RISK – RETURN ANALYSIS OF OPTIMAL PORTFOLIO USING THE SHARPE RATIO

BY

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MAY, 2012

DECLARATION

I hereby certify that this thesis submitted to the Graduate School of Kwame Nkrumah University of Science and Technology (Institute of Distance Learning) is my own work towards the Msc. degree and that, except for references to other researchers' work which have duly been acknowledged, this thesis has not been submitted to any other university for the award of a degree.

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(Head of Department) (Signature) Date

AND CERTIFIED BY

DEAN, IDL ………………………….. …………………
(Signature) Date

DEDICATION

This piece of work is dedicated to the Omnipotent God for seeing me through this work successfully and also to my wife Mrs. Lovia Antwi Boamah and my first born Abronne Antwi Edmund (Kobee) and also my mother Mad. Lydia Owusu Bemah for her assistance throughout my education carrier and finally Nana Barfi Adomako (Balme Library **–** University of Ghana).

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I wish to take this opportunity to acknowledge my supervisor Mr. V.K Dedu for his immense contributions for the success of this work. I also thank all the Lecturers of the Mathematics Department especially Mr. F. K Darkwa (Head of Department) for their encouragement and motivation and also extend my gratitude to Dr. Lord Mensah for his assistance and finally my brother Samuel Darko of Asokore Rural Bank, Micro Finance manager, Oforikrom Branch.

ABSTRACT

The Modern portfolio Theory is based on Harry Markowitz's 1952 work on mean – variance portfolios. He stated that a rational investor should either maximize his expected return for a given level of risk, or minimize his risk for a given expected return.

These two principles lead to an efficient frontier of portfolios, among which the investor is free to choose.

Fifty years on, there are no widely accepted practical implementations of mean – variance portfolio theory. The mean – variance approach puts excessive weights on assets with large excess returns, regardless of possible estimation errors. It yields unstable portfolios and extra gains do not make up for the excess transaction costs.

The goal of this project is to develop robust portfolio optimization methods. We develop a multi - factor objective function reflecting our investment preferences and solve the subsequent optimization problem using the Sharpe's ratio.

Many investors and portfolio managers always seek maximum returns with relative low risk or conversely, minimum risk with maximum expected returns. Which model or approach best meets investor's investment decisions and portfolio selection. The Markowitz model in 1952 and subsequently 1959, amongst other things seeks to address such dilemma faced by investors. In this thesis, we shall explore the Markowitz model in constructing optimal stock portfolio, analyze its modern relevance, and also predict its future durability in finance theory. We explore the mean – variance approach by Harry M. Markowitz in portfolio selection in single – period index model, and provides the basis for many important financial economic advances, including the Sharpe single – index model (Sharpe, 1964). This thesis highlights the concept of utility function in determining the risk preference of investors. Again, we analyze diversification under Markowitz portfolio construction and its impact on risk minimization, given unsystematic risk of a corporate organization. We shall be dealing with optimization problem like maximize expected return of a portfolio subject to a given level of risk, or conversely minimize risk subject to a given expected return (Markowitz, 1952, 1959, 1991), Merton (1972), Kroll, Levy and Markowitz (1984). In addition, we shall use some statistical parameters such as mean, variance (standard deviation), co – variance and correlation for our model formulation of the Markowitz framework.

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LIST OF ABBREVIATIONS

CAL ………………………… CAL BANK LTD .

EBG ……………………………. ECOBANK GHANA

EIC ……………………………. ENTERPRISE INSURANCE COMPANY

ETI ……………………………..ECOBANK TRANSNATIONAL INCORPORATED

GCB …………………………….GHANA COMMERCIAL BANK

SIC ……………………………. STATE INSURANCE COMPANY

HFC ……………………………..HFC BANK LTD.

SG – SSB …………………………….. SG – SSB BANK LTD.

SCB …………………………….. STANDARD CHARTERED BANK LTD.

TBL ……………………………THE TRUST BANK LTD.

UT ……………………………UNIQUE TRUST BANK LTD.

SCB PREF …………………………….. STANDARD CHARTERED BANK PREFERENCE SHARES.

CAPM …………………………… CAPITAL ASSET PRICING MODEL

SML ……………………………. SECURITY MARKET LINE

SCL ……………………………. SECURITY CHARACTERISTIC LINE

CML …………………………….. CAPITAL MARKET LINE

MPT ………………………………MODERN PORTFOLIO THEORY

CAL ** ……………………………..CAPITAL ALLOCATION LINE

UNIL ………………………………... UNILEVER COMPANY LIMITED

FML ……………………………… FAN MILK LIMITED

CHAPTER ONE

INTRODUCTION

RISK – RETURN ANALYSIS OF OPTIMAL PORTFOLIO

1.1 Background of the study:

A portfolio is simply a collection of financial assets involving investment tools such as bonds, foreign exchange, stocks, gold, asset-backed securities, real estate certificates and bank deposits which are held simultaneously by one person or a group of persons. If you own a home and household furnishings and a savings account you already have a portfolio.

Risk is the probability of the losses one incurred on portfolio investment and the return is the profit or benefit one derives from portfolio investment. Investment is the net worth on long term financial assets such as bonds, shares and mutual funds. Investment risk is most properly understood when it is expressed in statistical terms that consider the entire range of an investment's possible returns.

Markowitz again states that, the expected return (mean) and variance or standard deviation (risk) of return of a portfolio are the whole criteria for portfolio selection and construction. These parameters can be used as a possible maxim for how investors need to act. It is interesting to note that, the whole model is based on an economic fact of "Expected Utility". The concept of utility here is based on the fact that different investors have different investment goals and can be satisfied in different ways.

Consequently, every investor seeks to maximize their utility (satisfaction) by maximizing expected return and minimizing risk (variance).

Prior to Markowitz article in 1952, Hicks mentioned the necessity of improvement on theory of money in 1935. He introduced risk in his analysis and stated that "risk -factor" comes into our problem in two ways: First, as affecting the expected period of investment and second as affecting the expected net yield of investment. On his work William Sharpe (1964) and Litner (1965) almost simultaneously developed a model to price capital asset, popularly known as Capital Asset Pricing Model (CAPM). This model relates expected return to a measure of risk that incorporate what some consider to be the "only free lunch in finance economics", Diversification. This measure now known as beta, use theoretical result that, diversification allows investors to escape company's specific risk. The Markowitz model could be summarized as follows (Fabozzi, 1999), one needs to

- Calculate the expected return rates for each stock to be included in the portfolio.
- Calculate the variance or standard deviation (risk) for each stock to be included in the portfolio.
- Calculate the co- variance or correlation coefficients for all stocks, treating them as pairs.

Later studies by Sharpe(1964), Litner (1965) and Mossin (1966) on portfolio construction further investigated the trend of prices in cases where all savers invest in financial assets and particularly in share certificates in accordance with modern portfolio theory (Zorlu, 2003).

Although, it is no secret that Markowitz mean – variance model has empirical set backs or challenges, it is undisputable fact that it is the most widely used model in academic and real world application (Fama – 2004)[2]

The Markowitz optimization problem can be summarized as:

Maximize:

Stock Exchange List was investigated by mathematical and statistical methods for normality of assets' returns. We will investigate which data series is better to construct a portfolio in our investments.

In addition, Sharpe's ratio is also used to analyse risk and return of portfolio investment.

1.2 Problem Statement:

Most investors and portfolio managers seek to optimally construct their stock portfolio in order to satisfy their investment goals. However, the problem invariably remains " which combination of sets of portfolio must he select for him to get maximum return given a level of risk. Or conversely, which sets of portfolio would yield a minimum risk given a level of return".

The Modern Portfolio Theory (MPT) is appreciated by Scientists and it is the most practical investment model ever introduced by Harry M. Markowitz. The model and its components shall be fully introduced, also covering the mathematical development of the model.

The first part of this thesis "Theory of the Modern Portfolio Theory (MPT) gives a broad view on the theory to the reader, almost all the parameters and components of the basic model defined in this part. Some historical facts, the risk and return analysis, mathematical development of the model, diversification which is an important aspect of portfolio management – that is, you do not want to put all your eggs in one basket. For example, it is a lot more risky to have all your funds in one share than to have your funds spread over say five shares.

The second part of this thesis under title "Construction of the Model using Sharpe's ratio" for further investigations. This part is brief and references introduced can help the reader to get a better understanding of the process of using the Sharpe's ratio provided by this study can also help the reader for these calculations. The parameters such as Sharpe's ratio, time series (Daily, weekly, monthly, quarterly and yearly) shall be used for the analysis of historical and future data.

The final section under title "Empirical investigation" is the main part of this research. In the first part, the validity of one of the critical assumptions of the model was questioned and by some statistical test, this claim was supported, then a new ratio was introduced to handle this inefficiency regarding the model and finally these two ratios are tested against each other by different combination of some extra parameters introduced during the process.

1.3 Objectives of the Study / Research:

- To discover the best portfolio for investments chosen from the financial companies on the Ghana Stock Exchange.
- To analyse the ratio of risk to return on portfolio investment using the Sharpe's ratio.
- To determine how efficient diversification helps in investments.

Research Questions:

- What relationship exists between risk and return of portfolio?
- Why should investors diversify?
- What type of risk do investors care about? Is it "volatility"?

- What is the risk premium on any asset, assuming that investors are well diversified?

1.4 Methodology

In our effort to resolve the portfolio optimization problem by the use of Markowitz model, we shall make use of some statistical parameters such as mean, variance (standard deviation) covariance and correlation matrix in Markowitz model formulation. We shall also use Lagrange multiplier to solve problems such as maximize returns subject to risk constraint. We shall use the Markowitz model and Sharpe ratio to solve real world problems based on yearly for a four year period from 2007 – 2010 of financial companies on the Ghana Stock Exchange. This thesis shall also explore into the single index model as it reduces the computational burden of the Markowitz model as presented by Elton and Gruber (1987). A four year period data shall be collected from some financial institutions to enable us formulate our optimization model. Additional information shall be obtained from websites and journals on finance.

1.5 Justification of the Study:

The study of risk and return analysis of optimal portfolio is to provide adequate knowledge to financial assets allocation or financial institutions such as Banks, Insurance Companies, Credit Unions, Investors etc.

Also, to inform investors that efficient diversification reduces portfolio risk. To help investors appreciate the relation between risk and return of portfolio. This study shall also guide the investors in the Ghanaian economy who invest any how to select the best portfolio in the market to maximize returns with the same level of risk, which is the basis of Harry Markowitz theory on Modern Portfolio Theory (MPT).

1.6 Thesis organization:

This thesis is divided into five sessions.

(i) Chapter one talks about the introduction and also gives brief background of the study.

(ii) Chapter two presents the general literature review of the basic theory.

(iii) Chapter three discusses the mathematical and statistical estimates of the key input variables of portfolio theory.

(iv) Chapter four deals with the issues that arise when the theory is applied to Ghanaian financial institutions such as banks and insurance companies. A case study in Cal Bank Ltd.(CAL), Ecobank Ghana Ltd.(EBG), Enterprise Insurance Co. Ltd.(EIC), Ghana Commercial Bank(GCB), HFC Bank Ltd.(HFC), Standard Chartered Bank(SCB), SG – SSB Ltd.(SG-SSB), Ecobank Transnational Inc.(ETI), Trust Bank Ltd.(TBL), UT Bank Ltd.(UT), State Insurance Company(SIC) and SCB Preference Shares(SCB PRE).

(v) Chapter five discusses the conclusions, findings and the review of the model in the light of its limitation and suggested area of further research / study.

CHAPTER TWO

2.0 LITERATURE REVIEW

THE THEORY ON MODERN PORTFOLIO THEORY (MPT)

The Modern Portfolio Theory (MPT) is not as modern as it implies in first glance. Like other theorems and models, which went through mysteries, MPT has its own story too. But it is always so that one gets lucky and wins whole the pot. The insight for which Harry Markowitz (born August 24, 1927) received the Noble Prize was first published 1952 in an article entitled “portfolio selection”.

The article later expanded to a book by Markowitz in 1959, “Portfolio Selection”. “Efficient Diversification of Investments”. The quantitative approach of the model existed far back in time, and they were modeled on the investment trusts of the England and Scotland, which began in the middle of the nineteenth century. Where Galati cites a quote about diversification which showed that it happened also earlier in time, where in Merchant of Venice, Shake Spear put the words on Merchant Antonio who says:

> “My venture’s are not in one bottom trusted
>
> Nor to one place; nor is my whole estate.
>
> Upon the fortune of this present year
>
> Therefore, my merchandise makes me not sad”.

