OPTIMUM PRODUCTION PROBLEM

A CASE STUDY OF GHACEM- TAKORADI

BY

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DEDICATION

THIS WORK IS DEDICATED TO MY FATHER, MR. JAMES MENSAH AND MY SISTERS PHILIPPINE AND ROSEMOND, AND TO MY WIFE, JOANA, ALL OF WHOM HAVE DEDICATED SO MUCH OF THEIR LIVES, AND THEMSELVES, TO ME.

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ABSTRACT

The cement industry is the building block of the nation's construction industry. Few construction projects can take place without utilizing cement somewhere in the design. The most important use of cement is the production of mortar and concrete – the bonding of natural or artificial aggregates to form a strong building material, which is durable in the face of normal environmental effects. The main objective of this study is to develop cement production model at Takoradi-GHACEM using linear programming to minimize the monthly cost of cement production. A theoretical method used in solving models (graphical method and simplex method) and SixPap, a software for solving linear programming models was used. The study revealed among others, the factors that influence the cost of producing cement at Takoradi- GHACEM for which the production of a bag of cement becomes expensive. The factors include: cost of labour, cost of electricity, cost of fuel, cost of raw materials and cost of maintenance. Among the above listed factors, the most influential factors are: cost of raw materials and cost of electricity. The study revealed that, the average total cost GHS38500.40 and GHS1003585 for raw materials and electricity respectively can be optimized to GHS69667.04. This optimal solution can be achieved if the total quantity of electricity is reduced to 554859.45 kWh and the quantity of raw materials remains the same.

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CHAPTER 1

INTRODUCTION

1.1 Background to the Study

Advanced representation and solution techniques in production planning and scheduling received significant attention during the past decades, from both the part of research communities and the industry. This interest comes quite natural, regarding that these methods hold out a promise of increased productivity, better service level, higher flexibility, together with lower production costs. It is presumed that the above objectives can be reached by supporting the management to make smarter decisions on various levels of the planning hierarchy. Despite the attractive prospects, only a few of the recent research results has migrated into everyday practice. Although advances in operations research and artificial intelligence led to the development of novel modelling and solution techniques, industrial applications often require more, thus on the part of the researchers, richer models and more efficient algorithms. Production managers always have eyes on the future to identify actions they would take in order to reduce the cost of production. The systematic approach to reduce the cost of production is what is termed as production scheduling in optimization. (Kov´acs, 2005)

Ghana is a developing country with an average Gross Domestic product (GDP) growth rate of 7.7 percent. (Ghana statistical service, 2011) Investments have been made in different industries and numerous projects, especially in infrastructure section. The trend is positive and the number of construction projects will continue to grow. This means a growing demand for fundamental materials like cement and steel for the construction of needed facilities.

1.1.1 Early uses

The earliest construction cements are as old as construction and were non-hydraulic. (Hewlett, 1998) Wherever primitive mud bricks were used, they were bedded together with a thin layer of clay slurry. Mud-based materials were also used for rendering on the walls of timber or wattle and daub structures. Lime was probably used for the first time as an additive in these renders, and for stabilizing mud floors. A "daub" consisting of mud, cow dung and lime produces a tough and coating, due to coagulation, by the lime, of proteins in the cow dung. This simple system was common in Europe until quite recent times. With the advent of fired bricks, and their use in larger structures, various cultures started to experiment with higher-strength mortars based on bitumen (in Mesopotamia), gypsum (in Egypt) and lime (in many parts of the world). (http://en.wikipedia.org/wiki/Cement 2008)

It is uncertain where it was first discovered that a combination of hydrated non-hydraulic lime and a pozzolan produces a hydraulic mixture, but concrete made from such mixtures was first used on a large scale by the Romans. They used both natural pozzolans (trass or pumice) and artificial pozzolans (ground brick or pottery) in these concretes. Many excellent examples of structures made from these concretes are still standing, notably the huge monolithic dome of the Pantheon in Rome. The use of structural concrete disappeared in medieval Europe, although weak pozzolanic concretes continued to be used as a core fill in stone walls and columns. (Pierre-Claude, 2000)

1.1.2 Modern cement

Modern hydraulic cements began to be developed from the start of the Industrial Revolution (around 1700), driven by three main needs:

(i) Hydraulic renders for finishing brick buildings in wet climates

(ii) Hydraulic mortars for masonry construction of harbour works etc, in contact with sea water, and

(iii) Development of strong concretes.

In Britain particularly, good quality building stone became ever more expensive during a period of rapid growth, and it became a common practice to construct prestige buildings from the new industrial bricks, and to finish them with a stucco to imitate stone. Hydraulic limes were favoured for this, but the need for a fast set time encouraged the development of new cements. Most famous among these was Parker's "Roman cement". This was developed by Parker in the 1780s, and finally patented in 1796. It was, in fact, nothing like any material used by the Romans, but was a "Natural cement" made by burning septaria - nodules that are found in certain clay deposits, and that contain both clay minerals and calcium carbonate. The burnt nodules were ground to a fine powder. This product, made into a mortar with sand, set in 5-15 minutes. The success of "Roman Cement" led other manufacturers to develop rival products by burning artificial mixtures of clay and chalk (Francis, 1977).

(Smeaton, 1755) made an important contribution to the development of cements when he was planning the construction of the third Eddystone Lighthouse in the

English Channel. He needed a hydraulic mortar that would set and develop some strength in the twelve hour period between successive high tides. He performed an exhaustive market research on the available hydraulic limes, visiting their production sites, and noted that the "hydraulicity" of the lime was directly related to the clay content of the limestone from which it was made. Smeaton was a civil engineer by profession, and took the idea no further. Apparently unaware of Smeaton's work, the same principle was identified by Louis Vicat in the first decade of the nineteenth century. Vicat went on to devise a method of combining chalk and clay into an intimate mixture, and burning this, produced an "artificial cement" in 1817. James Frost, working in Britain, produced what he called "British cement" in a similar manner around the same time, but did not obtain a patent until 1822. In 1824, Joseph Aspdin patented a similar material, which he called Portland cement, because the render made from it was in colour similar to the prestigious Portland stone (Francis, 1977).

All the above products could not compete with lime/pozzolan concretes because of fast-setting (giving insufficient time for placement) and low early strengths (requiring a delay of many weeks before formwork could be removed). Hydraulic limes, "natural" cements and "artificial" cements all rely upon their belite content for strength development. Belite develops strength slowly. Because they were burned at temperatures below 1250 °C, they contained no alite, which is responsible for early strength in modern cements. The first cement to consistently contain alite was made by Joseph Aspdin's son William in the early 1840s. This was what we call today "modern" Portland cement. Because of the air of mystery with which William Aspdin surrounded his product, others (e.g. Vicat and Johnson, 1862) have claimed precedence in this invention, but recent analysis of both his concrete and raw cement have shown that William Aspdin's (1846) product made at Northfleet, Kent was a true alite-based cement. However, Aspdin's methods were "rule-of-thumb": Vicat is responsible for establishing the chemical basis of these cements, and Johnson (1880) established the importance of sintering the mix in the kiln.

Aspdin's 1841 innovation was counter-intuitive for manufacturers of "artificial cements", because they required more lime in the mix (a problem for his father), because they required a much higher kiln temperature (and therefore more fuel) and because the resulting clinker was very hard and rapidly wore down the millstones which were the only available grinding technology of the time. Manufacturing costs were therefore considerably higher, but the product set reasonably slowly and developed strength quickly, thus opening up a market for use in concrete. The use of concrete in construction grew rapidly from 1850 onwards, and was soon the dominant use for cements. Thus Portland cement began its predominant role (Hewlett, 1998).

1.1.3 Types of modern cement

Portland cement

Cement is made by heating limestone with small quantities of other materials (such as clay) to 1450°C in a kiln. The resulting hard substance, called 'clinker', is then ground with a small amount of gypsum into a powder to make 'Ordinary Portland Cement', the most commonly used type of cement (often referred to as OPC).

Portland cement is a basic ingredient of concrete, mortar and most non-speciality grout. The most common use for Portland cement is in the production of concrete. Concrete is a composite material consisting of aggregate (gravel and sand), cement, and water. As a construction material, concrete can be cast in almost any shape desired, and once hardened, can become a structural (load bearing) element. Portland cement may be gray or white.

Portland cement blends

These are often available as inter-ground mixtures from cement manufacturers, but similar formulations are often also mixed from the ground components at the concrete mixing plant (Kosmatka and Panarese, 1988).

