THE TRANSPORTATION PROBLEM

CASE STUDY :(GUINESS GHANA LIMITED)

BY

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Declaration

I hereby declare that this submission is my own work towards the MSc and that, to the best of my knowledge, it contains no material previously published by another person, nor material, which has been accepted for the award of any other degree of university, except where due acknowledgement has been made in the text

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Dedication

This study is dedicated to Almighty God, Asase family, my wife Olivia and my two lovely daughters Ohemaa and Awuraakua.

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Abstract

The proposed transportation model of manufacturing goods to customer (Key Distributors) is considered in this research. The data gathered were modelled as a Linear Programming model of transportation type and represent the transportation problem as tableau and solve it with the computer software solver to generate an optimal solution.

This transportation model will be useful for making strategic decisions by the logistics managers Guinness Ghana LTD in making optimum allocation of the production from the two plants (KAASI and ACHIMOTA) to the various customers(key distributors) at a minimum transportation cost.

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CHAPTER ONE

INTRODUCTION

1.1 A brief Review of the Transportation Problem

Business and Industries are practically faced with both economic optimization such as cost minimization of non-economic items that are vital to the existence of their firms.

The transportation models or problems are primarily concerned with the optimal (best possible) way in which a product produced at different factories or plants (called supply origins) can be transported to a number of warehouses or customers (called demand destinations). The objective in a transportation problem is to fully satisfy the destination requirements within the operating production capacity constraints at the minimum possible cost. Whenever there is a physical movement of goods from the point of manufacturer to the final consumers through a variety of channels of distribution (wholesalers, retailers, distributors etc.), there is a need to minimize the cost of transportation so as to increase profit on sales.

(i) The transportation problem is a special class of linear programming problem, which deals with shipping commodities from source to destinations. The objective of the transportation problem is to determine the shipping schedule that minimize that total shipping cost while satisfying supply and demand limits

The transportation problem has an application in industry, communication network, planning, scheduling transportation and allotment etc.

Considers a situation in which three points of origin (A_1, A_2, A_3) have suppliers available to meet needs at three destinations (N_1, N_2, N_3). The amount available at each origin is specified (a_1, a_2, a_3) as also the amounts needed at each destinations (n_1, n_2, n_3). Furthermore the cost of moving goods between origin and destination can be setup as a table of m_{ij} where the subscripts indicate the cell , given the cost of moving from the i'th origin to j'th destination: so for example m_{22} is the cost of moving goods from the origin A_2 to destination N_2. In real a life problem these quantities must be written in specific units: the supplies available and needs might be in tonnes, or even in thousands of tonnes; the movement costs will then be in cost units per tonne, for example $/tonne.

Transportation problem deals with the problem of how to plan production and transportation in such an industry given several plants at different location and larger number of customers of their products.

The transportation problem received this name because many of its applications involve in determining how to optimally transport goods.

Transportation problem is a logistical problem for organizations especially for manufacturing and transport companies. This method is a useful tool in decision-making and process of allocating problem in these organizations.

The transportation problem deals with the distribution of goods from several points, such as factories often known as sources, to a number of points of demand, such as warehouses, often known as destinations. Each source is able to supply a fixed number of units of products, usually called the capacity or availability, and each destination has a fixed demand, usually known as requirement.

Because of its major application in solving problems which involving several products sources and several destinations of products, this type of problem is frequently called "The Transportation Problem". The classical transportation problem is referred to as special case of Linear Programming (LP) problem and its model is applied to determine an optimal solution of delivery available amount of satisfied demand in which the total transportation cost is minimized

The transportation problem can be described using linear programming mathematical model and usually it appears in a transportation tableau.

There is a type of linear programming problem that may be solved using a simplified version of the simplex technique called transportation method.

The simplex method is an iterative algebraic procedure for solving linear programming problems" (Krajewski, et al., 2007).