Prior to Markowitz article, 1952, Hicks mentioned the necessity of improvements on theory of money in 1935. He introduced risk in his analysis, and he stated “The risk-factor comes into our problem in two ways: First, as affecting the expected period of investment, and second, as affecting the expected yield of investment.”

Galati also mentioned in his book that he could not demonstrate a formula relating risk of individual assets to risk of the portfolio as a whole. Since this work is based on MPT.

The model developed by Markowitz shall be considered and his work on mean-variance analysis. He states that the expected return (mean) and variance of returns of a portfolio are the whole criteria for portfolio selection. These two parameters can be used as a possible hypothesis about actual behaviour and a maxim for how investors ought to act.

It is essential to understand the intimates of Markowitz model. It is not all about offering a good model for investing in high return assets. It might be interesting to know that whole the model is based on an economic fact, "Expected utility". In economic term the concept of utility is based on the fact that different consumers have different ways. The two parameters risk and returns will make more sense to you when we go into the explanation of diversification of a portfolio. In behavioural finance we can explain it so; Investors are seeking to maximize utility.

Consequently if all investors are seeking to maximize the utility, so all of them must behave in essentially the same way. Which this consistency in behaviour can suggest a very specific statement about their aggregate behaviour. It helps us to reach some description for future actions.

Every model or theory is based on some assumptions, basically some simplification tools. Markowitz model relies on the following assumptions.

- Investors seek to maximize the expected return of total wealth.
- All investors have the same expected single period investment horizon.
- All investors are risk – averse, that is they will only accept a higher risk if they are compensated with a higher expected return.

- Investors base their investment decisions on the expected return and risk.
- All markets are perfectly efficient.

By having these assumptions in mind, we will go through some concepts and terminologies that will make us understand the model constructed in further part of this thesis.

Portfolio Theory (Harry M. Markowitz)

Past, Present and Future

“ Portfolio Theory” Before 1952

It is sometimes said that investors did not diversify much before 1952. Not true see:

Where Galati cites a quote about diversification which showed that it happened also earlier in time, where in Merchant of Venice, Shake Spear put the words on Merchant Antonio who says:

> “My venture’s are not in one bottom trusted
>
> Nor to one place; nor is my whole estate.
>
> Upon the fortune of this present year
>
> Therefore, my merchandise makes me not sad”.

Captain Long John Silver in Treasure Island

“I put it all away, some here, some there, none too much anywhere, by reason of suspicion”.

What was lacking in 1952 was adequate theory covering:

- The effect of diversification when risks are correlated.
- Risk/ Return trade off on the portfolio as a whole.

Importance of covariance

A security is likely to have a high return but has a small chance of going broke. Is a small investment in this security a reasonably safe bet?

Nor if the remainder of the portfolio consists of similar bets all of which will go broke at the same time.

Law of the Average Covariance (Markowitz 1959)

For an equal weighted portfolio, as the number of securities held increases Portfolio Variance approaches the Average Covariance.

Example 1 : For uncorrelated securities Portfolio Variance approaches zero.

Example 2 : If all securities have the same variance V_s, all distinct pairs of securities have the same correlation coefficient ρ, then Portfolio Variance V_p approaches correlation coefficient ρ times Security Variance V_s.

Portfolio Std Dev. σ_p approaches square – root of correlation ρ times Security Std Dev. σ_s.

I.e If ρ =	Then σ_p/σ_s =
0.25	0.500
0.10	0.316

Even with unlimited diversification.

Most of the ideas in Markowitz [1952] popped up one day in 1950 when Markowitz read Williams asserted:

The value of a stock should be the present value of its future dividends.

Markowitz thought:

Future is uncertain; must mean expected present value.

If one is only interested in the expected value of the portfolio. One maximizes expected value by putting all resources into the single security with greatest expected value. Diversification exists and is good. Clearly investors (and investment companies) seek return, avoid risk. Let's measure these expected value and standard deviation.

Portfolio return is a weighted sum of security returns.

Williams addresses the issues of risk and uncertainty.

"Whenever the value of a security is uncertain and has to be expressed in terms of probability, the correct value to choose is the mean value. The customary way to find the value of a risky security has always been to add a "premium risk" to the pure interest rate, and then use the sum as the interest rate for discounting future receipts. In the case of bond under discussion, which at 40 would yield 12 per cent to maturity, the "premium for risk" is 8 per cent when the pure interest rate is 4 per cent.

Markowitz, H.[1952] , " Portfolio Selection " The Journal of Finance, Vol. 7, number 1 pp.77 – 91. Proposes that mean, variance and covariance of securities to be estimated by a combination of statistical analysis and security analyst judgments, and the set of mean – standard deviation combinations implied by these be presented to investor for choice of desired risk – return combination.

Roy, A.D [1952], " Safety First and the holding of Assets " Econometrica 20

pp. 431 – 449.

Proposes portfolio choice based on mean and variance of portfolio as a whole. Specifically proposes choosing portfolio which maximizes

$$\frac{m - d}{\sigma}$$

where m is mean return, σ is standard deviation and d is a fixed disastrous return.

Markowitz formulated the portfolio problem as a choice of the mean and variance of a portfolio of assets. He proved the fundamental theorem of the mean variance portfolio theory, namely holding constant variance, maximize expected return, and holding constant expected return minimize variance. These two principles led to the formulation of an efficient frontier from which the investor could choose his or her preferred portfolio, depending on individual risk return preferences. The important message of the theory was that assets could not be selected only on characteristics that were unique to the security. Rather, an investor had to consider how each security co – moved with all other securities. Furthermore, taking these co – movements into account resulted in an ability to construct a portfolio that had the same expected return and less risk than a portfolio constructed by ignoring the interactions between securities.

Considering just the mean return and variance of return of a portfolio, of course, a simplification relative to including additional moments that might more completely describe the distribution of returns of the portfolio. Early work developed necessary conditions on either the utility function of investors or the return distribution assets that would result in mean variance theory being optimal (For example, Tobin, 1958). In addition researchers (Lee, 1977); Krass and Litzenberger, 1976) offered alternative portfolio theories that included more moments such as skewness or were accurate for realistic description of return (Fama, 1965; Elton and Gruber, 1974). Nevertheless, mean and variance theory has remained the cornerstone of modern portfolio theory despite these alternatives. This persistence is not due to the realism of the utility or return distribution assumptions that are necessary for it to be correct. Rather, we believe there are two reasons for its persistence. First, mean variance theory itself places large data requirements on the investor, and there is no evidence that adding additional moments improves the desirability of the portfolio selected. Second, the implications of mean variance portfolio theory are well developed, widely known, and have great intuitive appeal. Professionals who have never run an optimizer have learned that correlations as well as means and variances are necessary to understand the impact of adding a security to a portfolio. Risk measures such as beta, which have been developed based on mean variance analyses, add information and recognized and used by investors who have no idea of the theory behind them. The precepts of mean variance theory work. Thus, we will concentrate on mean variance portfolio theory.

Mean variance portfolio theory was developed to find the optimum portfolio when an investor is concerned with return distributions over a single period. An investor is assumed to estimate the mean return and variance of return for each asset being considered for the portfolio over a single period. In addition, the correlations or variances between all pairs of assets being considered need to be estimated. Once again, these estimates are for the single decision period. One of the major theoretical problems that has been analyzed is how the single – period problem should be modified if the investor's true problem is multi – period in nature. Papers by Fama (1970), Hakansson (1970, 1974) and Merton (1990), and Mossin (1969) have all analyzed this problem under various assumptions. The papers found that under several sets of reasonable assumptions, the multi – period problem can be solved as a sequence of single period problems. However, the optimum portfolio would be different from that selected if only one period was examined. The difference arises because the appropriate utility function in the multi –

period case is a derived utility function that takes into account multiple periods, and this differs from the utility function that is appropriate for decision – making over a single period. One assumption underlying most multi – period portfolio analysis is independence of returns between periods. There has been a substantial amount of research in the last decade showing that mean returns and variances are related over time and are functions of easily observable variables.

(Fama and French, 1989; Campbell and Shiller, 1988). A major research topic for the future will be how this empirical literature should effect optimum multi – period portfolio decisions. Another strand of theoretical research has been the study of separation theorems. It is easy to show that if an investor has access to a riskless asset, the choice of the optimum portfolio of risky assets is unequivocal and independent of the investor's taste for expected return or variance. This is the separation theorem. It has three implications. First, it facilitates calculation in that the portfolio problem can be stated as finding the tangency portfolio to a ray passing through the riskless asset in expected return standard deviation space. The tangency portfolio is a portfolio that maximizes the ratio of expected return minus the return on the riskless asset to the standard deviation. Second, it leads to a mutual fund theorem, namely that all investors can obtain their desired portfolio by mixing two mutual funds, one made up of the riskless asset and one representing the tangency portfolio. One of the areas of theoretical research deals with how many mutual funds are needed and what is the nature of the portfolios that comprise them under alternative assumptions about the nature of asset returns or utility functions. (For example, Ross, 1978). This is important because it provides guidance to investors and the mutual fund industry on what kinds of portfolios should be attractive.

Furthermore, financial institutions such as banks or insurance companies, mutual fund theorems provide guidance with respect to which types of commingled funds to offer. As the assumption of a constant riskless lending and borrowing rate is relaxed, other assumptions, more funds and new types of funds enter the decision set. For example, if there are different riskless lending and borrowing rates but short sales of risky assets are allowed, four funds are needed. Two of the portfolios are any two portfolios of risky assets that lie on the efficient frontier and the remaining two portfolios are the instruments which yield the riskless lending and borrowing rates.

There are two other types of theoretical research that have received substantial attention in the literature but have not had a major impact on the implementation of portfolio management. First, a number of articles have been written that analyze the portfolio problem in continuous time

(Merton, 1990). In the continuous time formulation, the portfolio problem and consumption investment problem are solved simultaneously.

In a series of papers, Elton et al. (1976, 1977, 1978a, b) showed how optimum portfolios could be selected by a simple ranking device. This ranking device clarified the characteristics of securities that would lead to their inclusion in an optimum portfolio. In portfolio evaluation, early studies employed a variety of evaluation techniques. These included the Sharpe ratio (Sharpe, 1966), the Treynor ratio (Treynor, 1965), the alpha of Jensen (1968, 1969), the use of randomly generated passive portfolios of the same risk of Friend et al. (1970). Each of these studies evaluated performance adjusting for a measure of risk. Some used total risk (Sharpe, and Friend et al.) as the correct measure. Others (Treynor, Jensen, and Friend et al.) used beta as the correct measure of risk.

Levy and Markowitz (1979) estimated utility by a function of mean and variance of return of 149 mutual funds and found that ordering portfolios by mean – variance rule was almost identical to the order obtained by using expected utility. Pulley (1981) indicated that the mean – variance formulation provides a very good local approximation to expected utility functions using both monthly and semi – annual return data. According to the study, investors can confidently rely on mean – variance optimization, with attitudes toward local changes in portfolio value reflected by the local relative risk – aversion (Pratt, 1964). The paper also suggested that the mutual fund should select portfolios which maximize utility for a wide class of individual investors having different utility function and wealth levels, regardless of the actual form of their utility function. The importance of Pratt – Arrow risk – aversion coefficient can be found in the work of Kallberg and Ziembia (1983) whose theorem was proved that under the assumption of jointly – normally – distributed security returns, maximizing expected utility with different global utility function will generate identical optimal portfolios if certain computed statistics such as resembling Pratt – Arrow risk – aversion are identical.

In a comparative study of the Markowitz model and the Sharpe's model, Affleck-Graves and Money (1976) noted interesting link between the two models. Their study used the expected index portfolio return and standard deviations, and observed that the result obtained with the Sharpe's model became progressively better with every index that was added. It further noted that if more portfolios are added to the point that each share was its own portfolios, the model simulates the Markowitz model. Again, it was found that if very low upper boundaries (in terms of percentage holding of any one share) were enforced on Markowitz model, the single- index model was a close approximation of the optimal invested in any one share to about 40 percent (if no upper boundary were enforced) and has in the region of six shares in the efficient portfolio which they felt gave it a natural diversification. In its simplest form the Markowitz model states that a portfolio that will give a minimum variance for a target expected return can be unambiguously selected from the collection assets. In other words, for every possible target portfolio return, there is a unique portfolio of assets that will give the required return at a minimum variance.

