) and performance than other factors. Age was a significant predictor of performance and is at the 10 percentile level.

(Timothy Rodgers 2005) in his paper titled Measuring value added in higher education came out that there is the natural tendency to what to keep measures simple, and this reflected in both tertiary and secondary education measures of the value added. However he said it has been shown that simplicity in this situation is at the expense of accuracy, from him the consequence of using simplistic measures of the "expected" school exam results is that the resulting measure of value added is not going to be very meaningful. The "exogenous" factors that influence school performance are involved and complex, and measuring their expected impact cannot be achieved by only examining the impact of previous academic achievement levels on performance but would be necessary to develop sophisticated modeling techniques if we are able to produce a credible measure of value added.

When performing an analysis, it is best to take a long term data to ensure that the performance improvement initiative ties in with the organization's vision, mission, and values. This connects each need with a metric to ensure that it actually does what it is supposed to do. This is best accomplished by linking performance analysis needed with other factors (Phillips, 2002):

By (Roberto G. Gutierrez, 2008) random effects are not directly estimated but summarized by their variance components, which are estimated from the data. As such, **xtmixed** is typically used to incorporate complex and multilevel random-effects structures into standard linear regression.

(Teck K *et al* 2009) in their paper that illustrates the analysis of longitudinal data using GEE (generalize estimation equations) and showing how output from SAS macros can be streamlined and organized to aid interpretation of analysis. They said though using GEE through procedures such as SAS PROC GENMOD is becoming increasingly common place, as far as model evaluation is concerned, its widespread use is somehow limited by the lack of easily accessible measures to evaluate the model goodness-of-fit directly from the default SAS output. Their study gives an example of how this can be done by first building three goodness-of-fit indices, namely the marginal R2, QIC and QICU, in a SAS macro, using various working correlation matrices, for model comparison. Their work specifically illustrate with a longitudinal data set, how four models with different working correlation matrices specification with a binomial logit link function, were generated using the macro. The results shows that estimated coefficients for the four models were largely similar; in their view they Support (Zeger and Liang, 1986) point that misspecification of working correlation would still give consistency result. Their study also illustrates the procedure of data management and preliminary data analyses work needed before carrying out similar analyses using several simple SAS macros these include carrying out statistical procedures such as factor analysis for examining constructs reliability, calculating reliability index.

From regression models for correlated data can be fit in several ways one of them is the use of SAS statistical software. According to them, in a recent contribution to the 27th SAS Users Group International Conference, Oliver Kuss described and illustrated several such methods (Kuss, 2002). Like Kuss we are predisposed to modeling correlated data with the NLMIXED procedure because it provides improved maximum likelihood (ML) estimates relative to appropriate ML estimates yielded by the GLIMMIX macro, and because, unlike the GENMOD procedure, it allows for the explicit modeling of random effects (*SAS/STAT® User's Guide*, *Version* 8, 2000). To them another drawback to the GLIMMIX approach is that its estimating method, penalized quasi-likelihood, has been shown-unless corrections are added to yield biased results for binary outcomes in some circumstances (Breslow and Clayton, 1993); (Breslow and Lin, 1995). In their view, modeling longitudinal data in which there is not a high degree of serial correlation the limitation may not be serious. When a random effect is used for the intercept if a compound symmetry covariance structure is induced and this provides a reasonable fit to the data in many applied problems. They added that random time effect might improve the fit in certain cases and they are currently working on developing the capacity to handle models with two random effects. To them SAS provides alternatives to some procedures; however, the alternatives are generally less robust and useful only in special circumstances.

(V.O. Oladokun, *et al*, 2008), stated that observed poor quality of graduates of some Nigerian Universities in recent times has been partly traced to inadequacies of the National University Admission Examination System. In their study of an Artificial Neural Network (ANN) model, for predicting the likely performance of students being considered for admission into the university was developed and tested. To them, the various factors that are likely to influence the academic performance of a student are; ordinary level subjects' score (O' Level), Subjects' combination, age on admission, parental background, types and location of secondary school attended (SHS), gender, etc were then used as input variables for the ANN model. A model based on the Multi layer Perception Topology was developed and trained using data spanning five generations of graduates from an Engineering Department of University. To them the first University test data evaluation shows that the ANN model is able to correctly achieve an accuracy of over 74%, prediction of the performance of more than 70% of prospective students which shows the potential efficacy of Artificial Neural Network as a prediction tool and a selection criterion for candidates seeking admission into a university. One limitation of this model is taking from the fact that not all the relevant performance influencing factors were obtainable from the pre-admission record forms filled by the students.

(Micky Shachar and Yoram Neumann, 2003) in their research on student's performance consider distance students (DS) and regular students (RS) using meta-analysis they use their final year grade/ score within the two year period to analyze their performance. To them 86 students (N= 86) met the established criteria for meta-analyses. In their view, a meta-analyses on a given research topic is directed toward the quantitative integration of findings from various studies. The data extraction and analysis from these works produced 86 calculated effect sizes, which yielded the final academic performance factor. These 86 effect sizes were the "basis" for the meta-analysis iterations conducted; DS have their mean at 50^{th} percentile while RS have their mean at 65^{th} percentile. They came with the conclusion that their null hypothesis defined as "there is no difference between DS and RS instruction for the final academic performance factor" should be rejected in favor of our alternative hypothesis.

(Stewart S M *et al*, 2006) in their longitudinal data analyses on the relationship between stress-related measures and academic performance during the first two years of medical school, medical students (n=121) were surveyed prior to beginning of classes and 8 months later variables predisposing to distress, stress response (depression and state anxiety), and stress management strategies were assessed. Pre-medical academic scores and grades at the end of five assessment periods over the course of the first 2 years of medical school were also obtained. The results shows that academic performance before and during medical school was negatively related to reported stress levels. On the bivariate correlations, there were numerous significant relationships between stress and academic performance.

(Micha Mandel and Rebecca A. Betensky, 2008) researching in Estimating time-to-event from longitudinal ordinal data using random-effects Markov models said Longitudinal ordinal data are common in many scientific studies, including those of multiple sclerosis (MS), and are frequently modeled using Markov dependency. They said several authors have proposed random-effects Markov models to account for heterogeneity in the population. In their paper, they went one step further and study prediction based on random-effects Markov models particular, they show how to calculate the probabilities of future events and confidence intervals for those probabilities, given observed data on the ordinal outcome and a set of covariates, and how to update them over time. They discuss the usefulness of depicting these probabilities for visualization and interpretation of model results and illustrate their method using data from a phase III clinical trial that evaluated the utility of interferon beta-1a (trademark Avonex) to MS patients of type relapsing-remitting. To them natural assumption of Markov dependency provides a convenient framework for the estimation of probabilities of various time-dependent events that are of biological interest (prediction). Their papers assume implicitly that the distribution of the first (baseline) state is independent of the random effect given covariate.

From (Umar and Co, 2010) social factors affect academic performance in terms of time demanded and the psychological state, these came as a result of their study on social effect on academic performance of student considering some social factors such as romantic relationships, organizations and clubs, and sports activities. The immediate question that comes out is how one strikes a balance between the stressful academic attainment and social activities. According to them all these factors have a direct or indirect relationship with students' performance. Their research find out that student cults are an academic impediment, romantic relationships having the highest impact, and may be a psychological barrier to an effective learning process also excessive sporting activities and involvement in clubs and organizations may pose a threat, but an insignificant one.

(Kenneth R. Garwood March, 2002) in his research of evaluating the predictive capability of the current criteria, other potential predictors, and determine an optimal set of predictors he examined the criteria used by the Air Force Institute of Technology (AFIT) to determine an applicant's academic eligibility to attend the in-residence Graduate Cost Analysis (GCA) program. He said academic performance in the GCA program was criterion variable and was measured by the cumulative graduate grade point average. Using predictive model developed by stepwise linear regression; some of the predictors use are age, gender, graduate grade point average (GGPA) etc which were functions of time. According to him, ideally, all criteria considered should be predictors of success of the institution and most of the criteria mentioned are believed to be just that. Their study shows that highest scores in graduate grade point average predicted higher performance, only the very lowest scores predicted poor performance, and even then, not to level of accuracy. In their view all indicated that the model may be a better measure of academic performance.

