KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY

COLLEGE OF SCIENCE

FACULTY OF DISTANCE LEARNING

DEPARTMENT OF INDUSTRIAL MATHEMATICS

OPTIMAL ALLOCATION OF FUNDS, THE LOAN PORTFOLIO:

(A CASE STUDY OF CHRISTIAN COMMUNITY MICROFINANCE LIMITED (CCML))

By

GYAKWA AUGUSTINE (B.Sc MATHEMATICS AND STATISTICS)

A Thesis Submitted to the School of Graduate Studies, Kwame Nkrumah University of Science and Technology (KNUST) in partial fulfillment of the requirements for the award of a Master of Science degree in Industrial Mathematics

DECLARATION

I hereby declare that this submission is my own work towards the Master of Science degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for award of any other degree of the university, except due acknowledgement has been made in the text.

Gyakwa Augustine, PG 6326211	…………………	..……..………
(Student's Name & ID)	Signature	Date

Certified by

Prof. S. K. Amponsah	………………….	……………...
(Supervisor)	Signature	Date
Mr. F.K. Darkwah	…………………	………………
(Head of Department)	Signature	Date
Prof. I.K. Dontwi	…………………	………………
(Dean, IDL)	Signature	Date

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DEDICATION

To the Glory of God

I like to dedicate this project to my late father, Mr. Kwasi Gyakwa and my mother, Christiana Korkor Gyakwa.

ABSTRACT

Many Ghanaians both in the informal and the formal sectors take loans for various reasons some for their wards education or being investment in their businesses. Others also take loans to acquire personal properties such as houses and cars. Most people go to the financial institutions for Loans. Due to poor allocation of funds by most financial institutions to prospective loan seekers the institutions are not able to maximize their profits. In view of this monies that can be used for social services in the community in which they operate go into bad debt. The main objective of this study is to develop Linear Programming model to help Christian Community Microfinance Limited (CCML), Eastern Zone, to allocate their funds to prospective loan seekers in order for them to maximize their profits. The problem was modeled as a linear programming problem. Simplex algorithm was used to solve the problem. It was observed that, if CCML Eastern zone, disbursed a total of GH¢120,000.00 a profit of GH¢32,068.48 will be realized.

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CHAPTER 1

1.0 OVERVIEW

This chapter present the brief introduction and the background of the study, the problem statement, the objectives and it justification of the study. It also include the methodology used, the scope and limitation and the theories organization.

1.1 INTRODUCTION OF THE STUDY

Microfinance encompasses the provision of financial services and the management of small amounts of money through a range of products and a system of intermediary functions that are targeted at low income clients. Microfinance also refers to provision of small loans and other facilities like savings, insurance, transfer services to poor low-income household and microenterprises. Microcredit also refers to as a small loan to a client made by a bank or other institutions.

Poverty reduction has been the agenda on the table of most developing countries. As a measure of reducing poverty, most of the developing countries have adopted the system of empowering the individual to be self-reliant. As a mean of helping the individual to be self-reliant, the 1Government, Non-Governmental Organization (NGOs), financial institutions have been giving these individuals some amount of help in the form of capital.

Ghana has a population of about 24 million, with an annual population growth of about 2.5% (GSS 2010 provisional results). Ghana has focused on poverty reduction as the core of its development strategy.

This approach was galvanized in 1995, the launch of the vision 2020 agenda, which is focused on promoting and analyzing reduction in Ghana.

The Poverty Reduction Strategy was also prepared in 2000 and has since prepared the Ghana Poverty Reduction Strategy 2002-2004: The Growth and Prosperity Agenda. From time to time governments have introduced major policies and mechanisms all to improve upon the livelihoods of the people of Ghana. However whether these policy frameworks have yielded the required result is another debate in all spheres of our social, economic and political systems. The budget statement to Ghana's parliament in 2011 by the Minister of Finance indicates that all economic indicators are positive and favours the expansion of business:

- GDP growth of 4.1 percent in 2009 compared to the sub-Saharan Africa growth of 2.0 percent;
- The fiscal deficit reduced significantly from 14.5 percent of GDP on cash basis at the end of 2008 to 9.7 percent of GDP in 2009;
- Inflation has trended downwards in sixteen (16) consecutive months from 20.74 percent at the end of June 2009 to reach 9.38 percent in October 2010, the lowest in the last two decades;
- Gross international reserves of US$3,973.0 million at the end of October 2010 has exceeded three months of import cover compared with reserves of US$2,036.2 million at end December 2008 which could barely cover 2 months of import.

- The Cedi has strengthened and appreciated by 0.1 percent, 2.2 percent and 5.4 percent against the US dollar, the pound sterling and the euro respectively.

The 2011 budget therefore focused on major growth-oriented programmes and projects that would improve and sustain Ghana's middle income status. The government of Ghana as part of the development agenda has indicated in the 2011 budget statement that significant investment

will be made in the areas of energy, road and rail transport to facilitate private sector expansion for employment generation. All these interventions will not be able to yield the expected results if the credit system in Ghana is not vibrant. Whiles commercial banks are lending at between 17

to 28 percent per annum, rural banks and microfinance institution are lending at between 28 to 40 percent per annum.

Lack of access to credit is generally seen as one of the main reasons why many people in developing economies which Ghana is one remain poor. Usually, the poor have no access to loans from the banking system, because they cannot put up acceptable collateral and or because the costs for banks of screening and monitoring the activities of the poor, and of enforcing their contracts, are too high to make lending to this group profitable. However, the poor in developing economies heaved a sigh of relief as they continually gained access to small loans with the help of so-called microfinance programmes. In a developing country context, credit is an important instrument for improving the welfare of the poor directly (consumption smoothing that reduces their vulnerability to short-term income shocks) as well as for enhancing their productive capacity through financing investment in human and physical capital.

The demand for credit for productive investments usually comes from those poor who are less risk-averse and enables them to overcome liquidity constraints, making it possible to undertake investments that can boost production, employment and income. Formal lenders normally provide this type of credit. Informal lenders usually provide credit for consumption purposes, which can have a long-term positive impact on household productivity, allowing acquisition of skills or improvement in health status if such loans are used for education or health care. These may enhance or at least preserve the productivity of the labour force. The credit market is also, at least potentially, an important instrument for consumption smoothing. An investigation of household credit thus has implications that link together micro-level analysis with factors that determine long-term macro-economic performance. Commercial Banks forms a greater percentage of formal lenders in Ghana and access to them are restricted to a small proportion of the population who can meet their stringent requirements, which include high minimum balances for account opening, onerous collateral requirements for loans, and long and costly administrative processes. Banks are, furthermore, mainly urban based, thereby adding the burden of transport costs if the predominantly rural population wishes to use bank facilities. Due to the lack of access to formal credit, the poor rely almost exclusively on the informal financial sector. Informal lenders innovatively seek to solve the problems of high risk, high cost and low returns that banks face when serving the poor. In practice households apply for credit, but lenders determine how much credit is allocated to them, based on their perception of the household's credit-worthiness. This often results in credit rationing that reflects the lender's perception of the household risk profile.

The failure of formal banks to serve the poor is due to high risks, high costs and consequently low returns associated with such business. In the credit market, the exchange between borrowers and lenders does not occur simultaneously. The delay involved in discharging the debt obligation exposes the credit transaction to considerable risk. To lower these risks, banks perform three tasks: they screen potential borrowers to establish the risk of default; they create incentives for borrowers to fulfill their promises; and they develop various enforcement actions to make sure that those who are able to repay, do so. When transacting with the poor these actions are difficult and costly to undertake. The scarcity of information results in information asymmetries between the poor and the banks. To address this problem, banks usually attach collateral requirements to loans. Collaterals do not only assist in determining creditworthiness, but also solve the incentive and enforcement problem.

Unfortunately, conventional collateral requirements usually exclude the poor who hardly ever have sufficient forms of conventional title, resulting in banks failing to meet the poor's demand for credit. Informal lenders have often, however, innovatively succeeded in limiting loan default. For instance, by lending to groups of borrowers, the joint liability and social collateral that are created ensure the strict screening of members, the incentive to honor commitments and members of the group monitoring each other's actions.

