Location of a Library in Sunyani Municipality using The Planar K-Centra Single-Facility Euclidean Location Algorithm.

By

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Industrial Mathematics

IDL

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DECLARATION

I hereby declare that this submission is my own work towards the Master of Science and that to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the university, except where due acknowledgement has been made in the text.

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ABSTRACT

The Planar k-Centra Single-Facility Euclidean Location Algorithm was used to strategically locate a library in the Sunyani Municipality which will be closer to the k-50 farthest towns and suburbs. One hundred rectangular co-ordinates were generated for 100 towns and suburbs in the Municipality as the inputs for the algorithm. Matlab codes were used to run the algorithm. The algorithm generated (406647.9km , 935589.2km) as the optimal point with 1811500km as its objective function value(The sum of the distances of 50 farthest locations away from the optimal location). Hence the proposed community for the library is Benu No. 2.

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DEDICATION

I wish to dedicate this thesis to my mother Cecilia Afrah, my sister Mercy Pormaa, my brothers Richard Asante and Daniel Bonsu and my closest friend Boahemaa Acheampong.

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Chapter 1

INTRODUCTION

1.1. Background Study

1.1.1. What is a Library?

A library is a collection of sources, resources, and services, and the structure in which it is housed; it is organized for use and maintained by a public body, an institution, or a private individual. In a more traditional sense, a library is a collection of books. It is also the collection, the building or room that houses such a collection, or both. The term "Library" has itself acquired a secondary meaning: "a collection of useful material for common use". This sense is used in fields such as Computer Science, Mathematics, Statistics, Electronics and Biology. It can be used by publishers in naming related books. E.g. The library of Anglo – Catholic Theology. A library often provides a place of silence for studying. (Wikipedia, 19/10/2010).

Public and institutional collections and services may be intended for use by people who choose not to, or can not afford to purchase an extensive collection themselves and who need material no individual can reasonably be expected to have, or who require professional assistance with their research. In addition to providing materials, libraries also provide services of librarians who are experts at finding and organizing information and at interpreting information need.

Libraries are regarded as one of the institutions that have a role in advancing literacy and education in the society.

1.1.2. The Early History of Libraries

The first libraries were composed, for the most part, of published records, a particular type of library called archives. Archaeological findings from the ancient city – states of sumer have revealed temple rooms full of clay tablets in cuneiform script. (Wikipedia, 19/10/2010). These archives were made up of the records of commercial transactions or inventories, with only a few documents touching theological matters, historical records or legends. Things were much the same in the government and temple records on papyrus of Ancient Egypt. (Wikipedia, 19/10/2010).

The earliest discovered private archives were kept at Ugarit; besides correspondence and inventories, texts of myths may have been standardized practice – texts for teaching new scribes. (Wikipedia, 19/10/2010). There is also an evidence of libraries at Nippur about 1900 B.C. and those at Nineveh about 700 B.C. (Wikipedia, 19/10/2010).

Over 30,000 clay tablets from library of Ashurbanipal have been discovered at Nineveh, providing archaeologists with an amazing wealth of Mesopotamian library, religious and administrative work. (Wikipedia, 19/10/2010).

1.1.3. Early Modern Libraries

Johannes Gutenberg's movable type innovation in the 15th century revolution bookmaking. From the 15th century in central and northern Italy, they assiduously assembled libraries of humanists and their enlightened patrons provided a nucleus around which an "academy" of scholars congregated in each Italian city of Consequence. Cosimo de Medici in Florence established his own collection, which formed the basis of the Laurentain library. In Rome, the papal collection were brought together by Pope Nicholas V, in separate Greek and Latin libraries, and housed by Pope Sixtus IV, who consigned the Bibliotheco Apostolic Vaticana to the cane of his librarian, the humanist Bartolomeo Platina in February 1475. (Wikipedia, 19/10/2010).

A lot of factors combined to create a "golden age of libraries" between 1600 and 1700. The quantity of books had gone up, as the cost had gone down, there was a renewal in the interest of classical literature and culture. Nationalism was encouraging nations to build great libraries. Universities were playing a more prominent role in education, and renaissance thinkers and writers were producing great works. Some of the more important libraries include the Bodlerian Library at Oxford, the Library of the British Museum, the Mazarine Library in Paris, and the National Central Library in Italy, the Prussian state Library, the M. E. Sattykov – Shchedrin State Public Library of St. Petersburg and many more. (Wikipedia, 19/10/2010).

1.1.4. Library Classification

A library classification is a system of coding and organizing library materials (books, serials, audiovisual materials, computer files, maps, manuscript, realia) according to their subject and allocating of call number to that information resource. Similar to classification systems used in biology, bibliographic classification systems group entities together that are similar, typically arranged in a hierarchical tree structure. A different kind of classification system, called a faceted classification system, is also widely used which allows the assignment of multiple classification to an object, enabling the classifications to be ordered in multiple ways. (Wikipedia, 19/10/2010).

1.1.5. Description of Libraries

Library classification form part of the field of library and information Science. It is a form of bibliographic classification (library classification are used in library catalogs, while "bibliographic classification" also covers classification used in other kinds of bibliographic database). It goes hand in hand with library (descriptive) cataloging under the rubric of cataloging and classification, sometimes grouped together as technical services. The library professional who engages in the process of cataloging and classifying library materials is called a cataloguer or catalog librarian. Library classification systems are one of the two tools used to facilitate subject access. The order consists of alphabetical indexing languages such as Thesauri and Subject Headings Systems.

Library classification of a piece of work consists of two steps. Firstly the "aboutness" of the material is asserted. Next, a call number (essentially a book's address), based on the time in use at the particular library will be assigned to the work using the notation of the system. It is important to note that unlike subject heading where multiple terms can be assigned to the same work, in library classification system, each work can only be placed in one class. This is due to shelving purpose.

1.1.6. Types of Libraries

1.1.6.1. Academic Libraries

These libraries are located on the campuses of colleges and universities and serve primarily the students and the faculty of that and other academic institutions. Some academic libraries, especially those at public institutions, are accessible to the members of the general public.

1.1.6.2. Public Libraries

These libraries provide services to the general public and make at least some of their books available for borrowing, so that readers may use them at home over a period of time. Typically, libraries issue library cards to community members wishing to borrow books.

1.1.6.3. Research Libraries

They are intended for supporting scholarly research, and therefore maintain permanent collections and attempt to provide access to all necessary material.

They are often academic libraries or national libraries, but many large special libraries have research libraries within their special filed and a very few of the large public libraries also serve as a research libraries.

1.1.6.4. School Libraries

Most public and private primary and secondary schools have libraries designed to support the school curriculum.

1.1.6.5. Special Libraries

All other libraries fall into this category. Many private businesses and public organizations, including hospitals, museums, research laboratories, law firms, and many government departments and agencies, maintain their own libraries for the use of their employees in doing specialized research, related to their work. Special libraries may or may not be accessible to some identified part of the general public.

1.1.7. **Importance of Libraries**

1.1.7.1. Inculcating the habit of reading

Reading is regarded as one of the most enriching habits for the simple reason that it is not just a hobby or a pass time that entertains you, but it is also an educational activity and hence brings to you a vast reservoir of knowledge. Reading increases the drive for knowledge and inspires people to gain more information. Thus a library is a treasure of valuable books for the people to use and gain from it.

1.1.7.2. Reference for Schools and Colleges

The quintessential library is a boon for the students in school and colleges. There exist a large number of reference books that provide information about wide ranging subjects are for students to understand the concepts in their curriculum. The reference books often provide an in-depth information about various subjects and thus help in the process of education.

1.1.7.3. Advice on Important Subjects

There are large number of books that provide advice about various topics like business, health, travel, food and careers. These books service as a great source of advice. People may make it a point to read and go through these books before taking important decisions in their life. Thus, libraries are also helpful to people who are looking for information about specific subjects. For example, a person who is planning to travel to a particular place would like to read about that destination.

1.1.7.4. Wholesome Information

A library usually has a good collection of encyclopedias, dictionaries and maps, which are a source of extensive information and references for people. The encyclopedias are a vast source of information about all the topics under the sky. These also exist specialized dictionaries like medical dictionaries, literature dictionaries, business dictionaries, which provide about specific terms used in specialized fields.

1.1.7.5. Entertainment and Fun

Libraries are also a host to large books that are a source of entertainment for us. Fiction books, which includes various genre like comedy, thriller, suspense, horror or drama, are tremendously popular within readers of varying age groups. Libraries are thus of entertainment and education for youngsters as well as adults. A library not only helps to inculcate the habit of reading but a thirst for knowledge, which makes a person humble and open to new ideas throughout his/her life. (Wikipedia, 19/10/2010).

1.1.7.6. Access to other big Libraries around the World

With the advent of the internet which has become part of the library, people are able to read books from the libraries of other libraries since most academic libraries have almost all their content on the internet, but one need a code before one can access. Research students can chat online with their supervisors and at times do conferencing on the information super highway. This is the virtual aspect of a library.

1.2. Internet: Importance and Usage for Library and Information Professionals.

One of the most significant achievements in the information and communication sector is the introduction of the advanced communication network i.e. the internet. The technology connecting a computer with millions of computers in the network. The internet today has become one of the most important mode of communication and its services are being expected by people. Libraries can also project their collections and activities and supplement their services by exploiting the internet. In fact the library professional is one that has been most intensively affected by the challenges of internet and the WWW. The shift from collection management to information management, from ownership to access, and the change in nature, boundaries and structure of information all call for a change in mind set of library professionals. The library professionals need to position themselves as leaders not only in information field but also in the field of the information technology (Information retrieval). (DESIDOC Bulietin of Information Technology, 2001).

1.2.1. Importance of Internet to Libraries

The internet has become part of library environment today. It has added a great value to the library and information service. According to Gryez, "With the expansion of internet a new class of electronic document has emerged, it was at once promising and attractive for its obvious advantage of speed and transmissibility and profoundly elusive and confounding to the library community because of its intangibility and malleability."

Within the last ten years, the internet has become global and ubiquitous. It reaches in hundreds of countries of all continents and is featured daily in the business sections of all major news papers.

The internet is playing important role in transforming the library system and the way in which we views the library resources and library services. With the help of Web based library services in developed countries, users are attended round the clock. Internet provides links to various library sites, specializing in almost every topic and they can be accessed directly from any part of the world. As the libraries are going web based more and more, libraries are becoming accessible via libraries' web pages. With an internet connection, a student in Indian University can browse through the documents in computers of US National libraries or else where on the globe. The net provides instant access to billions of information sources which include books, reports, journals, video films, sound recording and wide variety of other sources. The library and information professionals have a vital role to play in organizing the information and bridging the information gap.

1.2.2. The need of Library Professional in the Internet era

Librarians acting as custodians of information have gone through a dramatic change and from providing document to their clienteles have switched to be information providers. The role of librarians as information organizers and a navigator has gained importance in the internet era.

The library professionals need to focus and seize new opportunities and demonstrate how the tools of internet can be gainfully harnessed for improving library services.

Internet can be viewed as the biggest library in the world in which information is not properly structured and organized; there are no standardized rules of classification or access.

The librarians can play a greater role in identification, listing, and classifying information sources and providing systematic approach to accessing the required information. This way they can take rightful place as human agent alongside the search engine in searching the internet. In the due course of time, the librarians will have to develop new indexing methods and evaluation techniques to tap information from the internet and also establish the classification modes in an open way to allow for those additions of near categories of document that may differ from original priorities. Hence, the uncertainties raised by some people that internet may be a threat to library and library science profession is no longer true. The library and the internet are partners rather than enemies. In fact, the internet is a tool in the library setting.

