KWAME NKRUMAH UNIVERSITY OF SCIENCE

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MARKOV CHAIN ANALYSIS OF RAINFALL PATTERN IN THE SOUTH EASTERN COAST OF GHANA

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DECLARATION

This work was the result of my field research, except for references to other works carried out by other researchers which have been duly acknowledged. It has to be noted however that this work has not been submitted for the award of any other degree elsewhere apart from Kwame Nkrumah University of Science and Technology (KNUST). I am therefore responsible for the views expressed and the factual accuracy of its contents.

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DEDICATION

This work is dedicated to my loving wife Mrs. Tetteh Betty and my children Sedinam, Sedem and Priscilla.

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The greatest thanks go to the Almighty God for His love, grace and mercies bestowed upon me throughout my entire life and education career.

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ABSTRACT

This study develops an objective rainfall pattern assessment through Markov chain analysis using daily rainfall data from 1980 to 2010 for five towns along the south eastern coastal belt of Ghana namely Keta, Akatsi, Akuse, Accra and Cape Coast. The transitional matrices were computed for each town and each month using the conditional probability of rain or no rain on a particular day given that it rained or did not rain on the previous day. The steady state transition matrices and the steady state probability vectors were also computed for each town and each month. It was found that, the rainy or dry season pattern observed using the monthly steady state rainfall vectors tended to reflect the monthly rainfall time series trajectory. In particular, for Accra, the rainy season was observed to be in the months of May to June and September to October. Also, it was observed that the probability of rainfall tended to increase from east to west along the south eastern coast of Ghana.

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List of Abbreviations

UNFCCC: United Nations framework convention on Climate Change

CWSA: Community Water and Sanitation Agency

NMSA: Nation Meteorological Services Agency of Ghana

ITCZ: Inter Tropical Convergence Zone

GCM: Global Circulation Models

EPPM: Elliptical cell poison process model

GDSTM: Gausian Displacement Spatial- temporal model

RESTM: Radom ellipse spatial – temporal model

ANN: Artificial Neural Network

EfuNN: Evolving fuzzy Neural Network

ANN- SCGA: Artificial Neural Network with Scaled conjugate Gradient Algorithm

ENSO: Elni-no Southern Oscillation

SOI: Southern Oscillation Index

SSI: Sea Surface Temperature

ARMA: Auto- regressive moving average

JMA: Japan Meteorological Agency

CHAPTER ONE

INTRODUCTION

1.0 Background

A study has yielded the first confirmation that global warming is already affecting the world's rainfall patterns, bringing more precipitation to northern Europe, Canada and northern Russia but less to swathes of sub-Saharan Africa, southern India and South-East Asia. The changes "may have already had significant effects on ecosystems, agriculture and human regions that are sensitive to changes in precipitation, such as the Sahel". Scientists have long said that, global warming is bound to interfere with snow and rainfall patterns, because air and sea temperatures and sea-level atmospheric pressure - the underlying forces behind these patterns - are already changing. However, until now, evidence that, the interference was already happening, existed anecdotally or in computer models, rather than from observation. One problem for researchers has been lack of accurate, long-term rainfall data from around the world that would enable them to distinguish between regional or cyclical shifts in rainfall.

Zwiers (1999), a scientist with Environment; Canada, Toronto, found a way around these problems by using two data-sets of global rainfall pattern beginning, conservatively, in 1925 and ending in 1999. He compared these figures with 14 powerful computer models that simulate the world's climate system and found a remarkably close fit.

Over the 75-year period under study, global warming "contributed significantly" to increases in precipitation in the northern hemisphere's mid-latitudes, a region between 40 and 70 degrees north, he said. In contrast, the northern hemisphere's tropics and subtropics, a region spanning from the equator to 30 degrees latitude north, became drier.

Notwithstanding, torrential rains have hit the nation's capital creating floods in various parts that have caused great havoc to lives and property. The ravages and the ruins of the flood captured and highlighted on our television screens and newspaper pages are quite dreadful. According to BBC report, 23 people have died and several dozens were swept away on Sunday night by rapidly rising waters whiles others were stranded on the roofs of their houses. Transport links between the capital and other cities were disrupted. The 'Daily Graphic' on Tuesday 22nd June, 2010, put the death toll to 35. As we write this piece, there are scenes of collapsed buildings and fence walls, damaged roads, falling electricity and telephone poles with mangled wires, choked drains, gutters and ramshackle structures. It is deplorable and pathetic situation that is unbecoming of the status of Accra as the capital of Ghana.

The issue of floods has become annual ritual and it amazes me that the authorities wait till the worst happen before they start announcing their unexciting solutions. Are we reactive or proactive? Why must we wait for these things to happen before we find lasting solutions to them?

It is very disturbing to wake up every day after heavy a downpour to hear of loss of lives and property as a result of poor structuring of houses in the city. How many deaths do we expect to occur before our leaders take a critical look at the situation and come out with urgent solutions to mitigate this flooding problem? Are lives of innocent people precious to us? The inundated nature of Accra after down pour has always been disastrous and the nation spent millions of Ghana cedis to provide shelter, tents and relief items for the victims. Why can't our leaders re-structure the city to avoid the incident from occurring again?

You do not have to be a pilot to understand and appreciate the power of navigational system. These days most new cars are equipped with navigational system to save us time on a trip because it helps us to avoid needless delays by simply telling us where we are in relation to where we are going. This device is capable of showing the best route to take. so why are our leaders refusing to think out of the box.

As a nation, do we know where we're going? The apprehension is, how long does the nation have to wait to find solution to the flooding problems? How far can we see the problem? And whose job is it to protect us from this annual ritual of flooding and severe draught causing starvation like that which occurred in 1983? We are literally in crises situation as a result of persistent flooding; yet, we are busy building houses on water ways. So how far can we see the future? Who is responsible for the quality of our lives? To be able to see clearly, we have to think outside the box.

Climate change according to the United Nations Framework Convention on Climate Change (UNFCCC), is ''a change of climate which is attributed directly to human activity that alters the composition of the global atmosphere, and which in addition to natural climate variability, observed over a comparable time period". One of the effects of climate change is increased in precipitation which in most cases causes flooding.

On Friday, June 19, 2010, Accra, Kumasi, Takoradi and other towns were hit by a severe downpour of rain. In Accra, the rainfall which lasted for over four hours has been described by the Meteorological Services Department as one of the highest in the country's history. Accra has an average rainfall of 22.8mm for 15 "raining days." At Kaneshie, a commercial hot-spot in Ghana's capital, the activities of traders, commuters and pedestrians came to a halt. Drivers were compelled to pack their vehicles because they run the risk of falling into gullies and trenches that had developed. But this did not save them either, as many of such vehicles, except very heavy ones, were carried away by the running water.

Five days after this terrible incidence, the death toll was reported by the National Times newspaper as totaling 45. It is very sad to lose lives and properties through disasters like this. This year's flooding situation has not been the highest to hit only Ghana, but other countries alike all over the world have bitten part of this bitter cake.

In China, Nearly 3900 people have been killed or left missing in flood-related incidents. About 1750 people were also affected by devastating mudslides after long hours of rainfall on August 7 and 8, 2010.

According to Aljazeera, the recent flooding in Pakistan is the worst to hit the country in 80 years. The country's worst ever humanitarian disaster has ravaged an area roughly the size of England and affecting about 20 million people. "We had goats and buffalo and a wooden hut. We had grain to eat. The river ate everything, leaving the whole family hungry and empty-handed". This was narrated by a 50 year old victim. Should we sit down and watch without taking any cue from this? Certainly, no! We have to take lessons from the Twi adage which literally says that 'When you see a friend's beard burning you have to place water besides yours''. This therefore, brings into mind a very important question; what is our adaptation level so far as these natural disasters are concerned?

A water expert has attributed the decline of water bodies in Ghana to increasingly high temperatures in the country. The possible effect of world-wide climate change also had led to less rainfall. Dr Philip Gyau-Boakye, the CEO of a water agency that provides rural water supply in Ghana, Community Water and Sanitation Agency (CWSA), made the disclosure today Monday 6th (2010) as the world celebrates the International water Day. Dr Gyau-Boakye said rainfall patterns, for the past two decades in Ghana, had been changed into lower rainfall, which consequently dries up water bodies. This does not spare even rivers; he said environmental pollution also affected both the quantity and quality of water bodies. Dr Gyau-Boakye said stringent measures to protect water bodies from total extinction must be done. Some days back, geologists shocked the world with reports that the world's ten biggest rivers were at the risk of drying up.

Ghana is one of the poorest countries in the world whose economy is highly dependent on rain-fed agriculture. Climate variability is assumed to be the main cause for the frequently occurring drought in Ghana. Nowadays, famine and the name of Ghana are highly associated. This is because, for countries like Ghana, meteorological drought (deficiency of rain with respect to meteorological means) and agricultural drought (deficiency of rain with respect to crop water requirement) are immediate causes of famine.
Accordingly, the National Meteorological Services Agency of Ghana ((NMSAG), should build its capacity and tailor its services in the way that decision-makers get benefit from its services. In fact, the National Meteorological Services Agency of Ghana is actively participating in the National Early Warning System aimed at mitigating the effects of natural disasters, such as drought and flood. But the role of the Agency will be more beneficial if the information it provides is updated each time by results of various research activities.

As Ghana is located within the tropical region, it is influenced by weather systems of various scales, from mesoscale, such as thunderstorms, to large-scale ENSO related phenomena. The major rain-bearing system for the main rainy season (June to September) is the Inter Tropical Convergence Zone (ITCZ). On the other hand, the eastward moving mid-latitude troughs will facilitate the interaction between the mid-latitude cold air and the tropical warm air so that unstable conditions will be created for the moisture that comes into Ghana from the Atlantic ocean during the small rainy season (Feb. to May) (NMSA, 1996).

Most of the time, agricultural planning is difficult during the small rainy season due to the erratic nature of the rains. Moreover, in association with ENSO phenomena, a significant year-to-year variation in the performance of the rainy seasons has influenced the agricultural activities of the country. The forecasters of the NMSA of Ghana are aware of the problems associated with a reliance on forecasts using ENSO analogues. They have succeeded for the past several seasons during which such forecasts were issued. The government decision-makers are using their recommendations to alter agricultural practices on relatively short notices in order to maximize the value of the forecasted rains and minimize the impacts of forecasted droughts Nicholls & Katz (1991).

Recent advances in statistical methods have dramatically improved the range of techniques available for analysing data that are not from normal distribution. These new techniques, which are used in this study, parallel those used in the analysis of variance and regression for normally distributed data. This development is of considerable importance, since daily rainfalls are clearly not normally distributed (Stern *et al*., 1982).

Nowadays, rainfall pattern of Ghana has drastically changed leading to major disasters in the country. Such disasters include severe floods and draught, which affects most part of the country; especially the southern zones including Agbozome in Volta region and some part of Agona Swedru in central region which were affected by severe rains, which causes flood destroying lives and properties.

Furthermore, fitting and testing a wide range of models for daily rainfall data is easy due to the wide availability of computer packages associated with these new techniques, particularly, the Instat package developed by the Statistical Services Centre of the

University of Reading. Instat for windows (Version 1.3.1 test) is used for the most of the analysis done in this work.

Prediction of rainfall has remained an unsolved problem till this date. It has inevitable impact on crop production also influencing the socio-economic texture of the globe. In fact, it is one of the most important factors that govern the life of the earth. A mean temperature change of 10 near the earth surface leads to large change in rainfall. With the development of industrialization and the rapid growth of population, the management of water resources is becoming more important not only in Ghana but throughout the world.

The analysis of precipitation's behavior particularly in terms of amount of rainfall occurrence is beneficial for managing the consumption of water. Rain plays a major role in hydrology that finds its greatest applications in the design and operations of water resources, engineering works as well as agricultural systems, Srikanthan and McMahon (2005). Raiford et'al (2007) opines that; quantification of rainfall is generally done by using pluvial maps and Intensity-Duration-Frequency (IDF) curves.

Modeling rainfall data at useful time for different applications has been an important problem in hydrology for the last 30 years. A more recent interest in rainfall modeling is the perspective of using model parameters to characterize changes in the precipitation patterns because of the greenhouse effect and climate change. Available models have usually linked to the temporal and spatial scale required for the analysis.

In the spatial scale, the models at a single location and models that simultaneously represent rainfall at several locations (multisite models). There are considerable developments of models, which represent rainfall continuously in space. Cox and Isham (1994) presented an interesting classification of rainfall models in three types: empirical statistical models, dynamic models and intermediate stochastic models. The idea behind this classification is the amount of physical realism incorporated into the model structure. In the empirical case, there is no attempt to incorporate physical modeling of the atmosphere but to the empirical stochastic models to the available data. While the second type of models are pure physically based models, the third group is a combination of both method by which certain physical process of rainfall structure as for example, rain cells, rain bands and cell clusters, are described with a stochastic approach.

The probability estimation of rainfall states from available time series helps to obtain predictions for rainfall statistical parameters such as the averages, standard deviations and the first order autocorrelation coefficient. The transition probability estimations between the states of successive time instances are necessary for model construction.

Furthermore, theoretical Weibull, Gamma, Extreme Value Distribution functions are used most often in practice and for predicting the magnitude of rainfall. For accounting dependence in any time series, often a first order Markov Chain is used for modeling. For instance, large variety of weather events modeling and simulation were studied through Markov Chain Gringorten (1996). Markov Chain Racsko et al (1991) had achieved long time series of weather data generations also. For rainfall data, many authors have demonstrated that Markov Chain model is used to synthesize rainfall time series.

Gabriel and Neumann (1962) started the study on the sequence of daily rainfall occurrence. They found that the daily rainfall occurrence for the Tel Aviv data was successfully fitted with the first-order Markov chain model. Meanwhile, Kottegoda et al (2004) reported that the first order of the Markov chain model was found to fit the observed data in Italy successfully. The model is based on the assumption that there is a dependency of the daily rainfall occurrence to that of the previous day. Stern and Coe (1984) stated that the two most attractive features of the Markov chain models involved providing the ease in identifying the seasonality in daily rainfall occurrence and in most cases, the Markov chain of the first-order model can describe the daily rainfall occurrence; however, there are cases where this model failed to fit the observed data. As an alternative, the use of the Markov chain model of higher order often improved these inadequacies (Wilks, 1999; Hayhoe, 2000).