The commercial banks failure to help the poor get access to credit to help them come out of poverty, economists started thinking of new ways of helping the poor get access to loans. It was at this point that Microfinance came to mind. Micro-finance is generally an umbrella term that refers to the provision of a broad range of services such as deposits, loans, payment services, Money transfers and insurance to poor and low-income households and their micro-enterprises. In a much narrower sense though, micro-finance is often referred to as micro-credit for tiny informal businesses of micro-entrepreneurs. An outstanding feature of micro-finance programmes is that the end users of the services are by definition the poor, the ones who benefit. Similarly poor households will use a safe, convenient savings account to accumulate enough cash to buy assets such as inventory for a small business enterprise, to fix a leaky roof, to pay for health care, or to send more children to school. Microfinance also helps safeguard poor households against the extreme vulnerability that characterizes their everyday existence. Loans, savings, and insurance help smooth out income fluctuations and maintain consumption levels even during the lean periods. The availability of financial services acts as a buffer for sudden emergencies, business risks, seasonal slumps, or events such as a flood or a death in the family that can push a poor family into destitution. The center of this discussion lies the questions, is it true that Microfinance institutions really gain from using their limited resources on where the traditional bank do not want to operate?

1.2 BACKGROUND OF THE STUDY

Decision makers in all organizations continue to face the difficult task of balancing benefits against costs and the risks of realizing the benefits. Loan portfolio management is one of the responsibilities critical to the success of an institution. It is the dynamic process of managing an institution's primary earning assets to achieve the primary objectives of the boards strategic business and capital plans.

Loan portfolio encompasses all systems and processes used by management to adequately plan, direct, control and monitor the institutions lending operations. Loan portfolio ensures that all material aspect of lending operations are adequately controlled relative to the institutions risk bearing capacity. Loan portfolio helps management and decision makers in the analysis of how business results are achieved, whether such results will continue and how the institution can optimize its opportunity and provide great benefit to its members. Loan portfolio also helps decision makers to measure the portfolio risk both for short term returns and hold long term strategy. Finally, it helps mangers to minimize the funding of cost while lending against the market risk.

Experience with both for-profit and not- for-profit organizations shows that managers who must allocate resources are typically confronted with five challenges. Benefits are typically characterized by multiple objectives, which often conflict.

- This is nearly universal for organizations in the voluntary and public sectors and typical for those in the private sector.
- When decision makers are presented with a large number of opportunities they cannot know the details of each one sufficiently well to make informed decisions.
- If resources are allocated to each of several organizational units considered individually, the collective result appears not to make the best use of the total resource. That is, individually optimal decisions are rarely collectively optimal, giving rise to inefficient use of the available total resource.

- Many people are usually involved. Some provide expert judgment and advice to the decision maker, but that assistance inevitably reveals fundamental conflicts, which possibly creates competition. Others, with power to interfere or influence decision making, are often difficult to identify. Resolving those conflicts, and finding win-win solutions, often accompanies the process of resource allocation.

- Finally, implementation by those who disagree with the resource allocation can easily lead to the formation of small teams of people surreptitiously working on non-approved projects in which they are heavily invested personally.

These five characteristics of real-world resource allocation highlight the need for an approach that will enable decision makers to balance costs, risk and multiple benefits; to construct portfolios of investments across different areas such that the collective best use is made of the limited total resource; to consult the right people in a structured, coherent way, so that their multiple perspectives can be brought to bear on the issues; and to engage the key players to ensure their alignment to the way forward, while preserving their individual differences of approach. This can only be accomplished by blending a technical solution that captures the differing perspectives with a social process that engages those concerned.

The purpose of this paper is to present such a process, which combines technical elements of multi-criteria decision analysis (MCDA) with social aspects of decision conferencing, resulting in a tested approach to working with key players that creates shared understanding of the issues, a sense of common purpose and commitment to the way forward. The paper begins with a brief review of other technical approaches to resource allocation and then introduce the decision conference process.

An explanation of the MCDA socio-technical approach to resource allocation is followed by the presentation of lessons learned from applying it in real-world cases. It finishes with conclusions drawn from the experience of using the approach.

1.3 PROBLEM STATEMENT

Due to poor allocation of limited resources, some Microfinance Institutions and Rural Banks record marginal profits with some running at a lost. The main aim of this project is to propose a linear model subject to some constraints for a newly established Microfinance Institution – Christian Community Microfinance Limited (CCML), to enable them disburse their funds allocated for loans optimally leading to maximization of profits.

1.4 OBJECTIVES

The main objectives of this study are:

- To explore ways of disbursing funds allocated for loans effectively and efficiently in order to optimize profit margin of Christian Community Microfinance Limited.
- To serve as reference material in the libraries for students who wish to undertake research into similar field in the near future.
- To serve as a scientific method of providing executive with an analytical and objective basis for decision making.

1.5 METHODOLOGY

In order for the Christian Community Microfinance Limited (CCML) to maximize their profit, the proposed model will be based solely on the CCML's Loan Policy and its previous history on loan disbursement. The model will be solved using the Simplex Algorithm. The Linear Programming model has basic components, which are the objective function which is to be optimized (Maximized or minimized), the constraints or limitation and the non - negativity constraint.

In general, the formulation of Linear Programming model can be as follows:

Let $x_1, x_2, \ldots, x_n$ be n decision variables with m constraints, then

The objective function:

Maximize or Minimized

$$Z = \sum_{j=1}^{n} C_j X_j$$

Subject to the m constraints

$$\sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} x_j \, (\leq or \geq) b_1$$

The Non negativity constraints

$$x_j \geq 0$$

The simplex method is an iterative procedure for solving Linear Programming Problems in a finite number of steps. This method provides an algorithm which consists of moving from one vertex of the region of feasible solution to another in such a manner that the value of the objective function at the succeeding vertex is less or more than the previous vertex. This procedure is repeated and since the number of vertices is finite, the method leads to an optimal vertex in a finite number of steps or it may indicate the existences of unbounded solution.

1.6 JUSTIFICATION

Linear programming models are important tools for financial and Microfinance Institutions. The absence of a trusted model to help disburse funds allocated for loans has led to the collapse of Microfinance institutions in an economy such as Ghana. If loan limitations are not revised when circumstances change, a microfinance institution could be operating within guidelines that are too restrictive and if guidelines do not comply with current laws and rules, lending decisions may not reflect best practices or regulatory requirement. A loan policy that does not anticipate risks can lead to asset quality problem and poor earnings. The Microfinance Institution might run at a lost or even collapse if they are not able to retrieve all the loans they give out. Due to this, a more scientific and mathematical methods must be used to ensure adequate, effective and efficient distribution of funds they have available for loans to ensure constant growth and sustainability of the Institution. The proposed model will help Microfinance Institutions to efficiently distribute their funds for loans in order to maximize profit margin. In the case of CCML, much fund will be realized to engage in social intervention project for the community they work. The proposed model will also help decision makers to formulate prudent and effective loan policies.

1.7 SCOPE AND LIMITATION OF THE STUDY

This will cover the loan portfolio management policies of CCML for 2011 financial year. The constrains encountered include finance considerations, inaccessibility of data, limited time and unpreparedness and unreadiness of personnel to give out information necessary for the study.

1.8 ORGANIZATION OF THE STUDY

The study is organized in five chapters as follows. Chapter 1 provides general background issues to the study. It also provides the statement of problem. Again, it sets out the objectives of the study and provides justification for the objectives. Chapter 2 reviews relevant literature of the study. Both theoretical and empirical issues are reviewed in the literature. Chapter 3 discusses the methodological issues of the study. Chapter 4 also discusses the analysis of the empirical results presented for policy consideration. The final chapter, which is chapter 5, summarizes the main findings of the study and provides suggestions and policy recommendations.

CHAPTER 2

LITERATURE REVIEW

2.0 INTRODUCTION

This chapter presents some relevant literature in the field of optimal loan portfolio and in other field of study. It largely includes an abstract from projects under studied previously. A quite number of them make use of Linear Programming. Linear Programming is a structured procedure that incorporates defined decision variables that are significant in determining the maximum or minimum of an objective function. The objective function is subject to quantitative equality and linear inequality constraints.

2.1 LOANS

Loans in finance are the lending of sum of money. In common usage it is the lending of any piece of property. A loan may be secured by the charge on the borrower's property (as a house-purchase mortgage is) or be unsecured. There are number of conditions attached to a loan: for example, when it is to be repaid and the rate of interest to be charged on the sum of money loaned. Almost any person or any organization can make or receive a loan. The two major characteristics that vary among bank loans are the terms of the loan and the security or collate required to get the loan. For the loan term we have the long term and the short term, and of the security is secured or unsecured debt.

2.2 WHAT MOTIVATE PEOPLE TO GO FOR LOANS

Engaging in loan, gives one access to get a greater amount of money to fulfill ones project. Some clients find it difficult to pay for these loans but they still want to apply for it due to financial situations they find themselves. Most people apply for loans because of:

- Purchase a house or for renovation of a house.
- Pay for existing loan.
- Own a car.
- For personal, educational purposes and others.