1.2.3. Role of Internet in Discharging Library Functions

Internet is playing an important role in discharging the functions of libraries. It is changing the ways, the librarians organize manage and disseminate information. With more and more document getting missed – up electronically and internet resources growing at 18% a mouth, libraries of 21st century will have to shift towards electronic means of acquiring, processing and disseminating information.

Today all sorts of library services from membership registration to document delivery can be offered through the net. The trend is quite evident from the web sites of American Business School Libraries that are quite advanced in library and information services arena.

1.2.4. Some important library services that can be offered through the internet

Acquisition of documents, Technical processing / classification and cataloguing, circulation, Reference and Information Service, Communication, Resource sharing on the internet, Inter – Library Loan (ILL). Therefore, the internet has integrated nearly all the aspects of the library activities, the librarians can now use the internet for exploiting the catalogue of other institutions, ordering books and journals online, participation in ILL, use e-mail, and discuss, support reference services through remote database of most important established libraries. The scope is only limited to the imagination of the library professionals or the user on the line (Internet).

All that is required by the today's professionals is a through understanding of change in concept of librarianship and psychological willingness to look upon the internet and the WWW as an opportunity and respond to the challenges of information resource management and information infrastructural development for harnessing the benefit of the much talked about internet technology in context of the libraries. (DESIDOC Bulietin of Information Technology, 2001).

1.3. Sunyani Municipality (Sunyani East)

Currently, Sunyani East is one of the fastest growing municipalities in Ghana, with the human population of about 80,245 a growth rate of 3.4% per annum (Wikipedia, 2005) and a size of 2,488 kilometres square. The municipality shares boarders with the following districts; Sunyani West District to the north, Tano North District to the east, Berekum Municipal to the north-west, Dormaa Municipal to the south-west and lastly Asutifi District to the south. The following are the many schools and other important institutions in the municipality; Catholic University College, College Nature and Renewable Resources of Kwame Nkrumah University of Science and Technology, Sunyani Polytechnic, about twelve second cycle schools and hundreds of junior high schools and primary schools can be cited in this municipality. Sunyani township happens to be the capital of the Brong-Ahafo Region. Because of this most government agencies and most private companies have their offices in the township. All these confirm the necessity of a library complex in the municipality.

1.4. Problem Statement

Due to the shortage of libraries in our cities and sometimes the nonexistence of libraries, performance of our students is low. Where there are libraries, the locations are poorly sited.

The aim of this thesis is to propose the planar k-centra single-facility Euclidean location problem strategy to locate a library in Sunyani Municipal to minimize travel distances of k-50 farthest locations.

1.5. The objectives of the study

The objectives of the study are as follows:

1. To model the location of a central library in the Sunyani Municipality as Planar K-Centra Single Facility Euclidean Problem.
2. To determine the optimal location of the library by the Planar K-Centra Single Facility Euclidean Algorithm.

1.6. Methodology

The problem is to locate a library in Sunyani Municipality so as to minimize the travel distances between the new facility and the users using the Planer k-centra single facility Euclidean location strategy. The data type is a primary data recorded from the Sunyani Municipality map. The data will be taken from the Town and Country planning Department from the Sunyani Municipal Assembly. The data will be in the x-y coordinates. The algorithm we will use is the Weiszfeld's algorithm.

The scale for all the readings will be 1:50000. The software for the work will be the Matlab and the focus will be on locating a library to minimize the total distance of the k-50 farthest customer locations. The resource centres for the study are my personal laptop, the internet, the Sunyani Municipal library, the library of Kwame Nkrumah University of Science and Technology (Kumasi) and Sunyani Polytechnic library.

1.7. Justification for the study

The following reasons are why the study is significant:

It will give an effective coordinates for the computation distances and subsequently a central location for the library.

It will reduce the average travel time and distances from all the k-50 farthest costumer locations to the library location.

It will help to conserve fuel since the algorithm provides average shortest distances for all costumer locations.

It will help future researchers who would want to research into area navigation system for the municipality.

1.8. Organization of the thesis.

This thesis is structured as follows: Chapter one deals with the introduction and the background of the study. Chapter two

deals with the related literature relevant to the study. Chapter three discusses the methodology used for the study. Chapter four presents the results and the discussion of the case study. Chapter five which is the last chapter, deals the conclusion and the recommendations.

Chapter 2

2. LITERATURE REVIEW

This chapter reviews the literature existing in the area of facility location. Facility location problems date back to the 17th century when Fermat (1643) and Cavalieri, et al. (1647) simultaneously introduced the concept, although this theory is widely contested by location analyst experts. Late in 18th century, Pierre Varignon presented ”The Varignon Frame” which an analog solution to the planar minisum location problem. However, it started gathering more interest when Weber (1909) presented the Planar Euclidean single facility minisum problem in1909. The Weber problem considers the example of locating a warehouse in the best possible location such that the distance travelled between the warehouse and the customer is minimized. Weiszfeld (1937) conceptualised an iterative method to solve the minisum Euclidean problem today referred to as the Weiszfeld’s procedure.

Hakimis (1964) introduced a seminal paper on locating one or more points on a network with the objective to minimize the maximum distance. Francis et al. (1983) presented a survey paper in location analysis which defined four classes of location problems and described algorithms to optimize them. They are continuous planar, discrete planar, mixed planar and discrete network problems.

Daskin et al (1998) reviewed various strategic location problems where they emphasized that a good facility location decision is a critical element in the success of any supply chain. They explained median problems, centre problems, covering problems, and other dynamic location problem formulations in the context of a supply chain environment.

Facility location problems span many research fields like operation research, mathematics, statistics, urban planning designing, etc.

Location analysis goes back to the influential book of (the German industrial author Weber, A., 1909). The research was motivated by observing a warehouse operation and its inefficiencies. Weber considered the single warehouse location problem and evaluated it such that the travel distances for pickups and replenishment were reduced. Other notable work in this field was by Fermat (1643), who solved the location problem for three points constituting a triangle.

Another major concept in the field of location analysis was the concept of competitive location analysis introduced by (Hotelling, 1929). The paper discussed a method to locate a new facility considering already existing competition. The considered facilities were on a straight line. He proposed that the customers generally prefer visiting the closest service facility. He introduced the "Hotelling's Proximity Rule" which can be used to determine the market share captured by each facility. He just considered the distance metric during his analysis. The Hotelling model was extended by Drezner (1993) who introduced the concept of varying attractiveness among competing facilities. He analyzed cost and quality factors in addition to distance metric involved. Huff (1966) proposed the famous "Gravity Model" for estimating the market share captured by competitors. The gravity model states that existing customer locations attract business from a service in direct proportion to the existing locations and in inverse proportion to the distance between the service location and the existing customer locations.

The rectilinear distance location problem is a variant of the classic location problem. Rectilinear distance metric is the axial distance between two corresponding points taken at right angles from each other. Francis (1963) first considered the single facility location problem with rectilinear distances. The paper considered a simple substitution method for solving the proposed problem. Francis (1964) also solved the multi-facility rectilinear location problem with equal demand weights.

For collinear single facility location problems with Euclidean distances, a user can always obtain an optimal solution as discussed in the paper by (Rosen et al., 1993). This thesis assumes that the existing facilities are non-collinear. For non collinear location problems, Weiszfield (1937) was the first to propose a fixed-point iterative method that is known as the Weiszfield procedure. Weiszfield's alogarithm iteratively solves for the minisum location to the Weber problem based on the objective function.

Drezner (1985) in his paper conducted some sensitivity analyses for the single facility problem. He studied various variants of the problem with weight restrictions and location restrictions.

2.1. Location-Allocation Problems

Since 1963, when the first location-allocation model was formulated by Cooper (1963), there has been extensive research on the field. The simplest location-allocation problem is the Weber problem addressed by (Friedrich, 1929). This paper discussed the steps in locating a machine so as to minimize the sum of the weighted distances from all the raw materials sources. The seminal work in this area was on the p-median problem, initially formulated by (Hakimi, 964). The median problem was considered on a graph and the objective function was to reduce the average or the sum of the transportation costs from the service facility to the demand locations. It was derived that one of the optimal solutions locates the service facility on one of the nodes of the network.

2.2. K – Centra Problems

Most of the research in the field of location analysis focuses on the minimization of the average (or total) distance (median function) or the minimization of the maximum distance (center function) to some service facility. Approaches based on the median model are primarily concerned with achieving maximum spatial efficiency. This approach generally provides solutions in which remote and low – population density areas are neglected in favour of providing maximum accessibility to centrally situated and high – population density areas. To avoid this drawback, the center method can be applied which primarily addresses geographical equity issues compromising optimum spatial efficiency. Sometimes locating a facility using the center method doesn't satisfy all the complex underlying constraints a location problem is associated with. These factors led to more research in lieu of obtaining a compromise solution concept.

Halpern (1976) first introduced the cent – dian model as a parametric solution concept based on the bicriteria center/median model. Halpern modeled the problem in such a way that a compromise was achieved between median and centre objective functions such that the inherent objective function characteristics of both the problems are considered while solving. The two objective functions considered are total distance minimization and the maximum distance minimization criterions. The goal here was to find an optimal balance between efficiency (least - cost) and equity (worst - case). However, this particular method can sometimes fail to provide a solution to a discrete location problem mostly due to the limitations involved with direct combinations of two different functions.

Hansen et al. (1991) introduced a variation of the cent – dian problem in the generalized center problem, which minimizes the difference between the maximum distance and the average distance. This model can be extended to formulate solutions for multiple facility location problems on a plane as well as on a network. This model can also be applied to discrete location problems.

The k – centra problem concept was formulated by (Slater, 1978). The k – centra model combines both the center as well as the median concepts by minimization of the sum of the k largest distances. If $k = 2$ the model reduces to a standard center problem while with $k=n$ it becomes a standard median problem. This paper concentrated on the discrete single facility location problem on a tree graph.

Peeters (1998) studied the k – centrum model and introduced a full classification of the k – centrum criteria and some solution concepts. He proposed two different variations on the median and the center functions each. The functions considered were upper k – median where the sum of the k largest distances are minimized, lower k – median where the sum of the k smallest distances are minimized, upper k – center where the k largest distances are minimized, and lower k– center where the k smallest distance are minimized.

The k–centrum model is generally reserved for unweighted problems. However, significant research has been performed to show that satisfying the above criteria is not always necessary. Recently, Tamir (2001) solved a weighted multiple facility k – centrum problem on paths and tree graphs using simple polynomial time algorithms. In this method, weights are assigned to all the distances from the new location to the existing locations and the distances are scaled accordingly.

Ogryczak et al. (2002) in their paper introduced the conditional median method which is an extension of the k – centrum concept when applied to weighted problems. The paper proposes that a k – cenrum problem can be evaluated for optimality by just considering only that specific part of the demand which is in direct proportion to the existing largest distances. Thus this concept solves the objective function for the entire portion of the largest distances for a specified portion of the demand.

Less number of researches exists on a solution to the planar *k*- centra location problem. Indeed, it was this identified gap in the literature that led to this undertaking. Hence, this facility location literature review is relatively brief and will only address citations related to the *k*-centra location problem. For a large taxonomy and literature review on facility location in general, the reader is referred to (Brandeau et al., 1989). Francis et al. (1983) presented a survey paper in location analysis, which defined four classes of location problems and described algorithms to optimize them. For a more recent review of the facility location Landscape, Hale et at. (2003) wrote extensively on Location science research.

Saddath (2005) and Saddarth et al. (2008) also wrote extensively on the planar k-centra facility Euclidean problem in their various papers.

Fernandez et al. (2009) used the Planar Location Algorithm to locate a facility in the city of Murcia in South-East Spain.