The main purpose of this research is to show the use of first order Markov Chain modeling for daily basis of rainfall measurements over south eastern coast of Ghana.

1.1 Problem Statement

The recent change in climate is disturbing and has led to disaster in various parts of the world including Ghana. Untimely rainfall has destroyed lives and property in some part of the country like Agona Swedru in the central region, Agboxome in the Volta region, in and around Accra and the Bui dam in the northern region. Farmers also suffer losses from unexpected rainfall patterns.

1.2 Objectives

The objectives of the study are as follows;

1. To model the rainfall pattern in south eastern coast of Ghana; using Markov chain analysis.
2. To investigate the position of rainy or dry seasons within the year in selected meteorological stations.

3. To make recommendations for agricultural and commercial applications in south eastern coast of Ghana.

1.3 Methodology

The analysis of daily rainfall data shows that Markov Chain approach provides one alternative to modeling future variation in rainfall. These variations may either be in the form too much water, which will lead to flooding or too little water, which will lead to draught. Markov modeling is one of the tools that can be utilized to assist planners in assessing the rainfall.

The daily rainfall data used in this study were obtained from Meteorology Department, Accra, and cover the period, 1980-2010 in respect of Keta, Akatsi, Akuse, Accra and Cape Coast stations spinning the south eastern coast of Ghana. Microsoft Excel and Matlab were used to analyses the data.

1.4 Justification

The information on weather's wet and dry behaviour has vital importance to all allied fields like insurance, agriculture, and industry etc. Once the rainfall process is adequately and appropriately modeled, the model can then be used in agricultural planning, may be able to aid in draught, soil erosion and flood predictions, impact of climate change studies, rainfall runoff modelling, crop growth studies and other important fields

1.5 Limitations

Due to time constraints, the researcher considered data from 1980 to 2010 within Southern zone of Ghana. This could have been done using long term data from 1930 till now to make the work more reliable. The source and the reliability of the data is a source of problem since most departments were not willing to give out data. Further, the work was an academic work and needs to be done within a stipulated time frame. The analysis could have been done using other models, but the researcher dwelt with only Markov chain which may not be accurate but the researcher considered the advantages of Markov chain over other methods.

1.6 Organisation of Thesis

The first chapter of this thesis talks about the introduction to the topic which is, the background discusses a few findings about recent climate changes, the importance and effect of rain on lives and damages cause to humanity. The chapter two discusses literature review (work done by other researchers on the same or similar field and the method applied). The third chapter deals with the mathematical model (methodology) applied by the researcher in dealing with the problem or the topic (it is the actual work done by the researcher). The forth chapter is titled Data Analysis and discussion; it explains the meaning of the result obtained from the work and its relevance and application. The final chapter which is the fifth is given the heading conclusion. It is the summary of the piece of work done by the researcher. It also gives recommendation to areas that can be researched in the near future by other researchers and some techniques that can help others to do good work in the same or similar area of research.

CHAPTER TWO

LITERATURE REVIEW

2.0 Introduction

Early development of weather generators was motivated by desire to generate synthetic weather sequences that would capture essential statistical features of observed weather data, and to capture the effects of year-to-year weather variability on crop response, using biological simulation models.

Several applications have emerged that involve generating sequences of synthetic daily data that represent time series of climatic variables aggregated to, e.g., a monthly time scale – a procedure referred to as ‘‘stochastic disaggregation’’ or ‘‘temporal downscaling’’ – for use with hydrological or biological simulation models. These include: (a) predicting crop production impacts of climate change scenarios; Mearns et al. (1996), Semenov and Barrow (1997), Mavromatis and Jones (1998), (b) predicting crop yields based on seasonal climate forecasts; Hansen and Indeje (2004), (c) analyzing crop yield variability using long-term monthly meteorological records where the original daily observations have been lost or are otherwise unavailable Boer et al. (2004), and (d) interpolating between stations, e.g., to create gridded daily meteorological time series data sets; Kittelet et’al. (2004).

The need to preserve key statistical properties of the historic daily time series justifies the use of a stochastic model in each of these applications. Crops respond not to climatic averages, but to the dynamic, nonlinear interactions between daily sequences of weather, soil water, and nutrient balance. The statistical properties of rainfall are particularly important because of its influence on processes, such as solute leaching, soil erosion and crop water stress response, which depend on soil water balance dynamics. Any biases in variability of daily weather can seriously distort crop model prediction.

Osborn and Hulme, (1997), study spatial averaging or interpolation of daily weather data among stations that tends to distort day-to-day variability, biasing simulated crop response.

Hansen and Jones (2000), also study a particular problem for predicting crop response to the soil water balance is the tendency for spatial averaging to increase the frequency of days with rain and reduce the mean intensity of rainfall events. This distortion can result in either under-prediction of crop yields due to increased evaporative loss from the soil surface, or over prediction due to reduced dry spell duration, deWit and van Keulen (1987), Carbone (1993), Mearns et al., (1996), Riha et al., (1996), were also of the same view.

Mearns et al. (1995), Mavromatis and Jones (1999), opines that the same challenge arises when using the output of physically based global circulation models (GCMs) to predict crop response to either climate change scenarios or predicted seasonal climate variations. Although GCMs operate on sub-daily time steps, the spatial averaging that occurs within grid cells distorts the variability of daily weather sequences, generally resulting in too many rainfall events, with too little rain per event, suggested in the views of Goddard et al. (2001). Therefore, their predictions are typically aggregated into monthly or seasonal (i.e., _3 months) anomalies.

Wilks (1992), Katz (1996), Mearns et al. (1997), suggested two general approaches that are used with stochastic weather generators to disaggregate monthly climatic means into daily realizations. The most common is to adjust the input parameters of the stochastic model to match target means or other statistics. Understanding the statistical properties of a stochastic weather generator allows one to manipulate its input parameters to reproduce a wide range of statistical properties of interest, such as means, variances, and the relative influence of the number of storms (i.e., frequency) and the type of storm (i.e., the intensity distribution) on total rainfall,. This approach has been applied to climate change impact studies and disaggregation of seasonal climate Forecasts.

Multivariate techniques have been underlined as suitable and powerful tool to find hydrologically homogeneous region or to classify meteorological data such as rainfall. Principle component analysis, factor analysis and different cluster techniques have been used to classify daily rainfall patterns and their relationship to atmospheric circulation.

Romero et al. (1999), classify rainfall into spatio-temporal pattern in Iran, Singh (1999), classify flood and drought years and Stahl and Demuth (1999), classify streamflow drought. They used cluster analysis for regionalization involves grouping of various observations and variables into clusters, so that each cluster is composed of observations or variables with similar characteristics such as geographical, physical, statistical or stochastic behavior.

Mosely (1981), used hierarchical cluster analysis on rivers in New Zealand and Tasker(1982), compared methods of defining homogeneous regions including cluster analysis with a complete linkage algorithm. Acerman (1985), and Acerman and Sinclair (1986), concluded that clustering has some intrinsic worth to explain the observed variation in data. Gottschalk (1985), applied cluster and principal component analysis to the territory of Sweden and found that cluster analysis is an appropriate method to use on a national scale with heterogeneous hydrological regimes.

Nathan and McMahon (1990), performed hierarchical cluster analysis for the prediction of low flow of rain characteristics in southeastern, Australia. They found that Ward method with a similarity measure based on the squared Euclidean distance is the best method for cluster analysis.

Cox and Isham (1994), presented an interesting classification of rainfall models in three types: empirical statistical models, dynamic models and intermediate stochastic models. The idea behind this classification is the amount of physical realism incorporated into the model structure. In the empirical case, there is no attempt to incorporate physical modeling of the atmosphere but to the empirical stochastic models to the available data. While the second type of models are pure physically based models, the third group is a combination of both method by which certain physical process of rainfall structure as for example, rain cells, rain bands and cell clusters, are described with a stochastic approach.

Andrade et al (1998), Miranda and Andrade (1999), and Miranda et al. (2004), used concepts of graph theory to analyze spatial patterns in time correlation function among rain events, using recorded data from a set of stations in Northeast Brazil. In previous contributions they investigated properties of rain events in this region with concepts of statistical scale invariance within the data, which can be expressed in terms of temporal and spatial Hurst exponents. The method they used herein is similar to that proposed for the analysis of brain activity signals by Eguiluz et al. (2005). Within this approach, non-local spatial dependence is estimated by evaluating the Pearson coefficient between time series of pairs of stations.

Le Cam (1961), in his fundamental work, propane models for spatial-temporal precipitation based on stochastic point processes, this approach developed rapidly in the 1980s through a series of papers by Waymire et al. (1984), such models are based on a hierarchical structure in which rainfall fields occur in a temporal Poisson process, rain bands (storms) occur within each field in a spatial Poisson process (the rate of which may reflect orography and seasonality), and rain cells occur in each storm, clustering in space and time. Typically the cells, storms and fields move: in the simplest models, all components have a common velocity. They assume stochastic stationarity in both time and space. Thus, in fitting the models, they treat each month separately, and use data for a relatively homogeneous 6 spatial region.

Rodriguez-Iturbe et al. (1987, 1988), generalise that the spatial-temporal models that they developed were spatial analogues of models that they used successfully to represent the temporal process of rainfall at a single rain gauge, investigation in Cox and Isham (1988). The multi-site models similarly generalise the models of Cox and Isham (1994). All of these models have the desirable feature that they preserve the structure of the single-site models in their marginal properties.

2.1 Markov Chain Model

Liu *et al.* (2009), said Markov chain has been widely applied in the disciplines of natural science, engineering, economics and management. This approach has also been widely used in drought forecasting, Lohani and Loganathan, (1997); Lohani *et al.* (1998).

Paulo and Pereira (2007) stated that the Markov chain modeling approach is useful in understanding the stochastic characteristics of droughts and rainfall through the analysis of probabilities for each severity class, times for reaching the nondrought class from any drought severity state, and residence times in each drought class. They found that the approach can be satisfactorily used as a predictive tool for forecasting transitions among drought severity classes up to 3 months ahead

Lohani and Loganathan (1997) and Lohani *et al.* (1998) developed an early warning system for drought management using the Markov chain, in two climatic areas of Virginia (U.S.A.). The same approach was also adopted for developing a meteorological drought/rainfall forecasting model by Liu *et al.* (2009) in Laohahe catchment in northern China. In their study, spatio-temporal distributions were analyzed and forecasted by Markov chain.

Steinemann (2003) adopted six classes of severity, from wet to dry conditions, similar to those in PDSI, and used the Markov chain to characterize probabilities for drought class and duration in a class. The results obtained were used to propose triggers for early-activating of the drought preparedness plans at the basin scale.

Liu *et al.*(2009) demonstrated two advantages of the Markov chain technique for forecasting drought and rainfall conditions. They were: (1) the predictive performance increased greatly as the severity of drought increased, and (2) the predictive performance was always satisfactory for drought state transitions, and the prediction performance was acceptable for the successive and smooth states.

2.2 Spatial-temporal Models

Northrop (1996), generalised this model in the case where cells are elliptical rather than circular (it is referred to as the elliptical cell Poisson process model (EPPM). EPPM is likely to be more realistic, especially in the cases where banding is apparent in the radar images. These cells are also identifiable by the elliptical contours of their spatial autocorrelation plots. This model requires two extra parameters, the eccentricity and orientation of the cells, which are both assumed to be common to all cells.

Northrop (1996), have investigated a modified version of EPPM model, the temporal clustering of cells is achieved using a Bartlett-Lewis structure as above. Additionally, spatial clustering is incorporated using a Neyman-Scott-type mechanism in which the displacements of the cell origins from the storm centre follow a bivariate distribution in space. A range of storm shapes (e.g. bands and large masses) can be produced by variation of the parameters of the spatial clustering distribution. An important modification to the model of Cox and Isham (1988) is to have the storm centre moving with the same velocity as the cells so that cells are born within the existing structure of the storm. Two spatial clustering distributions are considered:

1. A bivariate Gaussian (normal) distribution. They refer to the resulting model as the Gaussian displacements spatial-temporal model (GDSTM);

2. A uniform distribution over a random ellipse. This gives rise to the random ellipse spatial-temporal model (RESTM).

2.3 Multi-site Models

Kakou (1997), suggested that multi-site models are reasonably parsimonious in their parametrization, requiring a single extra parameter, the cell duration scalar, for each new site that is included in the study were considered. The cross-correlation function of the rainfall intensity at a pair of sites were derived and has the implied functional form of the probability of a cell hitting two sites. It turns out that, for individual storms, this probability decays approximately exponentially with inter-site distance for sites which are well-separated and which are not aligned along the direction of the storm's movement; for sites which are closer together, the dependence is no longer exponential.

2.4 Single-site Models

The models described in the preceding sections were generalizations of models that have been used successfully to model the temporal evolution of rainfall at a single site. A first step towards improving the performance of these models involves studying ways in which the single-site models can be improved.

Rodriguez-Iturbe et al. (1987), is of the view that one of the most obvious ways in which the basic single-site models can be extended is by allowing for different types of stormto occur so as to randomize the cell duration parameter between storms in this approach; storms have a common structure but occur at different timescales. The main advantage of such models, in practical terms, lies in their ability to reproduce well the observed probability of no rainfall at various levels of aggregation. They have investigated an alternative to the randomization of the cell duration parameter for single-site models, instead allowing for different types of storm using an inverse relationship between the duration of an event and its intensity (the motivation being that intense convective events tend to be shorter-lived than shallower stratiform systems.

Cowpertwait (1994), adopted an explicit functional form for the dependence between cell depth and cell duration, it is possible to overcome the problems of over-parameterisation typically associated with attempts to model different cell types explicitly, Their work is based on the Neyman-Scott and Bartlett-Lewispoint process models, Rodriguez-Iturbe et al. (1987), which are modified to allow raincells with stochastically dependent duration and intensity, Kakou (1997).

