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Hedging Longevity Risk using Longevity Swaps: A case study of the Social Security and National Insurance Trust (SSNIT)

By

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Declaration

I hereby declare that this submission is my own work towards the award of the M.Sc degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the university, except where due acknowledgement had been made in the text.

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Dedication

I dedicate this work to my parents and siblings for their strong support and assistance throughout my study.

Abstract

Effective management of longevity risk is essential for every institution which is exposed to longevity risk. Defined benefit schemes in Ghana are especially exposed to longevity risk due to increasing life expectancy in Ghana. In this study we explore a hypothetical hedging strategy based on longevity swaps for the SSNIT pension scheme. We use the Cairns-Blake-Dowd model to forecast future mortality rates of pensioners from age 71 to 90. With the forecasted mortality rates we designed longevity swap contract whereby realized mortality rates would be swapped with the forecasted expected mortality rates. The payout structure under the swap ensures that the SSNIT's liability is completely hedged against longevity risk.

Acknowledgements

May the name of Almighty God be praised forever. It is His bountiful endowment of grace through Christ Jesus that kept me through.

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Chapter 1

Introduction

This study seeks to investigate how mortality derivatives could be used to hedge against longevity risks for pension providers in Ghana. Longevity risk refers to the risk that the actual survival rates and life expectancy will exceed expectations or pricing assumptions, resulting in greater-than-anticipated retirement cash flow needs. Pension providers are faced with the risk that pensioners will live longer than expected and since they have to pay monthly pension to the pensioners until their death; longevity risk may affect the annuity provider's solvency.

The need to manage longevity risk has become very important as employers and employees become aware of their exposure to longevity risk and their need to mitigate it. For individuals, longevity risk is the risk of outliving ones's income, resulting in a lower standard of living, reduced care, or a return to employment at old age. For those institutions providing covered individuals with guaranteed retirement income, longevity risk is the risk of undervaluing survival rates, resulting in increased liabilities to sufficiently cover promised payments.
According to the center for insurance policy and research of the National Association of Insurance Commissioners (NAIC, US), key drivers of the growing need to address longevity risk include an aging population, increasing life expectancy, a shift in who bears the responsibility of sufficient retirement income, uncertainty of government benefits and economic volatility.

1.1 Background

Lots of research have been carried out by a range of stakeholders (e.g., government actuarial or pension departments, academic institutions, through experience studies) across the world that is focused on the observed trend in mortality witnessed over the last century. The results of this researches points to the same undeniable conclusion. People are living longer today than they ever have in the past. Significant medical progress, improved hygiene and living standards, generally healthier lifestyles and the absence of both wars and major pandemic crises are some of the key factors responsible for the rising life expectancy. Crawford *et al* (2008)

This phenomenon has essential consequences for defined benefit plans, particularly those where payments to current retirees are in part funded by contributions from current employees. Government sponsored plans are one clear example. Governments of countries that are likely to experience "the demographic time bomb" will have to carefully consider future costs and weigh potential program modifications. Crawford *et al*(2008)

While the above observations discuss the population as a whole, Crawford *et al* (2008) also showed that historical mortality improvements have differed depending on time an an individual was born. This has been called the "cohort effect", which describes anomalies in observed mortality improvement for those born in a specific period of time.

Mortality Risk for Insurers: A life insurance policy promises to pay a specified amount of money upon death of a policy holder. In exchange for this payment, the policy holder pays a premium. The premiums could be a one-time upfront payment or could be paid in regular intervals (monthly, quarterly, etc).

Premiums are priced based on certain assumptions about future interest rates, mortality rates, expenses, investment returns etc. these assumptions are known as the basis for the pricing. Actual experience may not follow the assumptions made therefore there's a risk of the insurer making losses.

If mortality improves, people live longer than expected and the insurer's liability is deferred which translates into more profits for the insurer since the premiums would be paid and invested for a longer period and would more likely be more than the insurer's liability. On the other hand, if mortality deteriorates, people live shorter than expected and the Insurer's liability would be due much earlier than expected. The premiums collected and invested for the relatively short period, may not be enough to meet the insurer's liability resulting in a loss for the insurer.

Longevity Risk for pension providers: Unlike life insurers who gain when mortality improves, pension providers lose out when mortality improves. Annuity providers such as pension funds pay an amount to a life at regular intervals as long as the life remains alive. If mortality improves and people live longer, the annuity provider pays them for a longer period. Payments for longer periods than expected could cause an annuity provider to become insolvent. Cox and Lin (2007)

Various methods are used to hedge against undesired interest rates and investment returns but hedging against mortality risk is relatively uncommon. In this study, we develop a strategy to help annuity providers hedge their portfolio against longevity risk.

1.1.1 The need for Pension

Pensions, in a broad sense, are regular payments given to retired workers. At retirement, salaries are no more paid hence a decline or a complete cut off of income. To sustain a living after retirement for employees, most employers including government run a pension scheme. This pension scheme is meant to support employees who go on retirement for various reasons. Employees and employers make regular contributions to the scheme during their years of service and these contributions are invested.

It is obvious that pensions are necessary as in many cases it becomes the only source of livelihood for elderly people.

1.1.2 Types of Retirement Benefits

There are two major types of retirement plans, the defined benefit plan and the defined contribution plan.

Defined Benefit Plan

A defined benefit plan ensures that a certain amount is paid at retirement until death of the pensioner, according to a fixed formula which usually depends on the member's salary and the number of years of membership in the plan. A traditional defined benefit plan is a plan in which the benefit on retirement is determined by a set formula, rather than depending on investment returns. A traditional pension plan that defines a benefit for an employee upon that employee's retirement is a defined benefit plan (Davies, 1993).

Defined Contribution

Defined contribution plans allow the employer and/or employee to make contributions, so that the final benefits depend on how much was in the account and the rate earned by the account's investments. (Davies, 1993). An individual account must be set up for each participant in the plan. The plan does not guarantee a participant's benefits; instead, the plan is ”participant-directed”, meaning that the employee makes the investment decisions based on the employer's options.

1.2 Problem Statement

According to the world bank life expectancy data, Over the years, life expectancy have been increasing. Pension providers are obliged to pay a fixed amount to a pensioner on a monthly basis for as long as the pensioner remains alive.

Due to advances made in medical technology, people changing their lifestyles and other factors, life expectancy have increased continually since the 1960s.Long *et* *al* (2015). Coughlan *et al* (2007) found that each additional year of life adds 3-4% to the value of pension liability. The graph in fig1.1 shows how the life expectancy of Ghanaians has improved from 1960 to 2013. Life expectancy in Ghana increased by 16 years from 1960 to 2013 and by 3.3 years 10 years (2003 to 2013).

In addition to increasing life expectancy, contributions made to SSNIT has

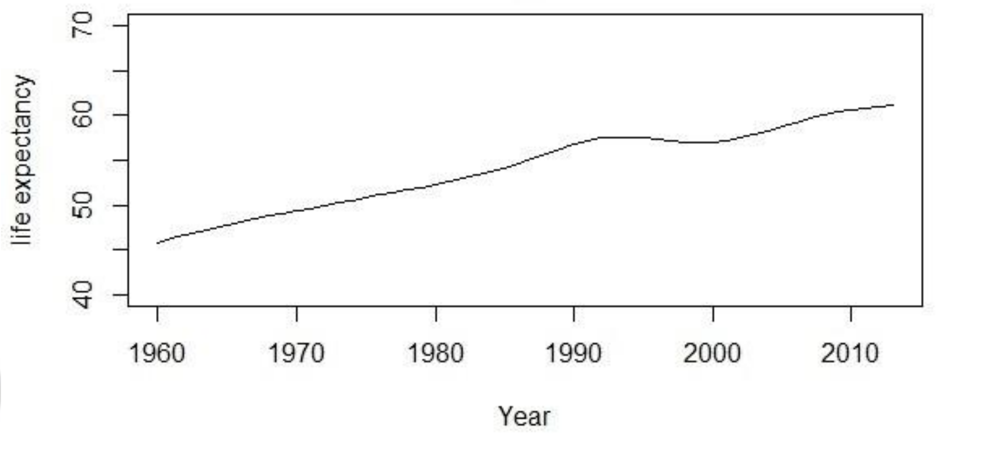

Figure 1.1: Life expectancy for Ghana from 1960 to 2013

decreased from 18.5 percent to 11.5 percent. Also, the guarantee period for pensioners has increased from 12 years to 15 years. National Pension Regulatory Authority (2010). As a result, SSNIT now receives less income but will pay out benefits for a longer period. This situation exposes SSNIT to the likelihood that at a future date, it may not be able to meet its financial obligations to pensioners. In this study we will explore ways which SSNIT can hedge against longevity risk and ensure that it is in a position to honor its financial obligation to pensioners. Pensions crisis is a predicted difficulty in paying for either corporate or state pensions or both due to a difference between pension obligations and the resources set aside to fund them.John Eatwell (2003) The ratio of workers to pensioners (the ”support ratio”) is declining. This is due to two demographic factors: increasing life expectancy coupled with a fixed retirement age, and a decrease in the fertility rate. Increased life expectancy (with fixed retirement age) increases the number of pensioners at any time, since individuals are retired for a longer fraction of their lives, while decreases in the fertility rate decrease the number of workers. In this study, our focus is on the increased life expectancy. Longevity risk cannot be precisely forecasted therefore it is necessary for pension funds to hedge their portfolio against this risk using one or more of various techniques.

1.3 Objectives of the study

In developing a mortality derivative such as a longevity swap, future mortality for higher ages must be estimated quantitatively. Bauer *et al* (2006)The objective of this study is to forecast the mortality pattern for male pensioners at higher ages (71-90). This forecasted mortality rates can then be used for a longevity swap transaction to hedge against unexpected shocks in life expectancy of pensioners.

1.3.1 Specific objectives

1. To forecast future mortality pattern at higher ages for male pensioners.
2. To design a longevity swap derivative to hedge against longevity risk.
3. Estimate the expected future lifetimes of pensioners.

1.4 Justification of the Study

At the end of this study, expected future mortality would be estimated for male pensioners under the SSNIT pension scheme. A reliable estimate for future mortality pattern would enable institutions exposed to mortality and longevity risk make better decisions. This study would be important on two main fronts:

1. The study provides a hedging strategy for pension providers to keep their portfolio immune to unexpected increases in life expectancy and thereby preventing insolvency.
2. To the pensioner, it guarantees that they don't outlive their income. Payments would be guaranteed as long as they are alive since the pension provider is immune to longevity risk.

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1.5 Limitation of the Study

The study did not take into account other risks to which pension funds are exposed to such as interest rate risk, adverse policy changes and unfavourable investment returns. Also in the proposed longevity swap, no counterparty risk was not taken into consideration. It was assumed that the would be no default.