One possibility to solve the optimal problem would be optimization method. The problem is however, formulated so that objective function and all constraints are linear and thus the problem can be solved.

1.2 BACKGROUND TO THE STUDY:

This chapter discusses background materials and concepts involved in the study.

Transportation problem is a particular class of linear programming, which is associated with day-to-day activities in our real life and mainly deals with logistics. It helps in solving problems on distribution and transportation of resources from one place to another. The goods are transported from a set of sources (e.g., factory) to a set of destinations (e.g., warehouse) to meet the specific requirements.

There is a type of linear programming problem that may be solved using a simplified version of the simplex technique called transportation method. Because of its major application in solving problems involving several product sources and several destinations of products, this type of problem is frequently called the transportation problem. It gets its name from its application to problems involving transporting products from several sources to several destinations. Although the formation can be used to represent more general assignment and scheduling problems as well as transportation and distribution problems. The two common objectives of such problems are either (1) minimize the cost of shipping m units to n destinations or (2) maximize the profit of shipping *m* units to *n* destinations.

The aim of this study is to look principally at a specific type of Linear Programming Problem, known as the Transportation Problem. Transportation theory is the name given to the study of optimal transportation and allocation of resources.

The model is useful for making strategic decisions involved in selecting optimum transportation routes so as to allocate the production of various plants to several warehouses or distribution centres.

The transportation model can also be used in making location decisions. The model helps in locating a new facility, a manufacturing plant or an office when two or more number of locations is under consideration. The total transportation cost, distribution cost or shipping cost and production costs are to be minimized by applying the model.

The transportation problem itself was first formulated by Hitchcock (1941), and was independently treated by Koopmans and Kantorovich .In fact Monge (1781) formulated it and solved it by geometrical means. Hitchaxic (1941) developed the basic transportation problem; however it could be solved for optimally as answers to complex business problem only in 1951, when George B. Dantizig applied the concept of Linear programming in solving the transportation model. Dantzing (1951) gave the standard LP-formulation TP and applied the simplex method to solve it. Since then the transportation problem has become the classical common subject in almost every textbook on operation research and mathematical programming.

The transportation problem can be described using linear programming mathematical model and usually it appears in a transportation tableau.

Linear programming has been used successfully in solution of problems concerned with the assignment of personnel, distribution and transportation, engineering, banking, education, petroleum, etc.

The classical transportation problem is the name of a mathematical model, which has a special mathematical structure. The mathematical formulation of a large number of problems conforms (or can be made to conform) to this special structure. So the name is frequently used to refer to a particular form of mathematical model rather than the physical situation in which the problem most natural originates.

The transportation problem is a special kind of the network optimization problem.

The transportation models play an important role in logistics and supply chains. The objective is to schedule shipments from sources to destinations so that total transportation cost is minimized. The problem seeks a production and distribution plan that minimizes total transportation cost. The problem can be formulated as the following mathematical program.

The function to be minimized (or maximised) is called Objective function. When the linear system model and the objective functions are both linear equations, we have a linear programming problem. Furthermore, LP algorithms are used in subroutines for solving more difficult optimisation problems. A widely considered quintessential LP algorithm is the Simplex Algorithm developed by Dantzig (1947) in response to a challenged to mechanise the Air Force planning process.

Linear Programming has been applied extensively in various areas such as transportation, construction, telecommunications, healthcare and public services to name but few areas.

The simplex algorithm was the forerunner of many computer programs that are used to solve complex optimization problems (Baynto, 2006). The transportation method has been employed to developed many different types of processes. From machine shop scheduling Mohaghegh (2006) to optimizing operating room schedules in hospitals (Calichman, 2005). The transportation method can be used to reduce the impact of using fossil fuels to transport materials.