2.6 Spectral method

The method of moments suffers from a number of disadvantages. In particular, the choice of features to incorporate into the fitting procedure is subjective, and the parameter values obtained can be quite sensitive to the features used in the fitting | hence model comparison can be difficult.

Brillinger and Rosenblatt (1967), makes inefficient use of available data, as only a few summary statistics are used in the fitting. In an attempt to overcome some of these difficulties, a spectral method has been developed. This method uses the sample Fourier coefficients rather than the original data, and makes use of the fact that, for large samples, small collections of the Fourier coefficients have a joint distribution which is approximately multivariate normal, This enables them to write down approximate likelihood functions for the mode parameters in terms of small subsets of the sample Fourier coefficients.

McCullagh and Nelder (1989), combined all approximate likelihood functions, an objective function is defined which can be interpreted as a log quasi-likelihood, Chandler (1997). This then provides a basis for objective model comparison procedures using standard statistical techniques such as likelihood ratio tests, The method has been developed for use in fitting single-site and spatial-temporal models. The reliance on second-order properties is a potential disadvantage in distinguishing between models whose main difference is in their wet/dry interval properties. More details may be found in Chandler (1996b, 1997).

Chandler (1997), describe spectral method so far as been used extensively in the fitting of single-site models, and some preliminary work on the fitting of spatial-temporal models has also been done. The main area of interest has been in the area of model comparison, as it is here that the apparent objectivity of the method is particularly useful. In the single-site case, numerous different models have been fitted to data from the HYREX raingauge network. Rigorous procedures for model comparison, such as likelihood ratio tests, are available which allow for the different numbers of parameters in the models. We conclude that the clustering models to the data is better than that of the Poisson model; also that storms tend to be asymmetric with more intense activity towards the beginning of a storm than at the end.

2.7 Artificial Neural Network

French et al. (1992), used Neural networks to estimate accurate information on rainfall as essential for the planning and management of water resources. Nevertheless, rainfall is one of the most complex and difficult elements of the hydrology cycle to understand and to model due to the complexity of the atmospheric processes that generate rainfall and the tremendous range of variation over a wide range of scales both in space and time,

Gwangseob and Ana, (2001),described Neural networks as been an accurate rainfall forecasting tool which is one of the greatest challenges in operational hydrology, despite many advances in weather forecasting in recent decades, Neural networks have been widely applied to model many of nonlinear hydrologic processes such as rainfall-runoff, Hsu et al. (1950), Shamseldin (1997), stream flow, Zealand et al. (1999), Campolo and Soldati (1999), Abrahart and See, (2000), groundwater management, Rogers and Dowla, (1994), water quality simulation, Maier and Dandy (1996), Maier and Dandy (1999), and rainfall forecasting.

Luk et al. (2000), studied and indicated that ANN is a good approach and has a high potential to forecast rainfall. The ANN is capable to model without prescribing hydrological processes, catching the complex nonlinear relation of input and output, and solving without the use of differential equations sited in Hsu et al. (1995), French et al. (1992). In addition, ANN could learn and generalize from examples to produce a meaningful solution even when the input data contain errors or is incomplete.

Luk et al. (2000), an artificial neural network (ANN) which is a mathematical model used for data processing inspired by the bioelectrical networks in the brain comprised of neurons and synapses. In an ANN, simple processing elements referred to as neurons are used to create networks that are capable of learning to model complex systems. For an introduction to the structure and design of Artificial Neural Networks the reader is referred to Hagan *et al*. (1996).

Karunanithi *et al*., (1994) has done a number of studies into the application of ANN in the field of rainfall-runoff modeling and flood forecasting sited in the work carried out by Lorrai and Sechi, (1995); Campolo *et al*., (1999).

Hsu *et al*. (1995) compared ANN models with traditional black box models, concluding that an ANN model is capable of giving superior performance over a linear ARMAX (autoregressive moving average with exogenous inputs) time series approach, when observed time series of flow rate and rainfall are used as input.

Smith *et al*. (2004), has an alternative to the ANN, genetic programming (GP) strategy introduced, an ANN can be considered for use in forecasting the error between the outputs of a physical rainfall runoff model and the observed runoff rates. A feed forward neural network has been used for this purpose and was found to provide similar accuracy to GP. An advantage of GP is that it is easier to use than an ANN approach in that it uses a function in the forecasting stage rather than a complicated network of neurons.

Gwangseob and Ana, (2001), developed an Artificial Neural Networks (ANN), which perform nonlinear mapping between inputs and outputs, has lately provided alternative approaches to forecast rainfall. ANN were first developed in the 1940s (Mc Culloch and Pitts, 1943), and the development has experienced a renaissance with Hopfield's effort Hopfield, (1982) in 5 iterative auto-associable neural networks.

Abraham et al. (2001) used an artificial neural network with scaled conjugate gradient algorithm (ANN-SCGA) and evolving fuzzy neural network (EfuNN) for predicting the rainfall time series. In the study, monthly rainfall was used as input data for training model. The authors analyzed 87 years of rainfall data in Kerala, a state in the southern part of the

Indian Peninsula. The empirical results showed that neuro-fuzzy systems were efficient in terms of having better performance time and lower error rates compared to the pure neural network approach. In some cases, the deviation of the predicted rainfall from the actual rainfall was due to a delay in the actual commencement of monsoon, El-Ni ˜no Southern Oscillation (ENSO).

Manusthiparom et al. (2003), has another study of ANN that relates to El-Ni ˜no Southern Oscillation was done and the authors investigated the correlations between El Nin˜o Southern Oscillation indices, namely, Southern Oscillation Index (SOI), and sea surface temperature (SST), with monthly rainfall in Chiang Mai, Thailand, and found that the correlations were significant. For that reason, SOI, SST and historical rainfall were used as input data for standard back-propagation algorithm ANN to forecast rainfall one year ahead. The study suggested that it might be better to adopt various related climatic variables such as wind speed, cloudiness, surface temperature, and air pressure as the additional predictors.

Toth et al. (2000) compared short-time rainfall prediction models for real-time flood forecasting. Different structures of auto-regressive moving average (ARMA) models, artificial neural networks, and nearest-neighbors approaches were applied for forecasting storm rainfall occurring in the Sieve River basin, Italy, in the period 1992–1996 with lead times varying from 1 to 6 h. The ANN adaptive calibration application proved to be stable for lead times longer than 3 h, but inadequate for reproducing low rainfall.

Koizumi (1999), has another application which employed an ANN model using radar, satellite, and weather-station data together with numerical products generated by the Japan Meteorological Agency (JMA) Asian Spectral Model for 1-year training data. Koizumi found that the ANN skills were better than persistence forecast (after 3 h), the linear regression forecasts, and numerical model precipitation prediction. As the ANN used only 1 year data for training, the results were limited. The author believed that the performance of the neural network would be improved when more training data became available. It is still unclear to what extent each predictor contributed to the forecast and to what extent recent observations might improve the forecast.

Coulibaly (2000) stated that ninety percent of ANN models applied in the field of hydrology used the back propagation algorithm. This algorithm involves minimizing the global error by using the steepest descent or gradient approach. The network weights and biases are adjusted by moving a small step in the direction of the negative gradient of the error function during each iteration. The advantage of this algorithm lies in its simplicity.
In the study, ANN model was applied for each of 75 rain gauge stations in Bangkok, to forecast rainfall from 1 to 6 h ahead as forecast point.

2.8 Conceptual rainfall-runoff Models

Franchini and Galeati (1997), Conceptual rainfall-runoff models (CRRMs) have become a basic tool for flood forecasting and for catchment basin management. These models permit calculation of the runoff generated by precipitation events by simulating the physical process that affect the movement of water over and through the soil. The accuracy of these calculations depends both on the structure of the model and on how the relevant parameters are defined. CRRMs generally have a large number of parameters which, because of their conceptual nature, cannot be measured directly and are therefore estimated on the basis of a calibration process which involves adjusting their values so that the simulated discharges fit the corresponding observed discharges as closely as possible. Measurement of the deviation between the two series represents the objective function. Therefore, the purpose of the calibration is ultimately to find the values of the parameters of the CRRM which reduce this deviation to a minimum or, in other words, those values which minimize the objective function.

Wang (1991), and Franchini (1996), also suggest a two step procedure which associates the GA with local-search optimization techniques for a subsequent “fine-tuning” process. In water resource management the applications address the optimization of aquifer monitoring systems, Cieniawsky et al. (1995), Wagner (1995), and their utilization, Mc Kinney and Lin (1994), the containment and recovery of polluted aquifers.

Rogers and Dowla, (1994), promogated the management of reservoir systems sited in Ritzel et al. (1994) and Esat and Hall (1994). The problems are addressed in complex single and multiple objective contexts and produce results which appear very promising.

Whitley and Hanson (1989), also suggested combined ways with other Artificial Intelligence methods (Neural Networks) as in Rogers and Dowla (1994). However, in all these applications the term “GA” indicates an algorithm that can be formulated in very many ways, Davis (1991), Michalewics (1992). It is interesting therefore to judge how the different GA structures affect the ability to find the region encompassing the optimum solution in the specific field of CRRM calibration, while considering that another different algorithm will perform the subsequent “fine-tuning” process.

Hendrickson et al. (1988), analyze the characteristics of the sequential use of two algorithms, the first based on the Pattern Search (PS) method, Hooke and Jeeves (1961), which is a direct search method, and the second (fine-tuning) based on the Newton method. This sequence, is shown to be a fairy good tool, primarily for the PS characteristics that make it less susceptible to irregularity of the response surface, thus less easily trapped on local minima, and, therefore, more efficient in the early stage of optimization.

Ibbirt and Donnel (1971), said; Conceptual rainfall-runoff models usually consist of a number of parameters. Most of the parameters have to be calibrated by examining the estimated and the measured discharge series. The use of function optimization methods for calibrating rainfall-runoff models has been studied by Johnston and pilgrim (1976), Jupta and Sorooshian (1985), Hendrickson et.al. (1988). They found that the standard optimization methods can be easily fooled into declaring convergence far short of the true optima because of high dimensionality and irregularities contained in the objective function response such as multiple optima, unsmoothness, discontinuity, elongated ridges, flat plateaus and so on.

CHAPTER THREE

METHODOLOGY

3.0 Introduction

A **Markov chain**, named for **Andrey Markov,** is a mathematical system that undergoes transitions from one state to another (from a finite or countable number of possible states) in a chain like manner. It is a **random process** endowed with the **Markov property**: the next state depends only on the current state and not on the past. Markov chains have many applications as **statistical model** of real-world processes.

Formally, a Markov chain is a discrete (discrete-time) random process with the Markov property. Often, the term "Markov chain" is used to mean a Markov process which has a discrete (finite or countable) state-space. Usually a Markov chain would be defined for a discrete set of times (i.e. a discrete-time Markov chain) although some authors use the same terminology where "time" can take continuous values. Also see continuous-time Markov process. The use of the term in Markov chain Monte Carlo methodology covers cases where the process is in discrete-time (discrete algorithm steps) with a continuous state space. The following concentrates on the discrete-time discrete-state-space case.

A "discrete-time" random process means a system which is in a certain state at each "step", with the state changing randomly between steps. The steps are often thought of as time, but they can equally well refer to physical distance or any other discrete measurement; formally, the steps are just the integers or natural numbers, and the random process is a mapping of these to states. The Markov property states that the conditional probability distribution for the system at the next step (and in fact at all future steps) *given* its current state depends only on the current state of the system, and not additionally on the state of the system at previous steps.

Since the system changes randomly, it is generally impossible to predict the exact state of the system in the future. However, the statistical properties of the system's future can be predicted. In many applications it is these statistical properties that are important.

The changes of state of the system are called transitions, and the probabilities associated with various state-changes are called transition probabilities. The set of all states and transition probabilities completely characterizes a Markov chain. By convention, we assume all possible states and transitions have been included in the definition of the processes, so there is always a next-state and the process goes on forever.

A famous Markov chain is the so-called "drunkard's walk", a **random walk** on the number line where, at each step, the position may change by +1 or −1 with equal probability. From any position there are two possible transitions, to the next or previous integer. The transition probabilities depend only on the current position, not on the way the position was reached. For example, the transition probabilities from 5 to 4 and 5 to 6 are both 0.5, and all other transition probabilities from 5 are 0. These probabilities are independent of whether the system was previously in 4 or 6.

Another example is the dietary habits of a creature who eats only grapes, cheese or lettuce, and whose dietary habits conform to the following rules:

- It eats exactly once a day.
- If it ate cheese yesterday, it will not today.
- It will eat lettuce or grapes with equal probability.
- If it ate grapes yesterday, it will eat grapes today with probability 1/10, cheese with probability 4/10 and lettuce with probability 5/10.

- If it ate lettuce yesterday, it will not eat lettuce again today but will eat grapes with probability 4/10 or cheese with probability 6/10.

This creature's eating habits can be modeled with a Markov chain since its choice depends solely on what it ate yesterday, not what it ate two days ago or even farther in the past. One statistical property that could be calculated is the expected percentage, over a long period, of the days on which the creature will eat grapes.

A series of independent events—for example, a series of coin flips—does satisfy the formal definition of a Markov chain. However, the theory is usually applied only when the probability distribution of the next step depends non-trivially on the current state.

3.1 Formal definition

A Markov chain is a sequence of **random variables** X_1, X_2, X_3, ... with the Markov property, namely that, given the present state, the future and past states are independent. Formally,

$$\Pr(X_{n+1}=x|X_1=x_1,X_2=x_2,\ldots,X_n=x_n)=\Pr(X_{n+1}=x|X_n=x_n).$$

The possible values of X_i form a countable set S called the **state space** of the chain.

Markov chains are often described by a directed graph, where the edges are labeled by the probabilities of going from one state to the other states.

3.2 Variations

- Continuous-time Markov processes have a continuous index.
- **Time-homogeneous Markov chains** (or **stationary Markov chains**) are processes where

$$\Pr(X_{n+1}=x|X_n=y)=\Pr(X_n=x|X_{n-1}=y)$$

for all *n*. The probability of the transition is independent of *n*.