Portland Blastfurnace Cement contains up to 70% ground granulated blast furnace slag, with the rest Portland clinker and a little gypsum. All compositions produce high ultimate strength, but as slag content is increased, early strength is reduced, while sulfate resistance increases and heat evolution diminishes. Used as an economic alternative to Portland sulfate-resisting and low-heat cements. (U.S. Federal Highway Administration, Ground Granulated Blast-Furnace Slag, 2007)

Portland Flyash Cement contains up to 30% fly ash. The flyash is pozzolanic, so that ultimate strength is maintained. Because flyash addition allows lower concrete water content, early strength can also be maintained. Where good quality cheap flyash is available, this can be an economic alternative to ordinary Portland cement (U.S. Federal Highway Administration, Fly Ash, 2007).

Portland Pozzolan Cement includes fly ash cement, since fly ash is a pozzolan, but also includes cements made from other natural or artificial pozzolans. In countries where volcanic ashes are available (e.g. Italy, Chile, Mexico, and the Philippines) these cements are often the most common form in use.

Portland Silica Fume cement. Addition of silica fume can yield exceptionally high strengths, and cements containing 5-20% silica fume are occasionally produced. However, silica fume is more usually added to Portland cement at the concrete mixer (U.S. Federal Highway Administration, Silica Fume, 2007).

Masonry Cements are used for preparing bricklaying mortars and stuccos, and must not be used in concrete. They are usually complex proprietary formulations containing Portland clinker and a number of other ingredients that may include limestone, hydrated lime, air entrainers, retarders, waterproofers and colouring agents. They are formulated to yield workable mortars that allow rapid and consistent masonry work. Subtle variations of Masonry cement in the US are Plastic Cements and Stucco Cements. These are designed to produce controlled bond with masonry blocks.

Expansive Cements contain, in addition to Portland clinker, expansive clinkers (usually sulfoaluminate clinkers), and are designed to offset the effects of drying shrinkage that is normally encountered with hydraulic cements. This allows large floor slabs (up to 60 m square) to be prepared without contraction joints.

White blended cements may be made using white clinker and white supplementary materials such as high-purity metakaolin.

Coloured cements are used for decorative purposes. In some standards, the addition of pigments to produce "coloured Portland cement" is allowed. In other standards (e.g. American Society for Testing and Materials - ASTM), pigments are not allowed constituents of Portland cement, and coloured cements are sold as "blended hydraulic cements".

Very finely ground cements are made from mixtures of cement with sand or with slag or other pozzolan type minerals which are extremely finely ground. Such cements can have the same physical characteristics as normal cement but with 50% less cement particularly due to the increased surface area for the chemical reaction. Even with intensive grinding they can use up to 50% less energy to fabricate than ordinary Portland cements (Justnes et al, 2004).

1.1.4 Non-Portland hydraulic cements

Pozzolan-lime cements. Mixtures of ground pozzolan and lime are the cements used by the Romans, and are to be found in Roman structures still standing (e.g. the Pantheon in Rome). They develop strength slowly, but their ultimate strength can be very high. The hydration products that produce strength are essentially the same as those produced by Portland cement.

Slag-lime cements. Ground granulated blast furnace slag is not hydraulic on its own, but is "activated" by addition of alkalis, most economically using lime. They are similar to pozzolan lime cements in their properties. Only granulated slag (i.e. water-quenched, glassy slag) is effective as a cement component.

Supersulfated cements. These contain about 80% ground granulated blast furnace slag, 15% gypsum or anhydrite and a little Portland clinker or lime as an activator. They produce strength by formation of ettringite, with strength growth similar to a slow Portland cement. They exhibit good resistance to aggressive agents, including sulfate.

Calcium aluminate cements are hydraulic cements made primarily from limestone and bauxite. The active ingredients are monocalcium aluminate $CaAl_2O_4$ (CA in Cement chemist notation) and Mayenite $Ca_{12}Al_{14}O_{33}$ ($C_{12}A_7$ in CCN). Strength forms by hydration to calcium aluminate hydrates. They are well-adapted for use in refractory (high-temperature resistant) concretes, e.g. for furnace linings. Calcium sulfoaluminate cements are made from clinkers that include ye'elimite ($Ca_4(AlO_2)_6SO_4$ or C_4A_3 in Cement chemist's notation) as a primary phase (Bye, 1999).

They are used in expansive cements, in ultra-high early strength cements, and in "low-energy" cements. Hydration produces ettringite, and specialized physical properties (such as expansion or rapid reaction) are obtained by adjustment of the availability of calcium and sulfate ions. Their use as a low-energy alternative to Portland cement has been pioneered in China, where several million tonnes per year are produced. Energy requirements are lower because of the lower kiln temperatures required for reaction and the lower amount of limestone (which must be endothermic ally decarbonated) in the mix. In addition, the lower limestone content and lower fuel consumption leads to a CO_2 emission around half that associated with Portland clinker. However, SO_2 emissions are usually significantly higher.

"Natural" Cements correspond to certain cements of the pre-Portland era, produced by burning argillaceous limestones at moderate temperatures. The level of clay components in the limestone (around 30-35%) is such that large amounts of belite (the low-early strength, high-late strength mineral in Portland cement) are formed without the formation of excessive amounts free lime. As with any natural material, such cements have very variable properties.

Geopolymer cements are made from mixtures of water-soluble alkali metal silicates and aluminosilicate mineral powders such as flyash and metakaolin.

1.1.5 Environmental and social impacts of cement

Cement manufacture causes environmental impacts at all stages of the process. These include emissions of airborne pollution in the form of dust, gases, noise and vibration when operating machinery and during blasting in quarries, and damage to countryside from quarrying. Equipment to reduce dust emissions during quarrying and manufacture of cement is widely used, and equipment to trap and separate exhaust gases are coming into increased use. Environmental protection also includes the re-integration of quarries into the countryside after they have been closed down by returning them to nature or re-cultivating them.

1.1.6 Climate

Cement manufacture contributes greenhouse gases both directly through the production of carbon dioxide when calcium carbonate is heated, producing lime and carbon dioxide, also indirectly through the use of energy, particularly if the energy is sourced from fossil fuels. The cement industry produces 5% of global man-made CO_2 emissions, of which 50% is from the chemical process, and 40% from burning fuel. The amount of CO_2 emitted by the cement industry is nearly 900 kg of CO_2 for every 1000 kg of cement produced (Kaya, et al., 2003).

1.1.7 Fuels and raw materials

A cement plant consumes 3,000 to 6,500 MJ of fuel per tonne of clinker produced, depending on the raw materials and the process used. Most cement kilns today use coal and petroleum coke as primary fuels, and to a lesser extent natural gas and fuel oil. Selected waste and by-products with recoverable calorific value can be used as fuels in a cement kiln, replacing a portion of conventional fossil fuels, like coal, if they meet strict specifications. Selected waste and by-products containing useful minerals such as calcium, silica, alumina, and iron can be used as raw materials in the kiln, replacing raw materials such as clay, shale, and limestone. Because some materials have both useful mineral content and recoverable calorific value, the distinction between alternative fuels and raw materials is not always clear. For example, sewage sludge has a low but significant calorific value, and burns to give ash containing minerals useful in the clinker matrix (World Business Council for Sustainable Development, 2005).

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1.1.8 History of Cement Production in Ghana

Ghana's cement industry is a duopoly of two firms. Until the late 90s, there was a monopoly of cement production in Ghana, held by a State Owned Enterprise (SOE). However, it was privatised in 1999, and another firm started to import cement about the same time, and then established a manufacturing plant in 2002. The increased competition resulted in falling prices (unfortunately no price data exists to substantiate this assertion), as the new entrant strived to undercut the incumbent in order to increase its market share (which it succeeded in doing), and this reportedly forced the incumbent to reduce prices also. The new entrant also introduced transportation and credit incentive schemes to entice distributors of the other cement company's product, to stock their cement.

As already noted, a high degree of concentration is common in the cement industry, and reflects the relative efficiency of large scale production in the industry, given the cost structure and significant economies of scale that large producers enjoy. However, Ghana's two cement producers have a proportionately large market to divide up between them compared with cement producers in other countries, which suggests there may be room for more cement firms in Ghana. In any case, the scope to export cement means that domestic market size should not necessarily constrain the number of cement firms that can viably operate within a country (Ellis and Singh, 2010).

Imported bags of cement are not widely available, and various stakeholders, for example in the construction industry, have alleged that one of the domestic cement producers blocks their attempts to import cement bags into the country through informal means. For example, cement shipments sometimes got waylaid at the port. Thus construction companies sometimes gave up in their attempts to import cement from other sources in the end, so these unofficial import barriers appeared to reduce competition from potentially cheaper imported cement.

Prices of cement doubled in 2007 causing great concern in the building industry. Although there had been some electricity load shedding which contributed to the price increases, in the view of many analysts, the price hikes had continued beyond the load shedding period. The dominant cement company attributed the increase in prices to the higher prices associated with imported raw materials. However, stakeholders held a contrary view and pointed to the domestic producer as a major factor in the price rise. They alleged that because the company enjoyed strong market power, it was able to set up higher prices without fear of losing customers (Ellis and Singh, 2010).