(Sano Paulo 2006) also research about growth status on academic achievement using multiple regression model analysis and univariate analysis of variance after assessing academic performance and measuring growth using standard procedure and height-for-age of 277 student selected randomly from his department. He came out that student whose growth move with their height and age perform better than those who do not grow well with their height and age (have retarded growth), which indicate that growth retardation have negative impact on academic performance.

(Chin W. Yang and Rod D. Raehsler 2007) in their research of academic performance using principles of microeconomics student as a case study said studying factors influencing academic performance has been very extensive in recent years beginning with a significant number of articles devoted to the economics discipline and expanding to a large number of other disciplines. They use an ordered-probit model on a sample of 488 students who enrolled in principles of microeconomics. They did their analysis on the estimated model and further study into the marginal impact of each explanatory variable some of which were gender, the student performance in other subjects etc, the analysis shows that a phenomenon of persistence can be used to describe final grades in microeconomics. It indicates in their result that student who does well in mathematics perform better in principles of microeconomics there by helping them to get good final grade. They also realized that gender has no effect on academics.

(Cakiroglllu Erdinc 2005) in his article aim to investigate the effort of gender and efficiency on academic performance on university grade level stated that self-efficiency has been define as an individual’s judgment of his or her capability to organize and execute the courses of action required to attain designated types of performances. To him strong sense of efficiency increase human accomplishment and people with high confidence in their capabilities approach difficult task as challenge to mastered rather than as threats to be avoided. Using stochastic rasch modeling came out with this conclusion, there was significant relationship between personal efficiency and academic performance and there was no significant effect on gender on academic performance.

(Diana F. *et al,* 2005) used longitudinal data to investigate how food insecurity (both non nutritional and inadequate amount of food) over time related to changes in academic performance. The data was collected over three year's period; the food insecurity was measured by interview parent and canteen workers verbally. To them food insecurity result in grade repetition, absenteeism, tardiness, aggression among others which are all factors that impaired the progress of academic performance. They concluded that food insecurity have negative impact on academic performance.

(Carlos E. Godoy Rodriquez, 2006). The research which aim to analyze the relations among the levels of skills in the technology handling (ICT Skills Index), educative and professional level of the father of the student, surrounding or the environment in which the student live, individual behavior and academic performance of the Business university students, in order to propose a model of causal relations that represents suitably, the effects of these factor use with academic aims on the results in the studies. It was a random, cross-sectional and anonymous study: they begin by being a descriptive research, and finishes as explanatory study. The statistical analysis used was carried out in two phases: descriptive univariado and with structural equations modeling. The result shows that the education and professional level of the father have the strongest effect on academics. Surrounding or the environment in which the student live and individual behavior was the second, ICT skills index had significant effect on academic performance.

CHAPTER THREE

METHODOLOGY

3.1 Data description

The data set used for this thesis was obtained from the exams office at department of mathematics KNUST. It consist of 62 students' semester weighted average (SWA) results for each semester, from first year first semester to third year second semester. The response of interest is the SWA, and variables of interest are age, gender, aggregate of entry, admission mode and the type of school attended by the student. In this data set, all students are pursuing the same course with the same number of credit hours. Ghana Education Service (GES) grade the schools based on the facilities in the school and the number of pupils who want to attend that particular school.

In this chapter we introduce the methodology that was used to implement a model to analyze the data .The model to be used is a random effect model that would capture the randomness on the data. With the advent of modern powerful computer software, one can easily analyze complex data without much knowledge of mathematical concepts behind it. However to be able to effectively analyze data using statistical models it is very important to understand the theoretical and conceptual framework of the model. This concept describes the rationale for using random-effect models for longitudinal data analysis and the statistical notation for defining these models; provides SAS syntax for random effects models; and illustrates the application of this SAS syntax.

Longitudinal data arise when repeated measurements are obtained for an individual (or unit of analysis) on one or more outcome variables at successive time points. Longitudinal data require the most elaborate modeling of the random variability. (Diggle, Liang and Zeger, 1994) distinguish among these components of variability, in longitudinal settings, where each individual typically has a vector Y_i of responses with a natural (time) ordering among the components.

We are interested in describing the semester weighted average (SWA) trend over semesters which is a function of time as well as whether there are significant differences in the trend across groups or characteristics of subjects defined by such characteristics as age, gender, average grade of entering the university, type of SHS (senior high school) attended etc. For example, student SWA may increase over time because student may score high marks in their various courses as they advance in the duration of their programs. However, there may be differences in the rates of increment for individuals from different age groups.

3.2 Random Effects Models

The random effect model is given by the equation:

$$Y_{ij} = X_i\beta + Z_i b_i + \varepsilon_i \tag{3.1}$$

Matrix formulation

A more solid representation of the model is afforded using matrices and vectors. This is very useful particularly in summarizing statistical aspect of the model. For individual or subject i,

$$y_i = x_i \beta + Z_i \upsilon_i + \boldsymbol{\varepsilon}_i \tag{3.2}$$

$(n_i \times 1) \quad (n_i \times \rho) \; (\rho \times 1) \; (n_i \times \boldsymbol{r}) \; (r \times 1) \quad (n_i \times 1)$

where y_i is the response variable X_i is the $n_i \times \rho$ covariate matrix for individual i, β is the $\rho \times 1$ vector of regression parameters, Z_i is the $n_i \times \boldsymbol{r}$ design matrix for random effect, υ_i is the $\boldsymbol{r} \times 1$ vector of random individual effects and $\boldsymbol{\varepsilon}_i$ is the $n_i \times 1$ error vector which is normally distributed $N \sim (0, \sigma^2)$.

Random effects models are also known as multi-level models or mixed models. The model is selected because Random effect models are regression models in which the regression coefficients are allowed to vary across the subjects. Here the subjects are the students from ACT IV. These models have two components:

$$Y_{ij} = \beta_0 + Z_i \upsilon_i + \varepsilon_i \tag{3.3}$$

- Within-individual component: A change of a student SWA's over semesters are described by a regression model with a population-level intercept and slope

$$Y_i = \beta_0 + Z_i \beta_i + \varepsilon_i \tag{3.4}$$

- Between-individual component: Variation in the SWA's over semester's base on individual differences.

For longitudinal studies, random effects models enable the analyst not only to describe the trend over time while taking into account of the correlation that exists between successive measurements, but also to describe the variation in the baseline measurement and in the rate of change over time. There are a number of techniques for analyzing longitudinal data, unlike including univariate and multivariate anovas and generalized linear models with generalized estimating equations in random affects analyses.

- Subjects are not assumed to be measured on the same number of time points, and the time points do not need to be necessarily equally spaced;
- Analyses can be conducted for subjects who may miss one or more of the measurement occasions, or who may be lost to follow-up at some point during study. In our example a student may fall sick during the entire duration of a semester examination.

Random effects models however allow for the inclusion of time-varying and time-invariant covariates. Time-varying covariates are independent variables that co-vary with the dependent variable over time. For example, a researcher studying trends in student Y_i performance over time, might also want to capture data on the highest and lowest marks of the group or degree of performance of co-student at each measurement occasion. The background of the student is likely to be an important predictor for performance assessment. It may also vary over time. Other covariates, like gender and fee paying status either do not change over time, or are less likely to change over time.

Random effects allow the analyst to model the correlation structure of the data. Thus, the analyst does not need to assume that measurements taken at successive points in time are equally correlated, which is the correlation structure that underlies the anova model. The analyst also does not need to assume measurements taken at successive points in time have an unstructured pattern of correlations, which is the structure that underlies the multivariate analysis of variance model. The former pattern is generally too restrictive, while the latter is too generic. With random effects model, the analyst can fit a specific correlation structure to the data, such as an autoregressive structure, which assumes a decreasing correlation between successive measurements over time. This can result in a more efficient analysis, with improved power to detect significant changes over time.

With student SWA data, varying numbers of measurement occasions and missing observations are typically not of great concern. Very few individuals are lost to follow-up in population-based studies. Loss to follow-up will occur when individuals leave the institution, or when they die. Moreover, time points of measurement will typically be equally spaced because time is often defined in terms of fiscal or calendar years, months or weeks. However, analyses of student SWA data may include both time varying and time-invariant covariates.