1.6 Organization of the Study

The entire study is presented in five chapters. Chapter one highlights the rationale of the study, objectives, justification and limitations of the study. In Chapter two, we review existing literature on Pensions in Ghana, longevity risk and financial instruments used in hedging longevity risk. Chapter three explains the methodologies used in the study, including nature and source of data, analytical tools used in the study and we present the results of the study in chapter four. In the concluding chapter, we make conclusions and recommendations based on our findings. We also make recommendations for future studies.

Chapter 2

Literature Review

2.1 Introduction

In this chapter, we review existing literature on longevity risk, hedging longevity risk and forecasting future mortality.

2.2 Longevity Risk

Longevity risk is any potential risk emanating from an increase in life expectancy of pensioners and annuity policy holders, which can eventually translate in higher than expected pay-out-ratios for many pension funds and insurance companies. Antolin(2007)

Longevity risk is present in any product where the issuer is exposed to financial losses if the policyholders live longer than expected. This often occurs when payments from the issuer are dependent on the survival of the policyholders. Traditionally, these products have been issued by insurance companies and have been used to protect individuals against outliving their income or assets. In recent times, there have been an increase in the number and types of financial instruments that are exposed to longevity risk.This can occur despite the fact that longevity risk transfer may not be the primary aim of the transaction.

Longevity risk is one of the main challenges facing life annuity providers and pension schemes. Life annuity and pension providers have to pay the pensioner and annuity holder respectively for life. This could threaten the financial stability of the paying institution if the assumptions made about mortality are underestimated. Mortality has been shown to improve over time, due to technological advancement in medical sciences and increased knowledge about healthy lifestyles.

If mortality in the future is better than had been expected, the liabilities of the life insurer will be reduced since payments would delay beyond the initial estimated time. Cox and Lin (2007) However, the annuity providers will have to pay benefits for a longer period than had expected there would be a loss on the annuity portfolio relative to their initial expectations. If future mortality is worse than was expected, the benefits to insurers and annuity providers are reversed. The life insurer makes more losses than expected and the annuity provider makes more profits than expected.

Few researchers investigated the issue of natural hedging. Most of the prior research explores the impact of mortality changes on life insurance and annuities separately, or investigates a simple combination of life and pure endowment life contracts (Frees *et al*.(1996); Marceau and Gaillardetz,1999; Milevsky and Promislow, 2001; Cairns *et al* (2004). Studies on their pact of mortality changes on life insurance focus on ”bad” shocks while those on annuities focus on ”good” shocks. Wang *et al.* (2003) analyze the impact of the changes of underlined factors guiding the process of the mortality hazard rates and propose an immunization model to calculate the optimal level of product mix between annuity and life insurance to hedge longevity risks based on the mortality experience in Taiwan. However, they do not use separate mortality tables to explore life insurance and annuity mortality experience. In practice, life insurance and annuity mortality experience can be very different, so there is ”basis risk” involved in using annuities to hedge life insurance mortality risk. Their model cannot pick up this basis risk.

2.2.1 Consequencies of Underestimating Longevity

Although longevity risk develops and reveals itself slowly over time, if left unaddressed it can affect financial stability by building up significant vulnerabilities in public and private balance sheets. IMF (2012)

Exposure to longevity risk may also expose a pension scheme to other related risks such as:

- Interest rate risk: As people live longer, the pension fund fund must invest for longer periods to meet its liabilities and hence is exposed to volatilities in interest rate.
- Increased inflation risk: The pension fund may also become exposed to unfavorable inflation rates in the long term.

2.3 Pension in Ghana

In Ghana, the pensions industry is regulated by the National Pensions Regulation Authority (NPRA) through the National Pensions Act. There are a few pension providers of which the Social Security and Insurance Trust (SSNIT) is the largest. For many years, Ghana operated a pension scheme known as CAP 30 which was created in 1950 for all public servants.The name ”CAP 30” was coined from chapter 30 of the pension ordinance of 1946. CAP 30 is a defined benefit scheme which gives members the option to choose between a lump sum payment on retirement or a monthly pension until death, Berkoh Nketiah (2005). To qualify for a pension under the CAP 30 scheme, one must serve continuously for 10 years in the public service. Upon retirement, a member gets 80% of his final salary as pension. The CAP 30 was a non-contributory scheme so members made no contributions to the scheme. It was entirely funded by the government.

In 1961, a compulsory savings Act (Act 70) was instituted to encourage National savings in Ghana and provide social security on a national scale. This was later replaced by the Social Security Act of 1965 (Act 279) which covered all establishment with five or more employees except those already covered by the CAP 30 scheme. The benefits under the scheme varied from 50% to 80%.
In 2008, Act 766 was passed and was implemented in 2010. The Act establishes a three tier Pension scheme. Tiers one and two are mandatory for formal sector workers and tier three was optional. However, the CAP 30 scheme

is still in force but limited to a few section of public servants such as the security agencies.Employers and employees contribute to the scheme. Employees make a mandatory contribution of 5.5% of salary whiles the Employers contribute 13% of the employee's salary.

The tier one is a defined benefit scheme which is mandatory for all formal sector workers both in the public and private sectors. The benefits depends on the average of the best three years salary of the member. The tier one scheme is managed by the Social Security and National Insurance Trust (SSNIT) and the benefit ranges from 37.5% to 50% of the member's pensionable salary. Tier two is a mandatory defined contributions plan being managed privately by a chosen pension trustee. Upon retirement, a member is paid a lump sum which is the contributions made and investment returns for the entire period.

The tier three is a voluntary occupational fund.The funds in tier three can also be accessed after ten years of contribution and can also be used as a collateral for mortgages.Tier two is also for workers in the informal sector.

Contributions by both employers and employees are exempted from tax. National Pension Regulatory Authority (NPRA)

2.3.1 Pensionable Age

According to the Organization for Economic Cooperation and Development (OECD) (2011), Pensionable age is defined here as the age at which people can first qualify for full pension benefit without actuarial deduction for early retirement. Normal pensionable age in most countries are clearly set out in legislation. However, it may be possible to retire before the the normal age without an "actuarial" reduction in pension benefits (to reflect the longer duration of benefit payment).

2.4 Benefits under SSNIT Pension Scheme

According to SSNIT, benefits are paid to members of the scheme when they qualify.There are three main contingencies under which benefits can be paid. These categories are listed below.

Old Age Pension: This is a monthly payment made to retired members of the scheme. Members who retire at the normal pensionable age (age 60) and have made contributions of at least 180 months qualify for a full pension. Members who retire earlier than the normal pensionable age but have made contributions to the scheme for at least 180 months qualify for a reduced pension.

Invalidity Pension: Members who for one reason or the other are incapable of working for a living and have contributed 12 months within the last 36 months before the unfortunate incidence. The member must provide a medical certificate to prove he or she is unable to be gainfully employed due to a disability (physical or mental).

Survivor's Lump sum Benefit: This is a lump sum paid to the beneficiary of a member of the scheme if the member dies in service or dies after retirement but before the age of 75. If a pensioner dies after the age of 75, nothing is paid to the beneficiary.

Other Benefits: With the three tier scheme, members would have access to multiple retirement income for members.

2.4.1 Old Age or Retirement Pension

Qualifying Conditions

Full Pension: A member of the SSNIT pension scheme qualifies for a full pension if he or she is at least 60 years old and have made contributions to the scheme for at least 15 years.

Reduced Pension: Members who are between 55 and 59 years old and have contributed to the scheme for at least 15 years can apply for a reduced pension.

Table 2.1: Reduced pension. Source: SSNIT

Age	55	56	57	58	59
% of full pension	60	67.5	75	82.5	90

(SSNIT website)

Basis of Calculation of Old Age Pension

- Age
- Average of best three years salary
- Earned Pension Right - Earned pension right is determined by the number of months of contribution and it ranges between 37.5% and 60%. A 15 years service period guarantees a 50% pension rights and each additional month of contribution earns an additional 0.125%.

  Reduced Pension

  For early retirement from 55 and below 60, the pension is computed as follows:

2.5 Quantification and Management of Longevity Risk

According to Crawford *et al* (2008), To ensure that pension funds are able to effectively manage the exposure to longevity risk, actuaries should be able to measure and quantify longevity risk as well as its impact. It is only when we fully comprehend the nature of longevity risk that we can design effective risk management tools to mitigate it.

Various attempts including Blake *et al* (2006), Cox and Lin (2007), Brouhns *et al* (2002) and Bauer *et al* (2006) have been made to manage longevity risk.

Some researchers have proposed an increase in the normal retirement age. Some countries such as Belgium are in the process of reviewing the retirement age in line with life expectancy. The current retirement age is 65 but will be increased to 67 by 2030. In Germany, the retirement age currently 65 years, 3 months would be gradually increased to 67 years in 2029 Berkel *et al* (2004). In Ghana, the retirement age was increased from 50 to 60 in 1965.

Blake *et al* have proposed transferring longevity risk to the capital market or to a third party. This can be done through different types of financial instruments. These financial instruments are usually in the form of financial derivatives with some kind of longevity index as the underlying asset. These are traded as special agreements between the parties since they are not standardized.

In order to quantify longevity risk, a model is needed to predict future mortality pattern which can be compared to current mortality pattern.

2.6 Hedging Longevity Risk

Various attempts have been made to hedge longevity risk using different financial instruments. Much work has been done in this area in recent years as life expectancy has been increasing for most countries. However, the increment in life expectancy in itself is not a problem but its unpredictable nature is the source of worry for businesses which are exposed to longevity risk.

In this section, we review five financial products that have been designed to help hedge longevity risk. They are listed below:

Survival bonds,

Longevity swaps, q-

forwards,

Natural hedge and

Buy in and buy out

2.6.1 Survival Bonds

Survival bonds or longevity bonds are longevity-indexed bonds. Survival bonds are analogous to inflation-indexed bonds. The coupon payments at time t, is dependent on the number of survivors at time t from the cohort. Blake and Burrow (2001) suggested that governments should issue longevity bonds thereby taking the risk from the pensions and annuity providers. The starting point of the Blake-Burrows argument is the familiar problem of how an insurance company should hedge (or otherwise manage) its exposure to mortality risk. Blake and Burrow pointed out that insurance companies' profitability on annuity portfolios is heavily dependent on subsequently realized mortality, therefore companies stand to make considerable losses if mortality improves unexpectedly. Insurance companies are thus naturally short mortality improvement risk, and they have no particularly good hedges against this risk. BB go on to argue that insurance companies are generally unable to absorb this risk themselves and that managing their asset portfolios to match these risks is costly and, in any case, provides an imperfect hedge (Blake and Burrow, 2001).

Their proposed solution is for the government to issue Survival Bonds, that is, bonds whose future coupon payments depend on the percentage of the whole population of retirement age (say 60) on the issue date still alive on the future coupon payment dates. The coupon payments on these bonds would be very highly correlated insurance with the companies' annuity payments, so the bonds should provide a very good hedge against mortality improvement risk: if annuitants live longer, the insurance companies would then make annuity payments for longer periods, but they would also receive greater offsetting coupon payments on their Survival Bond asset positions.