In conclusion, mean-variance optimization has under the banner of modern portfolio theory (see for example, Rudd and Classing 1982), gained widespread acceptance as a practical tool for portfolio construction. This has occurred over the last decade primarily as a result of the technological advances made in estimating covariance of portfolio return. Many investment advisory firms and pensions plan sponsors (and their consultants) today routinely compute mean-variance efficient portfolio allocation process. Specific applications include asset allocation (allocation across the broad asset classes such as stock and bonds), multiple money managers' decisions (allocation across money manager with different strategies and objectives), index matching (finding a portfolio whose returns will closely track those of a predetermined index such as the S & P 500), and active portfolio management (optimizing risk – return trade off assuming superior judgment).

In 2007, Sharpe justified that all relevant probability distributions do not necessarily have the same form and investor may not consider only quadratic utility function. Thus, mean and variance may not be sufficient statistics to identify the full distribution of returns for the portfolio. In this thesis, Sharpe presented alternative approach of mean – variance and expected utility optimization base on the assumption of asset price that would prevail if there were single representation investor who desired to maximize expected utility. His work suggested that when investor has quadratic utility function then non – quadratic portfolio may not be. Given diverse investor preferences, there should be diverse portfolio holdings and the expected utility optimization procedure allows one to find preferred asset combination for investors whose utility functions are different.

Using the equivalent martingale measures and martingale representation theorem as a tool to study portfolio selection problem began with Harrison and Kreps(1979), shortly after this, Pliska(1982, 1986) applied martingale methods, for the first time, in solving portfolio selection problem under the expected utility maximization framework with Cox and Haung (1986, 1989), Kazatzas, Lehoczky and Shreve(1987) followed.

Pliska (1982) solved discrete time stochastic control problem by martingale method was applied in solving continuous time portfolio optimization problem in complete markets, see Karatzas (1989) with unconstrained portfolio and Korn and Trauntmann (1995) with constrained portfolio.

In addition, the implementation of mean downside portfolio model is much more tedious since there are no simple means in computing portfolio risk (Grootveld and Hallerbach, 1999). Consequently the mean – variance model has remained the most robust portfolio framework in the recent years. Numerous researchers such as Haung and Litzenberger (1988), Elton and Gruber(1995), Elliot and Kopp(1999), Jorion(2003), Mercurio and Torricelli(2003), Prakash et al(2003), Ehrgolt et al (2004), Ambachtsheer (2005), Campell and Ulucan (2007), Biggs and Kane(2009) have successfully continued to study and revised the mean – variance model.

The research by Ulucan (2007) investigated the optimal holding period (investment horizon) for the classical mean – variance portfolio model. He used the historical transaction record of Istanbul Stock Exchange ISE – 100 index stock and Athens Stock Exchange FTSE – 40 index stocks data for empirical analysis. The results of the study showed that portfolio returns with varying holding period had a convex structure with an optimal holding period.

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Although, it is no secret that Markowitz mean – variance model has empirical set backs or challenges, it is undisputable fact that it is the most widely used model in academic and real world application (Fama – 2004) [2]

The SCL is a line of best fit through some data point. But statisticians call it a time series regression line. The model uses a one period rate of return from some market index in time period t, it is denoted as $R_{m,t.}$ Then to explain some rate of return from some asset, we denote it by the index i for the ith asset. The characteristics line is used by many security analysts, in the form of estimating the undiversifiable and diversifiable risk of an investment.

The security characteristic line (SCL) is denoted by the following regression equation.

operator, however, is not linear. This means that the risk of a portfolio, as measured by the variance, is not equal to the weighted sum of risk of the individual assets.

Before any further steps in analyzing the data we will examine the distributions' normality of our stream of data. There exist different statistical methods to do such a test. Some of them are computational and it is easier to construct a Null Hypothesis Testing with the help of them, and some others can only confirm our claim by visual evidence.

Another interesting results for constructing a portfolio with Markowitz model was the amazingly unrealistic result for the Sharpe ratio maximization. The problem with Sharpe ratio is that it is accentuated by investments that do not have a normal distribution of returns. As it is clear here, for a risk manager that tries to guard against large losses, the deviation from the normality cannot be neglected. "In the case of testing the hypothesis that a sample has been drawn from a normally distributed population, it seems likely that for large samples and when only small departures from normality are in question, the most efficient criteria will be based on the moment coefficients of the sample, e.g. on the values of

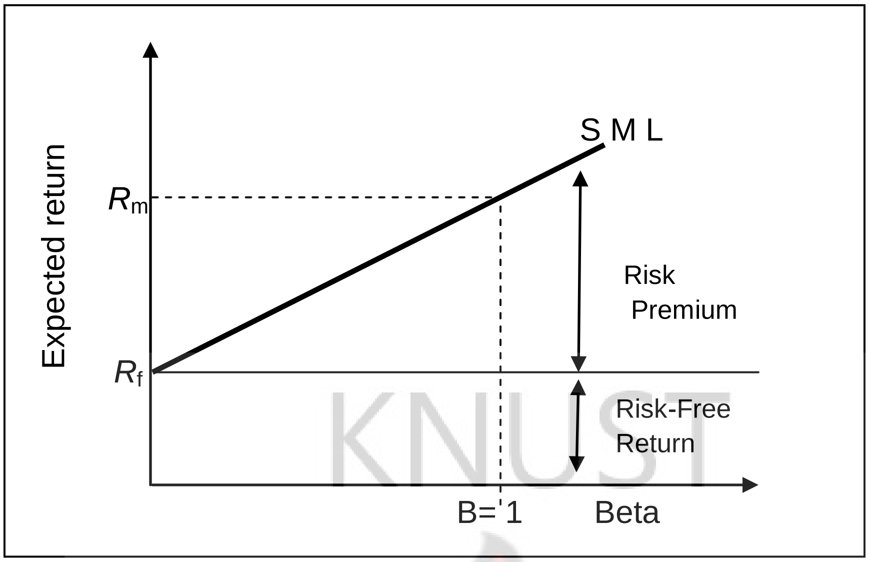


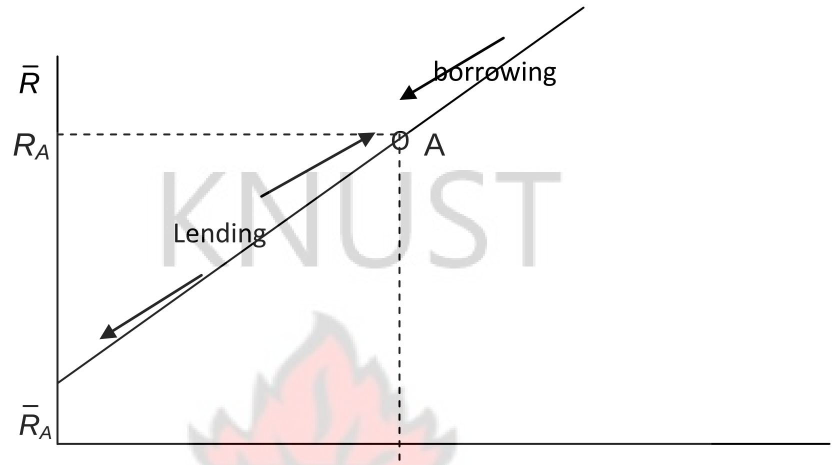
Figure 1 – The Security Market Line

Source: (Gruber et. al).

Here in the graph we can see that as the expected return increases so does the risk (Beta). The SML line is based on the risk free rate R_F. We can then also see that R_F is risk free it has a zero beta. When you go to the right of the graph, you will come to the market portfolio(M). The market portfolio is a hypothetical portfolio, consisting of all the securities that are available for an investor. That is why we have a beta of 1. The market's risk premium is determined by the slope of the SML line.

The difference in the Capital Market Line compared to the SML line is that all investors on the market are taking some position on the CML line by lending, borrowing or holding the market portfolio (A). The market value for the equity an investor holds is the same as for any other investor. Both of them own the same portfolio namely the market portfolio. The CML line considers the equity risk, standard deviation

The CML line also represents the highest possible Sharpe ratio. The CML line is derived by drawing a tangent line from the intercept point of the efficient frontier (or the optimal portfolio) to the point where the expected return equals the risk – free rate



CHAPTER THREE

RESEARCH METHODOLOGY

3.1.1 DATA COLLECTION METHODS

Secondary data were the one used for this thesis. They are already compiled data for statistical analysis. They are not collected especially for the investigation under consideration but have been collected for some other purpose(s). Secondary data are cheaper and easier to obtain.

Extraction from Administrative Records: This method is solely used to collect secondary data from published sources such as administrative files, libraries, print/electronic media, internet etc. For example the study on births and deaths in Ghana, data can easily be obtained from Births and deaths Department, Ghana Statistical Services and Ministry of Health. In our study secondary data were obtained from some selected financial institutions and Ghana Stock Exchange.

3.1.0 MATHEMATICS OF THE MARKOWITZ MODEL

The Markowitz model involves some mathematics, which makes it possible to construct stock portfolio with different combinations where short sale and lending or borrowing might be allowed or not. The Markowitz model is all about maximizing return, and minimizing risk, but simultaneously. We should be able reach a single portfolio of risky assets with the least possible risk that is preferred to all other portfolio with the same level of return. Our optimal portfolio will be somewhere on the ray connecting risk free investment to our risky portfolio and where the ray becomes tangent to our set of risky portfolios. This point has the highest possible slope. The classical Markowitz model is a form of quadratic programming problem.

Before proceeding any further on the mathematics of the Markowitz model, let us have a short calculations on the following statistical parameters:

3.1.1 Mean calculation

The mean is a measure of an average return of a portfolio. The mean of a portfolio can be calculated with several methods, but mainly arithmetic and geometric. In this work, we have chosen geometric over arithmetic for the reason which shall be proven mathematically in our subsequent sections. Let us look at them briefly.

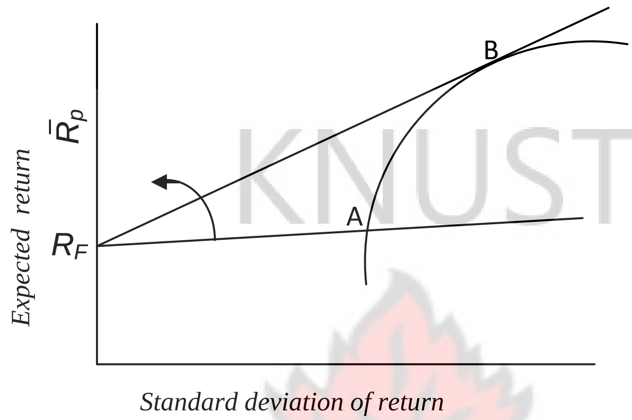
Definition 3.1.2 Arithmetic Mean.

The arithmetic mean of a list of numbers (observation) is the sum of all the members of the list divided by the number of items in the list.

Definition 3.1.3 Geometric mean.

The geometric mean of a collection of positive data is defined as the

Different points on the ray between tangent point and interception with expected return coordinate represents combination of different amounts possible to lend or borrow to combine with our optimal risky portfolio on intersection of tangent line and efficient set.



sales. It assumes that when a stock sold short, cash did receive but held as collateral. The constraint with Lintnerian definition of short sales is

Yields

λ is called a Lagrange multiplier. Now we show how it proceeds and then its application on our case:

1. Form the vector equation.

Where

EXAMPLE 1

Calculating optimal level of portfolio return

Supposing a financial institution has a portfolio of three securities (b). By putting the values of

By probability theory the mean of a weighted sum (in equation

Where,

In the original model by Markowitz, short sales were excluded, thus

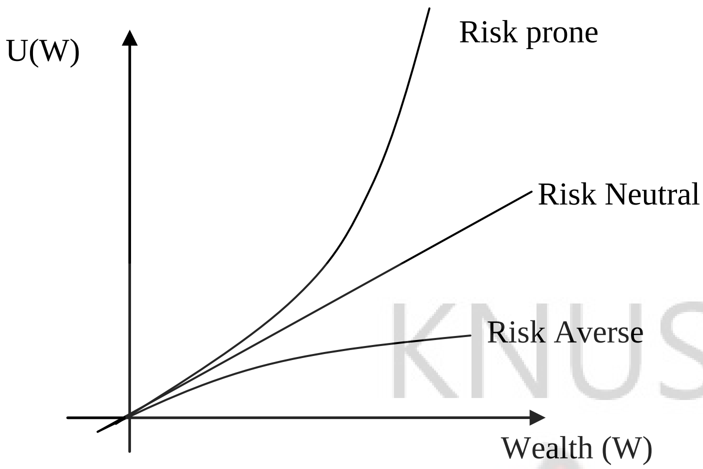


Figure 4- Utility Curve for Investors with Different Risk Preferences

Source: Kheirollah Bjarnbo (2007, p.4)

A further assumption is that risk and return preferences of an investor can be described via a quadratic utility function. This means when plotted on a graph, your utility function is a curve with decreasing slope, for larger risk. Where w is an indicator for wealth and U is a quadratic utility function. We have

$$U(w) = w - w^2$$

A consumer's utility is hard to measure.

