**KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY**

# OPTIMAL LOCATION OF A LIBRARY FACILITY

# (CASE STUDY: NKORANZA COMMUNITY)

**BY**

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## CERTIFICATION

I hereby declare that this submission is my own work towards the MSc., and that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the University, except where due acknowledgement has been made in the text.

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## ABSTRACT

Facility location is a branch of Operations Research and computational geometry concerning itself with mathematical modeling and solution of problems concerning optimal placement of facilities in order to minimize transportation costs, avoid placing hazardous facilities near housing, etc. One of the greatest problems facing both the public and the private sector enterprises is how to locate facilities. People site their facilities anywhere and anyhow without first considering how close that facility will be to people in the community and whether the facility is desirable, semi desirable or undesirable by the community. This work attempts to use p-median model to find a suitable site to locate a library facility at Nkoranza community. Two different methods were used to locate the facilities; the myopic algorithm and Lagrangian algorithm. Results of both algorithm suggest that we locate the facility at Kassadjan. The optimal objective function value was 92674 kilometers. This gave an average demand-weighted distance of 2.06 km.

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## DEDICATION

This work is dedicated to my lovely mother Lydia Fofie who is the source of my strength, wisdom and inspiration.

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CHAPTER 1

# INTRODUCTION

## 1.0 INTRODUCTION

Facility location is a branch of operations research and computational geometry concerning itself with mathematical modeling and solution of problems concerning optimal placement of facilities in order to minimize transportation costs, avoid placing hazardous materials near housing, outperform competitors' facilities, etc (Wikipedia, 2009). Location problems deal with finding the right site where one or more new facilities should be placed, in order to optimize some specified criteria, which are usually related to the distance (performance measure) from the facilities to the demand points. Suppose that a media company plans to place newspaper stands in a city. The company has already identified potential stand sites in a number of different neighborhoods and knows the cost of placing and maintaining a stand at each potential site. Further assume that the demand for newspapers in each neighborhood of the city is known. If the company wants to open any number of stands, where should they be located in order to minimize the sum of the total placing and maintaining cost and the average traveling distance of the customers? The preceding question is an example of a facility location problem.

Location problems have occupied a central place in Operations Research since the early 1960's. They model design situations such as deciding placements of factories, warehouses, fire stations or hospitals and clustering analysis. A facility is considered as a physical entity that provides services. Facility location problems arise in a wide set of practical applications in different fields of study: management, economics, production planning and many others (Peton, 2002). Facility is classified into three categories: non – obnoxious (desirable), semi – obnoxious and obnoxious (non-desirable), (Welch et al., 1997).

A desirable facility includes supermarket, shops, banks, fire stations, schools, libraries, post offices, warehouses, etc. as the customer needs access, of some sort, to the facility providing the service, it is beneficial if these facilities are sited close to the customers that they will be serving. This implies that the customer has better access to the facility. Undesirable (obnoxious) facilities are those facilities that have adverse effects on people or the environment.

A facility is defined as obnoxious facility if its undesirable effect far outweighs its accessibility. Erkut and Neuman (2000) defined undesirable facility as one that generates a disservice to the people nearby while producing an intended product or service. They generate some form of pollution, nuisance, potential health hazard, or danger to nearby residents; they also may harm nearby ecosystems. Some examples are nuclear power stations, military installations nuclear or chemical plants, incinerators, and pollution-producing industries. Although necessary to society, these facilities are undesirable and often dangerous to the surrounding inhabitants so lowering local house prices and quality of life (Amponsah, 2003). Although they provide some disservice to nearby residents, these facilities are necessary to societies. In addition, there is often some travel involved to and from these facilities and an associated transportation cost that increases with distance from the population, which in turn suggests that they should be placed away but not very far away. The terms semi – obnoxious and semi – desirable have also been used for some of these facilities, but the undesirable features (perceived or real) of these facilities dominate the desirable ones.

Brimberg and Juel introduced the term semi-desirable facility in 1998. They argued that the facilities cannot be classified as being purely desirable or purely obnoxious. Sometimes though a facility produces a negative or undesirable effect, this effect may be present even though a high degree of accessibility is required by the facility. For example, a stadium provides entertainment and so requires a large amount of access to enable supporters to attend a game. On the other hand, on match days, local non – football fans would have to contend with the noise and the traffic generated. This generation of noise is unpleasant for locals and therefore undesirable. The combination of the two makes this facility a semi-obnoxious. Another example is the gabage dump sites. Here, access is needed to deposit the waste produced by local population. Conversely, the disposal site may be offensive to look at, and also it emits offensive odour. These two contradicting points cause the disposal site to be defined as a semi – obnoxious facility. Other examples of semi – obnoxious facilities are ambulance and fire stations, airports, hospitals, power plants etc.

This thesis aims to locate a library as an example of non – obnoxious facility. Libraries are useful and necessary for the communities and schools hence its location should not be placed very far from the people to make it easily accessible hence it is classified as non – obnoxious facility.

## 1.1 BACKGROUND OF STUDY

A library is an organized collection of resources made accessible to a defined community for reference or borrowing. It provides physical or digital access to material, and may be a physical building or room, or a virtual space, or both. Collections can include books, periodicals, newspapers, manuscripts, films, maps, prints, documents, microform, CDs, cassettes, videotapes, DVDs, video games, e - books, audio books and other formats. Libraries range in size from a few shelves of books to several million items.

The first libraries consisted of archives of the earliest form of writing – the clay tablets in cuneiform script discovered in summer, some dating back to 2600 BC. These written archives mark the end of prehistory and the start of history. The earliest discovered private archives were kept at Ugarit. There is also evidence of libraries at Nippur about 1900 BC and at Nineveh about 700 BC showing a library classification system. Private or personal libraries made up of written books (as opposed to the state or institutional records kept in archives). The first library classification system was set up during the Han Dynasty. In North America, it is believed that personal collections of books were brought over to the continent by French settlers in the 16th century. The oldest non-personal library on the North American continent was founded at The Jesuit College in Quebec City in 1635. The first textbook on library science was published 1808 by Martin Schrettinger.

A library is organized for use and maintained by a public body, an institution, a corporation, or a private individual. Public and institutional collections and services may be intended for use by people who choose not to-or cannot afford to-purchase an extensive collection themselves, who need material no individual can reasonably be expected to have, or who require professional assistance with their research. In addition to providing materials, libraries also provide the services of librarians who are experts at finding and organizing information and at interpreting information needs. Libraries often provide quiet areas for studying, and they also often offer common areas to facilitate group study and collaboration. Libraries often provide public facilities for access to their electronic resources and the internet. Modern libraries are increasingly being redefined as places to get unrestricted access to information in many formats and from many sources. They are extending services beyond the physical walls of a building, by providing material accessible by electronic means, and by providing the assistance of librarians in navigating and analyzing very large amounts of information with a variety of digital tools.

Many institutions make a distinction between a circulating or lending library, where materials are expected and intended to be loaned to patrons, institutions, or other libraries, and a reference library where material is not lent out. Modern libraries are often a mixture of both, containing a general collection for circulation, and a reference collection which is restricted to the library premises. Also, increasingly, digital collections enable broader access to material that may not circulate in print. We have various forms of libraries, they are; national libraries, research libraries, reference libraries, public lending libraries, academic libraries, children libraries and special libraries.

### 1.1.1 PROFILE OF STUDY AREA

Nkoranza is a town in Ghana. It is in the state/region of Brong-Ahafo. The population is between 10,000 and 20,000. The town is located in the mid-north of Ghana and is the capital of Nkoranza south district, a district in the Brong Ahafo Region of Ghana. The Nkoranza south district is one of 22 administrative districts of the Brong Ahafo Region of Ghana. It covers a total area of 2,300 square km and is made up of some 120 mainly rural settlements. The total population of the district was estimated in 2010 at 100,929 (Ghana Statistical Service, 2010).

The district lies within the wet semi-equatorial region, in the transitional zone between the savannah woodland of the north and the forest belt of the south of the country. The main occupation of the inhabitants is agriculture, which employs about 95 percent of the economically active population of the district. Food crop farming is the main source of cash for the rural dwellers, and maize farming is the main cash crop grown (26 percent of total cultivated land), followed by yams (19 percent of cultivated land). In addition, other food crops such as vegetables, cassava, rice, groundnuts, cowpea, cocoyam and plantain are cultivated. Cotton and tobacco are also grown in parts of the district. Nevertheless, 34 percent of the population is also engaged in small scale industry. For purposes of comparison, the corresponding percentages of the regional (i.e., Brong

Ahafo) and national labour forces engaged in agriculture for the same period are 71 and 57 respectively.

Nkoranza has a population made up mainly of traders, civil servants and other government employees, transport operators, small scale industry operators and the like. Many urban dwellers, however, still take up agriculture as at least a minor activity. Currently there is no library facility in the Nkoranza Town and the municipality as a whole which when constructed would serve as resource centres for schools and decentralized departments in the Municipality. The municipality can boost of so many schools from basic level to the senior high school level. Only few schools in the municipality have a library of their own.

## 1.2 PROBLEM STATEMENT

Provision of a library facilities are of major concern to every community in the country as far as research and education is concerned and for that matter location of such facility is of keen interest to the state and the communities involved. One of the greatest problems facing both the public and the private sector enterprises is how to locate facilities. People site their facilities anywhere and anyhow without first considering how close that facility will be to people in the community and whether the facility is desirable, semi desirable or undesirable by the community.

Unlike Nkoranza north, Nkoranza south district has no library facility therefore this work seeks to find the optimal sites to locate a library facility at Nkoranza Township which is the capital of Nkoranza south district, using the p – median model.

### 1.3 OBJECTIVES

The objectives of this study are:

- To model location of library faculty as p – median problem.
- To find the optimal location using lagrangian method.
- To locate the facilities at suitable sites so that the average distance covered by students and people from their communities to the library will be minimized.

### 1.4 METHODOLOGY

The location problem was modeled as p – median problem. Data on road distances between suburbs were obtained and the total populations of each suburbs of Nkoranza Township were obtained from the 2010 population and housing census and used.

Floyd – Warshall's algorithm was used to find the distance matrix, $d(i, j)$ for all pairs shortest path. Myopic algorithm was used to estimate the demand-weighted distance which was then used as the upper bound (UB) for the Lagrangian algorithm. Lagrangian algorithm was used in optimal location for the p – median problem to find the site for the library facility. We solve both myopic and the Lagrangian algorithm manually.

Materials were obtained from 2010 population and housing census, KNUST library and internet

## 1.5 JUSTIFICATION OF THE STUDY

Research suggests that Library programs and resources play a vital role in the development of information-literate students. Research studies continue to show that an active school library program makes a significant difference to student learning outcomes. Library provides information and ideas that are fundamental to functioning successfully in today's information and knowledge-based society. Libraries are very important for the progress and development of a society. They are storehouse of knowledge. Libraries are significant for a civil society, they ensure accessibility to vast ocean of knowledge to common man. There are people in every society who have craving for reading but have no money to satisfy their urge. Libraries are blessing for such people. The main aim of this work is to find the optimal sites to establish a library facility at Nkoranza Township using the p-median model.

## 1.6 ORGANISATION OF THE THESIS

This thesis is organized into five main chapters. Chapter one presents the introduction of the thesis. This consists of the background of the study, the research problem statement, objectives of the research, methodology, and organization of the thesis. Chapter two is the literature review, which looks at briefly work done by other researchers on the topic. Chapter three is the formulation of the mathematical model. Chapter four contains the data collection and analysis. Chapter five looks at Summary, Conclusions and Recommendation of the analyzed data.

# CHAPTER 2

# LITERATURE REVIEW

## 2.0 INTRODUCTION

This chapter looks at other works done by researchers on facility locations problems.

## 2.1 FACILITY LOCATION PROBLEMS



Albareda-Sambola et al. (2008), introduce the multi – period incremental service facility location problem where the goal was to set a number of new facilities over a finite time horizon so as to cover dynamically the demand of a given set of customers. They prove that the coefficient matrix of the allocation subproblem that results when fixing the set of facilities to open is totally unimodular. This allows solving efficiently the Lagrangean problem that relaxes constraints requiring customers to be assigned to open facilities. They proposed a solution approach that provides both lower and upper bounds by combining subgradient optimization to solve a Lagrangean dual with an ad hoc heuristic that uses information from the Lagrangean subproblem to generate feasible solutions. The numerical results obtained in the computational experiments showed that the obtained solutions were very good. In general, we get very small percent gaps between upper and lower bounds with little computation effort.

