KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY

COLLEGE OF SCIENCE

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DEPARTMENT OF MATHEMATICS

OPTIMAL CONNECTION OF GAS PIPELINES FROM TEMA TO ALL REGIONAL CAPITALS OF GHANA

(AN APPLICATION OF PRIM'S ALGORITHM)

A THESIS SUBMITTED TO THE DEPARTMENT OF MATHEMATICS, IN PARTIAL FULFILMENT OF THE REQUIREMENT FOR THE AWARD OF A MASTER OF SCIENCE (MSc.) DEGREE IN INDUSTRIAL MATHEEMATICS.

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Declaration

I, the undersigned, hereby declare that the work contained in this essay is my original work, and that any works done by others have been acknowledged and referenced accordingly.

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Acknowledgement

My greatest gratitude goes to God Almighty whose grace and loving kindness has brought me to the dawn of another era of my life.

I am very much indebted to my parents whose strenuous effort and love has seen me through all these years of my academic career. To them I say, God richly bless you for your impact in my life.

I also appreciate the effort of my supervisor, Dr. S.K. Amponsah for his immense support and incalculable contribution towards this project.

Am deeply grateful to Mr Kwame Boateng of BOST Company Limited who took the trouble to educate me on the gas infrastructural network systems in Ghana and also helped me access information from the company

Am also grateful to Mr Patrick Osei-Darko of the Geology Department of KNUST who educated me on Grid Co-ordinates and its applications and assisted me gather data.

Finally, I appreciate all those who in one way or the other contributed to this project. God bless you all.

Dedication

To my lovely and ever supportive parents, Mr & Mrs Obiri-Danso

Abstract

In 2007, Ghana found oil in commercial quantities and a major by-product of it's exploration is natural gas. The WAGP Company has also laid pipelines from Nigeria through Togo and Benin to Tema and Takoradi in Ghana to meet the country's natural gas needs. There is therefore the need to establish an elaborate and extensive transportation system to efficiently and effectively move natural gas from producing regions to consumption regions. Natural gas transportation requires a continuous pipeline network from well-head to burner- tip. Until Ghana builds a national gas pipeline network, introduction of natural gas into the economy will not necessarily displace all fuel-oil demand. On the basis of its mandate in the natural gas sector as the developer and operator of the natural gas transmission system in Ghana, the Bulk Oil Storage and Transportation Company Limited intends to develop a nationwide network of natural gas transmission pipelines and regulatory metering stations in accordance with the proposed Natural Gas transmission master plan. This research therefore analyses Network Analysis Techniques to the pipeline design problem. The main objective of study is to determine an optimal natural gas pipeline connection route to distribute natural gas throughout the entire nation by extending pipeline lines from the receiving point at Tema and using the regional capitals as nodes. Data on National Grid Co-ordinates was collected and analysed. Euclidean distance formula was used to compute the distances between all pairs of node. Concept of minimal Spanning tree using Prim's algorithm is used in an analytical algorithm for generating near-optimal pipeline networks. An optimal pipeline length of 382.73km was obtained.

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CHAPTER ONE

INTRODUCTION

1.1 Background of Study

Natural gas which is commercially extracted from oil fields and natural gas fields have emerged as a cheaper and efficient source of energy. It has become a fuel of choice for various uses. It is most extensively used by domestic consumers for cooking and heating, by power producers for generating electricity and by petrochemical and fertilizer industries as both fuel and feedstock.

Natural gas is the cleanest of all fossil fuels. It is a source of energy that has become an imperative part of our world today and all economies of the world are driven to a large extent by their energy sector. In today's world, the struggle among nation's for the control of oil and gas fields shows how crucial this energy source is to the sustenance of every nation. While there are substitutes for natural gas in the industry (e.g. naphtha for fertilizer plants), market prices makes natural gas more competitive. Worldwide trade in natural gas is also growing rapidly, and it's reserves are more distributed throughout the world than those of oil. The proven reserves have more than doubled in the last 20 years, increasing the world R/P to 64.1 years in 1997 (BP, 1998).

Ghana as a nation for so many years has been importing natural gas from neighbouring Nigeria to meet her energy needs. In 2007 however, Ghana found oil in commercial quantities and a major by-product of this crude oil is natural gas. It therefore requires an elaborate and extensive transportation system to efficiently and effectively move the natural gas from producing regions to consumption regions.

Until Ghana builds a national gas pipeline network, introduction of natural gas into the economy will not necessarily displace all fuel-oil demand.

In view of its quest to develop gas infrastructure, the Government of Ghana has appointed the Bulk Oil Storage and Transportation Company Limited (BOST) as the developer and operator of the natural gas transmission system in accordance with the provision of EC Act 541, 1997.

On the basis of its mandate in the natural gas sector, BOST intends to develop a nationwide network of natural gas transmission pipelines and regulatory metering stations in accordance with the proposed Natural Gas transmission master plan.

Some advantages of liquefied natural gas to the nation include;

(i) Natural gas exploration will help rejuvenate some of the nation's most important industries.

(ii) New Natural gas development/ increased investments in Natural gas increased economic growth and more job creation.

(iii) Affordable, stable prices make natural gas a fuel source Ghana can depend on.

(iv) Natural gas use in the long run will reduce or end the impact of harmful gas flaring hence reducing the impact of global warming.

A major challenge associated with the extraction and use of the natural gas is the problem of transporting. There is the need to effectively transport gas from the gas fields to the processing units and from the processing unit to the end users. Over the years, pipelines has shown to be the safest, most efficient and economical mode of transporting natural gas.

In developed nations, gas pipelines has extensively been constructed to supply gas from oil fields to processing units and from these processing units to factories and homes where they are consumed.

The proposed connection of gas pipelines from Tema through the ten regional capitals is being designed to ensure that natural gas from the West African Gas Pipeline (WAGP) and the Jubilee Oil Fields is abundantly and cheaply transported to the consuming public and industries.

Ghanaians are increasingly depending on natural gas for their energy needs. In 2010 alone, the nation consumed 124.1 billion cubic metres of natural gas and demand has been projected to hit 165.2 billion cubic metres by 2015. (Business Monitor International (BMI))

Large portions of the gas consumed are transported by road from Tema and Takoradi to the rest of the regions. Ghana is yet to have a major natural gas production basins and the pipeline network of the gas is not fully developed to other regional capitals. Most of the natural gas consumed are imported and few supply from the West African Gas pipeline (WAGP). Of the natural gas consumed in Ghana, 0% is produced locally (BOST, 2010). There is no transmission of pipeline system connecting the West African Gas Pipeline from Takoradi or Tema to the regional capitals. The primary function of the transmission pipeline is to move huge amounts of natural gas hundreds of miles from the producing region to the local natural gas utility delivery points. This delivery points, called "city gates stations" are usually owned by distributions companies, although some are owned by transmission companies.

The transmission segment of the gas industry is responsible for transporting natural gas from the producer to the market areas via pipelines. The transmission segment is composed of pipelines, compressor stations, city gate stations, and storage facilities. The energy sector of Ghana is faced with urban and rural areas, transport sector, and industries' total dependence on natural gas. This gas is either not readily available or transportation is very high hence high cost of the gas. The connection of gas pipelines from Tema to the other regional capitals seek to solve the transportation problem. The installations of these pipelines across these regional capitals are expensive and disruptive to the natural environment. Thus, this research seeks to find the shortest-path through which these pipelines can be installed so as to reduce the cost of installation, minimize the number of pipelines needed and also reduced the destructive impact on the environment.

1.1.1 Gas Pipelines

A natural gas pipeline is a pipeline used to transport natural gas.

Energy Pipelines – oil, natural gas, gasoline and many other chemicals as well are part of the subterranean world, along with water lines, sewer lines, storm sewers, telephone lines, television cables and many other electric lines. Crude oil and natural gas which are the raw materials for energy the world consumes, are found in vastly different locations than where they are processed and refined into fuels, and in different locations where they are consumed.

Over the years, many forms of transportation have been used to move these natural resources to market places. Of all the transportations used, pipelines remain the safest, most efficient and economical way to move these natural resources.

Pipeline networks are largely directed networks. A directed network is composed of links which flow in only one direction whereas an undirected network encompasses flow in both directions on its links. An urban road system which utilizes one street is an example of an undirected network.

Pipelines systems tend to be directed because they connect supply and demand centres, and flow is generally from supply to demand.

The United States has the largest network of gas pipelines than any nation of the world. Gas pipeline network alone in the United States is ten times larger than that in Europe (Wikipedia, 2008). In Africa however there only two gas pipeline systems namely; The West African Gas Pipe line and The Trans –Sahara Gas Pipeline system (Wikipedia, 2008).

There are two general types of energy pipelines- oil pipelines and natural gas pipelines. Within each group are subsets that serve very specific portions of the energy market place. Within the oil pipeline network are both crude oil lines and refined product lines (Wikipedia, 2008).

Crude oil is also subdivided into Gathering Lines and Trunk Lines. These pipelines vary in diameter sizes as well as depth of oil reach. Natural gas pipeline systems however, is organised somewhat differently. Natural gas unlike oil is delivered directly to homes and business through Local distribution lines.

1.1.2 Safety of Pipelines

In advanced countries where major gas pipeline systems have been constructed, the safety records of these pipelines are excellent. Pipeline systems are recognized as both the safest transportation mode and the most economical way of distribution the vast quantity of oil from production fields to refineries and from refineries to consumers. Pipelines are usually the only feasible way to transport significant volumes by land over long distances. Without pipelines, our streets and highways will be overwhelmed by the trucks trying to keep up with the nation's demand for natural gas and petroleum products as demand rises.

In the United States for instance, the safety of the pipeline industry has improved substantially over the last 30 years with even greater gains in the last few years. All oil pipelines in the United States for instance had 129 spills larger than 50 barrels (2100gallons) for the 2001 data year (http://ops.dot.gov/stats.htm). Death and injuries resulting from oil pipelines transportation are very rare but do occur occasionally.

1.1.3 The West African Gas Pipeline

The West African Gas Pipeline (WAGP) is a 681 kilometre onshore and offshore pipeline with a diameter of 20inches and it's designed to carry natural gas from Nigeria's Escravos region of the Niger Delta area to Benin, Togo and Ghana. It is the first regional natural gas transmission system in sub-Saharan Africa.

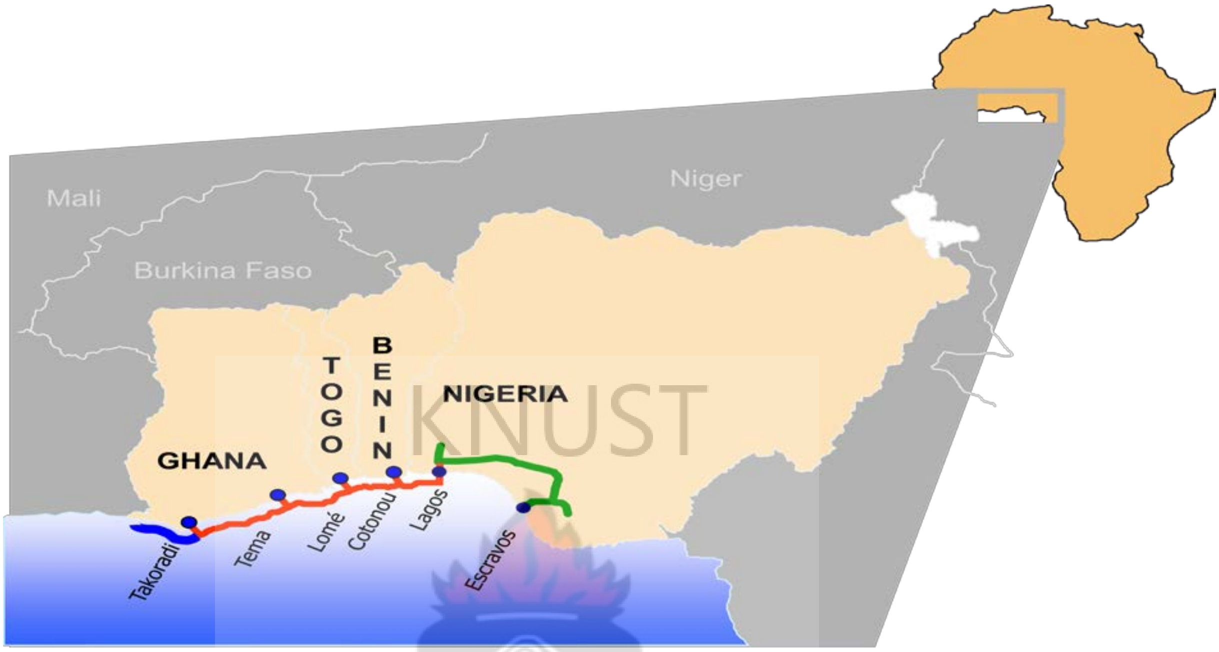
The project history began in 1982, when the Economic Community of West African States (ECOWAS) proposed the development of a natural gas pipeline throughout West Africa. In 1991, a feasibility report conducted by the World Bank on supplying Nigerian gas on West African markets deemed that the project was commercially viable. In September 1995, the governments of four African countries signed a Heads of State Agreement. The feasibility study was carried out in 1999. On 11 August 1999, a Memorandum of Understanding was signed by participating countries in Cotonou. In February 2000, an Inter-Governmental Agreement was signed. The WAGP implementation agreement was signed in 2003. Groundbreaking ceremony for the project was held at Sekondi-Takoradi, Ghana, on 3 December 2004. The construction started in 2005. The offshore pipeline was completed in December 2006. It was scheduled to start operating on 23 December 2007 but was delayed after leaks were detected in supply pipelines in Nigeria. The WAGP is expected to make a supply of 100 million standard cubic feet of natural gas per day upon completion of the project but currently its supply capacity is 40MSCF/day.

The Pipeline System is owned and operated by West African Gas Pipeline Company limited (WAPCo), a limited liability company. The company has its headquarters in Accra, Ghana, with an office in Badagry, Nigeria, and field offices in Cotonou - Benin, Lome - Togo, Tema and Takoradi, both in Ghana.

The West African Gas Pipeline Company (WAPCo) is in charge of both construction and operation of the pipeline. WAPCo is a limited liability company owned by Chevron West African Gas Pipeline Limited (36.9%); Nigerian National Petroleum Corporation (24.9%); Shell Overseas Holdings Limited (17.9%); and Takoradi Power Company Limited (16.3%), Societe Togolaise de Gaz (2%) and Societe BenGaz S.A. (2%).

Diagram 1.0

Diagram showing the connection of the West African Gas Pipeline from Nigeria to Ghana, Benin and Togo



Source: www.wagpa.org

1.1.4 Thermal Power Stations

(a) The Takoradi Thermal Plant

The Takoradi Thermal Power Station (TTPS) is located at Aboadze, 17 kilometres east of Sekondi-Takoradi in the Western Region of Ghana. The Power Station which started operation in 1997 was initiated by the Volta River Authority to complement the existing Hydro Plant at Akosombo and Kpong. TTPS is therefore a facility of strategic importance for meeting Ghana's energy needs.

The current installed capacity of the Takoradi Power Plant is 550MW and is to be upgraded to 660MW by the CMS/VRA joint Venture. The Plant has dual firing capacity and has been converted to run on natural gas.

The contractual agreement is to supply the Volta River Authority with 100million standard cubic feet of natural gas per day. However, current capacity of the WAGP is 40million standard cubic feet per day (www. ghanaweb.com)

(b) Sunon Asogli Thermal Plant

The Sunon Asogli Power (GH) Ltd is an enterprise jointly established by the Shenzhan Energy Group Limited and the China Africa Development Fund, the former having 60 per cent shares and the latter 40 per cent.

The project is the first private venture of it's kind in Ghana. It is environmentally friendly and has a natural gas fuelling combine cycle power plant situated on a 50-acra land.