1.2.1 BACKGROUND OF COMPANY

1.2.1.1 COMPANY PROFILE

The Guinness Ghana Breweries Ltd-Kaasi (GGBL-Kaasi) a leading producer of both alcoholic and non-alcoholic drinks in the country has been in existence for the past 40years and currently forms part of the Guinness Ghana Breweries Group. The brewery was established in 1970 under the name Guinness Ghana Limited until 30 November 2004 when it acquired 99.7% interest in Ghana Breweries Limited and subsequently changed its name to Guinness Ghana Breweries Ltd.

Ghana Breweries Limited was incorporated on 30 April 1992 under its previous name, 'ABC Brewery Limited. On 26 October 1994, it acquired the assets of Achimota Brewery Company Limited, a state-owned enterprise operating at Achimota, Accra. In October 1997, Heineken International acquired 90% of the outstanding ordinary shares of ABC Brewery Limited and subsequently renamed the company Ghana Breweries Limited. Ghana Breweries then merged with Kumasi Brewery Limited, a brewing company established in May 1959, with effect from 1 January 1998. Before this merging, Heineken and its wholly owned subsidiary, Limba Ghana Limited, held 50.26% of the issued shares of Kumasi Brewery Limited. Guinness Ghana Breweries Limited (GGBL) emerged out of a merger of Guinness Ghana Limited (GGL) and Ghana Breweries

The GGBL is a subsidiary of Diageo Plc, a company incorporated in Holland.

Guinness Ghana Breweries Limited has three sites, namely Achimota, in Accra, Ahensan in Kumasi and Kaasi also in Kumasi.

1.2.1.3 COMPANY BRANDS

GGBL-Kaasi produces and markets many brands of products and these include Guinness Foreign Extract Stout, Ready-to-drink (RTD) Gordon's Spark and Smirnoff Ice, non-alcoholic Malta Guinness, Malta Guinness Quench and Carbonated Soft Drink (CSD) Alvaro. .

As at 30th October 2009 the range of Guinness Ghana Brand products include : Mini Star, Gordon Spark ,Star Large , Malta Guinness Quench , Amstel Malta , Malta Guinness Can , Malta Guinness , Malta Guinness Quench Can , Guilder Large , Heineken Can/bottle , Guinness FES , Star Draft 30L Keg , Smirnoff Ice , Guinness FES can , Alvaro, Smirnoff /J& B / Gordon's) .

The branded products that are being imported and sold on behalf of other companies are Johnny Walker (Red or Black), Baileys/J&B

1.2.1.4 MANUFACTURING:

The Kaasi site operates as an installed and target capacity of eleven million hectolitres per annum. The total plant capacity for Kaasi and Achimota per day is 1800 and 1600 crates dependants on the plant efficiency.

The site operates an ultra modern brewing department, a modern and highly automated Packaging unit and distribution operations. The Brewery consumed a total of about 4,977,140Hl of water in 2009, representing an increase of about 85% when compared to the annual consumptions of 4,530,230hl for 2003. Production levels within the period also increased by about 42%

1.2.1.5 WAREHOUSE:

Raw materials, semi-finished goods and finished are kept at the warehouse at Ahinsan and Kaasi Store House.

The distribution of raw materials, semi-finish and the finish product is outsourced to third party contractors. Thus GGBL operates in 3 party logistics, which ensures materials, and finished goods are delivered at the right time to the right place in accordance with the planning schedule and at a minimum cost. There few registered transporters that are responsible for loading, packing, off loading and movement of raw material from port to warehouse, movement of finished products from Production warehouse to distributors.

1.2.1.6 **DISTRIBUTION:**

Finished products are sold directly to registered distributors. The distributors are the main agent who sells to retailers. The practice of exclusive distribution where only specially registered or authorised distributors (typically at least 5 distributors per a region) is the order of the day. These distributors act as wholesalers that sell directory to the public and so called “Beer Bars”.

1.3 PROBLEM STATEMENT

The thesis seeks to address the problem of determine the optimal transportation schedule that will minimizes the total cost of transporting beverage from the two production sites Kaasi and Achimota to the various key distributors geographically scatter in Ghana .