SWA assessments provide detailed data about each student on their unique learning path in each semester. Since measurement of student performances are very essential to any institution and parents, assessment item that interest student and help them capture details about what they know and what they are doing and also help lecturers to know what and how they are doing is use in almost all the institution in Ghana. Semester and time would be used interchangeably; also individual and students would be used interchangeably.

3.3 Random Intercept Model (RIM)

The simplest regression model for longitudinal data is one in which measurements are obtained for a single dependent variable at successive time points. Let Y_{ij} represent the measurement for the *i*th individual at the *j*th point in time,

$$Y_{ij} = \beta_0 + \beta_1 t_{ij} + \varepsilon_{ij} \tag{3.5}$$

β_0 is the intercept, β_1 is the slope, that is the change in the outcome variable for every one-unit increase in time (semester) and ε_{ij} is the error component. In this simple regression the ε_{ij}'s are assumed to be correlated, and to follow a normal distribution (i.e., $\varepsilon_{ij} \sim N(0, \Sigma)$). β_0 represents the average value of the dependent variable when time = 0, and β_1 represents the average change in the dependent variable for each one-unit increase in time (semester). There is a possibility that a student may start with low SWA and then increase over semesters as shown in figure (a) or no change in SWA over semesters as shown in figure (b) or start with high SWA and decrease over semester as in (c).

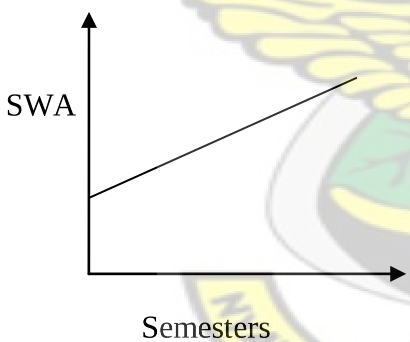

Figure 3.1 (a)

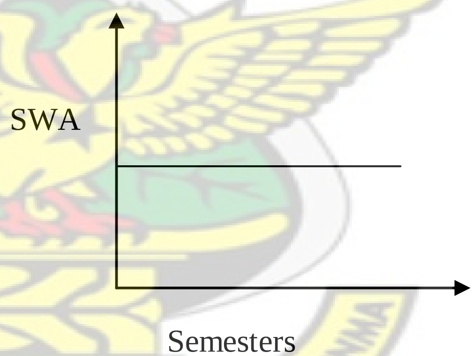

Figure 3.1 (b)

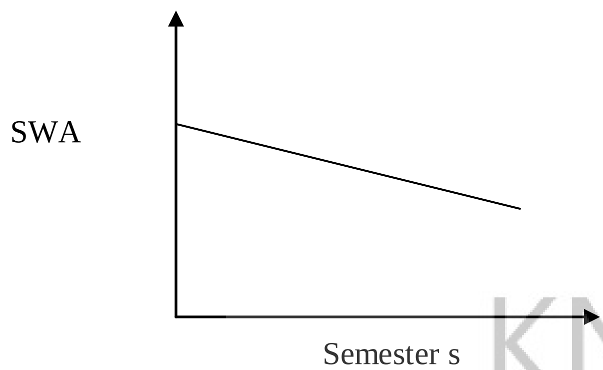

Figure 3.1(c)

Figure 3.1(a), 3.1(b) and 3.1(c): Demonstrating the possible average change of SWA's over semesters.

The implication was that on average student who perform well is depicted by fig 3.(a). fig 3.(c) depicts that on the average such students is performing poorly.

The simple random effects model is the one which the intercept is allowed to vary across individuals (students):

$$y_{ij} = \beta_0 + \beta_1 t_{ij} + \upsilon_{0i} + \varepsilon_{ij} \tag{3.6}$$

Where υ_{0i} represent the influence on individual *i* on his/her repeated observations. Note that if the individuals have no influence on their repeated outcome (SWA), then all the υ_{0i} will be equal to zero ($\upsilon_{0i} = 0$), but that may not be true, therefore υ_{0i} may have negative or positive impact on their SWA's therefore υ_{0i} may deviate from zero. For better reflection of this model on the characteristic individual the model is partition into within-subjects and between-subjects.

Within-subjects $\quad y_{ij} = b_{0i} + b_1 t_{ij} + \varepsilon_{ij}$ (3.7)

between subjects $\quad b_{0i} = \beta_0 + \upsilon_{0i}$ (3.8)

$$b_{1i} = \beta_1 \tag{3.9}$$

Equation 3.8 indicates that the intercept for the *i*th individual is a function of a population intercept plus unique contribution for individual. We assume $\upsilon_{0i} \sim N\left(0, \sigma^2_{\upsilon 0}\right)$. This model also indicates that each individual's slop is equal to the population slope, β_1, equation 3.9.

When both the slope and the intercept are allowed to vary across individual the model is:

$$\mathrm{Y}_i = \beta_0 + \beta_1 t_{ij} + \upsilon_{0i} + \upsilon_{1i} t_{ij} + \varepsilon_i \tag{3.10}$$

The within –subjects model is the same as

$$Y_i = b_{0i} + b_{1i} t_{ij} + \varepsilon_i \tag{3.11}$$

and between-subjects model is:

$$b_{0i} = \beta_0 + \upsilon_{0i} \tag{3.12}$$

$$b_{1i} = \beta_1 + \upsilon_{1i} \tag{3.13}$$

The within- subject model indicates that the individual *i*th SWA at time *j* is influence by their initial level b_{0i} and the time trend or the slope b_{1i}. The between –subject indicate that the individual *i*'s initial level is determined by the population initial level β_0 plus the unique contribution of υ_{0i} .Thus each individual has their own distinct initial level. Intercept for the *i*th individual is a function of a population intercept plus unique contribution for that individual. As well, the slope for the *i*th individual is a function of the population slope plus some unique contribution for that subject. We assume

$$D = \begin{pmatrix} \sigma^2_{\upsilon_0} & \sigma_{\upsilon_0}\sigma_{\upsilon_1} \\ & \sigma_{\upsilon_1}^{\ 2} \end{pmatrix} \tag{3.15}$$

is the variance-covariance matrix of random effects. Correlation exists between the random slope and the random intercept, so that individuals who have higher values for the intercept (i.e. higher or lower values on the dependent variable at the baseline time point) will also have higher or lower values for the slope . The resulting linear model can now be written as :

$$\begin{aligned} Y_i &= X_i\beta + Z_i b_i + \varepsilon_{1i} \\ b_i &\sim N(0, D) \\ \varepsilon_{1i} &\sim N(0, \sigma^2 I_{ni}) \end{aligned} \tag{3.16}$$

Assumptions:

$b_1, ..., b_N, \varepsilon_1, ..., \varepsilon_N$ b's are independent

$\varepsilon_1 \sim N\left(0, \sigma^2 I_{ni}\right)$ Is the measurement error

The variance of the measurement is given below:

$$V\left(y_i\right) = Z_i \Sigma_\upsilon Z_i^{'} + \sigma^2 I_{ni} \tag{3.17}$$

This model implies that conditional on the random effects, the errors are uncorrelated, as is displayed. This is seen in the above equation (3.17) since the error variance is multiplied by the identity matrix (i.e., all correlations of the error equal zero).

3.4 Restricted Maximum Likelihood Estimation

In restricted maximum likelihood estimation we consider the case where the variance of a normal distribution $N \;\; \mu,\sigma^2 \;\; i$ to be estimated based on a sample Y_i , ..., Y_N of N observations. Where the mean μ is known, the maximum likelihood estimation (MLE) for σ^2 equals $\sigma^2 = \sum_i (Y_i - \mu)^2/N$, which is unbiased for σ^2. When μ is not known, we get the same expression for the MLE but with μ replace by the sample mean $\overline{Y} = \sum_i Y_i /N$.

$$E(\sigma^2) \qquad = \qquad \frac{N-1}{N}\,\sigma^2 \tag{3.18}$$

The equation (3.18) indicate that the MLE is now biased downward due to the estimation of μ, the unbiased estimation of (3.18) yield the classical sample variance

$S^2 = \sum_i (Y_i - \overline{Y})^2/(N-1)$

To obtained an unbiased estimate for σ^2 directly we should use following; let Y = (Y_1…….Y_N)

denote the vector for all measurement and I_N be N-dimensional vector containing only ones and zero. The distribution of Y is then N (μ I_N, $\sigma^2 I_N$) where I_N equals the identity matrix, if A is N × (N− 1) any matrix with N− 1 linear independent columns orthogonal to the vector I_N, vector U of N − 1 which is the error contrasts is defined by $U = A^T Y$ following the normal distribution with mean vector zero and covariance matrix $\sigma^2 A^T A$. Maximizing the corresponding likelihood with respect to the only remaining parameter σ^2 yields $\sigma^2 = Y^T A\,(A^T A)^{-1} A^T Y/\,(N-1)$ which is equal to classical sample variance S^2. The resulting estimator is the RMLE since it’s restrict

(N− 1) error contrasts.