Ebery et al., (1998) considered and presented formulations and solution approaches for the capacitated multiple allocation hub location problems. They presented a new mixed integer linear programming formulation for the problem. They also constructed an efficient heuristic algorithm, using shortest paths. They incorporated the upper bound obtained from this heuristic in a linear – programming – based branch – and – bound solution procedure. They presented the results of extensive computational experience with both the heuristic and the exact methods.

Fonseca and Captivo (1996; 2006; 2007) studied the location of semi obnoxious facilities as a discrete location problem on a network. Several bi-criteria models were presented considering two conflicting objectives, the minimization of obnoxious effect and the maximization of the accessibility of the community to the closest open facility. Each of these objectives is considered in two different ways, trying to optimize its average value 13 over all the communities or trying to optimize its worst value. The Euclidean distance is used to evaluate the obnoxious effect and the shortest path distance is used to evaluate the accessibility. The obnoxious effect is considered inversely proportional to the weighted Euclidean distance between demand points and open facilities, and demand directly proportional to the population in each community.

Yapicioglu et al. (2005) introduced a new model for the semi-obnoxious facility location problem. The new model is composed of a weighted minisum function to represent the transportation costs and a distance – based piecewise function to represent the obnoxious effects of the facility. A single-objective particle swarm optimizer (PSO) and a bi-objective PSO are devised to solve the problem. Results are compared on a suite of test problems and showed that the bi-objective PSO produces a diverse set of non-dominated solutions more efficiently than the single-objective PSO and is competitive with the best results from the literature. Computational complexity analysis estimates only a linear increase in effort with problem size.

Ghodsi (2012), studied a two-stage stochastic facility location problem in the context of disaster response network design. The uncertainty inherent in disaster occurrence and impact is captured by defining scenarios to reflect a large spectrum of possible occurrences. In the first stage (pre-event response), planners should decide on locating a set of facilities in strategic regions. In the second stage (post-event response), some of these facilities are to be activated to respond to demand in the disaster affected region. The second-stage decisions depend on disaster occurrence and impact which are highly uncertain. To model this uncertainty, they defined a large number of scenarios to reflect a large spectrum of possible occurrences. In this case, facility activation and demand allocation decisions are made under each scenario. The aim was to minimize the total cost of locating facilities in the first stage plus the expected cost of facility activation and demand allocation under all scenarios in the second stage while satisfying demand subject to facility and arc capacities. They proposed a mixed integer programming model with binary facility location variables in the first stage and binary facility activation variables and fractional demand allocation variables in the second stage. They proposed two Lagrangian relaxations and several valid cuts to improve the bounds. They experimented with aggregated, disaggregated and hybrid implementations in calculating the Lagrangian bound and developed several Lagrangian heuristics. They perform extensive numerical testing to investigate the effect of valid cuts and disaggregation and to compare the relaxations. The second relaxation proved to provide a tight bound as well as high quality feasible solutions.

Albareda-Sambola et al. (2003), considered a combined location – routing problem. They defined an auxiliary network and give a compact formulation of the problem in terms of finding a set of paths in the auxiliary network that fulfill additional constraints. The LP solution to the considered model provides an initial lower bound and is also used in a rounding procedure that provides the initial solution for a Tabu search heuristic. Additionally, they proposed a different lower bound based on the structure of the problem. The results of computational testing on a set of randomly generated instances were promising.



Goldengorin et al, (1999) considered the simple plant location problem. This problem often appears as a sub-problem in other combinatorial problems. Several branch and bound techniques have been developed to solve these problems. Their thesis considered new approaches called branch and peg algorithms, where pegging refers to assigning 15 values to variables outside the branching process. An exhaustive computational experiment shows that the new algorithms generate less than 60% of the number of sub-problems generated by branch and bound algorithms, and in certain cases requires less than 10% of the execution times required by branch and bound algorithms. Firstly, for each sub-problem generated in the branch and bound tree, a powerful pegging procedure is applied to reduce the size of the sub-problem. Secondly, the branching function is based on predictions made using the Beresnev function of the sub-problem at hand. They saw that branch and peg algorithms comprehensively out perform branch and bound algorithms using the same bound, taking on the average, less than 10% of the execution time of branch and bound algorithms when the transportation cost matrix is dense. The main recommendation from the results of the experiment is that branch and peg algorithms should be used to solve SPLP instances.

Farahani et al. (2011) reviewed the covering problems in facility location. Here, besides a number of reviews on covering problems, a comprehensive review of models, solutions and applications related to the covering problem is presented after Schilling, Jayaraman, and Barkhi (1993). This survey tries to review all aspects of the covering problems by stressing the works after Schilling, Jayaraman, and Barkhi (1993). They first presented the covering problems and then investigated solutions and applications.

Cortinhal and Captivo (2003), considered the single source capacitated location problem. Given a set of potential locations and the plant capacities, it must be decided where and how many plants must be open and which clients must be assigned to each open plant. A Lagrangean relaxation is used to obtain lower bounds for this problem. Upper bounds are given by Lagrangean heuristics followed by search methods and by one tabu search metaheuristic. Computational experiments on different sets of problems were presented.

Mahdian et al. presented a 1.52-approximation algorithm for the metric uncapacitated facility location problem, and a 2-approximation algorithm for the metric capacitated facility location problem with soft capacities. Both these algorithms improve the best previously known approximation factor for the corresponding problem, and their soft-capacitated facility location algorithm achieves the integrality gap of the standard LP relaxation of the problem. Furthermore, they showed, using a result of Thorup, that the algorithms can be implemented in quasi-linear time.

Wu et al. (2004) presented an extension of the capacitated facility location problem (CFLP), in which the general setup cost functions and multiple facilities in one site are considered. The setup costs consist of a fixed term (site setup cost) plus a second term (facility setup costs). The facility setup cost functions are generally non-linear functions of the size of the facility in the same site. Two equivalent mixed integer linear programming (MIP) models are formulated for the problem and solved by general MIP solver. A Lagrangian heuristic algorithm (LHA) is also

Developed to find approximate solutions for this NP – hard problem. Extensive computational experiments are taken on randomly generated data and also well-known existing data (with some necessary modifications). The detailed results are provided and the heuristic algorithm is shown to be efficient.

Geoffrion and Me Bride (2007), lagrangean relaxation, a technique of quite general applicability, is studied in the particular context of the capacitated facility location problem with arbitrary additional constraints. For this class of problems they were able to obtain a reasonably complete algebraic and geometric understanding of how and why Lagrangean relaxation works. Extensive computational results are also reported. Although this work finds immediate application to improved computational procedures for the class of problems studied, our longer term aim is to encourage similar in-depth studies of Lagrangean relaxation for other important classes of problems.

Most of the time in a distribution system, depot location and vehicle routing are interdependent, and recent studies have shown that the overall system cost may be excessive if routing decisions are ignored when locating depots. The location-routing problem (LRP) overcomes this drawback by simultaneously tackling location and routing decisions. Prins et al. (2007) presents a cooperative metaheuristic to solve the LRP with capacitated routes and depots. The principle is to alternate between a depot location phase and a routing phase, exchanging information on the most promising edges. In the first phase, the routes and their customers are aggregated into supercustomers, leading to a facility - location problem, which is then solved by a Lagrangean relaxation of the assignment constraints. In the second phase, the routes from the resulting multidepot vehicle-routing problem (VRP) are improved using a granular tabu search (GTS) heuristic. At the end of each global iteration, information about the edges most often used is recorded to be used in the following phases. The method is evaluated on three sets of randomly generated instances and compared with other heuristics and a lower bound. Solutions are obtained in a reasonable amount of time for such a strategic problem and show that this metaheuristic outperforms other methods on various kinds of instances.

Due to the popularity of hub-and spoke networks in the airline and telecommunication industries, there has been a growing interest on hub location problems and related routing policies. Jaillet et al. (1996) introduced flow - based models for designing capacitated networks and routing policies. No a priori hub-and-spoke structure is assumed. The resulting networks may suggest the presence of “hubs”, if cost efficient.

The network design problem is concerned with the operation of a single airline with a fixed share of the market. They presented three basic integer programming models, each corresponding to a different service policy. Due to the difficulty of solving (even small) instances of these problems to optimality, they propose heuristic schemes based on mathematical programming. The procedure is applied and analyzed on several test problems consisting of up to 39 U.S cities. They provided comments and partial recommendations on the use of hubs in the resulting network structures.



Facility Location can be seen as a whole family of problems which have many obvious applications in economics. They have been widely explored in the Operations Research community, from the viewpoints of approximation, heuristics, linear programming, etc. Fellows and Fernau added a new facet by initiating the study of some of these problems from a parametric point of view. Moreover, they exhibited some less obvious applications of these algorithms in the processing of semistructured documents and in computational biology.

Efrat et al. (2003) studied two problems that arise in optimization of sensor networks: First, they devise provable approximation schemes for locating a base station and constructing a network among a set of sensors each of which has a data stream to get to the base station. Subject to power constraints at the sensors, their goal was to locate the base station and establish a network in order to maximize the life-span of the network. Secondly, they studied optimal sensor placement problems for quality coverage of given domains cluttered with obstacles. Using line – of - site sensors, the goal was to minimize the number of sensors required in order to have each point "well covered" according to precise criteria (e.g., that each point is seen by two sensors that form at least angle, or that each point is seen by three sensors that form a triangle containing the point).

Drezner et al. (2001) proposed five heuristic procedures for the solution of the multiple competitive facilities location problem. A franchise of several facilities is to be located in a trade area where competing facilities already exist. The objective is to maximize the market share captured by the franchise as a whole. They perform extensive computational tests and concluded that a two-step heuristic procedure combining simulated annealing and an ascent algorithm provides the best solutions.

Blelloch and Tangwongsan (2010) presented the design and analysis of parallel approximation algorithms for facility-location problems, including NC and RNC algorithms for (metric) facility location, k-center, k-median, and k-means. These problems have received considerable attention during the past decades from the approximation algorithms community, which primarily concentrates on improving the approximation guarantees. In their work, they ask: Is it possible to parallelize some of the beautiful results from the sequential setting? Their starting point was a small, but diverse, subset of results in approximation algorithms for facility-location problems, with a primary goal of developing techniques for devising their efficient parallel counterparts. They focused on giving algorithms with low depth, near work efficiency (compared to the sequential versions), and low cache complexity.

Most researchers have applied the technique of the p-median to solve location problems. The p – median problem is a powerful tool in analyzing facility location options when the goal of the location scheme is to minimize the average distance that demand must traverse to reach its nearest facility. It may be used to determine the number of facilities to site, as well as the actual facility locations. Demand data are frequently aggregated in p – median location problems to reduce then computational complexity of the problem. Demand data aggregation, however, results in the loss of locational information. This loss may lead to suboptimal facility location configurations (optimality errors) and inaccurate measures of the resulting travel distances (cost errors). Hillsman and Rhoda (1978) have identified three error components: Source A, B, and C errors, which may result from demand data aggregation. Current and Schilling proposed a method to measure weighted travel distances in p-median problems which eliminates Source A and B errors. Their test problem results indicate that the proposed measurement scheme yields solutions with lower optimality and cost errors than does the traditional distance measurement scheme.

Zhao and Batta, (2000) considered the p-median problem on a general network with link demands. They constructed the network in such a way that only transportation intersections are taken to be nodes and, therefore, both continuous and discrete link demands are allowed in their model. For such a model, they showed that a nodal solution can be used to approximate the true optimal solution and an error bound which involves only the demands on a single link is given for the error caused by such an approximation. Based on nodal solutions, they demonstrated that a model with continuous link demands can be transformed into an equivalent discrete link demand model. Further, they propose a method to aggregate demands on each link in solving the p-median problem on a general network without introducing any aggregation errors to the problem solution.