The project is aimed at providing 5,600 megawatts of power for Ghana and it's currently producing at a capacity of 200megawatts of power with it's second phase of 360 megawatts earmarked to be completed by the end of 2012.

1.1.5 NETWORK ANALYSIS

Natural gas network analysis is used to determine the amount of gas, flow rate of the gas and the pressure used in the pipeline system. Besides that, it can also be used to determine the usage of gas by consumers especially during the peak times. By making the network analysis, time of maintenance and failure of pipeline can be measured. The effect of new load on the pipeline can also be predicted. Generally, network analysis can be divided into three types; via Network Tracing, Network Routing and Network Allocation. Network tracing is done in other to trace the allocation of the particular pipeline in the system. Network routine is done in order to find the optimum path or the shortest path which cost less for the system. Network allocation is done in order to locate all the pipelines into the main support. Our analysis of the network shall be focused in networking routing.

1.1.6 NETWORK ROUTING

Route networks are composite entities that frequently model complex distribution and logistics decisions. The traditional operation research transportation problem is illustrative. A shipper with an inventory of goods at its warehouse must ship to geographically dispersed retail centres, each with a given customer demand. Each arc connecting a supply point to a retail centre incurs cost based upon some physical network, in this case the transportation network. Rather than solving the problem directly on the physical network, we pre-process the data and construct transportation routes. Hence, in effect an arc connecting a supply point and a retail centre might represent distribution channel with varied legs:

(i) From a warehouse(by truck) to a rail station,

(ii) From the rail station to a rail head elsewhere in the system,

(iii) From the rail head (by truck) to a distribution centre and

(iv) From the distribution centre (on a local delivery truck) to the final consumer.

If we assign the arc the composite distribution cost of all the intermediary legs, as well as the distribution capacity for this route, this problem becomes a classic network transportation model defined over a bipartite graph: find the flow from the gas reserves to consumers that minimizes overall costs.

Similar application arises in all of the problem settings in physical networks. In some applications, however the network might contain intermediary points that serve as transhipment nodes; they neither generate flow nor consume flow. Pumping stations in a natural gas transportation network would be an example. In this application, the network model is a general minimum cost flow problem, rather than a classic transportation problem.

1.2 Problem Statement

The demand for natural gas in Ghana has increased significantly as it is a cheaper alternative fuel used in our homes, industries, power generation as well as the mining and transportation sectors of the country. Natural gas transportation requires a continuous pipeline network from well-head to burner- tip.

This study researches into how Ghana, a developing oil and gas economy can effectively and efficiently transport the vast natural gas resources available to her from producing centres to the consumption regions. In this regard, this study is concerned with the optimal design of gas pipeline connection from Tema; the hub of Ghana's industrial drive through the ten regional capitals of the country. This will ensure effective and efficient distribution of the resource and will also curb the menace of frequent shortages experienced in the country.

1.3 Objectives of Study

The objectives of this study is to

(i) Extend gas pipelines from Tema to all Regional Capitals in Ghana by employing Prim's algorithm to design an optimal pipeline network to reduce cost in length of pipelines used and also to reduce environmental pollution.

(ii) Provide information which will serves as a guide for provision of medical services (immunization programmes), transportation accessibility, etc in terms of the shortest route to take in order to make the services cost effective.

(iii) Serve as a basis and a guide for further research to be conducted in the field of optimal pipeline design in Ghana.

1.4 Scope of Study

Ghana is located in West Africa. Politically, it is bounded to the east by Togo, to the west by Cote d'Ivoire, to the north by Burkina Faso and to the south of it is the Gulf of Guinea. Geographically, Ghana is divided into ten regions with their capital cities.

For the purpose of this study, the regional capitals will serve as the nodes in the network whereas the distances between the capital cities will be the edges or the arcs. The regions and their capital towns to which the gas pipelines are to be connected from Tema are; Greater Accra-Accra, Western –Takoradi, U

pper West-Wa, Upper East-Bolgatanga, Northern –Tamale, Brong -Ahafo -Sunyani, Ashanti-Kumasi, Eastern-Koforidua, Volta-Ho and Central- Cape Coast.

National Grid Co-ordinates are employed in the optimal pipeline design problem. The grid co-ordinates make use of specific points within the capital cities as nodes.

1.5 Significance of Study

With the popularity of natural gas, the scale of natural gas transmission and distribution pipeline network is continuously enlarged. The natural gas pipeline network must be optimized in order to increase economic efficiency and pipeline network utilization. Stake holders of the industry; consumers, producers as well as the government are required to monitor and adapt to the continuous changes in the political, economic, social and technological environment. Consequently, it can be said that a proper path and distances are crucial for the overall production and supply of natural gas from the source to different destinations.

It is envisioned that the findings from the study will enable the Bulk Oil Storage and Transportation Company (BOST), the Ghana National Gas Company (GNGC), Ghana National Petroleum Corporation (GNPC), together with other stakeholders to know the minimum length of pipelines needed to connect the entire nation and will also know the shortest route to use in connecting these pipelines.

Economically, Individual consumers as well as Industries will get access to the commodity in an economical and timely manner which in the long term has significant improvement on productivity.

An efficient and effective gas pipeline system will result in a switch from crude oil to clean-burning natural gas for industrial use, power generation as well as domestic use in Ghana. This will improve the environment for people in the region through measurable long-term reductions in air pollutants.

The study will also serve as a guide for further research in other areas such as network design in transportation (rail lines, water pipelines roads, etc), telecommunication (fibre-optic networks, computer networks etc) and rural and urban electrification in a township or country.

1.6 Methodology

Since the problem of study; minimum connection of gas pipelines from Tema to all capital cities in Ghana is a transportation problem, Shortest Path methods are employed in this research.

Shortest path problems are the most fundamental and commonly encountered problems in the study of transportation and communication network. All the networks are made up of nodes (vertices) and arcs. One can move from one node along some edges to some other node. Since all edges have length/weight associated with it, a shortest path is the one that has the minimum weight of edges among all the paths. Similarly, a longest path is the one for which the sum of its length is maximum. Shortest path methods include; Djikstra's algorithm, Floyds-Warshall algorithm, Prim's algorithm etc. This research seeks to determine the shortest distances (and paths) from the start node to connect all other nodes in the network.

The researcher accessed information on the national grid coordinates of Regional capital towns of Ghana from data provided by the Energy Research Centre of the Kwame Nkrumah University of Science and Technology, Kumasi. Series of interviews was conducted by the researcher with personnel of the Survey Department as well as Research Students of the department of Geology in KNUST to acquaint himself with the coordinate systems of the world and that of Ghana in particular. The researcher also contacted the engineer at BOST Company Limited and data on the demand levels as well as diagrams on proposed infrastructure developments were obtained. An interview section was conducted with the engineer of the company (Mr. Kwame Boateng) for the researcher to be abreast with developments of the gas infrastructure. Map of the road network system of the regional capitals and the distance they cover within the region was also obtained from the Ghana Highway Authority.

Prim's algorithm was employed to find an optimal pipeline design network. In addition to standard pipeline construction, special connection techniques are used when warranted by site-specific conditions such as when constructing across paved roads, highways, railroads, steep terrain, water bodies and wet lands, and when blasting through rock. All these are done so as to ensure safety and minimize cost and distance covered.

1.7 Justification

Over the years, the nation has been bedevilled by the rampart shortages of natural gas products. Households have been most affected in this regard since they need the commodity in cooking as well as other uses. It also takes several days for gas to be transported to consumers in the various regional capitals even if the gas is imported into the country. This makes the system unfair to those who are far from the delivery point.

Fortunately, Ghana is now oil and gas economy and the West African Gas Pipeline Company has laid pipelines from Nigeria to Tema and Tarkoradi to supply natural gas to the nation. It is therefore incumbent on the Government of the day to better explore this vast natural gas resource in an effective, efficient and timely manner so as to benefit the entire nation. There is therefore the need to find an effective distribution channel through which refined natural gas product can be transported from the Thermal Plant to the numerous consumers throughout the nation in an efficient and timely manner. This study seeks to find the minimum connection of gas pipelines from Tema to all capital cities in Ghana.

By this study, Stakeholders will be in a better position to know the minimum length of pipelines needed to connect the entire nation and will also know the minimum route to use in connecting these pipelines.

1.8 Limitations of Study

The main limitation to this study was

(i) Unavailability of literature relating to gas pipeline connections in Ghana. The effect of this was that there is no available research document for the researcher to make comparison.

(ii) Difficulties encountered in accessing information from the Ghana Survey Department and the Energy centre.

(iii) Financial constraints since the study were solely financed by the student.

1.9 Organisation of Study

The review of literature in chapter two summarizes a number of works in the field of networks which have application to pipelines. The chapter begins by reviewing publications related specifically to developments in the pipeline industry. The study of networks, however, encompasses more than just pipelines. Hence, the literature also contains summaries of published articles relating to transportation network optimization and optimal facility location Chapter three of the study discusses the methodology used in the study. It examines the mathematical treatment and presentation models, variant, formulation etc.

Chapter four basically deals with the data collection, analysis, as well as the computational algorithm used. It also discusses the results and significance of the results to the study.

Chapter five discuss the conclusion and summarizes the results obtained. Based on the finding, recommendations are also made.

CHAPTER TWO

LITERATURE REVIEW

2.1 Shortest Path Problems

In the study of transportation and communication networks, shortest path problems are fundamental and commonly encountered.

In graph theory, the shortest path problem is the problem of finding a path between two vertices (or nodes) in a graph such that the sum of the weight of its constituent edges is minimised. (Wikipedia, 2009),

There are many types of shortest path problems. For example we may be interested in determining the shortest path (i.e. the most economical path or fastest path or minimum-fuel consumption path) from one specific node in the network to another specific node; or we may need to find the shortest paths from a specific node to all other nodes. Shortest path between all pairs of nodes in a network are required in some problems. Sometimes one wishes to determine the shortest path from one given node to another given node that passes through certain specified intermediate routes. In other applications, one requires not only the shortest path but also the second and third shortest paths. There are also instances when the actual shortest path is not required but only the shortest distance is required. (Amponsah S.K. and Darkwa F.K. (2007).

There are also several variations according to whether the given graph is directed, undirected or mixed. For undirected graphs, the shortest path problem can be formally defined as follows. Given a weighted graph (that is, a set V of vertices, a set E of edges, and a real-valued weigh function f), and elements v and v' of V so that

$$\sum_{p \in p} f(p), \qquad \text{is minimal among all paths connecting } v \text{ to } v'$$

The problem is also sometimes called the single –pair shortest path problem, to distinguish it from the following variations.

(i) The single-source shortest path problem, in which we have to find shortest paths from a source vertex v to all other vertices in the graph.

(ii) The single-destination shortest path problems, in which we have to find shortest paths from all vertices in the directed graph to a single destination vertex v. This can be reduced to the single-source shortest path problem by reversing the arcs in the directed graph.

(iii) The all-pairs shortest path problem, in which we have to find shortest paths between every pair of vertices v, v' in the graph.

2.1.1 Shortest Path Algorithms

The most important algorithms for solving these problems are:

(i) Dijkstra's algorithm which solves the single-source shortest path problems.

(ii) Bellman-Ford algorithm which solves the single source problem if edge weights may be negative.

(iii) $A*$ Search algorithm which solves for single pair shortest path using heuristics to try to speed up the search.

(iv) Floyd-Warshall algorithm solves all pairs shortest paths.

(v) Johnson's algorithm which solves all pairs shortest paths, and may be faster than Floyd-Warshall on sparse graphs.

(vi) Perturbation theory finds (at worst) the locally shortest path.

2.2 Minimum Spanning Tree Problems. (MST)

The problem of finding a minimum spanning tree of a weighted graph is one of the best studied problems in the area of combinatorial optimization since its birth (Martin Mares, 2008).

The Minimum Spanning Tree (MST) problem is one of the most typical and well known problems in combinatorial optimization. Its colourful history begins in 1926 with the pioneering work of Boruvka who studied primarily an Euclidean version of the problem related to planning of electrical transmission lines but gave an efficient algorithm for the general version of the problem.

It is standard practice among authors discussing the minimum spanning tree problem to refer to the work of Kruskal (1956) and Prim (1957) as the sources of the problem and its first efficient solutions, despite the citation by both of Boruvka (1926) as a predecessor.

Schrijver (1996) cited Boruvka as the first to consider the MST. The MST algorithms that are commonly used are the Prim's (1957) and Kruskal's (1956) algorithms (cited in Jayawant and Glavin, 2009). However, Kruskal algorithm finds the minimum spanning forest if the network is not connected (Agarwal, 2010).

Zachariasen (1998) noted that all known exact algorithms for the Euclidean Steiner tree problem require exponential time. The general consensus is to use heuristics and approximation algorithms. However, one of the first and easiest methods involves the use of minimal spanning trees as approximation to the Steiner tree algorithm.

2.3 Optimal Networks

Network optimization techniques have been studied extensively and applied in various industries such as transportation, manufacturing, communication and circuit board design and among others. The problem is usually this nature; how can a set of points be connected together with a system of links for the least cost? In the case of pipeline networks, the points are supply and demand locations and the links are pipelines. The similarities and differences between this problem and others which utilize network optimization techniques are discussed. The overview highlights and development which have significance to pipelines.

Donkor et.al, (2011), used Prim's Algorithm to determine the minimum spanning tree of length 712km of the West African Gas Pipeline from Nigeria through Benin and Togo to Ghana. This was a reduction over the original 788.9km WAGP project design. The author also employed factor rating method to find an alternative path of length 723.29km. Steiner Tree algorithm and geometry were used to obtain an optimal pipeline length of 707.75km. This was a 10.3% reduction of the WAGP length.

Arogundade and Akinwale (2009) successfully used Prim's algorithm at a profit-oriented transportation systems in rural areas of Nigeria. The authors used Odeda Local Government map which has 88 villages connected by 96 roads. The project requires 388270 meters to be covered. Their findings shows that given the cost of fuel for travelling over 8000 meters to be 70.00 Naira, a transport company will make more profit with the implementation and use of Prim's algorithm; taking into an account the cost of travelling over 8000 meters by an averagely good Toyota 14 sector bus. The solution obtained was reported and compared with the result obtained before they apply Prim's algorithm on the existing network in both Table 2.1 and Table 2.2. For example, the value of the objective function from Oguntoke to Olajugun was 544 Naira as shown in table 2.2 without using Prim's algorithm as against 711 in table 2.1 when the algorithm was applied. The same situation is true when travelling from

Obantoko to Nitoji with objective function of 507 Naira as against 186 Naira without using the algorithm. They reported that the value of the objective function in Table 2.2 of column *h* is very low compared to that of table 2.1 where Prim's algorithm is used. This, they affirm that the company will make more profit with the implementation of the algorithm taking into account the cost of travelling over 8000 meters is estimated. In the course of their work, they discovered that the algorithm is very effective in providing shortest distances between two set of villages. In addition, Arogundade and Akinwale found that in one way it reduces the cost of fuel and time for transportation of passengers from one town to another which normally determines prices in Nigerian transportation.

Table 2.1: Results of Objectives Function using Prim's Algorithm

A	B	C	D	E	F=*e*/8000*70.00	*g*=axb	*h*=*g*-*f*
14	80.00	Oguntoko	Olajogun	46,730	40888	1120	711
14	50.00	Obantoko	Nitoji	21,960	192.15	700	507
14	130.00	Kangudu	Lokoji	60,270	527.36	1820	1292
14	130.00	Oguntoke	Alatibaba	64,710	566.21	1820	1253
14	80.00	Odeda	Akitoye	43,270	378.61	1120	741
14	60.00	Asawo	Mosafeyo	22,170	193.98	840	746

Table 2.2: Results of Objectives Function without using Prim's Algorithm

A	B	C	D	E	F=*e*/8000*70.00	*g*=axb	*h*=*g*-*f*
14	80.00	Oguntoko	Olajogun	64,690	566	1120	554
14	50.00	Obantoko	Nitoji	58,730	513.8	700	186
14	130.00	Kangudu	Lokoji	71,530	625.8	1820	1194
14	130.00	Oguntoke	Alatibaba	85,670	749.6	1820	1070
14	80.00	Odeda	Akitoye	47,710	417.4	1120	702
14	60.00	Asawo	Mosafeyo	46,920	410.55	840	429

A= number of passengers B= cost of travelling per head C= starting village

D= destination village E= total distance generated in meters F= total edge cost

G= total profit H= value of objective function.

