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OPTIMAL RESOURCE ALLOCATION OF A COCOA PROCESSING COMPANY: APPLICATION OF INTEGER PROGRAMMING WITH IMPLICIT ENUMERATION

BY

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DECLARATION

I hereby declare that this submission is my own work towards the Master of Science degree and that, to the best of my knowledge it contains no material previously published by another person nor material which has been accepted for award of any other degree of the university except where due acknowledgement has been made in the text.

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ABSTRACT

Most cocoa producing countries are moving from the traditional export of raw cocoa beans to cocoa processing countries due to the hike in profit at the finished end of the production chain. For example; cocoa processing Company Ghana Limited has been increasing in production tonnage consistently from two thousand (2,000) tonnes in 1983 to sixty two thousand five hundred (62,500) tonnes, currently. This thesis describes the resource allocation problem of Cocoa Processing Company Ltd. as an integer programming problem. We applied integer programming algorithm to solve the company's resource allocation problem. Our research focused on the use of the integer programming problem for resource allocation given limited available funds for Cocoa Processing Company Ltd. in Ghana.

The objective of this study is to mathematically model resource allocation problem of the Ghana cocoa processing company in the processing of the cocoa beans as a means of adding value to Ghana's cocoa to earn higher foreign exchange. In our methodology, we used the implicit enumeration algorithms in solving our problem. First, the algorithm was presented. A real life computational study was performed to evaluate the algorithms. We used the Quantitative Method (QM 32) to analyze our data. Cocoa Processing Company Ltd. resource allocation problem for the processing of the cocoa beans into various consumable products was modelled as integer programming problem. The Quantitative Method (QM 32) software was used to analyze the data collected from the Company Processing Company Ltd. The optimal value was GH¢770,000.00. It was observed that the solution that gave this achievable value was the utilization of the available resources to produced 3,000kg Royale, 7,000kg Altime, and 1,000kg Vitaco

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CHAPTER ONE

1.0 INTRODUCTION

Established in 1965, the Cocoa Processing Company Limited (CPC) based in Tema, near Accra in Ghana, was incorporated in November 1981 as a Limited Liability Company. CPC, whose main objective is to process cocoa beans by adding value to them, comprises two factories, namely the Cocoa Factory and the Confectionery Factory. The former processes raw cocoa beans in to semi-finished products - cocoa liquor, butter, natural/alkalized cake or powder whilst the Confectionery Factory manufactures the Golden Tree chocolate bars, couverture, pebbles(chocolate coated peanut), VITACO and ALLTIME drinking chocolate Powder, Chocó Delight(Chocolate spread), Chocó Bake and Royale natural cocoa powder. The CPC factories process only the choicest premium Ghana cocoa beans without any blending, probably the only factory in the world, which can make such a claim. Through intensive research and product development, CPC turns out products, which meet international quality standards and also consumer satisfaction (Shaski and Vigneri, 2003).

In the year 2002 CPC's position as one of the world's best chocolate producers was re-confirmed at the Monde Selection Competition held in Paris, France. At this Competition all the seven brands of chocolate and ALLTIME Drinking chocolate powder presented won gold medals on account of their distinctive quality. As a good corporate citizen, Cocoa Processing Company supports educational programmes in Ghana. The Company has established a GH₵500million-Endowment Fund to assist in equipping the distressed laboratories of the Science Faculties of the universities in Ghana. The Company is currently engaged in an expansion programme, which is aimed at increasing its cocoa throughout capacity from the original twenty five thousand (25,000) metric tonnes per annum to sixty five thousand (65,000) metric tonnes per annum. The expansion programme is in two phases. Phase one, which involved the construction of a new state-of-the-art factory with a capacity to process thirty thousand (30,000) metric tonnes of cocoa beans into cocoa liquor. Phase Two of the programme entails the rehabilitation of the existing factory into a modern factory with an increased capacity capable of processing thirty five thousand (35,000) metric tonnes of cocoa beans into cocoa butter cake and powder.

The Ghana Cocoa Board has overtaken the Government of Ghana as the largest shareholder of Cocoa Processing Company Limited. This follows a conversion of an outstanding debt of US$14,087,120.50 owed to Ghana Cocoa Board into equity. As a result of the conversion, COCOBOD's total holding in CPC has increased from 21.74% to 57.73%. In March 2011, shareholders of the confectionery maker at an Annual General Meeting ratified the decision to convert the US$14,087,120.50 owed to Ghana Cocoa Board. Subsequent to the approval, the corporate action was executed at an agreed exchange rate of US$1.00 to GH₵1.65 which represents GH₵23, 243,748.83 for 937,247,936 ordinary shares. A price of GH₵0.0248 per share was used in the transaction representing a 26-week average of CPC share price between June 6, 2011 and December 5, 2011. By this action, COCOBOD has exceeded the 30% mandatory takeover threshold under the Ghana Stock Exchange's Listing Rules. However, there are indications that COCOBOD has no such plan to takeover CPC since that will go contrary to the principle behind the divestiture of CPC in 2002 from COCOBOD to the public via the Government of Ghana. Government of Ghana's stake in CPC was reduced from 48.38% to 26.13% while SSNIT maintains its spot as the third largest shareholder though its holding was diluted from 18.78% to 10.15%. The stake of the investing public has also been diluted from

11.10% to 5.99% as a result of the conversion. In a related development, CPC has listed an additional 937,247,936 ordinary shares on the GSE, following the conversion of the US$14,087,120.50 owed to Ghana Cocoa Board. In relation to this, the total issued shares of CPC have been increased from 1,100,826,240 to 2,038,074,176.

In this chapter of the thesis, an overview of Ghana cocoa would be given; a brief description of the problem statement of the thesis is also presented together with the objectives, the methodology, the justification and the organization of the thesis.

1.1.0 COCOA IN GHANA

No other country comes to mind more than Ghana when one speaks of cocoa. Likewise, one cannot think of Ghana without thinking of its cocoa sector, which offers livelihoods for over seven hundred (700,000) farmers in the southern tropical belt of the country. From long time immemorial, Ghana's main exports, cocoa has been central to the country's debates on development, reforms, and poverty alleviation strategies since independence in 1957. The cocoa sector in Ghana has not been an unmitigated success, however.

After emerging as one of the world's leading producers of cocoa, Ghana experienced a major decline in production in the 1960s and 1970s, and the sector nearly collapsed in the early 1980s. Production steadily recovered in the mid-1980s after the introduction of economy wide reforms, and the 1990s marked the beginning of a revival, with production nearly doubling between 2001 and 2003. These ups and downs offer interesting lessons.

Various administrations in Ghana, including the colonial one, have used cocoa as a source of public revenue, and in so doing the Ghanaian experience offers a recurrent example of a policy practice followed by many other African countries: taxing the country's major export sector to finance public expenditure (Herbst, 1993). Revenue extraction by the state has had varying effects on production depending on global prices, marketing costs, explicit taxes on the sector, and macroeconomic conditions such as inflation and overvaluation of exchange rates and inelasticity of cocoa supplies. Regardless of the level of extraction, the need for sound macroeconomic management, of inflation and exchange rates in particular, becomes evident for continuing to offer incentives for production. The other is the need for Ghana's cocoa pricing policy to arrive at a marketing arrangement that does not kill the goose that lays the golden eggs. Ghana appears to have achieved such as arrangement without fully liberalizing the sector as other producers in West Africa have.

1.1.1 Achievements in the cocoa sector in Ghana

Since the introduction of cocoa in Ghana in the late 19th century, the crop has undergone a series of major expansions and contractions. Cycles are intrinsic to cocoa production because cocoa is influenced by environmental factors such as availability of forest land; ecological factors such as deforestation, outbreaks of disease, and geographic shifts in production; and economic and social factors such as migration (Ruf and Siswoputranto, 1995). Four distinct phases can be identified in regard to cocoa production in Ghana: introduction and exponential growth (1888–1937); stagnation followed by a brief but rapid growth following the country's independence (1938–64); near collapse (1965–82); and recovery and expansion, starting with the introduction of the Economic Recovery Program (ERP) (1983 to present).

Cocoa was introduced in the southern region of the Gold Coast in the mid-19th century by commercial farmers from the Eastern region districts of Akuapem and Krobo, who had moved west toward the adjacent district of Akyem to purchase mostly unoccupied forest land from the local chiefs for cocoa cultivation (Hill, 1963).

As Beckam (1976) noted, the Convention People's Party (CPP), founded by Kwame Nkrumah, benefited from extremely favorable postwar market conditions and accumulated cocoa income on a massive scale: following the sharp increase in market prices in the 1950s, farmers were paid two to three times more than they received before the war, and between 1947 and 1965 the government collected almost one-third of the total value of cocoa export as export duties. In 1950/51 the government increased export duties and began to take a much larger share of cocoa revenue by means of a graduated ad valorem tax that increased with the increase of the average selling price per ton of cocoa. To extend its influence to the rural sector, in 1953 the Nkrumah regime also created the United Ghana Farmers' Council (UGFCC), which was mainly concentrated in the cocoa growing regions despite its remit to cover the interest of farmers all over the country. The UGFCC was made the monopoly buyer of cocoa to create a platform for organizing the farmers behind the government and its administration.

Following the second elections in 1954, the cocoa export tax was further increased while the producer price remained at the same level for four years. This generated unrest and political agitation among cocoa farmers, ultimately forcing the government to increase the producer prices and to stabilize them during 1956–57 despite declining world cocoa prices. As a result, the share of government revenue in cocoa sales dropped from 60 percent to 13 percent between 1954/55 and 1956/57. After its third political victory of

1957, the government increased its share of cocoa revenues by reducing producer prices to the 1954 levels. It also obtained a "voluntary contribution," announced by the UGFCC on behalf of cocoa farmers, to share the burden of the Second Development Plan at a time when the government was also receiving soft loans from the CMB. These events made it obvious that by then the CMB had been transformed into an instrument of public finance. The capturing of windfall profits from high cocoa prices had important fiscal implications. Government expenditures grew dramatically over the 1950s: in real terms total consolidated public expenditures increased almost sixfold during this period. The share of government expenditure in GDP grew from 7 percent to 18 percent over the decade, and the share of extraordinary and development expenditure grew from 27 percent to 36 percent. In 1961, a cooperative society was given the monopoly right to purchase cocoa. From 1957 to 1964 exports grew steadily, and production reached an unprecedented level of four hundred and thirty thousand (430,000) tons despite the significant decline in world prices between 1960 and 1962.

In the early 1960s, when world prices plummeted, farmers were required to save 10 percent of their earnings in National Development Bonds, redeemable after 10 years. In

1963 this scheme was replaced by a farmers' income tax charged at a flat rate equal to previous saving deductions.