However, we can determine it directly with consumer behaviour theories, which assume that consumers will strive to maximize their utility. Utility is a concept that was introduced by Daniel Bernoulli. He believed that for the usual person, utility increased with wealth but at a decreasing rate. Figure 4 shows the utility curve for investors with different risk preferences.

Risk aversion can be determined through defining the risk premium, which by Markowitz defined to be maximum amount that an individual is prepared to give up to avoid uncertainly. It is calculated as the difference between the utility of the wealth and the expected utility of the wealth.

Thus ,

The Markowitz mean variance can be formulated mathematically as three equivalent optimization problems as follows:

(1) Maximizing the expected return for an upper limit on variance: thus

Max.

EXAMPLE 2

Suppose that an investor wishes to construct a portfolio, drawing from three (3) independent candidate stocks. The rates of return of these investments are given in the table below along with the associated probabilities. We demonstrate how the Markowitz model can determine the best combination of stocks in order to maximize the expected rate of return and to minimize the risk-adjusted by the investor's attitude towards the risk. Assume

Max;

mutual funds is because they are said to be well diversified. In order to have a diversified portfolio it is important that the assets chosen to be included in a portfolio do not have a perfect correlation, or a correlation coefficient of one.

Diversification reduces the risk on a portfolio, but not necessarily the return, and though it is referred as the only free lunch in finance. Diversification can be loosely measured by some statistical measurement, intra-portfolio correlation. It has a range from negative one to one and measures the degree to which the various asset in a portfolio can be expected to perform in a similar fashion or not.

Portfolio balance can be measured by some of these intra-portfolio correlations. As the sum approaches negative one the percentage of diversifiable risk eliminated reaches 100%. That is why it is called weighted average intra portfolio correlation. It is computed as

Intra-portfolio correlation	Percentage of diversifiable risk eliminated
1	0.0%
0.75	12.5%
0.5	25.0%
0.25	37.5%
0	50.0%
-0.25	62.5%
-0.5	75.0%
-0.75	87.5%
-1	100.0%

Table 3.2 - percentage of the diversifiable risk eliminated.

Now let's come back again to diversification. In order to understand how to diversify a portfolio we should understand the risk. According to Ibbotson et al. risk has two components, systematic and unsystematic. Where market forces affect all assets simultaneously in some systematic manner it generates systematic risk or what so called undiversifiable risk. Examples are Bull markets, Bear markets, wars, changes in the level of inflation. The other component of risk is unsystematic one, or so called diversifiable risk. These are idiosyncratic events that are statistically independent from the more wide spread forces that generate undiversifiable or unsystematic risk are Acts of God (Hurricane or flood), inventions, management error, lawsuits and good or bad news affecting one firm.

As defined above, total risk of a portfolio is the result of summation of systematic and unsystematic risk. On average, the total risk of a diversified portfolio tends to diminish as more randomly selected common stocks are added to the portfolio. But, when more than about three dozen random stocks are combined, it is impossible to reduce a randomly selected portfolio's risk below the level of undiversifiable risk that exists in the market, Figure 3 shows the graphical interpretation of this.

The straight line separates the systematic risk from unsystematic one, the systematic or undiversifiable risk lies under the straight line.

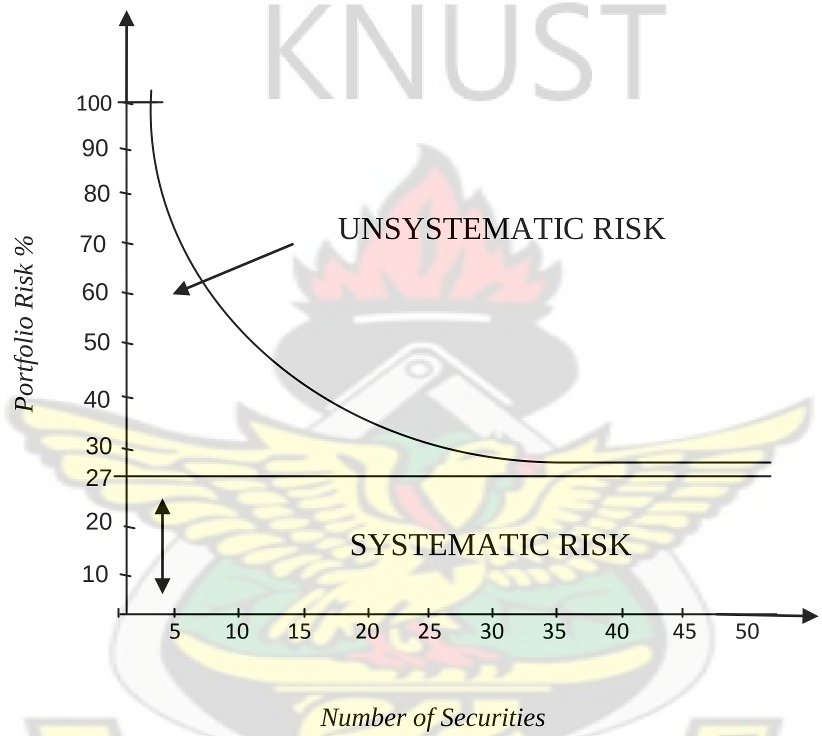


Figure 5 – The effect of number of securities on risk of the portfolio in Ghana.

3.5.1 The Markowitz efficient frontier

Every possible asset combination can be plotted in a risk-return space, and the collection of all such possible portfolios defines a region in this space. The line along the upper edge of this region is known as the efficient frontier. Combinations along this line represent portfolio (explicitly excluding the risk- free alternative) for which there is a lowest risk for a given level of return, or conversely, for a given amount of risk, the portfolio lying on the efficient frontier represents the combination offering the best possible return. Mathematically, the efficient frontier is the intersection of the set of portfolios with minimum variance and the set of portfolios with maximum return.

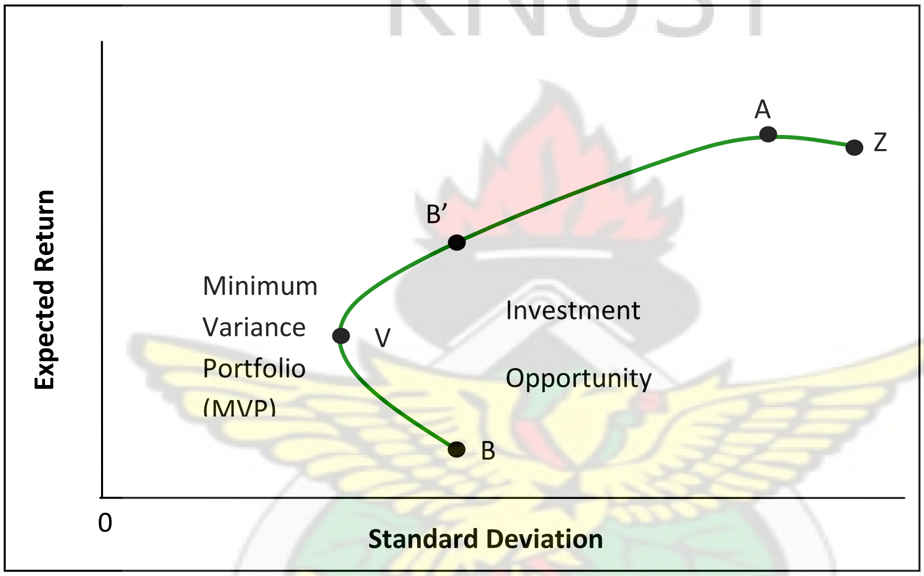


Figure 6 - Investment opportunity set for asset A and asset B

Figure 6 above shows the entire investment opportunity set, which is the set of all attainable combinations of risk and return offered by asset A and B in different proportions. Investors desire portfolios that lie to the northwest in fig 6. These are portfolios with high return and low volatility. The area within the curve BVAZ is the feasible opportunity set representing all possible portfolio combinations. Portfolios that lie below the minimum-variance portfolio (point V) on the curve can therefore be rejected as being inefficient. The portfolios that lie on the frontier VA would not be likely candidates for investor to hold, since the portfolios do not meet the criteria of maximizing expected return for a given level of risk, or minimizing risk for a given level of return. This is easily seen by comparing the portfolio represented by points B and

Standard Deviation

Figure 7 - The minimum variance portfolio. (The efficient frontier)

3.5.2 The Tangency portfolio

Suppose an investor has other preferences than taking the least possible amount of risk (thus investing in minimum variance portfolio), but rather investing in the portfolio with maximum Sharpe ratio. The Sharpe ratio is defined as the return-risk ratio, thus:

Equation (4.3) represents the expected return per unit of risk, so the portfolio with maximum Sharpe ratio gives the highest expected return per unit of risk, and is thus the most risk-efficient portfolio,

Graphically, the portfolio with maximum Sharpe ratio is the point where a line from the risk- free rate is tangent to the efficient frontier in mean standard deviation space, because this point has the property that it has the highest possible mean-variance (standard deviation) ratio. That is why it is called the tangency portfolio.

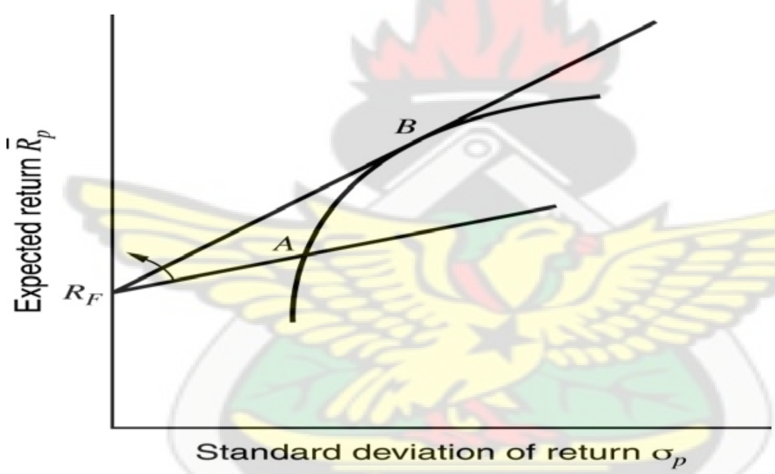


Figure 8 - The tangency portfolio

3.6.1 THE SHARPE RATIO

This ratio is a measurement for risk – adjusted returns and was developed by William F. Sharpe. This is where the name Sharpe ratio comes from. The Sharpe ratio is defined by;

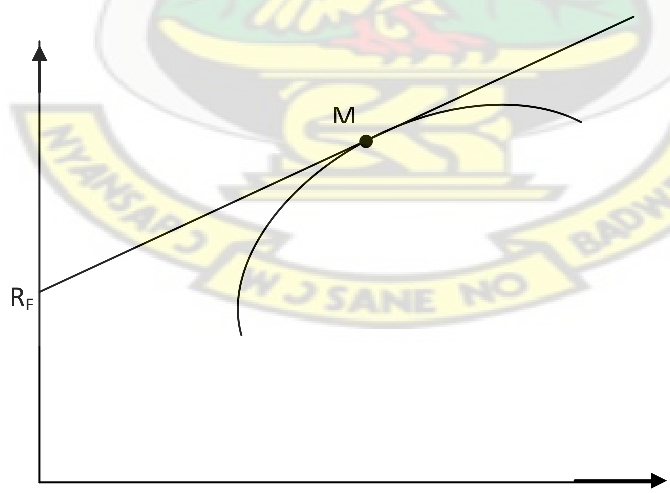
performance, considering both return and risk simultaneously, is the Sharpe index of portfolio performance. It is defined by

SOLUTION

In this figure we can see that the efficient frontier will be convex. The explanation is that there is a risk and return characteristics of the portfolio that will change in a non-linear fashion as the weighting of the component assets change.