2.3 FACTORS AFFECTING THE REPAYMENT OF LOANS

Lending is a risky enterprise because repayment of loans can seldom be fully guaranteed. Generally inspite of the importance of loan acquisition its repayment is fraught with a number of problems. Interest rates on loans are the most important factor affecting repayment of agricultural loans. In agriculture, large rate of default in loan repayment has been a perennial problem, farming experience, and total application costs. Most of the default arises from par management procedures, loan diversion and unwillingness to repay loans. Credit market in developing countries work ineffectively due to a number of market imperfections. These imperfections lead to loan default which includes:

- The interest rate ceiling usually imposed by the government.
- Monopoly power in credit market often exercised by informal lenders.

- Moral hazards.
- Low money borrowers.

2.4 PORTFOLIO MANAGEMENT

A new principle for choosing portfolio is based on historical returns data, the optimal portfolio based on this principle is the solution to a simple linear programming problem. This principle uses minimum return rather than variance as measure of risk. In particular, the portfolio chosen minimizes the maximum loss over all past observation periods, for a given level of return. The function avoids the logical problems of a quadratic utility function implied by mean-variance portfolio selection rules. The resulting minima portfolios are diversified; for normal return data, the portfolios are nearly equivalent to those chosen by a mean-variance rule.

Loan management is very complex and yet a vitally important aspect of any commercial bank operations. The balance sheet positions show the main sources of funds as deposits and shareholders contributions. In order to operate profitably, remain solvent and consequently grow, a bank needs to properly manage its excess cash to yield returns in the form of loans. The above are achieved if the bank can honour depositors withdrawals at all times and also grant loans to credible borrowers. This is so because loans are the main portfolios of a bank that yield the highest returns.

In the world of investment, investors want to earn the highest expected return from the portfolio. The rate of expected return depends on the level of tolerance. The expected return from a Portfolio of stocks is a combination of dividend and price yields. Portfolio selection and security analysis always becomes a vital area for decision making.

One of the basic problems of applied finance is the optimal selection of stocks, with the aim of maximizing future returns and constraining risks by appropriate measure. The problem was formulated by finding the portfolio that maximizes the expected return, with the risk constraints by the worst conditional expectation. Optimal portfolio selection problem can be formulated as a linear programming instance, but with exponential number of constraints.

The portfolio selection problem faced by a mutual fund manager can be formulated as a linear programming problem. This is to find those portfolio that are efficient in terms of predicted expected return and standard deviation of return, subject to legal constraints in the form of upper bounds on the proportion of the fund invested in any single security. Linear programming allows the use of an extremely simple and efficient special purpose solution algorithm.

Credit investing is a strange beast. The question “how much risk am I taking” is not easily answered. Traditionally with an equity portfolio the answer is usually expressed as a volatility or tracking error number. For fixed interest on credit portfolio the answer might be a duration number, an average credit rating or even a tracking error number or value of risk. Unfortunately, all these measures for credit portfolios can be significantly deficient by failing to capture the true risk profile of credit investments. Linear programming is used to measure the credit risk of a portfolio. It seeks to highlight the benefit, flaws and assumptions of each of these approaches. Leading global credit portfolio managers are implementing risk measurement and management approaches using linear programming techniques.

Any investor in a credit portfolio face non-diversifiable estimation driven uncertainty about two parameters. Probability of default and asset return correlation. Bayesian inference reveals that for realistic assumptions about the portfolio's credit and the data underlying parameter estimates, this uncertainty substantially increase the tail risk perceived by the investor. Since incorporating parameter uncertainty in a measure of tail risk is computationally demanding, linear programming derives and analyzes a closed form approximation to such a measure.

New approaches exist to measure the return-risk, trade-off in portfolio of risky debt instruments, whether bonds or loans. The use of complex, statistically based portfolio techniques to manage assets of financial institutions and fixed income portfolio is very much in its early phase and will continue to evolve, perhaps more quickly in the near future. Linear programming using the Simplex method substitutes the concept of unexpected loss for the more traditional variance of return measure used in equity securities analysis.

The manager of a bank operating in a competitive environment faces the standard goal of maximizing shareholders wealth specifically , this attempts to maximize the net worth of the bank, which in turn involves maximizing the net interest margin of the bank (among other factors, such as non-interest income). At the same time, there are significant regulatory constraints place on the banks, such as the maintenance of adequate capital, interest rate risk exposure etc.

Portfolio selection problem is usually considered as a bi-criteria optimization problem where a reasonable trade-off between expected rate of return and risk is sought. In the classical

Markowitz model the risk is measured with variance, thus generating a quadratic programming model.

The Markowitz model is frequently criticized as not consistent with axiomatic models of preferences for choice under risk. Models consistent with the preference axioms are based on the relation of stochastic dominance or on expected utility theory. The former is quite easy to implement, for comparison of given portfolios, it does not offer any computational tool to analyze the portfolio selection problem. The latter, when used for the portfolio selection problem, is restrictive in modeling preferences of inventors. A linear programming model of the portfolio selection problem is developed. The model is based on the preference axioms for choice under risk.

Methodology for measuring and optimizing the credit risk of a loan portfolio taking into account the non-normality of the credit loss distribution. Particular emphases were placed on modeling accurately joint default events for credit assets. In order to optimize portfolio credit risk, the authors minimized the conditional value at risk, a risk measure both relevant and treatable, by solving a simple linear programming problem subject to the traditional constraints of balance, portfolio expected return and trading. The outcomes, in terms of optimal portfolio compositions, assumed different default dependence structures were compared with each other. The solution of the risk minimization problem suggested how to restructure the inefficient loan portfolio in order to obtain the best risk or return profile.

The banking industry is one of World's leading industries. Being a Commercial Bank, giving Loans is the primary activity and Bank's managers in a competitive environment, make decisions about distribution of financial assets. Specifically, this attempts to maximize return and minimize the risk of the investment projects in a portfolio management structure. These categories involve banks in a Multi-objective Decision Making process. The main characteristics of these problems are that decision makers need to achieve multiple objectives in conflict with each other. This paper proposes a MOLP modeling approach for finding the best combination of loans portfolio to support Bank's managers in their related decision making. According to this model, we can obtain the corresponding optimal loan portfolio.

Many Ghanaians, both in the formal sector and the informal sector take loans for various reasons some being investment in businesses or their wards education. Others also take loans to acquire personal properties such as houses and cars. Most people rely on Banks for Loans. Due to poor allocation of funds by most banks to prospective loan seekers the banks are not able to maximize their profits. In view of this monies that can be used for social services in the community in which they operate go into bad debt

2.5 LINEAR PROGRAMMING

Linear programming (LP) is a highly versatile quantitative technique, which has found wide use in management and economics. It is used both as a research technique and as a planning tool, particularly at the individual firm and industry levels. In general, LP is designed to maximize or minimize a linear objective function subject to a set of linear constraints. Other related techniques are goal programming, mixed integer programming and quadratic programming. Some typical applications of linear programming include:

- Determining the most profitable combination of enterprise or activity levels for a business firm with limited supplies of various resources.

- Determining the most profitable investment portfolio, given the amount of investment capital available, rates of return on various stocks, bonds and other 'paper assets', and limits on high-risk investments.

- Formulating mixtures to combine ingredients such that a required overall composition of the mix is satisfied at least cost. Important applications are fuel and fertilizer blending and determination of livestock rations or supplementary feeds.

- Scheduling the various tasks in a construction project so as to complete the overall project in minimal time or at minimal cost and

- Determining the location and size of storage facilities and processing together with the distribution pattern, so as to minimize the total of transport, storage and processing costs.

The dynamics of the Danish mortgage loan system several models are prepared to reject the choices of a mortgage,). The models were formulated as multi stage stochastic integer programs, which are difficult to solve for more than ten (10) stages. Linear Programming was used to obtain near optimal solutions for large problem instances.

Cash-flow matching is an important and practical tool for managing interest rate risk. Interest rate fluctuations are major risk for the insurance and pension industry. If assets are invested shorter than the corresponding liabilities reinvestment risk arises because interest rate can fall.

On the other hand if assets are longer than the liabilities, then liquidation risk or market risk exist. An insurer or pension fund faces the problem of constructing from the current available universe of non callable and default free fixed income securities on investment portfolio that will meet the future liability payments. With a finite amount of resources, the decision maker seeks an initial investment portfolio with minimum cost such that it cash flow will at least meet the projected liability payment for each and every period in the planning horizon. Duality theory of linear programming provides insight for generalizing and solving cash-flow matching problem.

An optimal loan allocation mix policy from the steady state distribution of loan disbursement process. Using monthly data on actual loan disbursement of four loan types for a period of twenty-four months, by using a transition matrix. From the estimated probability transition matrix, the study state distribution indicated that in a long run, trade loan should constituted 77.3% of the total loan, 10.3% for service loan, 2.0% for production loan and 10.4% for Susu loan.