Even the presence of another domestic player in the market did not act as a sufficiently strong competitive restraint, as the market was a duopoly and the incumbent acted as a price leader, with the other firm following. As there was no Competition Authority in the country and the consumer associations were not strong, there was little or no investigation into price increases such as this. High cement prices represented a constraint to construction and infrastructure development, which underpinned growth, and thus had negative repercussions for the wider economy. A third cement plant was commissioned, which was to be established in Northern Ghana.

The history of Ghana's cement industry dates back to about 50 years ago. The government of Ghana in collaboration with NORCEM, in Norway, on August 30, 1967 established GHACEM to produce cement in commercial quantities for use in Ghana. In 1999, Heidelberg Cement took over Scancem, thus making it a subsidiary. Scancem International, which is a subsidiary of Heidelberg Cement producers, which owns GHACEM is among the world's biggest cement producers and currently active in 50 countries. It has about forty- two thousand (42,000) employees and annual sales of about 70 million tonnes. Now, total production of this product has been increased to more than 1.2 million tonnes per annum (http://www.heidelbergcement.com).

GHACEM uses imported raw materials such as clinker, limestone and gypsum in the manufacture of cement. For the first time in their 37 years operation in Ghana, GHACEM has started using local raw materials like limestone in 2004 with supplies from the Eastern Region to replace the imported limestone. Its cement has been produced with a local component of about 25 percent since 2004.

The cement market has always experienced lack of supply and suppliers have been able to sell their products easily. Because of scheduling important of cement and possible effects of price increase on project's progress, government has tried to control the market during these years. The controls are imposed by setting some limitation on cement trading. Pricing and rationing are two main mechanisms used for this purpose. These policies were not successful and finally let a black market which was not beneficial, neither to producers not end customers.

Most of cement companies use old technologies for production and their production processes are not efficient enough and consume notable amount of electricity. Other than it, pricing has caused a big problem. At current prices cement factories are not able to remain competitive for a long time. Actually, cement companies show profits in their financial statements because they use historical prices in calculating depreciation. If the replacement costs of machineries were considered in calculating production costs, the profitability of old factories would decrease so that the production would not be feasible anymore.

1.1.9 Geographical Area of the Study

GHACEM – Takoradi is the geographical area of study. Takoradi is in Western Region of Ghana on the coast. It includes Ghana's Southern most location, Cape Three Points. GHACEM – Takoradi is near Takoradi Harbour. GHACEM – Takoradi is chosen as a case study because it is one of the two cement producing industry in Ghana. Optimizing cement production would mean success in its operation.

The cement industry encounters several environmental changes that alter the market conditions. The production capacity will pass the market demand and consequently rivalry among competitors will grow subsidies on energy and electricity will be reduced in near future and will eventually be eliminated. The pricing and rationing of cement will be eliminated because of government's overall policy to reduce its interferences in the economy.

Although all pre-mentioned changes could have serious impacts on market and competition, the most important one is certainly the paradigm shift that occurs in this industry from production – oriented paradigm to customer oriented paradigm.

The Takoradi plant of the Ghana cement works (GHACEM) produced six hundred thousand (600,000) tonnes of cement in 1998, representing an increase of about 12 percent over that for 1997. The plants produced five hundred and fifty thousand (550,000) tonnes of cement in 1997. Production has been increasing at an annual rate of about twenty (20) percent since 1986; twenty (20) percent of the cement produced last year was purchased by the mining companies. The company has put in place health, safety and environmental awareness programme for its workers in addition to measures to minimized pollution.

1.1.10 Projects Undertaken With GHACEM Cement

GHACEM cement has been used for construction of big and small projects such as Tema Harbour, Takoradi Harbour, Akosomobo Dam, Adomi Bridge, Tema Motorway, Kotoka International Airport, Aboadze Thermal Plant, West African Gas Concrete Piping, Construction of new stadia at Takoradi, Tamale and the rehabilitation of Accra, Kumasi and Tema Stadia, Presidential Palace, Construction of Government affordable houses for workers, the construction of Schools and other social amenities (as HIPIC projects under President Kuffour's administration) and many other residential buildings in the country which has stood the test of time.

The Ghana cement Foundation is to regulate the equitable donation of cement to needy health clinics and educational institutions in deprived communities in the country. It is an important part of GHACEM'S corporate social responsibility programme to assist deprived communities to improve their health and educational infrastructures or build new ones.

In addition to this social gesture, it is a fact that GHACEM continues to contribute substantially to the economic development of Ghana and it is currently on record that GHACEM's financial contributions to the government of Ghana as at the end of 2009 in terms of direct and indirect taxes was fifty-eight million Ghana cedis (GH¢58,000,000).

1.2 Problem Statement

Today, most factories apply material requirements or manufacturing resources planning systems for medium-term production planning. The cost of production largely depends on the cost of the materials used in the production and cement is not exception to this. The production manager of GHACEM – Takordi explained that consumers pay much for products of GHACEM due to the high cost of production. He accepted the fact that there was waste in producing cement with the materials at hand which in turn increase the cost of production.

The problem that relates to the study of the cost of production is under experimentally controlled conditions. The problem has been examined from a relatively short run point of view, such that certain conditions can be considered as fixed; for example, a fixed cost of electrical units consumed and previously established plant technological capacities.

The cement industry, and especially a totally integrated cement industry that manufacturers the product from basic raw material to the completion of the final product, presents some interesting and complex problem in production scheduling. In the cement industry, the production characteristics are such that the production system falls somewhere between the general flow-type industry and the batch-type manufacturers. The problem at hand is to find out the ways of producing cement at the least cost. That is whether it is possible to produce more bags of cement from given quantities of raw materials than is currently produced or other factors of production can be varied to reduce the cost of production.

1.3 Objectives of the study

The study is aimed at seeking ways of producing cement at the least possible cost. This when achieved, would reduce the selling price on the market as total cost of production is always passed on to consumers. The main objective of this research is to develop a cement production cost model using optimization that minimizes the total production cost.

1.4 Methodology

This section presents the methodology used for developing the proposed production problem model. Investigations would be conducted to determine the variables involved in the production of cement. Data would be collected from GHACEM – Takoradi and the linear programming model would be made. A theoretical method used in solving models (graphical method and simplex method) and SixPap, a software for solving linear programming models will be used. The result obtained would be analyzed and discussed. Conclusion and recommendations would then be made.

1.5 Justification of the study

Ghana being a developing country, there is the need for the building of infrastructure such as roads, schools, hospitals, offices and residential facilities. Though this can be undertaken by government as well as individuals, the paste is very slow, compared to the population growth rate. This could account for reasons whys school children in certain part of the country hold classes in dilapidated buildings and under trees. Due to the high cost of building materials especially cement, it becomes impossible for some and very difficult for others to undertake construction projects. It has therefore become necessary to seek a way of making the product affordable to even to a low income earner. This would be possible if the commodity is produced at a lower cost and subsequently reducing the price to consumers.

1.6 Limitations of the Study

This research did not cover all aspect that goes into the cement production. Example: Cost of civil structures and maintenance of the production plant, cost of labour, ground rent and insurance premiums covering plant and personnel.

1.7 Organization of the study

This study is organized as follows: the introduction background of the study, objectives, justification of the study are captured in chapter one. Chapter two provides a review of existing theoretical and empirical literature. Chapter three

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discusses the methodology. Chapter four presents data collection and analysis of the results. Chapter five, the final chapter presents the summary of findings, conclusions and recommendations of the study.

1.8 Summary

This chapter considered the background of the study, stated the problems of the study, outlined the objectives of the study, justified the study, discussed briefly the method to be used and lastly discussed how the study would be organised. In the next chapter, the researcher shall put forward some pertinent literature in the field of linear programming.

Chapter 2

Literature Review

2.0 Introduction

This chapter reviews related literature and specifically discusses operational research as a broader topic of which linear programming is part. It proceeds with recent research on mathematical modelling. The chapter is organized under various relevant headings.

According to Brandimarte and Villa (1995), production management problems can be split up into different categories. They differentiated between production management problems as production planning, production scheduling, and production control. Production planning is the highest level. A decision at this level could be to decide for the total amount to produce in the next quarter. In this level capacity constraints can be dealt with as variables. Production scheduling has a shorter time horizon. The allocation of the different resources is done in this level. Production control, the lowest level, is a real time task. This level makes sure that the planning produced in upper levels is carried out. The problem of production management can often be cast in the form of a linear program with uncertain parameters and risk constraints. Typically, such problems are treated in the framework of multi-stage stochastic programming. Recently, a Robust Counterpart (RC) approach has been proposed, in which the decisions are optimized for the worst realizations of problem parameters. However, an application of the RC technique often results in very conservative approximations of uncertain problems. To tackle this drawback, an adjustable robust counterpart (ARC) approach has been proposed in (Ben-Tal et al., 2003). In ARC, some decision variables are allowed to depend on past values of uncertain parameters. A restricted version of arc, introduced in (Ben-Tal et al., 2003), which can be efficiently solved, is referred to as Affinely Adjustable Robust Counterpart (AARC).