Kevin Dowd (2003) faulted Blake and Burrows (2001) approach. His argument was that if the principal purpose of survival bonds is to hedge aggregate mortality risks, then the relevant base should not be the surviving proportion of the original population as Blake and Burrow suggested but should rather be the surviving proportion of the original annuitants. The reason being that the annuitants, not the population is the Insurance Company's underlying risky variable against which they seek a hedge. Dowd concluded that survival bonds conditional on the proportion of surviving annuitants would therefore provide a better hedge than contracts conditional on the surviving proportion of the original population.

The European Investment Bank and BNP Paribas Longevity Bond

In November 2004, the European Investment bank issued a new 25 year 540 million pounds bond indexed to life expectancy. The bond was intended for the UK life insurance companies and pension funds with exposure to longevity risk for the male population of England and Wales. The structure was initiated with BNP Paribas and Partner RE. The security was to be issued by the European Investment Bank (EIB), with BNP Paribas as the designer and originator and Partner Re as the longevity risk reinsurer. The face value of the issue was 540 million pounds and had a 25-year maturity. It was an amortising bond with floating coupon payments which was linked to a cohort of survivor index based on the realised mortality rates of English and Welsh males aged 65 in 2002. The initial coupon was set at 50 million pounds.

The Swiss Re Mortality Bond

In December 2003, Swiss Re issued a three-year life catastrophe bond, maturing on January 1, 2007, which helps to reduce Swiss Re's exposure to a catastrophic mortality deterioration. The issue size was USD400m. Investors receive quarterly coupons set at 3-month US dollar LIBOR + 135 basis points. However, the principal is unprotected and depends on what happens to a specifically constructed index of mortality rates across five countries namely the USA, the UK, France, Italy and Switzerland. The principal is repayable in full if the mortality index does not exceed 1.3 times the 2002 base level during any of the three years of the bond?s life. The principal is reduced by 5 percent for every 0.01 increase in the mortality index above this threshold and is completely exhausted if the index exceeds 1.5 times the basis level. The payoff schedule is shown below:

2.6.2 Longevity Swaps

Another instrument that could be used to hedge longevity risk is survival swaps. Dowd (2003) is one of the first to describe survivor swaps. He describes a swap based on the mortality experience of a reference population, where the populationdependent payments form the floating leg of the swap, with the fixed leg being the expected amount of those payments assessed at the time of the swap. Such an instrument could be of spectacular interest to a pension scheme because the main aim of a pension scheme is to invest such that the investment returns are sufficient to meet the liabilities which are usually long term in nature. A longevity swap transfers the risk of pension scheme members living longer than expected from pension schemes to an insurer or bank provider. The trustees of the pension scheme agree to pay a fixed series of payments, representing the expected benefits payable under the pension scheme plus a fee, in return for the swap provider paying the benefits that in fact fall due, based on actual scheme mortality. The trustees therefore have certainty over the payments that they are expected to make, even if scheme members live longer than expected.

A pension fund could use a survival swap to produce a series of payments that broadly reflect changes to the longevity of its members. All that is required is that the scheme’s investment returns are enough to meet the series of fixed payments making up one side of the swap. Since the pension scheme will be making fixed payments, the managers of the scheme could also purchase bonds or other fixed income instruments with maturity that coincides with their payments under the swap arrangement.

2.6.3 Natural Hedging

The values of life insurance and annuity (or pension) liabilities move in opposite directions in response to changes in the underlying mortality Cox and Lin (2007). Natural hedging utilizes this to stabilize aggregate liability cash flows. Insurance companies can find an optimal mix of their life insurance and annuity portfolios that will compliment both mortality and longevity risk.

Nan *et al* (2013) used a non parametric model to forecast future mortality. Their model circumvented the assumptions that all mortality rates are driven by the same factors. They concluded that the performance of natural hedging may be significantly affected by higher order variations in mortality rates.

2.6.4 q-forwards

q-forwards are simple capital markets instruments for transferring longevity risk. They are derivatives involving the exchange of the realized mortality rate of a population at some future date, in return for a fixed mortality rate agreed at inception. They are called q-forwards because the letter q is the symbol used by actuaries to denote mortality rates.

A portfolio of q-forwards can be used to provide an effective hedge of the longevity risk of a pension scheme, or the mortality risk of a life assurance portfolio.

Coughlan *et al* (2007) defined q-forwards as an agreement between two parties to exchange at a future date (the maturity of the contract) an amount proportional to the realised mortality rate of a given population, in return for an amount proportional to a fixed mortality rate that had been fully agreed at inception. In other words, q-forward is a zero-coupon swap that exchanges fixed mortality for realized mortality at maturity.

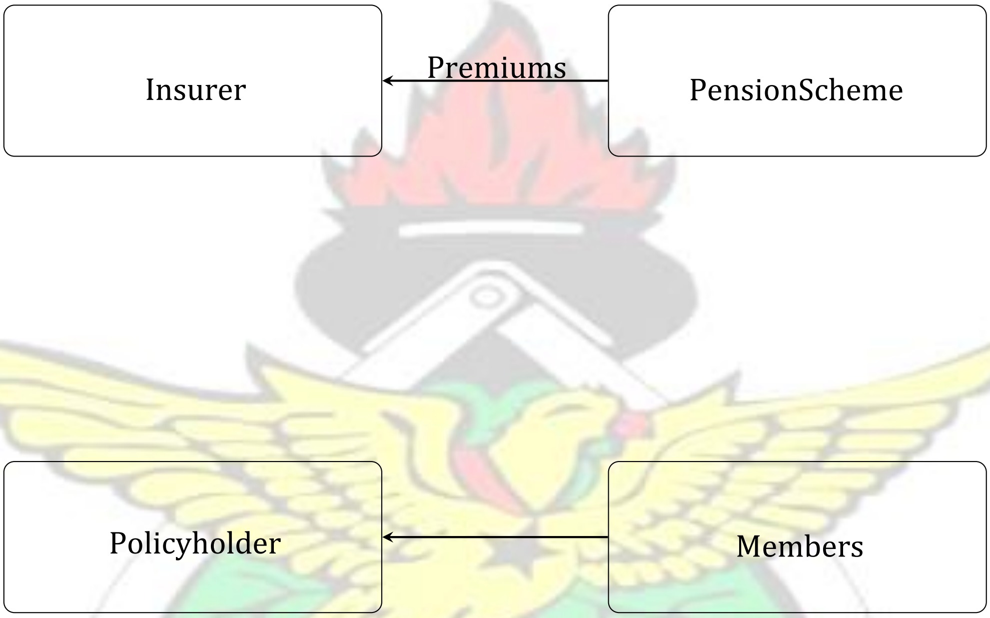
JP morgan developed a LifeMetrics index which is used to determine the value of the payout under q-forwards contracts. To hedge longevity risk of its pension liabilities, a pension plan could enter into a portfolio of q-forward contracts in which it receives fixed mortality rates and pays realized mortality rates. Such portfolio would involve q-forwards referencing both males and females across a range of different ages and maturities. At maturity, the hedge will pay out to the pension scheme an amount that increases as mortality rates falls to offset the corresponding higher value of pension liabilities. Pricing q-forwards is similar to the pricing of other forward-rate contracts, such as interest rate forwards or foreign exchange forwards.

The Life and Longevity Markets Association (LLMA), formed in 2010, is a nonprofit organisation founded and funded by members, these being Aviva, AXA, Deutsche Bank, J.P. Morgan, Munich Re, Legal and General, Morgan Stanley, Pension Corporation, Prudential PLC, RBS, Swiss Re and UBS. The LLMA aims to promote the development of a liquid traded market in longevity and mortalityrelated risk. The association supports the development of consistent standards, methodologies and benchmarks to help build a liquid trading market of the type that exists for Insurance Linked Securities (ILS), and other large trend risks like interest rate and inflation.

2.6.5 Buy-outs and Buy-ins

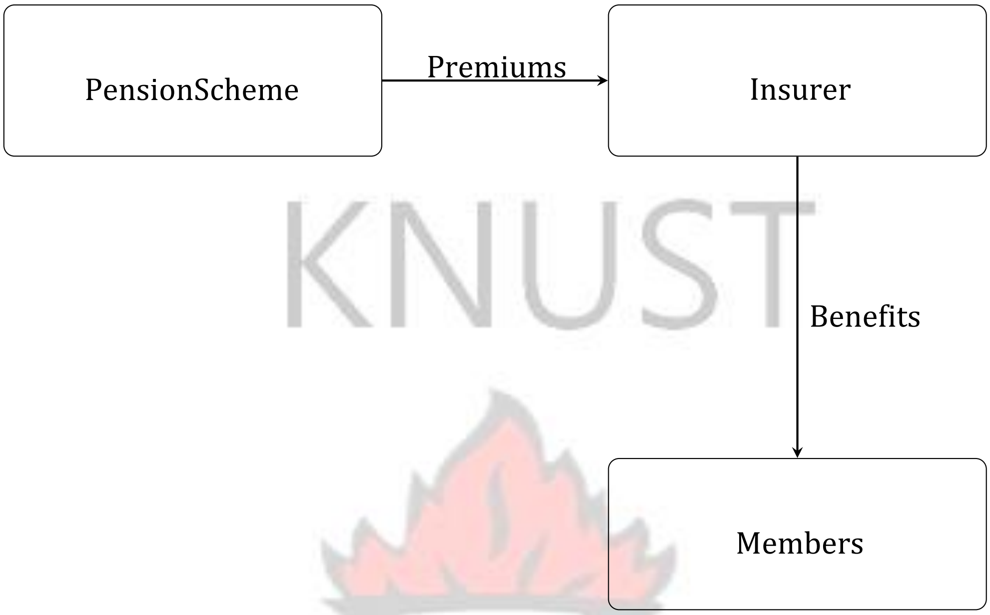
In order to curtail costs from rising further, pension funds may insure either part or their entire pension scheme liabilities with an insurer. (usually a specialist insurance comnpany). The pension scheme transferring its liabilities pays a premium to the insurance company accepting the liability. The premiums are usually paid up-front. These transactions are called bulk annuity policies and are structured in one of two ways , a buy-out or a buy-in.

In a buy-out transaction, the pension scheme's entire liabilities is transfered to the insurance company therefore the scheme is no longer obligated to pay benefits to members under the scheme.The buy-out transaction is structured such that the members of the scheme do not lose any contractual benefit. Their benefits are exactly incorporated into the pricing of the buy-out.After a buy-out, the members ceases to be part of the pension scheme. The insurance company takes full responsibility of their benefits. The members of the pension scheme becomes policyholders of the insurance company.



A pension scheme will usually opt for a buy-out before winding up its operation. A buy-out could be negotiated for part of the membership of the scheme or for the entire membership but partial buy-outs are rare because the trustees could be perceived to be bias towards a section of the membership.

In a buy-in, membership to the pension scheme is retained unlike a buy-out. A pension scheme transfers the responsibility of paying pension benefits to a regulated insurer.The insurer then ensures a stream of income to the pension funds which is paid out as benefits to the members of the scheme. The benefit of a buy-in to a pensioner is that the insurer assuming the liabilities is required as a regulatory requirement, to hold enough capital to meet its financial obligation.