The next step is finding the optimal market portfolio by connecting some chosen risk – free asset to the frontier, and then applying the Sharpe ratio which should be maximized. These two properties will give you two points on the graph, which you then make a straight line from. This line represents the lending part of a possible investment on the left side of the market portfolio. If you draw the line straight to the right also, you will be able to borrow and invest more in the market portfolio. This line that is connected to the efficient frontier is called the capital allocation line (CAL).

The following figure shows a graphical view of what was just described. Here the E(R) stands for return,



3.7.2 Description of Standard Deviation in Portfolio Theory

In portfolio theory the standard deviation measures how much the return of a portfolio or the stock moves around the average return. The standard deviation grows as returns move further above or below the average. This is considered as a measure of risk, where most investors only care about the standard deviation of a stock in one direction, above or below the mean. For investors who are long stocks do not want returns to dip below mean, but would be happy with returns that exceed it. If the returns on a portfolio or stock are normally distributed, then the standard deviation is a valid measure of the returns that are below the mean. If returns are not normal but skewed, then the standard deviation is less meaningful.

When it comes to calculation variance – covariance matrix for a large sample of different categories, in our case different equality returns, there exists a fast and simple way in excel to do this. This you do by accessing "Tools" in the excel work sheet, and then choose " Data Analysis", if it not enabled, you need to go to the "Tools" > Add – Ins" and add it. In the "Data Analysis" screen, you pick "covariance" to generate a variance covariance matrix.

3.7.3 Using Solver to optimize efficient points

When focusing on the efficient sets of portfolios, we want to find some split across the asset that achieves the target return by minimizing the variance of return. This problem is a standard optimization problem which excel's solver can solve. It contains a range of iterative search methods or optimization. Then for this case of the portfolio variance which is a quadratic function of the weights, and for we will be using solver for quadratic programming.

The solver requires changing cells, a target cell for minimization and the specification of constraints, which acts as which acts as restrictions on feasible values for the changing cells. The target cell to be minimized is the standard deviation of return, for the portfolio. Also that the changing cells should be the cells containing the weights.

The steps in using solver are:

1. Excel sover by choosing
   Tools > options > solver

2. Specify in the solver Parameter Dialog Box
   - The target cell to be optimized
   - Specify max or min
   - Choose changing cells

3. Choose Add to specify the constraints then OK. This constraint ensures that it must meet the target cell selected.

4. Click on options and ensure that Assume Linear Model is not checked.

5. Solver and get the results in the Spreadsheet.

The following figure shows how the solver screen looks like.

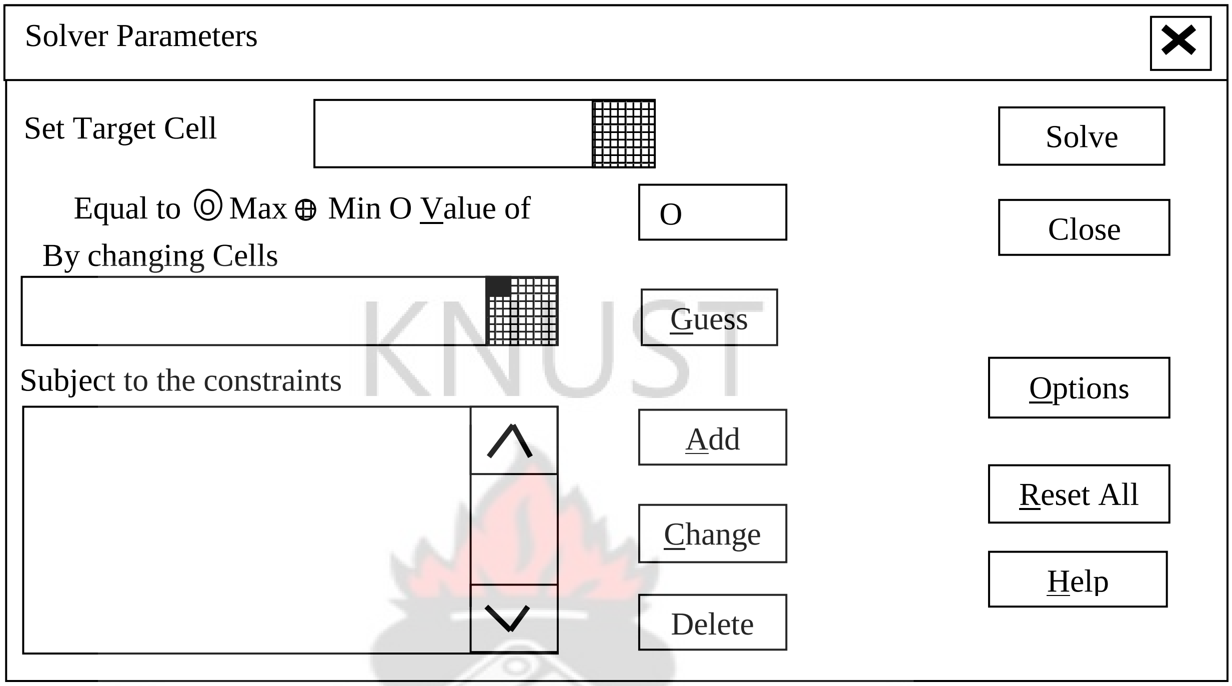


Figure 10 – Excel solver screen

Further excel implementations

For the other parts of the portfolio modeling implementation, we choose to edited formulae directly into the cells. E.g. when we implemented the CAL we applied the formula in the cell, since it is linear model, there will be o complications for excel to compute it. This is similar for the computations of the stock returns from prices, calculated by taking today's price minus yesterday's price divided by yesterday's price.

When the risk – free assets is added in to the model we will start working with the CAL. At this point one will start combining the efficient frontier and the risk – free asset. To do so one uses the Sharpe ratio by maximizing it. To maximize the Sharpe ratio we use again the solver in Excel. The different here and the previous, is that now set the target cell to be the Sharpe ratio, by changing the cells "weights". This will give you the optimal weighted portfolio reachable on the efficient frontier. Since the Sharpe ratio is the slope of the tangent portfolio, we can then draw a line from the risk – free asset and through the tangent portfolio on the efficient frontier. We do this by writing a linear equation for these combinations. Also by using Solver, which is connected to the risk and return of the portfolio, will give you the best possible return and the lowest risk for the market portfolio.

3.8.1 DIVERSIFICATION IN MARKOWITZ MODEL

Markowitz model suggests that it is possible to reduce the level of risk below the undiversifiable risk. Ibbotson categorized Markowitz diversification on five basic interrelated concepts:

1. The weights sum to one: The first concept requires that weights of the assets in the portfolio sum to 100%. Simply the investment weights are a decision variable, which is the main task for portfolio manager to determine them.

Where

securities whose rates of return have low enough correlation, according to Markowitz, he or she can reduce a portfolio's risk below the undiversifiable level.

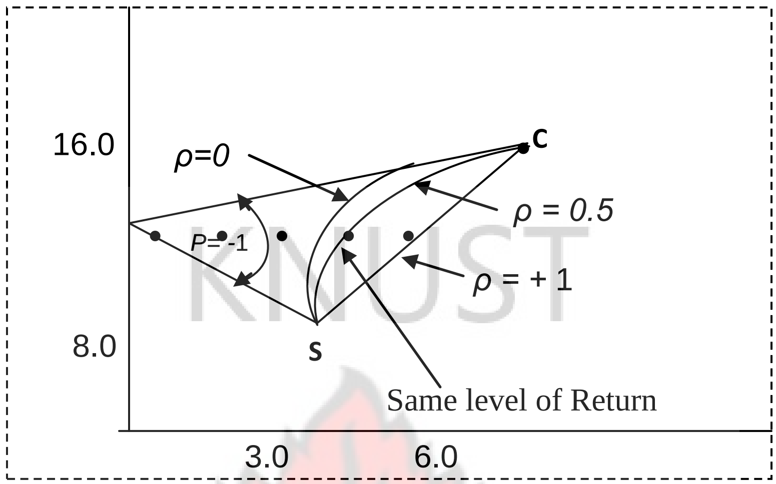


Figure 11 – Relationship between expected return and standard deviation of return for various correlation coefficients . Source: (Gruber et al).

5. The Capital Allocation Line: The last concept to consider on diversification by Markowitz is the Capital Allocation Line. This concept discusses the possibility of lending and borrowing at a risk free rate of interest provided by Markowitz model.   An example can be a government treasury bill, whereas the phrase risk free interest rate suggests the variance is zero.

   Markowitz model gives the opportunity risky asset manager to combine a risky asset or a set of risky assets (a portfolio of risky assets) with a riskless asset. In next parts this concept will be more clarified when we explain all concepts in MPT one by one.

3.8.2 The Risk Free Asset:

This asset is said to be a hypothetical asset which pays a risk – free return to the investor, with a variance and standard deviation equal to zero. Usually this type of assets is issued by the government and can be referred to as government bond or Treasury bill (T-bill). But then it is also assumed that government does not go bankrupt. In reality we can also conclude that there is no such thing as a risk-free asset, all financial instruments carry some degree of risk. But also that these risk free – rates are subject to inflation risk. The common notation of the risk – free asset is R_F.

3.8.3 MEASURING INVESTMENT RISK

We often think of risk as something bad happening to us. Investment risk is most probable understood when it is expressed in statistical terms that consider the entire range of an investment's possible returns. Moreover risk often depends on the number of individual assets held. In general, you can reduce risk by holding a reasonably diversified portfolio of individual assets. To understand how portfolio reduces risk, we must begin by looking at the risk of individual assets. You can measure an investment's risk with statistical measures called variance and standard deviation. These measures give us an indication of how likely it is that the actual return next year will deviate from the mean (or average) return expected. Suppose that the mean return on securities A and B for one year is 25%. However, suppose that during the year security A's returns ranged between -50% and 50% while security B's returns ranged between 10% and 35%. It is obvious that security A's returns are more widely dispersed around the average than security B. In statistical terms security A will have a higher variance and standard deviation than security B. The standard deviation is the square root of the variance. The formulae are:

S^2

Table 3.4 shows returns in the U.S.A for the period 1982 to 2006 and for the Ghana Stock Exchange during 1991-2006.

Using the standard deviation as a measure of risk, we see that from the U.S experience, shares that have exhibited the most risk are small company shares followed by the average company share. Long term bonds are much less riskier than shares and short term instruments such as Treasury Bills exhibit the lowest level of risk.

The standard deviation per year for the GSE for the period is 55.3%. When we compare returns and risk, it is not so obvious that the GSE is riskier than the average share in the United States. GSE shares have higher risk but also have higher average returns. A statistic called the coefficient of variation (CV) helps us to relate risk to return.

3.8.4 THE COEFFICINT OF VARIATION

The coefficient of variation (CV) is the amount of risk (standard deviation) per unit of return. The coefficient of variation is therefore the measure of relative variability that combines risk and return and is defined as follows:

CV =

3.8.6 MEASURING PORTFOLIO RISK

When individual securities are combined into a portfolio, there are two sources of variation in the returns on the portfolio. First each security has its own variance. Secondly, there is an additional variation introduced by the degree to which the securities vary among themselves. For example, suppose we combine the shares of UNIL and FML into a portfolio. The first element of variation is introduced by the fact that UNIL and FML have their individual variances. However, suppose that FML shares increase (decrease) in price whenever UNIL shares increase (decrease) in price. Then there is an additional source of volatility introduced by the co – movement of the two securities. The statistical measure of the co- movement between two securities is called the covariance. The covariance among two securities A and B or Cov (A, B) is:

Cov (

Year	r_1	r_2
1	0.05	0.20
2	-0.02	0.10
3	0.15	-0.10
4	0.18	0.05
5	0.20	0.10
Mean	0.11200	0.07000
Variance(S^2)	0.00952	0.01058
Standard deviation(S)	0.09760	0.10286

Year	r_1	r_2	(r_1
1	-0.062	0.13	-0.00806
2	-0.132	0.03	-0.00396
3	0.038	-0.17	-0.00646
4	0.068	-0.02	-0.00136
5	0.088	-0.02	0.0026
Total			-0.0172
Covariance			-0.0043

Table3.6 - Calculation of Mean, Variance and Covariance.

Source: Utility Market and Investment (Sam Mensah, 2008)

3.9.0 THE CORRELATION COEFFICIENT

The covariance measure indicates whether the returns on two securities have a positive, negative or zero association. However, the magnitude of the covariance measure is difficult to interpret. In order to be able to compare degrees of association between two variables, a standardized measure called the correlation coefficient or simply correlation is used. The correlation between two variables is usually depicted with the Greek letter ρ ("rho"). Thus, the correlation between the returns on security 1 and 2 is:

The risk of the two – security portfolio as measured by the standard deviation is much lower than the risk of the individual securities 1 and 2. Notice that the covariance between the two securities was negative thus, causing a reduction in the overall risk of the portfolio. The variance of a portfolio can also be expressed in terms of the correlation coefficient.