The government started to rely heavily on the CMB's reserves, and the producer price was reduced from 224 to 187 new cedi per ton between 1961 and 1964.With foreign exchange reserves declining and the budget deficit rising sharply, the government introduced a number of strong restrictive measures, an increase in taxes, foreign exchange controls, and comprehensive import licensing. The austerity of these measures lost Nkrumah much of his political consent, especially from cocoa farmers who had been aggravated by declining producer prices and by the conversion of the compulsory saving scheme into an explicit export tax. In the second half of 1964 the world cocoa price collapsed with a bumper crop in West Africa—Ghana alone reaching an unprecedented production record of five hundred and thirty eight thousand (538,000) tons. After the purchasing and marketing costs of the CMB and UGFCC were covered, virtually nothing was left for the government, and the CMB's liquidity resources were nearly exhausted.

To meet its expenses, the government started printing money, which ignited a 35 percent rise in inflation between October 1964 and July 1965. In the face of such pressure, cocoa producer prices were reduced to their lowest levels in years. The introduction of such highly restrictive measures represented a turning point in the fortunes of the Nkrumah government, which was overthrown in February 1966 and replaced by the National Liberation Council (NLC). The collapse of world cocoa prices in 1965 triggered another downturn (Stryker, 1990).

Real producer prices dropped consistently through the 1960s because of inflation fueled by the government's printing of money to compensate for loss of revenue from cocoa and the introduction of an exchange rate policy that led to the heavy overvaluation of the *cedi*, the local currency. By 1983, market exchange rates were nearly 44 times the official rate. Between 1970s and early 1980s, it is estimated that as much as 20 percent of Ghana's cocoa harvest was smuggled into Côte d'Ivoire (Bulír, 2002).

Meanwhile, an aging tree stock and the continued spread of disease made investment in cocoa unattractive. Farmers in old cocoa production areas, who found that sales prices barely covered their costs, increasingly turned from cocoa to food production

(Amanor 2005). Ghana's cocoa production dipped to a low of one hundred and fifty nine thousand (159,000) tons in 1982/83, a mere 17 percent of the total world volume, down from the 36 percent in 1964/65. The National Liberation Council dissolved the UGFCC and established the Producing Buying Company as a subsidiary of the CMB. Producer prices were raised and farmers were paid a bonus for top grade cocoa beans to upgrade the quality of cocoa being exported. Shortly before the Busia government came to power the cedi was devalued by 43 percent and cocoa prices were raised by 30 percent. Cocoa production stagnated in the face of unchanged real producer prices that remained at their 1950s levels. The Busia administration took advantage of windfall profits from high cocoa prices in 1970 to enable a rapid expansion of public expenditure. In 1971 the Busia regime was replaced by the Acheampong led National Redemption Council. Because of high world cocoa prices, this administration was initially able to offer higher prices to farmers without cutting public revenues, creating positive incentives to production. But a progressively worsening balance of payments situation fueled inflation and undermined subsequent increases in real wages, producer prices, and other real incentives. With the fall in world cocoa prices in the mid 70's, the general macroeconomic picture began to worsen: the government budget deficit rose to 127 percent of total government revenue and inflation accelerated to 116 percent. The strong overvaluation of the cedi implied that little was left of export revenues to divide between the government and the farmers. Cocoa revenue went from 46 percent in 1974 to 23 percent in 1979 and into negative figures between 1980 and 1981 because of the exchange rate misalignment. The rising costs of the CMB further reduced government revenues. In July 1978 the government underwent another regime change, and the cedi was devalued again, an austerity budget was introduced, and interest rates and cocoa producer prices were raised. Cocoa production sunk to its lowest level ever in 1980–81; the world price at the official exchange rate was lower than the producer price plus marketing costs. The domestic conditions that led to the downturn in Ghana's cocoa sector took place against an international backdrop of increasing supply of cocoa from new producers such as Indonesia and Malaysia and expanded production in Côte d'Ivoire and Brazil. By the early 1970s Ghana had also lost much of its cheap labor supply from Burkina Faso and Côte d'Ivoire, as migrant farmers, reluctant to work in the old cocoa-producing areas that had become less productive, were attracted to the neighbouring Ivorian regions, where policies granted migrants access to land at favorable terms. The recovery and second expansion phase (1983–2008).The turnaround in Ghana's cocoa sector began with the implementation of the ERP in 1983, which included a special program to revive the sector (the Cocoa Rehabilitation Project). Policy changes included increasing the farm gate prices paid to Ghanaian farmers relative to those paid in neighbouring countries, thus minimizing the incentive to smuggle, and devaluing the cedi, thus reducing the level of implicit taxation of farmers. As part of the Cocoa Rehabilitation Project, farmers were also compensated for removing trees infected with swollen shoot virus and planting new ones. This effort led to substantial rehabilitation, with a large number of farms planting higher-yielding cocoa tree varieties developed by the Cocoa Research Institute of Ghana. Production rebounded to four hundred thousand (400,000) tons by 1995/96 and productivity increased from 210 to 404 kilograms per hectare. Another important reform took place in 1992, when Cocobod (as CMB was renamed in 1984) shifted responsibility for domestic cocoa procurement to six privately licensed companies (commonly known as licensed buying companies or LBCs) and reduced its staff by 90 percent between 1992 and 1995. Growth in cocoa production became more pronounced starting in 2001, possibly driven by a combination of record-high world prices, increased share being passed onto farmers, and a set of interventions rolled out by the Cocobod to improve farming practices: mass spraying programs and high-tech subsidy packages to promote the adoption of higher and more frequent applications of fertilizer (Vigneri and Santos, 2008). Some of the growth during this period may also have been due to the influx of cocoa smuggled from Côte d'Ivoire, estimated between one hundred and twenty thousand (120,000) and one hundred and fifty thousand (150,000) tons in 2003/4 (Brooks et al.,2007).

It has been over 30 years since Ghana was the world's reigning king of cocoa, the throne long since usurped by West Africa neighbour Côte d'Ivoire. But because of a series of policies by the

Ghana Cocoa Board, production exploded in 2011, reaching a record over one million metric tons. That number translated into US $1.7 billion in government revenues, up nearly 30 per cent from a record of US $1.2 billion in fiscal 2004 (Paul Carlucci, 2012).

1.1.2 Technical change in the cocoa sector

Since 2001 a significant share of Ghana's agricultural productivity gains have been generated by export crops, with cocoa accounting for 10 percent of total crop and livestock production values (World Bank, 2007) and contributing to 28 percent of agricultural growth in 2006, up from 19 percent in 2001. At the same time, economic growth has been solid, averaging more than 5 percent since 2001 and reaching 6 percent in 2005–06. Coupled with the effects of greater access to education, health services, and land ownership (World Bank, 2008), this rate of growth has contributed to the near halving of the national poverty rate since the beginning of the 1990s, from 51.7 percent in 1991/92 to 28.5 percent in 2005/06 (Breisinger et al., 2008).

1.1.3 Cocoa's contribution to economic growth and poverty reduction in Ghana

In the Southern Forest Belt, where cocoa is produced, aggregate figures suggest that through the 1990s, cocoa-farming households, along with those engaged in mining or timber (the other predominantly export-oriented activities) and other commercial activities, experienced improvements in their living conditions compared with food crop farmers (McKay and Coulombe, 2003). Poverty reduction among cocoa farmers is clear. Household surveys indicate that poverty among cocoa-producing households dropped to

23.9 percent in 2005, down from 60.1 percent at the beginning of the 1990s (World Bank, 2007).

1.1.4 Reputation for high-quality cocoa for Ghanaian society and economy

Cocoa, like many other commodities, is often differentiated by country of origin, and this in turn is associated with a reputation based on average quality. The reputation, a national public good, enables the country to earn a premium in the global market for the crop it is producing. Generally, Ghana receives a price premium for its cocoa in world markets because of the slightly higher-than-average fat content; low levels of debris, which results in higher cocoa butter yields than beans containing high levels of debris; and low levels of bean defects, which generate a cocoa liquor flavor preferred by some end users. In addition to these attributes, the reputation of the Cocoa Marketing Company (the government division in charge of all exports) in ensuring the consistency and reliability of cocoa-related shipments and documents has played a central role in establishing the country's reputation for high-quality beans (Agrisystems Ltd., 1997). Using trade NYSE

Liffe cocoa market information, Gilbert (2009) suggests that Ghanaian cocoa draws a premium of 3 to 5 percent relative to Côte d'Ivoire, currently the world's largest producer of cocoa.

Characteristics that determine the quality of cocoa include content and quality of fat, consistency in the size of the beans, and their moisture content. These characteristics determine the quality of cocoa butter and cocoa liquor produced from the beans, the two ingredients that control texture, aroma, color, and flavor of chocolate. The fermentation, drying, storage, and evacuation of wet beans can alter the quality of cocoa beans dramatically, particularly in the development of the flavour of cocoa liquor. The classic

“West African” cocoa flavour is obtained by fermenting beans in a heap under banana leaves for about six days with frequent manual turning and thorough drying in the sun.

Drying beans slowly on raised platforms is very important for the quality of flavour because it quickly decreases the acidity level of the beans. Quality is also maintained by quickly collecting properly fermented and dried beans from smallholder farmers and promptly shipping them to avoid the build-up of moisture, mould, and free-fatty acids that can rapidly deteriorate the quality of the bean. Partly because of its reputation for high-quality cocoa, Ghana is able to sell most of its annual production through forward contracts, which fix the price farmers are given for their cocoa for the entire crop year. The value that international firms place on Ghana’s cocoa is also reflected by the amount of investment they have made in processing facilities in the country. Ghana’s export earnings from processed cocoa products more than tripled between 1991 and 2004, from $32 million to $105 million. However, because of the limited conditions under which semi-processed cocoa can be transported effectively (Fold, 2002), it is not clear whether local value-adding efforts will be sufficiently profitable for international companies to expand their operations in Ghana. Thus far, informal discussions with the private sector participants indicate that the net benefits from processing locally may not be significant, particularly because the government allows only a limited quantity of low-quality beans to be used for local processing, which has resulted in considerable underutilization of existing capacity in the country.

1.1.5 SUSTAINABILITY OF THE COCOA SECTOR

Ghana’s cocoa sector faces a number of challenges. For one, productivity levels are lower than they are in other countries. Ghana also faces the possibility that its quality advantage may disappear in the coming years. In addition, Ghana must determine how to keep its cocoa sector competitive as cocoa-producing households change. Finally, the environmental impact of current farming practices may soon constrain cocoa production expansion. On the other hand, however, Ghana has been quite successful in taking advantage of niche cocoa markets.

1.2 PROBLEM STATEMENT

The specific form of problem that this thesis seeks to solve is to mathematically model resource allocation problem of the Ghana cocoa processing company in the processing of the choicest premium cocoa beans as a means of adding value to Ghana's cocoa to earn higher foreign exchange.

1.3 OBJECTIVES

The objective of this study is to solve the resource allocation problem of the Ghana cocoa processing company in the processing of the cocoa beans as a means of adding value to Ghana's cocoa to earn higher foreign exchange.