Rosing and Hodgson (2001) mapped certain combinatorial aspects of the *p*-median problem and explore their effects on the efficacy of a common (1- opt) interchange heuristic and of heuristic concentration (HC) for the problem's solution. Although the problem's combinatorial characteristics exist in abstract space, its data exist in two-dimensional space and are therefore mappable. By simultaneously analysing the problem's patterns in geographic space and its combinatorial characteristics in abstract space, they provided new insight into what demand node configurations cause problems for the interchange heuristic and how HC overcomes these problems.

Narula and Ogbu (1976) presented a branch – and – bound algorithm for solving the p – median problem. The bounds are obtained by solving the Lagrangian relaxation of the p – median problem using the subgradient optimization method. The proposed algorithm is simple, requires small core storage and computational time, and can be used for solving large problems. Compararative results are also reported.

The p-median problem, like most location problems, is classified as NP-hard, and so, heuristic methods are usually used for solving it. The p-median problem is a basic discrete location problem with real application that has been widely used to test heuristics. Metaheuristics are frameworks for building heuristics. Mladenović et al.

examined the p-median, with the aim of providing an overview on advances in solving it using recent procedures based on metaheuristic rules.

Locating p facilities to serve a number of customers is a problem in many areas of business. The problem is to determine p – facility locations such that the weighted average distance travelled from all the demand points to their nearest facility sites is minimized. A variant of the p-median problem is the one in which a maximum distance constraint is imposed between the demand point and its nearest facility location, also known as the p-median problem with maximum distance constraint. Chaudhry et al (2003), applied genetic algorithms to solve relatively large sized constrained version of the p-median problem. They presented their computational experience on the use of genetic algorithms for solving the constrained version of the p-median problem using two different data sets. Their comparative experimental experience shows that this solution procedure performs quite well compared with the results obtained from the existing techniques.

The Capacitated p-median problem (CPMP) seeks to solve the optimal location of p facilities, considering distances and capacities for the service to be given by each median. Lorena and Senne, presented a column generation approach to CPMP. The identified restricted master problem optimizes the covering of 1- median clusters satisfying the capacity constraints, and new columns were generated considering knapsack subproblems. The Lagrangean / surrogate relaxation has been used recently to accelerate subgradient like methods. In their work the Lagrangean / surrogate relaxation is directly identified from the master problem dual and provides new bounds and new productive columns through a modified knapsack subproblem. The overall column generation process is accelerated, even when multiple pricing is observed. Computational tests are presented using instances taken from real data from São José dos Campos' city.

There has been some instances where researchers have used p – centre model to solve location problems. The p-center problem is one of the location problems that have been studied in operations research and computational geometry. Davoodi et al. (2011) introduced a compatible discrete space version of the heuristic Voronoi diagram algorithm. Since the algorithm gets stuck in local optimums in some cases, they applied a number of changes in the body of the algorithm with regard to the geometry of the problem, in a way that it can reach the global optimum with a high probability. Finally, a comparison between the results of these two algorithms on several test problems and a real-world problem are presented.

Let $G(V, E, W)$ be a graph with $n$-vertex-set $V$ and $m$-edge-set $E$ in which each edge $e$ is associated with a positive distance $W(e)$. The $p$-Center problem is to locate some kind of facilities at $p$ vertices of $G$ to minimize the maximum distance between any vertex and the nearest facility corresponding to that vertex. Yen and Chen (2007) proposes an additional practical constraint. They restrict that the $p$ vertices where the facilities are located must be connected, i.e., the subgraph induced by the $p$ facility vertices must be connected. The resulting problem is called the connected $p$-Center problem (the C$p$C problem). They first show that the C*p*C problem is NP-Hard on bipartite graphs and split graphs. Then, an $O(n)$ – time algorithm for the problem on trees is proposed. Finally, they extended this algorithm to trees with forbidden vertices, i.e. some vertices in $V$ cannot be selected as center vertices, and the time – complexity is also $O(n)$.

Matsutomi and Ishii (1997) considered a single facility location problem for an ambulance service station in a polygonal area X. Their objective was to locate an ambulance service station so as to minimize the maximum distance of the route which passes from the facility to the hospital by way of the scene of accident. They considered A-distance which is a generalization of rectilinear distance and was introduced by Widmayer *et al.* Assuming m hospitals at the points $H_1, H_2, \ldots\ldots\ldots, H_m$ and denoting the nearest hospital to a point $Q$ of $X$ With $S(Q)$, the following problem $P_M$ was considered.

$$P_M\colon Minimize \max_{P^*} \max_{Q\epsilon X} R(P^*, Q) = \{d_A(P^*, Q) + d_A(Q, S(Q))\},$$

Where $P^* = (x^*, y^*)$ is the location of an ambulance service station to be determined. Then they showed $P_M$ can be reduced to the messenger boy problem with A-distance. Utilizing this result, they proposed an efficient solution procedure by extending Elzinga and Hearn Algorithm to A – distance case.

Hsu and Nemhauser (1979), considered a bottleneck location problem on a graph and presented an efficient (polynomial time) algorithm for solving it. The problem involved the location of $K$ noxious facilities that are to be placed as far as possible from the other facilities, and the objective was to maximize the minimum distance from the noxious facilities to the others. They then showed that two other bottleneck (min – max) location problems, finding *K*-centers and absolute *K*-centers of a graph appear to be very difficult to solve even for reasonably good approximate solutions.

Megiddo and Supowit (1984) proved that the p – center and the p – median problems relative to both the Euclidean and the rectilinear metrics are NP - hard. In fact, they prove that it is NP-hard even to approximate the p – center problems sufficiently closely. The reductions are from 3 – satisfiability.

Huang (2005) studied facility location problems on networks with multiple types of facilities and multiple types of customers. The chapter 2 of his work focuses on the minisum Collection Depots Location problem. In this problem, a server has to visit the node requesting service as well as one of several collection depots. They proved that there exists a dominating location set for the problem on a general network. The properties of the solution on some simple network topologies were discussed. To solve the problem on a general network, they suggested a Lagrangian Relaxation embedded in a branch-and-bound algorithm.

## CHAPTER 3

## METHODOLOGY

### 3.1 INTRODUCTION

In this chapter we employ the methods used in formulating location models. We first discuss the various models and then propose one which is more appropriate for the facility under study. We have methods that do not make use of road links (which includes the centre of gravity method, Location break – even analysis and Factor rating method) and methods that make use of road links (they include p – centre models and p – median models).

### 3.2 LOCATION METHODS WITHOUT ROAD LINKS

There are many factors, both quantitative and qualitative, to consider in choosing a location to site a facility. Some of these factors are more important than the others, so we use weighting to make objective decisions. There are three (3) main location methods. These include the centre of gravity method, location break – even analysis and factor rating method.

### 3.2.1 CENTER OF GRAVITY METHOD

The centre of gravity method is a mathematical technique used for finding the location of a distribution center that will minimize distribution cost. For instance in the location of a market, the method takes into account the volume of goods shipped to those markets and shipping cost in finding the best location for the distribution centre. The first step in the centre of gravity method is to place the locations on a coordinate system. The coordinates of each location must be carefully noted. The origin of the coordinate system is arbitrary, just as long as the relative distances are correctly represented. This can be done easily by placing a grid over an ordinary map of the location in question. The centre of gravity is determined by equations $(3.10)$ *and* $(3.11)$ given below.

$$C_x = \frac{\sum W_i X_i}{\sum W_i} \qquad (3.10)$$

$$C_y = \frac{\sum W_i Y_i}{\sum W_i} \qquad (3.11)$$

Where

$C_x = x -$ Coordinate of the centre of gravity

$C_y =$ y – Coordinate of the centre of gravity

$X_i = x$ - Coordinate of location $i$

$Y_i =$ y – Coordinate of location $i$

$W_i =$ Volume of goods to or from location $i$

The centre of gravity is then determined by equation (3.10) and (3.11) above. If the centre of gravity $(C_x, C_y)$ does not fall in any of the city (coordinates), we locate it in the nearest city.

Table 3.1 below gives the map coordinates and shipping loads for a set of cities that we wish to connect through a central “hub”.

**Table 3.1 Map coordinates and shipping loads for a set of cities**

| Site | Map Coordinates $(x, y)$ | Shipping load |
|---|---|---|
| A | (5,12) | 15 |
| B | (6,10) | 10 |
| C | (4,14) | 15 |
| D | (9,7) | 22 |

$C_x = 6.3387$ $and$ $C_y = 10.3871$



Since the centre of gravity $(6.3387, 10.3871)$ does not fall into any of the map coordinates, we find the point which is closed to the centre of gravity. Since the coordinates $(6.3387, 10.3871)$ is closed to the map coordinate $(6, 10)$ the hub should be located near it.

### 3.2.2 LOCATION BREAK – EVEN ANALYSIS

The location break – even analysis is the use of cost – volume analysis to make an economic comparison of location alternative. By identifying fixed and variable cost and graphing them for each location, we can determine which location provides the lowest cost. Location break – even analysis can be done mathematically or graphically. The graphical approach has the advantage of providing the range of volume over which each location is preferable. The location break – even analysis method employs three steps, these are:

- Determine the fixed and variable cost for doing business at each location

- Plot the cost for each location, with cost on the vertical axis of the graph and volume on the horizontal axis.
- Select the location that has the lowest total cost for the expected volume of business.

The location break – even analysis is determined by equation (3.3);

$$Y = ax + b \qquad (3.12)$$

Where

$a =$ Variable cost

$b =$ Fixed cost

$x =$ Volume of business

$Y =$ Cost of business

Table 3.2 illustrates an example where the fixed and variable cost for three manufacturing sites for Juaben Oil Mill.

**Table 3.2: Fixed and variable cost for Juaben Oil Mill**

| Site | Fixed cost (b) | Variable cost (a) |
|---|---|---|
| 1 | 500 | 10 |
| 2 | 1000 | 6 |
| 3 | 1500 | 4 |

We relate Table 3.1 as

$Y_1 = 10x + 500$

$Y_2 = 6x + 1000$

$Y_1 = 4x + 1500$

For a volume of less than $100$ then management should choose site 1. Between $100 - 270$ site 2 is better and above $270$ site 3 is the best.

### 3.2.3 THE FACTOR RATING METHOD

The factor rating method is a method used to find a suitable location for a facility considering a number of factors. These factors may include; labour cost, labour availability, proximity to market, equipment supply, community desire etc. The factor rating method has six steps. These are;

i. Develop a list of relevant factors.
ii. Assign a weight to each factor to reflect its relative importance in management's objective.
iii. Develop a scale for each factor (for example, 1 to 10 or 1 to 100)
iv. Assign a score to each location for each factor, using the scale in step (iii),
v. Multiply the score by the weight assigned to each factor and total the score for each location.
vi. Make a recommendation based on the maximum point score; considering the result of quantitative approaches as well.

**Table 1.3 Factor rating method**

| Factor | Factor name | Rating weight | Ratio of Rating | Location A | Location B | Location C |
|---|---|---|---|---|---|---|
| 1 | Proximity to port | 3 | 0.15 | 13.05 | 12.30 | 12.0 |
| 2 | Power source | 4 | 0.20 | 16.0 | 14.0 | 13.0 |
| 3 | Workforce attitude | 4 | 0.20 | 6.0 | 14.0 | 14.0 |
| 4 | Distance | 2 | 0.10 | 5.8 | 8.0 | 6.0 |
| 5 | Community Desire | 2 | 0.10 | 9.0 | 6.0 | 8.0 |
| 6 | Equipment Supply | 3 | 0.15 | 7.5 | 9.0 | 13.5 |
| 7 | Economic Activity | 2 | 0.10 | 7.8 | 7.9 | 8.4 |
| | | | | $\sum A = 65.15$ | $\sum B = 71.2$ | $\sum C = 74.9$ |

From the aggregate scores location or site C will be recommended since it has the highest aggregate.

### 3.3 NETWORK LOCATION METHODS

Facility location concerns itself with optimal placement of facilities in order to minimize transportation costs, avoid placing hazardous materials near housing, etc. The points of placement of the facilities are called facility nodes and the population centres in which the facility will serve are called demand nodes. We have various facility location models classified according to their consideration of distance; The maximum distance models and total (or average) distance models.