A major difference between a buy-out and a buy-in is that in a buy-out, the relationship between the pension scheme and its members ceases to exist whiles in a buy-in, the relationship is maintained.

2.7 Mortality Index

Mortality indexes provide an objective method of measuring longevity risk. They broadly indicate the pace at which the mortality of a population is changing, enabling the measurement of longevity risk by comparing the difference between the expected and actual paths of the index. There have been a number of attempts by the industry to create indexes. Li and Li (2013).

- In 2006, Credit Suisse started a longevity index with the life expectancy at birth of the US population as its basis.
- In 2007, Goldman Sachs launched the QxX index, which is based on the number of survivors in the reference population.

- JP Morgan introduced the LifeMetrics in 2007, which renders death rates and period life expectancy figures.
- In 2008, Deutsche Borse released the Xpect Cohort Index, and it is linked to the number of survivors of a certain birth cohort.

2.8 Forecasting Future Mortality

As we have seen in various mortality derivatives discussed, a key component in designing a mortality derivative is forecasting future mortality. In this section, we will review models used in forecasting future mortality rates.

There's a need for the insurance and pension industries to forecasts of future mortality, since forecasts are required for pricing and reserving. Human mortality so far ahead depends on the impact of such unknown factors future medical advances, new infectious diseases, and even disasters, both natural and man-made. No attempts are made to take these underlying factors into account and future mortality forecasts are attempted by extrapolating past trends. There are a number of approaches to the problem. One of the oldest methods is based on the forecasting of parameters in some parametric model.

Age-Period-Cohort (APC) models are a widely used methods for smoothing mortality tables. The classic reference is Clayton and Schifflers (1987). Lee and Carter (1992) introduced a simple bilinear model of mortality in which the time dependent component of mortality is reduced to a single index which was then forecasted using time series methods. The model was fitted by ordinary least squares (OLS) with the observed log mortality rates as dependent variable. Brouhns *et al* (2002) improved on the OLS approach by modelling the number of deaths directly using a Poisson distribution and using maximum likelihood for parameter estimation. De Boor (2001) constructed a two-dimensional regression basis as the Kronecker product of B-splines but neither author considers nonnormal data or the forecasting problem. Gu and Wahba (1993) and Wood

(2003) use thin plate splines but again forecasting is not available. Curie *et al* (2004) used two-dimensional regression splines, specifically B-splines with penalties, usually known as P-splines (Eilers and Marx, 1996).

Curie *et al* (2004) extended this work by using B-splines to construct a basis for bivariate regression. This construction gives a basis in two dimensions with local support and hence a fully flexible family of fitted mortality surfaces. The regression approach leads to a generalized linear model which is fitted by penalized likelihood. An important feature of this method is that forecasting is a natural consequence of the smoothing process. They considered future values as missing values; the penalization then allows estimation of the future values simultaneously with the fitting of the mortality surface. We will see that the choice of penalty function, which can be of secondary importance in the smoothing of data, is now critical, since it is the penalty function that determines the form of the forecast.

Properties of the mortality indexes

Apart from being a good representative of varying age pattern of mortality improvement and being readily interpretable, the CBD indexes have other desirable properties that mortality indexes in general, should fulfil. Here we explain additional properties of the model.

Unambiguous: The population on which the mortality indexes are based must be defined in detail. In this study, the population used was males under the SSNIT pension scheme who retired at the normal retirement age of 60.

Transparent: The method used to calculate the index value must be clear. While there exist multiple methods for estimating the CBD model, the index provider can use one method. In this study, a computer program for fitting the model was used.

Objectivity: The method used to calculate the index should have as little as subjective input as possible. The CBD model used in this study meets this requirement because given our data and age range, the estimation of the parameters requires no subjective input.

Appropriateness: The indexes should reflect the compositions of the populations requiring the hedging. If the CBD mortality indexes are based on national populations, then this criterion may not be met as the mortality experience requiring hedging may be different from the mortality experience of the entire population. In this study, the reference populations are males under the SSNIT pension plan who retired at the normal retirement age of 60 and this is the population that requires the hedge hence the indexes are appropriate.

Chapter 3

Methodology

3.1 Introduction

In this chapter we shall consider the data used for the study and also discuss the models used.

3.2 Data

Secondary data was obtained from the Social Security and National Insurance Trust (SSNIT). SSNIT is the biggest pension provider in Ghana with investment across various sectors of the economy. The data obtained was well representative of Ghanaian Pensioners. It contains data on SSNIT pensioners from 1991 to 2013. For the purpose of the study, early retirements were ignored hence all analysis was carried out for pensioners who retire at the mandatory retirement age of 60. Also, disability retirement and ill health retirement were also ignored. The data was organised such that the mortality pattern for each cohort (a group of people retiring at the age of 60 in a particular year) was studied separately.

3.3 Stochastic Modelling For Mortality

In a longevity swap transaction, contracting parties would have to agree on the future mortality rates which determines the payment of the fixed leg of the transaction. The method used to estimate future mortality rates must also be agreed on. In this section, we present definitions and notations regarding mortality models. According to De Waegenaere *et al* (2010), The one-year death probability, is defined as $q_{x,t}^{(g)}$.This represents the probability that an individual belonging to the year group g,aged x in year t will not survive to age x+1 and the probability that the individual survives another year to age $x + 1$ is given by

$$p_{x,t}^{(g)} = 1 - q_{x,t}^{(g)} \tag{3.1}$$

The total number of deaths occurring in each year for each cohort was obtained from the data and the proportion of deaths in the cohort was obtained using the relation.

$$q_{x,t} = \frac{d_{x,t}}{l_{x,t}} \tag{3.2}$$

where $q_{x,t}$ = probability that a life aged x in year t dies before attaining age x+1 $d_{x,t}$ = the number of people aged x and dies in year t, $l_{x,t}$ = the number of people aged x in year t.

3.3.1 Guarantee Period

There's a guarantee period of 15 years (NPRA). If a pensioner dies before the guarantee period, a lump sum is paid to the employee's beneficiary. We obtained the proportion of people who survived beyond the guarantee period using the relation below.

Let ${}_tq_{x0}$ be the probability that a pensioner from a particular cohort, aged x_0 at time 0 will die before reaching the age x_{0+t}. Given the corresponding survival

probability ${}_tp_{x0}$, the stochastic number of survivors l_{x0+t} follows a Binomial distribution with parameters $(l_{x0}, {}_tp_{x0})$ and mean

$$E[l_{x0+t}] = l_{x0} \times {}_tp_{x0} \tag{3.3}$$

and the variance is given by

$$Var[l_{x0+t}] = l_{x0} \times {}_tp_{x0} \times (1 - {}_tp_{x0}) \tag{3.4}$$

where l_{x_0} is the number of members of the pension scheme retiring at age 60. If l_{x_0} is large, for example more than 30, according to the Central Limit Theorem, l_{x_0+t} is approximately distributed as Normal with the same parameters. (in this study,we take x_0 is taken to be 60 for homogeneity because the normal retirement age in Ghana is 60)

In a longevity swap transaction, the two parties involved would have to agree on the future mortality rates on which the payments would depend. Also, the method of estimating future mortality should be agreed on. We present notations and definitions as used by De Waegenaere *et al*(2010).

The one-year probability of death (mortality rate) is defined as $q_{x,t}^{(g)}$. This represent the probability that a pensioner belonging to group g (that is the cohort retiring in year g) aged x in year t will die before attaining the age $x+1$ for $x = x_0,...,x_m$ and $t = t_0,...,t_n$. Then the probability that the individual survives one year and attains age $x+1$ is given by

$$p_{x,t}^{(g)} = 1 - q_{x,t}^{(g)}, \tag{3.5}$$

This is known as the one year survival rate. The probability that a pensioner survives for a certain number of years (say τ years) is given by the product of the one-year probabilities

$${}_{\tau}p_{x,t}^{(g)} = \prod_{i=0}^{\tau-1} p_{x+i,t+i}^{(g)} \tag{3.6}$$

where ${}_1p_{x,t}^{(g)} = p_{x,t}^{(g)}$ is the one year survival probability.

The expected future life time of a pensioner aged x in year t belonging to group g can be estimated by

$$e_{(x,t}^{g)} = \sum_{\tau\geq 1} {}_{\tau}p_{(x,t}^{g)} \tag{3.7}$$

When modelling future mortality, the raw central death rate was used.it is defined as

$$m_{x,t}^{(g)} = \frac{D_{x,t}^{(g)}}{E_{x,t}^{(g)}} \qquad (3.8)$$

where $D_{x,t}^{(g)}$ is the number of individuals aged x from group g that died in year t and $E_{x,t}^{(g)}$ is the number exposed to risk, that is the number of individuals aged x from group g at the beginning of year t.

According to De Waegenaere *et al* (2010), the raw central rate of death is the instantaneous rate of death, that is, the probability that an individual of group g aged x dies in the next ε time units from t. where ε becomes small, the one-year death probabilities can be calculated from the central death rate.

$$q_{x,t}^{(g)} = 1 - exp(-m_{x,t}^{(g)}) \qquad (3.9)$$

In our study, τ = 15 is of importance since this is the guarantee period before which a lumps sum would be paid to the beneficiary of the pensioner if the pensioner dies. Therefore ${}_{15}P_{60}^{g}$ is the probability that a life from group g aged 60 survives for 15 years to age 75.

$${}_{15}P_{60} = \frac{l_{75}}{l_{60}} \qquad (3.10)$$

where l_{75} = the number of lives who survived to age 75. l_{60} = the number of individuals going on retirement at age 60.Dickson *et al* (2013)

3.3.2 Mortality Table

A mortality table was obtained for male pensioners who retired between 1991 and 2010. To obtain the mortality table, we counted the number of deaths in each year starting from the year of retirement. This was done for each cohort from 1991 to 2010. Due to the fact that as the years go by we get less data, the mortality table obtained was in a triangular shape. For example, taking the 1991 cohort, we have 20 years of data so we could obtain mortality rates for 20 years but for the 2010 cohort, we only have three years of data.

3.3.3 Forecasting Mortality Rates

In order to do a longevity swap, future mortality rates should be estimated. In this study, we use the Cairns-Blake-Dowd (2006) (CBD) model to forecast future mortality rates.

Logit Transformation

A logit is the defined as the logarithm of the odds. If P is the probability of an event, then $(1 - P)$ is the probability of not observing the event and the odds of the event are $\frac{P}{1-P}$. The logit transform is most frequently used in logistic regression and for fitting linear models to categorical data.

The Cairns-Blake-Dowd Model

The Cairns-Blake-Dowd model is a stochastic mortality model designed for modelling mortality at higher ages. It is therefore very useful in modelling longevity risk for pensions and annuity providers.