Gr¨owe-Kuska and R¨omisch (2005) noted that, the evolution of the uncertain parameters over the management period is modelled by a scenario tree and the goal is to minimize the expected production cost over this set of scenarios. In recent times solution methods which use Lagrangian relaxation and various nondifferentiable optimization methods and tools to solve the associated local sub problems have evolved. An example of such methods is the one produced by Bacaud *et al*. (2001).

Pinedo (2008) noted that scheduling is a decision-making process that is used on a regular basis in many manufacturing and services industries. It deals with the allocation of resources to tasks over given time periods and its goal is to optimize one or more objectives. The resources and tasks in an organization can take many different forms. The resources may be machines of a workshop, runways at an airport, crews at a construction sites, processing units in a computing environment, and so on. The tasks may be operations in a productions process take-offs and landing at an airport, stages in a construction project, executions of computer programs and so on. Each task may have a certain priority level, an earliest possible starting time and a due date. The objectives can also take many different forms. One objective may be the minimization of the number of tasks computed after their respective due dates.

Herrmann (2006) noted the ways that production scheduling has been done is critical to analyzing existing production scheduling systems and finding ways to improve upon them. The author covered not only the tools used to support decision-making in real-making in real-world production scheduling, but also the changes in the production scheduling systems. The author extended the work to the first charts developed by Gannt (1973) to advance scheduling systems that rely on sophisticated algorithms. Through his findings, the author was able to help production schedulers, engineers, and researchers understand the true nature of production scheduling in dynamic manufacturing systems and to encourage them to consider how production scheduling systems can be improved even more. The author did not only review the range of concepts and approaches used to improve production scheduling, but also demonstrate timeless importance.

2.1 Operational Research

One of the most commonly used models in operational research is mathematical programmes. They assist in analysing and understanding the problem by abstracting the problem to a certain level from its complex real world setting. However the applicability of the model to solve the problem is not always clear in the initial stage. A mathematical model essentially consists of different mathematical relationships. Such relationships can be equations, inequalities, logical dependencies and so on. These relationships represent relationships from the real world setting. Usually there is one objective function expressed in terms of variables and parameters, and many constraints on the variables.

2.2 Mathematical Programming

Mathematical programming then describes the minimization or maximization of the objective function subsidiary to the constraints (Williams, 2003). The different stages in practical mathematical programming are very similar to the operational research cycle.

Fourer et al., (2003) however specified a more detailed cycle for mathematical programming:

(i) formulate a model, the abstract system of variables, objectives, and constraints that represent the general form of the problem to be solved.

(ii) collect data that define a specific problem instance.

(iii) generate a specific objective function and constraint equations from the model and data.

(iv) solve the problem instance by running a programme, or solver, to apply an algorithm that finds optimal values of the variables.

(v) analyze the results.

(vi) refine the model and data as necessary.

There are different categories of mathematical programmes. A special case is called linear programming, which mathematically entails that the objective function and all constraints are linear equations and inequalities. The advantage of this case is that there exist a lot of fast methods to solve linear programmes and the guaranteed optimal solution. There are some models in which some of the variables are constraint to integer values. This more complicated case is called integer programming. But greater computational force of computers and development of advanced methods have also made integer programmes solvable.

2.3 Mathematical Model

A mathematical model is a description of a system using mathematical language. The process of developing a mathematical model is termed mathematical modelling. Mathematical models are used not only in the natural sciences (such as physics, biology, earth science, meteorology) and engineering disciplines (like computer science, artificial intelligence), but also in the social sciences (such as economics, psychology, sociology and political science); physicists, engineers, statisticians, operation research analysts and economists use mathematical models most extensively.

Mathematical models can be useful tools in exploring disease trends and health consequences of interventions in a population over time. In the case of cervical cancer, in which the time from acquisition of the Human papillomavirus (HPV) infection to development of invasive cancer can be two decades or more, models can be used to translate short-term findings from vaccine trials into predictions of long-term health outcomes.

The mathematical models which tell to optimise (minimize or maximise) the objective function Z subject to certain condition on the variables is called a Linear programming problem (LPP).

2.4 Linear Programming (LP)

Linear programming is the process of taking various linear inequalities relating to some situation, and finding the "best" value obtainable under those conditions. A typical example would be taking the limitations of materials and labour, and then determining the "best" production levels for maximal profits under those conditions.

In "real life", linear programming is part of a very important area of mathematics called "optimization techniques". This field of study (or at least the applied results of it) are used every day in the organization and allocation of resources. These "real life" systems can have dozens or hundreds of variables, or more.

The general process for solving linear-programming exercises is to graph the inequalities (called the "constraints") to form a walled-off area on the *x*, *y*-plane (called the "feasibility region"). The coordinates of the corners of this feasibility region are figured out (that is, the intersection points of the various pairs of lines are found), and these corner points are test in the formula (called the "optimization equation") for which the highest or lowest value is being sought.

2.4.0 Linear Programming Problems (LPP)

The standard form of the linear programming problem is used to develop the procedure for solving a general programming problem.

A general LPP is of the form

Max (or min) $Z = c_1x_1 + c_2x_2 + \dots + c_nx_n$

where $x_1, x_2, \dots x_n$ are called decision variable.

2.4.1 Application Areas of Linear Programming

The Application Areas of Linear Programming are: Transportation Problem, Military Applications, Operation of System of Dams, Personnel Assignment Problem and Other Applications such as manufacturing plants, distribution centres, production management and manpower management.

2.4.2 Basic Concept of Linear Programming Problem

Objective Function: The Objective Function is a linear function of variables which is to be optimised i.e., maximised or minimised. e.g., profit function, cost function etc. The objective function may be expressed as a linear expression.

Constraints: A linear equation represents a straight line. Limited time, labour etc. may be expressed as linear inequations or equations and are called constraints.

Optimisation: A decision which is considered the best one, taking into consideration all the circumstances is called an optimal decision. The process of getting the best possible outcome is called optimisation.

Solution of a LPP: A set of values of the variables $x_1, x_2, ..., x_n$ which satisfy all the constraints is called the solution of the LPP..

Feasible Solution: A set of values of the variables $x_1, x_2, x_3,\ldots,x_n$ which satisfy all the constraints and also the non-negativity conditions is called the feasible solution of the LPP.

Optimal Solution: The feasible solution, which optimises (i.e., maximizes or minimizes as the case may be) the objective function is called the optimal solution. Important terms Convex Region and Non-convex Sets.

2.4.3 *Mathematical Formulation of Linear Programming Problems*

There are mainly four steps in the mathematical formulation of linear programming problem as a mathematical model. We will discuss formulation of those problems which involve only two variables.

(i) Identify the decision variables and assign symbols x and y to them. These decision variables are those quantities whose values we wish to determine.

(ii) Identify the set of constraints and express them as linear equations/inequations in terms of the decision variables. These constraints are the given conditions.

(iii) Identify the objective function and express it as a linear function of decision variables. It might take the form of maximizing profit or production or minimizing cost.

(iv) Add the non-negativity restrictions on the decision variables, as in the physical problems, negative values of decision variables have no valid interpretation.

2.4.4 Advantages of Linear Programming

(i) The linear programming technique helps to make the best possible use of available productive resources (such as time, labour, machines etc.)

(ii) In a production process, bottle necks may occur. For example, in a factory some machines may be in great demand while others may lie idle for some time. A significant advantage of linear programming is highlighting of such bottle necks.

2.4.5 Limitations of Linear Programming

(a) Linear programming is applicable only to problems where the constraints and objective function are linear i.e., where they can be expressed as equations which represent straight lines. In real life situations, when constraints or objective functions are not linear, this technique cannot be used.

(b) Factors such as uncertainty, weather conditions etc. are not taken into consideration.

Solving linear programming problems efficiently has always been a fascinating pursuit for Computer Scientists and Mathematicians. The main reason behind this is the existence of a wide range of industrial applications that require highly efficient linear programming solvers. This pursuit has resulted in many different methods, which solve linear programming problems namely simplex methods and interior point methods.

Linear programming (LP) is a mathematical method for determining a way to achieve the best outcome (such as maximum profit or minimum cost) in a given mathematical model for some list of requirements represented as linear relationships. More formally, linear programming is a technique for the optimization of a linear objective function, subject to linear equality and linear inequality constraints.

Linear programmes are problems that can be expressed in canonical form:

Maximnze $\quad c^T x$

Subject to $\quad Ax \le b$

where **x** represents the vector of variables (to be determined), **c** and **b** are vectors of (known) coefficients and *A* is a (known) matrix of coefficients. The expression to be maximized or minimized is called the objective function ($\mathbf{c}^T\mathbf{x}$ in this case). The equations $Ax \le b$ are the constraints which specify a convex polytope over which the objective function is to be optimized. In this context, two vectors are comparable when every entry in one is less-than or equal-to the corresponding entry in the other. Otherwise, they are incomparable.