Dott (1997), in his study of Optimal Design of Natural Gas Pipeline of Amoco East Crisfield Gas Pipeline project, (Alberta in Canada) containing 22 wells and connected by a gathering system of approximately 66km pipeline uses Prim's and Steiner tree algorithm to design a MST that covered a 38km and less than 13.4% respectively of the original length of the pipeline. A similar study by Bolkan (1991) in the same year of the Palliser Natural Gas Pipeline Project Canada which was planned to connect 68 gas processing plant and compressor stations from South-eastern Alberta to Empress/McNeil export location on the Alberta Saskatchewan border via downstream TransCanada pipeline and Foothills pipelines to consumers all over North America. The project requires 709km laterals to a mainline approximately 243km in length for a total of 952km of installed pipe and a cost of $365 million.

The minimum spanning tree created by Prim's algorithm for the Palliser Pipeline design length of 952km, for a 13% total length reduction. The construction cost which corresponds to this 13% reduction may be approximated as 13% of $365 million or $46 million. The MST does not represent the shortest responsible network and up to a further 13.4% reduction may be achieved with a Steiner tree. It is also important to note that the Palliser Pipeline designers were probably unaware of or disinterested in any techniques which could achieve this global or local minimum and thus the MST length is probably the more appropriate measure.

Brimberg et.al, (2003) studied the optimal design of an oil pipeline network for the South Gabon oil field. This oil field has 33 nodes representing onshore platforms, onshore wells, several connection points and one port (Gamba) with 129 possible arcs having total distance of 188.2 miles. Their finding in concert with Prim's algorithm reduces the connection to 156.2 miles which is a reduction of 17% of the total distance to be covered. They reported that the government saved a substantial amount of money from the number of pipelines used, labour quantity, time and even minimize environmental destruction of the earth's topography.

The most recently published work in the field of network design promotes use of genetic algorithm to find optimal or near-optimal solutions. Genetic algorithms use Darwin's theory of evolution as a basis for their structure. Essentially, individuals seek out the fittest mate in order to procreate and give their likewise fit offspring a greater chance of survival.

Genetic algorithm solves problems by representing potential problem solutions as individuals. Each individual or solution has a unique genetic descriptor or chromosome. In this case, a chromosome is a string of characters or genes each of which represents one element of the solution it represents. An optimal or near-optimal solution is found through successive "crossovers" or mating of the chromosomes. Sometimes mutation occurs which can reduce or improve fitness of the offspring. Fitness of each solution is measured by a fitness function. Each generation of solutions is usually of the same size as the previous generation with the less fit solutions being replaced by more fit. The fit solution is always carried through until it is replaced or is found to be the best solution.

The genetic algorithm is superior to other heuristic algorithm because the crossing over and the mutation chromosomes enable the offspring which can be very different from either parent. Since a locally optimal tree network is likely to have a drastically different structure from the global optimum, this aspect of the genetic algorithm is valuable.

The history of the Steiner tree problem presented here indicates that determining spatially optimal networks is a complicated and imperfect science. In addition, the pipeline industry does not prescribe to many of the solution principles, even simple ones for design. In real route- design problems it is usually true that the finite number of routes is viable for consideration.

In computer science, Prim's algorithm is an algorithm that finds a minimum spanning tree for a connected weighted undirected graph. This means it finds a subset of the edges that forms a tree that includes every vertex, where the total weight of all the edges in the tree is minimized.

Prim's algorithm is an example of greedy algorithm. The algorithm was developed in the 1930 by Czech mathematician Vojtěch Jarnik and later independently by computer scientist Robert C Prim in 1957 and rediscovered by Edsger Dijkstra in 1959. Therefore it is also sometimes called the DJP algorithm, the Jarnik algorithm, or the Prim-Jarnik algorithm.

Gonnia et.al, (2007) describe parallel implementation of Prim's algorithm for finding a minimum spanning tree of a dense graph using MPI. Our algorithm uses a novel extension of adding multiple vertices per iteration to achieve significant performance improvements on large problems (up to 200,000 vertices). We describe several experimental results on large graphs illustrating the advantages of our approach on over a thousand processors.

Gloor et al, (1993) described a system of visualizing correctness proofs of graph algorithms. The system has been demonstrated for a greedy algorithm, Prim's algorithm for finding a minimum spanning tree of undirected, weighted graph. We believe that our system is particularly appropriate for greedy algorithms, though much of what we discuss can guide visualization of proofs in other contexts. While an example is not a proof, our system provides concrete examples to illustrate the operation of the algorithm. These examples can be referred to by the user interactively and alternatively with the visualization of the proof where the general case is portrayed abstractly.

McCarthy et al, (2009) presented the application of two well known graph algorithms, Edmonds' algorithm and Prim's algorithm, to the problem of optimizing distributed SPARQL queries. In the context of this paper, resolved by contacting any number of remote SPARQL endpoints. Two optimization approaches are described. In the first approach, a static query plan is computed in advance of query executions, using one of two standard graph algorithms for finding minimum spanning trees (Edmonds' and Prim's algorithm). In the second approach, the planning and the execution of the query are interleaved, so that as each potential solution is expanded it is permitted to an independent query plan. Our optimization approach requires basic statistic regarding RDF predicates which must be obtained prior to the user's query, through automated querying of the remote SPARQL endpoints.

Martel et al, (2002) studied the expected performance of Prim's minimum spanning tree (MST) algorithm implemented using ordinary heaps. We show that this implementation runs in linear or almost linear expected time on a wide range of graphs. This helps to explain why Prim's algorithm often beats MST algorithms which have better worst-case run times. Specially, we show that if we start with any n node edge graph and randomly permute its edge weights, then Prim's algorithm runs in expected O (m + nlogn log (2m/n)) time. Note that O (m + nlogn log (2m/n)) = O (m) when m = Ω (n log n log n). We extend this result to show that the same expected run times apply even when an adversary can select the weights of m/logn edges and the responsible weights of the remaining edges (which are then randomly assigned).

Chang et al, (2008) described the reason why it is beneficial to combine with graph theory and board game. Forbye, it also descants three graph theories: Dijkstra's, Prim's, and Kruskal's minimum spanning tree. Then it would describe the information about the game we choose and how to combine the game with before-mentioned three graph theories. At all times, we could account for the advantage of combining with these three graph theories and the game specifically.

Dijkstra's algorithm conceived by Dutch computer scientist Edsger Djikstra in 1959 is a graph search algorithm that solves the single-source shortest path problem for a path with nonnegative edge path costs, producing a shortest path tree. This algorithm is often used in routing. An equivalent algorithm was developed by Moore in 1957.

For a given source vertex (node) in the graph, the algorithm finds the path with lowest cost (i.e. the shortest path) between that vertex and any other vertex. It can also be used for finding cost paths from a single to a single destination vertex by stopping the algorithm once the shortest path to the destination vertex has been determined. For example, if the vertices of the graph represents cities and edge path cost represent driving distance between pairs of cities connected together by a direct road, Dijkstra's algorithm can be used to find the shortest route between one city and all other cities. As a result, the shortest path first is widely used in network routing protocols, most notably IS-SI and OSPF (Open Shortest Path First).

The literature makes it abundantly clear that the procedure commonly known today as Dijkstra's Algorithm was discovered in the late 1950s, apparently independently, by a number of analysts. There are strong indications that the algorithm was known in certain circles before the publication of Dijkstra's famous paper. It is therefore somewhat surprising that this fact is not manifested today in the "official" title of the algorithm.

Dijkstra, (1959) submitted his shorter paper for publication in Numerische Mathematik June 199. In November 1959, Pollack and Wiebenson (1960) submitted a paper entitled Solutions of the shortest route problem- a review to the journal Operations Research. This review briefly discusses and compares seven methods for solving the shortest path problem.

However, the review presents a ‘highly efficient’ method, attributed- as a dateless private communication- to Minty. The procedure is defined as follows (Pollack and Wiebenson (1960, p. 225): the objective is to find the shortest path from city A to city B.):

Sneidovich, (2006) described Dijkstra’s Algorithm as one of the most popular algorithms in computer science it is also popular in operations research. It is generally viewed and presented as a greedy algorithm. In this paper we attempt to change this perception by providing a dynamic programming perspective on the algorithm. In particularly, we reminded that this famous algorithm is strongly inspired by Bellman’s Principle of Optimality and that both conceptually and technically it constitutes a dynamic programming successive approximation procedure par excellence. One of the immediate implications of the perspective is that this popular algorithm can be incorporated in the dynamic programming syllabus and in turn dynamic programming should be (at least) alluded to in a proper exposition/teaching of the algorithm.

Goldberg et al, (2006) proposed shortest path algorithms that use a search in combination with a new graph-theoretic lower bounding technique based on landmarks and triangle inequality.

Our algorithms compute optimal shortest paths and work on any directed graph. We give experimental results showing that the most efficient of our new algorithms outperforms previous algorithms, in particular A* search with Euclidean bounds, by a wide margin on road networks and on some synthetic problem families.

Pangilinan et al, (2007) presented an overview of the multi-objective shortest path problem (MSPP) and a view essential and resent issue regarding the methods to its solution. The paper further explored a multi-objective evolutionary algorithm as applied to the MSPP and describes its behaviour in terms of diversity of solutions, computational complexity and optimally of solutions. Results show that the evolutionary algorithm can find diverse solutions to the MSPP in polynomial time (based on several networks instances) and can be an alternative when other methods are trapped by the tractability problem.

As in the case of single objective shortest path problem, the multi-objective shortest path problem has been studied extensively by various researchers in the fields of optimizations, route planning for traffic, transport design, information and communication networks design. The MSPP is an extension of the traditional shortest path problem and it is concerned with finding a set of efficient paths with two or more objectives that are usually in conflict. For example the problem of finding optimal routes in communication networks involve minimizing delay while maximizing throughput or finding efficient routes in transportation planning that simultaneously minimize travel cost, path length and travel time. The concept of optimization in the MSPP or in a multi-objective problem in general is different from the single-objective optimization problem wherein the task is not to find a solution that optimizes a single objective function. The task in a multi-objective problem is not to find an optimal solution for each objective function but to find an optimal solution that simultaneously optimizes .all objectives. And in most case, no single optimal solution exists, only a set of efficient and non- dominated solutions.

A variety of algorithms and methods as dynamic programming, label selecting, label correcting, interactive methods and approximation algorithms have been implemented and investigated with respect to the MSPP. The problem is known to be NP-complete. It has been shown that a set of problems exist wherein the number of Pareto-optimal solutions is exponential which implies that any deterministic algorithm that attempts to solve it is also exponential in terms of runtime complexity at least in the worst case. But some labels algorithm studies, disputes this exponential behaviour. They show that the number of efficient paths not exponential in practice. Other authors avoid the complexity problem by developing new methods that run in polynomial time. For instance, Hansen and Warburton separately develop Fully Polynomial Time Approximation Schemes (FPTAS) for finding paths that are approximately Pareto-optimal. Interactive procedures similarly avoid the problem of generating the complete set of efficient paths by providing a user-interface that assists the decision maker to focus only on promising paths and identifying solutions accordance to preferences.

Given the wealth of literature in multi-objective algorithms for the MSPP, there still seems to be a lack of reported review in Evolutionary Algorithm (EA) application in relation to the MSPP. Several of the most recent alternative methods focus mostly on execution comparisons speed of different MSPP algorithms but analysis of the salient issues in multi-objective performance analysis such as runtime complexity, diversity, and optimally of non- dominated solutions are almost omitted.

Divoky, (1990) presented a framework for solving the shortest-path, cost-flow problem with positive edge weights can be implemented by itself or as a subordinate process in a solution procedure for a bigger problems. The efficiency of such shortest-path frameworks depend on the technique employed to use the structure of the network from the solution of the bigger problem, as well as on the efficiency of the frameworks. The topology of the networks is characterized by arcs with positive weights and of sub-networks called zero-weight components. The techniques for using topology include; identifying the zero-weight edges and adding them to the graph; identifying and adding the basic sub-trees to the shortest-path tree; and interrupting the shortest-path frameworks scanning or fixing the labels of all the nodes in the sub-tree.

Pettie et al, (2002) evaluated the practical efficiency of a new shortest path algorithm for undirected graphs which was developed by the two authors. This algorithm works on the fundamental comparison-addition model. Theoretically, this new algorithm out-performs Dijkstra's algorithm on sparse graphs for the all-pairs shortest path-problem, and more generally, for the problem of computing single –source shortest path from different sources. Our extensive experimental analysis demonstrated that this is also the case in practice. The authors presented results which showed the new algorithm to run faster than Dijkstra's on a variety of sparse graphs when the number of vertices ranges from a few thousand to a few million, and when computing single-source shortest paths from a few as three different sources.

Wang et al, (2005) said multiple pairs of shortest path problem (MPSP) arise in many applications where the shortest paths and distances between only some specific pairs of origin-destination (OD) nodes in a network are desired. The traditional repeated single-source shortest path (SSSP) and all pair's shortest paths (APSP) algorithms often do unnecessary computation to solve the MPSP problem. Our method is especially suitable for applications with fixed network topology but changeable arc lengths and desired OD pairs. Preliminary computational experiments demonstrate our algorithms superiority on airline network problems over other APSP and SSSP algorithms.

Chang, (2009) described the shortest distance between two points as a straight line. But in the real world, if those two points are located at opposite ends of the country, or even in different neighbourhood, it is unlikely you will find a route that enables you to travel from origin to destination via one straight road. You might pull out a map to determine the fastest way to drive somewhere, but these days, you are just as likely to use a Web-based service or a handheld device to help with driving directions. The popularity of mapping applications for mainstream consumer use once again has brought new challenges to the research problem known as “shortest-path problem.”

The shortest path problem, one of the fundamental quandaries in computing and graph theory, is intuitive to understand and simple to describe. In mapping terms, it is the problem of finding the quickest way to get one location to another. Expressed more formally, in a graph in which vertices are joined by edges and in which each edge has a value, or cost, it is the problem of finding the lowest-cost path between two vertices. There are already several graph-search algorithms that solve this basic challenge and its variations, so why is shortest-path perennially fascinating to computer scientist?

Goldberg, (2001) principal researcher at Microsoft Research Silicon Valley, said there are many reasons why researchers keep studying the shortest-path problem.

“Shortest path in an optimization problem that’s relevant to a wide range of applications, such as network routing, gaming, circuit design, mapping,” Goldberg says. “The industry comes up with new applications all the time, creating different parameters for the problem. Technology with more speed and capacity, allows us to solve bigger problems, so the scope of the shortest-path problem itself has become more ambitious. And now there are Web-based services, where computing time must be minimized so that we can respond to queries in real time.”

Venkataraman et al, (2003) proposed a block version of Floyd’s all-pairs shortest-paths algorithm. The blocked algorithm makes utilization of cache than does Floyd’s original algorithm. Experiments indicate that the blocked algorithm delivers a speedup (relative to the unlocked Floyd’s algorithm) between 1.6 and 1.9 on a Sun Ultra Enterprise 4000/5000 for graphs that have between 240 and 1200 vertices is between 1.6 and 2.0.