- A **Markov chain of order *m*** (or a Markov chain with memory *m*) where *m* is finite, is a process satisfying

$$\begin{aligned}&\Pr(X_n=x_n|X_{n-1}=x_{n-1},X_{n-2}=x_{n-2},\dots,X_1=x_1)\\=&\Pr(X_n=x_n|X_{n-1}=x_{n-1},X_{n-2}=x_{n-2},\dots,X_{n-m}=x_{n-m})\text{ for }n>m\end{aligned}$$

In other words, the future state depends on the past *m* states. It is possible to construct a chain (Y_n) from (X_n) which has the 'classical' Markov property as follows:

Let $Y_n = (X_n, X_{n-1}, \dots, X_{n-m+1})$, the ordered *m*-tuple of *X* values. Then Y_n is a Markov chain with state space S^m and has the classical Markov property.

- An additive Markov chain of order *m* where *m* is finite, is where

$$\Pr(X_n=x_n|X_{n-1}=x_{n-1},X_{n-2}=x_{n-2},\dots,X_1=x_1)=\sum_{r=1}^{m}f(x_n,x_{n-r},r)$$

for $n > m$.

Example 3.1

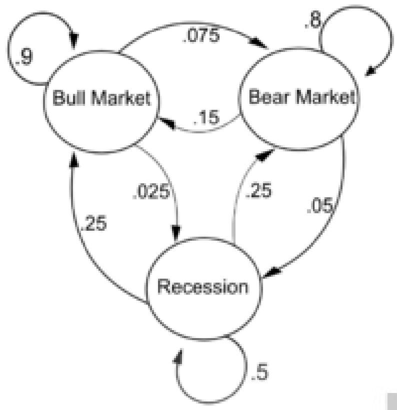

fig 3.1

A simple example is shown in the figure (3.1) above, using a directed graph to picture the state transitions. The states represent whether the economy is in a bull market, a bear market, or a recession, during a given week. According to the figure, a bull week is followed by another bull week 90% of the time, a bear market 7.5% of the time, and a recession the other 2.5%. From this figure it is possible to calculate, for example, the long-term fraction of time during which the economy is in a recession, or on average how long it will take to go from a recession to a bull market.

A thorough development and many examples can be found in the on-line monograph Meyn & Tweedie (2005). The appendix of Meyn (2007), also available on-line, contains an abridged Meyn & Tweedie.

A finite state machine can be used as a representation of a Markov chain. Assuming a sequence of independent and identically distributed input signals (for example, symbols from a binary alphabet chosen by coin tosses), if the machine is in state y at time n, then the probability that it moves to state x at time $n + 1$ depends only on the current state.

3.3 Markov chains

The probability of going from state i to state j in n time steps is

$$p_{ij}^{(n)} = \Pr(X_n = j \mid X_0 = i)$$

and the single-step transition is

$$p_{ij} = \Pr(X_1 = j \mid X_0 = i).$$

For a time-homogeneous Markov chain:

$$p_{ij}^{(n)} = \Pr(X_{k+n} = j \mid X_k = i)$$

and

$$p_{ij} = \Pr(X_{k+1} = j \mid X_k = i).$$

The n-step transition probabilities satisfy the Chapman-Kolmogorov equation, that for any k such that $0 < k < n$,

$$p_{ij}^{(n)} = \sum_{r\in S} p_{ir}^{(k)} p_{rj}^{(n-k)}$$

where S is the state space of the Markov chain.

The marginal distribution $\Pr(X_n = x)$ is the distribution over states at time n. The initial distribution is $\Pr(X_0 = x)$. The evolution of the process through one time step is described by

$$\Pr(X_n = j) = \sum_{r\in S} p_{rj} \Pr(X_{n-1} = r) = \sum_{r\in S} p_{rj}^{(n)} \Pr(X_0 = r).$$

Proof: from the chapman – kolmogorov equation, we have

$$p_{ij}^{(r+s)} = \sum_{k\in S} p_{ik}^{(r)} p_{kj}^{(s)} \quad \textit{for given } r \textit{ and } s$$

Set r =1, s =1, so that

$$p_{ij}^{(2)} = \sum_{k \in S} P_{ik} p_{kj}$$

Clearly $p_{ij}^{(2)}$ is the $(i, j)th$ element of the Matrix product $p.p = p^2$. Base on this result, assume that

$$p^{(r)} = p^r \quad r = 1,2,\ldots,n-1$$

Setting $r = n-1, s = 1$, we get

$$p_{ij}^{(n)} = \sum_{j \in S} P_{ik}^{(n-k)} p_{kj}$$

Which again can be seen as the $(i, j)th$ element of the matrix product $p^{n-1}.p = p^n$, which proves $p^{(n)} = p^n$. The result $p^{(n)} = p^{(0)} p^n$ is obtained by noting that

$$p(X_n = j) = \sum_{i \in S} p(X_n = j | X_0 = i) p(X_0 = i)$$

From this theory, the n-step transitions probabilities can be easily obtained by Simple matrix multiplication, for larger state space efficient of p^n are needed.

3.4 Reducibility

A state j is said to be **accessible** from a state i (written $i \to j$) if a system started in state i has a non-zero probability of transitioning into state j at some point. Formally, state j is accessible from state i if there exists an integer $n \geq 0$ such that

$$\Pr(X_n = j | X_0 = i) = p_{ij}^{(n)} > 0.$$

Allowing n to be zero means that every state is defined to be accessible from itself.

A state i is said to **communicate** with state j (written $i \leftrightarrow j$) if both $i \rightarrow j$ and $j \rightarrow i$. A set of states C is a **communicating class** if every pair of states in C communicates with each other, and no state in C communicates with any state not in C. It can be shown that communication in this sense is an **equivalence relation** and thus that communicating classes are the **equivalence classes** of this relation. A communicating class is **closed** if the probability of leaving the class is zero, namely that if i is in C but j is not, then j is not accessible from i.

That said, communicating classes need not be commutative, in that classes achieving greater periodic frequencies that encompass 100% of the phases of smaller periodic frequencies, may still be communicating classes provided a form of diminished, downgraded, or multiplexed cooperation exists within the higher frequency class.

Finally, a Markov chain is said to be **irreducible** if its state space is a single communicating class; in other words, if it is possible to get to any state from any state.

3.5 Periodicity

A state i has **period** k if any return to state i must occur in multiples of k time steps. Formally, the period of a state is defined as

$$k = \gcd\{n : \Pr(X_n = i | X_0 = i) > 0\}$$

(where "gcd" is the greatest common divisor). Note that even though a state has period k, it may not be possible to reach the state in k steps. For example, suppose it is possible to return to the state in {6, 8, 10, 12, ...} time steps; k would be 2, even though 2 does not appear in this list.

If $k = 1$, then the state is said to be **aperiodic**: returns to state i can occur at irregular times. Otherwise ($k > 1$), the state is said to be **periodic with period *k***.

It can be shown that every state in a communicating class must have overlapping periods with all equivalent-or-larger occurring sample(s).

It can be also shown that every state of a bipartite graph has an even **period**.

Recurrence

A state i is said to be **transient** if, given that we start in state i, there is a non-zero probability that we will never return to i. Formally, let the random variable T_i be the first return time to state i (the "hitting time"):

$$T_i = \inf\{n \geq 1 : X_n = i | X_0 = i\}.$$

Then, state i is transient if and only if:

$$\Pr(T_i = \infty) > 0.$$

If a state i is not transient (it has finite hitting time with probability 1), then it is said to be **recurrent** or **persistent**. Although the hitting time is finite, it need not have a finite expectation. Let M_i be the expected return time,

$$M_i = E[T_i].$$

Then, state i is **positive recurrent** if M_i is finite; otherwise, state i is **null recurrent** (the terms **non-null persistent** and **null persistent** are also used, respectively).

It can be shown that a state is recurrent if and only if

$$\sum_{n=0}^{\infty} p_{ii}^{(n)} = \infty.$$

A state i is called **absorbing** if it is impossible to leave this state. Therefore, the state i is absorbing if and only if

$$p_{ii} = 1 \text{ and } p_{ij} = 0 \text{ for } i \neq j.$$

3.6 Ergodicity

A state i is said to be ergodic if it is aperiodic and positive recurrent. If all states in an irreducible Markov chain are ergodic, then the chain is said to be ergodic.

It can be shown that a finite state irreducible Markov chain is ergodic if it has an aperiodic state. A model has the ergodic property if there's a finite number N such that any state can be reached from any other state in exactly N steps. In case of a fully-connected transition matrix where all transitions have a non-zero probability, this condition is fulfilled with N=1. A model with just one out-going transition per state cannot be ergodic.

3.7 Steady-state analysis and limiting distributions

If the Markov chain is a time-homogeneous Markov chain, so that the process is described by a single, time-independent matrix p_{ij}, then the vector $\boldsymbol{\pi}$ is called a **stationary distribution** (or invariant measure) if its entries π_j are non-negative and sum to 1 and if it satisfies

$$\pi_j = \sum_{i \in S} \pi_i p_{ij}.$$

An irreducible chain has a stationary distribution if and only if all of its states are positive recurrent. In that case, π is unique and is related to the expected return time:

$$\pi_j = \frac{1}{M_j}.$$

Further, if the chain is both irreducible and aperiodic, then for any *i* and *j*,

$$\lim_{n\to\infty} p_{ij}^{(n)} = \frac{1}{M_j}.$$

Note that there is no assumption on the starting distribution; the chain converges to the stationary distribution regardless of where it begins. Such π is called the **equilibrium distribution** of the chain.

If a chain has more than one closed communicating class, its stationary distributions will not be unique (consider any closed communicating class in the chain; each one will have its own unique stationary distribution. Any of these will extend to a stationary distribution for the overall chain, where the probability outside the class is set to zero). However, if a state *j* is aperiodic, then

$$\lim_{n\to\infty} p_{jj}^{(n)} = \frac{1}{M_j}$$

and for any other state *i*, let f_{ij} be the probability that the chain ever visits state *j* if it starts at *i*,

$$\lim_{n\to\infty} p_{ij}^{(n)} = \frac{f_{ij}}{M_j}.$$

If a state *i* is periodic with period $k > 1$ then the limit

$$\lim_{n\to\infty} p_{ii}^{(n)}$$

does not exist, although the limit

$$\lim_{n\to\infty} p_{ii}^{(kn+r)}$$

does exist for every integer *r*.

3.8 Steady-state analysis and the time-inhomogeneous Markov chain

A Markov chain need not necessarily be time-homogeneous to have an equilibrium distribution. If there is a probability distribution over states $\boldsymbol{\pi}$ such that

$$\pi_j = \sum_{i\in S} \pi_i \, \Pr(X_{n+1} = j \mid X_n = i)$$

for every state *j* and every time *n* then $\boldsymbol{\pi}$ is an equilibrium distribution of the Markov chain. Such can occur in Markov chain Monte Carlo(MCMC) methods in situations where a number of different transition matrices are used, because each is efficient for a particular kind of mixing, but each matrix respects a shared equilibrium distribution.

3.8 Finite state space

If the state space is finite, the transition probability distribution can be represented by a matrix, called the **transition matrix**, with the (*i*, *j*)th element of **P** equal to

$$p_{ij} = \Pr(X_{n+1} = j \mid X_n = i).$$

Since each row of **P** sums to one and all elements are non-negative, **P** is a right stochastic matrix.

3.9 Time-homogeneous Markov chain with a finite state space

If the Markov chain is time-homogeneous, then the transition matrix **P** is the same after each step, so the k-step transition probability can be computed as the k-th power of the transition matrix, $\mathbf{P}^k$.

The stationary distribution $\boldsymbol{\pi}$ is a (row) vector, whose entries are non-negative and sum to 1, that satisfies the equation

$$\pi = \pi \mathbf{P}.$$

In other words, the stationary distribution $\boldsymbol{\pi}$ is a normalized (meaning that the sum of its entries is 1) left eigenvector of the transition matrix associated with the eigenvalue 1.

Alternatively, $\boldsymbol{\pi}$ can be viewed as a fixed point of the linear (hence continuous) transformation on the unit **simplex** associated to the matrix **P**. As any continuous transformation in the unit simplex has a fixed point, a stationary distribution always exists, but is not guaranteed to be unique, in general. However, if the Markov chain is irreducible and aperiodic, then there is a unique stationary distribution $\boldsymbol{\pi}$. Additionally, in this case $\mathbf{P}^k$ converges to a rank-one matrix in which each row is the stationary distribution $\boldsymbol{\pi}$, that is,

$$\lim_{k\to\infty} \mathbf{P}^k = \mathbf{1}\pi$$

where **1** is the column vector with all entries equal to 1. This is stated by the Perron-Frobenius theorem. If, by whatever means, $\lim_{k\to\infty} \mathbf{P}^k$ is found, then the stationary distribution of the Markov chain in question can be easily determined for any starting distribution, as will be explained below.

For some stochastic matrices **P**, the limit $\lim_{k\to\infty} \mathbf{P}^k$ does not exist, as shown by this example:

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Because there are a number of different special cases to consider, the process of finding this limit if it exists can be a lengthy task. However, there are many techniques that can assist in finding this limit. Let **P** be an $n \times n$ matrix, and define $\mathbf{Q} = \lim_{k\to\infty} \mathbf{P}^k$.

It is always true that

$$\mathbf{QP} = \mathbf{Q}.$$

Subtracting **Q** from both sides and factoring then yields

$$\mathbf{Q}(\mathbf{P} - \mathbf{I}_n) = \mathbf{0}_{n,n}$$

where $\mathbf{I}_n$ is the identity matrix of size n, and $\mathbf{0}_{n,n}$ is the zero matrix of size $n \times n$. Multiplying together stochastic matrices always yields another stochastic matrix, so **Q** must be a stochastic matrix. It is sometimes sufficient to use the matrix equation above and the fact that **Q** is a stochastic matrix to solve for **Q**.