Since

3.9.2 **Zero correlation** ($\rho = 0$)

Zero correlation means that there is no correlation between the securities.

3.9.3 **Perfectly negative Correlation** ($\rho = -1$)

When $\rho = -1$, the expression for the variance of a portfolio takes its lowest value. Intuitively, this means that the gains and losses of the securities cancel out, keeping the value of the portfolio stable.

It is rare that correlations between security returns are perfectly positive or negative. Most security returns are positively correlated. However, whenever the correlation is less than perfectly positive, risk reduction can be obtained through diversification.

CHAPTER FOUR

The data on share prices for financial index were collected from the Ghana Stock Exchange on the financial companies for a four year period from 2007 – 2010. These are monthly data from January, 2007 – December, 2010.

		STOCK PRICES FOR FINANCIAL INDEX					2007 - 2010
		1	2	3	4	5	6
	DATE	CAL	EBG	EIC	ETI	GCB	HFC
2007	Jan	0.2205	1.3561	0.8824	2.27	0.62	0.54
	Feb	0.2119	1.3634	0.8865	1.135	0.612	0.54
	Mar	0.2331	1.41	0.8903	1.135	0.67	0.54
	Apr	0.25	1.417	0.9407	1.135	0.6784	0.54
	May	0.262	1.432	0.9503	1.16	0.6806	0.54
	Jun	0.272	1.4415	0.9834	1.16	0.6808	0.54
	Jul	0.3203	1.4817	1.081	1.165	0.687	0.5
	Aug	0.34	1.5156	1.2002	1.165	0.9502	0.54
	Sep	0.3425	1.61	1.2002	1.165	0.95	0.54
	Oct	0.3515	1.6508	1.2003	1.165	0.9601	0.5398
	Nov	0.4152	1.6804	1.2025	1.2815	0.971	0.5398
	Dec	0.442	2	1.3	1.2815	0.995	0.5398
2008	Jan	0.506	2.175	1.44	1.2815	1.0056	0.54
	Feb	0.7	2.47	1.69	1.285	1.16	0.54
	Mar	0.700	2.630	1.800	1.6	1.31	0.540
	Apr	0.62	3	1.87	2.2	1.38	0.61
	May	0.66	3.33	2.03	2.32	1.38	0.62
	Jun	0.7	3.95	2.34	0.47	1.35	0.62
	Jul	0.7	4.33	2.44	0.47	1.35	0.62
	Aug	0.7	4.8	3.01	0.45	1.3	0.62
	Sep	0.69	4.8	3.3	0.45	1.3	0.62
	Oct	0.67	4.8	3.28	0.45	1.29	0.62
	Nov	0.6	4.7	3.14	0.45	1.14	0.62
	Dec	0.6	4.5	3.14	0.45	1.1	0.62
2009	Jan	0.6	4.19	3	0.45	0.98	0.62
	Feb	0.45	3.8	3	0.45	0.62	0.62
	Mar	0.34	3.48	2.66	0.42	0.45	0.62
	Apr	0.3	2.3	2.66	0.41	0.6	0.62
	May	0.26	2	2.66	0.32	0.48	0.62
	Jun	0.3	2.06	2.63	0.16	0.54	0.62
	Jul	0.22	2.11	2.63	0.14	0.75	0.62

	Aug	0.25	3.01	2.5	0.17	0.8	0.62
	Sep	0.24	3.02	2.3	0.2	0.8	0.62
	Oct	0.22	2.8	2.29	0.13	0.9	0.62
	Nov	0.2	2.9	2.2	0.13	0.82	0.62
	Dec	0.2	2.8	2.2	0.15	0.74	0.62
2010	Jan	0.2	2.850	2.200	0.150	0.73	0.62
	Feb	0.17	3.160	2.170	0.140	0.8	0.62
	Mar	0.19	3.850	2.020	0.150	0.95	0.55
	Apr	0.23	3.570	1.640	0.170	1.13	0.6
	May	0.28	3.340	1.400	0.190	1.78	0.62
	Jun	0.27	3.340	1.690	0.150	1.62	0.59
	Jul	0.26	2.950	1.190	0.150	1.56	0.56
	Aug	0.3	3.200	1.700	0.150	1.9	0.52
	Sep	0.29	3.350	1.6100	0.1400	2	0.49
	Oct	0.28	3.050	1.8500	0.1500	1.9	0.47
	Nov	0.3	3.00	0.58	0.150	2.3	0.47
	Dec	0.31	3.000	0.500	0.150	2.7	0.44
	Share code	CAL	EBG	EIC	ETI	GCB	HFC

7	8	9	10	11		All-Share
SCB	SIC	SG-SSB	TBL	UT	SCBPREF	Index
16.00	0	0.6	1.325	0	0.520	5,012.16
16.601	0	0.6115	1.325	0	0.520	5,044.79
16.8016	0	0.6321	1.325	0	0.520	5,092.25
18.0007	0	0.6356	1.325	0	0.520	5,139.50
18.065	0	0.7151	1.325	0	0.520	5,224.47
18.0616	0	0.7566	1.325	0	0.520	5,294.58
18.08	0	0.901	1.325	0	0.500	5,341.76
20	0	0.9321	1.325	0	0.520	5,557.36
21	0	1.0503	1.325	0	0.520	5,676.77
24.1	0	1.091	1.325	0	0.500	5,839.62
24.1256	0	1.1108	1.325	0	0.520	6,387.16
26	0	1.25	1.325	0	0.520	6,599.77
26.601	0.33	1.305	1.325	0	0.520	6,718.88
26.65	0.42	1.35	1.33	0	0.520	7,005.29
26.770	0.39	1.49	1.33	0	0.520	
26.8	0.42	1.48	1.33	0	0.520	9,349.59

27.14	0.54	1.35	1.33	0	0.520	9,815.22
30.83	0.63	1.35	1.33	0	0.520	10,346.30
36.12	0.63	1.35	1.33	0	0.520	10,658.72
38	0.63	1.35	1.33	0	0.520	10,790.95
38	0.58	1.35	1.33	0	0.520	10,890.80
38	0.56	1.35	1.33	0	0.520	10,781.02
38	0.54	1.35	1.33	0.33	0.520	10,573.43
38	0.5	1.35	1.33	0.33	0.520	10,431.64
38	0.46	1.15	1.33	0.3	0.520	
38	0.42	1.14	1.33	0.29	0.520	9,836.84
38	0.35	1.14	1.33	0.29	0.520	9,247.17
32.3	0.3	0.56	1.33	0.2	0.520	8,822.91
29	0.31	0.54	1.33	0.2	0.520	7,496.02
29	0.26	0.46	1.33	0.2	0.520	5,423.98
29	0.22	0.4	1.33	0.18	0.520	5,230.49
29	0.23	0.46	1.33	0.17	0.520	5,900.41
29	0.3	0.56	1.33	0.18	0.520	6,291.94
29.25	0.3	0.45	1.33	0.18	0.520	5,378.72
30	0.3	0.45	1.33	0.2	0.520	5,378.72
30	0.27	0.45	1.33	0.21	0.520	5,572.34
30.2	0.3	0.45	1.33	0.23	0.520	
31	0.3	0.45	1.33	0.24	0.520	
32.75	0.27	0.45	1.33	0.25	0.520	
39.71	0.34	0.51	1.33	0.21	0.520	
41.5	0.32	0.67	1.33	0.29	0.520	
42.9	0.33	0.62	1.33	0.29	0.520	
43	0.39	0.55	1.33	0.23	0.520	
43.11	0.38	0.7	1.33	0.27	0.520	
44	0.38	0.65	1.33	0.25	0.520	
44.41	0.38	0.64	1.33	0.3	0.520	
45.13	0.42	0.6	1.33	0.3	0.520	
45.16	0.43	0.64	1.33	0.3	0.520	
SCB	SIC	SG-SSB	TBL	UT	SCB PREF	

DIVIDEND PER SHARE

		1	2	3	4	5
	DATE	CAL	EBG	EIC	ETI	GCB
2007	Jan	0.00550	0.06740	0.01000	0.02760	0.04000
	Feb	0.00750	0.06740	0.01000	0.02760	0.04000
	Mar	0.00750	0.06740	0.01500	0.02760	0.05500
	Apr	0.00750	0.06740	0.01500	0.02760	0.05500
	May	0.00750	0.06740	0.01500	0.02760	0.05500
	Jun	0.00750	0.06740	0.01500	0.02760	0.05500
	Jul	0.00750	0.06740	0.01500	0.02760	0.05500
	Aug	0.00750	0.06740	0.01500	0.02760	0.05500
	Sep	0.00750	0.06740	0.01500	0.02760	0.05500
	Oct	0.00750	0.06740	0.01500	0.02760	0.05500
	Nov	0.00750	0.06740	0.01500	0.02760	0.05500
	Dec	0.00750	0.06740	0.01500	0.02760	0.05500
2008	Jan	0.00750	0.06740	0.01500	0.02760	0.05500
	Feb	0.00750	0.06740	0.01500	0.02760	0.05500
	Mar	0.00750	0.06740	0.01500	0.02760	0.05500
	Apr	0.00750	0.06740	0.01500	0.02760	0.05500
	May	0.00750	0.06740	0.01500	0.02760	0.05500
	Jun	0.00750	0.06740	0.01500	0.02760	0.05500
	Jul	0.00750	0.06740	0.01500	0.02760	0.05500
	Aug	0.00750	0.06740	0.01500	0.02760	0.05500
	Sep	0.00750	0.06740	0.01500	0.02760	0.05500
	Oct	0.00750	0.06740	0.01500	0.02760	0.05500
	Nov	0.00750	0.06740	0.01500	0.02760	0.05500
	Dec	0.00750	0.06740	0.01500	0.02760	0.05500
2009	Jan	0.00750	0.06740	0.01500	0.02760	0.05500
	Feb	0.00750	0.06740	0.01500	0.02760	0.05500
	Mar	0.01450	0.16480	0.01500	0.02760	0.05500
	Apr	0.01450	0.16480	0.01500	0.02760	0.05500
	May	0.01450	0.16480	0.01500	0.02760	0.05500
	Jun	0.01450	0.16480	0.01500	0.02760	0.05500
	Jul	0.01450	0.16480	0.01500	0.02760	0.05500
	Aug	0.01450	0.16480	0.01500	0.02760	0.05500
	Sep	0.01450	0.16480	0.01500	0.02760	0.05500
	Oct	0.01450	0.16480	0.01500	0.02760	0.05500
	Nov	0.01450	0.16480	0.01500	0.02760	0.05500
	Dec	0.01450	0.16480	0.01500	0.02760	0.05500

2010	Jan	0.014500	0.164800	0.015000	0.027600	0.055000
	Feb	0.000000	0.164800	0.015000	0.027600	0.055000
	Mar	0.012000	0.000000	0.000000	0.000000	0.035600
	Apr	0.012000	0.000000	0.000000	0.000000	0.035600
	May	0.012000	0.000000	0.025000	0.000000	0.035600
	Jun	0.012000	0.000000	0.025000	0.004200	0.035600
	Jul	0.012000	0.000000	0.025000	0.004200	0.035600
	Aug	0.012000	0.000000	0.025000	0.004200	0.035600
	Sep	0.012000	0.000000	0.025000	0.004200	0.035600
	Oct	0.012000	0.000000	0.025000	0.004200	0.035600
	Nov	0.012000	0.000000	0.025000	0.004200	0.035600
	Dec	0.012000	0.000000	0.025000	0.004232	0.035600
	Share code	CAL	EBG	EIC	ETI	GCB