Arulselvan et al, (2008) considered the problem of maximizing the total connectivity for a set of wireless agents in a mobile ad hoc network. That is, given a set of wireless units having a start point and a destination point, our goal is to determine a set of routes for the units which maximize the overall connection time between them. Known as the Cooperative Communication Problem in Mobile Ad Hoc Networks (CCPM), this problem has several military applications including coordination of rescue groups, path planning for unmanned air vehicles, geographical exploration and target recognition. The CCPM is NP-hard; therefore heuristic development has been the major focus of research. In this work, we survey the CCPM examining first some early combinatorial formulations and solution techniques. Then we introduce new continuous formulations and compare the results of several case studies. By removing the underlying graph structure, we are able to create a more realistic model of problem as supported by the numeric evidence.

Razzaque et al, (2009) said a fast algorithm is proposed to calculate k^{th} power of an $n \times n$ Boolean matrix that requires addition operations were p is the probability that an entry of the matrix is 1. The algorithm generates a single set of inference rules at the beginning. It then select entries (specified by the same inference rule) from any matrix (Ak-1) and adds them up for calculating corresponding entries for AK. No multiplication operation is required. A modification of the proposed algorithm 1 can compute the diameter of any graph and for massive random graph, it requires only

The Floyd algorithm is essentially equivalent to the transitive closure algorithm independently discovered by Roy (1959) and Warshall (1962) (Pemmaraju and Skiena 2003),

Hougardy, (2010) the Floyd Warshall algorithm is a simple and widely used algorithm to compute shortest paths between all pairs of vertices in an edge weighted directed graph. It can also be used to detect the presence of negative cycles. Hougardy, (2010) show that for this task, many existing for the implementation of the Floyd- Warshall algorithm will fail because exponentially large numbers can appear during its execution.

Chen et al, (2007) focused on the optimization problems about complicated network, this paper present an algorithm KSPA to solve the K-shortest paths problem complicated networks based on algorithm, in which the time cost is taken as target function and the establishment of the target function model is given. Experimental results show that the proposed KSPA maintains an excellent efficiency on certain public traffic data. It can be used to solve the K-shortest paths problems in multi-graph.

Lysgaard, (2000) presented a new algorithm for finding the shortest path from a source to a single sink in a network, in which the location in the plane of each node is known. The algorithm consists of two phases. In the first phase a heuristic solution is to the shortest path problem is found. In the second phase the upper bound provided by the heuristic solution is utilized in a modification of a standard shortest path algorithm. Estimates based on computational test show that on average the computation time of the presented algorithm is on the order of 40-60% of the computation time is required if the information on node locations is not utilized.

Rabbani et al, (2008) presented a distribution network design problem in a multi-product supply chain system that involves the locating production plants and distribution warehouse as well as determining the best strategy for distributing the product from plants to warehouse and from warehouse to customers. The goal is to select the optimum numbers, locations and capacities of plant and warehouses to open, so that all customers demand of all product types are satisfied at minimum total costs of the distributing network. Unlike most of the previous researches, our study considered a multi-product supply chain system. We develop a mixed-integer mathematical programming model for designing a supply chain distribution network. Finally, this paper represents a real-case study to investigate designing a pharmaceutical supply chain distribution network. A possible extension is offered in the conclusion.

Multi pairs shortest path problem (MPSP) that arises in many applications where the shortest paths and distances between only some specific pairs of origin destination (OD) nodes in a network are desired. The traditional repeated single-source shortest path (SSSP) and all pairs shortest path (APSP) algorithm often do unnecessary computation to solve the MPSP problem. Sokol et al, (2005) proposed a new shortest path algorithm to save computational work when solving the MPSP problem. Our method is especially suitable for applications with fixed network topology but changeable arc length and desired OD pairs. Preliminary computational experiments demonstrate our algorithm superiority on airline network problems over other APSP and SSSP algorithm.

Hseih et al, (2004) designed shortest path routing algorithms is in general more difficult than designing simple routing algorithms. In his paper, he derived a shortest path routing algorithm for pyramid networks. The proposed algorithm takes O (1) time to determine the shortest path between any two nodes in a pyramid network. We also designed a distributed routing algorithm such that an intermediate node takes O (1) time to confirm the next node along the shortest path without any centralized controller.

Algorithms for finding the shortest path from one point to another have been researched for years. Applications abound, but let’s keeping things simple by saying we want to find the shortest path from one point A to point B in a city with just a few streets and interactions.

There are quite a few algorithms that have been developed to solve such problems, all with different C; C^N becomes astronomical when N gets even moderately large.

One of the fastest algorithms for solving this problem has a runtime of

CHAPTER THREE

METHODOLOGY

3.1 Introduction

In this chapter, we consider the modelling of the gas pipeline problem into a shortest path problem for easy computation. The chapter focuses on the algorithms used for solving the problem of designing gas pipeline network. The methods used here are useful for getting several alternative systems of near-optimal arrangements which can be compared using economic and qualitative analysis mostly used in industry to determine the most efficient network route.

A Minimum Spanning Tree (M.S.T) problem is the problem of finding a spanning tree of a connected weighted graph with minimum cost, which is one of the most well-known problems of combinatorial optimization. The M.S.T problem has obvious applications in communication networks such as connecting a set of computers in a network, or linking a set of cities with minimum cost. It sometimes occurs as a sub-problem in finding a solution to a bigger problem such as the Travelling Salesman Problem or the Minimum Weight Matching Problem.

3.2 Concept of Graphs and Trees

A graph G is a pair (V ; E), where V is a set of elements called vertices, and E is a set of pairs (u; v) called edges, such that u and v belong to V , we write e = (u; v) (Nahla M.A.I, 2011).

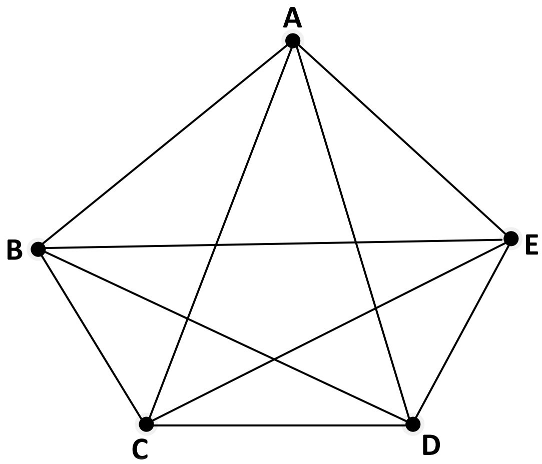


Figure 3.1: depicts of a graph G with 5 vertices and 10 edges.

Trees are important class of graphs. A tree T is a connected graph with no cycles. The edges of trees are called branches. A forest is disjoint union of trees (Nahla M.A.I, 2011).

Suppose T is a graph with n vertices, then the following statements are equivalent

(i) T is connected, and has $n-1$ edges

(ii) T contains no cycles.

(iii) There is a unique path between every pair of vertices in T.

(iv) T contains no cycles, and adding a new edge to T creates a unique cycle.

(v) T is connected, and removing any edge from T makes it disconnected.

A spanning tree of a connected graph G is a sub-graph of G that includes all the vertices of G and has the properties of a tree.

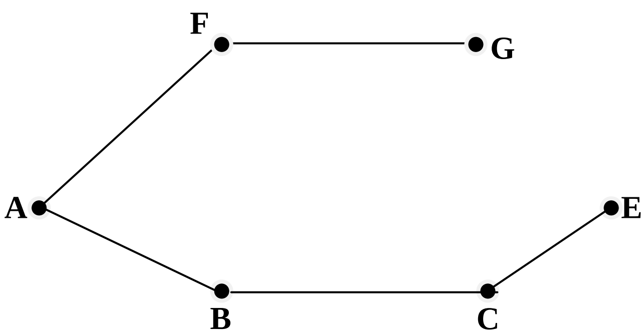


Figure 3.2: shows an example of a Tree.

3.3 Problem Formulation

Consider a company; the Ghana National Gas Company installing a system of pipelines to connect all regional capitals of Ghana to enhance the distribution of gas products to consumers. We assume the company requires the following features:

(i) A unique path which passes through every region exactly once

(ii) The company wants to start from a particular region (receiving point/processing plant) and ends in a specific destination.

(iii) A minimum cost path must be determined

To formulate this problem, we let W= (A, B, C, D, E, F, G, H, I, J) be an undirected graph. Generally, we define a graph G as an ordered pair G = (V, E) where

(i) V is a set of vertices (nodes)

(ii) E is a set of edges (links).

(iii) Each edge is a pair of vertices. In other words, each element of E is a pair of elements of V.

The minimum spanning tree problem consists of finding a connected acyclic sub graph $T = (A_T, B_T, C_T, D_T, E_T, F_T, G_T)$ of G, such that $A_T \subseteq A, B_T \subseteq B, C_T \subseteq C$ and so on and minimises $w(Q) = \sum_{(h,k)\in Q} w(h,k)$, which is the weight of the graph.

Since the problem is a minimum connection problem, it can be solved by the shortest path method but because of its special nature, it can be solved more easily using Prim's algorithm, which is more efficient for the minimum-connector problems.

The primary focus of this paper is a construction of an efficient algorithmic implementation rather than a theoretical analysis of the problem.

3.4 Techniques for Solving Shortest path problems

In this subsection, we discuss various techniques for solving shortest path problems. Notable among them are the Dijkstra's Algorithm, Floyd-Warshall Algorithm, Kruskal's Algorithm and Prim's Algorithm etc.

3.4.1 Single Source Paths, Non-Negative Weights (Dijkstra's Algorithm)

Dijkstra's algorithm, conceived by Dutch computer scientist Edsger Dijkstra in 1959, is a graph search algorithm that solves the single-source shortest path problem from a graph with nonnegative edge path costs, producing a shortest path tree. This algorithm is often used in routing. An equivalent algorithm was developed by Edward F. Moore in 1957.

For a given source vertex (node) in the graph, the algorithm finds the path with the lowest cost (i.e. the shortest path) between that vertex and that of every vertex. It can also be used for finding costs of shortest paths from a single destination vertex by stopping the algorithm once the shortest path to the destination vertex has been determined. For example, if the vertices of the graph represent cities and edge path costs represent driving distances between two pairs of cities connected by a direct road, Dijkstra's algorithm can be used to find the shortest route between one city and all other cities. As a result, the shortest path first is widely used in network routing protocols, most notably IS-SI and OSPF (Open Shortest Path First).

3.4.1.1 Algorithm

Let the node we are starting be called the initial node. Let a distance of the node Y be the distance from the initial node to it. Dijkstra's algorithm will assign more initial distance values and will try to improve them step-by-step.

(i) Assign to every node the distance value. Set it to zero for our initial node and infinity for all other nodes.

(ii) Mark all nodes as unvisited. Set initial node as current.

(iii) For current node, consider all its as unvisited neighbours and calculate their distance (from the initial node). For example, if current node (A) has distance of 8, and an edge connecting it with another node (B) is 2, the distance to B through A will be 8+2=10. If the distance is less than the previously recorded distance (infinity in the beginning, zero for the initial node), overwrite the distance.

(iv) When we are done considering all neighbours of the current node, mark it as unvisited. A visited node will not be checked ever again; its distance now recorded is final and minimal.

(v) Set the unvisited node with the smallest distance (from the initial node) as the next "current node" and continue from step 3.

Dijkstra's algorithm finds the shortest path from the source node S to all other node in a network with nonnegative arc lengths. Dijkstra's algorithm maintains a distance label d (i) with each node i, which is an upper bound on the shortest path length from the source node to each node i at any intermediate step, the algorithm divides the nodes of the network under considerations into two groups: those which it designates as permanently labelled (or permanent) and those which from it designates are temporarily labelled (or temporal). The distance label to any permanent node represents the shortest distance from the source node to that node.

The basic idea of the algorithm is to find out from the source node S and permanently labelled nodes in the order of their distance from the node S. initially, node S is assigned a permanent label of zero, and each other; node j a temporary label equally to infinity. At each iteration, the label of a node I is its shortest distance from the source node along the path whose internal node (i.e. nodes other than S or I itself) are permanently labelled. The algorithm selects a node i with minimum temporary label (breaking ties arbitrarily), makes it permanent, and reaches out from node- that is, scans all edges/arcs emanating from node I to update the distance labels of adjacent nodes. The algorithm terminates when it has designated all nodes permanent.

We can now express Dijkstra's algorithm as a set of steps.

Step 1: assign the permanent label O to the starting vertex.

Step 2: assign temporary labels to all the vertices that are connected directly to the most recently permanent labelled vertex.

Step 3: choose the vertex with the smallest temporary label and assign a permanent label to that vertex.

Step 4: Repeat step 2 and 3 until all vertices have been permanently labelled.

Step 5: find the shortest path by tracing back through the network.

Note: Recording the order in which we assign permanent labels to the vertices is an essential part of the algorithm.

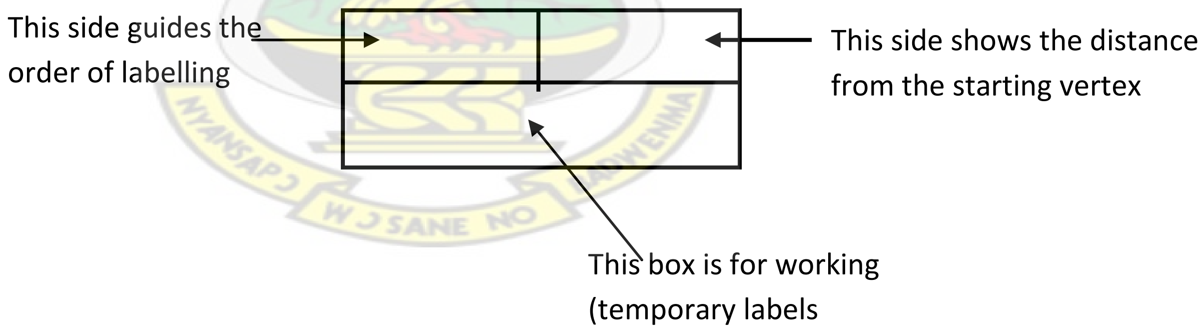


Figure 3.3

The algorithm gradually changes all temporary into permanent ones.

3.4.2 All-Pairs Shortest Path Problem

The shortest path between two nodes might not be a direct edge between them, but instead involve a detour through other nodes. The all-pairs shortest path problem requires that we determine shortest path distance between every pair of nodes in the network.

3.4.2.1 Floyd-Warshall Algorithm

The Floyd- Warshall algorithm obtains a matrix of shortest path distances within $O\{n^3\}$ Computations. The algorithm is based on inductive arguments developed by an application of a dynamic programming technique.

Let $d^k(i, j)$ represents the length of the shortest path from node i to node j subject to the condition that this path uses the node $1, 2, 3..., k-1$ as internal nodes clearly, $d^{n+1}(i, j)$ represents the actual shortest path distance from i to j. the algorithm first computes $d^1(i, j)$ for all node pairs i and j. Using $d^1(i, j)$, it then computes $d^2(i, j)$ for all pairs of nodes i and j. It repeats the process until it obtains $d^{n+1}(i, j)$ for all node pairs i and j when it terminates. Given $d^k(i, j)$, the algorithm computes $d^{k+1}(i, j) = \min\{d^k(i, k), d^k(i, j)\}$. The Floyd-Warshall algorithm remains of interest because it handles negative weight edges correctly.

3.4.3 Minimum-Connector Problems

Minimum – connector problems arises in so many applications in real life especially when one wants to find the minimum way to connect a set of nodes/points. Its numerous applications are seen in the area of rural electrification networks, telecommunication networks and transportation networks.

There are two classic best known greedy algorithms for solving minimum-connector problems namely; Prim's algorithm and Kruskal's algorithm.

The following points are to be noted about minimum-connector problems;

(i) A connected graph which contains no cycles is called a tree.

(ii) A spanning tree is a sub graph that includes all the vertices in the original graph is also a tree. A graph will have several trees.