The mean-variance framework has been used to analyze the effects of bank capital regulation on the asset and bankruptcy risk of insured, utility-maximizing banks. This literature claims that more stringent capital regulation will increase asset risk and can increase bankruptcy risk. These conclusions are notable because they are opposite to those obtained for insured, value-maximizing banks. In this paper, they showed that the utility-maximization literature does not support its conclusions regarding the effects of bank capital regulation because it has mischaracterized the bank's investment opportunity set by neglecting the option value of deposit insurance.

Linear programming is recognized as a powerful tool to help decision making under uncertainty in financial planning. It shows how portfolio optimization problems with sizes measured in millions of constraints and decision variables featuring constraints on semi-variance, skewness or nonlinear activity functions in the objective can be solved.

The usefulness of Lagrange multipliers for optimization in the presence of constraints is not limited to differentiable functions. They can be applied to problems of maximizing an arbitrary real valued objective function over any set whatever, subject to bounds on the values of any other finite collection of real valued functions denned on the same set. While the use of the Lagrange multipliers does not guarantee that a solution will necessarily be found for all problems, it is “fail-safe” in the sense that any solution found by their use is a true solution. Since the method is so simple compared to other available methods it is often worth trying first, and succeeds in a surprising fraction of cases. They are particularly well suited to the solution of problems of allocating limited resources among a set of independent activities.

Most of the time in a distribution system, depot location and vehicle routing are interdependent, and recent studies have shown that the overall system cost may be excessive if routing decisions are ignored when locating depots. The location-routing problem (LRP) overcomes this drawback by simultaneously tackling location and routing decisions. The principle is to alternate between a depot location phase and a routing phase, exchanging information on the most promising edges. In the first phase, the routes and their customers are aggregated into super customers, leading to a facility-location problem, which is then solved by a Lagrange relaxation of the assignment constraints.

In the second phase, the routes from the resulting multi depot vehicle-routing problem (VRP) are improved using a granular tabu search (GTS) heuristic. At the end of each global iteration, information about the edges most often used is recorded to be used in the following phases. The method is evaluated on three sets of randomly generated instances and compared with other heuristics and a lower bound. Solutions are obtained in a reasonable amount of time for such a strategic problem and show that this metaheuristic outperforms other methods on various kinds of instances.

Air traffic flow management in Europe has to deal as much with capacity constraints in en route airspace as with the more usual capacity constraints at airports. The en route sector capacity constraints, in turn, generate complex interactions among traffic flows. We present a deterministic optimization model for the European air traffic flow management (ATFM) problem. The model designs flow management strategies involving combinations of ground and airborne holding. The paper illustrates the complex nature of European (EU) ATFM solutions, the benefits that can be obtained by purposely assigning airborne holding delays to some flights and the issues of equity that arise as a result of the interactions among traffic flows. In particular, we show that, in certain circumstances, it is better, in terms of total delay and delay cost, to assign to a flight a more expensive airborne holding delay than a ground delay. We also show that in the EU ATFM context, fundamental conflicts may often arise between the objectives of efficiency and equity (“fairness”). This finding may have profound implications for the possibility of developing a “collaborative decision-making” environment for air traffic flow management in Europe.

Many Ghanaians both in the formal sector and the informal sector take loans for various reasons some being investment in businesses or their wards education. Others also take loans to acquire personal properties such as houses and cars. Most people rely on Banks for Loans. Due to poor allocation of funds by most banks to prospective loan seekers the banks are not able to maximize their profits. In view of this monies that can be used for social services in the community in which they operate go into bad debt. Linear Programming model was developed to help the Atweaban Rural Bank at Duayaw Nkwanta in the Tano North District of the Brong Ahafo Region to allocate their funds to prospective loan seekers in order for them to maximize their profits.

How bankers choose the riskiness of their individual assets is an important question. It is well known that fixed-premium deposit insurance leads a bank to prefer a high-variance asset portfolio, but its effect on individual asset choice has not been carefully evaluated. The paper demonstrates how bank examination procedures and capital adequacy standards can make the value of a bank's deposit insurance contract concave in individual asset risks. Insured bankers may therefore have a rational preference (ceteris paribus) for relatively safe individual loans, even while they prefer risky portfolio returns. The model's implications for loan securitization and the Federal regulators new risk-based capital standards are discussed.

We study bank capital regulation using a two-dimensional moral-hazard model. Banks choose capital and portfolio risk. They also choose their level of costly screening, which determines their mean portfolio return. Screening and risk are private information. Deposit insurance gives low franchise-value banks an incentive to choose a sub optimally high level of risk and a sub optimally low level of screening.

Ex ante capital regulation can mitigate this problem. Optimal capital requirements are generally non-monotonic in franchise value. Adding ex post fines to capital requirements improves welfare by significantly reducing the use of costly capital. Optimal fine schedules are characterized by fines on the extreme right-hand tail of the return distribution.

2.6 SUMMARY

This chapter presents some relevant literature in the field of optimal loan portfolio and in other field of study. It largely includes an abstract from projects under studied previously. A quite number of them make use of Linear Programming. The next chapter will discusses the profile of Christian Community Microfinance Limited (CCML), the research methodology and linear programming.

CHAPTER 3

RESEARCH METHODOLOGY

3.0 INTRODUCTION

This chapter discusses the profile of Christian Community Microfinance Limited (CCML), the research methodology and linear programming.

3.1.0 PROFILE OF CCML

3.1.1 ORGANIZATIONAL PROFILE

Christian Community Microfinance Limited (CCML) was established in March 2011 as a company limited by shares by the Christian Council of Ghana (CCG) to engage in the business of microfinance, and specifically to take over the existing business of Ecumenical Christian Loan Fund (ECLOF Ghana). It has its core business as follows.

- **Credit**: CCML's principal activity is the granting of credit facilities to micro entrepreneurs, farmers, poor and needy churches, private schools and a few small businesses;
- **Savings**: CCML also offers savings services to her clients. This service is offered to customers to enable them accumulate funds to meet collateral requirements on loans, and also to meet future financial needs;

Financial literacy: CCML believes in the empowerment of her clients. One of the main tools towards the achievement of this goal is financial literacy. CCML offers financial literacy training with the aim of helping clients to become

- Financially smart and aware of the consequences of their financial choices, which in the long term makes them creditworthy and better clients than they were before joining the scheme.

3.1.2 CCML'S VISION, MISSION AND VALUES

3.1.2.0 MISSION

To deliver sustainable returns by offering excellent customer-centered financial services which support the community.

3.1.2.1 VISION

To be the leading and preferred financial institution contributing to the transformation of the whole nation and beyond.

3.1.2.2 CORE VALUES

- Social responsibility
- Excellence
- Integrity
- Innovation
- Sustainability

3.2 RESEARCH DESIGN

The main sampling technique was purposive sampling procedure. Purposive sampling is when the people selected are the key individuals who can give the information require from CCML for the study. Questionnaire was designed for the CCML loan officers to obtain information on the loan type, interest rate and probability of bad debt. The head office was served with questionnaire. The questionnaire focused on how CCML adjust the loan condition to reflect the risk of lending and how to select a loan facility that would maximize profit with limited resources and minimize risk on the loan portfolio in other to obtain an optimal portfolio. A case study research design was used to conduct this study, which consisted of two credit officers and an operations manager of CCML at head office.

3.3 LINEAR PROGRAMMING

Linear programming (LP) is the most commonly used tool for quantitative technique, which has found wide use in management and economics. It is mostly used as a research technique and as a planning tool, particularly at the individual firm and industry levels. In general, LP is designed to maximize or minimize a linear objective function. This function is subject to a set of linear constraints.

In other words, linear programming is a mathematical technique that maximizing or minimizing a linear function known as objective function. This objective function is subject to a set of linear equations or inequalities known as constraints. It is a mathematical technique which involves the allocation of resources in an optimum manner, on the basis of a given criterion of optimality. The technique used here is linear because the decision variables in any given situation generate straight line when graphed. It is also programming because it involves the movement from one feasible solution to another until the best possible solution is attained.

Variables called decision variables usually represent things that can be adjusted or controlled. An objective function can be defined as a mathematical expression that combines the variables to express a goal and the constraints are expressions that combine variables to express limits on the possible solutions.

Generally the two constrains are:

- Unconstrained optimization
- Constrained optimization.

3.3.0 UNCONSTRAINED OPTIMIZATION

Unconstrained optimization seeks to find the lowest point or highest point on an objective function. For optimization to be required there must be more than one solution, any point on the function is a solution, and because the single variable is real - valued function, there are an infinite number of solutions. Some kind of optimization process is then required in order to choose the very best solution from among those available. The best solution can be the solution that provides the most profit or consumes the least of some limited resource.