3.5 Inference for the random effect

The random effect $\mathbf{b_i}$ reflects on how much the subject-specific profiles deviate from the overall average profile therefore it needs to be estimated. Such estimation can be interpreted as residuals which may be helpful for detecting special profiles (i.e. outliers) or group of individuals evolving differently in time. The variability in the data can be explained by random effects b_i. This section would use Bayessian techniques since the random effects in the model are assumed to be random variables. the distribution of the vector Y_i of responses for the *i*th individual condition on that individual's specific regression coefficients b_i is multivariate normal with mean vector $X_i\beta + Z_ib_i$ and with covariance matrix $\sum_i$. Also the marginal distribution of b_i is $N(0, D)$, D = covariance matrix. The distribution is usually called the prior distribution of the parameter b_i since it does not depend on the data Y_i. Once observed values y_i for Y_i have been collected, the posterior distribution of b_i is defined as conditional on $Y_i = y_i$, can then be calculated.

Denoting the density function of Y_i conditional on b_i, and the prior density function of b_i by $f(y_i/b_i)$ and $f(b_i)$, respectively, the posterior density function of b_i given $Y_i = y_i$ is given by

$$f(b_i/y_i) \equiv f(b_i/Y_i = y_i) = \frac{f(y_i/b_i)f(b_i)}{\int f(y_i/b_i)f(b_i)db_i} \qquad (3.19)$$

Suppressing the dependency on all above density functions on certain components of θ. Using the theory on general Bayesian linear models (Smith 1973) and (Lindley and Smith 1972) it can be shown that(3.19) is the density of the multivariate normal. b_i is estimated by the mean of this posterior distribution, called the posterior mean of b_i. This estimate is then given by

$$b_i(\theta) = E(b_i/Y_i = y_i) \qquad (3.20)$$

$$= \int b_i f \quad b_i / y_i \quad db_i \tag{3.21}$$

$$= DZ_i^{'} W_i \quad \alpha \quad y_i - X_i \beta \tag{3.22}$$

And the covariance matrix of the corresponding estimator equals

$$\text{Var}(b_i \quad \theta \quad) = DZ_i^T \{W_i - W_i X_i (\sum_{i=1}^{n} X_i W_i X_i)^{-1} X_i W_i \} Z_i D \tag{3.23}$$

Where W_i equals ε_i^{-1} (Laird and Ware 1982), the equation (3.23) underestimates the variability in $b_i \quad \theta$ since it ignores the variation of b_i. The influence for b_i is usually base on

$$\text{var} \quad b_i \quad \theta \quad - b_i \quad = D - \text{var} \quad b_i \quad \theta \tag{3.24}$$

Where θ = density function on certain components.

The unknown parameters α and β in the above equation can be replace by their restricted maximum likelihood estimates. The influences can now be drowning base on appropriate t-test or F-test.

3.6 The random intercepts model

We consider the random-intercepts model. The random-effects covariance matrix D is now scalar and it will be denoted by $\sigma^2{}_b$, the matrix Z_i are of the form I_{n_i}, a n_i- dimensional vector of ones. We will assume that all residual covariance matrices are of the form

$\sum_i = \sigma^2 I_{n_i}$, i.e. we assume conditional independent. The random intercept of subject i is given by

$$b_i = \sigma_b{}^2 I_{n_i}{}' \ \sigma_b{}^2 I_{n_i} I_{n_i}{}' - \sigma_b{}^2 I_{n_i} {}^{-1} \ y_i - X_i \beta \tag{3.25}$$

$$= \sigma_b{}^2 I_{n_i}{}' \left(\frac{\sigma^2{}_b}{\sigma^2} I_{n_i}{}' - \frac{\sigma^2{}_b}{\sigma^2 + n_i \sigma^2{}_b} I_{n_i} I_{n_i}{}' \right) \ y_i - X_i \beta \tag{3.26}$$

$$= \frac{n_i \sigma^2{}_b}{\sigma^2 + n_i \sigma^2{}_b} \frac{1}{n_i} \sum_{j=1}^{n_i} (y_i - x_{ij}{}' \beta) \tag{3.27}$$

Where the vector x_{ij} consists of the jth row in the design matrix X_i and $\frac{1}{n_i} \sum_{j=1}^{n_i} (y_i - x_{ij}{}' \beta)$ is equal to the average residual. If n_i is large for subject *i* a weight is put on the average residual yielding less shrinkage, the within-subject variability is large in comparison to the between-subject variability if more shrinkage is obtained.

3.7 Selecting a Correlation Structure for the Repeated Measurements

Fitting the Correct correlation structure to the data will ensure that model parameters and their standard errors are estimated correctly. A number of different covariance structures may be selected in PROC MIXED. The most common choices are:

- <u>Exchangeable or compound symmetric</u> - assumes that correlation between all pairs of repeated measurements are equal irrespective of the length of the time interval.

- Table 3.1: Exchangeable or compound symmetry

Compound symmetric	*Swa1*	*SWA2*	*SWA3*	*SWA4*	*SWA5*	*SWA6*
SWA1	1	*p*	*p*	*p*	*p*	*p*
SWA2		1	*p*	*p*	*p*	*p*
SWA3			1	*p*	*p*	*p*
SWA4				1	*p*	*p*
SWA5					1	*p*
SWA6						1

<u>Unstructured</u> - with this structure, all correlations are assumed to be different.

Table 3.2: Unstructured

Exchangeable	Swa1	Swa2	Swa3	Swa4	Swa5	Swa6
Swa1	1	p_1	p_2	p_3	p^4	p^6
Swa2		1	p^7	p^8	p^9	p^{10}
Swa3			1	p^{11}	p^{12}	p^{13}
Swa4				1	p^{14}	p^{15}
Swa5					1	p^{16}
Swa6						1

3.8 Data Structure

To use SAS for a random effects analysis of longitudinal data, the data set must be correctly structured. A longitudinal data set may have a multivariate structure or a univariate structure. In a multivariate structure each individual has a single data record which contains all of the repeated measurements. In a univariate structure, which is required for PROC MIXED, each individual has as many data records as there are measurement occasions.

As defined previously, let Y_{ij} represent the dependent variable value for the *i*th individual at the *j*th point in time and let X_{ij} be the vector of predictor variable values for the *i*th individual at the *t*th time point. That is, $X_{ij} = [X_{ij1}\ X_{ij2}\ \ldots\ X_{ijK}]$. The ID variable is a unique identifier for each individual in the data set. The univariate data structure is:

ID	$\boldsymbol{Y_{ij}}$	$\boldsymbol{X_{ij1}}$	$\boldsymbol{X_{ij2}}$	**…**	$\boldsymbol{XijK}$
1	Y_{11}	X_{111}	X_{112}	…	X_{11K}
1	Y_{12}	X_{121}	X_{122}	…	
…	…	…	…	…	…
1	Y_{1J}	X_{1J1}	X_{1J2}	…	X_{1JK}
2	Y_{21}	X_{121}	X_{122}	…	X_{12K}
…	…	…	…	…	…
N	Y_{NJ}	X_{NJ1}	X_{NJ2}	…	X_{1NJK}

In this univariate structure, each value of the dependent variable and the associated independent variable values are contained in a single record.