### 3.3.1 MAXIMUM DISTANCE MODELS

In some locations problems, an acceptable distance is set a priori. In the facility location literature, a priori acceptable distances such as these are known as “covering” distances. Demand within the covering distance of its closest facility is considered “covered.” An underlying assumption of this measure of covering distance is that demand is fully satisfied if the nearest facility is within the coverage distance and is not satisfied if the closest facility is beyond that distance. That is, being closer to a facility more than the covering distance does not improve satisfaction.

1. Set covering location model: the objective of this model is to locate the minimum number of facilities required to “ cover” all of the demand nodes (Toregas et al., 1971)
2. Maximal covering location problem (MCLP): the objective of this model is to locate a predetermined number of facilities, p, in such a way as to maximize the demand that is covered. Thus, the MCLP assumes that there may not be enough facilities to cover all of the demand nodes. If all nodes cannot be covered, then the model seeks the siting scheme that covers the most demand (Church and ReVelle, 1974).
3. The p-dispersion problem: The p-dispersion problem (PDP) is only concerned with the distance between new facilities and the objective is to maximize the minimum distance between any pair of facilities. Potential applications of the PDP include the siting of military installations where separation makes them more difficult to attack or locating franchise outlets where separation reduces cannibalization among stores (Kuby, 1987).

4. The p – center problem: this model requires the model to minimize the coverage distance such that each demand node is covered by one of the facilities to be sited within the endogenously determined coverage distance. The center problem is a minimax problem.

### 3.3.2 TOTAL OR AVERAGE DISTANCE MODELS

Many facility location planning situations are concerned with the total travel distance between facilities and demand nodes. Thus minimizing the maximum distance between facility and demand nodes.

1. The Maxisum Location Problem: The maxisum location problem seeks the locations of p facilities such that the total demand-weighted distance between demand nodes and the facilities to which they are assigned is maximized.
2. P – median problem: this model minimizes the average response time/distance between a demand site and the facilities to which they are assigned (Hakimi, 1964; 1965).

The *p* – median problem is the problem of locating *p* “facilities” relative to a set of “customers” such that the sum of the shortest demand weighted distance between “customers” and “facilities” is minimized. The model considered in this piece of work is the *p*-median. This is because the objective of this work is to minimize the average distance / time that students and people would travel from their schools and homes halls/hostel to the library facilities.

## 3.4 THE P – MEDIAN PROBLEM

The p – median problem may be formulated using the following notation:

inputs

$h_i$ = customer $i$ demand

$d_{ij}$ = distance between customer $i$ and candidate facility $j$

$P$ = number of facilities to be located



Decision variables.

$$X_j = \begin{cases} 1 & \text{if we locate at candidate site j} \\ 0 & \text{if not} \end{cases}$$

$$Y_{ij} = \begin{cases} 1 & \text{if customer i is served by facility j} \\ 0 & \text{if not} \end{cases}$$

Minimize

$$\sum_i \sum_j h_i d_{ij} Y_{ij} \quad \text{......(3.13)}$$

Subject to

$$\sum_j Y_{ij} = 1 \quad \forall i \text{ ......(3.14)}$$

$$\sum_j X_j = P \text{......(3.15)}$$

$$Y_{ij} - X_j \leq 0 \quad \forall i, j \text{ ......(3.16)}$$

$$X_j = 0, 1 \quad \forall j \text{ ......(3.17)}$$

$$Y_{ij} = 0, 1 \quad \forall j \text{ ......(3.18)}$$

The objective function (3.13) minimizes the total demand – weighted distance between each demand node. The constraints insure that the various properties of the problem are enforced. Specifically: Constraint (3.14) requires that, each demand node $i$ be assigned to exactly one facility $j$ . Constraint (3.15) requires that exactly $P$ facilities are located. Constraint (3.16) links the location variables, and the allocation variables. Constraints (3.17) and (3.18) insure that the location variables $(X)$ and the allocation variable $(Y)$ are binary.

The median formulation given above assumes that facilities are located on the nodes of the network. (Hamiki, 1995). Because of the binary constraints (3.17) and (3.18), the p – median formulation above cannot be solved with standard linear programming technique.

From the time when Kariv and Hakimi (1979) realized that P-median problems could be solved on a general graph as well as a tree, a number of heuristic algorithms have been proposed. These types of heuristics can be classified into what Golden (et al 1980) calls construction algorithms and improvement algorithms. Daskin (1995) discusses three heuristics: a myopic algorithm, an exychange heuristic and a neighborhood search algorithm. The myopic algorithm "constructs" a solution by locating the first facility at the one location that minimizes demand weighted total distance. This objective is calculated through total enumeration of the possible solutions. Subsequent facilities are located in a similar fashion, while holding the previously located facilities constant. The myopic heuristic is simple and thus easy to understand and apply.

### 3.5 MYOPIC ALGORITHM FOR THE P – MEDIAN

**Step 1:** Initialize $k = 0$ ($k$ will count the number of facilities we have located so far) and $X_k = \emptyset$, the empty set ($X_k$ will give the location of the $k$ facilities that we have located at each stage of the algorithm).

**Step 2:** Increment $k$, the counter on the number of facilities located.

**Step 3:** Compute ${Z_j}^k = \sum_i h_i d(i, j \cup X_{k-1})$ for each node $j$ which is not in the set $X_{k-1}$. Note that ${Z_j}^k$ gives the value of the $P$ – median objective function if we locate the k^th facility at node $j$, given that the first $k - 1$ facilities are at the locations given in the set $X_{k-1}$ (and node $j$ is not part of that set).

**Step 4:** Find the node $j^*(k)$ that minimizes ${Z_j}^k$ that is, $j^*(k) = arg \min_j \{{Z_j}^K\}$. Note that ${Z_j}^k$ gives the best location for the k^th facility, given the location of the first $k - 1$ facilities. Add node $j^*(k)$ to the set $X_{k-1}$ to obtain the set $X_{k-1}$ that is, set $X_k = X_{k-1} \cup j^*(k)$ .

**Step 5**: If $k = P$ (i.e., we have located $P$ facilities), STOP; the set $XP$ is the solution to the myopic algorithm. If $k < P$, go to Step 2.

### 3.6 LAGRANGIAN ALGORITHM FOR THE P – MEDIAN PROBLEM

Lagrangian relaxation is an approach to solving difficult problems such as integer programming problems. This technique gives us a clue as to whether the solution to the p – median problem is optimal or close to optimal. The Lagrangian relaxation is base on the premise that removing constraints from a problem makes the problem easier to solve. Therefore, to solve a problem, Lagrangian relaxations remove a constraint but introduce a penalty for violating the removed constraint. This revised problem is then optimized accordingly.

The Lagrangian relaxation approach under the p – median problem involves the following steps:

1. We remove constraint (3.14) and add the constraint and a vector of variables called Lagrange multipliers to the objective function.
2. We solve the resulting relaxed problem to find the optimal values of the original decision variables (in the relaxed problem).
3. Use the resulting decision variables from the solution to the relaxed problem found in step (3) to find a feasible solution to the original problem. Update the lower bound (LB) on the best feasible solution known for the problem.
4. Use the solution obtained in step (2) to compute a lower bound on the best value of the objective function.
5. Examine the solution obtained in step (2) and determine which of the relaxed constraints are violated. Use, for example, the subgradient optimization method to modify the Lagrange multipliers in such a way that the violated constraints are less likely to be violated on the subsequent iteration.

### 3.6.1 TERMINATION OF THE LAGRANGIAN ALGORITHM

The Lagrangian algorithm is terminated when one/more of the following conditions are met:

1. When a number of specified iterations is done.

2. The lower bound equals the upper bound $(i.e\ L^n = UB), or\ L^n$ close enough to UB.

3. $a^n$ becomes very small. When $a^n$ is very small, the changes are not likely to help solve the problem (Daskin, 1952).

4. When there is no violation of the relaxed constraints $\left(i.e., Q = \sum_i \{\sum_j Y_{ij}{}^n - 1\}^2 = 0\right)$.



### 3.6.2 FORMULATION OF THE LAGRANGIAN ALGORITHM

Restating the p – median problem

Minimize

$$\sum_i \sum_j h_i d_{ij} Y_{ij} \text{......}(3.4)$$

Subject to

$$\sum_j Y_{ij} = 1 \qquad \forall i \text{......}(3.5)$$

$$\sum_j X_j = P \text{......}(3.6)$$

$$Y_{ij} - X_j \leq 0 \qquad \forall i, j \text{......}(3.7)$$

$$X_j = 0\,,1 \qquad \forall j \text{......}(3.8)$$

$$Y_{ij} = 0\,,1 \qquad \forall j \text{......}(3.9)$$

If constraint (3.5) is relaxed, the following problem is then obtained

$$\underset{\lambda}{MAX} \quad \underset{X,Y}{MIN} \quad \sum_i \sum_j h_i d_{ij} Y_{ij} + \sum_i \lambda_i \left[1 - \sum_j Y_{ij}\right] \tag{3.10}$$

Subject to:

$\sum_j X_j = P$.............................................................................................(3.11)

$Y_{ij} - X_j \leq$

$0 \quad \forall i, j$ … … … … … … … … … … … … … … … … … … … … … … … … … (3.12)

$X_j =$

$0, 1 \qquad \forall j$ … … … … … … … … … … … … … … … … … … … … … … … … … (3.13)

$Y_{ij} = \ 0, 1 \qquad \forall j$ … … … … … … … … … … … … … … … … … … … … … … … … … … (3.14)

Solving the above problem. For fixed values of the Lagrange multipliers, $\lambda_i$ , the objective function in the previous step is minimized by computing the value of setting each of the location variables (X) to 1. This value is given by:

$$V_j = \sum_i \min(0, h_i d_{ij} - \lambda_i)$$

(3.15)

For each candidate location j. The P smallest values of V is then determined and the corresponding location variables (X) are set to 1 and all other location variables (X) to 0. The allocation variables (Y) are then set to:

$$\begin{cases} 1, \quad \text{if } X_j = 1 \text{ } and \text{ } h_i d_{ij} - \lambda_i < 0 \\ \end{cases}$$

$$Y_{ij} = \quad 0, \quad \text{if not} \tag{3.16}$$

For each iteration of this process, an upper bound of the objective function (estimate of a worst case scenario) and a lower bound (an optimistic estimate of the best case scenario) need to be determined. An upper bound is a solution that has been discovered which meets the constraints of the original unmodified problem. The lowest upper bound (best guess for the worst case) is sought for the purposes of this algorithm. The upperbound can be determined by simply determining the location closest to each customer. The corresponding allocation variables(Y) are then set to 1 while all others are set to 0. We then evaluate the P-Median objective function as stated originally.

Note that a solution to the simplified problem as outlined in Step 2, may or may not meet the constraints of the original problem. Since the modified problem need not meet the constraints of the original, the modified problem will produce an answer which will always be better or equal to the solution of the original problem. Thus, a lower bound on the P-Median problem can be determined by simply evaluating the original P-Median objective function using the values for the variables determined in Step 2.

A technique that drives the iterations to an optimal solution that meets the constraints of the original problem called subgradient optimization is used to update the value of the Lagrange Multipliers; the details of which are beyond the scope of this section. Based on subgradient optimization, a new variable t is introduced and defined as follows:

$$t^n = \frac{A^n(UB - L^n)}{\sum_i \left\{\sum_j Y_{ij} - 1\right\}} \tag{3.17}$$

Where

$t^n$ = the stepsize at the $n^{th}$ iterations of the Laagrangian procedure

$a^n$ = a constant on the $n^{th}$ iteration, with $a^1$ generally set to 2

UB = The best (smallest) upper bound on the p – median objective function.

$L^n$= the objective function of the Lagrangian function on the $n^{th}$ iteration

${Y_{ij}}^n$= the optimal value of the allocation variable, ${Y_{ij}}^n$ on the $n^{th}$ iteration.

The Lagrange multipliers are then updated according the following equation:

$$\lambda_i^{n+1} = \max\left\{0, \lambda_i^{n} - t^n\left(\sum_j {Y_{ij}}^n - 1\right)\right\} \tag{3.18}$$

**3.7 NETWORK BASED ALGORITHMS**

**3.7.1 SHORTEST PATH PROBLEM**

Shortest path problems are the most fundamental and most commonly encountered problems in the study of transportation, communication, and computer networks. In graph theory, the shortest path problem is the problem of finding a path between two vertices (or nodes) in a graph such that the sum of the weights of its constituent edges is minimized. An example is finding the quickest way to get from one location to another on a road map; in this case, the vertices represent locations and the edges represent segments of road and are weighted by the time needed to travel that segment.