3.3.1 CONSTRAINED OPTIMIZATION

Constrained optimization is much harder than unconstrained optimization. In constrained optimization you still have to find the best point of the function, but have to respect various constrains while doing so. Unlike unconstrained problems the best solution may not occur at the top of the peak or at the bottom of the valley, the best solution might occur halfway up a peak when a constraint disallow movement further up.

3.4 FORMS OF LINEAR PROGRAMMING PROBLEMS

A linear programming may be in one of these forms:

- Matrix form
- General form
- Standard form

3.4.0 A LINEAR PROGRAMMING IN THE MATRIX FORM

Linear programs are problems that can be expressed in a form as:

Maximize $C^T x$

Subject to $Ax \le b$

The objective function $c^T x$ in this case is to be maximized or minimized. The inequalities $Ax \le b$ are the constraints which the objective function is to be optimized.

A Linear programming model may simply be presented in the matrix vector form as:

Maximize (Minimize) $c^T x$ (objective function)

Subject to: $Ax \le b$ (constraints)

$x \ge 0$ (non - negativity constraints)

3.4.1 A LINEAR PROGRAM IN THE GENERAL FORM

A linear programming in the general form may be presented as;

Maximize or Minimize Z

Subject to:

$$g_i(x) \leq b_i \quad ,1 \leq i \leq p$$

$$g_i(x) = b_i \quad , p+1 \leq i \leq m$$

$$x_i \geq 0 \qquad 1 < i < n$$

The function $f(x)$ being minimized or maximized is the objective function. The conditions

$$g_i(x) \leq b_i \quad ,1 \leq i \leq p$$

$$g_i(x) = b_i \quad , p+1 \leq i \leq m$$

$$x_i \geq 0 \qquad 1 < i < n$$

are the constraints of the problem. The constraints of type $g_i(x) \leq b_i$ are the inequality constraints and $g_i(x) = b_i$ are the equality constraints, whereas $x_i \geq 0$ are the non – negativity constraints.

3.4.2 LINEAR PROGRAM IN THE STANDARD FORM

The standard form is the usual and most insightful form of describing a linear programming problem. It consists of the following parts:

- A linear function to be maximized

That is, Maximize:

$$Z = x_1 + x_2 + \cdots + x_n$$

- Problem constraints of the form

$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1$

$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq b_2$

$$\vdots$$

$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \leq b_m$

- Non-negativity variables

$$x_i \geq 0$$

- Non-negativity right hand side constant

$b_1 \geq 0$

Other forms, such as minimization problems, problems with constraints as well as problems involving negative variables can always be rewritten into an equivalent problem in standard form.

3.5 METHODS OF SOLVING LINEAR PROGRAMMING

Basically, the two methods of solving a linear programming problem are:

- The graphical (Geometrical) Method
- The simplex (Algebraic) Method

3.5.0 THE GRAPHICAL METHOD

This method of solving Linear Programming Problem is well applicable to problems involving only two decision variables. This method will be partially looked at with an illustration. The following steps have to be followed in solving Linear Programming Problem using the graphical approach;

- Definition of the decisions variables in accordance with problem given.
- Formulate the problem in a standard Linear Programming model. The standard Linear Programming model consists of the objective function which is to be minimized or maximized. The constraints are either inequalities or equalities. In general, if the problem is to be minimized, the inequality used is the greater than or equal to $(\geq)$ unless otherwise specified. On the other hand, the maximization problem goes with, the less than or equal to$(\leq)$, unless it is stated. The non - negativity constraint must also be stated.

- Each of the inequality is considered as an equation and are plotted on the graph as each will represents a straight line, geometrically.

- Mark the appropriate regions. In this case, if the inequality constraint corresponding to the line, less than or equal to, then the region below the line lying in the first quadrant is considered (due to the non - negativity of the decision variables). Otherwise if the inequality constraint corresponding to the line is, greater than or equal to, then the region above the line in the first quadrant is also considered.

The points lying in common region will satisfy all the constraints simultaneously. The common

- region obtained is called the feasible region (Feasibility Polygon). That is the region common to all constraints in the given problem. It contains all the feasible or possible solutions to the problem. Points in the feasible region do not flout any of the constraints. There may be a situation where a constraint may not touch the feasible region; such constraint is known as redundant constraint. The edges or vertex of the feasible region is called extreme points or corner points and these are the points used to obtain the optimal solution. The optimal solution is the solution that maximizes or minimizes the objective function as the case may be.
- In practice, we determine the coordinates of the feasibility polygon and then substitute these coordinates into the objective function. If the problem is to be maximized, the coordinate that gives the maximum value is the optimum solution; otherwise the one that gives the minimum value will give the optimal solution.
- Draw the necessary conclusion.

3.5.1 EXAMPLE OF A GRAPHICAL METHOD SOLUTION

A company produces two kinds of items, Calculator and Handset. The company wishes to determine the rates at which each type of item should be produced in order to maximize profits on the sales of the items on the assumption that all the items produced will be sold. Two Calculators and three Handset are produced per day and producing each type requires the same amount of time on the finishing machine, this machine can process at most a total of four items a day of either type. The profit generated on the Calculator and the Handset are GH¢15.00 and GH¢10.00 respectively.

The above problem is formulated as:

Let x_1 = Number of Calculator produce per day

x_2 = Number of Handset produced per day

Maximize $f(x) = 15x_1 + 10x_2$ (in GH¢ per day).

$x_1 \leq 2$ (Constraint for Calculator per day)

$x_2 \leq$ (Constraint for Handset per day)

$x_1 + x_2 \leq 4$ (Production limit for finishing machine per day)

$x_1 \geq 0$ and $x_2 \geq 0$

A graph of the constraints is plotted below:

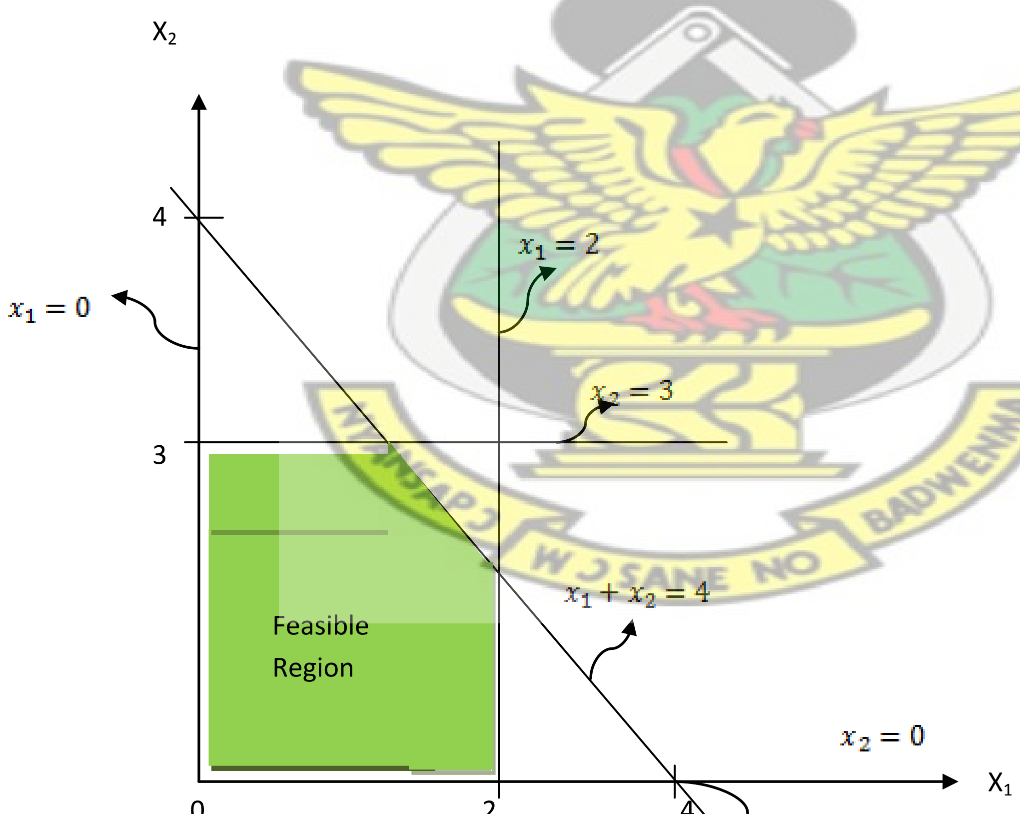

Figure 3.1: The feasible region of the company.