1.4 METHODOLOGY

In our methodology, we shall propose the implicit enumeration algorithms in solving our problem. First, the algorithm would be presented. A real life computational study would be performed to evaluate the algorithms. We shall employ Quantitative Method (QM 32) Analyzer to analyze our data

1.5 JUSTIFICATION

Ghana Cocoa even though considered as a high-quality cocoa in the world market and the second largest exporter in the world market, its contribution to the GDP is not encouraging because of its exportation being raw in nature. Between 2009, a survey conducted by Gilbert established cocoa unit values and terminal market differentials seeing a minimal increase from 4.8 % to 4.9 %. These trends are indicative of a high and persistent dependency on raw export of cocoa beans. Adding quality value to the cocoa before export could help to address the above problem drastically, hence the reason for solving the proposed problem.

1.6 LIMITATIONS OF THE STUDY

We used only one year's data in our analysis. Further study could be done by considering data of different years.

Another limitation is the unavailability of other cocoa processing companies in Ghana. For this study, data was collected from only one company, Cocoa Processing Company (Gh) Ltd.

1.7 ORGANIZATION OF THE THESIS

In chapter one, we presented a background study of cocoa situation in Ghana.

In chapter two, related work in implicit enumeration problems in resource allocation would be discussed. In chapter three, the implicit enumeration algorithms for solving our proposed problem would be introduced and explained. Chapter four presents data collection and analysis of data from CPC. Chapter five will conclude this thesis with the conclusion and recommendations of the study.

CHAPTER TWO

LITERATURE REVIEW

Dynamic command and control and battle management functions require fast and effective decision aids to provide optimal allocation of resources (object/sensor pairing, weapon/target assignment) for effective engagement and real-time battle damage assessment. The basic Weapon Target Assignment (WTA) problem considers the assignment of a set of platforms/weapons to a set of targets such that the overall expected effect is maximized.

Rosenberger et al., (2005) studied the basic WTA problem by allowing for multiple target assignments per platform, subject to the number of weapons available and their effectiveness. The authors formulated the problem as a linear integer programming problem and investigate two solution methods. The first method is a greedy approach based on the sequential application of the auction algorithm that was generalized for assigning n assets/resources to m targets. The second method is built on a implicit enumeration framework that enumerates feasible tours of assets/resources – a process that can become computationally intensive with increasing number of sources and targets but will find an optimal solution. The authors provided results of Monte Carlo experiments and provide comparative evaluation of the two solution methods. Finally, the authors extended the implicit enumeration technique to assigning multiple platforms per target and thereby demonstrate its utility for collaborative asset planning. While this study focuses on weapon target pairing for illustration purposes, the methods and results herein are readily applicable to sensor tasking and similar resource allocation problems.

The problem of optimal policy formulation for teams of resource-limited agents in stochastic environments is composed of two strongly-coupled sub-problems: a resource allocation problem and a policy optimization problem.

Dolgov and Durfee (2004) presented a model that combines the two problems into a single constrained optimization problem that yields optimal resource allocations and policies that are optimal under these allocations. The authors modelled the system as a multi-agent implicit enumeration process problem (MIEP), with social welfare of the group as the optimization criterion. The straightforward approach of modeling both the resource allocation and the actual operation of the agents as a multiagent Markov decision process (MDP), on the joint state and action spaces of all agents is not feasible, because of the exponential increase in the size of the state space. As an alternative, the authors described a technique that exploits problem structure by recognizing that agents are only loosely-coupled via the shared resource constraints. This allows us to formulate a constrained policy optimization problem that yields optimal policies among the class of realizable ones given the shared resource limitations.

Although our complexity analysis shows the constrained optimization problem to be NP-complete, our results demonstrate that, by exploiting problem structure and via a reduction to a mixed integer program, the authors were able to solve problems orders of magnitude larger than what is possible using a traditional multi-agent MDP formulation.

Sherali et al., (1991) developed both a planning and an operational computer-based tool through a particular location-allocation model. This model selects a set of candidate shelters from among a given set of admissible alternatives in a manner feasible to available resources, and prescribes an evacuation plan which minimizes the total congestion-related evacuation time. An extraneous flow is also superimposed on the network in order to represent the traffic of evacuees not using the designated shelters as destinations. The model formulated is a linear mixed-integer programming problem, for which the authors develop a heuristic and (two versions of) an exact implicit enumeration algorithm based on the generalized Benders' decomposition method. Computational experience is provided against a set of realistic test problems formulated on the Virginia Beach network.

Chang and Kyu (1991) presented an analytic implicit enumeration models for optimizing bus services with time dependence and elasticity in their demand characteristics. The major results consist of closed form solutions for the route spacings, headways, fares and costs for optimized feeder bus services under various demand conditions. A comparison of the optimization results for the four cases was also presented. When demand and bus operating characteristics are allowed to vary over time, the optimal functions are quite similar to those for steady demand and supply conditions. The optimality of a constant ratio between the headway and route spacing, which is found at all demand densities if demand is steady, is also maintained with a multi-period adjustment factor in cyclical demand cases, either exactly or with a relatively negligible approximation. These models may be used to analyze and optimize fairly complex feeder or radial bus systems whose demand and supply characteristics may vary arbitrarily over time.

D'Ouville and Mcdonald (1988) formulated an implicit enumeration algorithm model of optimal road capacity in the presence of unpriced congestion. The model includes the assumption that the peak traffic volume may fall below the capacity of the highway because of a large level of traffic density. The analysis in the model showed that the optimal equilibrium outcome falls in this range if the cost of road capacity is relatively high and/or if travel demand is price elastic. A numerical example was included to illustrate the use of the model.

Kim and Ralph (2005) presented an optimal algorithm for a resource allocation model, which was implemented into a framework for the development of an integration model. Unlike existing heuristic-based resource allocation models, the model does not depend solely on a set of heuristic rules, but adopts the concept of future float to set the order of priority when activities compete for resources. The model determines the shortest duration by allocating available resources to a set of activities simultaneously. Implicit enumeration methods were adopted to search optimal solutions. The results obtained from a case example indicate that the model is capable of producing optimal scheduling alternatives, compared to a single solution that is produced by either the total float model or the least impact model.

Scaparra and Cappanera (2008) developed a game theoretic approach for allocating protection resources among the components of a network so as to maximize its robustness to external disruptions. Specifically, the authors considered shortest-path networks where disruptions may result in traffic flow delays through the affected components or in the complete loss of some elements. The authors developed a multi-level program which identifies the set of components to harden so as to minimize the length of the shortest path between a supply node and a demand node after a worst-case disruption of some unprotected components. The authors proposed an implicit enumeration algorithm to solve the multi-level problem to optimality and streamline the approach by solving the lower level interdiction problem heuristically at each node of the enumeration tree. The authors also proposed some variable fixing rules which reduce the dimension of the lower level problems. A thorough computational investigation demonstrates that the proposed solution method is able to identify optimal protection strategies for networks of significant size. The authors also studied the sensitivity of the proposed approach to variations of the problem parameters, such as the level of offensive and defensive resources, and the distribution of the arc lengths and delays.

Tuma et al., (1998) developed an implicit enumeration method for dynamic allocation of transmission resources to a plurality of communications between a base station and a plurality of mobile terminals includes generating a pseudo-random sequence and performing an allocation. Each resource includes a plurality of possible values, and an allocation controller associated with the base station, referred to as the fast allocation controller, is able to allocate to the communications only certain combinations of possible values, referred to as available resources. The fast allocation controller generates a pseudo-random sequence and performs the allocation by selecting at least one available resource for each communication according to a value of the pseudo-random sequence.

Kondo (1998) presented a model for time-slot allocation for a communication in a time division multiple access (TDMA) communication system which allocates one or more time-slots in a TDMA frame and proposed an implicit enumeration method for solving the model. When new call request occurs, availability of idle time-slot for the communication is examined. If there is not enough idle time-slot to be allocated for the new call, it is examined whether there is high transmission speed communication using a plurality of time-slots. When any of the high transmission speed communications exist, time-slots are shared by newly requested communication and the high transmission speed communication using a maximum number of time-slots among the high transmission speed communications. Transmission speed adjustment is performed for such high transmission speed communications sharing a part of time-slot. The probability of occurrence of call loss is decreased by this method.

In a communications system, communications resources are allocated in a dynamic, "as needed" fashion. No explicit signaling is needed to exchange information pertaining specifically to an allocated communications resource. Jamal et al., (2004) presented a model in which resources are implicitly allocated by using one or more parameters known to both the radio access network and mobile station that are more or less unique to the mobile station. Such parameters are used to generate or address a communications resource for use by the mobile station. The parameters might, for example, be information readily acquired or communicated as a result of a synchronization procedure, a mobile station registration procedure, a mobile station access procedure, a paging procedure, etc. Example parameters may include a system frame number, system identification, radio access network identification, base station identification, cell identification, a mobile station-associated signature, an access reference number corresponding to the mobile station, a time instant when an acknowledgment message is received, etc.

Radhakrishnan (2007) presented a novel implicit branch and bound method for mixed integer linear programming problems. The integer variables are driven to their non-basic variables, which can be fixed, at either lower or upper bound, remaining non-basic variables as such. The concept of twin models is exploited to get more number of nodes to get the optimal solution. This method eliminates non-optimal solutions and thus reduces the number of iterations. A different and simple method of adding a new constraint is formulated. Many typical test problems are solved on a micro computer to highlight the efficacy of this method. The accuracy obtained was good. The fact that it required many branching to solve even a small model is a strong evidence of its invulnerability to round off error. Its memory requirement is also modest. It is highly effective in solving a very large class of integer programming models, mixed integer, pure integer and the special class involving only binary variables. This does not add any new constraints to the original constraint set, since it merely manipulates upper and lower bounds on the integer variables. Hence, it can be said that it is immune to round-off errors. Experience indicates that the use of surrogate constraint is effective in improving the computation time. The solution time varies almost exponentially with the number of variables. So problems up to 100 variables can be solved in a reasonable amount of computation time. The most important advantages are its immunity to round off errors and its modest memory requirement in comparison with other methods like branch and bound and cutting plane method.

Over the last two decades, our military forces have been working to incorporate the latest computer technology into the combat planning process. The earliest efforts use word processors, spreadsheets, and databases to organize planning data and to display high level summaries for commanders. Later efforts perform feasibility checks as missions are planned to insure that the necessary resources are available and that the assets requested are capable of meeting the assigned scheduling requirements. Some of the most recent computer planning tools has included the capability to automatically plan individual missions or groups of missions. These automated efforts have been heuristic in nature due to the time limitations inherent to real-time combat planning.