There are many types of shortest paths problems. The two most important shortest path problems are;

- How to determine shortest path distance (a shortest path) from a specific node S to another specific node T
- How to determine shortest distances (and paths) from every node to every other node in the network (S. K. Amponsah, 2009)

We discuss few of the network algorithm problems.



### 3.7.1.1 DIJKSTRA'S ALGORITHM

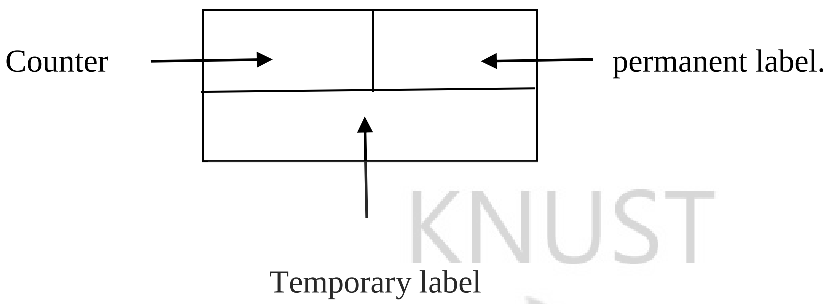
The Dijkstra's algorithm is one of the algorithms for finding the shortest path problem. The Dijkstra's algorithm finds the shortest path from a source $s$ to all other nodes in the network with nonnegative lengths. It maintains a distance label $d(i)$ with each node $i$, which is an upper bound on the shortest path length from the source node to any other node $j$. At any intermediate step, the algorithm divides the nodes of the network under consideration into two groups: those which it designates as permanently labeled (or permanent), and those which it designates as temporarily labeled (or temporal). The distance label to any permanent node represents the shortest distance from the source node to that node. The algorithm involves the following steps:

1. Assign the permanent label 0 to the starting vertex.
2. Assign temporary labels to all the vertices that are connected directly to the most recently permanent labeled vertex.
3. Choose the vertex with the smallest temporary label and assign a permanent label to that vertex.

4. Repeat step 2 and 3 until all vertices have permanently labels.

5. Find the shortest path by tracing back through the network.

It is important to notice that, recording the order in which permanent labels are assigned to the vertices is an essential part of the algorithm.






**The algorithm gradually changes all temporary labels into permanent ones (Comerford et al., 2004).**

### 3.7.1.2 FLOYD – WARSHAL'S ALGORITHM

The Floyd – Warshall algorithm is used to find shortest paths in a weighted graph. The algorithm obtains a matrix of shortest path distance within $0\{n^3\}$ computations. The algorithm is based on inductive arguments developed by an application of a dynamic programming technique.

Let $d^k(i,j)$ represent the length of the shortest path from node $i$ to node *j* subject to the condition that this path uses the nodes $1,2, \dots\dots, k-1$ as internal nodes. Clearly, $d^{k+1}(i,j)$ represent the actual shortest path distance from the node *i* to *j*. The algorithm first computes for all node pairs *i* and *j*. Using $d^1(i,j)$ , it then computes $d^2(i,j)$ for all node pairs *i* and *j*. It repeats this process until it obtains $d^{k+1}(i,j)$ for all node pairs *i* and $j$, then it terminates. Given $d^k(i,j)$, the algorithm computes $d^{k+1}(i,j)$ using $d^{k+1}(i,j) = \min\{d^k(i,k), d^k(k,j)\}$. The Floyd – Warshall algorithm remains of interest because it handles negative weight edges.

## CHAPTER 4

## DATA COLLECTION AND ANALYSIS

### 4.0 INTRODUCTION

In this chapter, we analyse data of population and road distances of the suburbs of Nkoranza obtained from the Municipal Statistical Service Department and town and country planning, Nkoranza respectively. The shortest path between connecting suburbs is of interest in this study, hence we considered the ten (10) suburbs of Nkoranza Township. The 10 suburbs with their respective populations and nodes is shown in Table 4.1 below.



**Table 2.1 Data of 2010 population of various Nkoranza suburbs**

| NODE | LOCATION | POPULATION |
|---|---|---|
| A | SESSIMAN | 5022 |
| B | NKORANZA FIE | 2230 |
| C | KOKOFU KOASE | 4866 |
| D | KRANSIESO | 5602 |
| E | ADINKRA AKYI | 5882 |
| F | ESTATE | 3860 |
| G | KASSADJAN | 6602 |
| H | NEW TOWN | 3426 |
| I | BREMAN | 4087 |
| J | AKUMSA DUMASE | 3445 |

The set of distances of roads linking the suburbs is shown in Table 4.2

**Table 4.2: Distances of roads connecting the suburbs (nearest meters)**

| From \ To | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| A | - | 1 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 4 | ∞ |
| B | 1 | - | 1 | 1 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ |
| C | ∞ | 1 | - | 1 | 1 | ∞ | 1 | ∞ | 3 | ∞ |
| D | ∞ | 1 | 1 | - | 1 | ∞ | ∞ | ∞ | ∞ | ∞ |
| E | ∞ | ∞ | 1 | 1 | - | 1 | 1 | ∞ | ∞ | ∞ |
| F | ∞ | ∞ | ∞ | ∞ | 1 | - | 1 | 1 | ∞ | ∞ |
| G | ∞ | ∞ | 1 | ∞ | 1 | 1 | - | 1 | 2 | ∞ |
| H | ∞ | ∞ | ∞ | ∞ | ∞ | 1 | 1 | - | 3 | 5 |
| I | 4 | ∞ | 3 | ∞ | ∞ | ∞ | 2 | 3 | - | 1 |
| J | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 5 | 1 | - |

The above data has been develop into a network and is shown in Figure 4.1. Numbers in the boxes next to the nodes are the total population of each corresponding suburb of Nkoranza. These numbers represent the demand $(h_i)$

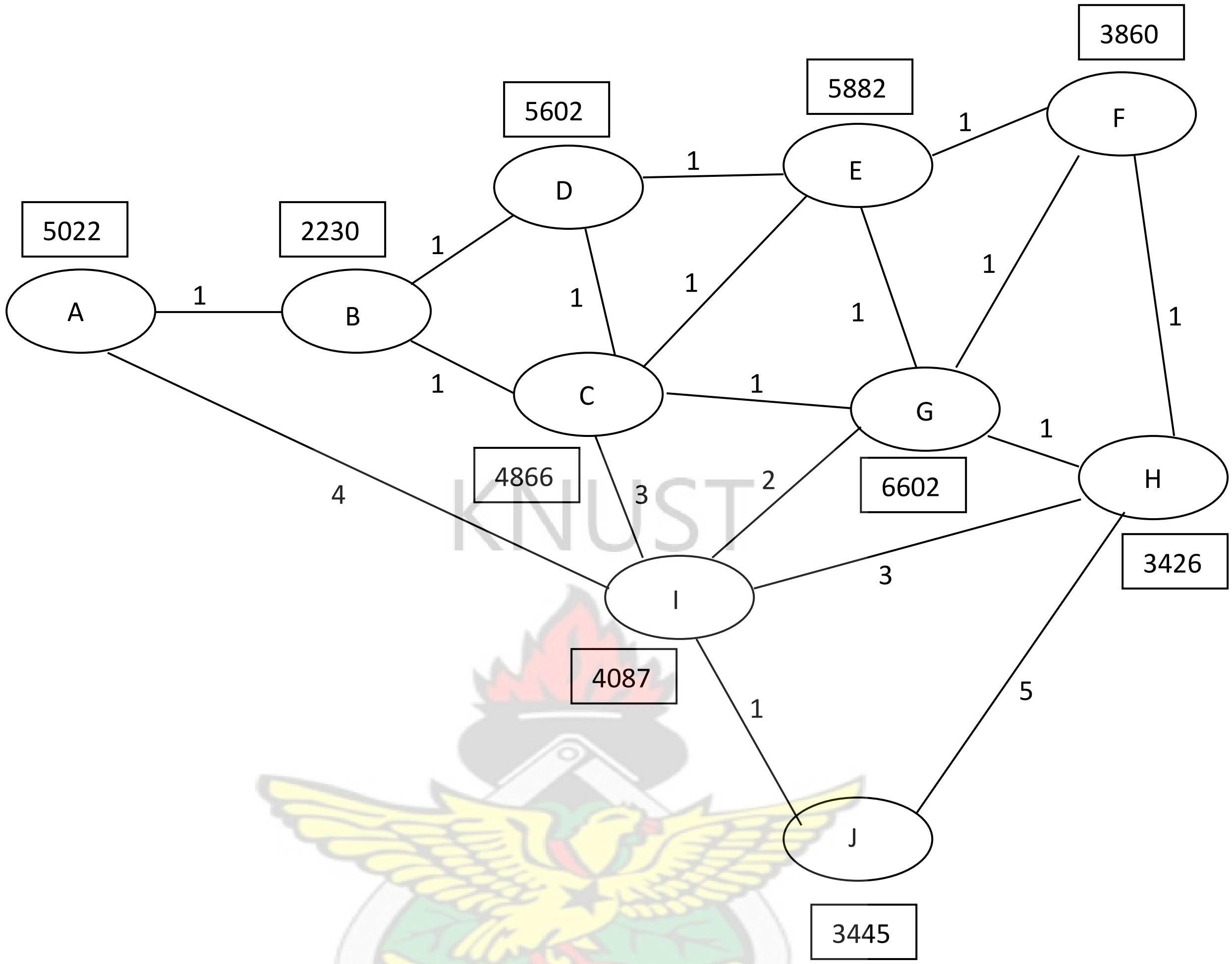



**Figure 4.1 Road network of Nkoranza Township with populations**

By using the Floyd's algorithm we obtain the shortest path matrix or distance matrix for Figure 4.1 and this is shown in Table 4.3

**Table 4.3: Shortest path distance matrix $d(i,j)$ of roads connecting the suburbs (in kilometers)**

| To / From | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| A | - | 1 | 2 | 2 | 7 | 7 | 6 | 7 | 4 | 5 |
| B | 1 | - | 1 | 1 | 2 | 7 | 2 | 8 | 4 | 6 |
| C | 2 | 1 | - | 1 | 1 | 2 | 1 | 6 | 3 | 7 |
| D | 2 | 1 | 1 | - | 1 | 2 | 2 | 7 | 4 | 7 |
| E | 7 | 2 | 1 | 1 | - | 1 | 1 | 2 | 3 | 8 |
| F | 7 | 7 | 2 | 2 | 1 | - | 1 | 1 | 3 | 6 |
| G | 6 | 2 | 1 | 2 | 1 | 1 | - | 1 | 2 | 6 |
| H | 7 | 8 | 6 | 7 | 2 | 1 | 1 | - | 3 | 4 |
| I | 4 | 4 | 3 | 4 | 3 | 3 | 2 | 3 | - | 1 |
| J | 5 | 6 | 7 | 7 | 8 | 6 | 6 | 4 | 1 | - |

Table 4.4 shows the shortest path distance matrix together with the demands at the various nodes.