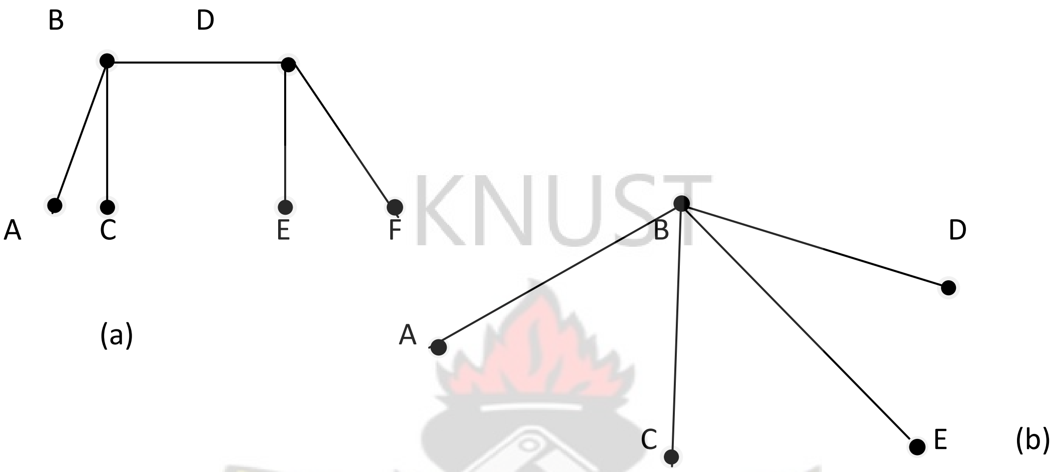


Figure 3.4: shows examples of spanning trees.

3.4.3.1 Prim's Algorithm

Prim's algorithm solves the shortest path problem for a directed graph with non-negative edge weights. For example, if the vertices of the graph represents gas well or roads connecting the towns and edge weights represent distances between pair of wells (or towns) connected by a pipeline, this algorithm can be used to find the shortest route between two gas wells (or towns). The algorithm was discovered in 1930 by mathematician Vojtech Jarnik and later independently by computer scientist Robert Prim in 1957 and rediscovered by Dijkstra in 1959. Therefore it is sometimes DJP algorithm or Jarnik algorithm. The algorithm builds a spanning tree from scratch by fanning out from a single node and adding arc one at a time. It means a tree spanning on a subset S of nodes and a nearest neighbour to S. the algorithm does so by identifying an arc (i, j) of minimum cost in the cut$[S, S]$. It adds arc (i, j) to the tree, node *j* to S repeats this basic steps until all nodes have been connected. This algorithm is said to be “greedy” since it picks the immediate best option available without taking into account the long-term effects of the choices made. The algorithm is;

Step 1: Start with any vertex.

Step 2: Join this vertex to the nearest vertex.

Ste p 3: Join on the vertex which is nearest to either of those already connected.

Step 4: Repeat until all vertices are connected.

Step 5: Add the lengths of all the edges included in the minimum length spanning tree.

The correctness of the algorithm follows directly from the fact that each arc that is added to the tree is contained in some minimum spanning tree with the arcs that have been selected in the previous step. The matrix formulation of Prim’s algorithm implemented in this thesis is given below.

3.4.3.1.1 Matrix Formulation of Prim’s Algorithm

Step 1: choose a start vertex and delete all elements in that vertex’s row and arrow its column.

Step 2: Neglecting all deleted terms, scan all arrowed columns for the lowest element and circle that element.

Step 3: Delete the circled element’s row and arrow its column.

Step 4: Repeat step 2 and 3 until all rows deleted.

Step 5: The spanning tree is formed by the circled arc.

Step 6: Write down the arcs in the order you selected them.

3.4.3.2 Kruskal’s Algorithm

Kruskal's algorithm is an iterative method for solving the M.S.T problem of a connected weighted graph G with n vertices. The algorithm starts by initializing a graph T with all the vertices in G and no edges. After that, it repeats the operation of adding an edge with a minimum weight to T such that no cycle is created, otherwise it ignores the edge. This repetition terminates when T has (n - 1) edges.

3.5 A Minimum Connector Problem

A cable TV company is installing a system of cables to connect all the towns in a region. The numbers in the following diagram show the distances in miles between the towns.

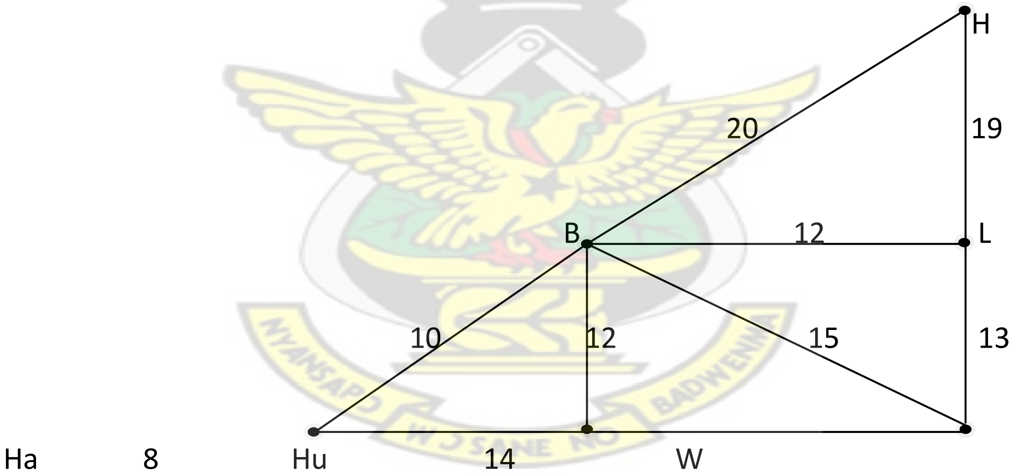


Figure 3.5: Depicts a Network of Towns in a Region

We are to find:

(i) A layout that will connect all the towns in the region.

(ii) A different layout which uses less cable.

(iii) The least amount of cable needed?

We can model this kind of problem using graph theory by generating a weighted graph G whose vertices represent the towns we want to connect and edges with their weights representing the direct paths with their cost.

Prim's algorithms works from a start point and builds up the spanning tree step by step, connecting edges into the existing solution. It can be applied directly to the distance matrix, as well as to work the network itself, so it is more suitable for using a computer if the network is large.

Solution by Prim's Algorithm

Choose a starting vertex, say L

L-B is the smallest edge joining L to the other vertices. Put LB into the solution.

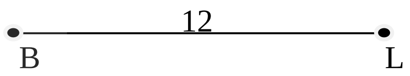


Figure 3.6

B-Ha is the smallest edge joining L and B to the other vertices. Put edge B-Ha into the solution.

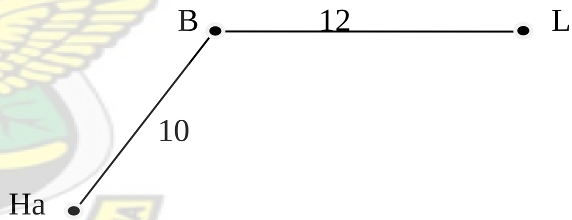


Figure 3.7

Ha-Hu is the smallest edge joining L, B and Ha to the other vertices. Put Ha-Hu into the solution.

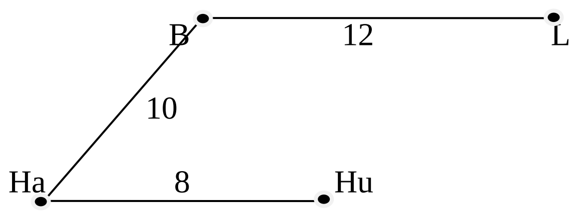
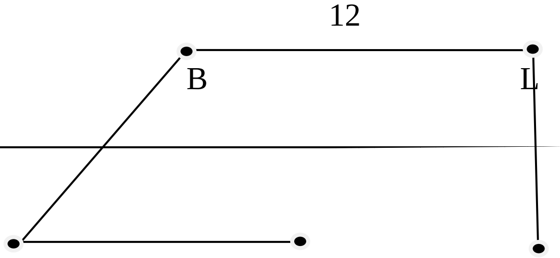


Figure 3.8

L-W is the smallest edge joining L, B



Ha and Hu to the other vertices.
Put L-W into the solution.

Figure 3.8

L-H is the smallest edge joining L,B
Ha, Hu and W to the other vertices.
Put L-H into the solution.

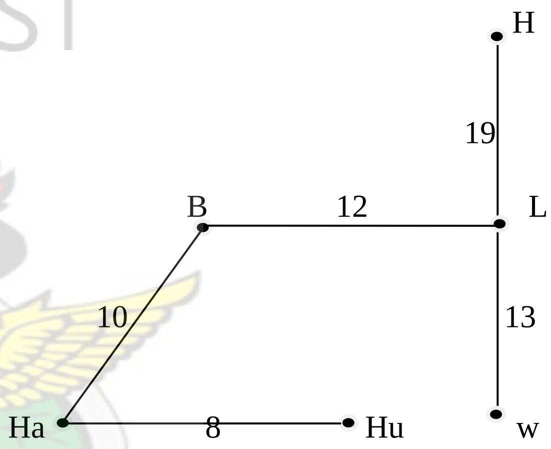


Figure 3.9

We have now connected all the vertices into the spanning tree.
The least length of cable needed is 62 miles.

Solution by Kruskal's Algorithm

Choose a starting vertex
Say Ha

Figure 3.10

Join Ha to Hu
since it has the smallest
distance in the network
put Hu into the solution

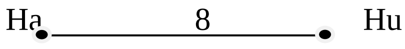


Figure 3.11

Join Ha to vertex
since that is the next
available smallest distance
put B into the solution.

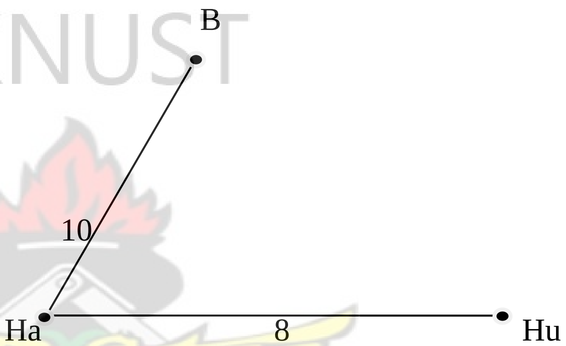


Figure 3.12

the next available
small distance is vertex B-L
we join B to L and put
L into the solution

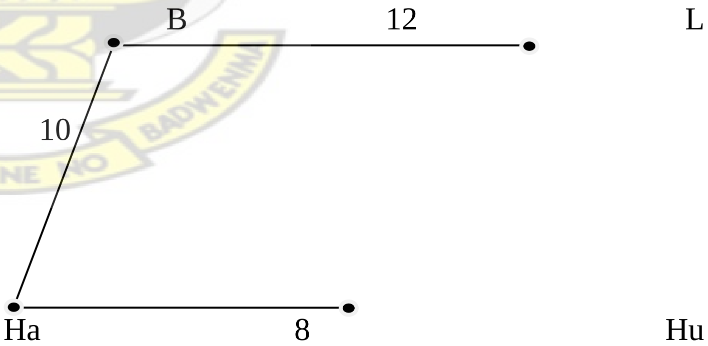


Figure 3.13

The next available small
Distance is L – W. We now
Join L to W and put W into
the solution

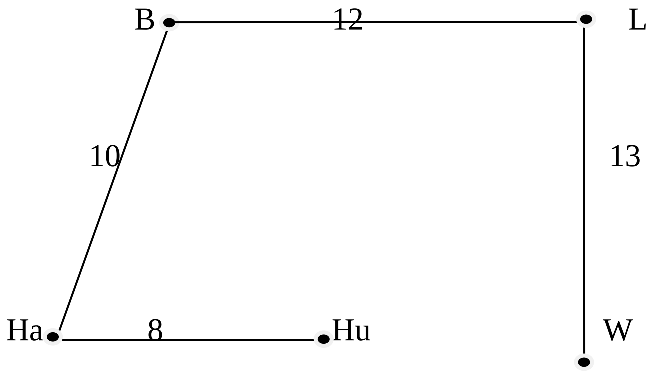


Figure 3.14

Finally we connect to the
Next small distance L-H
And put H into the solution

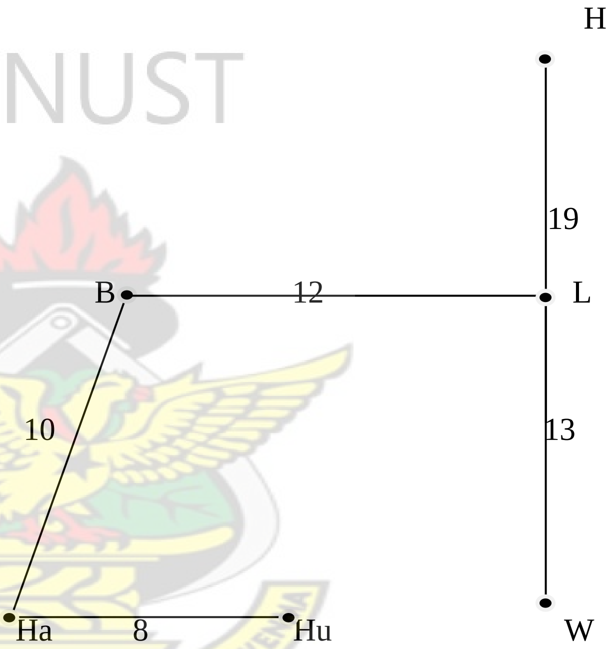


Figure 3.8

We have now connected all the vertices into the spanning tree.
The least length of cable needed is 62 miles.

3.6 Data Collection

Maps and field works are required for pipeline routine, pipeline design and construction. The information and data needed for the analysis was gathered from the BOST Company limited, Ghana Survey Department and the Energy Research Centre of the Kwame Nkrumah University of Science and Technology, Kumasi.

3.7 Summary

In this chapter we have formulated the pipeline design problem into a shortest path problem and also illustrated the algorithms used in solving these problems. We have also applied Prim's algorithm and Kruskal's algorithm in illustrating with examples.

In the next chapter, we put forward the data collection, analysis as well as the results of the research.

CHAPTER FOUR

DATA COLLECTION, ANALYSIS AND RESULTS

4.1 Introduction

This chapter deals primarily with the organization of the data used in the research, the analysis and algorithm used to obtain the desired results. It also discusses the processes for gas infrastructure development as well as the supply and demand balances of natural gas in the nation.

4.2 Data Collection

Data for the research was organized from three main sources; Ghana Survey Department, Energy Research Centre (K.N.U.S.T) and the Bulk Oil Storage and Transport (BOST) Company Limited.

National geographical co-ordinates of the nation were obtained from the Energy Commission. Interview sections were also held with personnel of the commission for the researcher to better understand the data and how it's applied.

The researcher also contacted the engineer at BOST Company Limited and data on the demand levels as well as diagrams on proposed infrastructure developments were obtained. An interview section was conducted with the engineer of the company (Mr. Kwame Boateng) for the researcher to be abreast with developments of the gas infrastructure.

4.3 National Natural Gas Transmission Utility

Until Ghana builds a national gas pipeline network, introduction of natural gas into the economy will not necessarily displace all fuel-oil demand.

In view of its quest to develop gas infrastructure, the Government of Ghana has appointed BOST Company Limited as the developer and operator of the natural gas transmission system in accordance with the provision of EC Act 541, 1997.