The CBD model was built on the observation that log mortality rates are approximately linear at ages above 40. The model uses two period-effect parameters to capture the trend improvement in mortality and the differential higher age dynamics.

$$\ln \frac{q(x,t)}{1-q(x,t)} = k_t^{(1)} + k_t^{(2)}(x - \bar{x}) \qquad (3.11)$$

$$\frac{q(x,t)}{1-q(x,t)} = exp(k_t^{(1)} + k_t^{(2)}(x - \bar{x})) \qquad (3.12)$$

$$q(x,t) = (1 - q(x,t))(exp(k_t^{(1)} + k_t^{(2)}(x - \bar{x}))) \qquad (3.13)$$

$$q(x,t) = exp(k_t^{(1)} + k_t^{(2)}(x - \bar{x})) - q(x,t)(exp(k_t^{(1)} + k_t^{(2)}(x - \bar{x}))) \qquad (3.14)$$

$$q(x,t) + q(x,t)(exp(k_t^{(1)} + k_t^{(2)}(x - \bar{x}))) = exp(k_t^{(1)} + k_t^{(2)}(x - \bar{x})) \quad (3.15)$$

$$q(x,t)(1 + exp(k_t^{(1)} + k_t^{(2)}(x - \bar{x}))) = exp(k_t^{(1)} + k_t^{(2)}(x - \bar{x})) \quad (3.16)$$

$$q(x,t) = \frac{exp(k_t^{(1)} + k_t^{(2)}(x - \bar{x}))}{1 + exp(k_t^{(1)} + k_t^{(2)}(x - \bar{x}))} \quad (3.17)$$

The first CBD mortality index, $k_t^{(1)}$, represents the level of the mortality curve (the curve of $q(x,t)$ in year t) after a logit transformation. A reduction in $k_t^{(1)}$, that is, a parallel downward shift of the logit-transformed mortality curve, represents an overall mortality improvement.

The second CBD mortality index, $k_t^{(2)}$, represents the slope of the logit-transformed mortality curve. An increase in $k_t^{(2)}$ that is, an increase in the steepness of the logit-transformed mortality curve, means that mortality (in logit scale) at younger ages (below the mean age $\bar{x}$improves more rapidly than at older ages (above the mean age).

The two parameters $k_t^{(1)}$ and $k_t^{(2)}$ would be obtained using a stochastic simulation. The simulation was done using a software written in r but embedded into Microsoft excel as an excel add-in.

Recall that the CBD model can be expressed as

$$\ln \frac{q(x,t)}{1 - q(x,t)} = k_t^{(1)} + k_t^{(2)}(x - \bar{x})$$

which implies that for a given year, t, the value of $q(x,t)$ after a logit transformation is linearly related with age. Given this structure, the CBD mortality indexes, that is, parameters $k_t^{(1)}$ and $k_t^{(2)}$ in the equation above can be easily interpreted. Although each CBD mortality index has its own meaning, it is important to consider them jointly because the association between them has a significant impact on the longevity of risk exposure of a portfolio.

3.4 Proposed Longevity Swap

Considering the fact that longevity risk is real, and the adverse effect it could have on the financial liability of the pension fund, we propose a longevity swap transaction for pension funds. For each cohort of pensioners from 1999, we have forecasted the future mortality (or survival rates). We denote the probability that a pensioner aged x dies before reaching year t by ${}_tq_x$ and the probability of survival to age $x + 1$ by p_x. Therefore the expected number of survivors to age $x+n$ from the cohort retiring in year i will be $l_x \times p_{x,t}$ where l_x is the number of pensioners who retired in year i and will be denoted by $E[l_{x+n}]$. Also, we denote the actual number of survivors from retirement year i to age $x + n$ by l_{x+n}. A longevity swap transaction for the pension fund will be structured such that the pension fund pays the investor (Insurance company or investment back) taking the other end of the swap deal a notional amount multiplied by $E[l_x]$ whiles the investor pays the pension fund the same notional amount multiplied by l_x. By this transaction, the pension fund is assured that all surviving pensioners are paid their pension since the longevity risk has been taken by a thirds party.

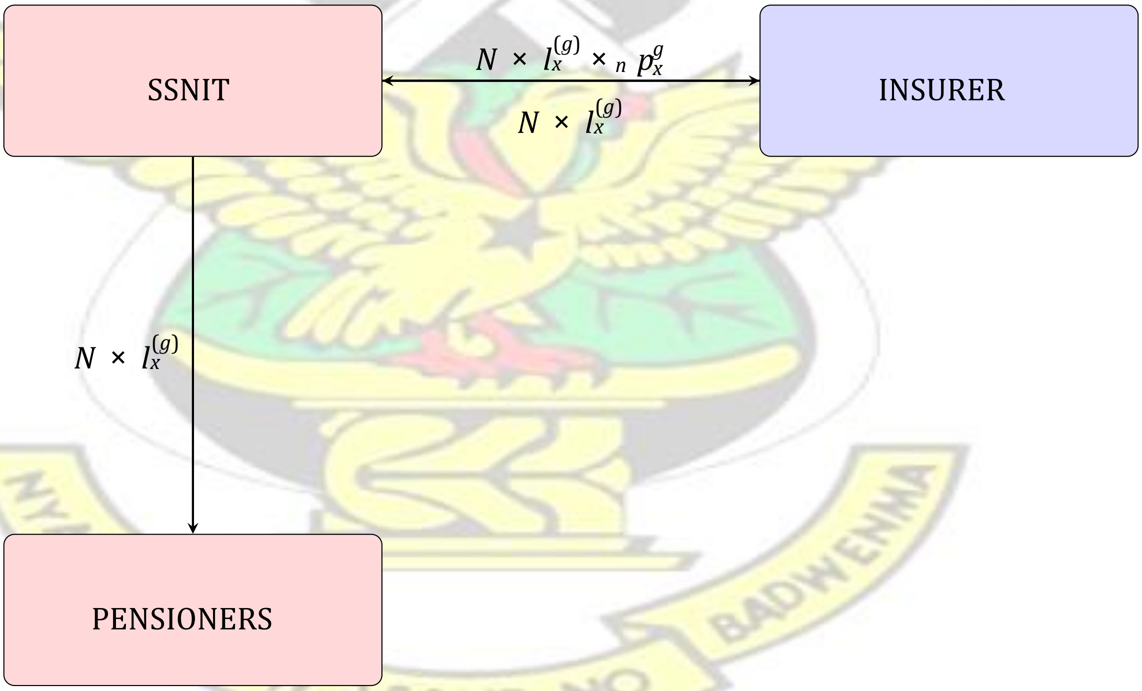
Table 3.1: Cash flows from a longevity swap

Year	SSNIT to Insurer	Insurer to SSNIT	SSNIT to Pensioners	SSNIT's profit/loss
0	premiums	0	l_x	0
1	$l_x \times P_x$	l_{x+1}	l_{x+1}	$l_x \times P_x - l_{x+1}$
2	$l_x \times {}_2P_x$	l_{x+2}	l_{x+2}	$l_x \times {}_2P_x - l_{x+2}$
3	$l_x \times {}_3P_x$	l_{x+3}	l_{x+3}	$l_x \times {}_3P_x - l_{x+3}$
4	$l_x \times {}_4P_x$	l_{x+4}	l_{x+4}	$l_x \times {}_4P_x - l_{x+4}$
.		.	.	.
.		.	.	.
.		.	.	.
n	$l_x \times {}_nP_x$	l_{x+n}	l_{x+n}	$l_x \times {}_2P_x - l_{x+1}$

Table 3.1 shows a cash flow for the proposed longevity swap. The profit or loss is the difference between the amount paid to the insurer and the amount the insurer pays to SSNIT. Like every other financial hedge, the primary purpose of the hedge is not for profits but to get rid of uncertainties in future cash flows.

In our proposed hedge, the amount SSNIT pays to pensioners is the same amount the insurer pays to SSNIT. At the beginning of the contract, SSNIT pays a single premium to the insurer. Consequently, the payments made by SSNIT depends on the expected number of survivors which in turn depends on the fore-casted mortality rates. The payments made by the insurer to SSNIT depends on the actual number of survivors.

The flowchart below describes the direction of the cash flows under a longevity risk. It can be seen that the amount paid to SSNIT by the insurer (the fixed leg of the contract) is equal to the amount SSNIT pays to the pensioners. N represents the notional amount. With this arrangement, SSNIT has completely hedged against unexpected shocks in longevity.



Chapter 4

Analysis

4.1 Introduction

The purpose of this chapter is to present and discuss the empirical findings of the study. The chapter is in four sections. We start with the descriptive statistics of pensioners of the SSNIT pension scheme to determine information on the number of people who retire in a year, the proportion of those people who survive beyond the guarantee period of 15 years. The second section presents results obtained from the mortality forecasts for male pensioners and the third section describes a simple longevity swap transaction based on the forecasted mortality rates.

4.2 Descriptive Statistics of the data

The summary statistics of the data is presented here to point out salient features of the data.The study used secondary data obtained from the Social Security and Insurance Trust (SSNIT). SSNIT have members all over the country and from different geographical, educational, cultural and professional background. Therefore the data is a good representation of pensioners in the country.

The graph above shows male retirement pattern between 1991 and 2013. The general trend is an increase in the number of males retiring at age 60 between 1991 and 2013 with the steepest increase occurring between 2001 and 2010. The pattern is similar for females retiring from active service at age 60 between 1991

Table 4.1: summary statistics for males retiring at age 60

Minimum	First quartile	Median	Mean	Third quartile	Maximum
1042	1564	2331	2459	2982	4901

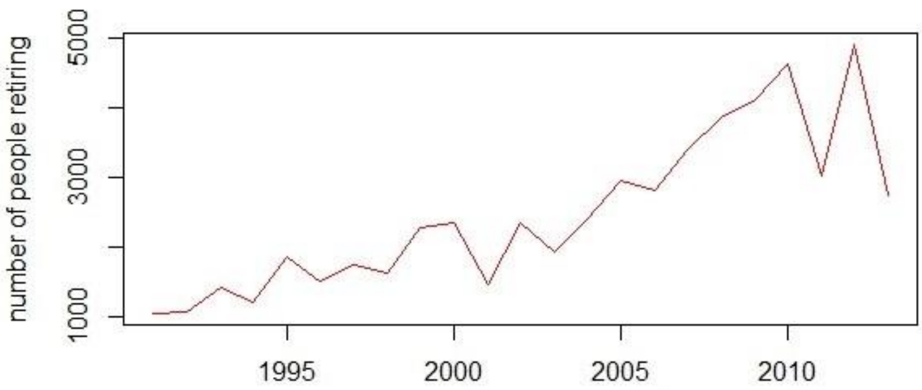


Figure 4.1: The number of males who retire at age 60

and 2013. The period recording the highest number of retirements was also between 2001 and 2010 where the trend peaks and declined the next year 2011 before a sharp increase again in 2012. The trend could be attributed to factors such as employment rates from the late 70s and the number of people who leave active service due to other decrements including early retirements.