Chapter 3

3. METHODOLOGY

3.1. Problem formulation and the Weiszfeld’s Algorithm

Model Formulation

The Weiszfeld’s algorithm is the objective function used to minimize the sum of the distances between the existing demand locations and the new facility location. Weiszfeld’ algorithm can be used to solve planar as well as network problems considering two – dimensional or three – dimensional coordinates. Weiszfeld’s algorithm iteratively solves for the minisum location to the Weber problem based on its objective function as shown in Equation 3.1.

Minimize $f(x,y) = \sum w_i \sqrt{(x - a_i)^2 + (y - b_i)^2}$ ………………………(3.1)

where,

w_i = Weights associated with the existing locations

x = x coordinate of the starting solution and later obtained by successive iterations

y = y coordinate of the starting solution and later obtained by successive iterations.

A_i = x coordinate of existing locations, b_i = y coordinate of existing locations.

Equation 3.1 is the sum of the weighted distance.

The Weiszfeld’s Algorithm

The steps followed in Weiszfeld’s Algorithm to solve a planar Euclidean location problem are as listed below:

Step1: Input initial coordinates (x,y).

Step 2: Solve for every γ_i as per Equation 3.2.

$$\gamma_i = \frac{w_i}{\sqrt{(x - a_1)^2 + (y - b_1)^2}} \qquad \text{………………….(3.2)}$$

Where, x=x-coordinates of starting point for the iterative algorithm.

y=y-coordinates of starting point for the iterative algorithm.

Step 3: sum for every γ_i for Γ(x,y) shown by Equation 3.3.

$$\Gamma_1(x,y) = \sum_{i=1}^{m} \gamma_i\,(x,y) \qquad \text{...……(3.3)}$$

Step 4: determine all $\lambda_i = \gamma_1/\Gamma$ as shown in Equation 3.4.

$$\lambda_i(x,y) = \frac{\gamma_i\,(x,y)}{\Gamma(x,y)} \qquad \text{…………..(3.4)}$$

Step 5: Weiszfeld's (x) = WFx $= \sum_{i=1}^{m} \lambda_i . a_i$ ……………(3.5)

Weiszfeld's (y) = WFy = $= \sum_{i=1}^{m} \lambda_i . b_i$ …………….(3.6)

Step 6: Determine the objective function value by summing the individual objective function values for all i as per Equation 3.7.

$$f_{euc} = \sum w_i \sqrt{(\text{WFx} - a_i)^2 + (\text{WFy} - b_i)^2} \qquad \text{………(3.7)}$$

Step 7: Repeat until stopping conditions are met.

Considering a random starting solution, the algorithm is evaluated to obtain a second set of trial values (WFx, WFy). The obtained coordinates are substituted to obtain

the third set of trial values and so on. By reiterating the Weisfeld's values for x, y, we can find the Euclidean solution close to or equal to the optimal solution. The stopping conditions are met when the difference between subsequent objective function values obtained is less than or equal to zero.

3.1.1. Solved Example for Weiszfeld's Algorithm

The objective of the Weiszfeld's algorithm is to find the minisum solution to a location problem. The example problem considered here is a non – weighted planar problem with coordinates listed in Table 3.1.

Column one is the points, column two is the x-coordinates and column three is the y-coordinates.

Table 3.1. Weiszfeld's Example Coordinates

Points	$\mathbf{a}_i$	$\mathbf{b}_i$
P1	2	3
P2	0	8
P3	9	3
P4	6	2
P5	7	2
P6	1	5

Considering the initial solution as (x,y) = (3,2). This value could be selected as any value depending on the user's discretion. To reduce the number of iterations required to solve the object function, the starting point based on visual judgment.

$(x,y) = (3,2)$

By substituting the value of (x,y) in Equation 3.2 we get the following γ_i values:

$$\gamma_i\,(x,y) = \frac{1}{\sqrt{(3-2)^2 + (2-3)^2}} = 0.707$$

γ_2 (x,y) = 0.149

γ_3 (x,y) = 0.164

γ_4 (x,y) = 0.333

γ_5 (x,y) = 0.25

γ_6 (x,y) = 0.277

The above γ values are summed up to obtain the Γ value.

$\Gamma(x,y)$ = (0.707+0.149+0.164+0.333+0.25+0.277) = 1.881

Using Γ and the corresponding γ value, λ values were calculated for each of the existing facility locations.

$$\lambda_1 (x,y) = \frac{0.707}{1.881} = 0.375$$

λ_2 (x,y) = 0.0792

λ_3 (x,y) = 0.0873

λ_4 (x,y) = 0.177

λ_5 (x,y) = 0.132

λ_6 (x,y) = 0.147

Using the obtained values, Weiszfeld coordinates (WFx, WFy) are calculated using the Equations 9 and 10.

$$WF(x) = \sum_{i=1}^{m} \lambda_i * a_i$$

= (0.375*2) + (0.0792*0) + (0.0873*9) + (0.177*6) + (0.132*7) + (0.147*1)

= 3.14

$$WF(y) = \sum_{i=1}^{m} \lambda_i * b_i$$

$= (0.375*3) + (0.0792*8) + (0.177*2) + (0.132*2) + (0.147*5)$

$= 3.201$

By using the trial solution coordinates, calculate the individual function values for all *i*. The sum of the obtained individual function values would give the objective function value for trial solution 2 as shown in Equation 3.7.

$f_{eucPi} = \sqrt{(WF(x) - a_i)^2 + (WF(y) - b_i)^2}$

$f_{eucP1} = \sqrt{(3.14 - 2)^2 + (3.201 - 3)^2} = 1.157$

$f_{eucP2} = 5.734$

$f_{euP3} = 5.863$

$f_{euP4} = 3.101$

$f_{euP5} = 4.042$

$f_{euP6} = 2.795$

$f_n = \sum f_{euP1} = 22.695$

The procedure described above is repeated until stopping conditions are achieved. Stopping condition is defined below;

$$\left| f_{eucn+1} - f_{eucn} \right| \leq 0$$

3.2. Application of Weiszfeld Algorithm to k – Centra Problem

The objective of this thesis is to solve a single facility, unweighted, planar k – centra problem with an aim to locate a library in the Sunyani Municipality to minimize the distance to the k – farthest customer points among several existing points. The input required for the problem includes the coordinates for the existing customer locations as well as *k* value. This *k* value depends on the type of problem, the type of coverage required, the existing customer locations on the plane or the discretion of the user.

The solution methodology used in this thesis is an iterative heuristic method utilizing Weiszfeld's algorithm. The solution steps are listed below:

- Step 1: Find the minimax solution for all the existing customer locations. This minimax solution is used as starting point for following iterations.
- Step 2: The next step is to find the distances of all existing locations from the minimax solution. The top k distances are considered to find the k farthest locations points.
- Step 3: the next step is to use the minisum approach to evaluate the obtained k farthest points. Weiszfeld's algorithm is applied to solve the minisum aspect. After the first iteration, a new location point is obtained.
- Step 4: the step is to check whether the same k points still are the points lying farthest away.

    - If so, the solution obtained is the k – centra solution.
    - If not, reiterate by considering the new k-largest distances and the corresponding existing points.
    - Stopping conditions are met when you end up with the same k-farthest points you started with.

The procedure is illustrated with a flowchart as shown in figure 3.1. However there are chances that the algorithm may go in a loop. This happens when the solution comes up with a different set of k points and this keeps repeating. This leads to an infeasible solution. Generally it is observed that the infeasibility is due to a single point getting displaced from the initial set of k points. The only solution possible here is a semi-optimal solution which neglects the coordinates of the rogue point. This point is better explained in the later part of the thesis where a MATLAB code for the discussed k- centra approach is provided.

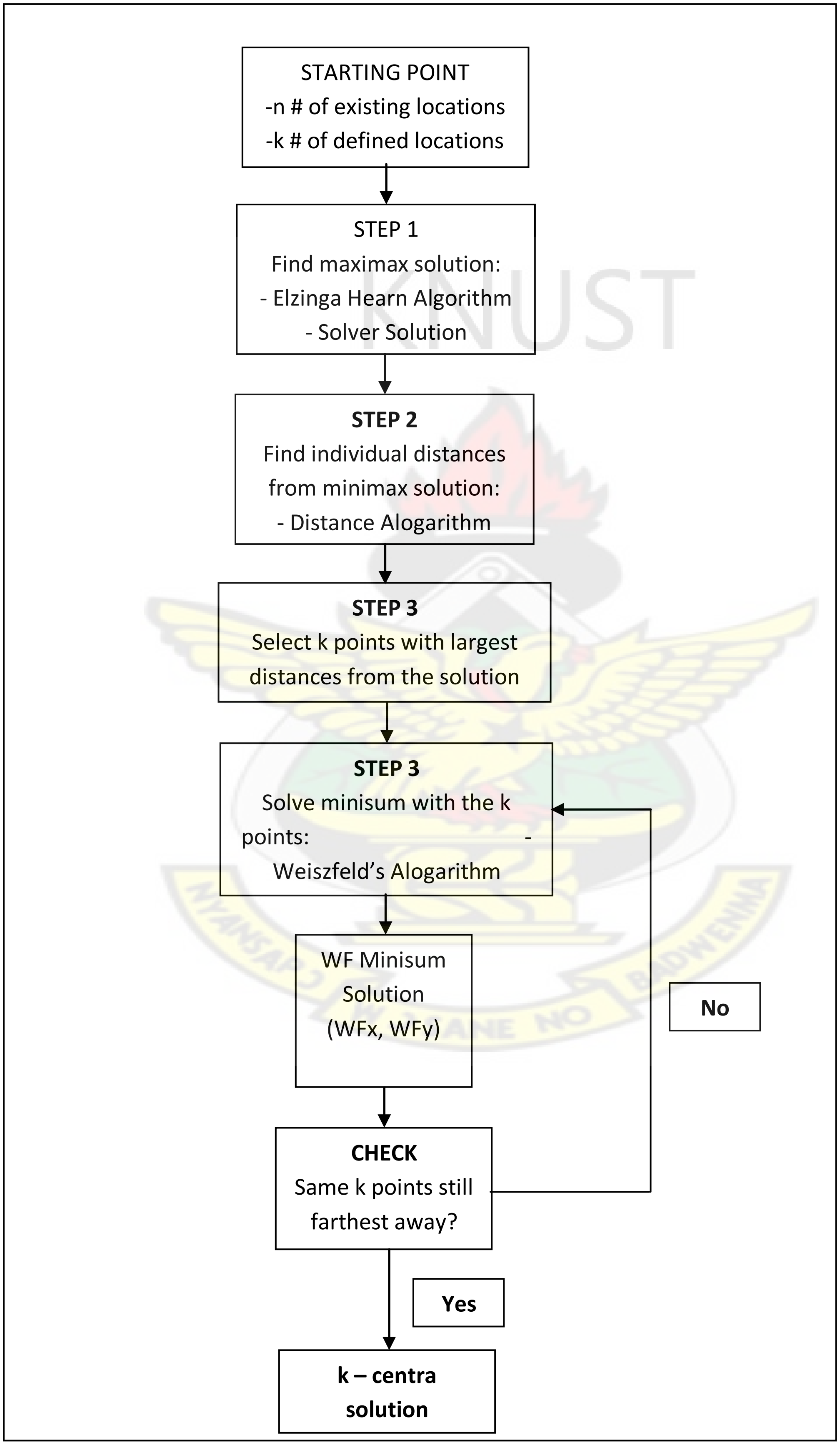

STARTING POINT
-n # of existing locations
-k # of defined locations
STEP 1
Find maximax solution:
- Elzinga Hearn Algorithm
- Solver Solution
STEP 2
Find individual distances from minimax solution:
- Distance Alogarithm
STEP 3
Select k points with largest distances from the solution
STEP 3
Solve minisum with the k points: - Weiszfeld's Alogarithm
WF Minisum Solution (WFx, WFy)
No
CHECK
Same k points still farthest away?
Yes
k – centra solution

Figure 3.1. A Flowchart Showing k-centra Solution Approach

3.3. Solved Examples Illustrating Real Life Location Scenarios.

Three small location problems were solved in this thesis using the k – centra methodology as discussed in the previous section. The analogy used was real life area planning problems in cities. There are numerous examples of planned and unplanned city layouts. Older cities generally fall in the unplanned category while newer cities are methodically planned depending on population density, topographical features and business sectors. Solution methodology for one of the considered examples is discussed in detail while the other two problems are explained briefly.