CHAPTER FOUR

DATA ANALYSIS

4.0 Introduction

This chapter is devoted to the demonstration of Random Effect Model as the tool for analyzing data with the help of SAS version 9.1 software. The data obtained consist of student's semester weighted average results from first year first semester to third year second semester of actuarial students in KNUST. The SWA is calculated by multiplying the marks obtained for each course by the credits of the particular course adding up the products and dividing by the total number of units of credit for the courses. (e.g if a student is offering 6 courses, then this is how the SWA would be calculated; marks obtained in the six courses = 62.86, 59.86, 56.39, 62.50, 62.22, 61.05 with respective credit hours = 2, 2, 3, 3, 3,3

62.86 *2 + 59.86 *2 + 56.39*3 + 62.50*3 +62.22 *3 + 61.05 *3 = 971.92

SWA = 971.92/16 = 60.75). The SWA is calculated for the semester while the CWA is for the number of semesters covered.

4.1 Exploratory Data Analysis

In general, graphical tools give a pictorial view of the distribution of the data while at the same time giving information on shape, location and spread of the data. In this section, some objective graphical techniques such as the individual profile and mean structure were used to explore the data set. The variance and the correlation structure were also considered for the data set

Individual profile describes how the profile of the SWA of students evolves over time (semester). Individual profiles augment the average plot with a suggestion of the variability seen within the data. The mean structure which is the average evolution describes how the profile for students with respect to SWA on the average evolves over semester conditioned on the type of school, admission type etc. The results of this exploration will be useful in order to choose a fixed-effects structure for the random –effect model.

Variance structure: the variance is important to build an appropriate longitudinal model. Here it is necessary that we correct the measurements for the fixed-effects structure. Correlation structure describes how SWA obtained are correlated over a semester. The correlation function depends on a pair of SWA scores over semesters and only under the assumption of stationary does this pair of semesters simplify to the semester lag.

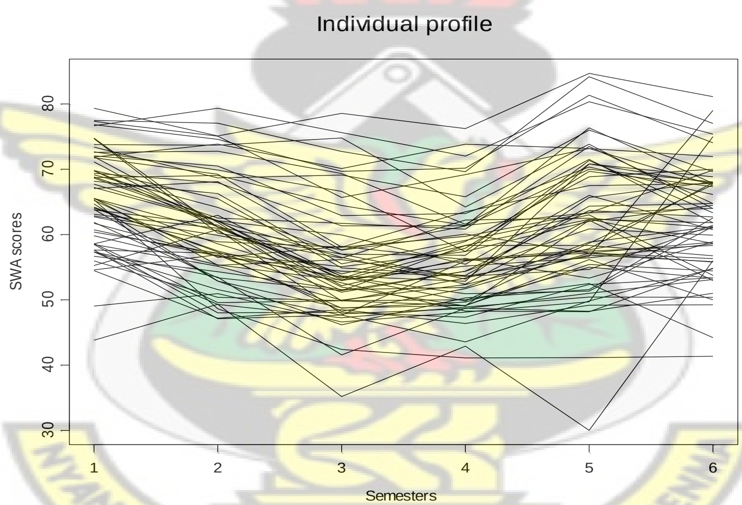

Figure 4.1: The individual profiles of 62 students with respect to their SWA's (%) over six semesters.

4.2.1 Individual Profiles

Figure 4.1 above shows the individual profiles for each of the student's SWA's at the end of each semester. These profiles are the summarized form of the total observations collected for the observational study. It is observed that most of the students start with a different SWA at semester one and that most students' start with SWA above 50%. From figure 4.1 we can deduce that there exists some variability between and within SWA's for each student.

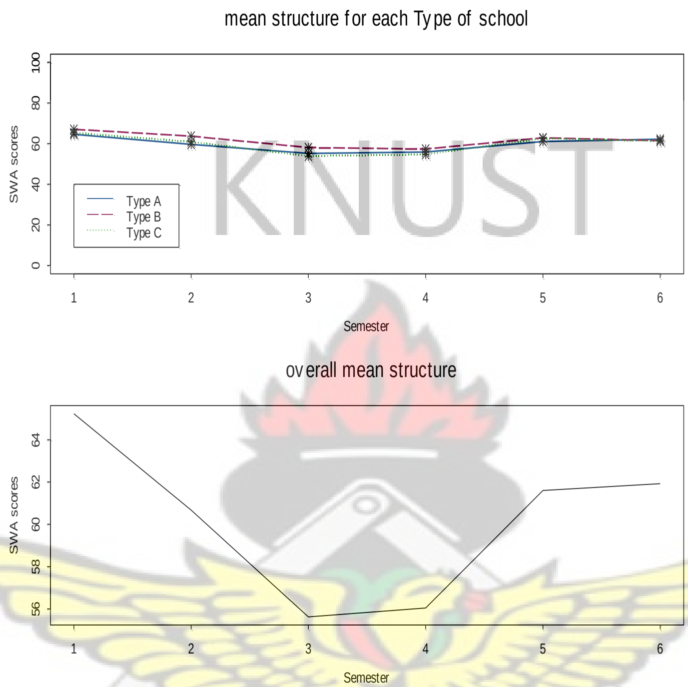

*Figure 4.2 Top panel; mean structure for each type of school. Bottom panel: the overall mean structure***.**

4.2.2: Mean profile for the various types of school.

The mean profiles in Figures 4.2 are the means of the students SWA conditional for the three types of graded schools as classified by GES. From the top panel we observe that on the average, SWAs' of student from type B is higher relative to type A and type C over semesters. Type C follows then type A as indicated at the beginning of graph. From the top panel in figure 4.2 we observed that the three types of schools average SWA is slightly above 60%, at the beginning of the semester one. The mean SWA decreases moderately and increases slightly after semester four.

The bottom panel of figure 4.2 shows the overall mean of all the SWA at each semester. Here we observe that on the average all students SWA scores decreases from semester one to semester three and then increases moderately after semester four. Appendix B shows the means at each semester for various classes of schools and the overall mean structure.

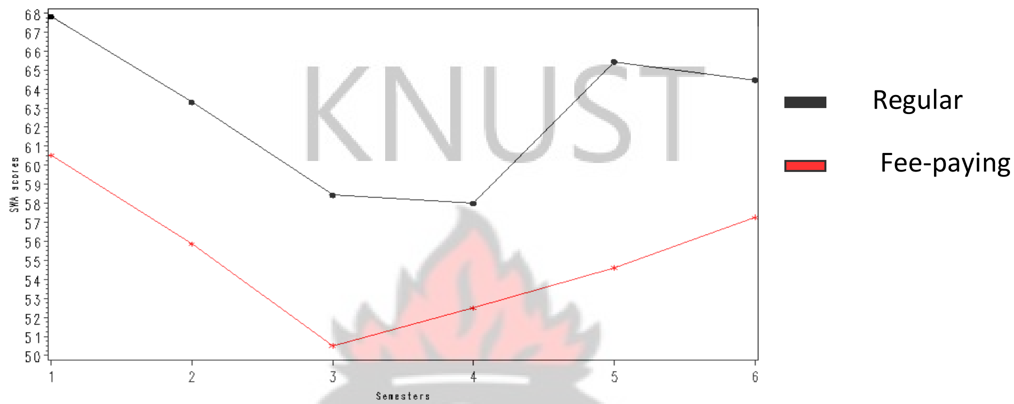

Figure 4.3: The Mean profile for regular and fee-paying student.

From figure 4.3 above shows the mean profile of SWA scores for both the regular and fee-paying students. From the figure, it is noted that regular students have higher SWA scores than fee- paying students on the average. We can also observe that the mean SWA for regular student was about 68% at semester one while that of fee- paying was around 60.5 %. The trend shows sharp decrease after semester one to semester three and increases moderately after semester four. This observation is similar to the overall mean structure.

Table 4.1: The Correlation Structure for the SWA scores

	Swa1	Swa2	Swa3	Swa4	Swa5	Swa6
Swa1	1.0000	0.8202	0.8067	0.7718	0.8398	0.5729
Swa2		1.0000	0.8828	0.8672	0.8258	0.6279
Swa3			1.0000	0.8884	0.8814	0.6885
Swa4				1.0000	0.8682	0.6484
Swa5					1.0000	0.7042
Swa6						1.0000

The correlation structure describes how SWAs correlate within semester. The correlation function depends on a pair of semester (time), does this pair of time simplify to the time lag. This is important since many exploratory and modeling tools are based on these assumptions. Since the structure varies with time, the variation may be captured by random effect model. A different way of displaying the correlation structure is using a scatter plot.