Here is one method for doing so: first, define the function $f(\mathbf{A})$ to return the matrix **A** with its right-most column replaced with all 1's. If $[f(\mathbf{P} - \mathbf{I}_n)]^{-1}$ exists then

$$\mathbf{Q} = f(\mathbf{0}_{n,n})[f(\mathbf{P} - \mathbf{I}_n)]^{-1}.$$

One thing to notice is that if **P** has an element $\mathbf{P}_{i,i}$ on its main diagonal that is equal to 1 and the *i*th row or column is otherwise filled with 0's, then that row or column will remain unchanged in all of the subsequent powers $\mathbf{P}^k$. Hence, the *i*th row or column of **Q** will have the 1 and the 0's in the same positions as in **P**.

3.10 Reversible Markov chain

A Markov chain is said to be **reversible** if there is a probability distribution over states, $\boldsymbol{\pi}$, such that

$$\pi_i \Pr(X_{n+1} = j \mid X_n = i) = \pi_j \Pr(X_{n+1} = i \mid X_n = j)$$

for all times *n* and all states *i* and *j*. This condition is also known as the detailed balance condition (some books refer the local balance equation). With a time-homogeneous Markov chain, $\Pr(X_{n+1} = j \mid X_n = i)$ does not change with time *n* and it can be written more simply as p_{ij}. In this case, the detailed balance equation can be written more compactly as

$$\pi_i p_{ij} = \pi_j p_{ji} \ .$$

Summing the original equation over *i* gives

$$\sum_i \pi_i \Pr(X_{n+1} = j \mid X_n = i) = \sum_i \pi_j \Pr(X_{n+1} = i \mid X_n = j)$$
$$= \pi_j \sum_i \Pr(X_{n+1} = i \mid X_n = j) = \pi_j \ ,$$

so, for reversible Markov chains, $\boldsymbol{\pi}$ is always a steady-state distribution of $\Pr(X_{n+1} = j \mid X_n = i)$ for every *n*.

If the Markov chain begins in the steady-state distribution, *i.e.*, if $\Pr(X_0 = i) = \pi_i$, then $\Pr(X_n = i) = \pi_i$ for all *n* and the detailed balance equation can be written as

$$\Pr(X_n = i, X_{n+1} = j) = \Pr(X_{n+1} = i, X_n = j)\,.$$

The left- and right-hand sides of this last equation are identical except for a reversing of the time indices *n* and *n* + 1.

Reversible Markov chains are common in Markov chain Monte Carlo (MCMC) approaches because the detailed balance equation for a desired distribution $\boldsymbol{\pi}$ necessarily implies that the Markov chain has been constructed so that $\boldsymbol{\pi}$ is a steady-state distribution. Even with time-inhomogeneous Markov chains, where multiple transitions matrices are used, if each such transition matrix exhibits detailed balance with the desired $\boldsymbol{\pi}$ distribution, this necessarily implies that $\boldsymbol{\pi}$ is a steady-state distribution of the Markov chain.

EXAMPLE 3.2;

A markov chain is a chain of events for which the probabilities of outcomes or states depend on what has happened previously e.g the probabilities of it being rainy or dry on a particular day depend on whether it was rainy or dry on the previous day.

The researcher knowing very well that the states “rainy” and “dry” do no overlap and cover all probabilities.

A state matrix or state vector is a row matrix which shows the probability of each state e.g take X= (0.15 0.85) to mean that the probability of it being rainy on a certain day is 0.15 and probability of it being dry on that day is 0.85. The sum of the elements of a state matrix is 1.

A transition probability matrix or transition matrix is a square matrix that shows the probabilities of moving from each state to every other state. The sum of the elements in each row is 1. An example is shown below with values that were used to find the conditional probability;

- Given that it is rainy, let the probability that it will be rainy a day later be

  $$\frac{5}{31} = 0.16$$

- Given that it is dry, let the probability that it would be rainy a day later be

  $$\frac{4}{31} = 0.13$$

This yields the table (transition matrix) below;

To

From

	Rainy	Dry
rainy	0.16	0.84
dry	0.13	0.87

or $P = \begin{bmatrix} 0.16 & 0.84 \\ 0.13 & 0.87 \end{bmatrix}$

The study state value of the power of the transition matrix

The study state transition matrix, the value of p^n approaches a fixed square matrix as n increases. This is the case in the table below;

p	p^2
$P = \begin{bmatrix} 0.16 & 0.84 \\ 0.13 & 0.87 \end{bmatrix}$	$= \begin{pmatrix} 0.1348 & 0.8652 \\ 0.1339 & 0.8661 \end{pmatrix}$
p^3	p^4
0.134044 0.865956 0.134017 0.865983	0.134044 0.865979 0.134044 0.865979

Each row (0.134044 0.865979) represents the long term probabilities of each state. Whatever the weather is today, in the long term the probability that it will be rainy on any day is 0.13 and the probability that it will be dry is 0.87.

The state matrix approaches a fixed matrix (0.134 0.866) which is called the steady state or stable matrix. The steady state matrix represents the long-term probabilities of each state. Whatever the weather is today, in the long term the probability that it will be rainy on a day is 0.134 and the probability that it will be dry on a day is 0.866. The state matrix approaches the steady state matrix (0.134 0.866) regardless of the value of the initial state matrix. The steady state matrix (0.134 0.866) shows that in the future, the rainfall pattern will follow the pattern of the given probability. Thus, in future, the probability that it will rain on a particular day is 0.134 and the probability that it will be dry on a particular is 0.866.

CHAPTER FOUR

ANALYSIS OF DATA

4.0 Introduction

Rainfall data was gathered to cover 5 towns in the south eastern coast of Ghana, precisely Keta, Akatsi, Akuse, Accra and Cape Coast (refer to map at appendix A(i)). According to the data gathered from 1980 to 2010 for the five towns, rainfall distribution is not evenly distributed. Looking at the data, the rain pattern is centralized from May to September in all the five towns without any certainty. This brings to the fore the doubt in mines of peoples whether to invest at a particular time. This research work is to create awareness in people to be able to know the certainty level of rainfall along the south eastern coast of Ghana.

4.1 Data Source

The time sequenced rainfall data used in this study were obtained from Department of Agricultural and Meteorology department of Ghana, for the period 1980-2010. From each year the maximum amount of rainfall were derived on daily basis and in this way, the length of time series will be at least 60 of extreme maximum rainfall amount for the five towns (twelve month), which is to be analyzed in the present study. Refer to all data at Appendix B and C.

4.2 Study Area

The distribution will cover only the south eastern coast of Ghana constituting the areas specifically Keta, Akatsi, Akuse, Accra and Cape Coast (refer fig. 4.1)

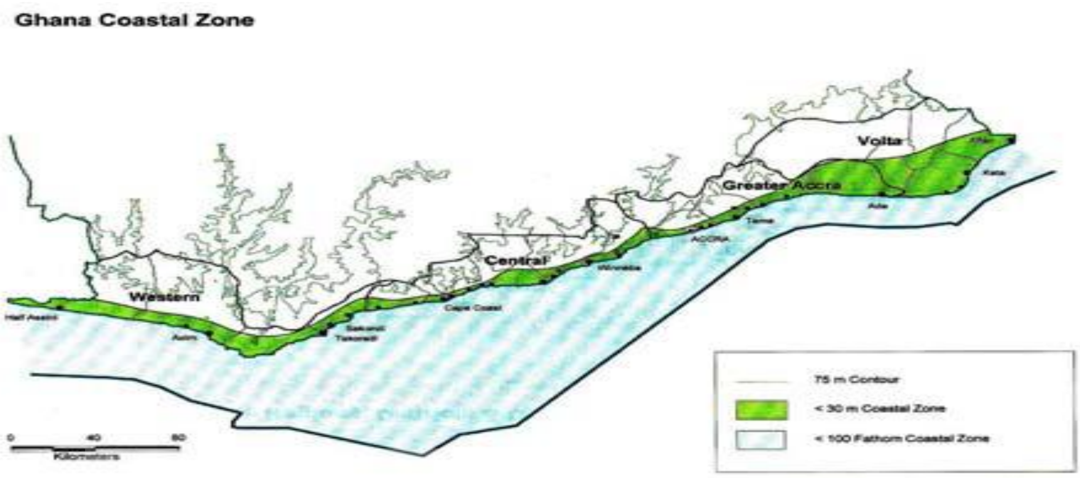

Fig. 4.1: Map of Study Area

4.3 Instrument

The data was analysed with Microsoft excel and Matlab. This was programmed with rainfall data of south eastern coast of Ghana from 1980 to 2010.

4.4 Preliminary Discussions on each town

Following the trend and pattern of rainfall in each town, the observations below were made on each town.

4.4.1 Preliminary discussion on Accra

Accra is the capital city of Ghana situated on the central coast part of Ghana. The time series graph at fig 4.2 represent the daily rainfall pattern of Accra from January day one to the last day of December. The graph shows daily pattern of rain in Accra over period of 30 years duration. The average values was used to draw the graph (refer data at appendix B(i))

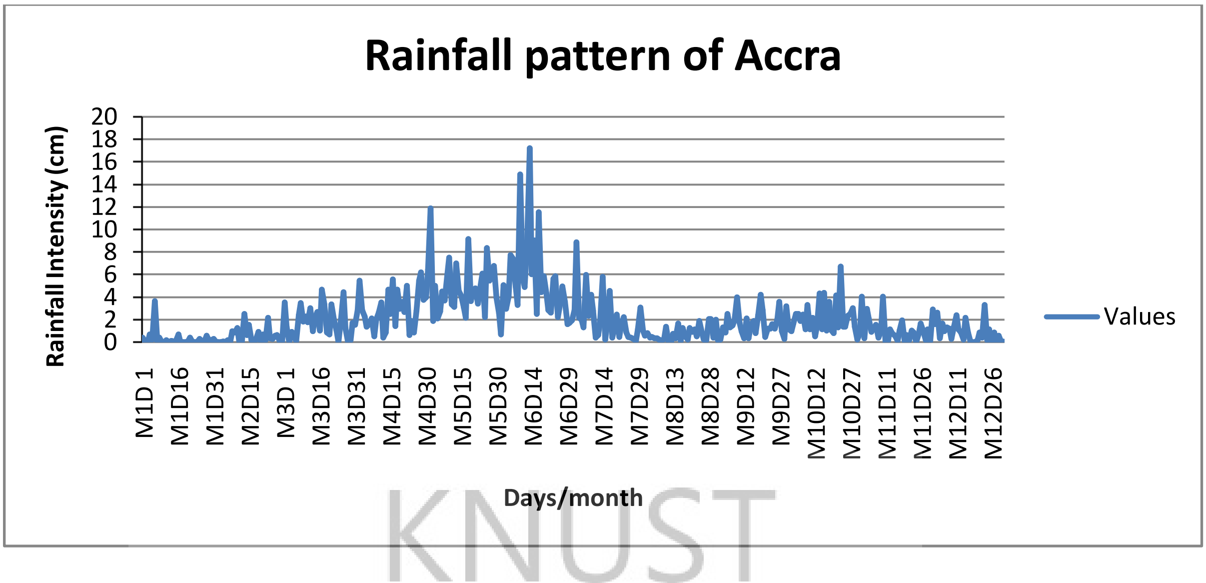

Fig. 4.2: Rainfall pattern of Accra

Looking at the rainfall data gathered from 1980 to 2010, January to April seems dry with a little heavy rain during specific days in January and March. The rains increase as the year gets into the middle part of the year (May to September) recorded a normal rainfall whilst June 14th to 18th recorded the highest rainfall. Towards the end of the year, the rainfall reduces from October to December.

4.4.2 Preliminary discussion on Akuse

Akuse is located a little away from the immediate coast. The daily rainfall pattern of Akuse is shown in fig. 4.3. The data used was an average data for a 30 year period, (refer to appendix B(ii))

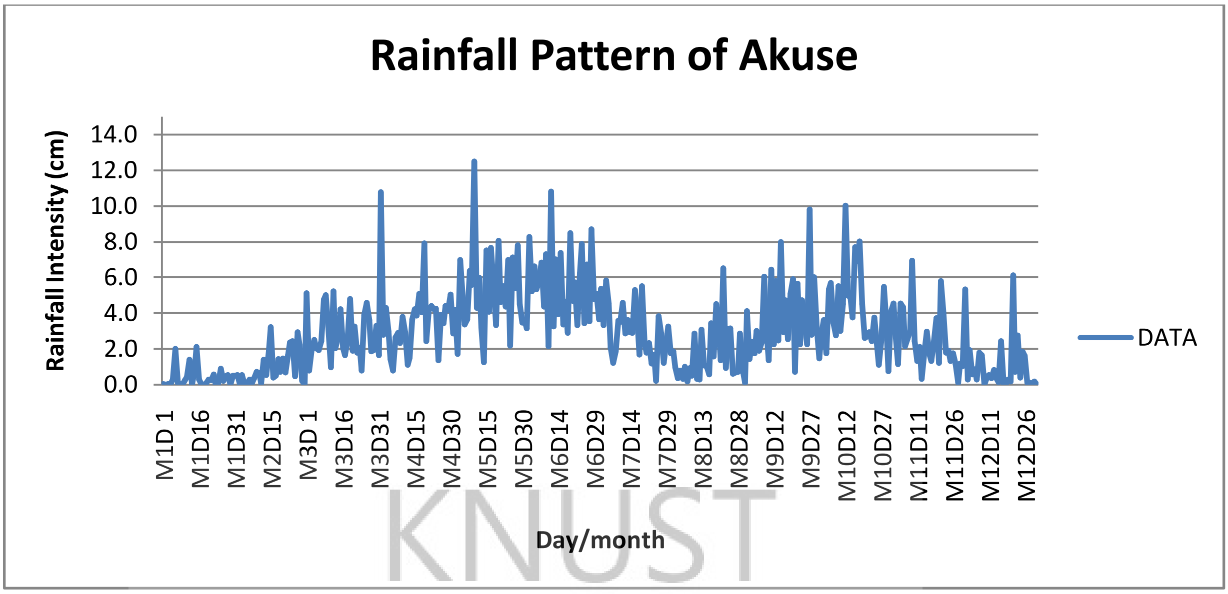

Fig 4.3: Rainfall Pattern of Akuse

In view of the time series graph at fig 4.3, Akuse experiences the driest moment in the month of January to February. The rains set in from March to September with the highest rainfall occurring in May 16th and average rains in the month of October. November and December recorded a little rain than January and February. The highest rainfall occurs in May to August.