3.3.1. Problem with Random Existing Locations

This is most common layout seen in most cities all over the world. A single – facility planar location problem associated with such a layout presents one of the most complex and time – consuming problems in the field of location analysis. However, using the k- centra approach discussed in this thesis, only a selective few points are randomly evaluated out of all the existing locations. Thus the computing time as well as complexity of the problem is reduced. The problem considered is a 15 point problem as shown in table 3.2 The problem is evaluated for different values of k (k=1to 15). The flowchart as shown in figure 3.1 is used as the solution guideline for evaluating the problem.

Table 3.2. Problem of Random cities

Points	**a$_i$**	**b$_i$**
P1	2	2
P2	2	4
P3	5	2
P4	3.5	4
P5	7	4
P6	9.5	4.5
P7	10.5	2
P8	11.5	5.5
P9	9.5	8
P10	7	6.5
P11	5	6.5
P12	2	9
P13	5.5	8.5
P14	8.5	9
P15	11	9.5

The graphical representation is shown in Figure 3.2. As the graph suggests, the existing points are randomly located on a planar layout. The aim is to locate a single service facility in the plane so as to minimize the sum of the distance to the *k* farthest points. The data required for this problem includes the coordinates for existing customer locations and a value for *k*. *k* value could depend on the nature of the problem or it could be based on logical reasoning and demand flow.

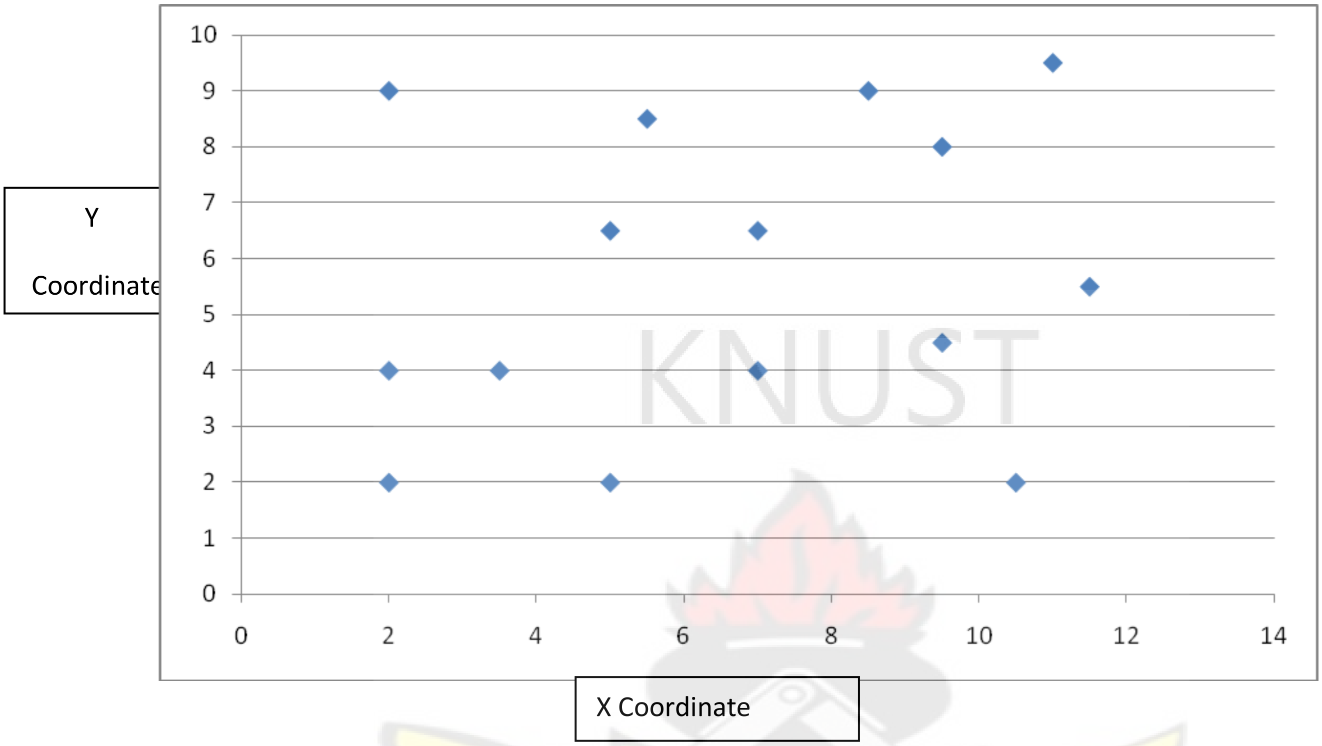


Figure 3.2. Graphical Representation of an Unplanned City layout

The first step in the discussed methodology is obtaining the minimax solution considering Euclidean metric. Two different methods were used to obtained the minimax solution. The first method is a graphical solution based on the Elzinga-Hearn algorithm. The Elzinga –Hearn algorithm is a graphical method for solving the minimax objective function for a location problem. The method funds is the smallest circle covering method where the objective is to find the smallest possible circle which encompasses all the existing locations in the plane. The method is shown in the flowchart below.

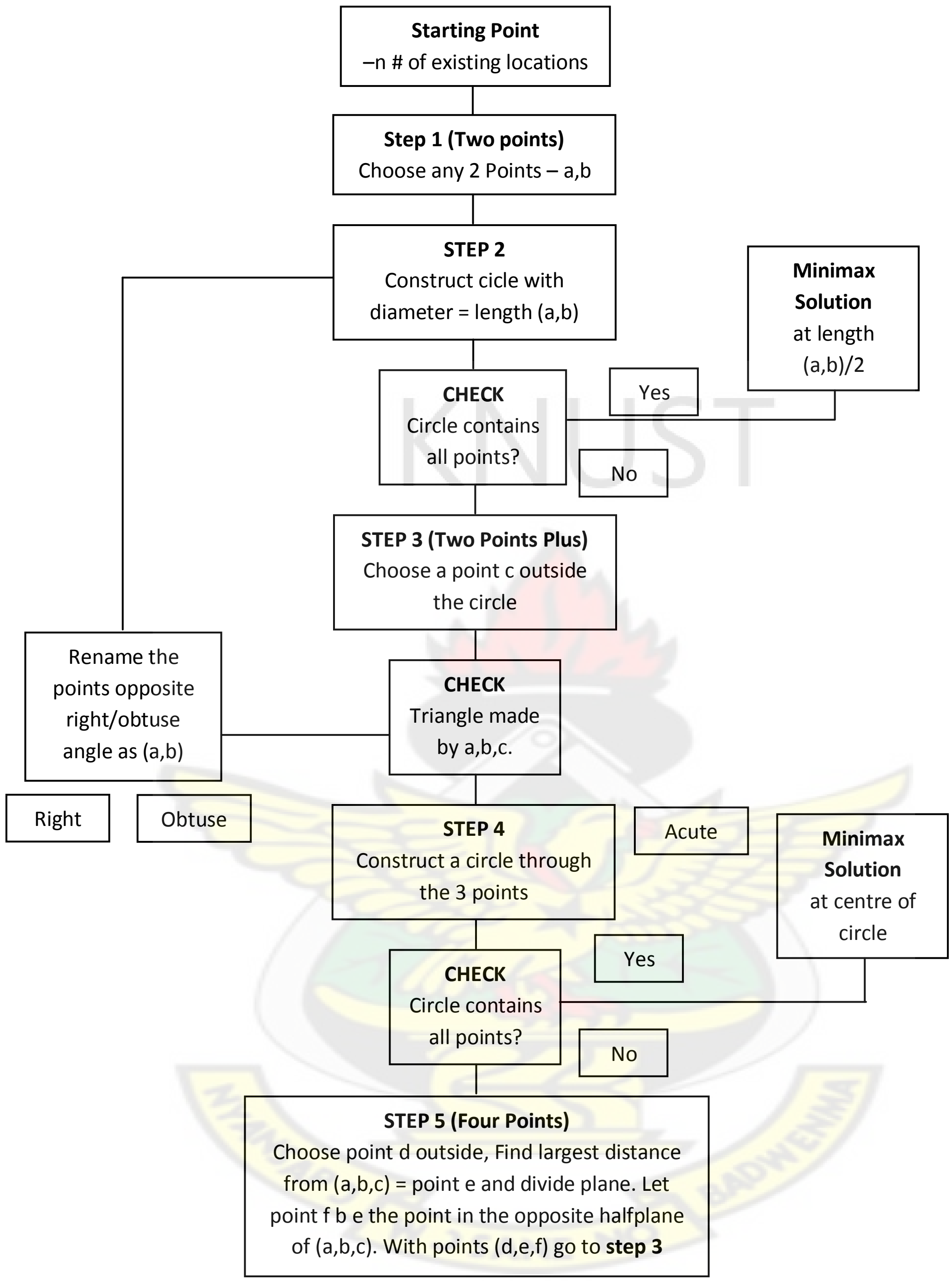


Figure 3.3. Flowchart for the Elzinga – Hearn Algorithm

Table 3.3. Elzinga – Hearn Algorithm

Procedure Called	Defining Points	Outside Point	Radius
Two points	1,3		1.5
Two Points	1,3	2	
Two Points Plus	1,3,2		1.8
Four Points	1,3,2	5	
Two Points Plus	1,2,5		2.75
Four Points	1,2,5	6	
Two Points Plus	1,2,6		4
Two Points	1,6	9	
Two Points Plus	1,6,9		4.9
Two Points	1,9	15	
Two Points Plus	1,9,15		6
Two Points	1,15	None	

The minimax solution is obtained at $x = 6.5$ and $y = 5.75$. Excel solver was also used to obtain the minimax solution and verify the results obtained by using the Elzinga – Hearn algorithm. The next step is to find the distance between the calculated minimax solution and the all the other existing location points. This provides the starting point for the k – centra algorithm. Distance algorithm is used for this computation. The distances are sorted in descending order as shown in Table 3.4. The top distances are considered and the problem is evaluated for just the obtained k farthest points.