The guarantee period is 15 years by then a person who retired at the age of 60 should be 75 years old. Therefore the graph shows the percentage of pensioners retiring in a given year who survived to the age of 75. In actuarial notation, this represents ${}_{15}P_{60}$ that is, the probability that a 60 year old survives to age 75. It can be seen clearly from the graph that the proportion of survivors beyond the guarantee period is greater for women than men in all years between 1991 and 1998. We stopped at 1998 because the cohort of normal retirees in 1999 will be 75 years old in 2014 but there wasn't enough data for 2014.

The crude death rates $m_{x,t}$, the one year death probabilities by year and age $q_{x,t}$ estimated as well as the number of deaths in each year, $D_{x,t}$ are all tabulated in the appendix.

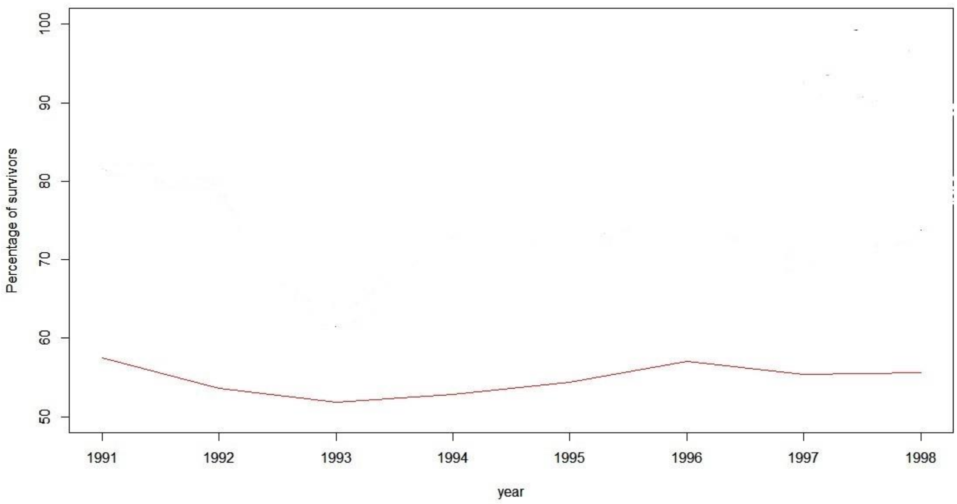


Figure 4.2: Percentage of pensioners who survive beyond the guarantee period

4.3 Results of the Cairns-Blake-Dowd Model

Using the Cairns-Blake-Dowd (CBD) model, mortality for each cohort was forecasted up to age 90. The parameters obtained are displayed in table 4.3 below. One of the most important factors needed to carry out a longevity hedge is a forecast for future mortality pattern. These forecasted values becomes the expected mortality with which a longevity hedge is done. In our longevity swap, the expected mortality is swapped with the realized mortality. The other leg of the swap deal pays SSNIT an amount which is dependent on the actual survivors. The longevity risk is thereby transferred to the other party who receives payments dependent on the expected mortality.

These parameters were derived using a software (an excel add-in) obtained from the CBD website (www.cbdmodel.com). Kappa1v and kappa2v are the two parameters for the CBD model and $\bar{x}$ is the mean age in the forecast data. The forecasted mortality table is attached in the Appendix.

The differences in projected mortality for the 1991 and 1992 cohorts may be due to cohort effects. Atingdui (2011) defined the cohort effect as the effect that having been born in a certain time, region, period or having experienced the same life experience (in the same time period) has on the development or perceptions of a particular group.

Table 4.2: Parameters of the Cairns-Blake-Dowd model

	$k_t^{(2)}$
	0.413964049
	0.23485895
	0.178653366
	0.159745102
	0.097051666
	0.073945206
	0.073284044
	0.025327022
	0.028617934
	0.038344897
$k_{t(1)}$	
-3.047378884	
-2.912521956	
-2.787168392	
-2.531943455	-2.97871478
-3.063075995	
-2.726411719	
-2.878547235	-2.94910655
-2.952760288	

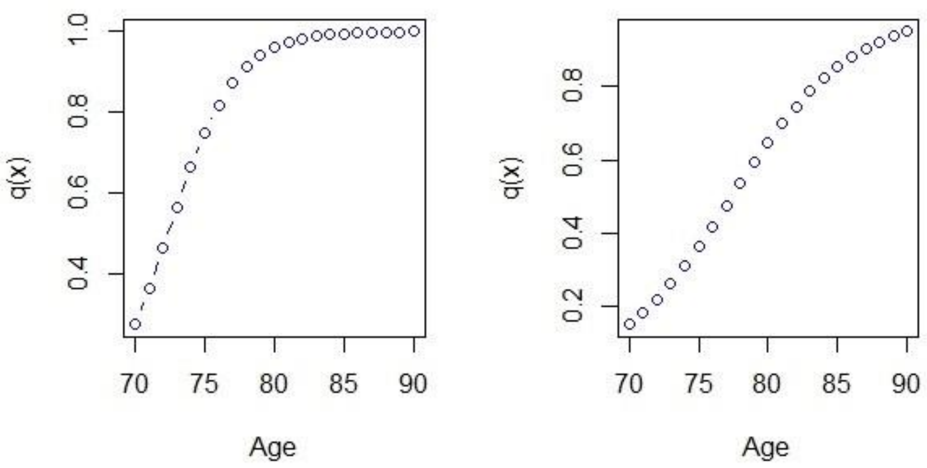


Figure 4.3: projected mortality for 1991 and 1992 cohorts

From the graphs, we can see that as the years go by, mortality rates at higher ages decreases. This suggests that in the future, mortality could significantly improve hence exposing the pension fund to longevity risk.

Using eqn 3.1 and eqn 3.7, we calculated the life expectancy, $e_{x,t}^{(g)}$ for pensioners from age 60 to age 90. The results are tabulated in table 4.4

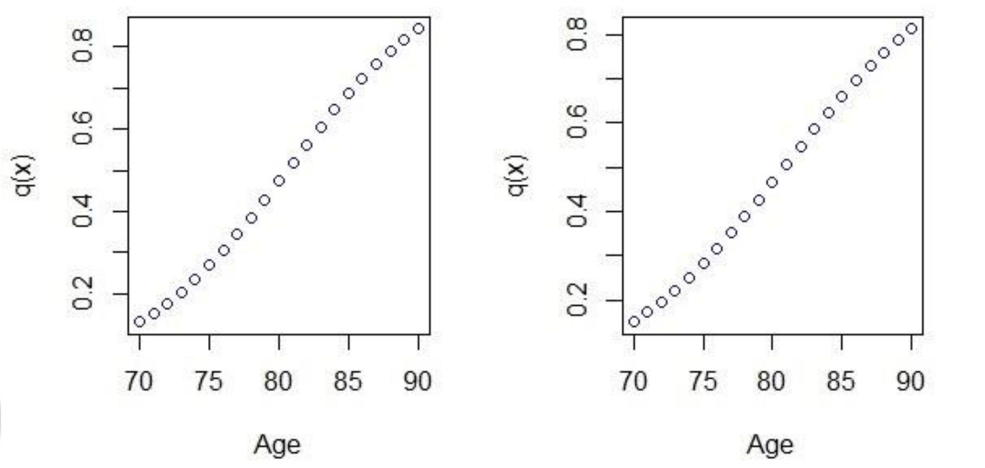


Figure 4.4: projected mortality for 1993 and 1994 cohorts

From the estimated life expectancy tables, we can see that the life expectancy of pensioners is expected to increase with time hence pension funds would need to set aside more funds to adequately meet their liabilities.