Linear programming can be applied to various fields of study. It is used most extensively in business and economics, but can also be utilized for some engineering problems. Industries that use linear programming models include transportation, energy, telecommunications, and manufacturing. It has proved useful in modelling diverse types of problems in planning, routing, scheduling, assignment, and design.

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2.4.6 The LP Formulation and the Underlying Assumptions

A Linear Programming problem is a special case of a Mathematical Programming problem. From an analytical perspective, a mathematical program tries to identify an extreme, thus, minimum or maximum point of a function $f(x_1, x_2, x_3, \ldots x_n)$, which furthermore satisfies a set of constraints, for example, $g(x_1, x_2, x_3, \ldots x_n) \geq b$. Linear programming is the specialization of mathematical programming to the case where both, function f to be called the objective function and the problem constraints are linear.

From an applications perspective, mathematical (and therefore, linear) programming is an optimization tool, which allows the rationalization of many managerial and/or technological decisions required by contemporary techno-socio-economic applications. An important factor for the applicability of the mathematical programming methodology in various application contexts is the computational tractability of the resulting analytical models. Under the advent of modern computing technology, this tractability requirement translates to the existence of effective and efficient algorithmic procedures able to provide a systematic and fast solution to these models. For Linear Programming problems, the Simplex algorithm, discussed later in the text, provides a powerful computational tool, able to provide fast solutions to very large-scale applications, sometimes including hundreds of thousands of variables (i.e., decision factors). In fact, the Simplex algorithm was one of the first Mathematical Programming algorithms to be developed (George Dantzig, 1947), and its subsequent successful implementation in a series of applications significantly contributed to the acceptance of the broader field of Operations Research as a scientific approach to decision making.

As it happens, however, with every modelling effort, the effective application of Linear Programming requires good understanding of the underlying modelling assumptions, and a pertinent interpretation of the obtained analytical solutions. Therefore, in this section we discuss the details of the LP modelling and its underlying assumptions, by means of the following example.

2.4.7 The general LP formulation

The general form for a Linear Programming problem is as follows:

Objective Function: max / min $f(x_1, x_2, x_3, \ldots, x_n) = c_1x_1 + c_2x_2 + c_3x_3 + \ldots c_nx_n$ (1)

Subject to technological Constraints:

$a_{i1}X_1 + a_{i2}X_2 + \ldots + a_{in}X_n \ (= \geq \leq) b_i, \ i=1,\ldots,m$ (2)

Sign Restrictions:

$(X_j \geq 0)$ or $(X_j \leq 0)$ or$(X_j$ urs$)$, $j = 1,...,n$ (3)

where “urs” implies unrestricted in sign.

The formulation of Equations 1 to 3 has the general structure of a mathematical programming problem. It is characterized by the fact that the functions involved in the problem objective and the left-hand-side of the technological constraints are linear. It is the assumptions implied by linearity that to a large extent determine the applicability of the above model in real-world applications.

To provide a better feeling of the linearity concept, let us assume that the different decision variables $x_{1},\ldots x_{n}$ correspond to various activities from which any solution will be eventually synthesized, and the values assigned to the variables by any given solution indicate the activity level in the considered plan(s). For instance, in the above example, the two activities are the production of items $P_1$and P_2, while the activity levels correspond to the daily production volume. Furthermore, assuming each technological constraint of Equation 2 imposes some restriction on the consumption of a particular resource, then referring back to the prototype example, the two problem resources are the daily production capacity of the two workstations $W_1$and W_2. Under this interpretation, the linearity property implies that:

Additively assumption:

> the total consumption of each resource, as well as the overall objective value are the aggregates of the resource consumptions and the contributions to the problem objective, resulting by carrying out each activity independently.

Proportionality assumption:

> these consumptions and contributions for each activity are proportional to the actual activity level.

It is interesting to notice how the above statement reflects to the logic that was applied when we derived the technological constraints of the prototype example:

(i) Our assumption that the processing of each unit of product at every station requires a constant amount of time establishes the proportionality property for our model.

(ii) The assumption that the total processing time required at every station to meet the production levels of both products is the aggregate of the processing times required for each product if the corresponding activity took place independently implies that our system has an additive behaviour.

It is also interesting to see how the linearity assumption restricts the modelling capabilities of the LP framework: As an example, in the LP paradigm, we cannot immediately model effects like economies of scale in the problem cost structure, and/or situations in which resource consumption by one activity depends on the corresponding consumption by another complementary activity. In some cases, one can approach these more complicated problems by applying some linearization scheme. The resulting approximations for many of these cases have been reported to be quite satisfactory.

Another approximating element in many real-life LP applications results from the so called divisibility assumption. This assumption refers to the fact that for LP theory and algorithms to work, the problem variables must be real. However, in many LP formulations, meaningful values for the levels of the activities involved can be only integer. This is, for instance, the case with the production of items $P_1$and P_2in our prototype example. Introducing integrality requirements for some of the variables in an LP formulation turns the problem to one belonging in the class of (Mixed) Integer Programming (MIP). The complexity of a MIP problem is much higher than that of LP's. Given the increased difficulty of solving IP problems, sometimes in practice, near optimal solutions are obtained by solving the LP formulation resulting by relaxing the integrality requirements, known as the LP relaxation of the corresponding IP and (judiciously) rounding off the fractional values for the integral variables in the optimal solution. Such an approach can be more easily justified in cases where the typical values for the integral variables are in the order of tens or above, since the errors introduced by the rounding-off are rather small, in a relative sense.

We conclude our discussion on the general LP formulation, by formally defining the solution search space and optimality. Specifically, we shall define as the feasible region of the LP of Equations 1 to 3, the entire set of vectors $X_1,X_2,...,X_n$ that satisfy the technological constraints of Eq.1 and the sign restrictions of Eq. 3. An optimal solution to the problem is any feasible vector that further satisfies the optimality requirement expressed by Eq. 1.

2.5 Recent Research on Mathematical Modelling

Sohier (2006) researched on the topic “Modelling a Complex Production Scheduling Problem - Optimization Techniques”. The purpose of this research was to evaluate different solution techniques used in optimization to potentially assist in possible improvements in other connected activities. The researcher concluded that operation research cycle could be iterated a number of times with an input feedback. During his implementation and demonstration, some opportunities to lower costs and increase productivity were suggested. Reasonably good results have been obtained in the final stage. The suggested combination is fast and can handle different types of costs. This is promising regarding the support of decomposition.

Sadhana (2002) researched on the topic "Efficient Presolving in Linear Programming". It was noted that a set of presolving techniques for linear programming was efficiently implemented. His aim was to remove the redundant constraints and variables and to identify the possible infeasibility and unboundedness conditions in the linear programming problems in as little time as possible. The results presented showed that linear programming solvers run faster with this efficient implementation of the presolving techniques than without it. It was concluded that presolving techniques are highly successful in reducing the size of the input matrices before they can be sent to an LP solver. Also the time taken by the presolving techniques was negligible when compared to the time taken by the LP solver. Further, the time taken by the LP solver to solve the presolved problem is significantly less than the time taken by it to solve the same unpresolved input problem.

Modelling in mathematics entails the creation of representations of reality. Ledder (2005) explained that modelling in mathematics to be the art and science of constructing mathematical models which can be used to gain insight into the physical processes or making predictions about the physical processes. He is of the view that constructing a mathematical model is the domain of science while that aspect of art deals with determining an appropriate conceptual model.

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CHAPTER THREE

METHODOLOGY

3.1 Introduction

Production problems involve a single product which is to be manufactured over a number of successive time periods to meet pre-specified demands. The demand for a product is high if the cost of that product is low. The cost of a product depends on the cost of production and a low cost of production can be achieved in the course of the minimizing of the total production cost. This chapter will focus on the formulation of a production problem and the development of an algorithm for minimizing cost of production using linear programming (simplex algorithm).

3.1 Mathematical formulation of production problems

The production problem involves the manufacturing of a single product which can either be shipped or stored. The cost of raw materials and the electricity cost of producing each bag of cement are known. Total cost of production is made up of total cost of raw material used plus total cost of electricity. The underlying assumptions of the mathematical formulation are:

(i) Cements produced cannot be allocated prior to being produced.

(ii) Cements produced in a particular month are allocated to the demand in that month or months ahead.

The production problem is modelled as a balanced transportation problem as follows: Since the production of cement involves raw materials and electricity, we consider the raw material for producing each bag of cement as sources S_1, S_2, S_3, …, S_n and electricity used to produce each bag of cement as destination W_1, W_2, W_3, …, W_n.