**Table 4.4: Demand ($h_i$) and shortest path distance matrix $d(i,j)$**

| | | To | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $h_i$ | From | A | B | C | D | E | F | G | H | I | J |
| 5022 | A | 0 | 1 | 2 | 2 | 7 | 7 | 6 | 7 | 4 | 5 |
| 2230 | B | 1 | 0 | 1 | 1 | 2 | 7 | 2 | 8 | 4 | 6 |
| 4866 | C | 2 | 1 | 0 | 1 | 1 | 2 | 1 | 6 | 3 | 7 |
| 5602 | D | 2 | 1 | 1 | 0 | 1 | 2 | 2 | 7 | 4 | 7 |
| 5882 | E | 7 | 2 | 1 | 1 | 0 | 1 | 1 | 2 | 3 | 8 |
| 3860 | F | 7 | 7 | 2 | 2 | 1 | 0 | 1 | 1 | 3 | 6 |
| 6602 | G | 6 | 2 | 1 | 2 | 1 | 1 | 0 | 1 | 2 | 6 |
| 3426 | H | 7 | 8 | 6 | 7 | 2 | 1 | 1 | 0 | 3 | 4 |
| 4087 | I | 4 | 4 | 3 | 4 | 3 | 3 | 2 | 3 | 0 | 1 |
| 3445 | J | 5 | 6 | 7 | 7 | 8 | 6 | 6 | 4 | 1 | 0 |

## 4.1 MYOPIC ALGORITHM FOR THE P - MEDIAN

We use myopic algorithm to find the first two medians. We first find demand time distance $[h_i * d(i,j)]$. Thus we multiply row A by $h_A$ and row B by $h_B$ and so on. By summing the entries in each column we obtain the values of $Z_j^{\,1}$. The smallest value of $Z_j^{\,1}$ gives the solution to the p – median problem. This is shown in Table 4.5.

**Table 4.5: Myopic median $[h_i * d(i, j)]$**

| | Node $j$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Node $i$ | A | B | C | D | E | F | G | H | I | J |
| A | 0 | 5022 | 10044 | 10044 | 35154 | 35154 | 30132 | 35154 | 20088 | 25110 |
| B | 2230 | 0 | 2230 | 2230 | 4460 | 15610 | 4460 | 17840 | 8920 | 13380 |
| C | 9732 | 4866 | 0 | 4866 | 4866 | 9732 | 4866 | 29196 | 14598 | 34062 |
| D | 11204 | 5602 | 5602 | 0 | 5602 | 11204 | 11204 | 39214 | 22408 | 39214 |
| E | 41174 | 11764 | 5882 | 5882 | 0 | 5882 | 5882 | 11764 | 17646 | 47056 |
| F | 27020 | 27020 | 7720 | 7720 | 3860 | 0 | 3860 | 3860 | 11580 | 23160 |
| G | 39612 | 13204 | 6602 | 13204 | 6602 | 6602 | 0 | 6602 | 13204 | 39612 |
| H | 23982 | 27408 | 20556 | 23982 | 6852 | 3426 | 3426 | 0 | 10278 | 13704 |
| I | 16348 | 16348 | 12261 | 16348 | 12261 | 12261 | 8174 | 12261 | 0 | 4087 |
| J | 17225 | 20670 | 24115 | 24115 | 27560 | 20670 | 20670 | 13780 | 3445 | 0 |
| Total | 188527 | 131904 | 95012 | 108391 | 107217 | 120541 | **92674** | 169671 | 122167 | 239385 |

When we sum all the columns, we have node G as the minimum with 92674 so for one facility we locate it at node G. To locate a second median, we compute $[h_i * \min\{d(i, G); d(i, j)\}]$ for each node location pair $(i, j)$. Hence we adjust the distance matrix and the results is shown in Table 4.6

**Table 4.6: The second Myopic median [$\boldsymbol{h_i} * \mathbf{min}\{\boldsymbol{d(i,j)}; \boldsymbol{d(i,G)}\}$]**

| | Node $j$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Node $i$ | A | B | C | D | E | F | G | H | I | J |
| A | 0 | 5022 | 10044 | 10044 | 30132 | 30132 | 30132 | 30132 | 20088 | 25110 |
| B | 2230 | 0 | 2230 | 2230 | 4460 | 4460 | 4460 | 4460 | 4460 | 4460 |
| C | 4866 | 4866 | 0 | 4866 | 4866 | 4866 | 4866 | 4866 | 4866 | 4866 |
| D | 11204 | 5602 | 5602 | 0 | 5602 | 11204 | 11204 | 11204 | 11204 | 11204 |
| E | 5882 | 5882 | 5882 | 5882 | 0 | 5882 | 5882 | 5882 | 5882 | 5882 |
| F | 3860 | 3860 | 3860 | 3860 | 3860 | 0 | 3860 | 3860 | 3860 | 3860 |
| G | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| H | 3426 | 3426 | 3426 | 3426 | 3426 | 3426 | 3426 | 0 | 3426 | 3426 |
| I | 8174 | 8174 | 8174 | 8174 | 8174 | 8174 | 8174 | 8174 | 0 | 4087 |
| J | 17225 | 20670 | 20670 | 20670 | 20670 | 20670 | 20670 | 13780 | 3445 | 0 |
| Total | **56867** | 57502 | 59888 | 59152 | 81190 | 88814 | 92674 | 82358 | 57231 | 62895 |

Node A gives the minimum value of 56867 when the columns of the above matrix are added, so for two facilities we have node G and node A with an objective value of 56867.

## 4.2 LAGRAGIAN RELAXATION FOR THE P – MEDIAN

We use the lagrangian algorithm formulated in section 3.6 to solve the 2 – median problem as follows.

Minimize

$$0Y_{AA} + 5022Y_{AB} + 10044Y_{AC} + 10044Y_{AD} + 35154Y_{AE} + 35154Y_{AF} + 30132Y_{AG} + 35154Y_{AH}$$

$$+20088Y_{AI} + 25110Y_{AJ} + 2230Y_{BA} + 0Y_{BB} + 2230Y_{BC} + 2230Y_{BD} + 4460Y_{BE} + 15610Y_{BF}$$

$$+4460Y_{BG} + 17840Y_{BH} + 8920Y_{BI} + 13380Y_{BJ} + 9732Y_{CA} + 4866Y_{CB} + 0Y_{CC} + 4866Y_{CD}$$

$$+4866Y_{CE} + 9732Y_{CF} + 4866Y_{CG} + 29196Y_{CH} + 14598Y_{CI} + 34062Y_{CJ} + 11204Y_{DA} +$$

$$5602Y_{DB} + 5602Y_{DC} + 0Y_{DD} + 5602Y_{DE} + 11204Y_{DF} + 11204Y_{DG} + 39214Y_{DH} + 22408Y_{DI}$$

$$+39214Y_{DJ} + 41174Y_{EA} + 11764Y_{EB} + 5882Y_{EC} + 5882Y_{ED} + 0Y_{EE} + 5882Y_{EF} + 5882Y_{EG}$$

$$+11764Y_{EH} + 17646Y_{EI} + 47056Y_{EJ} + 27020Y_{FA} + 27020Y_{FB} + 7720Y_{FC} + 7720Y_{FD}$$

$$+3860Y_{FE} + 0Y_{FF} + 3860Y_{FG} + 3860Y_{FH} + 11580Y_{FI} + 23160Y_{FJ} + 39612Y_{GA} + 13204Y_{GB}$$

$$+6602Y_{GC} + 13204Y_{GD} + 6602Y_{GE} + 6602Y_{GF} + 0Y_{GG} + 6602Y_{GH} + 13204Y_{GI} + 39612Y_{GJ} +$$

$$23982Y_{HA} + 27408Y_{HB} + 20556Y_{HC} + 23982Y_{HD} + 6852Y_{HE} + 3426Y_{HF} + 3426Y_{HG} + 0Y_{HH}$$

$$+10278Y_{HI} + 13704Y_{HJ} + 16348Y_{IA} + 16348Y_{IB} + 12261Y_{IC} + 16348Y_{ID} + 12261Y_{IE} +$$

$$12261Y_{IF} + 8174Y_{IG} + 12261Y_{IH} + 0Y_{II} + 4087Y_{IJ} + 17225Y_{JA} + 20670Y_{JB} + 24115Y_{JC}$$

$$+24115Y_{JD} + 27560Y_{JE} + 20670Y_{JF} + 20670Y_{JG} + 13780Y_{JH} + 3445Y_{JI} + 0Y_{JJ}$$

(4.1)

Subject to:

$$Y_{AA} + Y_{AB} + Y_{AC} + Y_{AD} + Y_{AE} + +Y_{AF} + Y_{AG} + Y_{AH} + Y_{AI} + Y_{AJ} = 1$$

$$Y_{BA} + Y_{BB} + Y_{BC} + Y_{BD} + Y_{BE} + Y_{BF} + Y_{BG} + Y_{BH} + Y_{BI} + Y_{BJ} = 1$$

$$Y_{CA} + Y_{CB} + Y_{CC} + Y_{CD} + Y_{CE} + Y_{CF} + Y_{CG} + Y_{CH} + Y_{CI} + Y_{CJ} = 1$$

$$Y_{DA} + Y_{DB} + Y_{DC} + Y_{DD} + Y_{DE} + Y_{DF} + Y_{DG} + Y_{DH} + Y_{DI} + Y_{DJ} = 1$$

$$Y_{EA} + Y_{EB} + Y_{EC} + Y_{ED} + Y_{EE} + Y_{EF} + Y_{EG} + Y_{EH} + Y_{EI} + Y_{EJ} = 1$$

$$Y_{FA} + Y_{FB} + Y_{FC} + Y_{FD} + Y_{FE} + Y_{FF} + Y_{FG} + Y_{FH} + Y_{FI} + Y_{FJ} = 1$$

$$Y_{GA} + Y_{GB} + Y_{GC} + Y_{GD} + Y_{GE} + Y_{GF} + Y_{GG} + Y_{GH} + Y_{GI} + Y_{GJ} = 1$$

$$Y_{HA} + Y_{HB} + Y_{HC} + Y_{HD} + Y_{HE} + Y_{HF} + Y_{HG} + Y_{HH} + Y_{HI} + Y_{HJ} = 1$$

$$Y_{IA} + Y_{IB} + Y_{IC} + Y_{ID} + Y_{IE} + Y_{IF} + Y_{IG} + Y_{IH} + Y_{II} + Y_{IJ} = 1$$

$$Y_{JA} + Y_{JB} + Y_{JC} + Y_{JD} + Y_{JE} + Y_{JF} + Y_{JG} + Y_{JH} + Y_{JI} + Y_{JJ} = 1$$

(4.2)

$$X_A + X_B + X_C + X_D + X_E + X_F + X_G + X_H + X_I + X_J = 2 \dots\dots\dots\dots\,(4.3)$$

$$Y_{AA},\ Y_{AB},\ Y_{AC},\ Y_{AD},\ Y_{AE},\ Y_{AF},\ Y_{AG},\ Y_{AH},\ Y_{AI},\ Y_{AJ} \le X_A$$

$$Y_{BA},\ Y_{BB},\ Y_{BC},\ Y_{BD},\ Y_{BE},\ Y_{BF},\ Y_{BG},\ Y_{BH},\ Y_{BI},\ Y_{BJ} \le X_B$$

$$Y_{CA},\ Y_{CB},\ Y_{CC},\ Y_{CD},\ Y_{CE},\ Y_{CF},\ Y_{CG},\ Y_{CH},\ Y_{CI},\ Y_{CJ} \le X_C$$

$$Y_{DA},\ Y_{DB},\ Y_{DC},\ Y_{DD},\ Y_{DE},\ Y_{DF},\ Y_{DG},\ Y_{DH},\ Y_{DI},\ Y_{DJ} \le X_D$$

$$Y_{EA},\ Y_{EB},\ Y_{EC},\ Y_{ED},\ Y_{EE},\ Y_{EF},\ Y_{EG},\ Y_{EH},\ Y_{EI},\ Y_{EJ} \le X_E$$

$$Y_{FA},\ Y_{FB},\ Y_{FC},\ Y_{FD},\ Y_{FE},\ Y_{FF},\ Y_{FG},\ Y_{FH},\ Y_{FI},\ Y_{FJ} \le X_F$$

$$Y_{GA},\ Y_{GB},\ Y_{GC},\ Y_{GD},\ Y_{GE},\ Y_{GF},\ Y_{GG},\ Y_{GH},\ Y_{GI},\ Y_{GJ} \le X_G$$

$$Y_{HA},\ Y_{HB},\ Y_{HC},\ Y_{HD},\ Y_{HE},\ Y_{HF},\ Y_{HG},\ Y_{HH},\ Y_{HI},\ Y_{HJ} \le X_H$$

$$Y_{IA},\ Y_{IB},\ Y_{IC},\ Y_{ID},\ Y_{IE},\ Y_{IF},\ Y_{IG},\ Y_{AH},\ Y_{II},\ Y_{IJ} \le X_I$$

$$Y_{JA},\ Y_{JB},\ Y_{JC},\ Y_{JD},\ Y_{JE},\ Y_{JF},\ Y_{JG},\ Y_{JH},\ Y_{JI},\ Y_{JJ} \le X_J$$

(4.4)