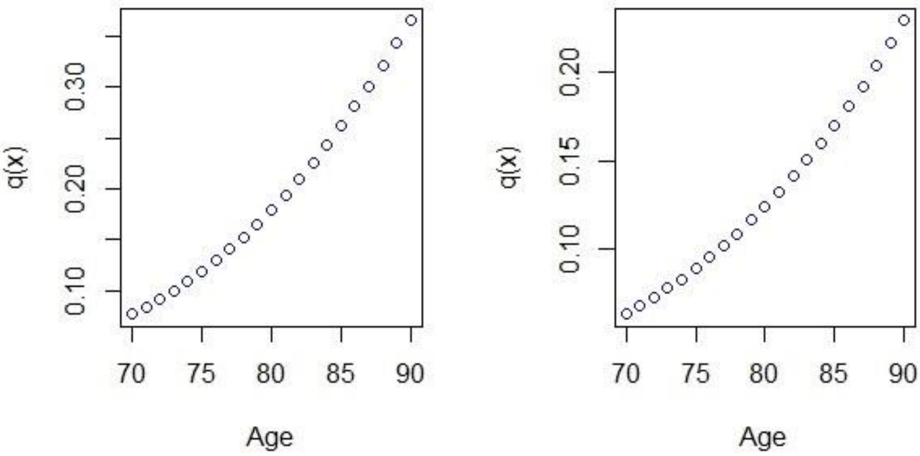


Figure 4.5: projected mortality for 1995 and 1996 cohorts

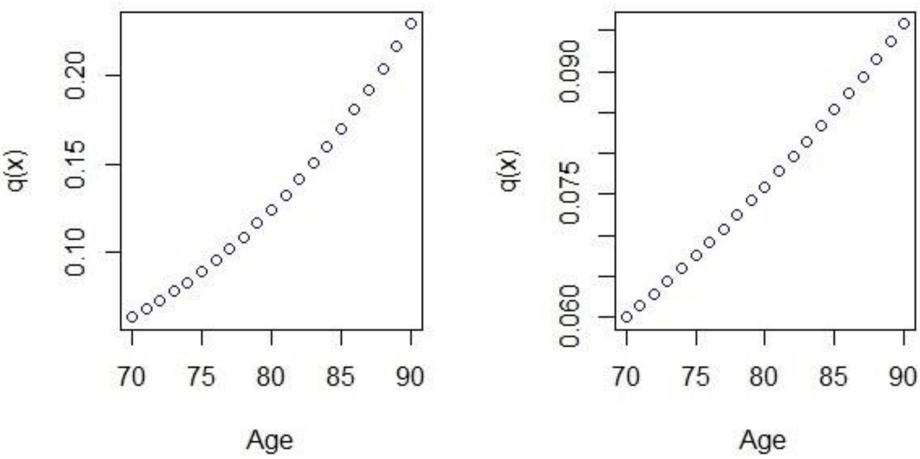


Figure 4.6: projected mortality for 1997 and 1998 cohorts

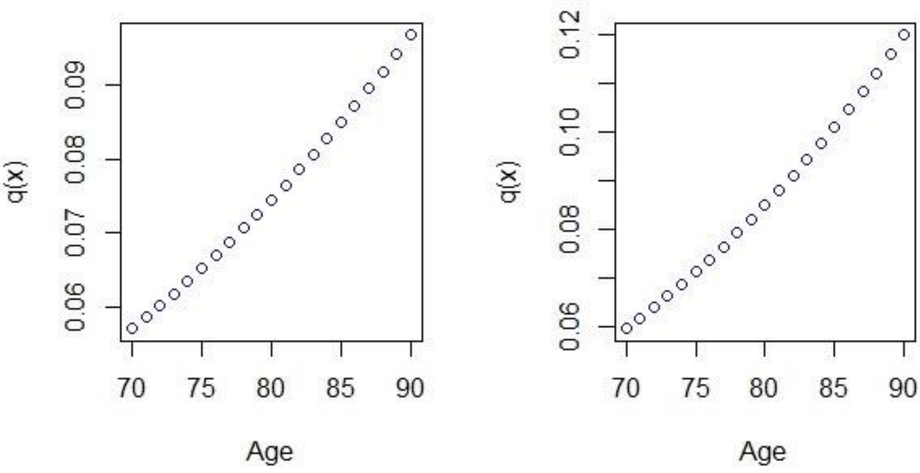


Figure 4.7: projected mortality for 1999 and 2000 cohorts

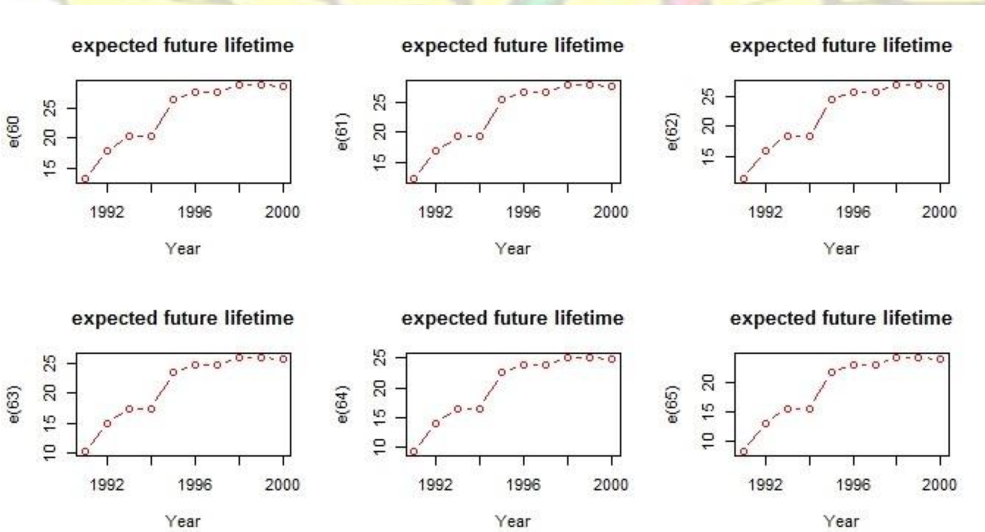


Figure 4.8: $e_x^{(g)}$ for $g = 1991, 1992, \ldots,$ 2000 and x = 60,61,...,65

Table 4.3: Life expectancy of pensioners by age and year of retirement

Age /Year	1991	1992	1993	1994	1995
60	13.15	17.86	20.39	20.34	26.42
61	12.15	16.86	19.39	19.35	25.43
62	11.15	15.87	18.42	18.38	24.46
63	10.16	14.91	17.46	17.46	23.50

64	9.20	13.96	16.51	16.52	22.56
65	8.25	13.00	15.58	15.60	21.62
66	7.29	12.06	14.64	14.67	20.67
67	6.34	11.13	13.71	13.75	19.71
68	5.38	10.19	12.80	12.85	18.78
69	4.44	9.27	11.87	11.96	17.84
70	3.50	8.33	10.94	11.08	16.91
71	2.78	7.48	10.07	10.23	15.99
72	2.14	6.66	9.22	9.40	15.07
73	1.60	5.88	8.40	8.60	14.16
74	1.17	5.14	7.61	7.82	13.26
75	0.83	4.45	6.84	7.07	12.37
76	0.58	3.81	6.11	6.35	11.49
77	0.40	3.23	5.42	5.67	10.62
78	0.27	2.71	4.76	5.02	9.76
79	0.18	2.24	4.15	4.41	8.91
80	0.12	1.84	3.57	3.84	8.08
81	0.08	1.48	3.05	3.30	7.25
82	0.05	1.18	2.57	2.81	6.45
83	0.03	0.93	2.13	2.35	5.66
84	0.02	0.72	1.73	1.94	4.88
85	0.01	0.54	1.38	1.56	4.13
86	0.01	0.40	1.07	1.22	3.39
87	0.01	0.28	0.79	0.92	2.67
88	0.00	0.19	0.55	0.64	1.97
89	0.00	0.11	0.34	0.40	1.29
90	0.00	0.05	0.16	0.19	0.63

Table 4.4: Life expectancy of pensioners by age and year - continued

Age /Year	1996	1997	1998	1999	2000
60	27.77	27.64	28.91	28.98	28.72
61	26.77	26.65	27.91	27.99	27.73
62	25.81	25.69	26.95	27.02	26.77
63	24.86	24.74	26.00	26.05	25.82
64	23.90	23.80	25.04	25.09	24.85
65	22.94	22.86	24.09	24.14	23.88
66	21.99	21.91	23.15	23.20	22.93
67	21.05	20.99	22.21	22.25	22.00
68	20.10	20.06	21.27	21.31	21.07
69	19.16	19.14	20.33	20.37	20.13
70	18.22	18.22	19.39	19.42	19.18
71	17.29	17.29	18.45	18.47	18.24

72	16.35	16.35	17.51	17.53	17.30
73	15.43	15.43	16.57	16.59	16.37
74	14.50	14.50	15.64	15.65	15.43
75	13.59	13.59	14.70	14.72	14.50
76	12.68	12.68	13.77	13.78	13.57
77	11.77	11.77	12.84	12.85	12.65
78	10.87	10.87	11.91	11.92	11.72
79	9.98	9.98	10.98	10.99	10.80
80	9.10	9.10	10.06	10.06	9.88
81	8.22	8.22	9.14	9.14	8.97
82	7.36	7.36	8.21	8.21	8.06
83	6.50	6.50	7.29	7.29	7.15
84	5.65	5.65	6.37	6.37	6.24
85	4.81	4.81	5.46	5.46	5.34
86	3.98	3.98	4.54	4.54	4.44
87	3.16	3.16	3.63	3.63	3.54
88	2.35	2.35	2.72	2.72	2.65
89	1.55	1.55	1.81	1.81	1.76
90	0.77	0.77	0.90	0.90	0.88

Chapter 5

Conclusion and Recommendation

5.1 Introduction

In this chapter, we present a summary of our findings and also draw conclusions from these findings. Furthermore, we also make recommendations.

5.2 Summary of Main Results

The forecasted mortality curves above were plotted using R. Forecasted future mortality rates for male pensioners who retired between 1991 and 2000 obtained was plotted against age for each of the cohorts. The curves obtained are consistent with mortality rates which increase with age.

Also we can observe that the mortality rates at higher ages is on the decrease. The forecasted mortality for the 1991 cohort is higher than the forecasted mortality for the 1992 cohort and continues in that order. This suggests that mortality is improving hence more pensioners are likely to survive beyond the guarantee period of 15 years. Also, it is recommended that SSNIT takes appropriate measures to hedge its longevity risk.

5.3 Conclusions

The objectives of the study was to forecast future mortality rates which would be used to determine the payments to be made under a longevity swap contract and also to design a longevity swap for the SSNIT pension scheme. We analyzed pensioners data from SSNIT to determine central mortality rates, the one-year probability of death. We used the Cairns-Blake-Dowd model to forecast future one-year death probability up to age 90 for each cohort.

A 5-year forecasted mortality table for ages 70 to 75 for males who retired from 1991 to 1995 is shown in Table 5.1 below The full forecasted mortality tables for

Table 5.1: Forecasted mortality for 1991 to 1995 cohorts

AGE	1991	1992	1993	1994	1995
70	0.273376472	0.149538252	0.130800255	0.150176432	0.076318182
71	0.36271615	0.181923698	0.152484239	0.171721873	0.083447261
72	0.462662004	0.219512541	0.177030832	0.195646759	0.09117655
73	0.565701265	0.262377019	0.204575072	0.222011486	0.099544008
74	0.663358624	0.310284108	0.235180222	0.250821162	0.108587616
75	0.748807358	0.362638054	0.268816644	0.282014185	0.118344856

ages 70 to 90 for males who retired in 1991 through to 2000 can be found in Table 5.2 and Table 5.3 in the appendix.

These future probabilities would be used to make payments for the floating leg of the longevity swap. The fixed leg of the swap will depend on the actual number of survivors.

The research also estimated the future lifetimes of pensioners from age 60 to 90 using the mortality rates obtained from the data and and future mortality rates estimated from the CBD model.

5.4 Recommendations

Based on the conclusions made, we make some recommendations to policy makers and institutions exposed to longevity risk.

- The government should help create an exchange where standardized longevity linked instruments can be traded for investors and institutions exposed to different kinds of mortality related risks can buy and sell securities that suit their needs.
- It is recommended that since life expectancy is on the increase, the normal retirement age should also be adjusted in accordance with the increasing trend.
- It is recommended that pension providers in Ghana such as SSNIT should take measures to hedge against longevity risk using a longevity swaps or other financial instruments.

We also recommend further studies in the area of pricing premiums for longevity swap.

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Appendix

Table 5.2: Forecasted mortality for 1991 to 1995 cohorts

AGE	1991	1992	1993	1994	1995
70	0.273376472	0.149538252	0.130800255	0.150176432	0.076318182
71	0.36271615	0.181923698	0.152484239	0.171721873	0.083447261
72	0.462662004	0.219512541	0.177030832	0.195646759	0.09117655
73	0.565701265	0.262377019	0.204575072	0.222011486	0.099544008
74	0.663358624	0.310284108	0.235180222	0.250821162	0.108587616
75	0.748807358	0.362638054	0.268816644	0.282014185	0.118344856
76	0.818501176	0.41846607	0.305343233	0.315453057	0.128852109
77	0.872159587	0.476463768	0.344494628	0.350918966	0.140143962
78	0.911666414	0.535103253	0.385877458	0.388111564	0.152252421
79	0.939806963	0.592788108	0.428978204	0.426654969	0.165206049
80	0.95938215	0.648023442	0.47318378	0.466110307	0.179029029
81	0.972775762	0.69956373	0.517813823	0.505994255	0.193740171
82	0.981836486	0.746509159	0.562161444	0.545802056	0.209351894
83	0.987919087	0.78833848	0.605537447	0.585032785	0.225869199
84	0.99198138	0.824884568	0.647312221	0.623214212	0.243288678
85	0.994685048	0.856270965	0.686950073	0.659924678	0.261597606
86	0.996480344	0.882830953	0.724032324	0.694809954	0.280773145
87	0.997670641	0.905027241	0.758267715	0.727593837	0.300781725
88	0.99845902	0.923383621	0.789490833	0.75808221	0.321578639
89	0.998980841	0.938433427	0.817650885	0.78616113	0.343107902
90	0.999326078	0.950684874	0.842794032	0.811790116	0.365302408