Hove and John (1998) presented an implicit enumeration method as optimal alternatives to the limited heuristics available in the current combat planning tools. This research formulates and solves a new class of project scheduling problems with applications to both military and civilian planning. It is shown that the solution space for this class of problems may be reduced in order to improve the effectiveness of both optimal and heuristic solution methodologies. In addition, a general method for extending implicit enumeration algorithms to obtain k-best solution sets is developed. The reduced solution space and the general k-best solutions methodology are exploited to develop several efficient solution approaches for this new class of problems; an implicit enumeration algorithm, a decomposition approach, an evolutionary algorithm, and a hybrid decomposition approach. The applicability and flexibility of the methodology are demonstrated with a case study that focuses on the force level planning of combat missions for an air campaign. While the focus of the case study is combat planning, the concepts illustrated are applicable to the general field of program and project management.

Chen and Askin (2009) formulated and analyzed the joint problem of project selection and task scheduling. The authors studied the situation where a manager has many alternative projects to pursue such as developing new product platforms or technologies, incremental product upgrades, or continuing education of human resources. Project return is assumed to be a known function of project completion time. Resources are limited and renewable. The objective is to maximize present worth of profit. A general mathematical formulation that can address several versions of the problem is presented. An implicit enumeration procedure is then developed and tested to provide good solutions based on project ordering and a prioritization rule for resource allocation. The algorithm uses an imbedded module for solving the resource-constrained project scheduling problem at each stage. The importance of integrating the impact of resource constraints into the selection of projects is demonstrated.

Resource availability constraints are a typical real-life construction scheduling problem; a problem that limits a constructor's ability to execute and deliver a project as originally planned. It is, thus, imperative that developed project schedules should have not only well-thought project logic networks (successor/predecessor information and activity durations) but also resource assignments (including cost) for each activity in the network so that the effects of resource constraints can effectively be accounted for.

Aslani et al., (2009) presented a new approach to resource constrained scheduling that allows for activity prioritization when a project is subject to limited resources. The methodology proposed is based on a utility index, hereby defined as the ratio of the number of required resources for a specific activity to the total number of required resources among competing activities. An implicit enumeration technique is adopted to maximize the utility value for each activity so that the resource allocation among competing activities, as suggested by the method, results in the minimum overall project duration.

A recurring problem in managing project activity involves the allocation of scarce resources to the individual activities comprising the project. Resource conflict resolution decisions must be made whenever the concurrent demand for resources by the competing activities of a project exceeds resource availability. When these resource conflict resolution decisions arise, project managers seek direction on which activities to schedule and which to delay in order that the resulting increase in project duration is the minimum that can be achieved with the given resource availabilities ties.

Patterson (1985) the procedures examined in his paper are all designed to provide for this type of decision support. Each procedure examined is enumerative based, methodically searching the set of possible solutions in such a way that not all possibilities need be considered individually. The methods differ in the manner in which the tree representing partial schedules is generated and is saved, and differ in the methods which are used to identify and discard inferior partial schedules.

Each procedure was found to be generally superior on a specific class of problems, and these classes are identified.

Ming and Heng (2003) presented an implicit enumeration model which addressed the fundamental matters and limitations of existing methods for critical-path method (CPM) based resource scheduling, which are identified by reviewing the prior research in resource constrained CPM scheduling and repetitive scheduling. The proposed method is called the resource-activity critical-path method (RACPM), in which (1) the dimension of resource in addition to activity and time is highlighted in project scheduling to seamlessly synchronize activity planning and resource planning; (2) the start/finish times and the floats are defined as resource-activity attributes based on the resource-technology combined precedence relationships; and (3) the ‘‘resource critical’’ issue that has long baffled the construction industry is clarified. The RACPM is applied to an example problem taken from the literature for illustrating the algorithm and comparing it with the existing method. A sample application of the proposed RACPM for planning a footbridge construction project is also given to demonstrate that practitioners can readily interpret and utilize a RACPM schedule by relating the RACPM to the classic CPM. The RACPM provides schedulers with a convenient vehicle for seamlessly integrating the technology/process perspective with the resource use perspective in construction planning. The effect on the project duration and activity floats of varied resource availability can be studied through running RACPM on different scenarios of resources. This potentially leads to an integrated scheduling and cost estimating process that will produce realistic schedules, estimates, and control budgets for construction.

Due to the resource-driven nature of construction management, the construction manager must develop a plan of action for directing and controlling resources of workers, machines, and materials in a coordinated and timely fashion in order to deliver a project within the limited funding and time available. Hence, aside from a technology and process focus (i.e., what is to be done and how), a resource-use focus (i.e., who is to do it with what) must be adequately considered in describing a construction method or operation in a project plan. Nevertheless, the most popular project planning methods - the critical-path method (CPM) and the related network diagramming techniques (PERT, Node Diagramming, and Precedence Diagramming) - fail to seamlessly synchronize activity planning and resource planning, the two integral functions in project planning. CPM assumes limitless availability of resources. This assumption is not valid in most practical situations, in which there exist definite limits on the amount of resources available and these resources are shared by a number of activities or even projects. To overcome this recognized drawback, which brings about unrealistic or impossible CPM schedules, analytical or heuristic techniques for resources allocating/levelling on CPM network plans have also been developed since the early 1960s. These techniques generally consist of two stages. First, the project is broken down into distinct activities that are logically or technologically related to one another according to the construction process/method without imposing resource constraints (e.g., the superstructure follows the substructure; the concrete pouring succeeds the formwork and reinforcement). Second, basic CPM scheduling calculations are made for early and late start and finish dates and total and free float times, based on which (1) the project is rescheduled so that a limited number of resources can be efficiently utilized while minimizing the unavoidable extension of project duration (also known as resource allocation); or (2) the start times of certain activities are adjusted within the float limits for a levelled resource profile (also known as resource levelling).

Limited-resource allocation algorithms aim to find the CPM schedule duration that is shortest as well as consistent with specified resource limits and essentially deal with the notorious ‘‘combinatorial explosion’’ problem in mathematics. Lee and Gatton (1994) presented an implicit enumeration approach to solve the problem optimally.

An alternative approach to resource-constrained CPM scheduling is the use of heuristic methods that apply priority rules based on activity characteristics, such as the ‘‘minimum total slack’’ rule, to prioritize activities that compete for limited resources. The resulting schedule satisfies the technological constraints and the resource constraints but is not optimal in terms of achieving the shortest project duration. The total floats, earliest start times, and latest start times, as calculated from the CPM analysis usually serve as part of the criteria in the heuristic priority rules.

Abeyasinghe et al., (2001) presented a new implicit enumeration approach that does not require CPM calculations but used Gantt charts combined with an intermediate tool called ancillary networks to facilitate the process of resource-loading CPM, instead of using priority rules. Their method also attempted to define a critical path in the sense of the classic CPM by identifying the path with the longest activity duration.

Network techniques are inefficient when applied to schedule repetitive projects, mainly because (1) using a large number of activities to represent repetitive activities in a CPM format makes the resulting CPM schedule difficult to visualize and analyze; and (2) the resource-leveled CPM networks do not guarantee work continuity of resources (Hegazy and Wassef, 2001). Special resource-constrained repetitive scheduling models have been developed based on the line of balance (LOB) technique, which accounts for precedence relationships, crew availability, crew work continuity constraints, etc. The resulting schedule is a time space chart with the space dimension representing a number of identical units (e.g., floor/road section) going through a series of repetitive activities (Halpin and Riggs, 1992).

Latest developments have coupled the resource-driven scheduling algorithm with the dynamic programming formulations and an automated interruption mechanism to optimize resource utilization and minimize the project cost and duration in scheduling repetitive serial activities (El-Rayes and Moselhi, 1997, 2001). By adding the space constraint to a repetitive scheduling system specially developed for multi-story building projects.

(Thabet and Beliveau, 1997) considered the limited space availability at the workface and the effect of space congestion on crew productivity during the generation of the schedule. Hegazy and Wassef (2001) integrated the CPM and LOB methodologies and used implicit enumeration methods to minimize the total construction cost in projects with repetitive non serial activities, achieving the optimum combination of construction methods, number of crews, and interruptions for each repetitive activity.

Special Ordered Sets (SOSs) of variables of types one and two are used in mathematical programming for formulating both multiple choice problems and non-convex separable problems, respectively.

Bricker (1977) presented problem formulations which may be more efficiently solved by mixed-integer programming implicit enumeration codes when special codes for handling SOSs are unavailable. A shipment planning problem is used to illustrate some of this formulation.

The general solution approach leading to the global optimum for multiple choice integer programming uses the implicit enumeration procedure designed for special ordered sets. Incorporated in such a solution procedure, the partitioning strategy based on a weighted mean method is most often adopted to calculate the pseudo penalties on the partitioned sub-constraints for branching and backtracking. Based on a reformulation and transformation technique, Lin and Bricker (1991) showed that the calculation of true penalties on these partitioned sub-constraints can be performed efficiently.

The problem of sequencing inspection operations subject to errors in order to minimize the expected sum of inspection and penalty costs is formulated. Three basically different types of sequences (complete, fixed and variable) are defined. Raz and Bricker (1993) presented methods for computing the optimal solution for each type, and a family of heuristics for the variable sequence problem. These methods are based on an implicit enumeration approach and involve recursive calls to a sequence evaluation function.

Sankaran et al., (1999) developed and test a strong fractional cutting-plane algorithm for the classical non-pre-emptive precedence- and resource-constrained project scheduling problem. While the authors basic approach is to formulate the problem as a 0-1 IP and solve it by LP-based implicit enumeration method, the authors enhanced the algorithm considerably through (a) an improved IP reformulation, (b) problem pre-processing techniques, and (c) on-the-fly tightening of the LP relaxation by generating strong and valid inequalities that are violated by the current (fractional) LP optimum. Preliminary computational results on a collection of test problems are encouraging.

Kawatra et al., (1999) studied the capacitated minimal spanning tree with unreliable links and node outage costs problem. Tree topologies appear in the design of centralized communication networks. In these topologies the number of nodes in a sub-tree rooted at the central node is limited to a predefined number due to polling, loading, and response time restrictions. The links in a communication network are prone to failure. Whenever a link in these networks fails, all the terminal nodes connected to the central node through that link are unable to communicate until the faulty link is repaired. In some networks such failures can have adverse economic effect on the network user. The economic effect on the network user due to inability of a terminal to communicate with the central node due to link failure is called node outage cost. The sum of expected yearly node outage costs for a network depends on the topology of the network. The authors suggested an implicit enumeration based approach to solve the integer programming formulation of the network topology problem. The objective of the problem is to minimize the sum of link costs and node outage costs. The authors computational results on a set of test data with up to 80 nodes show that compared to the previously developed greedy heuristic, their method gave solutions that are better by up to 6 percent. The gaps between the authors solutions and the lower bounds found as a by product of the solution procedure are in the 2-17 percent range.

The Multi-period Capacitated Minimal Spanning Tree Problem (MCMSTP) consists of scheduling the installation of links in a network so as to connect a set of terminal nodes S = [2,3...N] to a central node (node 1) with minimal present value of expenditures, where link capacities limit the number of terminal nodes. Some of the terminal nodes are active at the beginning of the planning horizon while others are activated over time. Kawatra and Bricker (2000) formulated this problem as an integer programming problem. An implicit enumeration exchange heuristic procedure for solving the problem is presented. The authors also presented a Lagrangian relaxation method to find a lower bound for the optimal objective function value. This lower bound may be used to estimate the quality of the solution given by the implicit enumeration exchange heuristic. Experimental results over a wide range of problem structures show that the implicit enumeration exchange heuristic method yields verifiably good solutions to this problem.