$$X_A,\ X_B,\ X_C,\ X_D,\ X_E,\ X_G,\ X_H,\ X_I,\ X_J\ \in \{0,1\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(4.5)$$

$$Y_{AA},\ Y_{AB},\ Y_{AC},\ Y_{AD},\ Y_{AE},\ Y_{AF},\ Y_{AG},\ Y_{AH},\ Y_{AI},\ Y_{AJ}, Y_{BA},\ Y_{BB},\ Y_{BC},\ Y_{BD},\ Y_{BE},\ Y_{BF},\ Y_{BG},\ Y_{BH},$$
$$Y_{BI}, Y_{BJ}, Y_{CA},\ Y_{CB},\ Y_{CC},\ Y_{CD},\ Y_{CE},\ Y_{CF},\ Y_{CG},\ Y_{CH},\ Y_{CI},\ Y_{CJ}, Y_{DA},\ Y_{DB},\ Y_{DC},\ Y_{DD},\ Y_{DE},\ Y_{DF},$$
$$Y_{DG},\ Y_{DH}, Y_{DI},\ Y_{DJ}, Y_{EA},\ Y_{EB},\ Y_{EC},\ Y_{ED},\ Y_{EE},\ Y_{EF},\ Y_{EG},\ Y_{EH},\ Y_{EI},\ Y_{EJ}, Y_{FA},\ Y_{FB},\ Y_{FC},\ Y_{FD},$$
$$Y_{FE},\ Y_{FF},\ Y_{FG}, Y_{FH},\ Y_{FI},\ Y_{FJ}, Y_{GA},\ Y_{GB},\ Y_{GC},\ Y_{GD},\ Y_{GE},\ Y_{GF},\ Y_{GG},\ Y_{GH},\ Y_{GI},\ Y_{GJ}, Y_{HA},\ Y_{HB},$$
$$Y_{HC},\ Y_{HD},\ Y_{HE},\ Y_{HF}, Y_{HG},\ Y_{HH},\ Y_{HI},\ Y_{HJ}, Y_{IA},\ Y_{IB},\ Y_{IC},\ Y_{ID},\ Y_{IE},\ Y_{IF},\ Y_{IG},\ Y_{AH},\ Y_{II},\ Y_{IJ}$$
$$, Y_{JA},\ Y_{JB},\ Y_{JC},\ Y_{JD},\ Y_{JE}, Y_{JF},\ Y_{JG},\ Y_{JH},\ Y_{JI},\ Y_{JJ} = \{0,1\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(4.6)$$



At this point, we relax constraint (4.2). We let the Lagrangian multiplier be $\lambda_i$, then we multiply the constraints through by the Lagrange multipliers, $\lambda_i$, and then bring them into the objective function. The end result, is shown in equation (4.1b).

$$\underset{\lambda}{MAX}\ \underset{X,Y}{MIN}$$
$$(0-\lambda_A)Y_{AA} + (5022Y_{AB} - \lambda_A) + (10044 - \lambda_A)Y_{AC} + (10044 - \lambda_A)Y_{AD} + (35154 - \lambda_A)Y_{AE}$$
$$+(35154 - \lambda_A)Y_{AF} + (30132 - \lambda_A)Y_{AG} + (35154 - \lambda_A)Y_{AH} + (20088 - \lambda_A)Y_{AI} +$$
$$(25110 - \lambda_A)Y_{AJ} + (2230 - \lambda_B)Y_{BA} + (0 - \lambda_B)Y_{BB} + (2230 - \lambda_B)Y_{BC} + (2230 - \lambda_B)Y_{BD} +$$
$$(4460 - \lambda_B)Y_{BE} + (15610 - \lambda_B)Y_{BF} + (4460 - \lambda_B)Y_{BG} + (17840 - \lambda_B)Y_{BH} + (8920 - \lambda_B)Y_{BI}$$
$$+(13380 - \lambda_B)Y_{BJ} + (9732 - \lambda_C)Y_{CA} + (4866 - \lambda_C)Y_{CB} + (0 - \lambda_C)Y_{CC} + (4866 - \lambda_C)Y_{CD}$$
$$+(4866 - \lambda_C)Y_{CE} + (9732 - \lambda_C)Y_{CF} + (4866 - \lambda_C)Y_{CG} + (29196 - \lambda_C)Y_{CH} +$$
$$(14598 - \lambda_C)Y_{CI} + (34062 - \lambda_C)Y_{CJ} + (11204 - \lambda_D)Y_{DA} + (5602 - \lambda_D)Y_{DB} + (5602 - \lambda_D)Y_{DC}$$
$$+(0 - \lambda_D)Y_{DD} + (5602 - \lambda_D)Y_{DE} + (11204 - \lambda_D)Y_{DF} + (11204 - \lambda_D)Y_{DG} + (39214$$
$$-\lambda_D)Y_{DH} + (22408 - \lambda_D)Y_{DI} + (39214 - \lambda_D)Y_{DJ} + (41174 - \lambda_E)Y_{EA} + (11764 - \lambda_E)Y_{EB}$$

$$+(5882-\lambda_E)Y_{EC}+(5882-\lambda_E)Y_{ED}+(0-\lambda_E)Y_{EE}+(5882-\lambda_E)Y_{EF}+(5882-\lambda_E)Y_{EG}$$

$$+(11764-\lambda_E)Y_{EH}+(17646-\lambda_E)Y_{EI}+(47056-\lambda_E)Y_{EJ}+(27020-\lambda_F)Y_{FA}+$$

$$(27020-\lambda_F)Y_{FB}+(7720-\lambda_F)Y_{FC}+(7720-\lambda_F)Y_{FD}+(3860-\lambda_F)Y_{FE}$$

$$+(0-\lambda_F)Y_{FF}$$

$$+(3860-\lambda_F)Y_{FG}+(3860-\lambda_F)Y_{FH}+(11580-\lambda_F)Y_{FI}+(23160-\lambda_F)Y_{FJ}+$$

$$(39612-\lambda_G)Y_{GA}+(13204-\lambda_G)Y_{GB}+(6602-\lambda_G)Y_{GC}+(13204-\lambda_G)Y_{GD}+$$

$$(6602-\lambda_G)Y_{GE}+(6602-\lambda_G)Y_{GF}+(0-\lambda_G)Y_{GG}+(6602-\lambda_G)Y_{GH}+(13204-\lambda_G)Y_{GI}$$

$$+(39612-\lambda_G)Y_{GJ}+(23982-\lambda_H)Y_{HA}+(27408-\lambda_H)Y_{HB}+(20556-\lambda_H)Y_{HC}+$$

$$(23982-\lambda_H)Y_{HD}+(3426-\lambda_H)Y_{HF}+(6852-\lambda_H)Y_{HE}+(3426-\lambda_H)Y_{HG}+(0-\lambda_H)Y_{HH}$$

$$+(10278-\lambda_H)Y_{HI}+(13704-\lambda_H)Y_{HJ}+(16348-\lambda_I)Y_{IA}+(16348-\lambda_I)Y_{IB}+$$

$$(12261-\lambda_I)Y_{IC}+(16348-\lambda_I)Y_{ID}+(12261-\lambda_I)Y_{IE}+(12261-\lambda_I)Y_{IF}$$

$$+(8174-\lambda_I)Y_{IG}$$

$$+(12261-\lambda_I)Y_{IH}+(0-\lambda_I)Y_{II}+(4087-\lambda_I)Y_{IJ}+\left(17225-\lambda_J\right)Y_{JA}+(20670-\lambda_I)Y_{JB}$$

$$+\left(24115-\lambda_J\right)Y_{JC}+\left(24115-\lambda_J\right)Y_{JD}+\left(27560-\lambda_J\right)Y_{JE}+\left(20670-\lambda_J\right)Y_{JF}+$$

$$\left(20670-\lambda_J\right)Y_{JG}+\left(13780-\lambda_J\right)Y_{JH}+\left(3445-\lambda_J\right)Y_{JI}+\left(0-\lambda_J\right)Y_{JJ}+\lambda_A+\lambda_B+\lambda_C+\lambda_D$$

$$+\lambda_E+\lambda_F+\lambda_G+\lambda_H+\lambda_I+\lambda_J \tag{4.1b}$$

Subject to:

Constraint (4.3), (4.4), (4.5), (4.6)

### 4.2.1 ALGORITHM

STEPS:

1. Use the myopic algorithm to determine the upper bounds (UB)
2. Input $\lambda_i$ , $\alpha = 2$ , $h_i$ , $d_{ij}$ $and\ UB\ for\ i, j = A, B, C, D, E, F, G.$
3. For each $j$ , compute $U_{ij} = \begin{cases} h_i d_{ij} - \lambda_i & if\ h_i d_{ij} < \lambda_i \\ 0, & if\ h_i d_{ij} > \lambda_i \end{cases}$
4. Calculate $V_j = \sum_i U_{ij}$
5. Pick the two least values of $V_j$
6. For such j values, assign $X_{j1} = 1, X_{j2} = 1\ and\ Y_{ij} = 1\ for\ U_{ij} < 0.$
7. Calculate sum of square violation, $Q = \sum_i \{\sum_j Y_{ij}{}^n - 1\}^2 = 0$
8. Calculate $L^n = \sum_i \sum_j \left(h_i d_{ij} - \lambda_i{}^n\right) Y_{ij} + \sum_i \lambda_i{}^n$
9. If the sum of square violation, Q=0 then stop.
10. Otherwise test, if $L^n - L^{n-1} \leq 0$ then use $a^n = \frac{1}{2} a^{n-1}$, if not use $a^n = a^{n-1}$
11. Calculate $t^n = \frac{\alpha^n (UB - L^n)}{\sum_i \left\{ \sum_j Y_{ij} - 1 \right\}^2}$
12. Compute $\lambda_i{}^{n+1} = \max\left\{0, \lambda_i{}^n - t^n \left( \sum_j Y_{ij}{}^n - 1 \right)\right\}$
13. Return to step 2

### 4.2.2 SOLUTION TO LAGRAGIAN ALGORITHM

**First Iteration**

Step 1: Compute $V_j = \sum_i U_{ij}$ But $U_{ij} = \min\{0,\ h_i d_{ij} - \lambda_i\}$. We also let $\lambda_i = 5000$ for $i = A, B, C, D, E, F, G, H, I\ and\ J$. The computation is shown in Table 4.7 below. The column totals gives the values of $V_j\ for\ j = A, B, C, D, E, F, G, H, I\ and\ J$.