The production capacities a_i at the sources S_i are taken to the supplies in a given period i and the demands at the warehouse W_j is d_j. The problem, which will meet all the demands at minimum total cost while satisfying all constraints of production capacity and demands

Let c_{ij} be the raw material cost per unit during the time period plus the electricity cost per unit from time period until time j. If we let x_{ij} denote the number of units to be produced during time period i from S_i for allocation during time period j to W_j then for all i and j, $x_{ij} \geq 0$ (since the number of units produced cannot be negative).

$i = 1, 2, 3,\ldots, m$ and $j=1, 2, 3,\ldots, n$.

For each i, the amount of cement produced at S_i is:

$$\sum_{j=1}^{n} x_{ij}$$

We shall consider a set of m supply points from which a unit of the cement is produced. Since supply point S_i can supply at most a_i units in any given period, we have:

$$\sum_{j=1}^{n} x_{ij} \leq a_i, \; i = 1, 2, \ldots, m$$

We shall also consider a set of n demand point which the cements are allocated. Since demand points W_j must receive d_j units of the shipped cements, we have:

$$\sum_{i=1}^{m} x_{ij} \geq dj, j = 1,2,\ldots,n$$

Since units are produced cannot be allocated prior to being produced, C_{ij} is prohibitively large for i > j to force the corresponding x_{ij} to be zero or if allocation is impossible between a given source and destinations, a large cost of M is entered. The total cost of production then is:

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij}$$

The objective is to determine the amount of cements allocated from source to a destination such that the total production costs are minimized.

The model is:

Minimize

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij}$$

Subject to:

$$\sum_{j=1}^{n} x_{ij} \leq a_i,\ i = 1, 2, 3, \dots, m \quad (Supply\ constraints)$$

$$\sum_{i=1}^{n} x_{ij} \geq d_i,\ j = 1,2,3, \dots, m\ (Demand\ constraints)$$

$$x_{ij} \geq 0, \quad j = 1, 2, 3, \dots, n \qquad i = 1, 2, 3, \dots, m$$

The non-negative condition $x_{ij} \geq 0$ is added since negative values for any x_{ij} have no physical meaning. Hitchcock (1947) formulated the production scheduling model and this was also considered independently by Koopmans (1947).

The formulation above is solved using a method known as the Linear Programming (LP).

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3.2 Linear Programming (LP)

Linear programming is a considerable field of optimization for several reasons. Many practical problems in operation research can be expressed as LP problems. Certain special cases of LP, such as network flow problems and multicommodity flow problems are considered important enough to have generated much research on specialized algorithms for their solution. A number of algorithms for other types of optimization problems work by solving LP problems as sub-problems. Historically, ideas from LP have inspired many of the central concepts of optimization theory, such as duality, decomposition, and the importance of convexity and its generalizations. Likewise, LP is heavily used in microeconomics and company management, such as planning, production, transportation, technology and other issues. Although the modern management issues are ever-changing, most companies would like to maximize profits or minimize costs with limited resources. Therefore, many issues can be characterized as LP problems.

Linear programming is a subclass of allocation modelling. It is a method of allocating scarce resources to competing activities under the assumptions of linearity. The structure of the problems it deals with is made up of variables with linear relationships with each other. In the LP problem, decision variables are chosen so that a linear function of the decision variables is optimized and a simultaneous set of linear constraints involving the decision variables is satisfied. LP is a generalization of Linear Algebra. It is capable of handling a variety of problems, ranging from finding schedules for airlines or movies in a theatre, to distributing oil from refineries to markets. The reason for this great versatility is the ease at which constraints can be incorporated into the model.

3.2.1 The Basic LP Problem

A LP problem contains several essential elements. First, there are decision variables (x_j), which denotes the amount undertaken of the respective unknowns, of which there are n (j=1, 2 ..., n).

Next is the linear objective function where the total objective value (Z) equals

$c_1x_1 + c_2x_2 + \dots + c_nxn.$

Here c_j is the contribution of each unit of x_j to the objective function. The problem is also subject to m constraints.

An algebraic expression for the i^{th} constraint is:

$$a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n \leq b_i \ (i=1, 2, ..., m)$$

where b_i denotes the upper limit or right hand side imposed by the constraint and a_{ij} is the use of the items in the i^{th} constraint by one unit of x_j. The c_j, b_i, and a_{ij} are the data (exogenous parameters) of the LP model.

Given these definitions, the LP problem is to choose $x_1, x_2, ..., x_n$ so as to

Maximize (Max) $Z = c_1x_1 + c_2x_2 + \dots + c_nx_n$

subject to (s.t.) $a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \leq b_1$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \leq b_2$$

$$. \quad . \quad . \quad .$$

$$. \quad . \quad . \quad .$$

.   .   .   .

$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \leq b_m$

$x_1 \geq 0, x_2 \geq 0, \ldots, x_n \geq 0$

3.2.2 Other forms of the LP Problem

Not all LP problems will naturally correspond to the above form. Other legitimate representations of LP models are:

Objectives which involves minimization instead of maximization i.e.,

Minimize $Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$. .... (3.1)

Constraints which are "greater than or equal to" instead of "less than or equal to"; i.e.,

$a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n \geq b_1$. .... (3.2)

Constraints which are strict equalities; i.e.,

$a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n = b_1$. .... (3.3)

Variables without non-negativity restriction i.e., x_j can be unrestricted in sign i.e.,

$x_j \geq 0$ or $x_j \leq 0$. .... (3.4)

Variables required to be non-positive i.e.,

$x_j \geq 0$. .... (3.5)

3.2.3 The Standard Form of LP

Linear programme can have objective functions that are to be maximized or minimized, constraints that are of three types (≤, ≥ or =), and variables that have upper and lower bounds. An important subset of the possible LPs is the standard form LP.

A standard form LP has these characteristics:

- The objective function must be maximized,
- All constraints are ≤ type,
- All constraint right hand sides are nonnegative,
- All variables are restricted to nonnegative.

In an algebraic representation, a standard form LP with *m* functional constraints and *n* variables looks like this:

- Objective function:

  Maximize $Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$

where the c_j, the coefficients in objective function represent the increase or decrease in Z, the objective function value per unit increase in x_j.

The *m* functional constraints take the form:

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_{1n} \leq b_1$$

$$a_{12}x_1 + a_{22}x_2 + \ldots + a_{2n}x_{2n} \leq b_2$$

$$\begin{array}{cccc} . & . & . & . \\ . & . & . & . \\ . & . & . & . \end{array}$$

$$a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n \leq b_m$$

where b_m are the resource limits, and the a_{mn} are the coefficients of the functional constraint equations, expressing the usage resource *m* consumed by activity *n*.

3.2.4 Terminology

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The function to be maximized or minimized is called the objective function. A vector, ***x*** for the standard maximum problem or ***y*** for the standard minimum problem, is said to be feasible if it satisfies the corresponding constraints. The set of feasible vectors is called the constraint set. A linear programming problem is said to be feasible if the constraint set is not empty; otherwise it is said to be infeasible. A feasible maximum (resp. minimum) problem is said to be unbounded if the objective function can assume arbitrarily large positive (resp. negative) values at feasible vectors; otherwise, it is said to be bounded. Thus there are three possibilities for a linear programming problem. It may be bounded feasible, it may be unbounded feasible, and it may be infeasible. The value of a bounded feasible maximum (resp. minimum) problem is the maximum (resp. minimum) value of the objective function as the variables range over the constraint set. A feasible vector at which the objective function achieves the value is called optimal.

3.2.5 Standard Minimization Problem

A standard minimization problem is a linear programming problem with an objective function that is to be minimized. The objective function is of the form

$$P = ax + by + cz + \cdots$$

where $a, b, c, \ldots$ are real numbers and $x, y, z, \ldots$ are decision variables.

The decision variables are constrained to nonnegative values. Additional constraints are of the form:

$$Ax + By + Cz + \cdots \geq M$$

where A, B, C,... are real numbers and M is nonnegative.

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3.2.6 Economic Assumptions of the Linear Programming Model

In formulating this problem as a linear-programming model, one must understand the economic assumptions that are incorporated into the model. Basically, one assumes that a series of linear (or approximately linear) relationships involving the decision variables exist over the range of alternatives being considered in the problem. For the resource inputs, one assumes that the prices of these resources to the firm are constant over the range of resource quantities under consideration. This assumption implies that the firm can buy as much or as little of these resources as it needs without affecting the per unit cost. Such an assumption would rule out quantity discounts. One also assumes that there are constant returns to scale in the production process. In other words, in the production process, a doubling of the quantity of resources employed doubles the quantity of output obtained, for any level of resources. Finally, one assumes that the market selling prices of the two products are constant over the range of possible output combinations. These assumptions are implied by the fixed per-unit profit contribution coefficients in the objective function. If the assumptions are not valid, then the optimal solution to the linear-programming model will not necessarily be an optimal solution to the actual decision-making problem. Although these relationships need not be linear over the entire range of values of the decision variables, the linearity assumptions must be valid over the full range of values being considered in the problem.