The limiting value of each of the constraint is shown as a line. Each constraint eliminates part of the plane. For example the vertical line labeled, $x_1 = 2$, is the limiting value of the inequality $x_1 \leq 2.$ All points to the right of the line violate the constraint (i.e. the infeasible region). The areas eliminated by the constraints are uncolored. The colored area represents points that are not eliminated by any constraint, and is called feasible region. To find a point in the feasible region gives the largest valued of the objective function. One way to do this is to randomly choose feasible points and to calculate the value of the objective function at those points, keeping the point that gives the maximum profit (best value of the objective.)

Because there are an infinite number of points in the feasible region, this is not very effective because there is no guarantee that the best point will be found, or even that an objective function value that is close to the best possible value will be found. An efficient search technique based on a couple of simple observations is developed.

A line of equal profits is drawn to represent the objective function after assigning a value say zero for the objective function so as to get a straight line passing through the origin. The objective function line is stretched to the extreme points of the feasible region. The important idea is that, the first contact between the objective function and the feasible region always involves at least one corner point. Hence, an optimum solution to the linear programming is always at a corner point or extreme point as shown in the figure 3.2 below. The extreme points for the feasible region are the five corners above, each of which is the intersection of two of the lines determined by the constraints. Now considering all of the points where the objective function would be equal to 10. These are the points satisfying the equation $15x_1 + 10x_2 = 10$ - all such points lie on a line, and in particular are the points on the first arrow line on figure 3.2 below starting from the origin. Consider the points where the objective function is 20 (the points lying on the line $15x_1 + 10x_2 = 20$), a similar picture, but with the arrow line simply shifted up one unit. The goal is to maximize the objective function while satisfying all of the constraints (staying within the feasible region). Geometrically, moving this line up as far as possible, as the objective function increases and stay in the feasible region, all possible values of the objective function take on something similar. ("something similar" because the picture would just be solid arrow if looked at all the objective contours). From this picture, the optimal solution occurs at the top-right extreme point. The extreme point is the intersection of two lines, simply calculate the intersection point of those two lines to see to find the precise point where optimality is obtained (these are called binding constraints since they are satisfied by equality).

This geometric argument is nice and intuitive, but falls apart if having more than three variables (since, in at least four dimensional space where such pictures cannot be drawn).

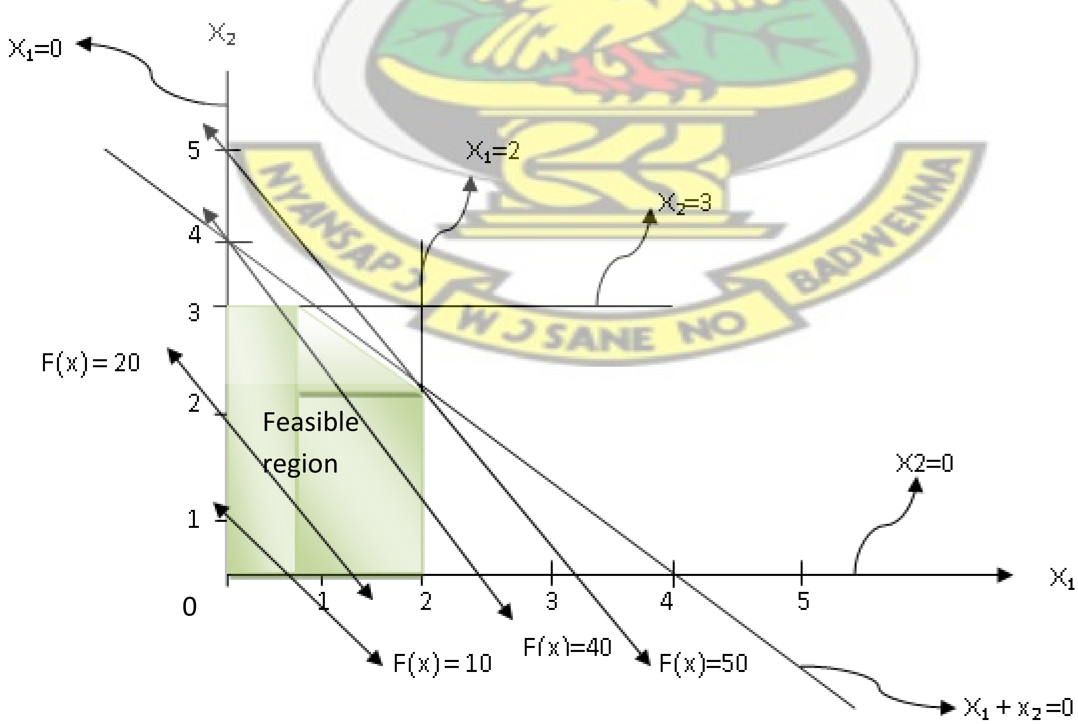

Figure 3.2: The feasible region showing objective contours (a line of equal profits).

3.6 SIMPLEX METHOD

The simplex method is the name given to the solution algorithm for solving linear programming problems developed by George Dantzig in 1947. A simplex is an n-dimensional convex figure that has exactly (n+1) extreme points. For example, a simplex in two dimensions is a triangle, and in three dimensions is a tetrahedron. The simplex method refers to the idea of moving from one extreme point to another on the convex set that is formed by the constraint set and non-negativity conditions of the linear programming problem.

The principle underlying the simplex method involves the use of the algorithm which is made up of two phase, where each phase involves a special sequence of number of elementary row operations known as pivoting. A pivot operation consist of finite number of m-elementary row operations which replace a given system of linear equations by an equivalent system in which replace a given system of linear equations by an equivalent system in which a specified decision variables appears in only one of the system and has a unit coefficient.

Generally, the algorithm has two phases, the first phase is finding an initial basic feasible solution (BFS) to the original problem and the second phase, consists of finding an optimal solution to the problem which begins from the initial basic feasible solution.

3.6.0 FORMULATION OF THE PROBLEM

The objective function to be Maximized or Minimized is given by

$$Z = C_1 x_1 + C_2 x_2 + \cdots + C_n x_n$$

Subject to the *m* constraints given by

$$a_{11} x_1 + a_{12} x_2 + \cdots + a_{1n} x_n \leq b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq b_2$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \leq b_m$$

The Non negativity constraints

$x_1 \geq 0, x_2 \geq 0 \ldots x_n \geq 0$

Where c_j, a_{ij} and b_j are all known constraints and greater than zero and i=1,2,3….,m and j=1,2,3, …, n.

3.6.1 ALGORITHM FOR SIMPLEX METHOD

A basic feasible solution to the system of m linear constraint equations and n variables is required as a starting point for the simplex method. From this starting point, the simplex successively generates better basic feasible solutions to the system of linear equation. We proceed to develop a tabular approach for the simplex algorithm. The purpose of the tableau form is to provide an initial basic feasible solution that is required to get simplex method started. It must be noted that basic variable appear once and have coefficient of positive one.

3.6.2 SETTING UP INITIAL SIMPLEX TABLEAU

In developing a tabular approach we adopt these notations as used in the initial simplex tableau.

c_j = objective function coefficients for variable j

b_j = right - hand side value for constraints *i*

a_j = coefficients variable j in constraints i

c_B = objective function coefficients of the basic variables

$C_j - Z_j$ = the net evaluation per unit of j^{th} variable

[A] matrix = the matrix (with m rows and n columns) of the coefficients of the variable in the constraint equations.

Table 3.1 (General from – Initial Simplex Tableau)

		Decision variables				Slack variables				
c_j		c_1	c_2	…	c_n	0	…	0	solution	Objective function coefficients
c_B	Basic variables	x_1	x_2	…	x_n	s_1	…	s_m		Headings
0	s_1	a_{11}	a_{12}	..	a_{1n}	1	…	0		Constraints coefficients
0	s_2	a_{21}	a_{22}	…	a_{2n}	0	…	0		
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮		
	s_m	a_{m1}	a_{m2}	…	a_{mn}	0	…	1		
		Z_1	Z_2	…	Z_{mn}	Z_{11}	…	Z_{1m}	Current value of objective function	
	$c_j - Z_j$	$c_1 - Z_1$	$c_2 - Z_2$	…	$c_{mn} - Z_{mn}$	$c_{11} - Z_{11}$	…	$c_{1m} - Z_{1m}$		Reduced cost

Illustration

$Maximize\ Z = 6x_1 + 8x_2$

Subject to:

$$5x_1 + 10x_2 \leq 60$$

$$4x_1 + 4x_2 \leq 40$$

$$x_1, x_2 \geq 0$$

The above example can be restated in the standard form as follows:

$Maximize\ Z = 6x_1 + 8x_2 + 0s_1 + 0s_1$

Subject to:

$$5x_1 + 10x_2 + s_1 = 60$$

$$4x_1 + 4x_2 + s_1 = 40$$

$$x_1, x_2, s_1, s_2 \geq 0$$

Transferring to the initial simplex tableau, we have Table 3.1.1

Table 3.1.1 (The Initial Tableau)

			Pivot element	Pivot column			
		c_j	6	8	0	0	
	c_B	Basic Variables	x_1	x_2	s_1	s_2	solution
Pivot row	0	s_1	5	10	1	0	60
	0	s_2	4	4	0	1	40
		Z_j	0	0	0	0	0
		$c_j - Z_j$	6	8	0	0	

The current basic variables always form an identity matrix within the simplex tableau. Note that the basic variables form a basis matrix that is an identity matrix (I). From the initial tableau, the solution values can be read directly in the rightmost column. The values of Z_j row are calculated by multiplying the elements in the C_n corresponding elements in the columns of the [A] matrix and summing them. Each value in the $C_j - Z_j$ column by the row represents the net or net contribution that is added by producing one unit of product (if $C_j - Z_j$, is positive) or the net profit or net contribution that is subtracted by producing one unit of product j (if $C_j - Z_j$ is negative).