Table 3.4. Distance Algorithm

Points	Distance
P1	5.857687
P15	5.857687
P12	5.550901
P7	5.482928
P8	5.006246
P2	4.828302
P3	4.038874
P14	3.816084
P9	3.75
P4	3.473111
P6	3.25
P13	2.926175
P5	1.820027
P11	1.677051
P10	0.901388

Considering k=6 for this illustration, points P1, P15, P12, P7, P8 and P2 are the points farthest away from the minimax solution (6.5,5.75). Considering the minimax solution as the starting solution, Weiszfeld's algorithm is used to solve the minisum objective function. The Weiszfeld's function is shown in Equation 3.10.

$$Minimize\ f(x,y) = \sum w_i \sqrt{(x - a_i)^2 + (y - b_i)^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(3.1)$$

The next step is for solve for every γ_i as shown in Equation 15. x and y are the starting solution coordinates.

$$\gamma_i = \frac{1}{\sqrt{(x - a_i)^2 + (y - b_i)^2}} \qquad \ldots\ldots\ldots\ldots\ldots(3.2)$$

The γ_i values obtained are shown in Table 3.5

Table 3.5. γ_i Values

γ1	0.17072
γ12	0.18015
γ2	0.20711
γ8	0.19975
γ7	0.18238
γ15	0.17072

The next step is to obtain the $\Gamma(x,y)$ values by summing for every γ_i. The value of $\Gamma(x,y)$ comes out to be 1.1108 in this case. The $\Gamma(x,y)$ value calculation is shown in Equation 3.3.

$$\Gamma(x,y) = \sum_{i=1}^{m} \gamma_i (x,y) \qquad \ldots\ldots\ldots(3.3)$$

The next step to determine all λ_i values as shown in Equation 3.4. The values obtained are listed in Table 3.6.

$$\lambda_i(x,y) = \frac{\gamma_i(x,y)}{\Gamma(x,y)} \qquad \ldots\ldots\ldots\ldots.. (3.4)$$

Table 3.6. λ_i values

λ_1	0.15368
λ_{12}	0.16218
λ_2	0.18645
λ_8	0.17982
λ_7	0.16419
λ_{15}	0.15368

The final step is to find the Weiszfeld's values for x and y and the objective function value. The formulae are listed in Equations 3.5, 3.6, and 3.7. The WF values are presented in Table 3.7.

Weiszfeld's (x) = WFx $= \sum_{i=1}^{m} \lambda_i . a_i$ ………… (3.5)

Weiszfeld's (x) = WFy = $= \sum_{i=1}^{m} \lambda_i . b_i$ …………. (3.6)

Objective function value: $f_{euc} = \sum w_i \sqrt{(x-a_i)^2 + (y-b_i)^2}$ ……….. (3.7)

Table 3.7. WF values

WF(x)	**WF(y)**
6.4876	5.29013

The objective function values are shown in Table 3.8.

Table 3.8. Objective Function Values

$f_{euc\mathrm{P1}}$	5.5641
$f_{euc\mathrm{P12}}$	5.8221
$f_{euc\mathrm{P2}}$	4.6688
$f_{euc\mathrm{P8}}$	5.0174
$f_{euc\mathrm{P7}}$	5.1893
$f_{euc\mathrm{P15}}$	6.1717
f_{euc}	32.4334

Using the distance algorithm to find the distance from the obtained minisum solution to all existing locations, it is observed that the same six points are farthest away. In this case, this solution is feasible and optimal for k = 6. Thus the k-centra solution for this problem lies at x = 6.4876 and y = 5.29013.

CHAPTER 4

DATA COLLECTION AND ANALYSIS

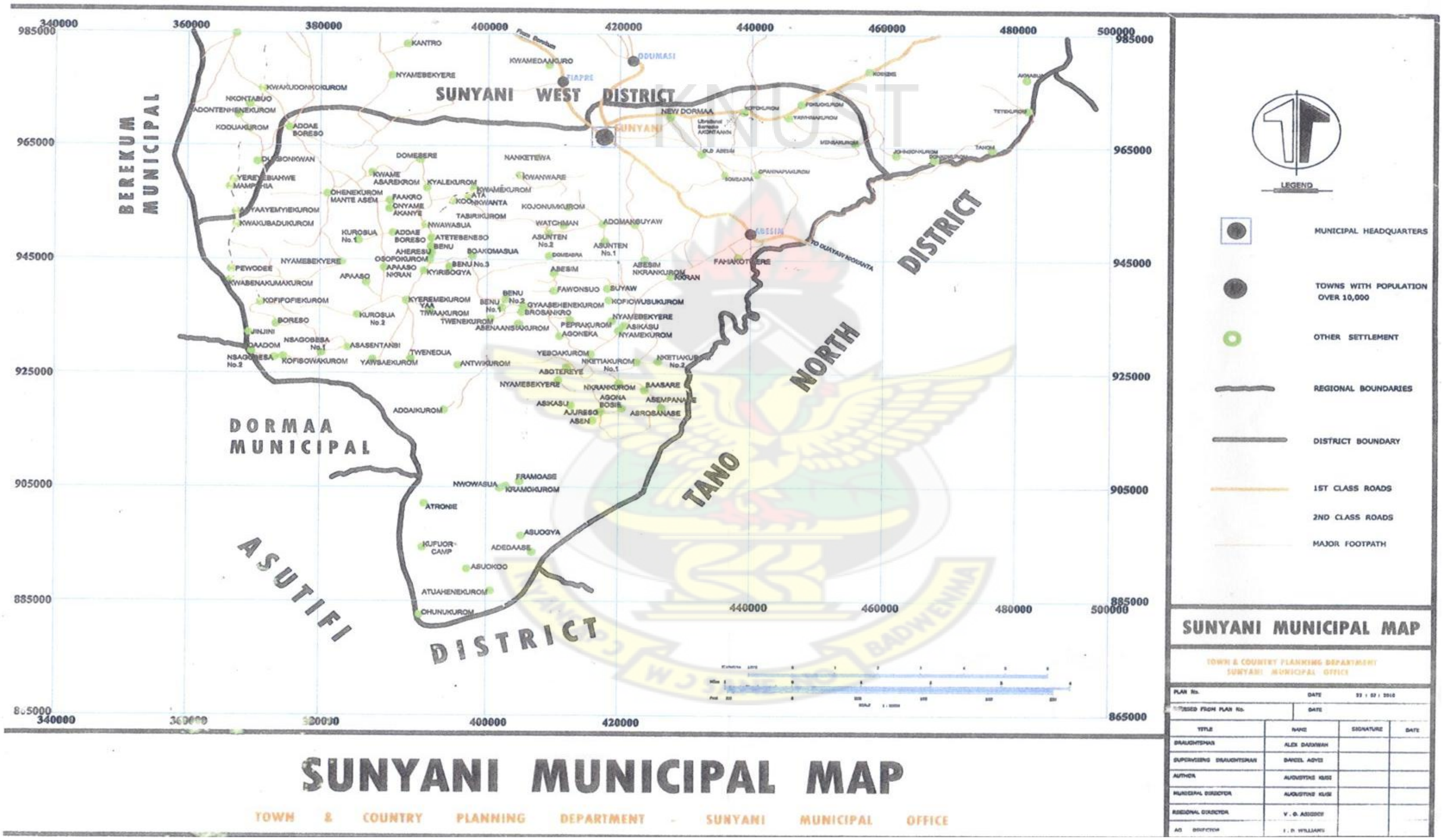
4.1. Data Collection

The Government Agencies that were contacted for the primary data and other important information for this thesis were the Regional Lands Office(Sunyani), Sunyani West District Assembly, Survey Department and Sunyani Municipal Town and Country Planning Department where the map for the Sunyani Municipal was obtained.

4.2. Sunyani Municipal Map with Its Towns And Suburbs

The Sunyani Municipal map obtained from the office of the Town and Country Planning Department which was used for the thesis has been captured showing its towns and suburbs. Figure 4.1 shows the Sunyani Municipal Map with its towns and suburbs.

Figure 4.1. The Map of Sunyani Municipal showing Towns and Suburbs.



4.3. Data Processing

The rectangular co-ordinates of the towns and suburbs were obtained with the help of a grid sheet since the co-ordinates could not be found on the internet (Google map and Microsoft Encarta).The towns and suburbs were coded as numbers from 1 to 100. The Liberation Barracks was labeled as a suburb. The co-ordinates were converted from hundred thousands to hundreds. E.g. The co-ordinate for Ohunukurom becomes (392 , 882) instead of (392000 , 882000).

Table 4.1 shows a list of 100 towns and suburbs with their number codes. The completed list of the 100 towns and suburbs are listed in appendix D.

Table 4.1; Table Showing number codes for towns and suburbs and their various x, y coordinates.

NUMBER	TOWNS AND SUBURBS	X(a_i) in km	Y(b_i) in km
1	OHUNUKUROM	392	882
2	ATUAHENEKUROM	401	887
3	ASUAKOO	397	891
4	KUFUOR CAMP	393	895
.	…….	…	…
99	FOKUOKUROM	448	974
100	LIBERATION BARACKS	434	941

4.4. Model Formulation

$f_{euc} = \sum w_i \sqrt{(WFx - x)^2 + (WFy - y)^2}$ Where ,

x values are in column 3 of Table 4.1.

y values are in column 4 of Table 4.1.

$W_i=1$ $i=1, 2, 3,\ldots,100$

4.4.1. The Planar k-centra Algorithm

The hundred rectangular co-ordinates were used as the inputs for the Planar k-Centra Single-Facility Euclidean Location Problem Algorithm coded in matlab. The steps for the algorithm is shown below;

Step1: Input initial coordinates (x,y), the minimax.

Step 2: Solve for every γ_i as per Equation 2.

$$\gamma_i = \frac{w_i}{\sqrt{(x - a_i)^2 + (y - b_i)^2}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.1)$$

Where,

x=x-coordinates of starting point for the iterative algorithm

y=y-coordinates of starting point for the iterative algorithm

Step 3: sum for every γ_i for $\Gamma(x,y)$ shown by Equation 3.

$$\Gamma_1(x,y) = \sum_{i=1}^{m} \gamma_i (x,y) \qquad \ldots\ldots\ldots(4.2)$$

Step 4: determine all $\lambda_i = \gamma_i/\Gamma$ as shown in Equation 4.3

$$\lambda_i(x,y) = \frac{\gamma_i (x,y)}{\Gamma(x,y)} \qquad \ldots\ldots\ldots\ldots..(4.3)$$

Step 5: Weiszfeld's (x) = WFx $= \sum_{i=1}^{m} \lambda_i.a_i$ ……………(4.4)

Weiszfeld's (y) = WFy $= \sum_{i=1}^{m} \lambda_i.b_i$ ……………(4.5)

Step 6: Determine the objective function value by summing the individual objective function values for all *i* as per Equation 3.7.

$$f_{euc} = \sum w_i \sqrt{(WFx - a_i)^2 + (WFy - b_i)^2} \qquad \ldots\ldots\ldots(4.6)$$

Step 7: Repeat until stopping conditions are met.

Considering a random starting solution, the algorithm is evaluated to obtain a second set of trial values (WFx, WFy). The obtained coordinates are substituted to obtain the third set of trial values and so on. By reiterating the Weisfeld's values for x, y, we can find the Euclidean solution close to or equal to the optimal solution. The stopping conditions are met when the difference between subsequent objective function values obtained is less than or equal to zero.

4.5. Computational Procedure

Matlab program software (Siddarth, 2005) was used for the coding of the Planar k-Centra Single-Facility Euclidean Location Problem algorithm.

The codes were developed and ran on the Intel(R) Pentium(R) Dual CPU T2370, 32 BG Operating system, 1014 MB RAM, 1.73 GHZ speed, with Windows Vista laptop computer. The code runs successfully on the windows vista.

The number of iterations was 100 and 4 test runs were carried out.

4.6. Results

The minimax location was (401500km , 944000km). The algorithm gave two locations as the optimal points. These are (399830.7km , 941547.6km) and (406647.9km , 935589.2km) with 1815,000km and 1811,500km as their respective objective function values (The sum of the distances of 50 farthest locations away from the optimal location).

4.7. Discussion of Results

This work is similar to the facility located by Fernandez, et al. (2009) in the city of Murcia in South-East Spain.

The algorithm gave two locations as the optimal points. These are (399830.7km , 941547.6km) and (406647.9km , 935589.2km) with 1815,000km and 1811,500km as their respective objective function values (The sum of the distances of 50 farthest locations away from the optimal location). Considering the fact that 1811500km < 1815000km the optimal point should be (406647.9km , 935589.2km).