3.2.7 LP Methods

There are several methods of solving LP Problems. These are:

The Graphic method

The Vector method

The Systematic Trial-and-Error method

The Interior Point methods (Primal-Dual)

The Simplex method

3.2.8 Graphical solution of the linear programming problem

Various techniques are available for solving linear-programming problems. For larger problems involving more than two decision variables, one needs to employ algebraic methods to obtain a solution. For problems containing only two decision variables, graphical methods can be used to obtain an optimal solution. For this approach, graph the feasible solution space and objective function separately and then combine the two graphs to obtain the optimal solution.

3.2.9 Primal-Dual Interior Point Methods for Linear Programming

In linear programming, the problem to solve in standard form is:

$$\text{Minimize} \quad c^T x$$

$$\text{subject to } Ax = b \qquad x \geq 0, \qquad \text{(PP)}$$

where c, x ε R^n, b ε R^m and A is an m × n matrix. This problem is called the primal problem. Associated with it, is the dual problem, which can be formulated as:

$$\text{maximize} \quad b^T y$$

$$\text{subject to:} \quad A^T y \leq c,$$

or, in standard form

$$\text{maximize} \quad b^T y$$

$$\text{subject to } A^T y + s = c, \qquad s \geq 0, \qquad \text{(DP)}$$

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The simplex method is used for this model because it is a step-by-step procedure for moving from corner point to corner point of the feasible solution space in such a manner that successively larger (or smaller) values of the maximization (or minimization) objective function are obtained at each step. The procedure is guaranteed to yield the optimal solution in a finite number of steps. The corner point is point in the feasible region which intersects at two or more boundary lines. It is acknowledged that if an optimal solution to the objective function exists, it must occur at a corner point of the feasible region. The simplex method determines:

- The combination of enterprises that maximizes profit or minimizes cost for the establishment
- The maximum profit or value of the establishment or the minimum cost
- Shadow prices for resources or inputs used on the establishment.

Steps

1. Formulate the problem in the standard manner. After, the inequalities have to be converted to equalities by introducing slack variables. These should be balanced or symmetrical so that each slack variable appears in each equation with a proper co-efficient.

2. Design an Initial Programme (a Basic Feasible Solution)

   Design the first programme so that only the slack variables are included in the solution. Place this programme in a simplex table. In the objective row above each column variable, place the corresponding coefficient of that variable from step 1.

Tableau (Basic Solution)

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	c_j	c_1	c_2	…	c_n	0	0…	0		
Basic		x_1	x_2	…	x_n	S_1	$S_{2\ldots}$	S_m	b_j	RHS(b_j/a_{ij})
S_1	0	a_{11}	a_{12}	…	a_{1n}	1	0 …	0		
S_2	0	a_{21}	a_{22}	…	a_{2n}	0	1 …	0		
.	.	.	.	.	.	.	.	.		
.	.	.	.	.	.	.	.	.		
.	.	.	.	.	.	.	.	.		
S_m	0	a_{m1}	a_{m2}	…	a_{mn}	0	0 …	1		
z_j	Z_1	Z_2	Z_{mn}	...	Z_{11}	Z_{12}	...	Z_{1m}		
$c_j - z_j$	$c_1 - z_1$	$c_2 - z_2$	$c_{mn} - z_{mn}$	...	$c_{11}-z_{11}$	$c_{12}-z_{12}$	...	$c_{1n}-z_{1n}$		

3. Test and revise the table

   Calculate the net-evaluation row. To get a number in the net-evaluation row under a column, multiply the entries in that column by the corresponding numbers in the objective column (Z_j), and add all the products (C_j), then subtract this sum from the number listed in the objective row (Z_j) at the top of the column. Enter the result in the net-evaluation row under the column.

   Test: Examined the entries in the net-evaluation row for the given simplex tableau. If all the entries are zero or negative, the optimal solution has been obtained.

4. Obtain the optimal solution

   Repeat step 3 until an optimal solution has been derived.

The general mathematical programming problem we will treat is:

Optimize $F(X)$

Subject To (s.t.) $G(X) \varepsilon S_1$

$X \varepsilon S_2$

Here X is a vector of decision variables. The level of X is chosen so that an objective is optimized where the objective is expressed algebraically as F(X) which is called the objective function. This objective function will be maximized or minimized. However, in setting X, a set of constraints must be obeyed requiring that functions of the X's behave in some manner. These constraints are reflected algebraically by the requirements that: a) G(X) must belong to S_1 and b) the variables individually must fall into S_2.

CHAPTER FOUR

DATA COLLECTION AND ANALYSIS

4.0 Introduction

This chapter deals with data collection, analysis of data collected, thus: formulation of the model and the sensitivity analysis.

4.1 Data Analysis

Raw materials usage in 2009 by the factory ranged from a low of 139974 tons in March to a high of 189497 in April. An average of 163366 tons was used each month. An average of 4006460 KWh of electricity per month was used. Among the months, electricity usage varies from a low of 2305020KWh in February to a high of 4946020KWh in March is given Table 4.1.

Table 4.1: SHOWING THE QUANTITY OF RAW MATERIALS AND ELECTRICITY FOR 2009

Month	Raw materials(Tons)	Electricity(kwh)
Jan	182679	4123160
Feb	163386	2305020
Mar	139974	4946020
Apr	189497	4539860
May	186947	4807380
Jun	144867	4243280
Jul	141007	3461650
Aug	169517	3951780
Sep	146035	3556670
Oct	164667	4229360
Nov	161832	4014930

Dec	169985	3898410
Total	1960392	48077520
Average	163366	4006460
Ratio/ Unit	0.117814	0.125558

The cost per month for raw materials electricity in 2009 is given in Table 4.2. The total average cost of raw materials and electricity are GH₵19246.80 and GH₵503043.92 respectively.

Table 4.2: SHOWING THE COST OF RAW MATERIALS AND ELECTRICITY IN PRODUCING CEMENT (2009)

Month	Raw material cost (GH₵)	Electricity cost (GH₵)
Jan	21522.10	518549.33
Feb	19249.20	356857.95
Mar	16490.90	603607.34
Apr	22325.30	561628.39
May	22024.90	590689.29
Jun	17067.30	530167.00
Jul	16612.60	433047.72
Aug	19971.40	488864.00
Sep	17204.90	442869.00
Oct	19400.00	517836.00
Nov	19066.00	497112.00

Dec	20026.60	495299.00
Total	230961.00	6036527.03
Average	19246.80	503043.92

Seasonal average cost of raw materials and electricity for the year 2009 was obtained by adding all the cost of raw materials and electricity, for the dry season (November to March) and dividing the total by the number of month (5), and for the wet season (April to October) and dividing the total by the number of months (7) is given in Table 4.3.

Table 4.3: SHOWING AVERAGE SEASONAL COST OF RAW MATERIALS AND ELECTRICITY FOR 2009

Season	Raw materials	Electricity
Dry	19270.80	494285
Wet	19229.60	509300
Total	38500.40	1003585

Seasonal average quantities of raw materials and electricity for the year 2009 was obtained by adding all the quantity of raw materials and electricity, for the dry season (November to March) and dividing the total by the number of month (5), and for the wet season (April to October) and dividing the total by the number of months (7) is given in Table 4.4.

Table 4.4: SHOWING AVERAGE SEASONAL QUANTITIES OF RAW MATERIALS AND ELECTRICITY FOR 2009

Season	Raw materials	Electricity
Dry	163571	3857508
Wet	163219	4112854
Total	326790	7970362

The seasonal unit cost of raw materials and electricity for the year 2009 was obtained by dividing the average cost of raw materials and electricity by the average quantity of raw materials and electricity for the dry season and the wet season is given in Table 4.5.

Table4.5: SHOWING UNIT SEASONAL COST OF RAW MATERIALS AND ELECTRICITY FOR 2009

Season	Raw materials	Electricity
Dry	0.117813	0.128136
Wet	0.058844	0.123831
Total	0.176657	0.251967

4.2 Model

4.2.0 Objective function: The objective function is formulated from the fact that, total monthly cost of production depends on total monthly cost of raw materials and total monthly cost of electricity.The objective is to minimize the total cost, C, of production of cement, where the total cost is equal to the sum of the product of unit cost of raw materials and quantity, and the product of the unit cost of electricity and quantity. Defining x_1 as the quantity of raw materials and x_2 as quantity of electricity, the objective function is

Min C = $0.117814x_1 + 0.125558x_2$

4.2.1 Constrains: The first constrains in this model is the sum of the product of unit cost of raw materials and quantity, and the product of the unit cost of electricity and quantity for the dry season. The Right Hand Side (RHS) is the proposed least dry season cost.