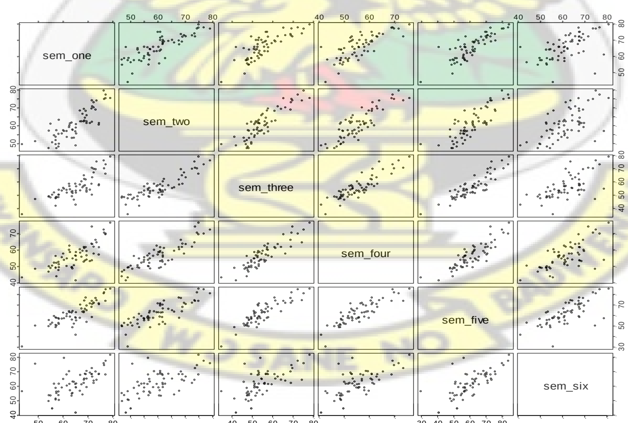

Figure 4.4: The scatter plot of the correlation structure

Figure 4.4 shows there is a high correlation between the pair-wise repeated SWA. This is due to the fact that the SWA's were taken repeatedly for the same student over semesters. This can also been seen from the pair wise scatter-plots between two repeated SWA over semester. In figure 4. 4, it's clearly seen that as the distance from the diagonal is increasing so also the degree of relationship is decreasing.

Table 4. 2: The overall sample variance for each semester

Semester	Variance
1	27.9I43
2	41.2807
3	41.3396
4	37.4799
5	82.5722
6	44.3194

Table 4.2 above shows the computed sample variance for each semester. We can observe that the variance increases from semester one to semester three, decrease at semester four, increases at semester five and sharp decrease at semester six. This may indicate that the variation of the SWA's are finite unstable and not homogeneous. However, the assumption of constancy of variance may not be too many because some of the variation may be accounted for by the individual effects.

4.3 Random intercept with no fixed effects

$$\begin{aligned} swa_{ij} &= \upsilon_{0i} + \varepsilon_{ij} \\ \upsilon_{0i} &\sim N(0,\sigma_{b0}^2), \quad \varepsilon_{ij} \sim N(0,\sigma^2) \end{aligned} \tag{4.0}$$

This model in equation (4.0) is called the null model. This typically the first model that we would used in deciding whether or not to select a random effects model for the data. The model contains only one parameter which is the random intercept effect. It partitions the total variation in the data into within-individual and between-individual component.

The intraclass correlation (ICC) coefficient computed from this null model is a useful tool for deciding whether a random effects model might be an appropriate choice for the data. The numeric formula for the ICC is

$$ICC = \frac{\hat{\sigma}^2_{\upsilon_0}}{\hat{\sigma}^2_{\upsilon_0} + \hat{\sigma}^2} \tag{4.1}$$

Where $\hat{\sigma}^2$ is the residual variance.

Table 4.3: Covariance parameter estimate

Cov Parm	Estimate	Standard Error	Z Value	Pr Z
Intercept	53.3370	10.5730	5.04	<.0001
Residual	30.0708	2.4153	12.45	<.0001

Here $\hat{\sigma}^2_{\upsilon_0}$ = 53.34 and $\hat{\sigma}^2$ = 30 .0

Therefore, ICC = 53.34/ (53.34 + 30.07) =0.64, indicating that 64% of the variation in the data is explained by allowing the intercept to vary across individuals. The statistically significant value for the within-individual variation suggests the data structure is best captured by using a random effects model.

4. 4: Random intercept and slope with no fixed effects

$$swa_{ij} = \upsilon_{0i} + \upsilon_{1i}t_{ij} + \varepsilon_{ij}$$
$$\upsilon_{0i} \sim N(0, \sigma_{b0}^2), \upsilon_{1i} \sim N(0, \sigma_{b1}^2),\ \varepsilon_{ij} \sim N(0, \sigma^2) \qquad (4.3)$$

Table 4.4: Covariance Parameter Estimate for null model in (4.3)

Cov Parm	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	43.8224	12.7400	3.44	0.0003
UN(2,1)	1.3903	1.3669	1.02	0.3091
UN(2,2)	0.0100	.	.	.
Residual	29.6605	2.3862	12.43	<.0001

The ICC was computed using variance estimates for the random intercept and slope model in (4.3), as well as their covariance. For this model, ICC = (43.82 + 1.39 + 0.01)/ (43.82 + 1.39 + 0.01+ 29.66) =0.604 indicating 60.4% of the variation in the data is accounted for by allowing the intercept and slope to vary across individuals.

Now that substantial proportion of the variation in the data can be explained by inclusion of both the intercept and slope we retained these parameters in the model. The next model also contains all of the fixed effect that we were interested in testing for statistical significance.

4.5 Formulation of the random effects models

In terms of the random effects for the analysis, the following data set from student's SWA would be considered. Both a linear trend over the six semesters and a quadratic effects from semesters one to semester six were considered we allowed a linear, quadratic trend for type of graded school and fee-paying status. This was due to the results obtained from the exploratory analysis.

$$swa_{ij} = \beta_1 admagg + \beta_2 gender + \beta_3 entryage + \beta_4 typeA + \beta_5 typeB + \beta_6 typeC +$$
$$(\beta_7 typeA + \beta_8 typeB + \beta_9 typeC + \beta_{10} R + \beta_{11} F)t_{ij} + (\beta_{12} typeA + \beta_{13} typeB + \beta_{14} typeC + \beta_{15} R + \beta_{16} F)t_{ij}^2$$
$$+ b_{0i} + \varepsilon_{1ij} \quad (4.4)$$

Here, admission aggregate, gender and entry age were not allowed to vary overtime.

Table 4.5 solution for fixed effects.

Effect	ntype	fee	Estimate	Standard Error	DF	t Value	Pr > |t|
type A	1		138.23	10.5305	257	13.13	<.0001
type B	2		143.27	10.9168	257	13.12	<.0001
type C	3		140.19	10.5960	257	13.23	<.0001
admagg			-1.2620	0.1560	257	-8.09	<.0001
entryage			-2.6027	0.5816	257	-4.47	<.0001
gender			0.9043	1.4149	257	0.64	0.5233
tme*regular		0	-8.1713	2.2930	257	-3.56	0.0004
tme*fee paying		1	-8.1600	2.2930	257	-3.56	0.0004
tme*type A	1		-0.5267	2.3653	257	-0.22	0.8240
tme*type B	2		0.5018	2.4906	257	0.20	0.8405
tme*type C	3		0	.	.	.	.
tme2*type A	1		1.2141	0.1490	257	8.15	<.0001
tme2*type B	2		0.9698	0.1436	257	6.75	<.0001
tme2*type C	3		1.0560	0.4904	257	2.15	0.0322
tme2*regular		0	-0.00610	0.1721	257	-0.04	0.9717

In this, the random –intercepts model is considered. It is a linear mixed model where only subject-specific effect is the intercept. From table 4.5, not all the parameters estimate were significant at 5% level of significant. The above model also assumes a constant correlation between any two SWA's from the same semester with constant variance and the corresponding random effect covariance matrix D been a scalar. We observed from table 4.5 that admission aggregate, entry age, type of school were all significant. However gender is not significant. On the average there is no significant difference in the SWA between types of schools over a linear trend but significant over quadratic trend. Fee-paying status is also significant over semesters.