The Scatter plot below indicates the diamond figured minimax (401.5, 944.0), the two asterisks points are the more optimal point (406.6479, 935.5892) and other optimal point (399.830, 941.5476). The dots are the 100 points for the Towns and Suburbs of the municipality.

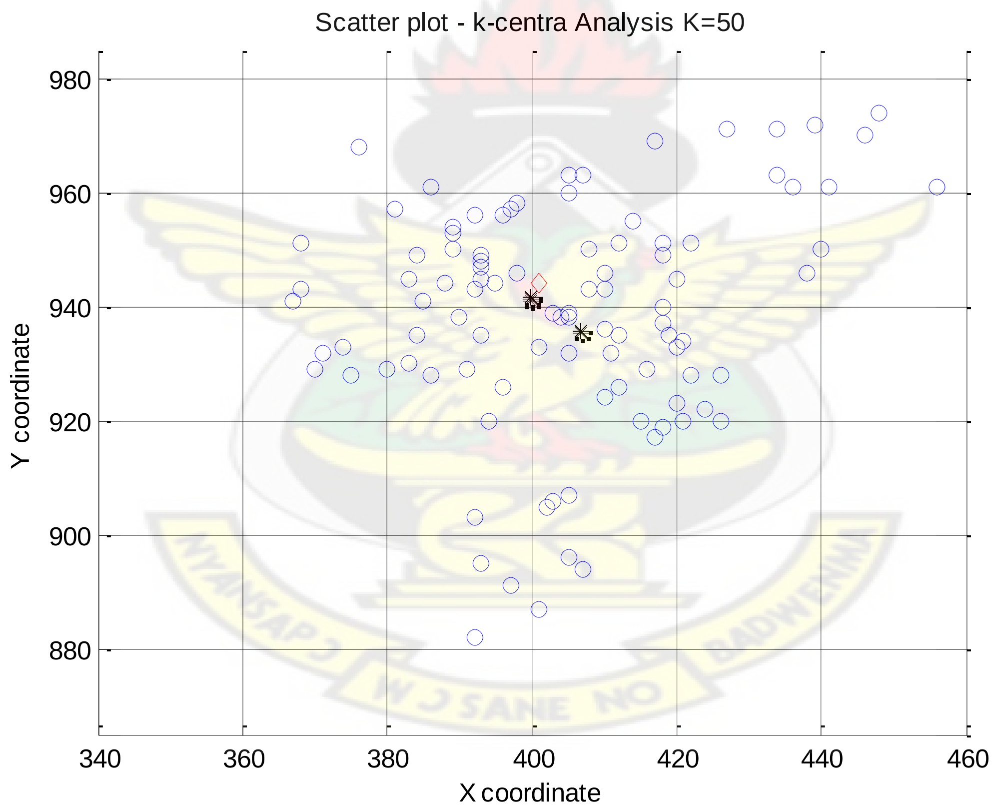


Figure 4.2. This figure is showing the 100 scatter points, the diamond figured point is the minimax and the two asterisks are the two different optimal points.

CHAPTER 5

CONCLUSION AND RECOMMENDATION

5.1. Conclusion

The Planar k-Centra Single-Facility Euclidean Location Problem Algorithm has been successfully applied using the rectangular co-ordinates of the towns and suburbs of the Sunyani Municipality to locate a library. This work is similar to the facility located by Fernandez, et al. (2009) in the city of Murcia in South-East Spain. Matlab codes were written to determine the strategic location. The algorithm generated two locations. These are (399830.7km , 941547.6km) and (406647.9km , 935589.2km) with 1815000km and 1811500km as their respective objective values. Here, there is the need to select one location as the more optimal location based on the one with the lowest objective function value, since we are minimizing the sum of the distances of the 50 farthest customer locations from the optimal location.

Considering the fact that 1811500km < 1815000km the optimal point should be (406647.9km , 935589.2km). Hence the proposed community for the library is Benu No. 2.

5.2. Recommendation

Based on the study, the following recommendations are made:

The library should be located in the town and suburb coded 55 (Beno No. 2 community).

This work should serve as basis for further research in the area of Planar k-Centra Single-Facility Euclidean Location Problem.

Since Sunyani Municipality was used as the case study, the researcher recommends Benu No. 2 to the Municipal Assembly, contractors, urban and feeder roads and other developers that the community is one of the most closest to the 50 farthest towns and suburbs in the municipality.

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APPENDIX A-MATLAB METHODOLOGY

1. MATLAB is a high-level computer language recognize worldwide in the mathematical commu Ogryczak, W. and Zawadzki, M. (2002) “Conditional Median: A Parametric Solution Concept for Location Problems.” Annals of Operations Research, 110: 167-181.

nity for its computational power in performing various complex mathematical tasks. The MATLAB package is available with many software add-ons to solve a multitude of problems in various application areas. It is an interactive tool with options for 2-D or 3-D visualization of input or output data. MATLAB can be integrated with various other software applications to suit every computational need.

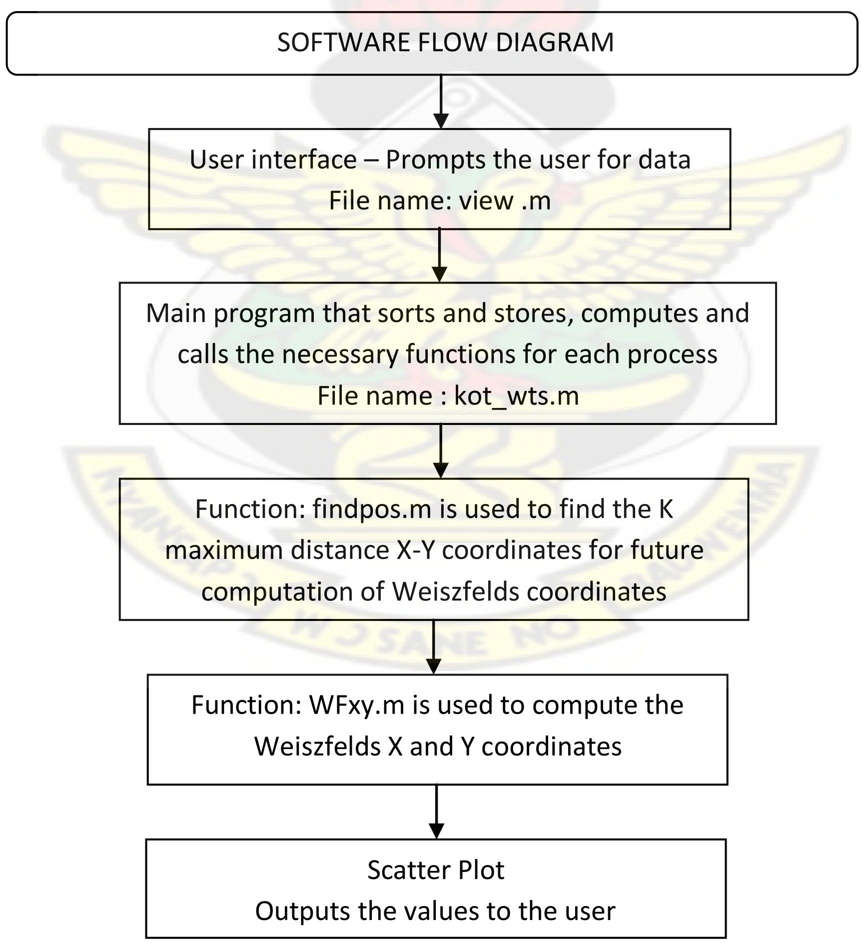
MATLAB Application to the k-Centra Problem

The advantage of using MATLAB to solve a location problem over traditional manual methods is better accuracy as well as increased computational speed. The manual method for evaluating k-centra problems is a very tedious one involving much iteration to get a more accurate solution. The other advantage is you can massage the code to suit your computational needs. The code discussed later in this appendix c has input options which are user-defined like the number of existing locations, *value* of *k*, number of iterations, search space boundaries, and search space increment. The code has been programmed to output the coordinate solution in a graphical representation. You can add a lot more features like inputting coordinates data directly from other applications, 3-D visualization, report generation, etc.

The same logic used for manually solving a k-centra problem is used to code the MATLAB program.

Data Flow Logic and MATLAB Functions

The k-centra example problems involved were solved in MATLAB 7.5.0(R2007B), in the following manner. The user interface is a basic MATLAB program that prompts the user to input various data point coordinates and other parameters. User input includes the number of existing locations (N) and respective coordinates entered one by one (x and y separately), maximum range of x and y, search space increment, k-term, and number of iterations. The added advantage is that this can be modified easily as per the user's requirement and future upgrades. This program then calls the main program which stores the user inputted data, saves and computes the various distances and k-centra parameters by calling the respective functions.



Software Flow Diagram

APPENDIX B-MINIMAX LOGIC

The minimax function forms an important part of the logic flow for this program. The minimax function is called upon after the “View” function and uses user input as its starting variables. Data used for the function includes “N” number of existing locations and corresponding coordinates. Consider the N data points to be Data 1, Data 2…….Data N. The user reference is initiated as Xref and Yref and is started at (0, 0) and is incremented on the basis of user input. The maximum range threshold is also set by the user (Xmax and Ymax). The distance or range for each data point from this reference is then computed using the distance formula as shown by Equation * below;

$$DN_{xy} = \sqrt{(x_{ref} - x)^2 + (y_{ref} - y)^2} \qquad \text{………(*)}$$

The distances are represented as $D1_{oo}$, $D1_{01}$…, $D1_{20\ldots}$, $D1_{x0\ldots}$, DN_{xy}. The tabular representation is shown in table below;

Minimax Data Representation

Xref	**Yref**	**Data 1**	**Data 2**	**.. Data ..**	**Data N**	**Max**
0	0	$D1_{00}$	$D2_{00}$	..	DN_{00}	$Dmax_{00}$
0	1	$D1_{01}$	$D2_{01}$	..	DN_{01}	$Dmax_{01}$
0	2	$D1_{02}$	$D2_{02}$	..	DN_{02}	$Dmax_{02}$
..	..	..	..	..	..	
0	Ymax=y	$D1_{0y}$	$D2_{0y}$	..	DN_{0y}	$Dmax_{0y}$
1	0	$D1_{10}$	$D2_{10}$	..	DN_{10}	$Dmax_{10}$
1	1	$D1_{11}$	$D2_{11}$	..	DN_{11}	$Dmax_{11}$
1	2	$D1_{12}$	$D2_{12}$	..	DN_{12}	$Dmax_{12}$
..	..	..	..	..	..	
2	0	$D1_{20}$	$D2_{20}$	..	DN_{20}	$Dmax_{20}$
2	1	$D1_{21}$	$D2_{21}$	..	DN_{21}	$Dmax_{21}$

..	..	..	..	..	..	
..	..	..	..	..	..	
Xmax = x	Ymax = y	$D1_{xy}$	$D2_{xy}$	..	DN_{xy}	$Dmax_{xy}$

After each increment, and progressively stepping through the search grid, the program computes the maximum of each of these distances and is represented in the program as $Dmax_{00}$…, $Dmax_{xy}$ shown in table below

Maximum Distance Matrix

$Dmax_{00}$	$Dmax_{01}$	…….	$Dmax_{oy}$
$Dmax_{10}$	$Dmax_{11}$	…….	$Dmax_{1y}$
$Dmax_{20}$	$Dmax_{21}$	…….	$Dmax_{2y}$
……..	…….	…….	…….
$Dmax_{x0}$	$Dmax_{x1}$	…….	$Dmax_{xy}$

The minimum values from each of the values are calculated and forms an intermediate step in the calculation. The matrix is shown in table;