Minimizing a non-decreasing separable concave cost function over a polyhedral set arises in capacity planning problems where economies of scale and fixed costs are significant, as well as production planning when a learning effect results in decreasing marginal costs. This is an NP-hard combinatorial problem in which the extreme points of the polyhedral set must be enumerated, each of them a local optimum. Implicit enumeration methods have been frequently used to solve these problems. Although it has been shown that in general the bound provided by the surrogate dual is tighter than that of the Lagrangian dual, the latter has generally been preferred because of the apparent computational intractability of the surrogate dual problem. Sohn and Bricker (2003) presented an implicit enumeration algorithm that exploits the superior surrogate dual bound in a branch-and-bound algorithm without explicitly solving the dual problem. This is accomplished by determining the feasibility of a set of linear inequalities.

Marek and Boguslaw (1985) formulated a generalized algorithm for graph coloring by implicit enumeration. A number of backtracking sequential methods were discussed in terms of the generalized algorithm. Some are revealed to be partially correct and inexact. A few corrections to the invalid algorithms, which cause these algorithms to guarantee optimal solutions, are proposed. Finally, some computational results and remarks on the practical relevance of improved implicit enumeration algorithms are given.

In the early days of integer programming, problems were often solved by “implicit enumeration.” This is a branching method that uses “pre-processing” to fix variables or simplify the problem, but it typically does not use the continuous relaxations associated with branch-and-bound methods. As examples one might cite Hansen's work on boolean problems (1969, 1970), or Gar_nkel and Nemhauser's (1970) solution of a political districting problem.

For large multivariate data sets the data analyst often wants to know the best set of independent regressors to use in a multiple linear regression model. Akaike’s Information Criteria (AIC) is one information criterion calculated in SAS that is used to score a model. For a small number of independent variables *p*, an explicit enumeration of all possible 2^p models is possible. However, for large multivariate data sets where *p* is large, an explicit enumeration of all possible models becomes computationally intractable. Beal (2010) presented SAS code that implements the exact implicit enumeration algorithm authored by Bao (2005) that has been shown to always arrive at the globally optimal minimum AIC value when let run to completion. The number of models evaluated to determine the optimal model with the smallest AIC score is minimal and shown to be much more efficient than an explicit enumeration of all possible models. A large multivariate data set is simulated with a known true model to demonstrate how fast the exact implicit enumeration algorithm arrives at the true model. The number of models evaluated is compared to an explicit enumeration algorithm and the REG procedure in SAS.

Hoang et al., (2003) proposed a new implicit enumeration procedure based on monotonicity for a general class of nonlinear complementarily problems. In contrast to most existing methods for this problem, our algorithm does not require any restrictive condition when applied to polynomial complementarily problems. Several test problems taken from the literature are solved to illustrate the procedure. In addition, the authors report on experiments on ninety randomly generated non-separable polynomial test problems with polynomial degree up to 41.

Zhiwei and Xianmin (2007) studied Radio resource allocation (RRA) problems in orthogonal frequency division multiple access (OFDMA) systems. By assuming perfect channel estimation for all users, fast implicit enumeration based optimal and suboptimal algorithms were proposed to solve the RRA problems in OFDMA systems. As demonstrated by simulation results, the proposed optimal algorithm offers the same performance as that achieved by using exhaustive full-search algorithm, but the computational complexity involved is significantly reduced relative to the full-search algorithm. The proposed suboptimal algorithm offers near- optimal performance whereas the associated computational complexity is much lower than that associated with the proposed optimal algorithm.

Lumpy demand forces capacity planners to maximize the profit of individual factories as well as simultaneously take advantage of outsourcing from its supply chain and even competitors. This study examines a business model of capacity planning and resource allocation in which consists of two profit-centered factories. Kung and Chen (2009) proposed an implicit enumeration algorithm for solving a set of non-linear mixed integer programming models of the addressed problem with different economic objectives and constraints of negotiating parties. An individual factory applies a specific resource planning policy to improve its objective while borrowing resource capacity from its peer factory or lending extra capacity of resources to the other. The proposed method allows a mutually acceptable capacity plan of resources for a set of customer tasks to be allocated by two negotiating parties, each with private information regarding company objectives, cost and price. Experiment results reveal that near optimal solutions for both of isolated (a single factory) and negotiation-based (between the two factories) environments are obtained.

Resources for activities in construction project are limited in the real-life problems. So resource allocation is of great importance to construction project management to avoid the waste and shortage of resources on a construction project. Yan et al., (2002) presented an implicit enumeration algorithm model for resource allocation. Compared to the traditional crossover methods, the proposed model develops a new operator to avoid producing illegal solution. The model can effectively provide the optimum solution to resource allocation problem. An illustrative example was presented to demonstrate the performance of the proposed approach.

Billionnet et al., (1992) presented an implicit enumeration algorithm to solve one of the task allocation problems. Task assignment in a heterogeneous multiple processors system is investigated. The cost function is formulated in order to measure the inter-task communication and processing costs in an uncapacited network. A formulation of the problem in terms of the minimization of a sub-modular quadratic pseudo-Boolean function with assignment constraints is then presented. The use of a branch-and-bound algorithm using a Lagrangean relaxation of these constraints was employed. The lower bound is the value of an approximate solution to the Lagrangean dual problem. A zero-duality gap, that is, a saddle point, is characterized by checking the consistency of a pseudo-Boolean equation. A solution is found for large-scale problems (e.g., 20 processors, 50 tasks, and 200 task communications or 10 processors, 100 tasks, and 300 task communications). Excellent experimental results were obtained which are due to the weak frequency of a duality gap and the efficient characterization of the zero-gap (for practical purposes, this is achieved in linear time). Moreover, from the saddle point, it is possible to derive the optimal task assignment.

Providing efficient workload management is an important issue for a large-scale heterogeneous distributed computing environment where a set of periodic applications is executed. The considered shipboard distributed system is expected to operate in an environment where the input workload is likely to change unpredictably, possibly invalidating a resource allocation that was based on the initial workload estimate. The tasks consist of multiple strings, each made up of an ordered sequence of applications. There is a quality of service (QoS) minimum throughput constraint that must be satisfied for each application in a string, and a maximum utilization constraint that must be satisfied on each of the hardware resources in the system. The challenge, therefore, is too efficiently and robustly manages both computation and communication resources in this unpredictable environment to achieve high performance while satisfying the imposed constraints. Vladimir et al., (2008) addressed the problem of finding a robust initial allocation of resources to strings of applications that is able to absorb some level of unknown input workload increase without rescheduling. The proposed hybrid two-stage method of finding a near-optimal allocation of resources incorporates two specially designed mapping techniques: (1) the Permutation Space Genitor-Based heuristic and (2) the follow-up implicit enumeration heuristic based on an Integer Linear Programming (ILP) problem formulation. The performance of the proposed resource allocation method is evaluated under different simulation scenarios and compared to an iteratively computed upper bound.

In systems involving multiple autonomous agents, it is often necessary to decide how scarce resources should be allocated. When agents have competing interests, they may have incentive to deviate from protocols or to lie to other agents about their preferences. Due to the strategic nature of such interactions, there has been a recent surge of interest in addressing problems in competitive multi-agent systems by bringing together techniques from computer science and game-theoretic economics. In some cases, the interesting issue is the application of ideas from computer science to make existing economic mechanisms practical. In other cases, selfish agents' conflicting demands of a computer system can best be understood and/or managed through game-theoretic analysis. Kevin (2003) studied problems that fall into both cases and proposed an implicit enumeration method for solving the problems.

A distributed system consists of, possibly heterogeneous, computing nodes connected by communication network that do not share memory or clock. One of the main benefits of distributed systems is resource sharing which speeds up computation, enhances data availability and reliability. However resources must be discovered and allocated before they can be shared. Virtual caching is a new caching scheme which allows a host node to grant authority of caching pages in some fraction of its own cache to nearby nodes. However the virtual caching protocol doesn't mention how a client node obtains virtual cache from remote host. To address this problem Nagaraj et al., (2009) formulated a resource discovery and allocation problem. The authors focused their attention on how to locate resources-surplus donor nodes and to determine how much of the request for resources of deficient nodes will be satisfied, efficiently in a connected network especially within a finite hop of the resource deficient node. The authors intended to minimize the amount of unfulfilled request of deficient nodes. Virtual cache allocation can be changed any time depending upon the requirement. The authors proposed implicit enumeration method for both in terms of time and amount of communication performed. The authors also estimated the quality of distribution achieved by comparing the distribution yielded by the heuristics and by the solution of ILP formulation of the problem. The authors also proposed and compare few implicit enumeration methods for minimizing the amount of unfulfilled request for resources, of deficient nodes when nodes look for resources within finite hops. For the bounded hops we restrict ourselves to the resource distribution within one hop. By using non-anonymous arbitrary topology with sequence number of request to resolve deadlocks and distributing resources over the original arbitrary network. Sequence number of the request is the unique ID of sender node. We proposed a implicit enumeration method to distribute resources over anonymous arbitrary topology by passing a token. The token is privilege to distribute the resources. Each resource - surplus node is giving its extra nodes in such a way so that it itself doesn't becomes resource-deficient in the process. Load is not infinitely divisible. We are focusing our attention only to determine how much of the request of each resource - deficient node will be satisfied.

A new trust-based model is developed for optimizing resource allocation in a distributed Grid computing environment. Highly shared resources in a Grid create the insecurity and dependability concerns that hinder distributed applications. Shanshan et al., (2003) modelled the Grid resource allocation process as a multi-objective integer-programming problem. Trusted Grid computing power is maximized over multiple resources. At the same time, the aggregate cost of Grid services is minimized to yield a high performance/cost ratio. Our new Security-

Assured Resource Allocation (SARA) scheme enables dynamic resources under the security constraints. The Grid resources are assured with a distributed security infrastructure. First, the authors presented the mathematical model for SARA optimization process. Then the authors used the Simplex method and implicit enumeration method to obtain optimal or suboptimal solutions. The authors illustrated the ideas with a working example of a pool of 11 host machines from 2 resource sites. For k resource sites with N machine hosts, our SARA algorithm has a search complexity O(2k-1N/k), which is much lower than O(2N) in exhaustive search for the optimal solutions. Our model can be applied to secure many Grid applications in scientific supercomputing or in cyberspace information services.