4.6: Random intercept and slope model

$$swa_{ij} = \beta_1 admagg + \beta_2 gender + \beta_3 entryage + \beta_4 typeA + \beta_5 typeB + \beta_6 typeC +$$
$$(\beta_7 typeA + \beta_8 typeB + \beta_9 typeC + \beta_{10} R + \beta_{11} F)t_{ij} + (\beta_{12} typeA + \beta_{13} typeB + \beta_{14} typeC + \beta_{15} R + \beta_{16} F)t_{ij}^2$$
$$+ b_{0i} + b_{1i} t_{ij} + \varepsilon_{1ij} \tag{4.5}$$

Table 4.6 Solution for Fixed Effects

Effect	ntype	fee	Estimate	Standard Error	DF	t Value	Pr > |t|
type A	**1**		138.97	10.6994	208	12.99	<.0001
type B	**2**		143.95	11.1563	208	12.90	<.0001
type C	**3**		140.63	10.8120	208	13.01	<.0001
admagg			-1.2679	0.1815	208	-6.99	<.0001
entryage			-2.6673	0.5932	208	-4.50	<.0001
gender			1.7992	1.3505	208	1.33	0.1842
tme*regular		**0**	-8.1847	2.2944	208	-3.57	0.0004
tme*feepaying		**1**	-8.1465	2.2944	208	-3.55	0.0005
tme*type A	**1**		-0.5225	2.3568	208	-0.22	0.8248
tme*type B	**2**		0.4987	2.4843	208	0.20	0.8411
tme*type C	**3**		0	.	.	.	.
tme2*type A	**1**		1.2127	0.1537	208	7.89	<.0001
tme2*type B	**2**		0.9690	0.1455	208	6.66	<.0001
tme2*type C	**3**		1.0550	0.4908	208	2.15	0.0327
tme2*regular		**0**	-0.00401	0.1841	208	-0.02	0.9826
tme2*feepaying		**1**	0	.	.	.	.

Table 4.6 shows the random-intercept and slope model with some regressors and individual slopes. From the table 4.6 we realized that student SWA's depends on type of school, entry age, admission aggregate and the status over semesters, the quadratic time effect of type of school. The remaining effects were not significant. Focusing first on the estimated regression parameters in appendix B the mean of various type of school, it indicates that student start off, on average, with SWA score of 65.23 and change by -4.56 points in semester 2, -5.05 points in semester 3, 0.43 points in semester 4, 5.55 points in semester 5 and 0.322 in semester 6. Lower scores on the SWA reflect lower academic performance, so students are not improving over the first three semesters. Students' SWA improves by about 2.1 points after semester 4. Both the intercept and slope are statistically significant in this analysis. The intercept being significant is not particularly meaningful; it just indicates that SWA's are different than zero at semester one. However, because the slope is significant, the rate of improvement is significantly different from zero based on this analysis..

Table 4.7: variance components using REML log-likelihood

Model	Covariance Structure	-2*l*	Comparison	Difference	Types	Critical values
1	Random intercept	2471.22				
2	Random intercept & slope	1931.24	(1) – (2)	539.98	$\chi^2_{2,0.05}$	5.99

The initial random effect model with the above-mentioned variance components was fitted using Restricted Maximum Likelihood (REML) method and using chi-square ($\chi^2_{2,0.05}$) for the fixed effects. Table 4.7 shows the reduction of these variance components. From the table 4.5, the presence of slope is clearly detected ($\chi^2_{2,0.05}$ = 5.99) and the random slope effect cannot be dropped from the model (p value <0.0001). Hence, the variance- covariance matrix for random effect model was not reduced. After deciding on the variance components, an approximate model was used to reduce the mean structure ie. the fixed effects. The gender, type A*tme, type B*tme, type C*tme, tme2*regular and tme2*fee paying effect could be dropped (p value > 0.05).

4.7: The final model:

$$swa_{ij} = \beta_1 admagg + \beta_2 entryage + \beta_3 typeA + \beta_4 typeB + \beta_5 typeC + (\beta_6 R + \beta_7 F)t_{ij} + (\beta_8 typeA + \beta_9 typeB + \beta_{10} typeC)t_{ij}^2 + b_{0i} + b_{1i}t_{ij} + \varepsilon_{1ij} \tag{4.6}$$

Table 4.8: Parameter estimates for final model using random intercept and slope

Effect	ntype	fee	Estimate	Standard Error	DF	t Value	Pr > |t|
type A	**1**		132.81	10.2144	213	13.00	<.0001
type B	**2**		139.51	10.5943	213	13.17	<.0001
type C	**3**		136.09	11.0803	213	12.28	<.0001
admagg			-1.2413	0.1507	213	-8.24	<.0001
entryage			-2.3339	0.5478	213	-4.26	<.0001
tme*regular		**0**	-8.5475	0.6647	213	-12.86	<.0001
tme*feepaying		**1**	-8.4833	0.6852	213	-12.38	<.0001
tme2*type A	**1**		1.1931	0.1039	213	11.48	<.0001
tme2*type B	**2**		1.0802	0.08330	213	12.97	<.0001
tme2*type C	**3**		1.0996	0.2155	213	5.10	<.0001

Table 4.9 : Covariance Parameter Estimates

Cov Parm	Subject	Estimate
UN(1,1)	**IndexNo**	22.1851
UN(2,1)	**IndexNo**	-0.8510
UN(2,2)	**IndexNo**	0.6212
Residual		17.1111

From table 4.8 the t-test for the fixed effects shows that all fixed effects in the model are statistically significant. Thus, the average SWA depends on the entry age of the student, the type of school and the admission mode (regular or fee-paying). This 'net-reduction' effect reduces over semester. This is indicated by the negative sign of the estimates of admagg, entryage, tme*fee-paying and tme*regular effects.

From table 4.9 the random intercept has a relatively large estimate with respect to the other variance-components, this supports the fact that there is high 'between student' variability based on SWA's at semester one. The negative estimate of the covariance implies that students who start with high SWA at semester one, have more tendency to exhibit reduction of SWA over the semesters

CHAPTER FIVE

DISCUSSION OF RESULTS AND RECOMMENDATION

The choice of the random effects model is driven by the results from the exploratory data analysis. In models where continuous/discrete covariates are believed to have an effect on the mean response, the choice of the most elaborate model becomes less obvious. Thus, the average trends and individual profiles were used to select a candidate mean structure. Since there appears to be some curvature in the average trend and individual profile plots, a quadratic time effect was fitted to the data.

From the individual profiles are the total observations collected for the analysis. From the profiles of the type of schools and admission type for students, it could be assumed that each profiles evolution follows a quadratic trend. In addition, there is some variability between and within students in each group. Also, it could be observed that most students who started with high SWA score for semester one had a low SWA score before semester six.

Further, the mean profile for GES graded type of school was explored. We observe that the students from type B school on the average seem to score high SWA than those from type A and type C school. The entry age, gender and admission aggregate of students were not varied over time, from table 4.5 and 4.6 we observe that gender is statistically insignificant but entry age and admission aggregate was shown to have significant effect on the students' SWA scores. This result is the same for and for all the models considered.

From table 4.1 and figure 4.4 we observed that the correlation structure decreases slightly across the semesters generally. Furthermore all the correlation co-efficient were above 0.5 which is an indication of strong correlation between pairs of SWA's over semesters. The highest correlation coefficient occurs at semester three and semester four which is 0.8884 and the lowest occurs at semester one and semester six Swa6 with the value 0.5729

Exploration of the variance structure indicated that the variance seems not be to be constant. Given the shape of the variance function, the random intercept and random intercept slope effects were considered for modeling. Taking into account all of these in building the random effect model.

From table 4.3, ICC indicated that about 64% of the variability in the data has been explained,

considering random intercept model, were equal slope, constant correlation, constant variance, were assumed. We observed that gender, type A, B, and C with time effect and regular with time squared effect are statistically insignificant. Also on average students' from type B school has the highest estimated value of 143.27 from the random intercept model.

From random intercept and slope model where the slope is varied. We observed that the change of student SWA is slightly different from zero. In this model gender, type A, B, and C with time effect and regular with time square effect are statistically insignificant. From table 4.7 it is clear that the random slope effect cannot be dropped from the model, hence the rejection of the null hypothesis.

The final model in table 4.8 shows that age of entry, type of school attended, admission aggregate and admission mode (regular or fee-paying) are all significant.

From the chosen model among all model, fitted to the data set, we conclude based on the results obtained from the final model that student's SWA depends on the entry age of student, admission aggregate, type of school attended and the admission mode (fee-paying or regular). The older student's score lower SWA and the younger student scores high SWA at semester one. Student with high admission aggregates scores low SWA and student with low admission aggregates score high SWA at semester one. The performance of type B and C schools students are better as compare to that of type A at semester one. Meanwhile, in all the models it appeared, student SWA's decrease from semester one and increases after semester four. Over semester there is no significant difference in SWA's for the fee paying status. We also conclude that for type of schools there is a significant quadratic effect.