On the basis of its mandate in the natural gas sector, BOST intends to develop a nationwide network of natural gas transmission pipelines and regulatory metering stations in accordance with the proposed Natural Gas transmission master plan:

(i) Tema-Accra Network

30 km (18" pipeline)

(ii) Western Gas Transmission System (Takoradi – Kumasi)

1,200 km (6" – 18" pipeline)

(iii) Coastal (Jubilee) Transmission System (Bonyire – Takoradi)

125 (20") pipeline

(iv) Takoradi – Aboadze WAGP Interconnection Line

15 – 20 km (18" – 20" pipeline)

126 Eastern Gas Transmission System

500 km (6" – 18" pipeline)

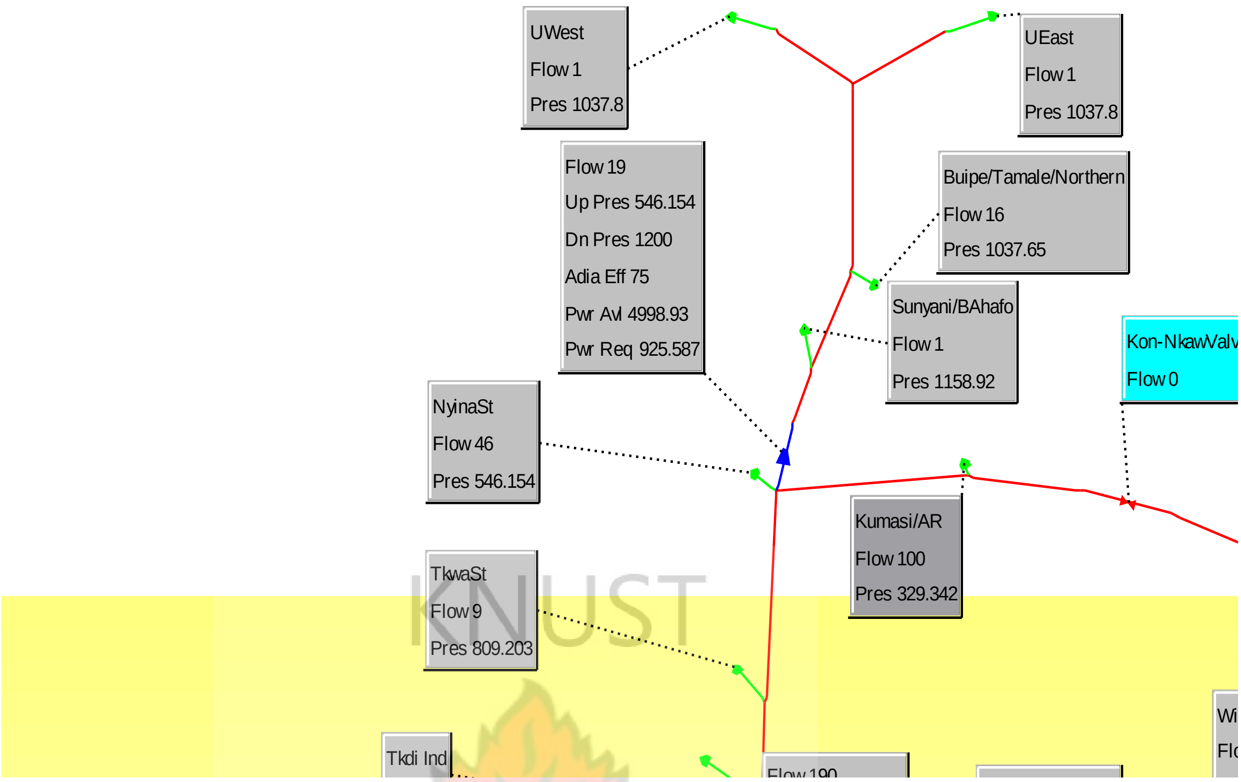


Fig. 4.1: National Natural Gas Transmission and Distribution System

Source: BOST Company Limited

4.4 Transmission Systems

BOST intend to establish a natural gas hub in Takoradi (similar to Louisiana) where gas from various fields would be gathered (or stored) for further delivery to the transmission system.

(i) Coastal (Jubilee) Transmission System (Bonyire – Takoradi)

125 Km (20”) pipeline - transport gas from Jubilee and the nearby fields to gathering point at Takoradi Petroleum terminal where various transmission networks will emanate from.

(ii) Takoradi – Aboadze WAGP Interconnection Line

15 – 20 km 18" – 20" pipeline – will interconnect coastal transmission line with WAGP system and ensure reverse flow through WAGP to Tema.

(iii) Western Gas Transmission System (Takoradi – Kumasi)

350 km (6" – 18" pipeline) – will initially terminate at Kumasi where there is a planned 600 MW natural gas fired thermal power plant and will pass through strategic project areas in Western and Ashanti regions namely: Nyinahin Alumina Plant (mining and processing of bauxite would require about 45.9MMSCFD); Tarkwa manganese project (production of electrolytic manganese dioxide would require about 9MMSFD); Iron and steel manufacture in Takoradi (about 9.5 MMSCFD); Opong Mansu (Iron Steel) and Abooso glass factory.

Possible future extension to North will help in Limestone and clinkerization at Buipe (about 16 MMSCFD) and Nouli clinkerization.

4.5 Natural Gas Infrastructure Requirement

In the short, medium to long term, BOST intends to install the following facilities as part of its infrastructure requirement.

4.5.1 The Tema Distribution System

The idea to develop a secondary Gas Market for Tema came about as a result of the West African Pipeline Development Process. The main objective at the time was to develop a market for for natural gas beyond the contracted amount to V.R.A. Thus other I.P.Ps and Industries in the Tema area were to benefit from standard gas supply in excess of gas supply to the foundation customer. The production of gas in Ghana and the ever growing demand for natural gas in the Tema area has made the development of this project a high priority for Government.

The Infrastructure in Tema will consist of:

(a) An RMS and connecting gas pipelines to serve IPPs including VRA and Sunon-Asogli and Industrial end users in Tema.

(b) Tema City-Gates Network which is 32km

(c) In the long term, an onshore Liquefied Natural Gas (LNG) Import and Receiving Terminal at Tema or a Floating Storage & Re-gasification Unit (FSRU) to complement the WAGP project.

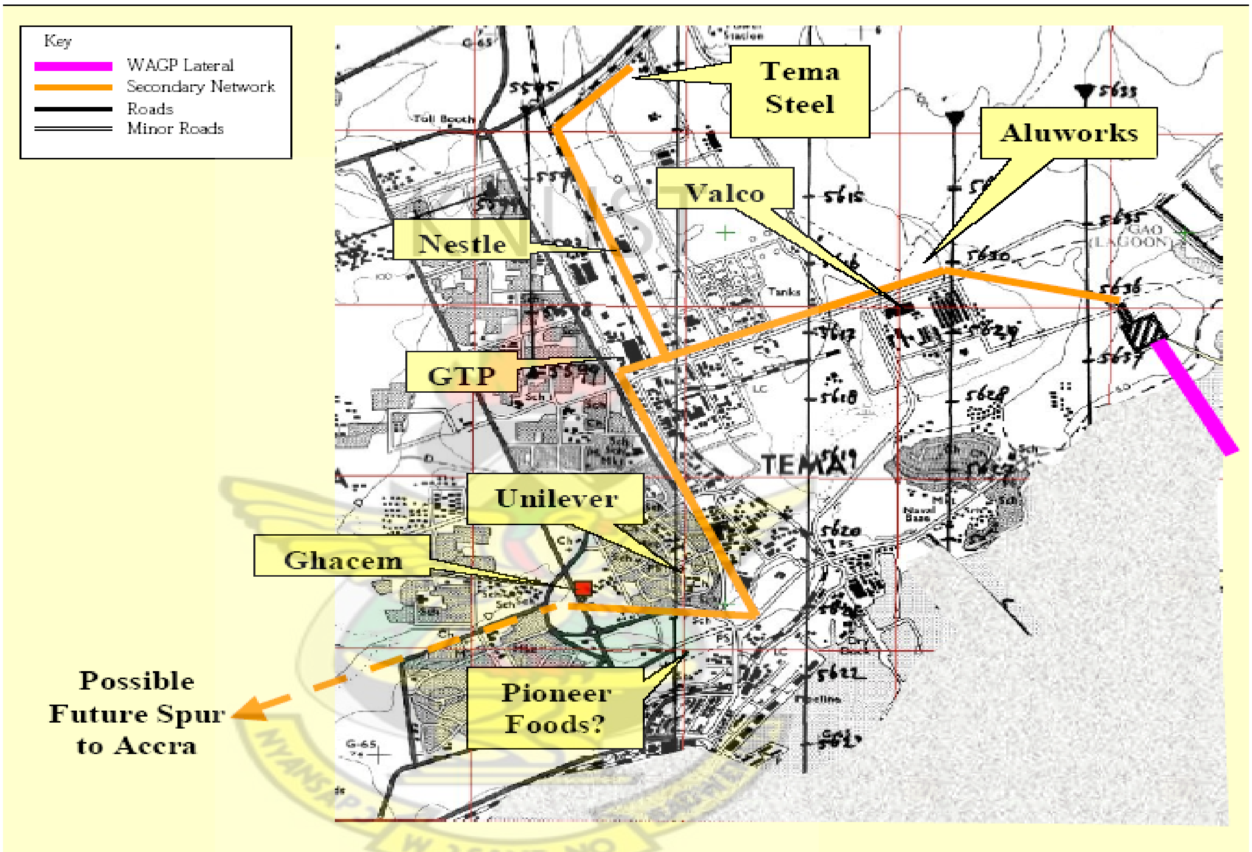


Fig 4.2: Proposed Natural Gas Pipeline System in Tema

Source: BOST Company Limited

4.5.2 Western Corridor Transmission System

The development of the Western Corridor Gas Pipeline is meant to support availability of relatively clean and affordable fuel to power industrial growth in the western part of the Country. The transmission pipeline will take its source from the terminus of the undersea pipeline at landfall which will bring gas from the Jubilee and other fields such as Tweneboah, Sankofa and Dzata fields.

This infrastructure is therefore seen as a critical project that will commercialize gas production for industrial development.

The transmission system will supply natural gas to independent power producers, mining companies, petrochemical industries and other economic interest in the Country.

Also the western corridor Gas Pipeline is to become the backbone for a national transmission pipeline which will transport gas to other economic zones in the Northern portions of the country in the long term.

In the short term, BOST will establish a natural gas hub at the BOST petroleum Terminal in Pumpuni that will gather and dispatch gas from the processing plant to end users. The following projects will be undertaken;

(a) 123km, 20” NG pipeline from Bonyere (processing plant) to BOST Petroleum Terminal.

(b) 123km, 6-8” LPG pipeline from Bonyere (processing plant) to BOST Petroleum Terminal.

(c) 15-20km, 6”-18” pipeline from BOST Petroleum Terminal to Kumasi.

Figure 4.3 shows a schematic Presentation of the Western Transmission System.

4.6 Potential Markets for Natural Gas

Natural gas is a very attractive fuel for many purposes. it provides a cleaning burning flame relatively unpolluted exhaust gases less harmful to the environment, easily controlled rates of heating and where required, high heating intensity. it is thus has many advantages over the inherent thermal efficiency hence its extensive market potential.

The estimation of potential gas demand over the planning period will be driven by the identification of base load users like power sector that will act as potential anchors for the development of viable gas demand centres.

The potential for gas demand arising from other sectors such as petrochemicals, fertilizers, transport and residential and commercial sectors is also a possibility.

4.6.1 Power Demand

The main driver of gas growth in Ghana is expected to be the power sector. Gas demand for electricity generation was assessed by looking at electricity demand and the capacity of generating plants required meeting this demand (Nexant, 2010). Firm generation capacity can be broken into three categories;

(i) Existing Facilities

(ii) Facilities under construction

(iii) planned facilities

4.6.2 Gas Demand from Industries

Energy commission estimated potential gas demand from industry by focusing on key industrial users. Potential industrial gas demand was divided into two coastal and non-coastal categories.

The coastal categories include;

(i) Tema industrial area

(ii) Tema Oil refinery and

(iii) Valco/Takoradi Industrial area- include a two million MT alumina refinery.

The non-coastal area covers the following extractive industries:

(i) Bauxite

(ii) Limestone

(iii) Manganese ore and

(iv) Iron ore

The non-coastal demand has generally been referred to as Strategic Projects.

By estimation, unconstrained demand for gas from industrial users is expected to reach 270MMSCFD by 2030 (Nexant, 2010)

Table 4.1: Projected Unconstrained Gas Demand from Power Plants and Industries.

Year	2010	2011	2012	2013	2014	2015	2020	2025	2030
Power (MMSCFD)	96.4	163.5	282.1	352.1	352.1	422.3	470.8	604.6	814.9
Industry (MMSCFD)	0	25.2	112.8	118.4	124.4	130.8	166.6	212.7	271.4
Total	**96.4**	**189.0**	**394.9**	**470.6**	**476.5**	**553.1**	**637.5**	**817.3**	**1086.4**

Source: BOST, 2010

By Nexant 2010 estimation, natural gas demand will be about, 1086 MMSCFD by the year 2030, assuming an annual increase of 13%.

The gas demand will depend on the volumes and cost of gas supply available to consumers as well as infrastructure availability.

4.7 Natural Gas Supply

Gas supply to Ghana is expected to come from two main sources: the West African Gas Pipeline (WAGP) and indigenous offshore sources. The initial capacity of the West African Gas Pipeline is 170MMSCFD with an ultimate capacity of 474MMSCFD. For the WAGP supply, custody of the gas will be taken at the regulating/metering station at the beach heads in Tema and Takoradi.

The indigenous supply is expected to come from the Jubilee field and later from the marginal discoveries of North and South Tano and Saltpond as well as the newly discovered Twenboah, Sankofa and Dzata fields in Tano and West Cape Three basins. The indigenous supply sources will be preferred because of their market area proximity and reliability of supply.

To date only jubilee and other previous minor discoveries in North and South Tano as well as Saltpond fields have been appraised, GNPC however, puts the conservative estimate of the country's gas reserves at well over 5TCF. The confirmed indigenous gas supply is expected to come from the offshore Jubilee field in the medium term.

4.7.1 Gas Supply Forecast

The Environmental Protection Agency's strategic environmental assessment (SEA) for oil and gas sector identified three scenarios for gas supply.

(i) Scenario 1: includes the gas production forecasts for various field development cases of the Jubilee Field. The estimated for Scenario 1 is based on Jubilee Field High production case.

(ii) Scenario 2: includes the gas production forecast for the Jubilee High case and for the other announced gas discoveries such as Tweneboah, Sankofa , Dzata, Ebony, North and South Tano.

(iii) Scenario3: includes Scenario 2 and possible future discovery onshore Voltaian basin.

Gas production based on scenario 2 is expected to reach 400MMSCFD by 2015 and 800MMSCFD in the year 2020, which could be sustained until the year 2030 (Nexant, 2010)

Table 4.2: Scenario 1: Base Case

Year	2010	2011	2012	2013	2014	2015	2018	2020	2025	2030
WAGP Supply (MMSCFD)	38	162	180	180	180	180	180	180	180	180
Domestic Production (MMSCFD)	0	0	59	61	61	113	113	113	81	27
Total (MMSCFD)	**38**	**162**	**239**	**241**	**241**	**293**	**293**	**293**	**261**	**207**

Source: BOST, 2010

Table 4.3: Scenario 2: Medium Case

Year	2010	2011	2012	2013	2014	2015	2018	2020	2025	2030
WAGP Supply (MMSCFD)	38	162	180	280	280	280	380	380	380	380
Domestic Production (MMSCFD)	0	0	59	61	61	113	435	435	137	91
Total (MMSCFD)	**38**	**162**	**239**	**341**	**341**	**393**	**815**	**815**	**517**	**471**

Source: BOST, 2010

Table 4.4: Scenario 3: High Case

Year	2010	2011	2012	2013	2014	2015	2018	2020	2025	2030
WAGP Supply (MMSCFD)	38	162	180	380	380	380	380	380	380	380
Domestic Production (MMSCFD)	0	0	59	61	61	113	435	435	696	696
Total (MMSCFD)	**38**	**162**	**239**	**441**	**441**	**493**	**815**	**815**	**1455**	**1455**

Source: BOST 2010

Table 4.5: Gas Demand & Supply Balance

Year	**2010**	**2011**	**2012**	**2013**	**2014**	**2015**	**2020**	**2025**	**2030**
Supply (MMSCFD)	38	162	239	441	441	493	815	1455	1455
Demand (MMSCFD)	96.4	189.0	394.9	470.6	476.5	553.1	637.5	817.3	1086.4

Source: BOST, 2010

4.8 Computational Experience

The distances between all pairs of nodes were computed using the Euclidean distance formula. The results are shown in Appendix C.