Hanif et al., (2007) presented a novel quantitative analysis for the strategic planning decision problem of allocating certain available prevention and protection resources to, respectively, reduce the failure probabilities of system safety measures and the total expected loss from a sequence of events. Using an *event tree optimization* approach, the resulting risk-reduction scenario problem was modeled and then reformulated as a specially structured nonconvex factorable program. The authors derived a tight linear programming relaxation along with related theoretical insights that serve to lay the foundation for designing a tailored implicit enumeration algorithm that is proven to converge to a global optimum. Computational experience is reported for a hypothetical case study, as well as for several realistic simulated test cases, based on different parameter settings. The results on the simulated test cases demonstrate that the proposed approach dominates the commercial software BARON v7.5 when the latter is applied to solve the original model by more robustly yielding provable optimal solutions that are at an average of 16.6% better in terms of objective function value; and it performs competitively when both models are used to solve the reformulated problem, particularly for larger test instances.

CHAPTER THREE

METHODOLOGY

3.0 INTRODUCTION

This chapter provides an in depth explanation of our algorithm.

In order to understand the value of implicit enumeration algorithms in integer programming, it is necessary to have a good understanding of some of the background polyhedral theory for general integer programming problems.

Integer programming problem can be formally stated as:

$$\text{Maximize: } c^T x$$

$$\text{Subject to: } Ax \leq b$$

$$x \geq 0 \text{ and integer.}$$

Where $A \in R^{n \times m}$, $x \in Z^{m}$, $b \in R^{1 \times m}$.

Every integer programming problem has a corresponding linear relaxation. This linear relaxation is used as part of various methods to solve integer programming problems, and is defined as:

$$\text{Maximize } c^T x$$

$$\text{Subject to } Ax \leq b$$

$$x \geq 0 \text{ and integer.}$$

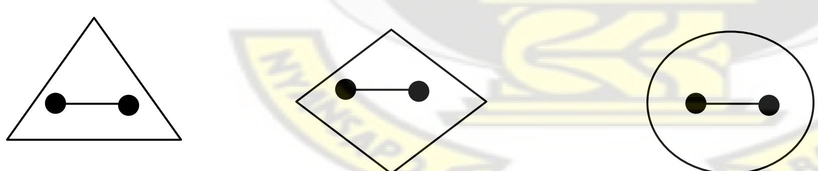
Where $A \in R^{n \times m}$, $x \in R^{m}$, $b \in R^{1 \times m}$.

Solving integer programming problem can be difficult and time consuming. A great deal of research has been performed to improve the solving times and ease of integer programming problem. One of the major areas which research has been done in is the polyhedral theory.

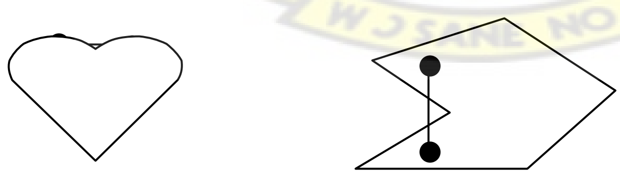
3.1 Polyhedral Theory

Polyhedral theory is an important body of knowledge that helps describe and develop solutions to both linear and integer programming problems. The basic idea behind polyhedral theory is to derive a good linear formulation of the set of solutions by identifying linear inequalities that can be proved to be necessary in the description of the convex hull of feasible solution. The feasible region of any linear programming problem can be represented as a polyhedron, and it is this polyhedron that polyhedral theory seeks to describe. The following basic definitions are noted.

A subset S of R^n ($S \subseteq R^n$) , is said to be convex if for any two elements x_1,x_2 in S , the line segment between the two points $[x_1 , x_2]$ in S is also contained in S. Thus, a set $S \subseteq R^n$ is convex if and only if for every $\lambda \in [0, 1]$ and for every point $[x_1, x_2]$ in S, $\lambda x_1 + (1 - \lambda) x_2 \in S$. We note that in the plane, the following sets are convex



Whereas



Are not convex, (Amponsah, 2008).

Similarly, given a set $S \subseteq R^n$, the convex hull of S, denoted by conv(S) is defined to be the set of all convex combinations of vectors from S. Thus, conv(S):= $\{S = \sum^k_{i=1}\lambda_i v_i : \lambda_i \geq 0, \sum^k_{i=1}\lambda_i = 1$ and $\forall_i \in S\}$. In other words if $S \subseteq R^n$, then the convex hull, conv(S), is defined as the intersection of all convex sets that contain S. The convex hull of two points is a line segment.

A half-space is the set of points that satisfy a linear inequality, $\{x \in R^n : a^T x \leq b\}$.

A polyhedron is a finite intersection of half-spaces and hyper planes. In other words, a polyhedron is a set of the form $\{x \in R^n : Ax \geq b\}$, where $A \in R^{nxm}$ and $b \in R^n$.

Clearly, a polyhedron is a convex set. Furthermore, the solution space of a linear programming problem is also a polyhedron.

A polyhedron $X \subset R^n$ is bounded if there exists a constant k such that $|X|\text{-}1 < k \ \forall\ x \in X, \forall\ i \in [1, n]$.

A polytope is defined as a bounded polyhedron.

A hyper plane is a set of points that satisfy a linear inequality at equality point,

$\{x \in R^n : a^T x = b\}$. It is also said to be the intersection of two closed half-spaces.

A line segment joining the points $[x_1, x_2] \in X$ is the set

$[x_1, x_2] = \{x \in R^n : x = \lambda x_1 + (1 - \lambda) x_2, \text{ for } 0 \leq \lambda \leq 1\}$.

A point on the line segment for which $0 < \lambda < 1$, is called an interior point of the line segment.

A point x in a non-empty convex set X is said to be an extreme point of X if it is not an interior point of any line segment in X. Thus, x is an extreme point of X if there are no two distinct points x_1, x_2 of X such that $x = \lambda x_1 + (1 - \lambda) x_2, 0 < \lambda < 1$.

Equivalently, x is an extreme point of X whenever $x = \lambda x_1 + (1 - \lambda) x_2$ for $[x_1, x_2]$ in X, and $0 \leq \lambda \leq 1$, $x_1 = x_2 = x$. Thus, an extreme point of a polyhedron is any point that is not a convex combination of two other points in the set.

Given an integer programming problem, max c^Tx, subject to $Ax \leq b$, $x \geq 0$ and integer, let P be the set of all feasible solutions. Thus, $P = \{x \in Z^n : Ax \leq b, x \geq 0\}$. The goal of polyhedral theory in integer programming is to completely describe the conv (P), which will be referred to as P^{ch}. The fact that P^{ch} is a polyhedron is vital to integer programming research.

Two types of polyhedron points are also critical to this research. Let $x' \in P^{ch}$, then x' is an extreme point if and only if there does not exist $[x_1, x_2] \in P^{ch}$, $x_1 \neq x_2$, such that

$x' = 0.5x_1 + 0.5x_2$. Any point that is not an extreme point is called an inner point. See Nemhauser and Wolsey (1988) for a complete discussion on this topic. It can be observed that the extreme points of P^{ch} are always integer and that an optimal solution to any linear integer programming problem will always occur at an extreme point.

To describe P^{ch}, knowledge of the dimension of a set of points is also critical. It is important to determine the dimension so that we can know what spaces are critical in P^{ch}. Affine independence can be used to determine the dimension of a space.

For any given vectors $x_1 \ldots x_m \in R^n$, and $\lambda_1 \ldots \lambda_m \in R^n$,

A linear combination is defined as $\sum^m_{i=1}\lambda_i x_i$.

An affine combination is a linear combination where $\sum^n_{i=1}\lambda_i = 1$.

A convex combination is an affine combination where $\lambda_i \geq 0 \ \forall i$.

The linear hull of $x_1 \ldots x_m$ is the set of vectors $\{\sum\lambda_i x_i : \lambda_i \in R\}$.

Similarly, the convex hull is the set of all vectors that are convex combinations of the x_i's. Thus if $X \subseteq R^n$, then the convex hull of X, denoted as conv(X) is the set of all points that are convex combinations of points in X. The convex hull of X is a polyhedron.

Given the points $x_1, x_2, \dots, x_k \in R^n$, if the equation $\sum^k_{i=1}\lambda_i x_i = 0$ has the unique solution of $\lambda_i = 0$, then this k points are linearly independent.

If the system $\{\sum^k_{i=1}\lambda_i x_i = 0, \sum^k_{i=1}\lambda_i = 0\}$ has the unique solution of $\lambda_i = 0$, then the k points are affinely independent.

The dimension of a space is equal to the maximum number of affinely independent points minus one. The problem with determining a polyhedron's dimension by the maximum number of affinely independent points is that, it may be challenging to know whether or not the maximum is obtained.

Two theorems are very important to this area of polyhedral theory. The first relates the dimension of a space to its rank, which is defined as the number of linearly independent vectors that can be found. The rank of a matrix A, denoted as rank (A), is the maximum number of linearly independent columns (= rows) of it. The second theorem relates the dimension of a face to space and is given further on in this thesis.

Theorem 3.1

If $P^{ch} \subseteq R^n$, then dim (P^{ch}) + rank $(A^=, b^=) = n$ where $A^=$ and $b^=$ are constraints that are met at equality by every $x \in P^{ch}$.

Proof: Nemhauser and Wolsey (Nemhauser and Wolsey, 1988)

If a polyhedron has dimension equal to n, then it is considered to be full-dimensional. In the case that the polyhedron is empty set (∅), the dimension is defined to be -1.

An inequality $\alpha^T x \leq \beta$ is a valid inequality for P^{ch} if and only if the inequality is satisfied by every point in P. In other words, the inequality $\alpha^T x \leq \beta$ is a valid inequality for $X = \{x \in R^n : Ax \leq b\}$ if and only if it is satisfied by all points of X. The inequality cannot eliminate a feasible point. If an inequality is valid for a feasible set X, then it is also valid for conv(X). A valid inequality is also called a cut or cutting plane.

Given $\alpha^T x \leq \beta$ being a valid inequality for X, then $F = \{x' \in X: \alpha^T x' = \beta\}$ is a face of X. Thus each cut or cutting plane $\alpha^T x \leq \beta$ induces a face of P^{ch} and the face takes the form $\{x \in P^{ch} : \alpha^T x' = \beta\}$. In other words, a face is a cut or cutting plane that intersects a feasible integer part of a polyhedron.

A face is proper if it is not ∅ or P^{ch}. A valid inequality that does not define a proper face of P^{ch} is redundant and can be removed.

If dim (F) = dim(X) – 1, then F is a facet of X. Thus a facet-defining inequality is an inequality that defines a face of dimensions one less than the dimension of P^{ch}.

Facet-defining inequalities dominate any other valid inequalities and hence it is important to be able to generate (explicitly or implicitly) at least some of them for more efficient solutions. Facets are important because of the role they can play in solving integer programming problems. If a facet is found, the portion of space in the linear relaxation above the facet will be completely cut off, and any other inequalities used to describe that face will be dominated by the facet. If all facets are found, an optimal solution to the linear relaxation is guaranteed to be an integer point and thus it will also be the optimal solution to the integer programming problem.

The following theorem is frequently used to prove that an inequality defines a facet.