4.4.3 Preliminary discussion on Cape Coast

Cape-Coast lies in the central part of the coastal region of Ghana. It consists of rainforest as well as swamp region covering a little part of the eastern coast of Ghana. The rainfall pattern of Cape Coast over the years has changed. Looking at the time series graph at fig 4.4; the rainfall pattern seems to be undulating and not stable. The data used was an average data for a 30 year period, (refer to appendix B(iii))

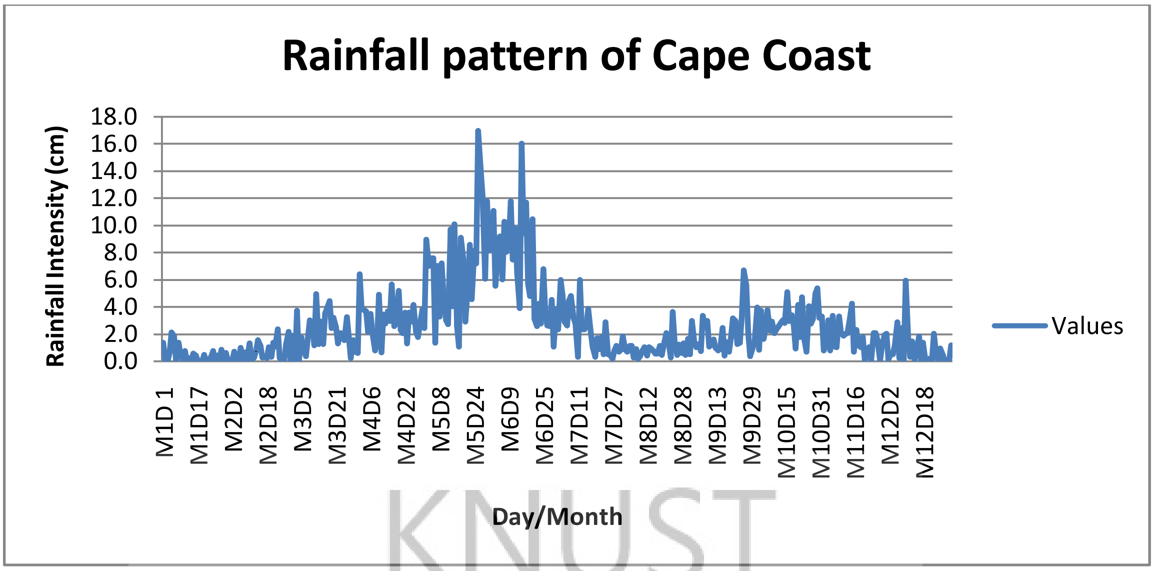

Fig. 4.4: Rainfall Pattern of Cape Coast

The graph reviewed that, January to March each year, the region experiences milled drought. During April to September and close to November, the rainfall pattern is averagely good. The highest rainfall occurs in May with peak on 26th May. The graph is a normal graph.

4.4.4: Preliminary discussion on Akatsi

Akatsi is the commercial hub along the Volta coast towards Aflao. The main occupation of the people are fishing and farming. The reliance of people of Akatsi on rains to cultivate most of their crops has over the years eluded the people. The graph below shows the average rainfall pattern of Akatsi over the period of 30 years; (refer to appendix B(iv) for data).

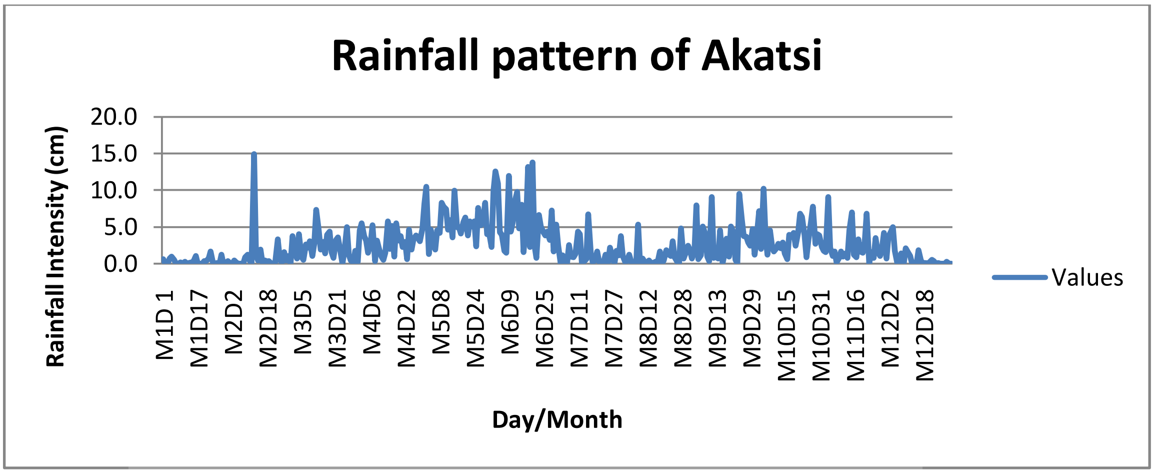

Fig 4.5: Rainfall Pattern of Akatsi

The observed rainfall graph at fig 4.5 for Akatsi from meteorological department of Ghana have revealed that over the years, the rains set in during the early part of February until August. The later part of August experiences a little drought as observed in the graph at fig 4.5. Critically observed, the formation of rainfall around Akatsi occurs in April to June with normal distribution of rains in October to December. The highest rainfall occurs around 15th February and between May and June.

4.4.5: Preliminary discussion on Keta

Keta lies along the coastal belt of Ghana precisely south-eastern coast of Ghana. The geographical location of Keta is south-eastern of Accra towards Aflao. The rainfall pattern of Keta is very unstable. The average rainfall data was used to graph the pattern of rainfall, (refer to appendix B(v) for data)

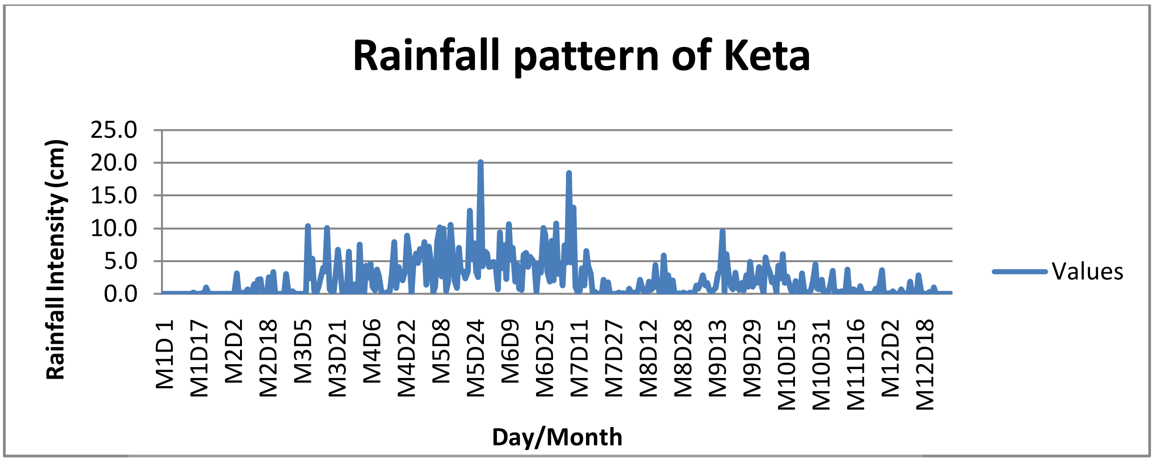

Fig. 4.6: Rainfall Pattern of Keta

The time series graph at fig. 4.6 shows that the rainfall in Keta starts in March to July. January to middle part of May has experienced little rain and some cases no rain occurring form 1st January to 3rd February. From July 14th to December experience low rainfall looking at the spread of rainfall in the area throughout the year. The highest rainfall occurs in May 27th to 14th July.

4.5 Markov chain Modeling

A markov chain is a chain of events for which the probabilities of outcomes or states depend on what has happened previously e.g. the probabilities of it being rainy or dry on a particular day depend on whether it was rainy or dry on the previous day.

The researcher knowing very well that the states “rainy” and “dry” do no overlap and cover all probabilities.

A state matrix or state vector is a row matrix which shows the probability of each state e.g take X= (0.15 0.85) to mean that the probability of it being rainy on a certain day is 0.15 and probability of it being dry on that dry is 0.85. The sum of the elements of a state matrix is 1.

4.5.1 Markov Chain analyses for rainfall pattern in Akuse

The average daily rainfall pattern was calculated for each day in each month. For each day a rainy day was picked to determine whether the next day is dry or rainy. The sum of the number of such days (rain or dry) was calculated and divided by the total number of days in the month. Similar computations were done for dry days. An example was taken from the daily rainfall data of Akuse (refer data at appendix c (i)) for the month January of 31 values which was used to find the conditional probability as shown below;

- Given that it is rainy, the probability that it would be rainy a day later is given by

$$\frac{4}{31} = 0.13$$

- Given that it is dry, the probability that it would be rainy a day later is given by

$$\frac{2}{31} = 0.06$$

This yields the table (transition matrix) below;

To

From

	Rainy	Dry
Rainy	0.13	0.87
Dry	0.06	0.94

or $P = \begin{bmatrix} 0.13 & 0.87 \\ 0.06 & 0.94 \end{bmatrix}$

The study state value of the power of the transition matrix

The study state transition matrix, the value of p^n approaches a fixed square matrix as n increases. This is the case in the table below;

p	p^2	p^3
0.13 0.87 0.06 0.94	0.691 0.9309 0.064 0.9358	0.064837 0.935 0.064494 0.935506
p^4	p^5	p^6
0.064539 0.935461 0.064515 0.935485	0.064518 0.935482 0.064516 0.935484	0.064516 0.935484 0.064516 0.935484

Each row (0.064516 0.935484) represents the long term probabilities of each state. Whatever the weather is today, in the long term the probability that it will be rainy on any day is 0.06 and the probability that it will be dry is 0.94.

The state matrix approaches a fixed matrix (0.064506 0.935333) which is called the steady state or stable matrix. The steady state matrix represents the long term probabilities of each state.

Whatever the weather is today, in the long term the probability that it will be rainy on a day is 0.045 and the probability that it will be dry on a day is 0.935. The state matrix approaches the steady state matrix (0.045 0.935) regardless of the value of the initial state matrix.

The steady state matrix (0.045 0.935) shows that in the future, the rainfall pattern of Akuse will follow the pattern of the given probability. Thus in future, the probability that it will rain on a particular day in Akuse is 0.045 and the probability that it will be dry on a particular in Akuse is 0.935.

The table below shows the conditional probability of either rainy or drought in a particular day for the various month in Akuse.

Table 4.1: Rainfall pattern of Akuse

Month	Initial matrix (p)	Long term prob.	Prob. for either rain/dry on a day	
	P	P^n	P(rain)	P(dry)
January	$\begin{pmatrix}0.13 & 0.87\\0.06 & 0.94\end{pmatrix}$	$p^6 = \begin{pmatrix}0.045 & 0.935\\0.045 & 0.935\end{pmatrix}$	0.045	0.935
February	$\begin{pmatrix}0.25 & 0.75\\0.07 & 0.93\end{pmatrix}$	$p^9 = \begin{pmatrix}0.085 & 0.915\\0.085 & 0.915\end{pmatrix}$	0.085	0.915
March	$\begin{pmatrix}0.26 & 0.74\\0.32 & 0.68\end{pmatrix}$	$p^6 = \begin{pmatrix}0.302 & 0.698\\0.302 & 0.698\end{pmatrix}$	0.302	0.608
April	$\begin{pmatrix}0.37 & 0.63\\0.33 & 0.67\end{pmatrix}$	$p^5 = \begin{pmatrix}0.344 & 0.656\\0.344 & 0.656\end{pmatrix}$	0.344	0.656
May	$\begin{pmatrix}0.32 & 0.68\\0.42 & 0.58\end{pmatrix}$	$p^7 = \begin{pmatrix}0.382 & 0.618\\0.382 & 0.618\end{pmatrix}$	0.382	0.618
June	$\begin{pmatrix}0.33 & 0.67\\0.20 & 0.80\end{pmatrix}$	$p^8 = \begin{pmatrix}0.230 & 0.770\\0.230 & 0.770\end{pmatrix}$	0.230	0.770
July	$\begin{pmatrix}0.26 & 0.74\\0.35 & 0.65\end{pmatrix}$	$p^6 = \begin{pmatrix}0.321 & 0.679\\0.321 & 0.679\end{pmatrix}$	0.321	0.679
August	$\begin{pmatrix}0.06 & 0.94\\0.35 & 0.65\end{pmatrix}$	$p^{12} = \begin{pmatrix}0.271 & 0.729\\0.271 & 0.729\end{pmatrix}$	0.271	0.729
September	$\begin{pmatrix}0.40 & 0.60\\0.33 & 0.67\end{pmatrix}$	$p^6 = \begin{pmatrix}0.355 & 0.645\\0.355 & 0.645\end{pmatrix}$	0.355	0.645
October	$\begin{pmatrix}0.39 & 0.61\\0.23 & 0.77\end{pmatrix}$	$p^9 = \begin{pmatrix}0.274 & 0.726\\0.274 & 0.726\end{pmatrix}$	0.274	0.726
November	$\begin{pmatrix}0.40 & 0.60\\0.10 & 0.90\end{pmatrix}$	$p^{13} = \begin{pmatrix}0.143 & 0.857\\0.143 & 0.857\end{pmatrix}$	0.143	0.857
December	$\begin{pmatrix}0.06 & 0.94\\0.23 & 0.77\end{pmatrix}$	$p^9 = \begin{pmatrix}0.197 & 0.803\\0.197 & 0.803\end{pmatrix}$	0.197	0.803

Discussions

Basically; the rainfall pattern of Akuse may be describe as average. Following the probability at table 4.1; in the long run, the rainfall is lower in January to April.