**Table 4.7: Values of** $V_j = \sum_i U_{ij}$

| | $U_A$ | $U_B$ | $U_C$ | $\boldsymbol{U_D}$ | $U_E$ | $U_F$ | $\boldsymbol{U_G}$ | $U_H$ | $U_I$ | $U_J$ |
|---|---|---|---|---|---|---|---|---|---|---|
| A | -5000 | 0 | 0 | **0** | 0 | 0 | **0** | 0 | 0 | 0 |
| B | -2770 | -5000 | -2770 | **-2770** | -540 | 0 | **-540** | 0 | 0 | 0 |
| C | 0 | -134 | -5000 | **-134** | -134 | 0 | **-134** | 0 | 0 | 0 |
| D | 0 | 0 | 0 | **-5000** | 0 | 0 | **0** | 0 | 0 | 0 |
| E | 0 | 0 | 0 | **0** | -5000 | 0 | **0** | 0 | 0 | 0 |
| F | 0 | 0 | 0 | **0** | -1140 | -5000 | **-1140** | -1140 | 0 | 0 |
| G | 0 | 0 | 0 | **0** | 0 | 0 | **-5000** | 0 | 0 | 0 |
| H | 0 | 0 | 0 | **0** | 0 | -1574 | **-1574** | -5000 | 0 | -6296 |
| I | 0 | 0 | 0 | **0** | 0 | 0 | **0** | 0 | -5000 | -913 |
| J | 0 | 0 | 0 | **0** | 0 | 0 | **0** | 0 | -1555 | -5000 |
| $V_j$ | -7770 | -5134 | -7770 | **-7904** | -6814 | -6574 | **-8388** | -6140 | -6555 | -5913 |

From Table 4.7 above If the demand at node $i$ is allocated to a facility at node $j$, then $Y_{ij} = 1$. The $V_j$ values suggest that, if a facility is located at node A, then $Y_{AA} = Y_{BA} =$

1. This means that, demands at nodes *A* and *B* would be allocated to the facility at node *A*, if the facility is at *A*. Similarly, if the facility is at B, then $Y_{BB} = Y_{CB} = 1$. Also, if the facility is at *C*, then $Y_{BC} = Y_{CC} = 1$. Again, if the facility is at *D*, then we have $Y_{BD} = Y_{CD} = Y_{DD} = 1$. If the facility is at *E*, then $Y_{BE} = Y_{CE} = Y_{EE} = Y_{FE} = 1$. Also if the facility is at F, then $Y_{FF} = Y_{HF} = 1$. If the facility is at *G*, then $Y_{BG} = Y_{CG} = Y_{FG} = Y_{GG} = Y_{HG} = 1$. If the facility is at *H*, then $Y_{FH} = Y_{HH} = 1$. Also if the facility is at I, then $Y_{II} = Y_{JI} = 1$. Lastly, if the facility is at J, then $Y_{HJ} = Y_{IJ} = Y_{JJ} = 1$. We want to locate two Library facilities, so we choose node G and D (the two nodes with the smallest $V_j$ values). Thus we set $X_G = X_D = 1$, and the rest $X_j = 0$. It means that $Y_{AA} = Y_{BA} = Y_{BD} = Y_{BG} = Y_{CD} = Y_{CG} = Y_{DD} = Y_{FG} = Y_{GG} = Y_{HG} = 1$, and the rest of $Y_{ij} = 0$.



Step 2: we find the first Lagrangian objective function value, $L^1$.

$$L^1 = \sum_i \sum_j (h_i d_{ij} - \lambda_i) Y_{ij} + \sum_i \lambda_i$$

$$= 10(5000) + (-8388 - 7904) = 33708$$

Next, we find the sum of square violation of constraint 4.2, which is

$$\sum_i (\sum_j Y_{ij}{}^1 - 1)^2 = (0-1)^2 + (2-1)^2 + (2-1)^2 + (1-1)^2 + (0-1)^2 + (1-1)^2 + (1-1)^2 + (1-1)^2 + (0-1)^2 + (0-1)^2 = 6$$

Step 3: we find the stepsize, $t^n = \dfrac{\alpha^n (UB - L^n)}{\sum_i \left\{ \sum_j Y_{ij} - 1 \right\}^2}$

$UB = 92674$ (i.e. myopic optimal value), $L^n = L^1 = 25938$, $\sum_i \left(\sum_j Y_{ij}{}^1 - 1\right)^2 = 6$,

$\alpha^n = \alpha^1 = 2$ (Daskin, 1952). It must be noted that, $\alpha^n$ is halved if $L^{i+1} - L^i \leq 0$. We have

$$t^1 = \frac{\alpha^1(UB - L^1)}{\sum_i \left\{\sum_j Y_{ij} - 1\right\}^2} = \frac{2(92674 - 33708)}{6} = 19655.3$$

Step 4: update the Lagrangian multiplier, $\lambda_i$,

$$\lambda_i^{\,n+1} = \max\left\{0, \lambda_i^{\,n} - t^n\left(\sum_j Y_{ij}^{\,n} - 1\right)\right\} \quad \text{Daskin(1952)}$$

$$\lambda_i^{\,2} = \max\left\{0, \lambda_i^{\,1} - t^n\left(\sum_j Y_{ij}^{\,n} - 1\right)\right\}$$

$$\lambda_A^{\,2} = \max\{0, 5000 - 19655.33(-1)\} = 24655.33$$

$$\lambda_B^{\,2} = \max\{0, 5000 - 19655.33(1)\} = 0$$

$$\lambda_C^{\,2} = \max\{0, 5000 - 19655.33(1)\} = 0$$

$$\lambda_D^{\,2} = \max\{0, 5000 - 19655.33(0)\} = 5000$$

$$\lambda_E^{\,2} = \max\{0, 5000 - 19655.33(-1)\} = 24655.33$$

$$\lambda_F^{\,2} = \max\{0, 5000 - 19655.33(0)\} = 5000$$

$$\lambda_G^{\,2} = \max\{0, 5000 - 19655.33(0)\} = 5000$$

$$\lambda_H^{\,2} = \max\{0, 5000 - 19655.33(0)\} = 5000$$

$$\lambda_I^{\,2} = \max\{0, 5000 - 19655.33(-1)\} = 24655.33$$

$$\lambda_J^{\,2} = \max\{0, 5000 - 19655.33(-1)\} = 24655.33$$

The results obtained from the various iterations of the Lagrangian algorithm are summarized in Table 4.8 below.

**Table 4.8 Computational Results of the Various Iterations.**

| Variable | 1st Iteration | 2nd Iteration | 3rd Iteration | 4th Iteration | 5th Iteration |
|---|---|---|---|---|---|
| $V_A$ | $-7770$ | $-108390$ | $-44386$ | $-225553$ | $-97464$ |
| $V_B$ | $-5134$ | $-119760$ | $-60654$ | $-282176$ | $-126164$ |
| $V_C$ | $-7770$ | $-151540$ | $\mathbf{-83899}$ | $\mathbf{-319068}$ | $\mathbf{-159538}$ |
| $V_D$ | $\mathbf{-7904}$ | $-138160$ | $-73946$ | $-305689$ | $146159$ |
| $V_E$ | $-6814$ | $\mathbf{-155640}$ | $\mathbf{-89737}$ | $-306863$ | $-159137$ |
| $V_F$ | $-6574$ | $-136510$ | $-69523$ | $-293539$ | $-143708$ |
| $V_G$ | $\mathbf{-8388}$ | $\mathbf{-159350}$ | $-80048$ | $\mathbf{-321406}$ | $\mathbf{-166553}$ |
| $V_H$ | $-6140$ | $-125580$ | $-52413$ | $-244409$ | $-112078$ |
| $V_I$ | $-6555$ | $-130270$ | $-55435$ | $-291913$ | $-132383$ |
| $V_J$ | $-5913$ | $-91346$ | $-35949$ | $-180343$ | $-73289$ |
| $L^n$ | $33708$ | $-64837$ | $-5836$ | $-226394$ | $-71541$ |
| $Q$ | 6 | 10 | 8 | 10 | 9 |
| $t^n$ | 19655.33 | 7875.6 | 24628 | 15953 | 36492 |
| $\alpha^n$ | 2 | 0.5 | 2 | 0.5 | 2 |
| ${\lambda_A}^{n+1}$ | 5000 | 24655.33 | 16780 | 25455 | 25455 |
| ${\lambda_B}^{n+1}$ | 0 | 16780 | 0 | 25455 | 0 |
| ${\lambda_C}^{n+1}$ | 0 | 16780 | 0 | 25455 | 0 |
| ${\lambda_D}^{n+1}$ | 5000 | 16780 | 0 | 25455 | 0 |
| ${\lambda_E}^{n+1}$ | 24655.33 | 16780 | 0 | 25455 | 0 |
| ${\lambda_F}^{n+1}$ | 5000 | 16780 | 0 | 25455 | 0 |
| ${\lambda_G}^{n+1}$ | 5000 | 16780 | 0 | 25455 | 0 |
| ${\lambda_H}^{n+1}$ | 5000 | 16780 | 16780 | 25455 | 0 |
| ${\lambda_I}^{n+1}$ | 24655.33 | 16780 | 0 | 25455 | 0 |
| ${\lambda_J}^{n+1}$ | 24655.33 | 16780 | 41408 | 25455 | 0 |

## 4.3 DISCUSSION OF RESULTS

The overall total demand (total population of Nkoranza) is 45022. From Table 4.5 the first myopic median correspond to node G with a total demand weighted distance of 92674 km. Thus, the optimal total demand – weighted distance if only one facility were to be located is 92674 km, resulting in an average distance of 2.06 km (92674 / 45022). This suggest that if one Library facility is to be located at Nkoranza township then it should be located at Kassadjan and each individual has to cover an average distance of 2.06 kilometers to reach the facility at Kassadjan.

For the second median, from Table 4.6 the facility is to be located at node A which is Sessiman. The total demand-weighted distance is 56867 km, resulting in an average distance of approximately 1.26 kilometers. This result also means that, if we locate two facilities at Nkoranza (Kassadjan and Sessiman), then the average distance that each person in Nkoranza would have to travel to the nearby facility is approximately 1.26 km.

In Table 4.8 there was an increase in the value of $L^n$ from the first iteration, then decreases in the second iteration and it also increases in the third iteration and so on. As a result the value of $\alpha^n$ has been fluctuating throughout the iterations. The Lagragian algorithm confirms the location at node G (Kassadjan) by the myopic algorithm.

# CHAPTER 5

# CONCLUSION AND RECOMMENDATION.

## 5.1 CONCLUSSION

The main objective of the thesis was to use the p – median model to find optimal site location for library facility at Nkoranza. We used the myopic algorithm and the Lagrangian Algorithm was used to find a suitable site. The results obtained from the myopic algorithm suggest that, if we want to locate the library facility then we need to locate the facility at Kassadjan, and if we want to locate a second facility, then it should be at Sessiman. The results obtain from the Lagrangian algorithm suggested that the facility should be located at Kassadjan (node G) with the upper bound being 92674km which is the total-demand weighted distance.

## 5.2 RECOMMENDATION

The following recommendations are made:

- Stakeholders, Corporate bodies, as well as private individuals, who wants to establish library at Nkoranza, are advised to establish it at Kassadjan.
- In this thesis we apply construction algorithm being the Myopic algorithm, and also used the Lagrangian algorithm, students and researchers can extend it by using the various improvement algorithms thus exchange algorithm and neighbourhood search algorithm.

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## APPENDICES

## APPENDIX A

**THE DISTANCE FROM ONE SUBURB TO THE OTHER**

| SUBURB | NKORANZA FIE | KOKOFU KOASE | KASSADJAN | ADINKRA AKYI | NEW TOWN | ESTATE | BREMAN | KRANSIESO | AKUMSA DUMASE | SESSIMAN |
|---|---|---|---|---|---|---|---|---|---|---|
| NKORANZA FIE | 0 | 1 | 2 | 2 | 3 | 3 | 4 | 1 | 5 | 1 |
| KOKOFU KOASE | 1 | 0 | 1 | 1 | 3 | 4 | 3 | 1 | 4 | 1 |
| KASSADJAN | 2 | 1 | 0 | 1 | 1 | 1 | 2 | 1 | 4 | 2 |
| ADINKRA AKYI | 2 | 1 | 1 | 0 | 1 | 1 | 3 | 1 | 5 | 2 |
| NEW TOWN | 3 | 3 | 1 | 1 | 0 | 1 | 3 | 2 | 5 | 3 |
| ESTATE | 3 | 4 | 1 | 1 | 1 | 0 | 3 | 2 | 5 | 3 |
| BREMAN | 4 | 3 | 2 | 3 | 3 | 3 | 0 | 3 | 1 | 4 |
| KRANSIESO | 1 | 1 | 1 | 1 | 2 | 2 | 3 | 0 | 4 | 1 |
| AKUMSA DUMASE | 5 | 4 | 4 | 5 | 5 | 5 | 1 | 4 | 0 | 6 |
| SESSIMAN | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 1 | 6 | 0 |

**NB:** *FIGURES ARE GIVEN TO THE NEAREST KILOMETRE*

## APPENDIX B

***POPULATION OF SUBURBS***

| NO. | SUBURB | POPULATION |
|---|---|---|
| 1. | NKORANZA FIE | **2230** |
| 2. | KOKOFU KOASE | **4866** |
| 3. | KASSADJAN | **6602** |
| 4. | ADINKRA AKYI | **5882** |
| 5. | NEW TOWN | **3426** |
| 6. | ESTATE | **3860** |
| 7. | BREMAN | **4087** |
| 8. | KRANSIESO | **5602** |
| 9. | AKUMSA DUMASE | **3445** |
| 10. | SESSIMAN | **5022** |