Recommendation

Based on the literatures on the students' academic performance, it is known that there are some factors such as the education back ground of the parents, financial status of parents etc may also have influence on students academic performance. Given that the study was an observed study, the effects of these factors on students' academic performance cannot be estimated from this study. Thereby, we recommended that these factors should be included in future if a similar study is conducted.

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Appendix

Appendix A: The total data

Obs	admagg	type	swa1	swa2	swa3	swa4	swa5	swa6	entryage	ta	tb	tc	feep
1	17	B	65.33	48.62	42.39	41.1	41.14	41.36	20	0	1	0	1
2	9	C	67.19	62.05	53.06	57.89	69.67	67.32	20	0	0	1	0
3	14	C	71.11	61.19	53.65	52.83	70.83	68.84	.	0	0	1	0
4	19	A	64.19	57.33	51.17	54.39	63.67	62.68	19	1	0	0	1
5	17	B	58.47	55.14	49.95	52.33	56.86	55.84	23	0	1	0	1
6	14	C	69.38	67.95	61.33	61.33	76.33	67.84	.	0	0	1	0
7	14	A	59.81	55.33	53	55.17	56.43	58.58	.	1	0	0	0
8	17	A	56.67	54.29	51.57	51.89	58.83	61.21	20	1	0	0	1
9	9	B	72.05	70.48	62.89	63.22	70.33	66.11	20	0	1	0	0
10	20	B	67.71	62.11	59.94	55.44	66	60.74	18	0	1	0	1
11	15	A	60.71	56.67	54.39	53.61	56.67	79	19	1	0	0	1
12	9	A	71.86	73.86	70.11	73.83	73.17	71.95	20	1	0	0	0
13	9	A	71.9	61.86	49.83	53.33	57.83	67.95	19	1	0	0	0
14	14	A	62.71	58.89	58.05	59.72	63.33	69.84	21	1	0	0	1
15	20	A	57.32	53.48	41.56	48.71	48.25	51	20	1	0	0	1
16	9	A	58.63	49.58	49.86	49.1	56.17	50	21	1	0	0	0
17	9	B	76.67	79.33	75.61	72	80.33	75.37	19	0	1	0	0
18	17	A	54.47	47.19	48.5	46.38	49.17	49.23	21	1	0	0	1
19	8	B	76.81	74.57	69.67	70.17	81.33	74.05	19	0	1	0	0
20	9	A	69.76	61.14	53.28	58.94	68.83	70	19	1	0	0	0
21	19	B	65.57	55.29	56.17	53.61	54.67	61.42	19	0	1	0	1
22	9	A	74.76	61.14	57.67	61.28	70	68.05	19	1	0	0	0
23	20	A	61.86	53.37	51.39	54.39	60.86	59.89	19	1	0	0	1
24	20	B	55.26	60.76	47.83	48.67	51	53.11	20	0	1	0	1
25	9	A	79.33	75.1	78.56	76.22	84.67	81.11	19	1	0	0	0
26	17	A	62.86	59.86	56.39	62.22	62.5	61.05	18	1	0	0	1
27	15	C	65.24	56.48	58.17	56.17	57.67	61	.	0	0	1	0
28	9	A	73.38	68.67	69.11	65.78	76	69.63	19	1	0	0	0
29	18	A	49.05	50.95	46.62	48.11	49.71	75	18	1	0	0	0
30	10	A	58.57	62.95	53.56	50.94	51.33	61.21	19	1	0	0	0
31	8	B	77.48	77.05	72.72	69.61	84.17	77	19	0	1	0	0
32	9	A	68.16	66.26	55.5	59.11	63	53.74	22	1	0	0	0
33	14	C	64.68	52.89	46.17	50.3	51.57	62.53	.	0	0	1	0
34	17	B	65.52	60.52	51.39	58.33	62.33	58.63	21	0	1	0	1
35	17	A	58.47	48	48.1	47.33	52.35	53.36	21	1	0	0	0
36	9	A	77.33	75.05	65.67	69.17	73.17	67.58	19	1	0	0	0
37	9	A	73.76	73.68	74.78	64.22	73.83	63.95	20	1	0	0	0
38	14	C	63.76	61.43	51.94	49.06	62.17	52.95	.	0	0	1	0
39	17	B	64.05	57.48	52.61	48.28	48.17	54.84	20	0	1	0	1
40	18	B	57.62	57.14	48.25	51.72	57.14	55.79	22	0	1	0	1
41	17	A	43.81	49.33	35.19	42.9	30	56	21	1	0	0	1
42	8	A	74.67	60.24	57	56.83	70.83	67.84	19	1	0	0	0
43	9	B	68.71	69.05	64.44	61.39	63.33	64	19	0	1	0	0

44	16	C	67.05	68.1	57.61	60.06	57.17	56.32	19	0	0	1	1
45	12	A	63.86	52.86	48.78	49.14	55.43	54.42	19	1	0	0	0
46	18	A	69.76	62.1	52.17	56.33	53.33	58.89	19	1	0	0	1
47	15	C	54.58	56.9	48.72	49.67	55	53.32	.	0	0	1	0
48	8	A	72.67	69.86	66.61	57.39	71.33	63.32	19	1	0	0	0
49	9	A	69.81	65.48	54.89	53.72	64.33	64.74	19	1	0	0	0
50	11	A	69.14	64.14	57	53.17	65.67	68.05	18	1	0	0	0
51	9	A	68.81	62.52	61.72	60.83	73.5	64.74	19	1	0	0	0
52	16	A	57.33	51.57	47.65	55.95	57.45	58.32	19	1	0	0	1
53	19	A	57.32	49.29	47.25	48.9	50.43	50.37	19	1	0	0	1
54	18	A	55.95	50.38	49.87	50	52.5	44.21	20	1	0	0	1
55	9	A	60.37	57.14	55.06	57.39	62	64.47	21	1	0	0	0
56	15	A	57.63	47.1	47.67	43.55	49.83	54.76	20	1	0	0	0
57	18	A	65.48	58.57	52.94	56.06	57.17	58.84	19	1	0	0	0
58	9	A	72.57	72.76	69.33	61.56	71.5	61.79	18	1	0	0	0
59	9	A	63.1	60.95	53.95	58.33	58.33	56.74	20	1	0	0	0
60	14	A	68.63	62.05	56.5	52.39	62.33	61.79	.	1	0	0	0
61	8	A	71.76	69.24	58.06	63.5	67.5	67.79	19	1	0	0	0
62	9	A	61.71	56.71	53.52	49.83	58.67	67.63	19	1	0	0	0

Appendix B: The mean of the various types of schools

.

Obs	tme	meant1	meant2	meant3
1	1	64.6327	67.0192	65.3738
2	2	59.6815	63.6569	60.8738
3	3	55.2154	57.9892	53.8313
4	4	55.8929	57.3746	54.6638
5	5	61.0220	62.8308	62.5513
6	6	62.2127	61.4046	61.2650

Appendix C: The mean of fee-paying and regular students.

	tme	meanfp	meannfp
1	1	60.5245	67.8160
2	2	55.8595	63.3140
3	3	50.5132	58.4263
4	4	52.4968	57.9965
5	5	54.6123	65.4410
6	6	57.2609	64.4840

Appendix D: The Covariance Matrix

Covariance Matrix,						
	swa1	**swa2**	**swa3**	**swa4**	**swa5**	**swa6**
swa1	55.1733	49.9838	51.9892	43.2976	65.5777	35.0389
swa2	49.9838002	67.3066102	62.8373408	53.7342500	71.2250967	42.4129
swa3	51.9892913	62.8373408	75.2718072	58.2155426	80.3890714	46.3208
swa4	43.2976689	53.7342500	58.2155426	57.0487402	68.9330205	42.8187
swa5	65.5777407	71.2250967	80.3890714	68.9330205	110.5127339	60.9502
swa6	35.0389521	42.4129323	46.3208114	42.8187164	60.9502572	67.7940187