Since all the Z_j values (j=1,…,4) are equal to zero in the simplex tableau, we proceed to generate a new basic feasible solution (extreme point) that yields a better value for the objective function. This is accomplished by selecting one of current non-basic variables to be made basic and one of the current basic variables to be made non-basic in such a fashion that the new basic feasible solution yields an improved value for the objective function. This process is called changing that basic or iterating.

3.6.3 IMPROVING THE SOLUTION

The criteria for which a variable should enter or leave basis is summarized as follows:

- Variable Entry Criteria: The variable entry criterion is based upon the value in the $C_j - Z_j$ row of the simple tableau. For a maximization problem, the variable selected for entry is the one having the largest (most positive) value of $C_j - Z_j$. When all values of $C_j - Z_j$ are zero or negative, the optimal solution has been obtained.

Variable Removing Criterion: The variable removal criterion is based upon the ratios formed as the values (bi) in the "right-hand-side" column are divided by the corresponding values (a_{ij} coefficients) in the column for the variable selected to enter the basis. Ignore any a_{ij} values in the column that are zero or negative (ie., do not compute the ratio). The variable chosen to be removed from the basis is the one having the smallest ratio. In the case of ties for the smallest ratio between two or more variables,

- break the tie arbitrarily (i.e. simply choose one of the variable for removal). This variable removal criterion remains the same for both maximization and minimization problems.

  Applying the variable entry and removal criteria to our present maximization problem x_2 is chosen as the variable to enter basis and S_1 leaves the basis. Thus, the current basic variable S_1 is replaced by non-basic variable x_2.

  Now that we have determined the new elements in basis and that not in basis, we proceed to determine the new solution through pivoting x_2 into basis and pivoting s1 out of basis. The pivoting process involves performing elementary row operations on the rows of the simplex tableau to solve the system of constrain equations in terms of the new set of basic variables. We initiate pivoting processing by identifying the variable, x_2 to the basis by denoting its corresponding column as the pivot column in Table 3.1.2.

  Similarly, we identify the variable, S_1 to be removed from the basis by specifying the pivot row which is the row it corresponds as in Table 3.1.2. The element at the intersection of the pivot column and pivot is referred to as pivot element. The two-step pivoting process proceeds as follows:

  a) Convert the pivot element to one by dividing all values in the pivot row by pivot element (10). This new row is entered in the next tableau, Table 3.1.3

The objective of the second step is to obtain zeros in all the elements of the pivot column, except, of course for the pivot element itself. This is done by elementary row operations involving adding or subtracting the appropriate multiple of the

a) new pivot row or from the other rows. Performing these calculations, the results are as presented in Table 3.1.3.

Table 3.1.2 (Second Simplex Tableau)

		Pivot element	Pivot column				
		c_j	6	8	0	0	
	c_B	Basic Variables	x_1	x_2	s_1	s_2	solution
	8	x_2	$1/2$	1		0	6
Pivot row	0	s_2	2	0		1	16
		Z_j	4	8		0	48
		$c_j - Z_j$	2	0		0	

The second simplex tableau can be construed as shown in Table 3.1. 3 Note that the columns that correspond to the current basic variables x_2 (real variable) and s_2 (slack variable) from a basis [B] which is identity matrix. The values in the Z_j row and $C_j - Z_j$ row are computed in the same way as in the initial simplex tableau. Observe that $C_j - Z_j = 2$ (>0) and so the optimal solution has not been obtained and continue the iteration since we are maximizing.

We continue the process by deterring the variables leaving the basis and which is entering the basis using the variable entry and removing criteria stated earlier. The outcome is summarized in Table 3.1.4

Table 3.1.4: (Third Simplex Tableau - Optimal Solution)

	c_j	6	8	0	0	
c_B	Basic Variables	x_1	x_2	s_1	s_2	solution
8	x_2	0	1			2
6	x_1	1	0			8
	Z_j	6	8		1	64
	$c_j - Z_j$	0	0		-1	

Observe that in this third simplex tableau all $C_j - Z_j$ values are either zero or negative. We have thus obtained the optimal solution with $x_1 = 8$ and $x_2 = 2$, Therefore the optimal value of Z is, Z = 64. The optimal solution suggests that the profit will be maximized when eight (8) products of x_1 and two (2) products of x_2 are produced.

3.6.4 SIMPLEX METHOD WITH MIXED CONSTRAINTS

Some Linear Programming problem may consist of a mixture of $\leq$, = and $\geq$ sign in the constraints and wish to maximized or minimized the objective function. Such mixture of signs in the constraints is referred to as mixed constraints.

The following procedure is followed when dealing with problem with mixed constraints.

- Ensuring that the objective function is to be maximized. If it is to be minimized then we convert it into a problem of maximization by Max W = -Min (-Z)
- For each constraints involving 'greater or equal to' we convert to 'less than or equal to' that is, constraints of the form

  $a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \geq b_2$

  Is multiplied by negative one to obtain

  $-a_{21}x_1 - a_{22}x_2 - \ldots - a_{2n}x_n \leq -b_2$

- Replace constraints

  $a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \geq b_2$

  and

  $a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \leq b_2$

  by

  $a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$

Where the latter is written as

$-a_{21}x_1 - a_{22}x_2 - \ldots - a_{2n}x_n = -b_2$

- Form the initial simplex tableau

- If there exist no negative appearing on the RIGHT HAND SIDE column of the initial tableau, proceed to obtain the optimum basic feasible solution.
- If there exist a negative entry on the Right Hand Side column of the initial tableau,

  (i) Identify the most negative at the Right Hand Side; this row is the pivot row.

  (ii) Select the most negative entry in the pivoting row to the left of the Right Hand Side; This entry is the pivot element.

  (iii) Reduce the pivot element to 1 and the other entries on the pivot column to 0 using elementary row operation.

- Repeat 6 as long as there is a negative entry on the Right Hand Side column. When no negative entry exists on the Right Hand Side column, except in the last row, we proceed to find the optimal solution.

3.7 SUMMARY

In this chapter, Linear Programming and Simplex method were discussed. The analysis on the Linear Programming and Simplex method also form part of the discussion in this section of the work. In the next chapter, the data collected from the microfinance company will be used to formulate the linear model and solve it using the Simplex method.

CHAPTER 4

DATA COLLECTION, MODELING AND ANALYSIS

4.0 INTRODUCTION

This chapter analyzed the data taken from Christian Community Microfinance Limited (CCML), A model is proposed and solved to help CCML maximize its net profit. For this study, data was collected from CCML with focus on the loan records for the 2011 financial years report. The institution, CCML, is in the process of formulating a loan policy involving a total of GH¢120,000. The institution is obligated to grant loans to different clientele. The table 4.1 below provides the type of loans, the interest rate charged by the bank, and the probability of bad debt as estimated from past years.

Table 4.1: Loans available to the CCML.

Type of loan	Interest rate	Percentage of bad debt
Church	0.28	0.02
Anidaso	0.3	0.12
Boafo	0.3	0.20
School	0.4	0.01
Korkorko	0.3	0.03

Source – CCML2011 eastern zone annual report

Bad debts are assumed unrecoverable and hence produce no interest revenue. For policy reasons, there are limits on how the institution allocates the funds.

- Allocate at least 50% of the total funds to Anidaso loan and Church loan.
- To assist the smaller scale business in the institution catchment areas Church loan must equal at least 50% of the School, Boafo and Korkorko loans.
- The sum of Church loan and Korkorko loan must be at least greater than 50% of Anidaso loan, Boafo loan and School loan.
- The sum of School loan and Boafo loan must be at least 25% of the total funds.
- The sum of Anidaso and Boafo loans must be at least 29% of the total funds.
- Allocate at least 5% of the total funds to Boafo loan.
- The institution also has a stated policy, that the total ratio for the bad debt on all loans may not exceed 0.05.