Intermediate Distance Matrix

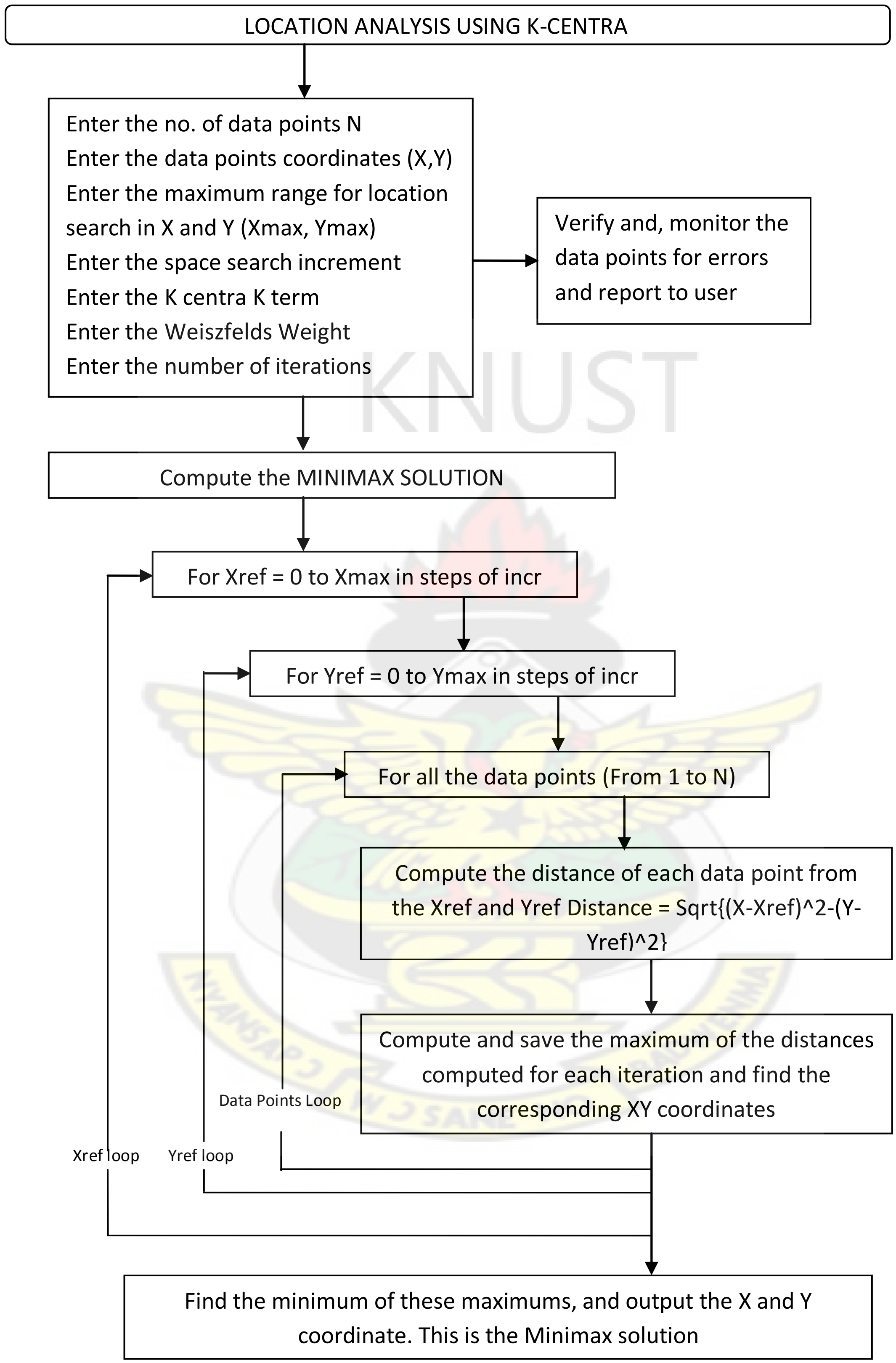
Min of $Dmax_{00}$: $Dmax_{x0}$	Min of $Dmax_{01}$: $Dmax_{x0}$	……..	Min of $Dmax_{0y}$: $Dmax_{xy}$

The next step is to find the minimum of these intermediate minimums. This is the minimax solution distance. We can trace the (Xref, Yref) coordinate where this occurs as the maximum distances matrix directly represents the search grid of (Xref, Yref) coordinates as shown in Table 14 below, and the user data points (D1, D2….DN) will also lie inside this grid.

Search Grid Coordinates

00	01	02	…….	0y
10	11	12	…….	1y
20	21	22	…….	2y
….	….	…..	…….	…..
x 0	x 1	x 2	…….	Xy

Thus the minimax solution, its coordinates and its distance from other existing locations can be calculated. The flow chart for the minimax function is provided in figure below. The function code is provided in this appendix.



Minimax Function Flowchart

Finding the k – Maximum x-y Coordinates

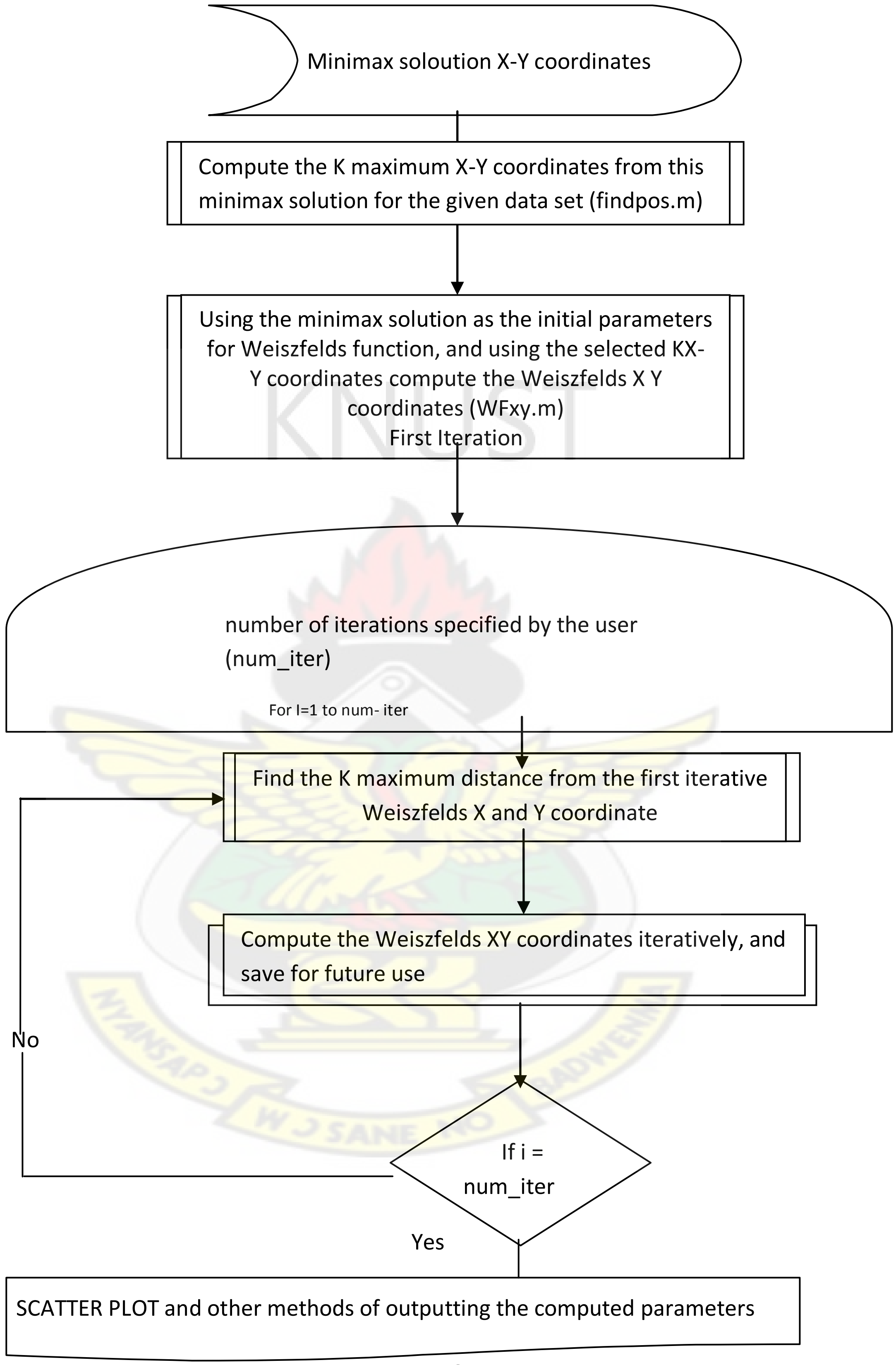
The next step in the flow is to find the coordinates associated with the k-maximum distances from the minimax solution. The user inputs the data coordinates and the value of *k*. The position for each of the coordinates from the minimax solution is computed. Depending on the value of *k*, the x-y coordinates farthest from this minimax solution is stored for future use. Say *k* is 5, the five farthest distances and their coordinates are considered. The MATLAB process uses inbuilt functions like 'find' (to find the position of the data if it's stored in an array), and "max" (to find the maximum distance).

Since the data was stored in an array, we zeroed the selected values to avoid repetitions while it uses these inbuilt functions.

Weiszfeld's Function

In this case, the only difference is that the minimax solution is used as the reference coordinates i.e., as the initialization parameters. In future iterations, the Weiszfeld's function (WFxy.m) are computed iteratively, and processed for the number of iterations prompted by the user. The Weiszfeld logic is coded similar to the method used in the manual solutions. The steps are repeated for as much iteration as requested but the user.

In every iteration, we use the findpos.m function to compute the farthermost *k* points for each iteratively computed WFxy. The flowchart is given below;



Weiszfeld's Function Flowchart

APPENDIX C-MATLAB CODE

The findpos.m

```
function [maxd,xx,yy,bdist,cnt]=findpos(bdist,k,data)

cnt=1;

while(k~=0)
   maxdist(cnt)=max(bdist);
   pos=find(bdist==maxdist(cnt));

   if(length(pos)>1)
      for n=1:length(pos)
         xx(cnt)=data(pos(n),1);
         yy(cnt)=data(pos(n),2);
         bdist(pos(n))=0;
         cnt=cnt+1;

         if(cnt==k+1)
            k=0;
         end
      end
      clear pos
   else

      for n=1:length(pos)
         xx(cnt)=data(pos(n),1);
         yy(cnt)=data(pos(n),2);
         bdist(pos(n))=0;
         cnt=cnt+1;

         if(cnt==k+1)
            k=0;
         end
      end
% %
      clear pos
   end
   maxd=0;
end
```

The WFxy.m

```
function [WFx,WFy]=WFxy(kpts,bestx,besty,K,Wt)

for g=1:1:K
   gamma(g)=Wt/sqrt((kpts(g,1)-bestx)^2+(kpts(g,2)-besty)^2);
end

tau=sum(gamma);

for l=1:1:K
   lamda(l)=gamma(l)/tau;
   lam_x(l)=lamda(l)*kpts(l,1);
   lam_y(l)=lamda(l)*kpts(l,2);
end

WFx=sum(lam_x);
WFy=sum(lam_y);
```