$0.117813x_1 + 0.128136x_2 \geq 66762.30$

The second constrain in this model is the sum of the product of unit cost of raw materials and quantity, and the product of the unit cost of electricity and quantity for the wet season. The Right Hand Side (RHS) is the proposed least wet season cost.

$0.058844x_1 + 0.123831x_2 \geq 68708.80$

Finally, negative production times are not possible. Therefore, each of the decision variables is constrained to be nonnegative:

$x_1 \geq 0, x_2 \geq 0.$

4.2.2 Slack (Surplus) Variables

Surplus variables are subtracted from the greater than or equal to inequality (≥) constraints to convert these constraints to equalities. Like the slack variables, surplus variables are given coefficients of zero in the objective function because they have no effect on the value. In the preceding cost-minimization problem, two surplus variables (S_1, S_2) are used to convert the two (greater than or equal to) constraints to equalities as follows:

Min C = $0.117814x_1 + 0.125558x_2 + 0s_1 + 0s_2$

Subject to:

$$0.117813x_1 + 0.128136x_2 - 0s_1 = 66762.30$$

$$0.058844x_1 + 0.123831x_2 - 0s_2 = 68708.80$$

$$x_1, x_2, s_1, s_2 \geq 0$$

4.3 Graphical Method:

Which is the objective of the function? Minimize

Function: 0.117814 X_1 + 0.125558 X_2

Restrictions:

0.117813 X_1 + 0.128136 X_2 = 66762.30

0.058844 X_1 + 0.123831 X_2 = 68708.80

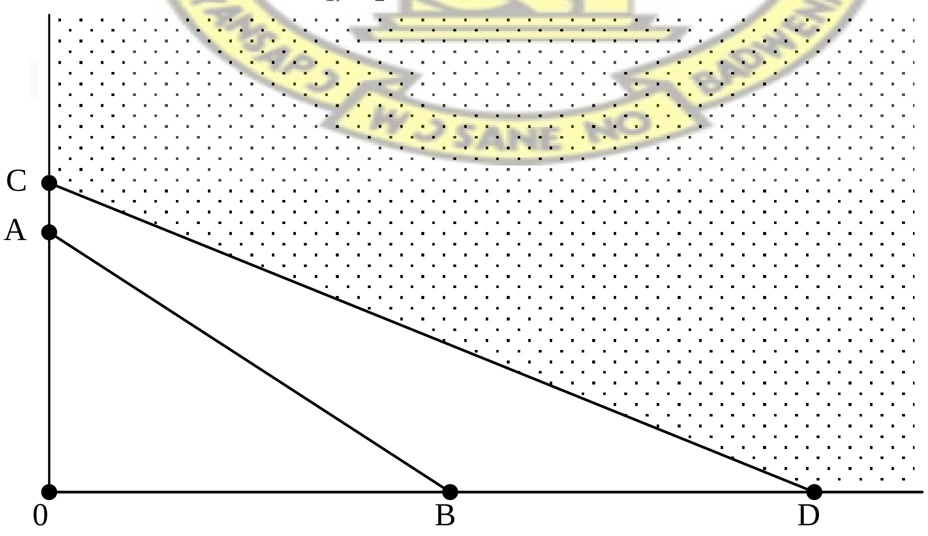
Minimize $Z = 0.117814\ X_1 + 0.125558\ X_2$

Subject to:

$$0.117813\ X_1 + 0.128136\ X_2 \geq 66762.3$$

$$0.058844\ X_1 + 0.123831\ X_2 \geq 68708.8$$

$$X_1, X_2 \geq 0$$



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The problem is unbounded as the problem is a minimizing problem; it is possible to find a solution.

Table 4.6: SHOWING VALVES OF X COORDINATE, Y COORDINATE AND F FOR THE POINTS O, A, B, C AND D.

Point	X coordinate	Y coordinate	F value
O	0	0	0
A	0	521026.877692	65419.0927093
B	566680.247511	0	66762.8666802
C	0	554859.445535	69667.0422624
D	1167643.26015	0	137564.723051

The fields where the solution coordinates are, appears in the shaded region. The fields where is not possible to find the solution, appears in unshaded region.

4.4 Two-Phase Simplex Method:

Which is the objective of the function? Minimize

Function: 0.117813 X_1 + 0.125558 X_2

Restrictions:

0.117813 X_1 + 0.128136 X_2 = 66762.30

0.058844 X_1 + 0.123831 X_2 = 68708.80

Minimize Z = $0.117814x_1 + 0.125558x_2$

Subject to:

$0.117813x_1 + 0.128136x_2 \geq 66762.3$

$0.058844x_1 + 0.123831x_2 \geq 68708.8$

Transforming the problem to standard form, adding slack, surplus and artificial variables as appropriate as:

Maximize $Z = -0.117814x_1 - 0.125558x_2 + 0x_3 + 0x_4 + 0x_5 + 0x_6$

Subject to:

$0.117813x_1 + 0.128136x_2 - 1x_3 + 1x_5 = 66762.3$

$0.0588044x_1 + 0.123831x_2 - 1x_4 - 1x_6 = 68708.8$

$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$

Will build the first board of Phase I from Two-Phase Simplex Method

Board 1

					0	0	-1	-1
Base	Cb	P0	P1	P2	P3	P4	P5	P6
P5	-1	66762.3	0.117813	0.128136	-1	0	1	0
P6	-1	68708.8	0.058844	0.123831	0	-1	0	1
Z		-135471.1	-0.176657	-0.251967	1	1	0	0

Board 2

			0	0	0	0	-1	-1
Base	Cb	$\mathbf{P_0}$	$\mathbf{P_1}$	$\mathbf{P_2}$	$\mathbf{P_3}$	$\mathbf{P_4}$	$\mathbf{P_5}$	$\mathbf{P_6}$
P2	0	521026.878	0.91943716	1	-7.804208	0	7.8042080	0
P6	-1	4189.52071	-0.05501082	0	0.9664029	-1	-0.96640288	1
Z		-4189.52071	0.05501082	0	-0.9664029	1	1.96640288	0

Board 3

			0	0	0	0	-1	-1
Base	**C_b**	**P_0**	**P_1**	**P_2**	**P_3**	**P_4**	**P_5**	**P_6**
P2	0	554859.4455	0.4751960333	1	0	-8.07552228	0	8.07552228
P3	0	4335.169913	-0.05692328108	0	1	-1.034765123	-1	1.034765123
Z		0	0	0	0	0	1	1

There is no possible solution for the problem, so we can pass to Phase II.

Board 1

			-0.117814	-0.125558	0	0
Base	**C_b**	**P_0**	**P_1**	**P_2**	**P_3**	**P_4**
P2	-0.125558	554859.44554	0.4751960333	1	0	-8.07552228
P3	0	4335.1699130	-0.0569232811	0	1	-1.034765123
Z		-69667.042262	0.0581493365	0	0	1.013946427

The optimal solution is Z = 69667.0422624

X1 = 0

X2 = 554859.445535

CHAPTER FIVE

SUMMARY OF FINDING CONCLUSIONS AND RECOMMENDATIONS

5.0 Introduction

This chapter focuses on the outcomes of the analysis in chapter four and considers the extent to which the objectives of the study have been realized. Items covered in this chapter include summary, conclusions and recommendations.

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5.1 Summary of Finding

This research involved a study that sought to develop a model that will minimize the total production cost of cement at GHACEM - Takoradi.

From the analysis done in chapter four using linear programming it shows that, the average total cost of GH¢ 38500.40 and GH¢ 1003585 for raw materials and electricity respectively can be optimized to GH¢69,667.04. This optimal solution can be achieved if the total quantity of electricity is reduced to 554859.45 and the quantity of raw materials remains the same.

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5.2 Conclusions

From the study and the analysis done, it was realized that cost of producing cement can be reduced with respect to the factors that influence the production of cement. It was also discovered that, the cost of raw materials does not have effect on production cost of cement.

From observation, it was noticed that presently on an average, 163366 tons of raw materials and 4006460 kwh of electricity was used at the production site which cost GH¢ 19246.80 and GH¢ 503043.92 respectively. Finally, it was observed that if the cost of electricity is reduced, the cost of cement on the market will go down.

5.3 Recommendations

Based on the finding of this research work, it is recommended that a production cost of GH¢ 69,667.04 can reduce cost of GH¢ 230961.00 less on raw material and GH¢ 6036527.03 less on electricity yearly at GHACEM - Takoradi. The cost of cement production can be reduced if the unit price of raw materials and the unit price of electricity do not exceed GH¢ 0.12 for raw materials and GH¢ 0.13 for electricity.

All machinery and equipments at the production site should be given regularly monitoring and maintenance. There should be proper arrangement with E.C.G. for adequate supply of power to the area where the plant is located. There should be constant repairs of damage equipments and machinery at the production site and there should be energy efficiency technology.

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