Theorem 3.2

Let $\alpha^T x \leq \beta$ be a valid inequality of P^{ch}. Then if there exists an $x \in P^{ch}$ such that $\alpha^T x < \beta$, then the dimension of the face induced by $\alpha^T x \leq \beta$ is at most dim $(P^{ch}) - 1$.

Proof: Hammer et al., (1975).

3.1.1 IMPLICIT ENUMERATION

Implicit enumeration, applied to integer programming, is a systematic evaluation of all possible solutions without explicitly evaluating all of them.

Implicit enumeration techniques try to enumerate the solution space in an intelligent way. The algorithm has the advantage that it requires no linear-programming solutions.

One way to solve such problems is complete enumeration, which list all possible binary combinations of the variables and select the best such point that is feasible. The approach works very well on a small problem such as this, where there are only a few potential 0–1 combinations for the variables. In general, though, an n-variable problem contains 2^n 0–1 combinations; for large values of n, the exhaustive approach is prohibitive. Instead, one might implicitly consider every binary combination, just as every integer point was implicitly considered, but not necessarily evaluated, for the general problem via branch-and-bound. Here, we adopt the opposite tactic of always maintaining the 0–1 restrictions, but ignoring the linear inequalities.

The idea is to utilize a branch-and-bound (or subdivision) process to fix some of the variables at 0 or 1. The variables remaining to be specified are called free variables. Note that, if the inequality constraints are ignored, the objective function is maximized by setting the free variables to zero, since their objective function coefficients are negative.

The simplicity of this trivial optimization, as compared to a more formidable linear program, is what is exploited.

The techniques used here apply to any integer-programming problem involving only binary variables, so that implicit enumeration is an alternative branch-and-bound procedure for this class of problems. In this case, subdivisions are fathomed if any of three conditions hold:

i) the integer program is known to be infeasible over the subdivision,

ii) the 0–1 solution obtained by setting free variables to zero satisfies the linear inequalities; or

iii) the objective value obtained by setting free variables to zero is no larger than the best feasible 0–1 solution previously generated.

These conditions correspond to the three stated earlier for fathoming in the usual branch-and-bound procedure.

If a region is not fathomed by one of these tests, implicit enumeration subdivides that region by selecting any free variable and fixing its values to 0 or 1.

The most common algorithm of this type is branch and bound.

3.1.2 BRANCH AND BOUND

Branch and bound uses the linear relaxation as starting point to search for the optimal integer solution. Every linear relaxation solution that is found during the branch and bound process is given a corresponding node on the branching tree. Once a node's relaxations point has been found, any variable with a fractional value may be chosen as the branching variable. Two child nodes with corresponding branches are created from this parent node. One branch requires the branching variable to be greater than or equal to its relaxation value rounded up to the nearest integer. The other branch requires the branching variable to be less than or equal to the relaxation solution rounded down to the nearest integer. Using these values, two new relaxation points are found and two more nodes are created in the tree. This process is repeated until all nodes have been fathomed.

A fathomed node is finished, and no more nodes or branches are created below any fathomed nodes. Fathoming a node in a branch and bound algorithm occurs under three circumstances. If a node is found that: (i) cannot produce a feasible solution to the linear relaxation, then that node is fathomed. (ii) returns an integer solution, then that node is fathomed. Although other feasible solutions may exist below that node, none will be better than that node's solution. (iii) has a linear relaxation solution with a value lower than the value of a previously discovered integer solution, then that node is fathomed.

An alternative to the branch and bound method is to use cutting planes to reduce the linear relaxation space. The basic idea of the cutting plane method is to cut off parts of the feasible region of the corresponding linear program, so that the optimal integer solution becomes an extreme point and can be found by the simplex algorithm. This method attempts to find a hyper plane that intersects the solution space below the current linear relaxation point without eliminating any integer solutions. Once such a hyper plane has been put in place, a new linear relaxation point is found, and branch and bound can be implemented or additional cutting planes can be added until an integer solution is returned as the solution to the linear programming problem.

3.1.3 BRANCH AND BOUND FOR 0-1 INTEGER PROGRAM

This can be solved by enumerating all 2^n possible variables x, with every node representing a problem

The algorithm

set $U = +\infty$, mark all nodes in the tree as active

1. select an active node k, and solve the corresponding LP relaxation problem

   let $\bar{x}$ be the solution of the relaxation

2. if $c^T \bar{x} \geq U$, mark all nodes in the subtree with root k as inactive

3. if all components of $\bar{x}$ are 0 or 1, mark all nodes in the subtree with root k as inactive; if moreover $c^T \bar{x} < U$, then set $U := c^T \bar{x}$ and save $\bar{x}$ as the best feasible point found so far

4. otherwise, mark node k as inactive

5. go to step 1

CHAPTER FOUR

DATA COLLECTION AND ANALYSIS

4.0 INTRODUCTION

In this chapter, we shall consider a computational study of implicit enumeration algorithm for solving the resource allocation problem.

The choice of the resource allocation model is a real life problem in Cocoa Processing Company of Ghana. The aim is to determine the optimal allocation policy in the business so that the business can obtain the optimum return of profit from the various combinations of resources needed to process the Cocoa beans into consumable products.

4.1 Data Collection and Analysis

Cocoa Processing Company of Ghana can add value to Ghana Cocoa beans to earn higher foreign exchange by processing the Cocoa beans into seven products, namely: Bake, Delight, Pebbles, Royale, Altime, Bar and Vitaco. The profit contributions of the seven products are respectively (in Ghana cedis) 45, 96, 64, 57, 76, 75 and 67. The seven products also require the combination of various ingredients, namely; Milk Powder, Sugar, Cocoa Butter, Cocoa Masse, Lacithin, Vanillin, Groundnuts, Corn, Cocoa Powder, and Cereal Powder. The expected quantities per 1000kg for different number of ingredients, as estimated from the past records, are given in Table 4.1

Table 4.1: Required Resources for the processing of the Seven Products per 1000kg

	PRODUCTS							
ADDITIVES	**BAKE**	**DELIGHT**	**PEBBLES**	**ROYALE**	**ALTIME**	**BAR**	**VITACO**	**AVAILLABILITY**

MILK POWDER	0	15	22	0	4	18	16	180
SUGAR	0	50	2	0	10	6	11	100
COCOA BUTTER	24	0	18	0	0	7	0	130
COCOA MASSE	6	0	10	0	0	9	0	90
LACITHIN	0	30	4	0	8	6	7	120
VANILLIN	0	40	0	0	14	19	10	110
GROUNDNUTS	0	32	28	0	0	7	0	128
CORN	0	0	40	0	0	0	0	118
COCOA POWDER	0	22	0	5	8	0	62	278
CEREAL POWDER	0	50	0	0	0	0	0	105

The corresponding unit of labour needed to process the seven products into finish goods is four, five, five, three, three, three and four respectively. The total unit of labour needed should not exceed thirty-five.

The problem is to find the optimal product mix that will maximize the profit.

The problem can be modelled as:

Maximize $Z = \sum_{i=1}^{n} P_i X_i$

Subject to $\sum_{i=1}^{n} C_i X_i \leq W$

$X_i \geq 0, integer.$

Where;

P_i : Profit of each product

X_i : Number of resources needed for each product

C_i : Cost of each resource

W : Resource limit

Thus,

Maximize Z = $45X_1 + 96X_2 + 64X_3 + 57X_4 + 76X_5 + 75X_6 + 67X_7$

Subject to

$$15X_2 + 22X_3 + 4X_5 + 18X_6 + 16X_7 \leq 180$$

$$50X_2 + 2X_3 + 10X_5 + 6X_6 + 11X_7 \leq 100$$

$$24X_1 + 18X_3 + 7X_6 \geq 130$$

$$6X_1 + 10X_3 + 9X_6 \geq 90$$

$$30X_2 + 4X_3 + 8X_5 + 6X_6 + 7X_7 \leq 120$$

$$40X_2 + 14X_5 + 19X_6 + 10X_7 \leq 110$$

$$32X_2 + 28X_3 + 7X_6 \leq 128$$

$$40X_3 \leq 118$$

$$22X_2 + 5X_4 + 8X_5 + 63X_7 \geq 279$$

$$50X_2 \leq 105$$

$$4X_1 + 5X_2 + 5X_3 + 3X_4 + 3X_5 + 3X_6 + 4X_7 \leq 35$$

The QM software was used for the data analysis. Table 4.2 is the summary of the data analyzed using the QM software.

TABLE 4.2 **SUMMARY OF RESULTS OF DATA ANALYSED**

ITERA-TION	LEVEL	SOLUTION TYPE	SOLUTION VALUE	X_1	X_2	X_3	X_4	X_5	X_6	X_7
		OPTIMAL	770	0	0	0	3	7	0	1

1	0	NONINTEGER	814.29	0	0	0	3.8	7.8	0	0
2	1	NONINTEGER	799.23	0	0	0.5	3	7.9	0	0
3	2	NONINTEGER	795.46	0.6	0	0	3	7.9	0	0
4	3	NONINTEGER	784.77	0	0	0	3	6.9	0	1.3
5	4	NONINTEGER	773.91	0	0	0	3	6	0.5	1.6
6	5	NONINTEGER	763.67	0	0.3	0	3	6	0	1.7
7	6	INTEGER	761	0	0	0	3	6	0	2
8	5	NONINTEGER	763.96	0	0	0	3	5.2	1	1.9
10	6	NONINTEGER	762.15	0	0	0	3	5	1.1	1.9
11	7	SUBOPTIMAL	760.45	0	0.04	0	3	5	1	1.9
12	7	SUBOPTIMAL	743.15	0	0	0	3	3.4	2	2.5
13	6	SUBOPTIMAL	748.9	0	0	0	3	6	1	0.7
14	4	NONINTEGER	783.4	0	0	0	3	7	0	1.2
15	5	NONINTEGER	780.86	0	0	0	3	7.1	0	1
16	6	NONINTEGER	777.89	0	0	0	3	7	0.12	1
17	7	NONINTEGER	774.8	0	0.05	0	3	7	0	1
18	8	INTEGER	770	0	0	0	3	7	0	1
19	8	INFEASIBLE								
20	7	INFEASIBLE								
21	6	INFEASIBLE								
22	5	INFEASIBLE								
23	3	NONINTEGER	783.29	1	0	0	2.5	7.86	0	0
24	4	NONINTEGER	772.21	1.36	0	0	2	7.86	0	0
25	5	SUBOPTIMAL	765.92	1	0	0	2	7.31	0	0.7