The codes for the Planer k-centra

```
clear all;
close all;
clc;

no=input('Number of data points ');

for i=1:no
   fprintf('Data Point%d\n',i);
   data(i,1)=input('X ');
   data(i,2)=input('Y ');
   fprintf('\n');
end

xmax=input('Max Range of X ');
ymax=input('Max Range of Y ');
incr=input('Search-Space Increment ');
K=input('K Centra K term ');
Wt=input('Weizfelds Weight ');
num_iter=input('Number of Iterations ');

% kot_wts
```

```
x=1;
y=1;

for xref=0:incr:xmax
   for yref=0:incr:ymax
      for i=1:length(data)
         dist(i)=sqrt((data(i,1)-xref)^2+(data(i,2)-yref)^2);
      end
      maxdist(x,y)=max(dist);
      y=y+1;
   end
   if(y>ymax+1)
      x=x+1;
      y=1;
   end
end

xcols=[0:incr:xmax];
ycols=[0:incr:ymax];

intmin=min(maxdist);
minimax=min(intmin);

[row,col]=find(maxdist==minimax);

bestX=xcols(row)
bestY=ycols(col)

  for i=1:1:length(data)
    bestdist(i)=sqrt((data(i,1)-bestX)^2+(data(i,2)-bestY)^2);
  end

 [maxd,xx,yy,bdist,cnt]=findpos(bestdist,K,data);

 kpts=[xx' yy'];

 [WFx,WFy]=WFxy(kpts,bestX,bestY,K,Wt)

 for z=1:1:num_iter

    clear i

    for i=1:1:length(data)
       newdist(i)=sqrt((data(i,1)-WFx)^2+(data(i,2)-WFy)^2);
    end
    [maxd_new,xx_new,yy_new,bdist_new,cnt_new]=findpos(newdist,K,data);
```

```
      kpts_new=[xx_new',yy_new'];
      [WFx,WFy]=WFxy(kpts_new,WFx,WFy,K,Wt);
      WFXY(z,:)=[WFx,WFy];
for l=1:1:K
       feucp(l)=Wt*sqrt((kpts_new(l,1)-WFx)^2+(kpts_new(l,2)-WFy)^2);
       feuc=sum(feucp)

       end

      clear newdist;

  end

  WFXY


  scatter(data(:,1),data(:,2))
  hold on
  grid
  xlabel('X coordinate')
  ylabel('Y coordinate')
  axis([340 460 865 985])
  title('Scatter plot - k-centra Analysis K=50')
   hold on
  scatter(bestX,bestY,48,'rd')
  hold on
 scatter(WFXY(:,1),WFXY(:,2),48,'k*')
  hold on

   %function [maxd,xx,yy,bdist,cnt]=findpos(bdist,k,data)

cnt=1;

while(k~=0);
    maxdist(cnt)=max(bdist);
    pos=find(bdist==maxdist(cnt));

    if(length(pos)>1)
        for n=1:length(pos)
            xx(cnt)=data(pos(n),1);
            yy(cnt)=data(pos(n),2);
            bdist(pos(n))=0;
            cnt=cnt+1;

            if(cnt==k+1)
                k=0;
```

```
         end
      end
      clear pos
   else
      xx(cnt)=data(pos,1);
      yy(cnt)=data(pos,2);
      bdist(pos)=0;
      cnt=cnt+1;
if(cnt==k+1)
         k=0;
      end


code modification here
      for n=1:length(pos)
         xx(cnt)=data(pos(n),1);
         yy(cnt)=data(pos(n),2);
         bdist(pos(n))=0;
         cnt=cnt+1;

         if(cnt==k+1)
            k=0;
         end
      end

      clear pos
   end
   maxd=0;
end
%function [WFx,WFy]=WFxy(kpts,bestX,bestY,K,Wt);
   gamma(g)=Wt/sqrt((kpts(g,1)-bestX)^2+(kpts(g,2)-bestY)^2);
%end
tau=sum(gamma);

for l=1:K
  lamda(l)=gamma(l)/tau;
  lam_X(l)=lamda(l)*kpts(l,1);
   lam_Y(l)=lamda(l)*kpts(l,2);
end

WFx=sum(lam_X);
WFy=sum(lam_Y);
```

APPENDIX D-THE DATA FOR THE CASE STUDY

NO.	TOWNS AND SUBURBS	X Km	Y Km		NO.	TOWNS AND SUBURBS	X Km	Y Km
1	OHUNUKUROM	392	882		2	ATUAHENEKUROM	401	887
3	ASUAKOO	397	891		4	KUFUOR CAMP	393	895
5	ADEDAESE	407	894		6	ASUAGYA	405	896
7	ATRONIE	392	903		8	NWOWASUA	402	905
9	KRAMOKUROM	403	906		10	FRAMOASE	405	907
11	ABEN	417	917		12	AJURESO	418	919
13	AGONA BOSIE	421	920		14	ASIKASU	415	920
15	ABROSANASE	426	920		16	ADDAIKUROM	394	920
17	NYAMEBEKYERE	410	924		18	NKRANKUROM	420	923
19	BAASARE	424	922		20	NSAGOBESA NO. 2	370	929
21	KOFISOWAKROM	375	928		22	NSAGOBESA NO. 1	380	929
23	JINIJINI	371	932		24	YAWSEAKUROM	386	928
25	TWENEDUA	391	929		26	ANTWIKUROM	396	926
27	ABOTAREYE	412	926		28	NKATIAKUROM NO. 1	422	928
29	NKATIAKUROM NO. 2	426	928		30	ASASENTANBI	383	930
31	BORESO	374	933		32	KOFIFOFIEKUROM	310	937
33	KUROSUA NO. 2	384	935		34	TIWAAKUROM	393	935
35	TWENEKUROM	401	933		36	ABENAANSIAKUROM	405	932
37	AGONEKA	411	932		38	YEBOAKUROM	416	929
39	NYAMEKUROM	420	933		40	ASIKASU	421	934
41	NYAMEBEKYERE	419	935		42	PEPRAKUROM	412	935

43	KOFIOWUSUKUROM	418	937		44	SUYAW	418	940
45	FAWONSUA	410	936		46	ABESIM	410	943
47	KYEREMEKUROM	390	938		48	APAASU	385	941
49	KWABENAKUMA KUROM	367	941		50	PEWODEE	368	943
51	NYAMEBEKYERE	383	945		52	PAASO NKRAN	388	944
53	KYIRIBOGYA	392	943		54	BENU NO. 3	395	944
55	BENU NO. 2	403	939		56	BENU NO. 1	404	938
57	GYASEHENEKUROM	405	939		58	BROSANKRO	405	938
59	NKRAN	408	943		60	ABESIM NKRANKUROM	420	945
61	FAHIAKOTWERE	438	946		62	DOMEABRA	410	946
63	BOAKOMASUA	398	946		64	OSOFOKUROM	393	945
65	AHERESO	393	947		66	BENU	393	948
67	ATETEBENESO	393	949		68	ASUNTEN NO. 1	418	949
69	ABOM	418	951		70	KENYAW	422	951
71	WATCHMAN	412	951		72	ASUNTEN NO. 2	408	950
73	KURASUA NO. 1	384	949		74	ADDAE BORESO	389	949
75	NWAWASUA	492	951		76	NWAKUBADUKUROM	368	951
77	KOJONUMKUROM	414	955		78	ABESIM TOWNSHIP	440	950
79	OHENEKUROM, MANTE ASEM	381	957		80	ONYAME AKANYE	389	953
81	FAAKUROM	389	954		82	KOONKWANTA	396	956
83	ATA	397	957		84	KWAMEKUROM	398	958
85	OPANINAPAKUROM	441	961		86	DOMEABRA	436	961

87	OLD ABESIM	434	963		88	KWALEKUROM	392	956
89	KWAME ASAREKUROM	386	961		90	KWANWARE	405	960
91	DOMESERE	405	963		92	NANKETEWA	407	963
93	ADDAE BORESO	376	968		94	MENSAKUROM	456	961
95	MAIN SUNYANI TOWNSHIP	417	969		96	NEW DORMAA	427	971
97	KOTOKUROM	439	972		98	YAWHIMAKUROM	446	970
99	FOKUOKUROM	448	974		100	LIBERATION BARACKS	434	941

APPENDIX E-MATLAB SOLUTION FOR THE CASE STUDY DATA (100 ITERATIONS)

NO.	X km	Y km	f km		NO.	X km	Y km	f km
1	399853	941108.5	1814700		2	406649.6	935398.4	1810700
3	399825.6	941458.3	1813200		4	406643.5	935550.2	1817800
5	399827.7	941529.2	1820800		6	406646	935581.1	1826100
7	399829.7	941543.7	1828400		8	406647.4	935587.5	1832400
9	399830.4	941546.8	1834300		10	406647.8	935588.8	1837800
11	399830.7	941547.4	1830600		12	406647.9	935589.1	1838000
13	399830.7	941547.5	1840800		14	406647.9	935589.1	1829300
15	399830.7	941547.6	1838500		16	406647.9	935589.1	1842300
17	399830.7	941547.6	1836400		18	406647.9	935589.2	1841400
19	399830.7	941547.6	1837500		20	406647.9	935589.2	1845500
21	399830.7	941547.6	1854900		22	406647.9	935589.2	1857500
23	399830.7	941547.6	1853900		24	406647.9	935589.2	1855700
25	399830.7	941547.6	1870200		26	406647.9	935589.2	1860900
27	399830.7	941547.6	1845300		28	406647.9	935589.2	1848700
29	399830.7	941547.6	1838700		30	406647.9	935589.2	1823200
31	399830.7	941547.6	1829500		32	406647.9	935589.2	1930200
33	399830.7	941547.6	1844700		34	406647.9	935589.2	1858400
35	399830.7	941547.6	1859700		36	406647.9	935589.2	1863000

37	399830.7	941547.6	1853100		38	406647.9	935589.2	1852300
39	399830.7	941547.6	1836000		40	406647.9	935589.2	1835500
41	399830.7	941547.6	1834800		42	406647.9	935589.2	1835300
43	399830.7	941547.6	1829300		44	406647.9	935589.2	1815700
45	399830.7	941547.6	1808700		46	406647.9	935589.2	1797800
47	399830.7	941547.6	1799100		48	406647.9	935589.2	1794000
49	399830.7	941547.6	1780300		50	406647.9	935589.2	1783800
51	399830.7	941547.6	1780600		52	406647.9	935589.2	1784600
53	399830.7	941547.6	1782100		54	406647.9	935589.2	1777500
55	399830.7	941547.6	1774500		56	406647.9	935589.2	1769200
57	399830.7	941547.6	1766900		58	406647.9	935589.2	1762900
59	399830.7	941547.6	1761000		60	406647.9	935589.2	1757500
61	399830.7	941547.6	1764700		62	406647.9	935589.2	1757300
63	399830.7	941547.6	1754500		64	406647.9	935589.2	1766000
65	399830.7	941547.6	1756700		66	406647.9	935589.2	1753000
67	399830.7	941547.6	1758900		68	406647.9	935589.2	1753900
69	399830.7	941547.6	1757800		70	406647.9	935589.2	1749700
71	399830.7	941547.6	1740300		72	406647.9	935589.2	1737800
73	399830.7	941547.6	1741300		74	406647.9	935589.2	1739600
75	399830.7	941547.6	1725100		76	406647.9	935589.2	1734300
77	399830.7	941547.6	1750000		78	406647.9	935589.2	1746500
79	399830.7	941547.6	1756600		80	406647.9	935589.2	1772100

81	399830.7	941547.6	1765800		82	406647.9	935589.2	1765000
83	399830.7	941547.6	1750500		84	406647.9	935589.2	1736900
85	399830.7	941547.6	1735600		86	406647.9	935589.2	1732200
87	399830.7	941547.6	1742200		88	406647.9	935589.2	1743000
89	399830.7	941547.6	1759300		90	406647.9	935589.2	1759800
91	399830.7	941547.6	1760400		92	406647.9	935589.2	1760000
93	399830.7	941547.6	1765900		94	406647.9	935589.2	1779500
95	399830.7	941547.6	1786600		96	406647.9	935589.2	1797400
97	399830.7	941547.6	1796200		98	406647.9	935589.2	1801300
99	399830.7	941547.6	1815000		100	406647.9	935589.2	1811500