Prim's algorithm was then used to find the internodal distances that form a minimum spanning tree. The internodal distances of the MST are shown in Appendix C.

Table 4.6: Matrix of intermodal distances showing edges of the MST

Node	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**
A	∞	8.65	51.40	34.90	45.04	64.54	60.49	88.08	125.71	166.27	171.90
B	8.65	∞	51.78	43.55	37.30	56.99	60.36	88.04	131.53	169.90	177.99
C	51.40	51.78	∞	62.40	51.33	59.49	9.45	36.69	94.96	122.03	141.02
D	34.90	43.55	62.40	∞	78.02	96.59	71.56	95.56	105.33	154.74	149.21
E	45.04	37.30	51.33	78.02	∞	19.74	55.38	78.73	145.45	171.99	191.88
F	64.54	56.99	59.49	96.59	19.74	∞	60.56	78.65	154.26	174.35	199.90
G	60.49	60.36	9.45	71.56	55.38	60.56	∞	27.69	94.13	116.66	139.40
H	88.08	88.04	36.69	95.56	78.73	78.65	27.69	∞	87.28	95.91	128.52
I	125.71	131.53	94.96	105.33	145.45	154.26	94.13	87.28	∞	59.67	46.72
J	166.27	169.90	122.03	154.74	171.99	174.35	116.66	95.91	59.67	∞	59.26
K	171.9	177.99	141.02	149.21	191.88	199.90	139.40	128.52	46.72	59.26	∞

Using prim's algorithm,

(i) Choose a starting vertex say, vertex A.

Figure 4.3

Delete row A and look for the smallest entry in column A. The smallest entry in column A is 8.65km. AB is the smallest edge joining A to other vertices.
Put edge AB into the solution and delete row B

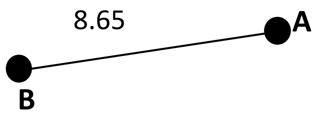


Figure 4.4

(ii) Look for the smallest entry in column A and B. The smallest entry in column A and B is 37.30km.

BE is the smallest edge joining A and B to the other vertices. Put edge BE into the solution and delete row E.

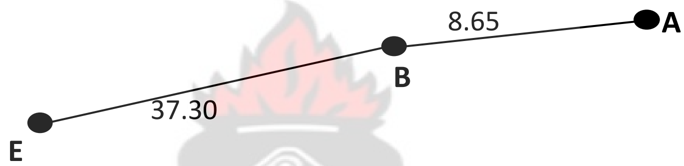


Figure 4.5

(iii) Look for the smallest entry in column A, B and E. The smallest entry in column A, B and E is 19.74km.

EF is the smallest edge joining A, B and E to the other vertices. Put edge EF into the solution and delete row F.

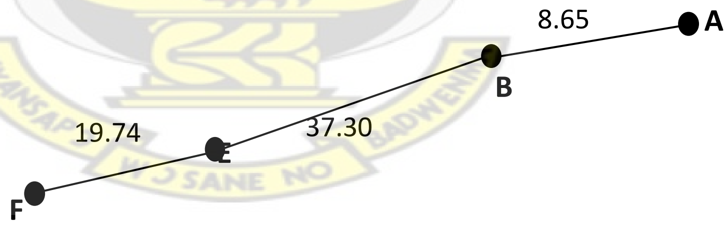


Figure 4.6

(iv) Look for the smallest entry in column A, B, E and F. The smallest entry in column A, B, E and F is 34.90km.

AD is the smallest edge joining A, B, E and F to the other vertices. Put edge AD into the solution and delete row D.

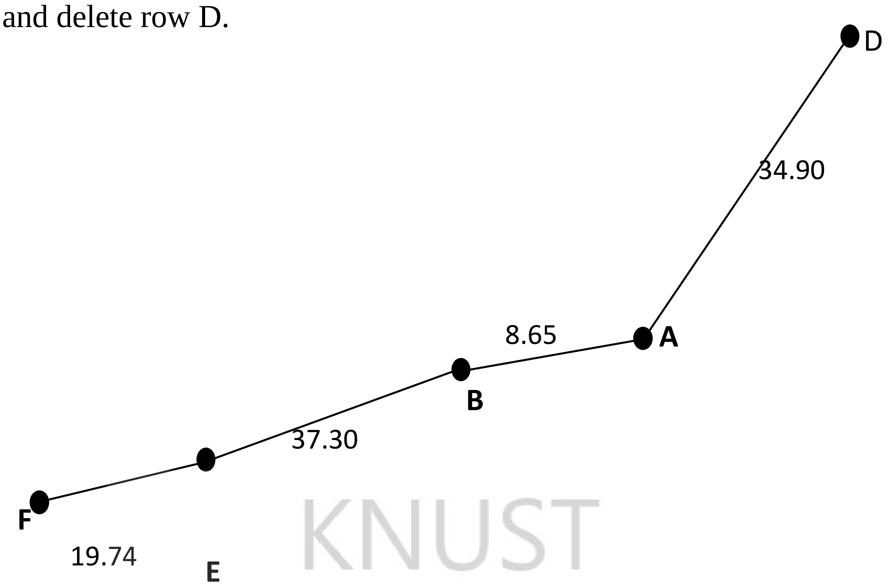


Figure 4.7

(v) Look for the smallest entry in column A, B, E, F and D. The smallest entry in column A, B, E, F and D is 51.33km.

EC is the smallest edge joining A, B, E, F and D to the other vertices. Put edge EC into the solution and delete row C.

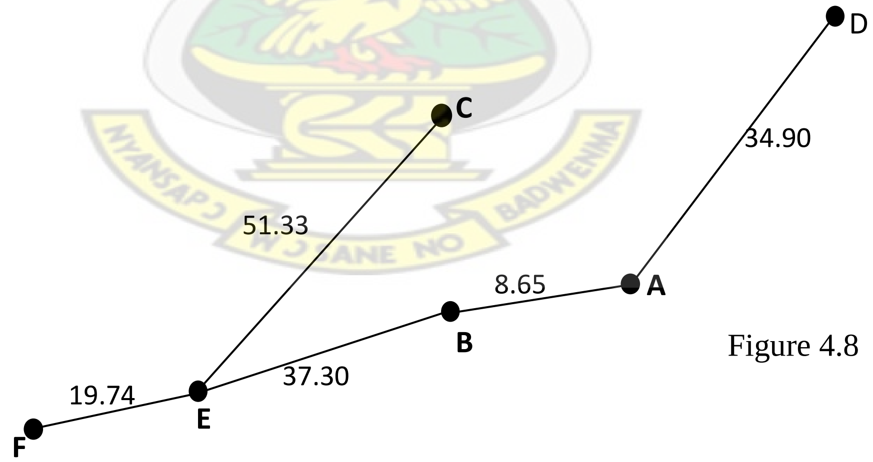


Figure 4.8

(vi) Look for the smallest entry in column A, B, E, F, D and C. The smallest entry in column A, B, E, F, D and C is 9.45km.

CG is the smallest edge joining A, B, E, F and D to the other vertices. Put edge CG into the solution and delete row G.

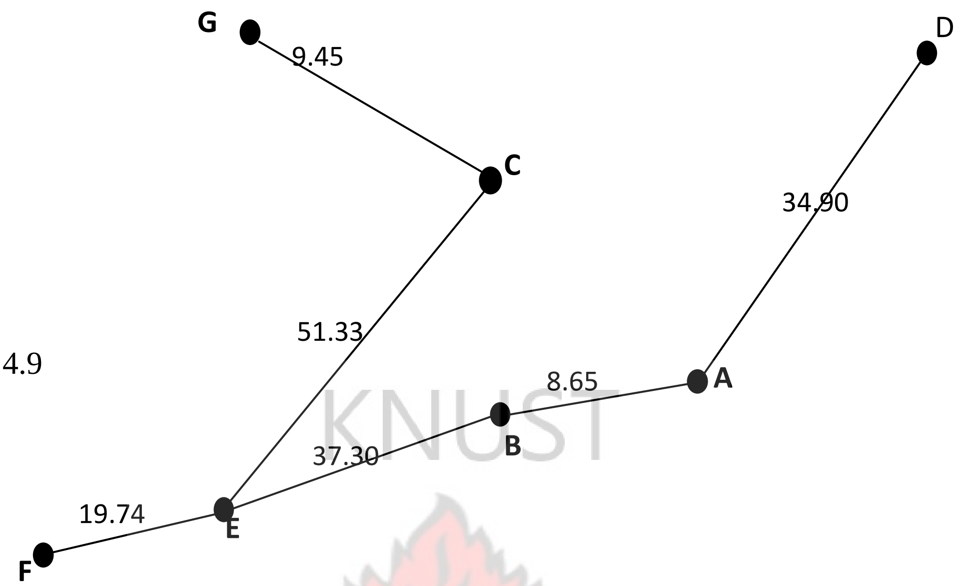


Figure 4.9

(vii) Look for the smallest entry in column A, B, E, F, D, C and G The smallest entry in column A, B, E, F, D, C and G is 27.69km.

GH is the smallest edge joining A, B, E, F, G and H to the other vertices. Put edge GH into the solution and delete row H.

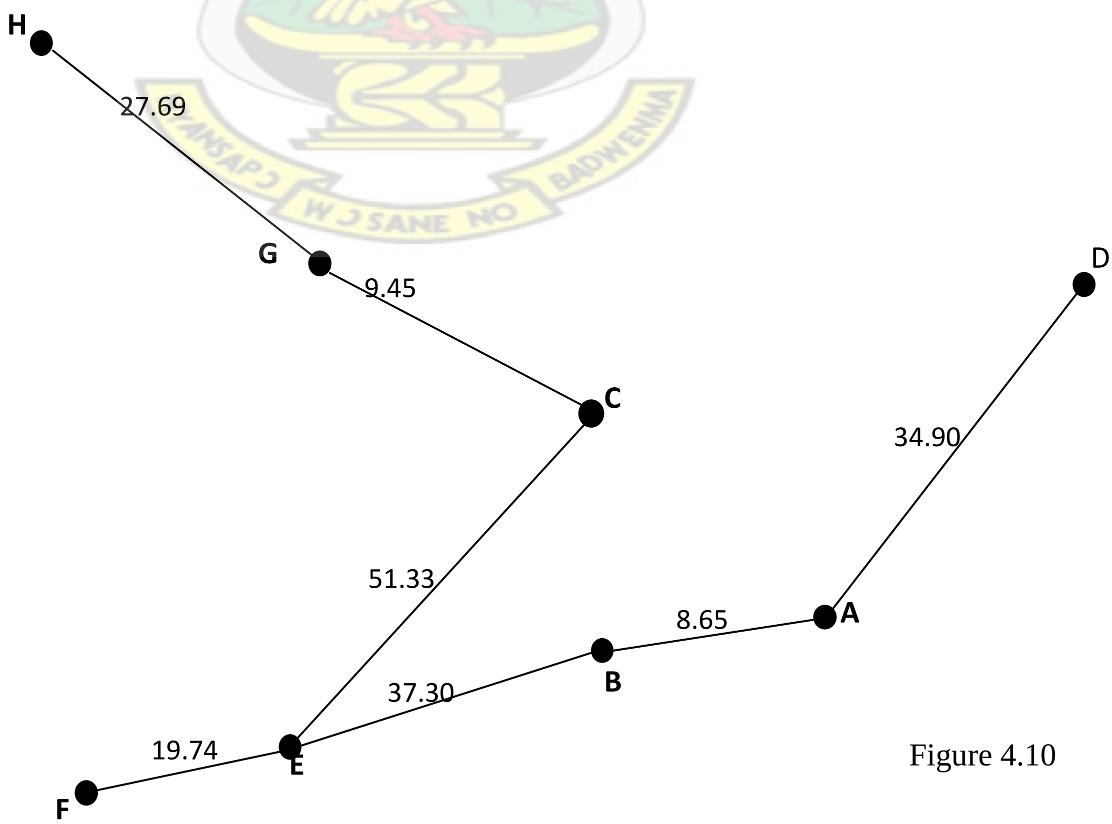


Figure 4.10

(viii) Look for the smallest entry in column A, B, E, F, D, C, G and H The smallest entry in column A, B, E, F, D, C, G and H is 87.28km.

GH is the smallest edge joining A, B, E, F, G, H and I to the other vertices. Put edge HI into the solution and delete row I.

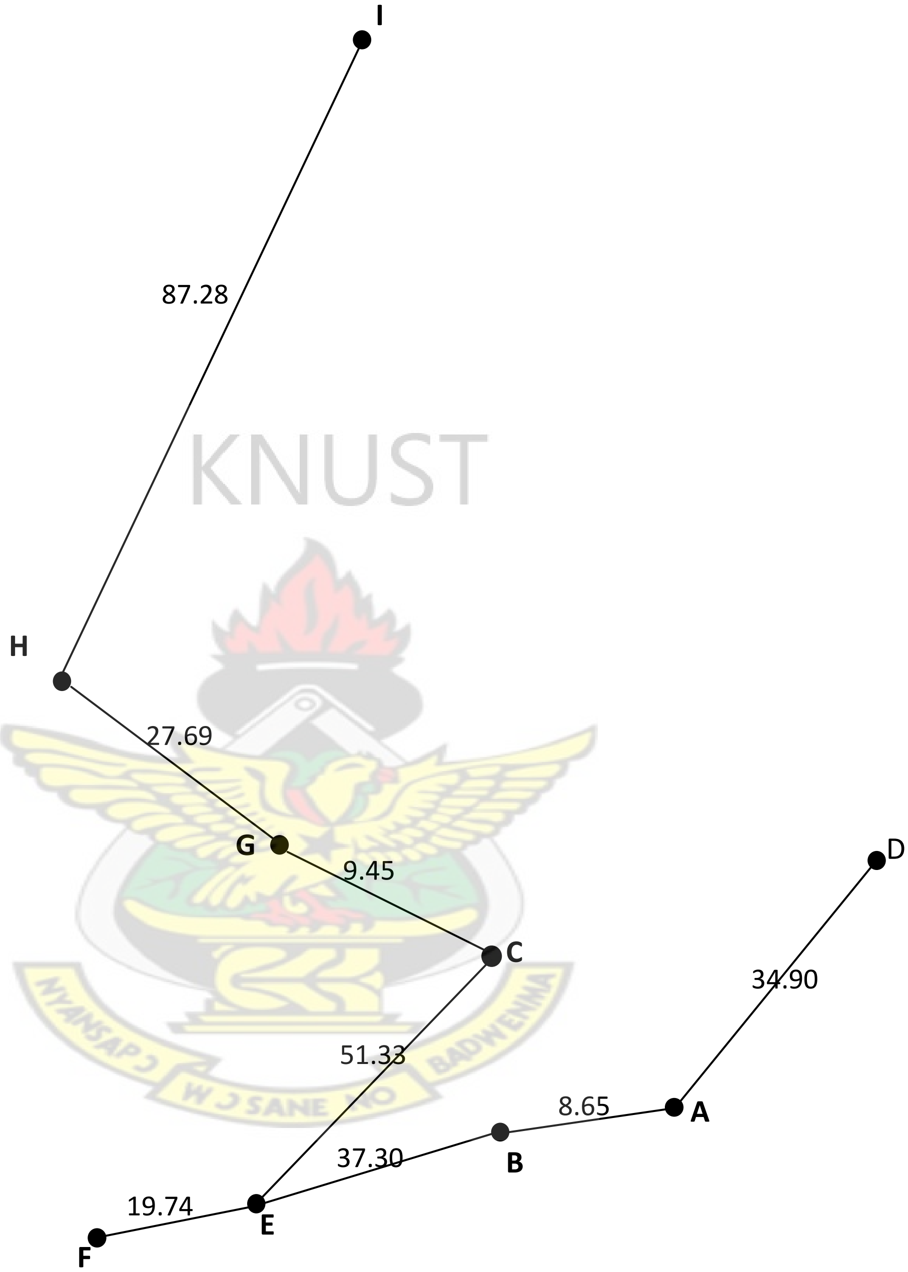


Figure 4.11

(ix) Look for the smallest entry in column A, B, E, F, D, C, G, H and I The smallest entry in column A, B, E, F, D, C, G, H and I is 56.67km.