										7
26	5	SUBOPTIMAL	752.29	2	0	0	1.14	7.86	0	0
27	4	NONINTEGER	773.33	1	0	0	3	7.33	0	0
28	5	NONINTEGER	773	1	0	0	3	7	0.33	0
29	6	SUBOPTIMAL	767.2	1	0.20	0	3	7	0	0
30	6	NONINTEGER	772.33	1	0	0	3	6.33	1	0
31	7	NONINTEGER	772	1	0	0	3	6	1.33	0
32	8	SUBOPTIMAL	765.75	1	0.16	0	3	6	1	0.0 5
33	8	SUBOPTIMAL	763.29	1.14	0	0	3	5.14	2	0
34	7	INFEASIBLE								
35	5	INFEASIBLE								
36	2	NONINTEGER	783.29	0	0	1	2.14	7.86	0	0
37	3	NONINTEGER	780.63	0	0	1.09	2	7.86	0	0
38	4	NONINTEGER	779.96	0.11	0	1	2	7.86	0	0
39	5	NONINTEGER	778.08	0	0	1	2	7.69	0	0.2 3
40	6	SUBOPTIMAL	769.93	0	0	1	3	7	0.39	0.4 6
41	6	INFEASIBLE								
42	5	SUBOPTIMAL	752.29	0.9	0	1	0.81	7.86	0	0
43	4	SUBOPTIMAL	752.29	0	0	2	0.48	7.86	0	0
44	3	SUBOPTIMAL	767	0	0	1	3	7	0	0

45	1	NONINTEGER	810.67	0	0	0	4	7.67	0	0
46	2	NONINTEGER	809.37	0	0	0	4.04	7	0.63	0
47	3	NONINTEGER	808.57	0	0	0	4	7	0.63	0.04
48	4	NONINTEGER	808.57	0	0	0	4	7	0.63	0
49	5	NONINTEGER	796.62	0	0.25	0	4	7	0	0.18
50	6	NONINTEGER	782.5	0	0	0	4	7	0	0.5
51	7	NONINTEGER	782.5	0.5	0	0	4	7	0	0
52	8	SUBOPTIMAL	760	0	0	0	4	7	0	0
53	8	SUBOPTIMAL	754.33	1	0	0	4	6.33	0	0
54	7	NONINTEGER	776.33	0	0	0	4	6.33	0	0
55	8	SUBOPTIMAL	767.75	0	0	0	4	6	0	1.25
56	8	INFEASIBLE								
57	6	SUBOPTIMAL	737.75	0.75	1	0	4	5	0	0

58	5	NONINTEGER	802.63	0.12	0	0	4	6.5	1	0
59	6	NONINTEGER	800.42	0	0	0	4	6.31	0.9	0.27
60	7	NONINTEGER	796.80	0	0	0	4	6	1.17	0.37
61	8	NONINTEGER	793.39	0	0.07	0	4	6	1	0.41

62	9	NONINTEGER	792.5	0	0	0	4	6	1	0.5
63	10	SUBOPTIMAL	759	0	0	0	4	6	1	0
64	10	NONINTEGER	775.33	0	0	0	4	5.33	1	1
65	11	SUBOPTIMAL	766.75	0	0	0	4	5	1	1.25
66	11	INFEASIBLE								
67	9	SUBOPTIMAL	700.81	0	1	0	4	2.24	1	1.69
68	8	NONINTEGER	779.62	0	0	0	4	4.54	2	0.85
69	9	NONINTEGER	773.28	0	0	0	4	4	2.3	1.02
70	10	SUBOPTIMAL	767.31	0	0.13	0	4	4	2	1.09
71	10	SUBOPTIMAL	758.81	0	0	0	4	2.77	3	1.42
72	9	NONINTEGER	771.4	0	0	0	4	5	2	0.2
73	10	SUBOPTIMAL	768.86	0	0	0	4	5.14	2	0
74	10	INFEASIBLE								
75	7	INFEASIBLE								
76	6	SUBOPTIMAL	753.33	1	0	0	4	5.33	1	0
77	4	SUBOPTIMAL	748	0	0	1	4	6	0	0
78	3	NONINTEGER	791.67	0	0	0	5	6.67	0	0
79	4	NONINTEGER	791	0	0	0	5	6	0.67	0

80	5	NONINTEGER	779.4	0	0.4	0	5	6	0	0
81	6	NONINTEGER	779	0	0	0	5.67	6	0	0
82	7	NONINTEGER	774.5	0	0	0	5	6	0	0.5
83	8	SUBOPTIMAL	766.6	0	0	0.4	5	6	0	0
84	8	SUBOPTIMAL	757.33	0	0	0	5	5.33	0	1

85	7	NONINTEGER	772.67	0	0	0	6	5.67	0	0
86	8	SUBOPTIMAL	760	0	0	0	6.67	5	0	0
87	8	INFEASIBLE								
88	6	SUBOPTIMAL	761	0	1	0	5	5	0	0
89	5	NONINTEGER	790.67	0	0	0	5	5.67	0.9	0
90	6	NONINTEGER	790	0	0	0	5	5	1.67	0
91	7	NONINTEGER	778.4	0	0.40	0	5	5	1	0
92	8	NONINTEGER	778	0	0	0	5.67	5	1	0
93	9	NONINTEGER	773.5	0	0	0	5	5	1	0.5
94	10	SUBOPTIMAL	765	0	0	0.4	5	5	1	0
95	10	SUBOPTIMAL	756.33	0	0	0	5	4.33	1	1
96	9	NONINTEGER	771.67	0	0	0	6	4.67	1	0
97	10	SUBOPTIMAL	759	0	0	0	6.67	4	1	0
98	10	INFEASIBLE								
99	8	SUBOPTIMAL	753.21	0	1	0	5.36	3.64	1	0
100	7	NONINTEGER	789.67	0	0	0	5	4.67	2	0
101	8	NONINTEGER	789	0	0	0	5	4	2.67	0

102	9	NONINTEGER	777.4	0	0.4	0	5	4	2	0
103	10	NONINTEGER	777	0	0	0	5.67	4	2	0
104	11	NONINTEGER	772.5	0	0	0	5	4	2	0.5
105	12	SUBOPTIMAL	764.6	0	0	0.4	5	4	2	0
106	12	SUBOPTIMAL	755.33	0	0	0	5	3.33	2	1
107	11	NONINTEGER	770.67	0	0	0	6	3.67	2	0
108	12	SUBOPTIMAL	758	0	0	0	6.67	3	2	0
109	12	INFEASIBLE								
110	10	SUBOPTIMAL	745.43	0	1	0	5.71	2.29	2	0
111	9	NONINTEGER	788.67	0	0	0	5	3.67	3	0
112	10	NONINTEGER	786.42	0	0	0	5.09	3	3.58	0
113	11	NONINTEGER	784.79	0	0	0.05	5	3	3.58	0
114	12	NONINTEGER	784.41	0	0	0	5	3	3.52	0.11
115	13	NONINTEGER	774.17	0	0.22	0	5	3	3	0.23
116	14	NONINTEGER	771.5	0	0	0	5	3	3	0.5
117	15	SUBOPTIMAL	760.5	0.5	0	0	5	3	3	0
118	15	SUBOPTIMAL	754.33	0	0	0	5	2.33	3	1
119	14	SUBOPTIMAL	712.73	0.80	1	0	5	0.93	3	0
120	13	NONINTEGER	777.61	0.18	0	0	5	2.43	4	0
121	14	NONINTEGER	774.46	0	0	0	5	2.15	4	0.38

122	15	NONINTEGER	772.65	0	0	0	5	2	4.09	0.43
123	16	NONINTEGER	770.95	0	0.04	0	5	2	4	0.45
124	17	NONINTEGER	770.5	0	0	0	5	2	4	0.5
125	18	SUBOPTIMAL	737	0	0	0	5	2	4	0
126	18	SUBOPTIMAL	753.33	0	0	0	5	1.33	4	1
127	17	INFEASIBLE								
128	16	SUBOPTIMAL	753.65	0	0	0	5	0.38	5	0.96
129	15	INFEASIBLE								
130	14	SUBOPTIMAL	731.33	1	0	0	5	1.33	4	0
131	12	SUBOPTIMAL	726	0	0	1	5	2	3	0
132	11	SUBOPTIMAL	769.67	0	0	0	6	2.67	3	0
133	10	INFEASIBLE								
134	8	INFEASIBLE								
135	6	INFEASIBLE								
136	4	INFEASIBLE								
137	2	INFEASIBLE								

From table 4.2:

The analysis went through about 130 iterations and 17 levels. Different solution types were obtained during the iterations e.g. Optimal, Suboptimal, Integer, Non-integer, and Infeasible Solutions.

The first iteration gave the solution value of 814.29 which is non-integer and also the values for X_4 and X_5 are non-integers.

The second iteration gave the solution value of 799.23 which is non-integer and also the values forX_3 and X_5 are non-integers.

The third iteration gave the solution value of 795.46 which is non-integer and also the values for X_1 and X_5 are non-integers.

The fourth iteration gave the solution value of 784.77 which is non-integer and also the values for X_5 and X_7 are non-integers.

The fifth iteration gave the solution value of 773.91 which is non-integer and also the values for X_6 and X_7 are non-integers.

The sixth iteration gave the solution value of 763.67 which is non-integer and also the values for X_2 and X_7 are non-integers.

The seventh iteration gave the solution value of 761 which is an integer and also the values for X_4 and X_5 and X_7 are all integers.

The eighth to tenth iterations gave non-integer value.

Iterations eleven, twelve, thirteen have solution values: 760.45, 743.15 and 748.9 which are all suboptimal, meaning they are not optimal solutions and also they are not integer solutions.

Iterations eighteen have an integer solution of 770 with X_4 , X_5 and X_7 having integer values of 3, 7 and 1 respectively.

All the other iterations gave either non-integer solutions or infeasible solutions.

So our optimal integer solution occurred at iteration eighteen which has the solution of 770 with X_4 , X_5 and X_7 having integer values of 3, 7 and 1 respectively.

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.0 INTRODUCTION

We have described the resource allocation problem of Cocoa Processing Company Ltd. as an integer programming problem. We applied integer programming algorithm to solve the company's resource allocation problem. Our research focused on the use of the integer programming problem for resource allocation given limited available funds for Cocoa Processing Company Ltd. in Ghana.

5.1 CONCLUSIONS

Cocoa Processing Company Ltd. resource allocation problem for the processing of the cocoa beans into various consumable products was modelled as integer programming problem. The Quantitative Method (QM 32) software was used to analyze the data collected from the Company Processing Company Ltd. The optimal value was seven hundred and seventy thousand Ghana Cedis (GH¢770,000.00). It was observed that the solution that gave this achievable value was the utilization of the available resources to produced three thousand kilogram (3,000kg) of Royale, seven thousand kilogram (7,000kg) of Altime, and one thousand kilogram (1,000kg) of Vitaco.

5.2 RECOMMENDATIONS

The use of computer application in computation gives a systematic and transparent solution as compared with an arbitrary method. Using the more scientific Quantitative Methods Software for the integer programming problem model for the Company Processing Company Ltd. gives a better result. Management may benefit from the proposed approach for their resource allocation to guarantee optimal utilization of resources. We therefore recommend that our integer programming problem model should be adopted by the Cocoa Processing Company for its resource planning.

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