From table 4.1, the rainfall increases from May towards the month October. The rains began to reduce again drastically from November to December. The conditional probability indicates that the highest rainfall would occur in May with probability of 0.382; that is if it will rain in any particular day in May.

4.5.2 Markov Chain analysis for rainfall pattern in Accra

The average daily rainfall pattern was calculated for each day in each month. For each day a rainy day was picked to determine whether the next day is dry or rainy. The sum of the number of such days (rain or dry) was calculated and divided by the total number of days in the month. Similar computations were done for dry days.. An example was taken from the daily rainfall data of Accra (refer data at appendix c (ii)) for the month January of 31 values which was used to find the conditional probability as shown below;

- Given that it is rainy, the probability that it would be rainy a day later is given by

  $$\frac{3}{31} = 0.10$$

- Given that it is dry, the probability that it would be rainy a day later by is given by

  $$\frac{4}{31} = 0.13$$

This can be seen in table below;

To

From

	Rainy	Dry
rainy	0.10	0.90
Dry	0.13	0.87

or $P = \begin{bmatrix} 0.10 & 0.90 \\ 0.13 & 0.87 \end{bmatrix}$

The study state value of the power of the transition matrix

The study state transition matrix, the value of p^n approaches a fixed square matrix as n increases. This is the case in the table below;

p	p^2	p^3
0.10 0.90 0.13 0.87	0.127 0.873 0.126 0.874	0.12619 0.87381 0.126217 0.873783
p^4	p^5	p^6
0.126214 0.873786 0.126213 0.873787	0.126214 0.873786 0.126214 0.873786	

Each row (0.126214 0.873786) represents the long term probabilities of each state. Whatever the weather is today, in the long term the probability that it will be rainy on any day is 0.126 and the probability that it will be dry is 0.874.

The state matrix approaches a fixed matrix (0.126214 0.873793) which is called the steady state or stable matrix. The steady state matrix represents the long term probabilities of each state.

Whatever the weather is today, in the long term the probability that it will be rainy on a day is 0.126 and the probability that it will be dry on a day is 0.874. The state matrix approaches the steady state matrix (0.126 0.873) regardless of the value of the initial state matrix.

The steady state matrix (0.126 0.874) shows that in the future, the rainfall pattern of Accra will follow the pattern of the given probability. Thus in future, the probability that it will rain on a particular day in Accra is 0.126 and the probability that it will be dry on a particular in Accra is 0.874.

The table below shows the probability of either rainy or drought in a particular day for the various month in Accra.

Table 4.2: Rainfall pattern of Accra

Month	Initial matrix (p)	Long term prob. matrix	Prob. for either rain/dry on a day	
	P	P^n	rain	dry
January	$\begin{pmatrix} 0.10 & 0.90 \\ 0.13 & 0.87 \end{pmatrix}$	$p^5 = \begin{pmatrix} 0.126 & 0.874 \\ 0.126 & 0.874 \end{pmatrix}$	0.126	0.874
February	$\begin{pmatrix} 0.11 & 0.89 \\ 0.14 & 0.86 \end{pmatrix}$	$p^5 = \begin{pmatrix} 0.136 & 0.864 \\ 0.136 & 0.864 \end{pmatrix}$	0.136	0.874
March	$\begin{pmatrix} 0.19 & 0.81 \\ 0.10 & 0.90 \end{pmatrix}$	$p^7 = \begin{pmatrix} 0.110 & 0.890 \\ 0.110 & 0.890 \end{pmatrix}$	0.110	0.890
April	$\begin{pmatrix} 0.33 & 0.67 \\ 0.23 & 0.77 \end{pmatrix}$	$p^7 = \begin{pmatrix} 0.256 & 0.744 \\ 0.256 & 0.744 \end{pmatrix}$	0.256	0.744
May	$\begin{pmatrix} 0.52 & 0.48 \\ 0.42 & 0.58 \end{pmatrix}$	$p^7 = \begin{pmatrix} 0.467 & 0.533 \\ 0.4677 & 0.533 \end{pmatrix}$	0.467	0.533
June	$\begin{pmatrix} 0.40 & 0.60 \\ 0.30 & 0.70 \end{pmatrix}$	$p^7 = \begin{pmatrix} 0.333 & 0.667 \\ 0.333 & 0.667 \end{pmatrix}$	0.333	0.667
July	$\begin{pmatrix} 0.19 & 0.81 \\ 0.29 & 0.71 \end{pmatrix}$	$p^7 = \begin{pmatrix} 0.264 & 0.736 \\ 0.264 & 0.736 \end{pmatrix}$	0.264	0.736
August	$\begin{pmatrix} 0.23 & 0.77 \\ 0.19 & 0.81 \end{pmatrix}$	$p^5 = \begin{pmatrix} 0.198 & 0.802 \\ 0.198 & 0.802 \end{pmatrix}$	0.198	0.802
September	$\begin{pmatrix} 0.37 & 0.63 \\ 0.27 & 073 \end{pmatrix}$	$p^7 = \begin{pmatrix} 0.30 & 0.70 \\ 0.30 & 0.70 \end{pmatrix}$	0.30	0.70
October	$\begin{pmatrix} 0.35 & 0.65 \\ 0.26 & 0.74 \end{pmatrix}$	$p^6 = \begin{pmatrix} 0.324 & 0.676 \\ 0.324 & 0.676 \end{pmatrix}$	0.324	0.676
November	$\begin{pmatrix} 0.23 & 0.77 \\ 0.13 & 0.87 \end{pmatrix}$	$p^8 = \begin{pmatrix} 0.144 & 0.856 \\ 0.144 & 0.856 \end{pmatrix}$	0.144	0.856
December	$\begin{matrix} 0.06 & 0.94 \\ 0.03 & 0.97 \end{matrix}$	$p^5 = \begin{pmatrix} 0.031 & 0.969 \\ 0.031 & 0.969 \end{pmatrix}$	0.031	0.969

Discussions

Looking at table 4.2; the rainfall pattern of Accra, it reveal that the rains are averagely distributed among the entire zone of Accra from April to October. The data reveals that the highest rainfall is recorded in the month May, June and October with probability of 0.467, 0.333 and 0.324 respectively. The rainfall reduces in December considering the probability value of rain in December of 0.031.

4.5.3 Markov Chain analyses for rainfall pattern of Cape-Coast

The average daily rainfall pattern was calculated for each day in each month. For each day a rainy day was picked to determine whether the next day is dry or rainy. The sum of the number of such days (rain or dry) was calculated and divided by the total number of days in the month. Similar computations were done for dry days. An example was taken from the daily rainfall data of Cape Coast (refer data at appendix c (iii)) for the month January of 31 values which was used to find the conditional probability as shown below;

- Given that it is rainy, the probability that it would be rainy a day later is given by

$$\frac{2}{31} = 0.06$$

- Given that it is dry, the probability that it would be rainy a day later is given by

$$\frac{5}{31} = 0.16$$

This can be seen in table below;

To

From

	Rainy	Dry
rainy	0.06	0.94
Dry	0.16	0.84

or $P = \begin{bmatrix} 0.06 & 0.94 \\ 0.16 & 0.84 \end{bmatrix}$

The study state value of the power of the transition matrix

The study state transition matrix, the value of p^n approaches a fixed square matrix as n increases. This is the case in the table below;

p	p^2	p^3
0.06 0.94 0.16 0.84	0.154 0.846 0.144 0.856	0.1446 0.8554 0.1456 0.8544
p^4	p^5	p^6
0.14554 0.85446 0.14544 0.85456	0.145446 0.854554 0.145456 0.854544	0.145456 0.854554 0.145454 0.854546
p^7	p^8	
0.145454 0.854546 0.145455 0.854545	0.145455 0.854545 0.145455 0.854545	

Each row (0.145455 0.854545) represents the long term probabilities of each state. Whatever the weather is today, in the long term the probability that it will be rainy on any day is 0.15 and the probability that it will be dry is 0.85.

The state matrix approaches a fixed matrix (0.145455 0.854545) which is called the steady state or stable matrix. The steady state matrix represents the long term probabilities of each state.

Whatever the weather is today, in the long term the probability that it will be rainy on a day is 0.145 and the probability that it will be dry on a day is 0.855. The state matrix approaches the steady state matrix (0.145 0.855) regardless of the value of the initial state matrix.

The steady state matrix (0.145 0.855) shows that in the future, the rainfall pattern of Cape Coast will follow the pattern of the given probability. Thus in future, the probability that it will rain on a particular day in Cape coast is 0.145 and the probability that it will be dry on a particular in Cape Coast is 0.855.

The table below shows the probability of either rainy or drought in a particular day for the various month in Cape Coast.

Table 4.3: Rainfall pattern of Cape Coast

Month	Initial matrix (p)	Long term prob. Matrix	Prob. for either rain/dry on a day
January	$\begin{pmatrix} 0.06 & 0.94 \\ 0.16 & 0.84 \end{pmatrix}$	$p^8 = \begin{pmatrix} 0.145 & 0.854 \\ 0.145 & 0.854 \end{pmatrix}$	0.145 0.854
February	$\begin{pmatrix} 0.14 & 0.86 \\ 0.07 & 0.93 \end{pmatrix}$	$p^6 = \begin{pmatrix} 0.075 & 0.923 \\ 0.075 & 0.923 \end{pmatrix}$	0.075 0.923
March	$\begin{pmatrix} 0.23 & 0.77 \\ 0.06 & 0.94 \end{pmatrix}$	$p^9 = \begin{pmatrix} 0.072 & 0.928 \\ 0.072 & 0.928 \end{pmatrix}$	0.072 0.928
April	$\begin{pmatrix} 0.23 & 0.77 \\ 0.30 & 0.70 \end{pmatrix}$	$p^6 = \begin{pmatrix} 0.280 & 0.720 \\ 0.280 & 0.720 \end{pmatrix}$	0.280 0.720
May	$\begin{pmatrix} 0.29 & 0.71 \\ 0.42 & 0.58 \end{pmatrix}$	$p^8 = \begin{pmatrix} 0.372 & 0.628 \\ 0.372 & 0.628 \end{pmatrix}$	0.372 0.628
June	$\begin{pmatrix} 0.60 & 0.40 \\ 0.27 & 0.43 \end{pmatrix}$	$p^{13} = \begin{pmatrix} 0.403 & 0.597 \\ 0.403 & 0.597 \end{pmatrix}$	0.403 0.597
July	$\begin{pmatrix} 0.45 & 0.55 \\ 0.55 & 0.45 \end{pmatrix}$	$p^7 = \begin{pmatrix} 0.50 & 0.50 \\ 0.50 & 0.50 \end{pmatrix}$	0.50 0.50
August	$\begin{pmatrix} 0.23 & 0.77 \\ 0.39 & 0.61 \end{pmatrix}$	$p^8 = \begin{pmatrix} 0.336 & 0.664 \\ 0.336 & 0.664 \end{pmatrix}$	0.336 0.664
September	$\begin{pmatrix} 0.33 & 0.67 \\ 0.10 & 0.90 \end{pmatrix}$	$p^{10} = \begin{pmatrix} 0.130 & 0.870 \\ 0.130 & 0.870 \end{pmatrix}$	0.130 0.870
October	$\begin{pmatrix} 0.32 & 0.68 \\ 0.42 & 0.58 \end{pmatrix}$	$p^7 = \begin{pmatrix} 0.382 & 0.618 \\ 0.382 & 0.618 \end{pmatrix}$	0.382 0.610
November	$\begin{pmatrix} 0.23 & 0.77 \\ 0.17 & 0.83 \end{pmatrix}$	$p^6 = \begin{pmatrix} 0.182 & 0.820 \\ 0.182 & 0.820 \end{pmatrix}$	0.182 0.820
December	$\begin{pmatrix} 0.03 & 0.97 \\ 0.13 & 0.87 \end{pmatrix}$	$p^7 = \begin{pmatrix} 0.118 & 0.882 \\ 0.118 & 0.882 \end{pmatrix}$	0.118 0.882

Discussion

The study state probability rainfall values for the region indicate that, the highest rainfall was recorded in the month of July with the probability of 0.50. This showed that the chances of both rain and drought indication is 50 percent. Looking at data from January to December; it is vividly clear that the rainfall increases gradually from May with the probability of 0.372 until August with the probability of 0.336 and began to decrease from September gradually until December. The maximum rainfall was recorded in the months of May to August. In general, Cape-Coast has a greater edge and better atmospheric conditions comparatively to the rest four (4) towns.

4.5.4 Markov Chain analyses for rainfall pattern of Akatsi

The average daily rainfall pattern was calculated for each day in each month. For each day a rainy day was picked to determine whether the next day is dry or rainy. The sum of the number of such days (rain or dry) was calculated and divided by the total number of days in the month. Similar computations were done for dry days. An example was taken from the daily rainfall data of Akatsi (refer data at appendix c (iv)) for the month January of 31 values which was used to find the probability as shown below;

- Given that it is rainy, the probability that it would be rainy a day later is given by

$$\frac{3}{31} = 0.10$$

- Given that it is dry, the probability that it would be rainy a day later is given by

$$\frac{2}{31} = 0.06$$

This can be seen in table below;

To

From

	Rainy	Dry
Rainy	0.10	0.90
Dry	0.06	0.94

or $P = \begin{bmatrix} 0.10 & 0.90 \\ 0.06 & 0.94 \end{bmatrix}$

The study state value of the power of the transition matrix

The study state transition matrix, the value of p^n approaches a fixed square matrix as n increases. This is the case in the table below;

p	p^2	p^3
0.10 0.90 0.06 0.94	0.064 0.936 0.0624 0.9376	0.06256 0.93744 0.062496 0.937504
p^4	p^5	p^6
0.062502 0.937498 0.0625 0.9375	0.0625 0.9375 0.0625 0.9375	0.0625 0.9375 0.0625 0.9375

Each row (0.0625 0.9375) represents the long term probabilities of each state. Whatever the weather is today, in the long term the probability that it will be rainy on any day is 0.06 and the probability that it will be dry is 0.94.