IJ is the smallest edge joining A, B, E, F, G, H and I to the other vertices. Put edge IJ into the solution and delete row J.

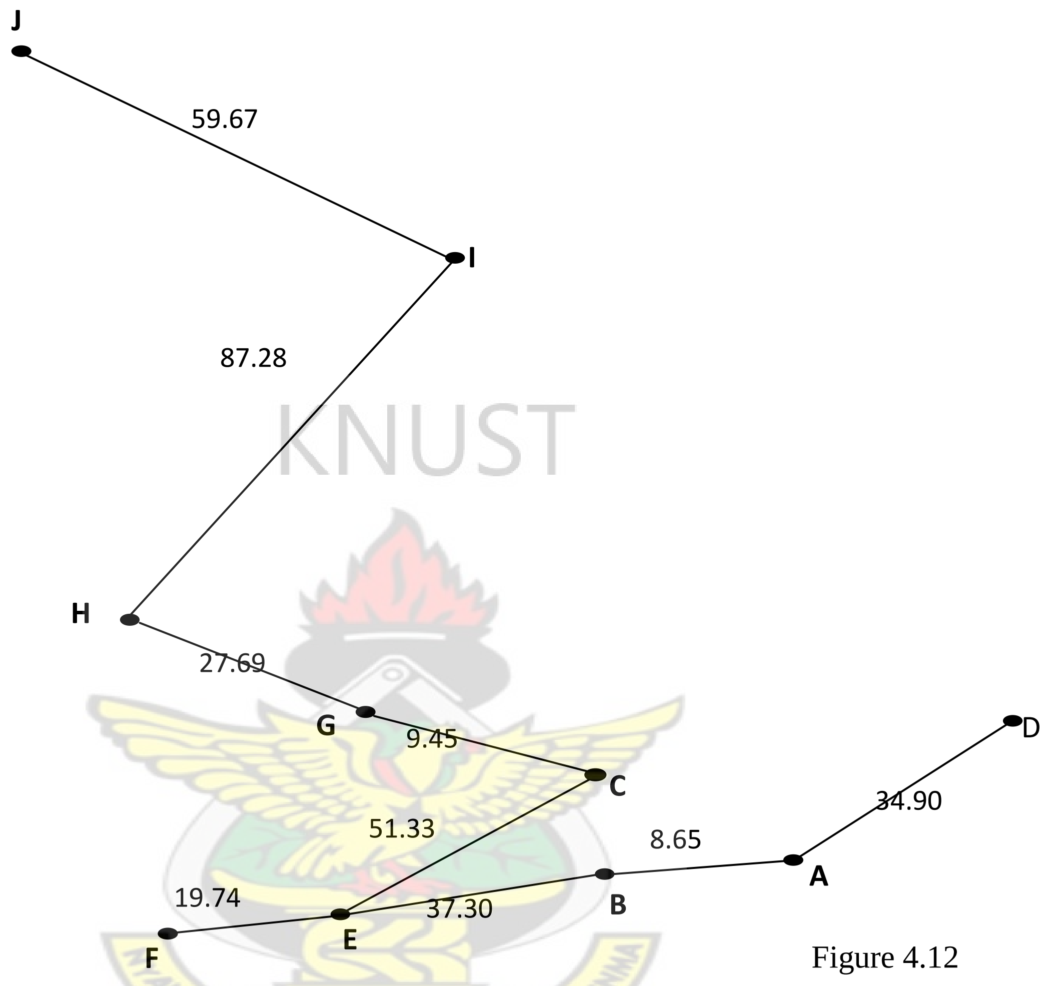


Figure 4.12

(x) Look for the smallest entry in column A, B, E, F, D, C, G, H, I and J The smallest entry in column A, B, E, F, D, C, G, H and I is 46.72km.

IK is the smallest edge joining A, B, E, F, G, H, I and J to the other vertices. Put edge IK into the solution and delete row K.

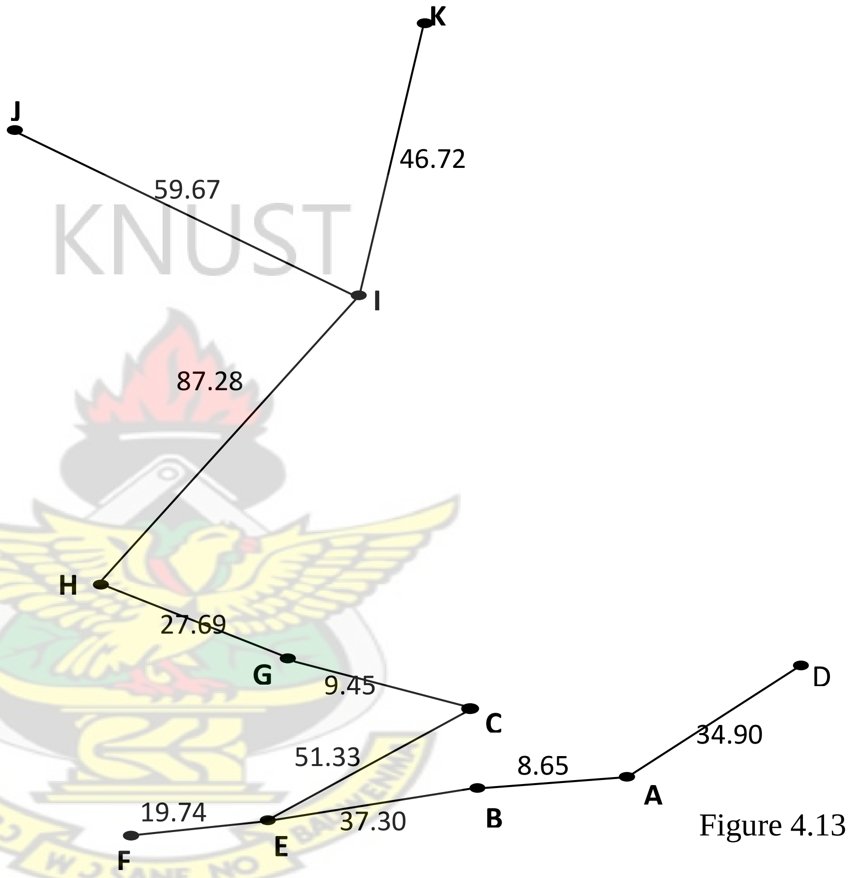


Figure 4.13

4.9 Results and Discussions

We have now connected all the vertices into a spanning tree as shown below.

Total Length of pipeline needed $= 8.65 + 37.30 + 19.74 + 34.90 + 51.33 + 9.45 + 27.69 + 87.28 + 59.67 + 46.72$

$$= 382.73 \text{ km}$$

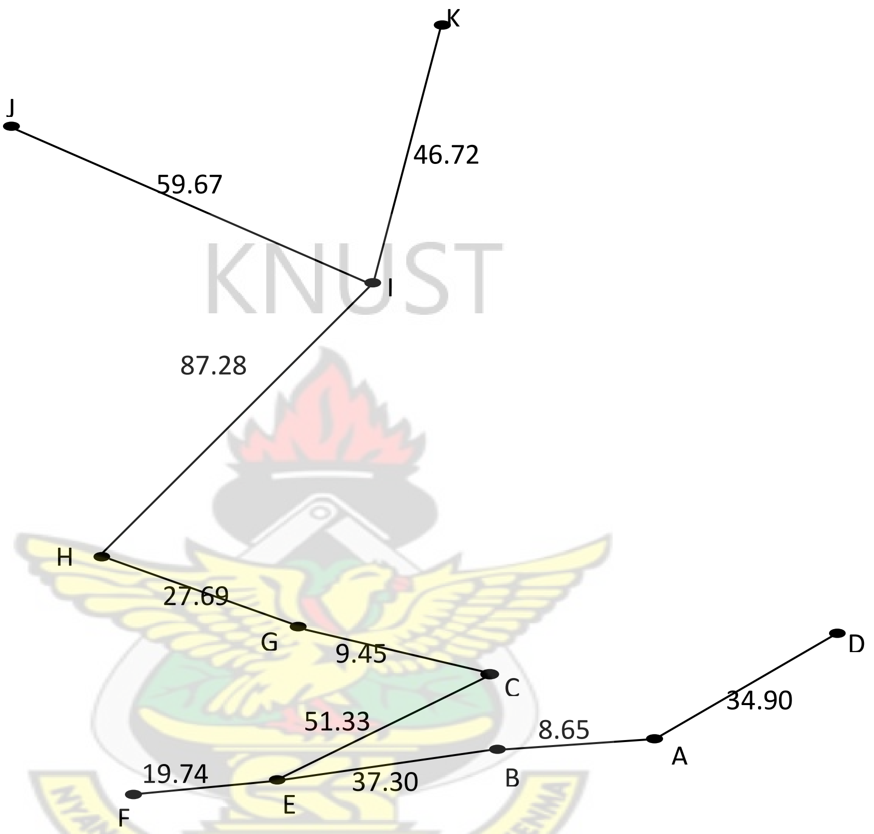


Figure 4.13: depicts the optimal connection of gas pipelines from Tema (node A) through all ten regional capitals.

From the results above, this study has established that it will cost the Bulk Oil Storage and Transportation Company 382.73km of pipelines to connect all regional capitals in Ghana.

4.10 Summary

We have now realised our optimal connection to be approximately 382.73Km. Unfortunately, there is no established work in the field of optimal pipeline design for the nation and therefore there cannot be any form of comparison with the result above.

The next chapter discusses the summary and conclusion of the research. Recommendations will also be made to all stakeholders of the industry.

CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATION

5.1 Summary

Starting from a fairly organized research space on the topic of spatial network optimization for pipelines, this work characterizes the basic pipeline network and proposes a number of analytical solution methods for obtaining near-optimal networks. The research brings together a common problem associated with the exploration of oil and gas with standard transportation engineering solution methods, to create a non-traditional approach to pipeline network design.

This work applies established network analysis techniques to the pipeline network design problem in an effort to develop straight-forward, analytical design tools for the pipeline industry.

Several network concepts and problems are presented in the literature review. This include the WAGP project, the Odeda Local Government area network problem in Nigeria, Amoco East Crisfield Gas Pipeline project, minimization of network length and using network medians and centres to locate central facilities or hub et cetera.

Each subsequent chapter puts forward one of these problems as it specifically relates to pipelines. Industry data and illustrations are used to exemplify the proposed network technique. The most significant finding of this work was that, as a developing oil and gas economy, there is little or absolutely no published research in the field of network optimization for pipelines in Ghana. This work therefore makes a significant contribution to the body of knowledge regarding pipeline network design in Ghana. This study therefore, is intended to be a basis for further research to be conducted in this field. It has also been established in the body of this work that the field of electricity transmission may contain some work which could be of value to the pipeline problem, due to similarities in between the two systems and should be reviewed for this purpose. It is the authors hope that the techniques used herein will be applied and expanded by industry practitioners and academic researchers to achieve greater efficiencies and economies, to the benefit of producers, transporters and consumers of natural gas.

5.2 Conclusion

As stated earlier the purpose for this research was to find an optimal pipeline design from Tema to all regional capitals of Ghana.

To arrive at this optimal length, grid co-ordinates of the various regional capitals were retrieved from the national grid co-ordinates. The distance between all pairs of nodes (regional capitals) was computed using the Euclidean distance formula. Prim's algorithm was then applied to determine optimal pipeline design.

The findings in this study provide an optimal pipeline design of 382.73km.

5.3 Recommendations

(i) Until Ghana builds a national gas pipeline network, introduction of natural gas into the economy will not necessarily displace all fuel-oil demand. The Government should therefore commit itself in building an extensive gas pipeline network to ensure effective and efficient distribution of natural gas.

(ii) Natural gas is an existent source of cheaper energy for industrial and commercial purposes with huge potential for economic growth. Government and all stakeholders should take pragmatic steps to ensure a well coordinated research into all aspect of the industry to ensure the effective and efficient utilization of our natural gas.

(iii) The author of this work recommends the critical examination of this work for the purpose of expanding the body of knowledge and workable solution techniques in the field of pipeline designs in Ghana.

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APPENDIX A

Grid and WGS-84 co-ordinates of the Ten Regional Capitals of Ghana

		WGS-84		National Grid Co-ordinates	
Node	**Location**	**Latitude**	**Longitude**	**Easterns** (X m)	**Northen** (Y m)
A	**Tema** (Kpone)	5°40'04.378N	0° 02' 17.112E	251,031.525	193,890.728
B	**Accra**	5° 36' 5.6 N	0°10'0.166667W	245,800.857	187,002.838
C	**Koforidua**	6°05'6.0833N	0°15'0.25W	208,898.739	223,331.019
D	**Ho**	6°36'6.6 N	0°28'0.466667E	271,351.286	222,268.191
E	**Cape Coast**	5°05'5.0833N	1°15'1.25W	211,620.156	172,078.64
F	**Takoradi** (Assaka)	4°53'4.8833N	1°46'1.76667W	192,798.496	166,057.4784
G	**Kumasi** (Kumasi Airport)	6°43'6.71667N	1°36'1.6W	199,898.934	226,202.852
H	**Sunyani** (Yamfo)	7°20'7.3333N	2°20'2.3333W	178,141.106	243,331.352
I	**Tamale**	9°30'9.5N	0°51'0.85W	225,100.381	316,897.221
J	**Wa**	10°03'10.05N	2°30'2.5W	169,624.682	338,864.129
K	**Bolgatanga**			223,481.578	363,592.538

APPENDIX B

Grid Co-ordinates of Nodes

Node	(X km)	(Y km)
A	251.031	193.890
B	245.800	187.002
C	208.898	223.331
D	271.351	222.268
E	211.620	172.078
F	192.798	166.057
G	199.898	226.202
H	178.141	243.331
I	225.100	316.897
J	169.624	338.864
K	223.481	363.592

APPENDIX C

Matrix of intermodal distances showing edges of the MST

Node	A	B	C	D	E	F	G	H	I	J	K
A	∞	8.65	51.40	34.90	45.04	64.54	60.49	88.08	125.71	166.27	171.90
B	8.65	∞	51.78	43.55	37.30	56.99	60.36	88.04	131.53	169.90	177.99
C	51.40	51.78	∞	62.40	51.33	59.49	9.45	36.69	94.96	122.03	141.02
D	34.90	43.55	62.40	∞	78.02	96.59	71.56	95.56	105.33	154.74	149.21
E	45.04	37.30	51.33	78.02	∞	19.74	55.38	78.73	145.45	171.99	191.88
F	64.54	56.99	59.49	96.59	19.74	∞	60.56	78.65	154.26	174.35	199.90
G	60.49	60.36	9.45	71.56	55.38	60.56	∞	27.69	94.13	116.66	139.40
H	88.08	88.04	36.69	95.56	78.73	78.65	27.69	∞	87.28	95.91	128.52
I	125.71	131.53	94.96	105.33	145.45	154.26	94.13	87.28	∞	59.67	46.72
J	166.27	169.90	122.03	154.74	171.99	174.35	116.66	95.91	59.67	∞	59.26
K	171.9	177.99	141.02	149.21	191.88	199.90	139.40	128.52	46.72	59.26	∞