4.1 MATHEMATICAL MODEL.

The variable of the model can be defined as follows:

x_1 = Church loans (in thousands of Ghana cedis)

x_2 = Anidaso loans

x_3 = Boafo loans

x_4 = School loans

x_5 = Korkorko loans

The objective of the CCML is to maximize its net return comprised of the difference between the revenue from interest and lost funds due to dad debts. Because bad debts are not recoverable, both as principal and interest, the objective function may be written as.

Maximize

$$Z = 0.28(0.98x_1) + 0.30(0.88x_2) + 0.30(0.80x_3) + 0.40(0.99x_4) + 0.30(0.97x_5)$$

$$-0.02x_1 - 0.12x_2 - 0.2x_3 - 0.01x_4 - 0.03x_5$$

This function is simplifies to

Maximize: $Z = 0.2544x_1 + 0.144x_2 + 0.04x_3 + 0.386x_4 + 0.264x_5$

The problem has nine constrains:

- Limit on total funds available

$$x_1 + x_2 + x_3 + x_4 + x_5 \leq 120{,}000.00$$

- Limit on Church and Anidaso loans.

$$x_1 + x_2 \geq 0.5(120{,}000.00)$$

$$\Rightarrow x_1 + x_2 \geq 60{,}000.00$$

- Limit on Church loans.

$x_1 \geq 0.5(x_3 + x_4 + x_5)$

$=> x_1 - x_3 - x_4 - x_5 \geq 0$

- Limit on Church and Korkorko loans Compare to School, Boafo and Anidaso Loans

$x_1 + x_5 \geq 0.5(x_2 + x_3 + x_4)$

$=> x_1 - 0.5x_2 - 0.5x_3 - 0.5x_4 + x_5 \geq 0$

- Limit on Boafo and School loans.

$x_3 + x_4 \geq 0.25(120{,}000.00)$

$=> x_3 + x_4 \geq 30{,}000.00$

- Limit on Anidaso and Boafo loans.

$x_2 + x_3 \geq 0.29(120{,}000.00)$

$=> x_2 + x_3 \geq 34{,}800.00$

- Limit on Boafo loans.

$x_2 \geq 0.05(120{,}000.00)$

$=> x_2 \geq 6{,}000.00$

- Limit on bad debt.

$$\frac{0.02x_1 + 0.12x_2 + 0.20x_3 + 0.01x_4 + 0.03x_5}{x_1 + x_2 + x_3 + x_4 + x_5} \leq 0.05$$

$$=> -0.03x_1 + 0.07x_2 + 0.15x_3 - 0.04x_4 - 0.02x_5 \leq 0$$

- Non-negativity

$x_1 \geq 0,\ \ x_2 \geq 0,\qquad x_3 \geq 0,\ \ x_4 \geq 0,\ \ x_5 \geq 0$

The following is the output returned by the Management Scientist solver for the model.

Maximize $Z = 0.2544x_1 + 0.144x_2 + 0.04x_3 + 0.386x_4 + 0.264x_5$

Subject to:

1. $x_1 + x_2 + x_3 + x_4 + x_5 \leq 120{,}000.00$
2. $x_1 + x_2 \geq 60{,}000.00$
3. $x_1 - x_3 - x_4 - x_5 \geq 0$
4. $x_1 - 0.5x_2 - 0.5x_3 - 0.5x_4 + x_5 \geq 0$
5. $x_3 + x_4 \geq 30{,}000.00$
6. $x_2 + x_3 \geq 34{,}800.00$
7. $x_2 \geq 6{,}000.00$
8. $-0.03x_1 + 0.07x_2 + 0.15x_3 - 0.04x_4 - 0.02x_5 \leq 0$
9. $x_1 \geq 0,\ \ x_2 \geq 0,\ \ x_3 \geq 0,\ \ x_4 \geq 0,\ \ x_5 \geq 0$

4.2 OPTIMAL SOLUTION

Objective Function Value = 32,068.48

Variable	Value	Reduced Costs
x_1	31200.0000	0.0000
x_2	28800.0000	0.0000
x_3	6000.0000	0.0000
x_4	45200.0000	0.0000
x_5	8800.0000	0.0000

Constraint	Slack/Surplus	Dual Prices
1	0.0000	0.3443
2	1200.0000	0.0000
3	0.0000	-0.0833
4	21200.0000	0.0000
5	0.0000	-0.2354
6	0.0000	-0.1106
7	0.0000	-0.0066
8	4.0000	0.0000

Objective Coefficient Ranges

Variable	Lower Limit	Current Value	Upper Limit
x_1	0.0190	0.2544	0.2610
x_2	0.0334	0.1440	0.3794
x_3	No Lower Limit	0.0400	0.1506
x_4	0.2610	0.3860	No Upper Limit
x_5	0.2544	0.2610	0.3860

Right Hand Side Ranges

Constraint	Lower Limit	Current Value	Upper Limit
1	119880.0000	120000.0000	122400.0000
2	No Lower Limit	0.0000	1200.0000
3	-13200.0000	0.0000	300.0000
4	No Lower Limit	30000.0000	51200.0000
5	26000.0000	34800.0000	34833.3333
6	4800.0000	6000.0000	6057.1429
7	59600.0000	60000.0000	68800.0000
8	-4.0000	0.0000	No Upper Limit

4.3 DISCUSSION OF RESULTS

There are several things to observe about this output data.

1. The reduced costs for x_1, x_2, x_3, x_4 *and* x_5 are zero. This is because the reduced costs are the objective function coefficients of the original variables, and since, x_1, x_2, x_3, x_4 *and* x_5 are basic at the optimum, their objective function coefficients must be zero when the tableau is put into proper form. This is always true, either the variable is zero (non-basic), or the reduced cost or dual price is zero.
2. It is also seen that the pattern holds for the slack and surplus variables too. The dual Prices for constraints (1), (3), (5), (6) and (7) are non-zero at the optimum because they correspond to the five active constraints at the optimum, hence their slack variables are non-basic (value is zero), so the dual prices can be non-zero.
3. When both the variable and the associated reduced cost or dual prices are zero, then we have either degeneracy if the variable is basic or multiple optima if the variable is non-basic.

It must be noted that the optimal solution with $x_1 =$ GH¢31,200.00, $x_2 =$ GH¢28,800.00, $x_3 =$ GH¢6,000.00, $x_4 =$ GH¢45,200.00 and $x_5 =$ GH¢8,800.00 show that the institution should allocate funds to all the loan products, since none of them with value of zero.

Table 4.2 below summarized the amount and the percentage of funds to be allocated to the various loans product of the financial institution, CCML.

Table 4.2: Summary of the result.

Type of loan	Amount to allocate (GH¢)	Percentage of funds to allocate (%)
Church	31,200.00	26.00
Anidaso	28,800.00	24.00
Boafo	6,000.00	5.00
School	45,200.00	37.70
Korkorko	8,800.00	7.30

4.4 SUMMARY

In this chapter, data collected from the institution (CCML) were used to formulate the proposed model and the output results were also discussed. The next chapter, which is to be the final chapter of the work, presents the conclusions and recommendations of the study.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

5.0 INTRODUCTION

This chapter presents the conclusions drawn from the study and makes some recommendations to help Christian Community Microfinance Limited (CCML) in order to optimize the profit margin.

5.1 CONCLUSIONS

Reading through this work, it is clear that most financial institutions in the country do not have any

Scientific method for given out loans. Due to this, most financial institutions are not able to optimize their profits, which intern affects their socio economic contributions in the areas in which they operate. A model has also been proposed to help CCML disburse their funds available for loans. The model shows that if CCML, Eastern zone, adapts to the model they can be able to make an annual profit of GH¢32,068.48 on loans alone. Hence the scientific method used to develop the proposed model can have a dramatic increase in the profit margin of the institution (CCML) should they adapt to it.

5.2 RECOMMENDATIONS

From the conclusion we realized that using scientific methods to give out loans helps Financial institutions to increase their profits. Hence it is recommended that Christian Community Microfinance Limited (CCML) should adapt this model in their allocation of funds reserved for loans.

Secondly, it is recommended also that managers of financial institutions like Bank be educated to employ mathematicians to use scientific methods to find an appropriate mathematical model to help them disburse funds of the banks more efficiently.

Lastly, it is again recommended that apart from loan disbursement, banks and other financial institutions should employ scientific methods and mathematical methods in most of the businesses they conduct.

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