The state matrix approaches a fixed matrix (0.0625 0.9375) which is called the steady state or stable matrix. The steady state matrix represents the long term probabilities of each state.

Whatever the weather is today, in the long term the probability that it will be rainy on a day is 0.0625 and the probability that it will be dry on a day is 0.9375. The state matrix approaches the steady state matrix (0.0625 0.9375) regardless of the value of the initial state matrix.

The steady state matrix (0.0625 0.9375) shows that in the future, the rainfall pattern of Akatsi will follow the pattern of the given probability. Thus in future, the probability that it will rain on a particular day in Akatsi is 0.062 and the probability that it will be dry on a particular in Akatsi is 0.938.

The table below shows the probability of either rainy or drought in a particular day for the various month in Akatsi.

Table 4.4: Rainfall pattern of Akatsi

Month	Initial matrix (p)	Long term prob. Matrix	Prob. for either rain/dry on a day
January	$\begin{pmatrix} 0.10 & 0.90 \\ 0.06 & 0.94 \end{pmatrix}$	$p^5 = \begin{pmatrix} 0.063 & 0.938 \\ 0.063 & 0.938 \end{pmatrix}$	0.063 0.938
February	$\begin{pmatrix} 0.11 & 0.89 \\ 0.04 & 0.96 \end{pmatrix}$	$p^6 = \begin{pmatrix} 0.043 & 0.957 \\ 0.043 & 0.957 \end{pmatrix}$	0.043 0.957
March	$\begin{pmatrix} 0.13 & 0.87 \\ 0.35 & 0.96 \end{pmatrix}$	$p = \begin{pmatrix} 0.287 & 0.713 \\ 0.287 & 0.713 \end{pmatrix}$	0.287 0.713
April	$\begin{pmatrix} 0.20 & 0.80 \\ 0.27 & 0.73 \end{pmatrix}$	$p^7 = \begin{pmatrix} 0.252 & 0.748 \\ 0.2532 & 0.748 \end{pmatrix}$	0.252 0.748
May	$\begin{pmatrix} 0.35 & 0.65 \\ 0.27 & 0.71 \end{pmatrix}$	$p^6 = \begin{pmatrix} 0.309 & 0.691 \\ 0.309 & 0.691 \end{pmatrix}$	0.309 0.691
June	$\begin{pmatrix} 0.47 & 0.53 \\ 0.53 & 0.47 \end{pmatrix}$	$p^5 = \begin{pmatrix} 0.50 & 0.50 \\ 0.50 & 0.50 \end{pmatrix}$	0.50 0.50
July	$\begin{pmatrix} 0.29 & 0.71 \\ 0.35 & 0.65 \end{pmatrix}$	$p^6 = \begin{pmatrix} 0.330 & 0.670 \\ 0.330 & 0.670 \end{pmatrix}$	0.330 0.670
August	$\begin{pmatrix} 0.26 & 0.17 \\ 0.19 & 0.81 \end{pmatrix}$	$p^6 = \begin{pmatrix} 0.204 & 0.796 \\ 0.204 & 0.796 \end{pmatrix}$	0.204 0.796
September	$\begin{pmatrix} 0.43 & 0.57 \\ 0.37 & 0.63 \end{pmatrix}$	$p^5 = \begin{pmatrix} 0.394 & 0.606 \\ 0.394 & 0.606 \end{pmatrix}$	0.394 0.606
October	$\begin{pmatrix} 0.42 & 0.58 \\ 0.39 & 0.61 \end{pmatrix}$	$p^4 = \begin{pmatrix} 0.402 & 0.598 \\ 0.402 & 0.598 \end{pmatrix}$	0.402 0.598
November	$\begin{pmatrix} 0.40 & 0.60 \\ 0.17 & 0.83 \end{pmatrix}$	$p^{11} = \begin{pmatrix} 0.221 & 0.779 \\ 0.221 & 0.779 \end{pmatrix}$	0.221 0.779
December	$\begin{pmatrix} 0.10 & 0.90 \\ 0.03 & 0.91 \end{pmatrix}$	$p^6 = \begin{pmatrix} 0.032 & 0.968 \\ 0.032 & 0.968 \end{pmatrix}$	0.032 0.968

Discussion

The observed data revealed that the rains would not be enough in the month of January to February. Gradually, the rains increase from March to July and decrease a little bit in the month of August. The month of September to October, see the increase of rainfall throughout the years. In June, it shows clearly that the probability of rains occurring has equal chance as that of drought or no rain. (Probability of 50%). Averagely, Akatsi seems to have a better edge over the rest of the four towns.

4.5.5 Markov Chain analyses for rainfall pattern of Keta

The average daily rainfall pattern was calculated for each day in each month. For each day a rainy day was picked to determine whether the next day is dry or rainy. The sum of the number of such days (rain or dry) was calculated and divided by the total number of days in the month. Similar computations were done for dry days. An example was taken from the daily rainfall data of Keta (refer data at appendix c (v)) for the month January of 31 extreme values which was used to find the probability as shown below;

- Given that it is rainy, the probability that it would be rainy a day later is given by

$$\frac{1}{31} = 0.03$$

- Given that it is dry, the probability that it would be rainy a day later is given by

$$\frac{2}{31} = 0.06$$

This can be seen in table below;

To

From

	Rainy	Dry
Rainy	0.03	0.97
Dry	0.06	0.94

or $P = \begin{bmatrix} 0.03 & 0.97 \\ 0.06 & 0.94 \end{bmatrix}$

The study state value of the power of the transition matrix

The study state transition matrix, the value of p^n approaches a fixed square matrix as n increases. This is the case in the table below;

p	p^2	p^3
0.03 0.97 0.06 0.94	0.591 0.9409 0.0582 0.9418	0.058227 0.941773 0.058254 0.941746
p^4	p^5	p^6
0.058253 0.941747 0.058252 0.941748	0.058252 0.941748 0.058252 0.941748	

Each row (0.058252 0.941748) represents the long term probabilities of each state. Whatever the weather is today, in the long term the probability that it will be rainy on any day is 0.06 and the probability that it will be dry is 0.94.

The state matrix approaches a fixed matrix (0.05825 0.941712) which is called the steady state or stable matrix. The steady state matrix represents the long term probabilities of each state.

Whatever the weather is today, in the long term the probability that it will be rainy on a day is 0.058 and the probability that it will be dry on a day is 0.942. The state matrix approaches the steady state matrix (0.058 0.942) regardless of the value of the initial state matrix.

The steady state matrix (0.058 0.942) shows that in the future, the rainfall pattern of Keta will follow the pattern of the given probability. Thus in future, the probability that it will rain on a particular day in Keta is 0.058 and the probability that it will be dry on a particular in Akuse is 0.942.

The table below shows the probability of either rainy or drought in a particular day for the various month in Keta.

Table 4.5: Rainfall pattern of Keta

Month	Initial matrix (p)	Long term prob. Matrix	Prob. for either rain/dry on a day
January	$\begin{pmatrix} 0.03 & 0.97 \\ 0.06 & 0.94 \end{pmatrix}$	$p^5 = \begin{pmatrix} 0.058 & 0.942 \\ 0.058 & 0.942 \end{pmatrix}$	0.058 0.942
February	$\begin{pmatrix} 0.14 & 0.86 \\ 0.04 & 0.96 \end{pmatrix}$	$p^8 = \begin{pmatrix} 0.044 & 0.956 \\ 0.044 & 0.956 \end{pmatrix}$	0.044 0.956
March	$\begin{pmatrix} 0.06 & 0.94 \\ 0.10 & 0.90 \end{pmatrix}$	$p^5 = \begin{pmatrix} 0.096 & 0.904 \\ 0.096 & 0.904 \end{pmatrix}$	0.096 0.904
April	$\begin{pmatrix} 0.2 & 0.8 \\ 0.23 & 0.77 \end{pmatrix}$	$p^5 = \begin{pmatrix} 0.223 & 0.777 \\ 0.223 & 0.777 \end{pmatrix}$	0.223 0.777
May	$\begin{pmatrix} 0.16 & 0.84 \\ 0.23 & 0.77 \end{pmatrix}$	$p^6 = \begin{pmatrix} 0.215 & 0.785 \\ 0.215 & 0.785 \end{pmatrix}$	0.215 0.785
June	$\begin{pmatrix} 0.33 & 0.67 \\ 0.20 & 0.80 \end{pmatrix}$	$p = \begin{pmatrix} 0.230 & 0.770 \\ 0.230 & 0.770 \end{pmatrix}$	0.230 0.770
July	$\begin{pmatrix} 0.29 & 0.71 \\ 0.03 & 0.97 \end{pmatrix}$	$p^{11} = \begin{pmatrix} 0.041 & 0.959 \\ 0.041 & 0.959 \end{pmatrix}$	0.041 0.959
August	$\begin{pmatrix} 0.23 & 0.77 \\ 0.19 & 0.81 \end{pmatrix}$	$p^5 = \begin{pmatrix} 0.198 & 0.802 \\ 0.198 & 0.802 \end{pmatrix}$	0.198 0.802
September	$\begin{pmatrix} 0.40 & 0.60 \\ 0.23 & 0.77 \end{pmatrix}$	$p^{10} = \begin{pmatrix} 0.277 & 0.723 \\ 0.277 & 0.723 \end{pmatrix}$	0.277 0.723
October	$\begin{pmatrix} 0.29 & 0.71 \\ 0.13 & 0.87 \end{pmatrix}$	$p^8 = \begin{pmatrix} 0.155 & 0.845 \\ 0.155 & 0.845 \end{pmatrix}$	0.155 0.845
November	$\begin{pmatrix} 0.10 & 0.90 \\ 0.13 & 0.87 \end{pmatrix}$	$p^5 = \begin{pmatrix} 0.126 & 0.874 \\ 0.126 & 0.874 \end{pmatrix}$	0.126 0.874
December	$\begin{pmatrix} 0.03 & 0.97 \\ 0.10 & 0.90 \end{pmatrix}$	$p = \begin{pmatrix} 0.093 & 0.907 \\ 0.093 & 0.907 \end{pmatrix}$	0.093 0.907

Discussion

The observed data indicates that, the rainfall pattern of keta is not favourable. Looking at the probability values of this town. It shows clearly that rainfall may occur a little in the month of April to May with even a downward trend. August and September may also hit a little rain where the rest of the month is likely not to experience any rain.

4.6 General Findings

The Markov chain analysis for the various towns showed that, the probability of rain in each town varies day by day. Also the long term probability differs from town to town. Observation made on the long term probability values for each town showed that all the areas tended to be dry. This is because in each case, the long term rainfall values are generally low.

Table 4.6, Maximum rainfall pattern of each town

	Keta	Akatsi	Akuse	Accra	Cape Coast
Maximum rainfall	0.277	0.50	0.382	0.467	0.50

From the table, we observe that from Keta to Akatsi to Akuse to Accra through to Cape Coast; the maximum rainfall probability increases (ie from east to west)

CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.0 Introduction

This chapter presents the general summary, conclusion and recommendations of the research work.

5.1 Summary

Rainfall pattern assessment has been a challenging task among rainfall researchers and professionals. There are many rainfall tools that have been developed around the world and are commonly used to quantify rainfall conditions as was discussed in chapter two. It was found that in most cases, Markov chain analysis developed for a specific region, and could not be directly applicable to other regions due to inherent complexity of rainfall phenomena, different hydro-climatic conditions and catchment characteristics.

Markov chain analysis has been employed to study different climatic regions around the world. However, little or no such study has been conducted to show a rainfall pattern in Ghana which has one of the highest recorded rainfall levels (per capita) on the Earth. In this study, an employment of Markov chain analysis for modeling historical daily rainfall within the south eastern coast of Ghana has been done.

Historical daily rainfall values recorded in south eastern coast during 1980-2010 were used in this study to investigate how well the Markov chain method was capable of defining rainfall conditions.

The study showed that Markov chain analysis is as good as other methods in predicting patterns of rainfall and detecting similarities in historical rainfall.

The summary of the highest recorded rainfall from the time series data and the highest expected rainfall probability for various month is indicated in the table below;

Table 5.1, Maximum rainfall and Maximum probability rainfall

TOWN	MONTH	MAXIMUM RAINFALL (mm)	MONTH	MAXIMUM PROBABILITY
ACCRA	JUNE	17	MAY	0.467
AKUSE	MAY	13	MAY	0.382
CAPE COAST	MAY	17	MAY & JULY	0.372 & 0.50
AKATSI	JUNE	15	JUNE	0.50
KETA	MAY-JULY	20	MAY-SEPT.	0.215 & 0.277

Indications from table 5.1 confirm that the highest rainfall occurring from the time series data tallies with the computed maximum probability from Markov chain analysis. This shows Markov chain analysis is a very good tool to use to investigate the rainfall pattern of those towns.

5.2, Conclusion

This study aimed to classify and assessed annual rainfall over south eastern coast of Ghana into rainy/ drier groups. It was found that a Markov chain analysis classifies this pattern.

The steady state transition matrices and the steady state probability vectors were computed for each town and each month.

It was found that, the rainy or dry season pattern observed using the monthly steady state rainfall vectors tended to coincide with the monthly rainfall time series trajectory. In particular for Accra, the rainy season was observed to be in the month of May to June and September to October.

The probability of rainfall tended to increase from east to west along the south eastern coast of Ghana.

5.3 Recommendations

It is recommended that, this work could be helpful to business organizations, Agro industries and Agricultural insurance practitioners to know at what time it is likely to rain or not. In this way they would be able to advice their clients as to what time to invest or not to invest. Further research could be done covering the whole nation since this study concentrated only on south-eastern coast of Ghana.

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APPENDIX

APPENDIX A

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