6	7	8	9	10	11	12
HFC	SCB	SIC	SG-SSB	TBL	UT	SCBPRE
0.00550	1.15000	0	0.04500	0.06410	0	0.08740
0.00550	1.30000	0	0.04500	0.06410	0	0.08740
0.00550	1.30000	0	0.04500	0.06410	0	0.08740
0.00550	1.30000	0	0.04500	0.06410	0	0.08740
0.00550	1.30000	0	0.04500	0.03420	0	0.08740
0.00550	1.30000	0	0.04500	0.03420	0	0.08740
0.00550	1.30000	0	0.04500	0.03420	0	0.08740
0.00550	1.30000	0	0.04500	0.03420	0	0.08740
0.00550	1.30000	0	0.04500	0.03420	0	0.08740
0.00550	1.30000	0	0.04500	0.03420	0	0.08740
0.00550	1.30000	0	0.04500	0.03420	0	0.08740
0.00550	1.30000	0	0.04500	0.03420	0	0.08740
0.00550	1.30000	0	0.04500	0.03420	0	0.08740
0.00550	1.30000	0	0.04500	0.03420	0	0.08740
0.00550	1.30000	0	0.04500	0.03420	0	0.08740
0.00550	1.45000	0.01140	0.04500	0.03420	0	0.08740
0.00550	1.45000	0.01140	0.04500	0.03420	0	0.08740
0.00550	1.45000	0.01140	0.04500	0.03420	0	0.08740
0.01000	1.45000	0.01140	0.04500	0.03420	0	0.08740
0.01000	1.45000	0.01140	0.04500	0.03420	0	0.08740
0.01000	1.45000	0.01140	0.04500	0.03420	0	0.08740
0.01000	1.45000	0.01140	0.04500	0.03420	0	0.08740

0.01000	1.45000	0.01140	0.04500	0.03420	0	0.08740
0.01000	1.45000	0.01140	0.04500	0.03420	0	0.08740
0.01000	1.45000	0.01140	0.04500	0.03420	0	0.08740
0.01000	1.45000	0.01140	0.04500	0.03420	0	0.08740
0.01000	1.50000	0.01140	0.04500	0.03420	0.01010	0.07510
0.01000	1.50000	0.01770	0.04500	0.03420	0.01010	0.07510
0.01000	1.50000	0.01770	0.04500	0.03420	0.01010	0.07510
0.01000	1.50000	0.01770	0.04500	0.03420	0.01010	0.07510
0.01000	1.50000	0.01770	0.04500	0.03420	0.01010	0.07510
0.01000	1.50000	0.01770	0.04500	0.03420	0.01010	0.07510
0.01000	1.50000	0.01770	0.04500	0.03420	0.01010	0.07510
0.01000	1.50000	0.01770	0.04500	0.03420	0.01010	0.07510
0.01000	1.50000	0.01770	0.04500	0.03420	0.01010	0.07510
0.01000	1.50000	0.01770	0.04500	0.03420	0.01010	0.07510
0.010000	1.500000	0.017700	0.045000	0.034200	0.010100	0.075100
0.010000	1.500000	0.017700	0.040000	0.034200	0.010100	0.075100
0.015000	0.000000	0.000000	0.040000	0.000000	0.010700	0.082200
0.015000	2.470000	0.000000	0.040000	0.236000	0.010700	0.082200
0.015000	2.470000	0.000000	0.040000	0.023100	0.010700	0.082200
0.015000	2.470000	0.000000	0.040000	0.023100	0.010700	0.082200
0.015000	2.470000	0.000000	0.040000	0.023100	0.010700	0.082200
0.015000	2.470000	0.000000	0.040000	0.023100	0.010700	0.082200
0.015000	2.470000	0.017700	0.040000	0.023100	0.010700	0.048200
0.015000	2.470000	0.017700	0.040000	0.023100	0.010700	0.048200
0.015000	2.470000	0.017700	0.040000	0.023100	0.010700	0.048200
0.015000	2.470000	0.017700	0.040000	0.023100	0.010700	0.048200
HFC	SCB	SIC	SG-SSB	TBL	UT	SCBPRE

RETURNS (Mean)

	DATE	CAL	EBG	EIC	ETI	GCB	HFC
2007	Jan						
	Feb	-0.01406	0.055084	0.015979	-0.48784	0.051613	0.010185
	Mar	0.135441	0.083614	0.015567	0.024317	0.160131	0.010185
	Apr	0.104676	0.052766	0.073458	0.024317	0.094627	0.010185
	May	0.078	0.058151	0.026151	0.046344	0.084316	0.010185
	Jun	0.066794	0.053701	0.050616	0.023793	0.081105	0.010185
	Jul	0.205147	0.074644	0.114501	0.028103	0.089894	-0.06389
	Aug	0.08492	0.068367	0.124144	0.023691	0.463173	0.091
	Sep	0.029412	0.106756	0.012498	0.023691	0.057672	0.010185
	Oct	0.048175	0.067205	0.012581	0.023691	0.068526	0.009815
	Nov	0.20256	0.058759	0.01433	0.123691	0.068639	0.010189
	Dec	0.082611	0.230302	0.093555	0.021537	0.081359	0.010189

2008	Jan	0.161765	0.1212	0.119231	0.021537	0.06593	0.010559
	Feb	0.398221	0.166621	0.184028	0.024268	0.208234	0.010185
	Mar	0.010714	0.092065	0.073964	0.266615	0.17674	0.010185
	Apr	-0.10357	0.166312	0.047222	0.39225	0.09542	0.139815
	May	0.076613	0.132467	0.093583	0.067091	0.039855	0.02541
	Jun	0.07197	0.206426	0.160099	-0.78552	0.018116	0.008871
	Jul	0.010714	0.113266	0.049145	0.058723	0.040741	0.008871
	Aug	0.010714	0.124111	0.239754	0.01617	0.003704	0.016129
	Sep	-0.00357	0.014042	0.101329	0.061333	0.042308	0.016129
	Oct	-0.01812	0.014042	-0.00152	0.061333	0.034615	0.016129
	Nov	-0.09328	-0.00679	-0.03811	0.061333	-0.07364	0.016129
	Dec	0.0125	-0.02821	0.004777	0.061333	0.013158	0.016129
2009	Jan	0.0125	-0.05391	-0.03981	0.061333	-0.05909	0.016129
	Feb	-0.2375	-0.07699	0.005	0.061333	-0.31122	0.016129
	Mar	-0.22778	-0.06647	-0.10833	-0.00533	-0.18548	0.016129
	Apr	-0.075	-0.29172	0.005639	0.041905	0.455556	0.016129
	May	-0.085	-0.05878	0.005639	-0.1522	-0.10833	0.016129
	Jun	0.209615	0.1124	-0.00564	-0.41375	0.239583	0.016129
	Jul	-0.21833	0.104272	0.005703	0.0475	0.490741	0.016129
	Aug	0.202273	0.504645	-0.04373	0.411429	0.14	0.016129
	Sep	0.018	0.058073	-0.074	0.338824	0.06875	0.016129
	Oct	-0.02292	-0.01828	0.002174	-0.212	0.19375	0.016129
	Nov	-0.025	0.094571	-0.03275	0.212308	-0.02778	0.016129
	Dec	0.0725	0.022345	0.006818	0.366154	-0.03049	0.016129
2010	Jan	0.0725	0.076714	0.006818	0.184	0.060811	0.016129
	Feb	-0.0775	0.166596	-0.00682	0.117333	0.171233	0.016129
	Mar	0.117647	0.270506	-0.06221	0.268571	0.25625	-0.09677
	Apr	0.273684	-0.07273	-0.18812	0.133333	0.226947	0.118182
	May	0.269565	-0.06443	-0.14634	0.117647	0.606726	0.058333
	Jun	0.007143	0	0.225	-0.21053	-0.06989	-0.02419
	Jul	0.007407	-0.11677	-0.28107	0.028	-0.01506	-0.02542
	Aug	0.2	0.084746	0.44958	0.028	0.240769	-0.04464
	Sep	0.006667	0.046875	-0.03824	-0.03867	0.071368	-0.02885
	Oct	0.006897	-0.08955	0.164596	0.101429	-0.0322	-0.0102
	Nov	0.114286	-0.01639	-0.67297	0.028	0.229263	0.031915
	Dec	0.073333	0	-0.09483	0.028	0.189391	-0.03191
	Share code	CAL	EBG	EIC	ETI	GCB	HFC
MEAN		**0.047943**	**0.056183**	**0.014234**	**0.03669**	**0.101443**	**0.012451**
ST.DEVIATION		0.126858	0.119988	0.154931	0.207879	0.168189	0.03746
VARIANCE		0.016093	0.014397	0.024004	0.043214	0.028288	0.001403

SCB	SIC	SG-SSB	TBL	UT	SCBPRE
0.109438	0	0.094167	0.048377	0	0.168077
0.090392	0	0.107277	0.048377	0	0.168077
0.148742	0	0.076728	0.048377	0	0.168077
0.075791	0	0.195878	0.048377	0	0.168077
0.071774	0	0.120962	0.025811	0	0.168077
0.072995	0	0.25033	0.025811	0	0.129615
0.178097	0	0.084462	0.025811	0	0.2148
0.115	0	0.175089	0.025811	0	0.168077
0.209524	0	0.081596	0.025811	0	0.129615
0.055004	0	0.059395	0.025811	0	0.2148
0.131578	0	0.165826	0.025811	0	0.168077
0.073115	0	0.08	0.025811	0	0.168077
0.050712	0	0.068966	0.029585	0	0.168077
0.053283	0	0.137037	0.025714	0	0.168077
0.049682	0	0.02349	0.025714	0	0.168077
0.066791	0.312857	-0.05743	0.025714	0	0.168077
0.189388	0.187778	0.033333	0.025714	0	0.168077
0.218618	0.018095	0.033333	0.025714	0	0.168077
0.092193	0.018095	0.033333	0.025714	0	0.168077
0.038158	-0.06127	0.033333	0.025714	0	0.168077
0.038158	-0.01483	0.033333	0.025714	0	0.168077
0.038158	-0.01536	0.033333	0.025714	0	0.168077
0.038158	-0.05296	0.033333	0.025714	0	0.168077
0.038158	-0.0572	-0.11481	0.025714	0	0.168077
0.038158	-0.06217	0.030435	0.025714	0	0.168077
0.038158	-0.13952	0.039474	0.025714	0	0.168077
-0.11053	-0.11029	-0.4693	0.025714	-0.27552	0.144423
-0.05573	0.092333	0.044643	0.025714	0.0505	0.144423
0.051724	-0.10419	-0.06481	0.025714	0.0505	0.144423
0.051724	-0.08577	-0.03261	0.025714	-0.0495	0.144423
0.051724	0.125909	0.2625	0.025714	0.000556	0.144423
0.051724	0.381304	0.315217	0.025714	0.118235	0.144423
0.060345	0.059	-0.11607	0.025714	0.056111	0.144423
0.076923	0.059	0.10000	0.025714	0.167222	0.144423
0.05	-0.041	0.10000	0.025714	0.1005	0.144423
0.056667	0.176667	0.10000	0.025714	0.143333	0.144423
0.076159	0.059	0.10000	0.025714	0.087391	0.144423
0.104839	-0.041	0.088889	0.025714	0.08375	0.144423
0.212519	0.259259	0.222222	0	-0.1172	0.158077
0.107278	-0.05882	0.392157	0.177444	0.431905	0.158077
0.093253	0.03125	-0.01493	0.017368	0.036897	0.158077

0.059907	0.181818	-0.04839	0.017368	-0.17	0.158077
0.06	-0.02564	0.345455	0.017368	0.220435	0.158077
0.07794	0	-0.01429	0.017368	-0.03444	0.158077
0.065455	0.046579	0.046154	0.017368	0.2428	0.092692
0.071831	0.151842	-5.4E-17	0.017368	0.035667	0.092692
0.055396	0.065952	0.133333	0.017368	0.035667	0.092692
SCB	SIC	SG-SSB	TBL	UT	SCBPRE
0.076348	**0.028866**	**0.071114**	**0.02918**	**0.025847**	**0.156301**
0.059523	0.105125	0.135074	0.023625	0.102603	0.023391
0.003543	0.011051	0.018245	0.000558	0.010527	0.000547

Shares code	Mean	Variance
CAL	0.047943	0.016093
EBG	0.056183	0.014397
EIC	0.014234	0.024004
ETI	0.03669	0.043214
GCB	0.101443	0.028288
HFC	0.012451	0.001403
SCB	0.076348	0.003543
SIC	0.028866	0.011051
SG-SSB	0.071114	0.018245
TBL	0.02918	0.000558
UT	0.025847	0.010527
SCBPRE	0.156301	0.000547

Table 4.1 Mean and variance of shares

Shares	Sharpe Ratio
CAL	-0.29211
EBG	-0.24016
EIC	-0.45676
ETI	-0.23239
GCB	0.097763
HFC	-1.93672
SCB	-0.14535
SIC	-0.53397
SG – SSB	-0.1028
TBL	-2.3627
UT	-0.57652
SCBPRE	3.048152