AGE	1991	1992	1993	1994	1995	1996
60	0.000959693	0.001877934	0.004926108	0.00918197	0.006458558	0.007915567
61	0.00192123	0.009407338	0.025459689	0.033698399	0.029252438	0.037234043
62	0.005774783	0.040835708	0.042089985	0.074978204	0.044642857	0.050414365
63	0.040658277	0.052475248	0.054545455	0.065975495	0.058995327	0.045818182
64	0.050454087	0.045977011	0.06650641	0.086781029	0.060831782	0.039634146
65	0.044633369	0.059145674	0.061802575	0.06519337	0.052214144	0.048412698
66	0.051167964	0.068684517	0.075937786	0.085106383	0.048117155	0.064220183
67	0.044548652	0.06375	0.093069307	0.107235142	0.068131868	0.048128342
68	0.062576687	0.082777036	0.073144105	0.112879884	0.063679245	0.062734082
69	0.060209424	0.059679767	0.073027091	0.135399674	0.073047859	0.064935065
70	0.080779944	0.092879257	0.09656925	0.126415094	0.064311594	0.045940171
71	0.051515152	0.071672355	0.068917018	0.144708423	0.074540174	0.041433371
72	0.075079872	0.069852941	0.092145015	0.181818182	0.058577406	0.04088785
73	0.07253886	0.081027668	0.131447587	0.145061728	0.052222222	0.035322777
74	0.061452514	0.129032258	0.218390805	0.137184116	0.037514654	0.027777778
75	0.111111111	0.155555556	0.257352941	0.10460251	0.03410475	
76	0.142857143	0.178362573	0.353135314	0.163551402		

77	0.177083333	0.288256228	0.984693878
78	0.246835443	0.88	
79	0.584033613		

Table 5.3: Forecasted mortality for 1991 to 1995 cohorts

AGE	1996	1997	1998	1999	2000
70	0.063366902	0.063366902	0.059978751	0.0569999	0.059466407
71	0.067899956	0.067899956	0.061422737	0.058557784	0.061647621
72	0.072732109	0.072732109	0.062899161	0.060155531	0.063903406
73	0.077879414	0.077879414	0.064408637	0.061794011	0.066235907
74	0.08335825	0.08335825	0.06595179	0.063474105	0.068647302
75	0.089185244	0.089185244	0.067529246	0.065196704	0.071139798
76	0.095377181	0.095377181	0.069141639	0.066962709	0.07371563
77	0.10195089	0.10195089	0.070789608	0.068773032	0.07637706
78	0.108923128	0.108923128	0.072473799	0.070628591	0.079126369
79	0.116310429	0.116310429	0.074194859	0.072530316	0.081965862
80	0.124128953	0.124128953	0.075953443	0.074479143	0.084897857
81	0.132394307	0.132394307	0.077750208	0.076476015	0.087924687
82	0.141121349	0.141121349	0.079585818	0.078521883	0.091048695
83	0.150323981	0.150323981	0.081460937	0.080617704	0.094272228
84	0.160014918	0.160014918	0.083376233	0.08276444	0.097597634
85	0.170205454	0.170205454	0.085332378	0.084963059	0.101027257
86	0.180905203	0.180905203	0.087330044	0.087214529	0.104563435
87	0.192121846	0.192121846	0.089369908	0.089519825	0.108208488
88	0.203860862	0.203860862	0.091452644	0.091879923	0.111964718
89	0.216125267	0.216125267	0.09357893	0.094295797	0.115834402
90	0.228915355	0.228915355	0.095749443	0.096768426	0.119819783

Table 5.4: Central mortality rates for male pensioners by age and year of retirement

AGE	2003	2004	2005	2006	2007	2008
60	0.007763975	0.0096517	0.007804547	0.009936125	0.008557096	0.008062419
61	0.033907147	0.03940678	0.032831737	0.030107527	0.029464286	0.025694809
62	0.048596112	0.042787825	0.043140028	0.042867701	0.029132168	0.029601722
63	0.046538025	0.041013825	0.036954915	0.031660232	0.034112445	
64	0.047619048	0.044690053	0.037221796	0.047448166		
65	0.054375	0.041247485	0.039059386			
66	0.053536021	0.055089192				
67	0.048882682					

Table 5.5: Central mortality rates for male pensioners by age and year of retirement- continuation

AGE	1997	1998	1999	2000	2001	2002
60	0.009837963	0.006819591	0.009275618	0.00986701	0.011572498	0.010204082
61	0.040911748	0.035580524	0.029424877	0.040727903	0.035812672	0.042955326
62	0.054235222	0.049838188	0.0404226	0.048328817	0.047142857	0.048025135
63	0.054768041	0.040871935	0.040210627	0.038917893	0.049475262	0.047147572
64	0.062031357	0.052556818	0.049875312	0.025185185	0.054416404	0.04453241
65	0.054505814	0.062968516	0.054068241	0.056737589	0.05087573	0.059036769
66	0.079169869	0.0624	0.05327414	0.063909774	0.056239016	0.059988993
67	0.075125209	0.059726962	0.067409144	0.075731497	0.052141527	0.050936768
68	0.084837545	0.068058076	0.057196732	0.063935444	0.061886051	0.054287477
69	0.085798817	0.0593963	0.050666667	0.051724138	0.069109948	
70	0.078748652	0.049689441	0.054073034	0.047552448		
71	0.078454333	0.020697168	0.047512992			
72	0.06480305	0.035595106				
73	0.057065217					

Table 5.6: Central mortality rates for male pensioners by age and year of retirement- continuation

Table 5.7: Number of deaths by year and age

AGE	1991	1992	1993	1994	1995	1996
60	1	2	7	11	12	12
61	2	10	36	40	54	56
62	6	43	58	86	80	73
63	42	53	72	70	101	63
64	50	44	83	86	98	52
65	42	54	72	59	79	61
66	46	59	83	72	69	77

67	38	51	94	83	93	54
68	51	62	67	78	81	67
69	46	41	62	83	87	65
70	58	60	76	67	71	43
71	34	42	49	67	77	37
72	47	38	61	72	56	35
73	42	41	79	47	47	29
74	33	60	114	38	32	22
75	56	63	105	25	28	
76	64	61	107	35		
77	68	81	193			
78	78	176				
79	139					

Table 5.8: Number of deaths by age and year continued

AGE	1997	1998	1999	2000	2001	2002
60	17	11	21	23	17	24
61	70	57	66	94	52	100
62	89	77	88	107	66	107
63	85	60	84	82	66	100
64	91	74	100	51	69	90
65	75	84	103	112	61	114
66	103	78	96	119	64	109
67	90	70	115	132	56	87
68	94	75	91	103	63	88
69	87	61	76	78	66	
70	73	48	77	68		
71	67	19	64			
72	51	32				
73	42					

Table 5.9: Number of deaths by age and year continued

AGE	2003	2004	2005	2006	2007	2008	2009
60	15	23	23	28	29	31	32
61	65	93	96	84	99	98	101
62	90	97	122	116	95	110	
63	82	89	100	82	108		

64	80	93	97	119
65	87	82	98	
66	81	105		
67	70			
68				

Table 5.10: One-year death rates for male pensioners by year and age: 1991-1995

Age	1991	1992	1993	1994	1995
60	0.000959233	0.001876172	0.004913995	0.009139944	0.006437746
61	0.001919385	0.009363227	0.025138324	0.033136933	0.028828727
62	0.005758141	0.040013164	0.0412165	0.072236292	0.04366103
63	0.039842818	0.051122192	0.053084534	0.063846195	0.057288826
64	0.049202418	0.044936083	0.064343082	0.083122157	0.059018483
65	0.043651955	0.057430548	0.059931539	0.06311372	0.050874405
66	0.049880929	0.066378825	0.073126131	0.081585424	0.046977871
67	0.043570933	0.06176047	0.08886965	0.101685583	0.065862717
68	0.060658975	0.079443625	0.07053312	0.106742052	0.061694083
69	0.058432674	0.057933834	0.070424353	0.12663322	0.070443659
70	0.077603352	0.088696473	0.09205298	0.118751031	0.062287232
71	0.050210741	0.06916417	0.066595868	0.13472545	0.071829815
72	0.072330612	0.067469053	0.08802711	0.166247082	0.056894764
73	0.069970395	0.077831823	0.123174769	0.135031102	0.050882072
74	0.0596024	0.121054388	0.196188753	0.128190302	0.036819697
75	0.105160683	0.144060477	0.226904692	0.099317537	0.033529739
76	0.1331221	0.163360974	0.297517868	0.150877149	
77	0.162290028	0.250430493	0.626446437		
78	0.218730754	0.585217088			
79	0.442355498				

Table 5.11: One-year death rates for male pensioners by year and age: 1996-2000

Age	1996	1997	1998	1999	2000
60	0.007884322	0.009789729	0.00679639	0.009232733	0.009818491
61	0.036549379	0.040086159	0.034954979	0.028996181	0.039909668
62	0.04916465	0.052790725	0.048616642	0.039616505	0.047179568
63	0.044784378	0.053295281	0.040047941	0.039412908	0.038170321
64	0.038858988	0.060146584	0.05119959	0.048651961	0.024870684
65	0.047259489	0.053046997	0.061026964	0.052632545	0.055158026
66	0.062201511	0.076117028	0.060492991	0.051879941	0.061910365
67	0.046988532	0.072372668	0.057978294	0.06518735	0.072934907
68	0.060806812	0.081338487	0.065793783	0.055591744	0.061934444

69	0.062871686	0.082221145	0.057666751	0.049404517	0.050409213
70	0.044900897	0.07572779	0.048475117	0.052637085	0.04643954
71	0.040586742	0.075455719	0.020484451	0.046401916	
72	0.04006322	0.062747963	0.03496905		
73	0.034706209	0.055467533			
74	0.027395523				

Table 5.12: One-year death rates for male pensioners by year and age: 2000-2005

Age	2001	2002	2003	2004	2005
60	0.011505794	0.010152197	0.007733913	0.009605271	0.007774171
61	0.035178986	0.042045816	0.033338742	0.038640432	0.032298626
62	0.046048891	0.046890169	0.047434218	0.041885344	0.042222735
63	0.048271299	0.046053389	0.045471736	0.040184139	0.036280416
64	0.052962326	0.043555398	0.046503045	0.043706164	0.03653758
65	0.049603231	0.057327892	0.052923114	0.040408384	0.038306404
66	0.054686836	0.0582251	0.052128203	0.053599267	
67	0.050805479	0.04966124	0.047707155		
68	0.060010008	0.052840219			
69	0.066775931				

Table 5.13: One-year death rates for male pensioners by year and age: 2006-2010

Age	2006	2007	2008	2009	2010
60	0.009886925	0.008520589	0.008030005	0.007778279	0.00796637
61	0.02965881	0.029034446	0.025367506	0.02453416	
62	0.041961871	0.028711917	0.029167883		
63	0.031164294	0.033537175			
64	0.046340096				