Table 4.2 - The Sharpe ratio of the shares

Shares	Coefficient of Variation
CAL	2.645996
EBG	2.135661
EIC	10.88444
ETI	5.665823
GCB	1.657974
HFC	3.008597
SCB	0.779629
SIC	3.641798
SG - SSB	1.899387
TBL	0.809644
UT	3.969648
SCBPRE	0.149657

Table 4.3 - Coefficient of variation of the shares

CORRELATION OF PORTFOLIOS

	CAL	EBG	EIC	ETI	GCB	HFC	SCB	SIC	SG-SSB	TBL	UT	SCBPRE
CAL	1	0.310764	0.068655	0.016282	0.393638	-0.0508	0.308415	0.197603	0.432625	0.225595	0.221522	0.02528
EBG	0.310764	1	0.239807	0.120902	0.059291	-0.09404	0.344831	0.152478	0.415495	-0.07607	0.136579	0.125152
EIC	0.068655	0.239807	1	-0.15705	-0.09212	-0.17993	0.049516	-0.23025	0.097025	-0.08243	0.123796	0.355561
ETI	0.016282	0.120902	-0.15705	1	0.068919	0.143803	-0.18463	0.06036	0.26676	-0.00017	0.132043	-0.07094
GCB	0.393638	0.059291	-0.09212	0.068919	1	0.174062	0.081684	-0.09742	0.066853	0.406664	0.104888	-0.05537
HFC	-0.0508	-0.09404	-0.17993	0.143803	0.174062	1	0.147542	0.142144	-0.01629	0.144131	-0.0709	0.298372
SCB	0.308415	0.344831	0.049516	-0.18463	0.081684	0.147542	1	0.213602	0.397354	0.065543	0.112382	0.166055
SIC	0.197603	0.152478	-0.23025	0.06036	-0.09742	0.142144	0.213602	1	0.227018	-0.21064	0.01632	-0.18863
SG-SSB	0.432625	0.415495	0.097025	0.26676	0.066853	-0.01629	0.397354	0.227018	1	0.337744	0.588702	0.005695
TBL	0.225595	-0.07607	-0.08243	-0.00017	0.406664	0.144131	0.065543	-0.21064	0.337744	1	0.549501	0.11333
UT	0.221522	0.136579	0.123796	0.132043	0.104888	-0.0709	0.112382	0.01632	0.588702	0.549501	1	-0.23243
SCBPRE	0.02528	0.125152	0.355561	-0.07094	-0.05537	0.298372	0.166055	-0.18863	0.005695	0.11333	-0.23243	1

Table 4.4 – The Correlation coefficients of the portfolios

P - VALUES OF PORTFOLIOS

	CAL	EBG	EIC	ETI	GCB	HFC	SCB	SIC	SG-SSB	TBL	UT	SCBPRE
CAL	0	0.033497	0.646563	0.913499	0.006191	0.734542	0.03493	0.183062	0.002389	0.127337	0.134538	0.866056
EBG	0.033497	0	0.104471	0.418216	0.692194	0.529535	0.017611	0.306221	0.00368	0.611329	0.359969	0.401921
EIC	0.646563	0.104471	0	0.291779	0.537988	0.226183	0.741004	0.119464	0.516474	0.581745	0.407078	0.014179
ETI	0.913499	0.418216	0.291779	0	0.645293	0.334875	0.214103	0.686925	0.069898	0.999115	0.376291	0.635622
GCB	0.006191	0.692194	0.537988	0.645293	0	0.241947	0.585184	0.514759	0.65525	0.00456	0.482898	0.711623
HFC	0.734542	0.529535	0.226183	0.334875	0.241947	0	0.322326	0.340537	0.913436	0.333761	0.635811	0.041638
SCB	0.03493	0.017611	0.741004	0.214103	0.585184	0.322326	0	0.149402	0.005681	0.661595	0.451994	0.264624
SIC	0.183062	0.306221	0.119464	0.686925	0.514759	0.340537	0.149402	0	0.124892	0.155266	0.913298	0.204155
SG-SSB	0.002389	0.00368	0.516474	0.069898	0.65525	0.913436	0.005681	0.124892	0	0.020244	1.35E-05	0.969697
TBL	0.127337	0.611329	0.581745	0.999115	0.00456	0.333761	0.661595	0.155266	0.020244	0	6.33E-05	0.448165
UT	0.134538	0.359969	0.407078	0.376291	0.482898	0.635811	0.451994	0.913298	1.35E-05	6.33E-05	0	0.115915
SCBPRE	0.866056	0.401921	0.014179	0.635622	0.711623	0.041638	0.264624	0.204155	0.969697	0.448165	0.115915	0

Table 4.5 – The P – values of the portfolios

PORTFOLIO OF SHARES	OPTIMUM VALUES
SCBPRE AND CAL	6.759245
SCBPRE AND EBG	6.760391
SCBPRE AND EIC	7.191688
SCBPRE AND ETI	6.786283
SCBPRE AND GCB	6.825663
SCB AND HFC	6.979591
SCBPRE AND SCB	6.756521
SCBPRE AND SIC	6.936527
SCBPRE AND SG - SSB	6.772213
SCBPRE AND TBL	6.771616
SCBPRE AND UT	7.009849

Table 4.6 – A table showing the optimum values of the portfolios

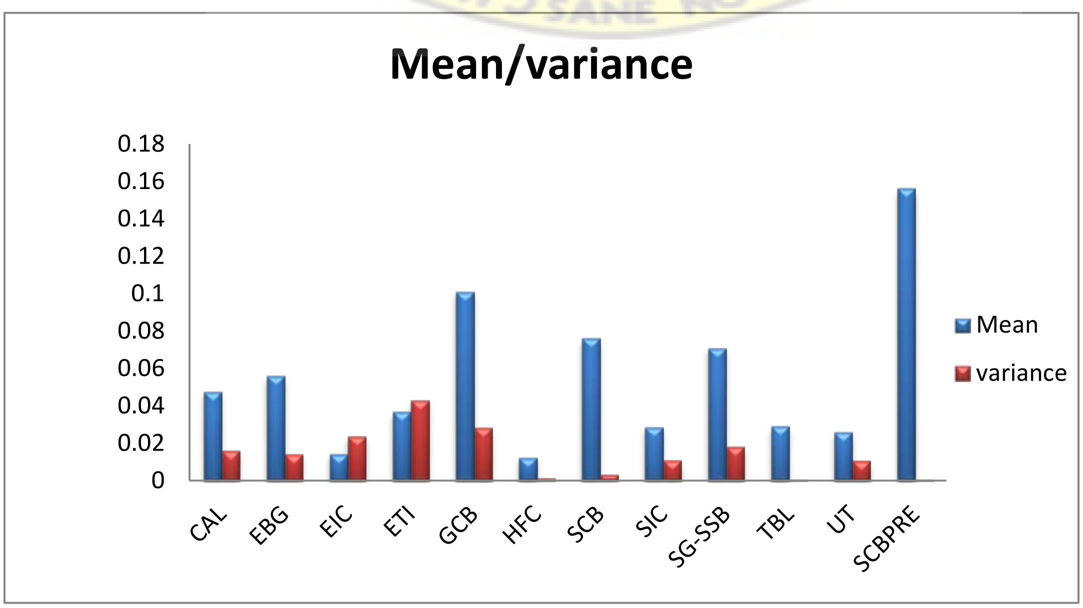


Figure 12 – Multiple Bar Chart for Mean and

Variance of the stocks

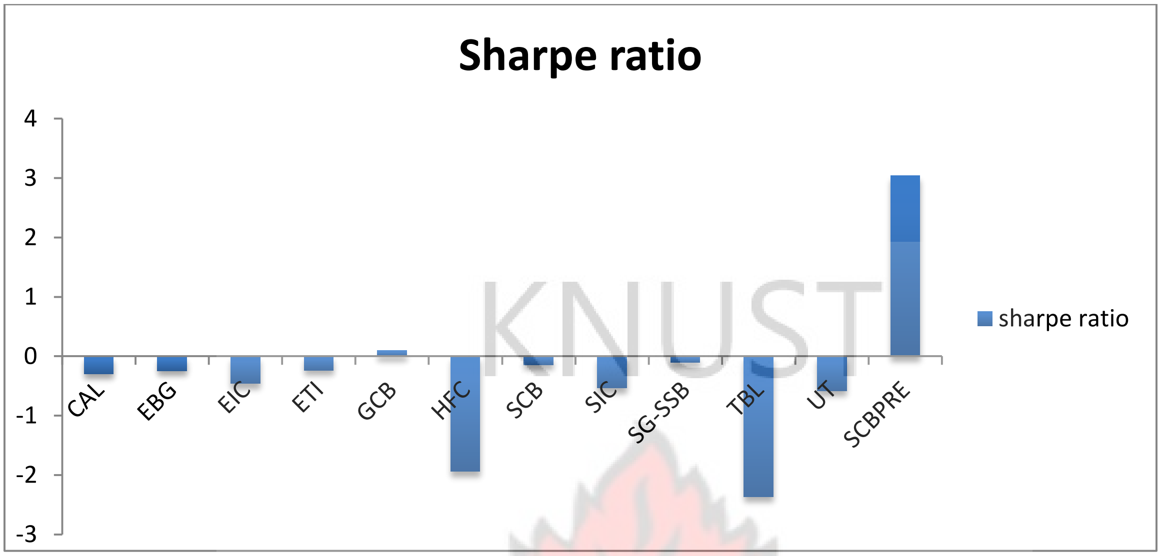


Figure 13 – A Bar chart of Sharpe ratio of the stocks

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

This chapter basically shows how the research objectives are achieved with the appropriate findings, conclusions and recommendations.

5.1 **Research Objectives**

Research Objective One

The two financial institutions which have largest Sharpe ratio values and least p – value between 0.01 and 0.1 would be appropriate to form a portfolio because they have a confidence level of about 90% to 99%. The optimal portfolio is the one which maximizes the Sharpe ratio.

When the Sharpe ratio is large then it is good for investment.

Research Objective Two

To establish the veracity of the fact that Sharpe ratio was used to analyze the ratio of return to risk. The one which has the largest Sharpe ratio is good for investment. Since that would maximize expected return for a given level of risk or minimize the risk for a given expected return.

Research Objective Three

The research has determined how efficient diversification helps in investments. It is appropriate to form a portfolio by using investments which reduces portfolio risk that is p – values between 0.01 and 0.1.

To draw conclusions and offer appropriate recommendations

5.2 CONCLUSIONS

From the analysis done so far one can infer that the higher the risk the higher the expected return and the larger the value of coefficient of variation the more riskier it is.

The financial company with largest Sharpe ratio value is the best for investment.

The companies which have largest Sharpe ratio values and with p – values between 0.01 and 0.1 are highly recommended to form portfolio.

Looking at Table 4.2 we can say that SCB Preference Shares have the largest Sharpe ratio value and followed by GCB shares but from table 4.5 their P – value of 0.711623 is not statistically significant because the confidence level is 28.84%.

Moreover, from Table 4.6 SCB Preference Shares and EIC shares is the optimal portfolio since that maximizes the Sharpe ratio with a value of 7.191688.

In addition from Table 4.5 SCB Preference Shares and EIC shares have P – value of 0.014179 hence 98.58% confidence level.

From Table 4.3 SCB preference Shares have the least value of coefficient of variation (CV) and hence have a least risk per unit of return.

RECOMMENDATIONS

1. From the research we want to recommend to investors with high capital to invest with financial companies on the Ghana Stock Exchange that form a portfolio by maximizing returns with the same level of risk or minimizing risk for a given expected returns.

2. To encourage individuals or investors to invest in two companies that form a portfolio with small P – values between 0.01 and 0.1 and hence at a high confidence level of about 90% to 99% and also maximizes the Sharpe ratio.

3. To help the investors to analyze the risk and returns of portfolio before investing in companies so that they will not be at a loss.

4. From the research we want to recommend to investors that the Standard Chartered Bank (SCB) Preference Shares and Enterprise Insurance Company (EIC) would be the best to

form a portfolio since that maximizes the Sharpe ratio and hence it is the optimal portfolio and also has a P- value of 0.041791 and at 98.58% confidence level.

SUGGESTED AREA FOR FURTHER STUDY/RESEARCH

In the future the research should involve both financial and non financial companies on the Ghana Stock Exchange (GSE) to get the best two companies to form an optimal portfolio.

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