1.4 OBJECTIVE

The study intended:

1. To model the distribution of GGBL products as a transportation problem
2. To minimize the transportation cost.

1.5 Methodology

The Management Science will be use for finding an optimal solution of transportation problem with equality constraints.

Source of information for the project are the internet, mathematical books from the KNUST Library and Mathematics department.

The problems of GGBL to be modeled as the linear programming model of transportation type, and represent the Linear Programming or the transportation problem as tableau and solve it with the management science application.

1.6 JUSTIFICATION

The profit gained as a result of minimising the transportation cost will enable Guinness Ghana LTD to contribute to its continuous projects and programmes such as:

1. Sports: key sponsor for Ghana Black Star team

2. Education: periodic contribution to Otumfo Education fund

3. Environmental support: funding for Okyemans reforestation project in the Eastern region of Ghana

4. Health: noted for its periodic contributions to the Ghana Heart Foundation

5. Culture and entertainment. It also sponsors festivals of several ethnic groups in Ghana such as Ga Homowo, Bakatue and etc.

1.7 ORGANIZATION OF THE THESIS

The chapters One introduces the thesis in general, the review of transportation problem, the background for Transportation Problem, the background of company (GGBL), the problem statement, objective, methodology, justification and the organization of the thesis.

Chapter Two is concern with the definition and the detailed literature review of the transportation problem/model.

Chapter Three discuss detailed methodology.

This includes the formulation of the transportation problem, the transportation tableau, the solutions for the transportation problem, and methods for solving transportation problems to optimality.

Chapter Four provides an over view of the computational platforms for implementation and solution of the model and introduces the real-life data sets used in the solution process.

Finally chapter Five summarises the conclusions with respect to overall aims of the project and proposed recommendation for future research/study. It reports the computational results and provides a comprehensive analysis of the outcome and performance of the proposed solution approaches.

CHAPTER TWO

LITERATURE REVIEW

2. 1 INTRODUCTION

The transportation problem (TP) is an important Linear Programming (LP) model that arises in several contexts and has deservedly received much attention in literature.

The transportation problem is probably the most important special linear programming problem in terms of relative frequency with which it appears in the applications and also in the simplicity of the procedure developed for its solution. The following features of the transportation problem are considered to be most important.

The TP were the earliest class of linear programs discovered to have totally unimodular matrices and integral extreme points resulting in considerable simplification of the simplex method.

The study of the TP's laid the foundation for further theoretical and algorithmic development of the minimal cost network flow problems.

2.2 LITERATURE REVIEW

The transportation problem was formalized by the French mathematician (Monge, 1781). Major advances were made in the field during World War II by the Soviet/Russian mathematician and economist Leonid Kantorovich. Consequently, the problem as it is now stated is sometimes

Known as the Monge-Kantorovich transportation problem. Kantorovich (1942), published a paper on continuous version of the problem and later with Gavurian, and applied study of the capacitated transportation problem (Kantorovich and Gavurin, 1949)

Many scientific disciplines have contributed toward analyzing problems associated with the transportation problem, including operation research, economics, engineering, Geographic Information Science and geography. It is explored extensively in the mathematical programming and engineering literatures. Sometimes referred to as the facility location and allocation problem, the

Transportation optimization problem can be modeled as a large-scale mixed integer linear programming problem.

The origin of transportation was first presented by Hitchcock, (1941), also presented a study entitled “The Distribution of a Product from Several sources to numerous Localities”. This presentation is considered to be the first important contribution to the solution of transportation problems. Koopmans, (1947), presented an independent study, not related to Hitchcock’s, and called “Optimum Utilization of the Transportation System“. These two contributions helped in the development of transportation methods which involve a number of shipping sources and a number of destinations. The transportation problem, received this name because many of its applications involve determining how to optimally transport goods.

However it could be solved for optimally as an answer to complex business problem only in 1951, when George B. Dantzig applied the concept of Linear Programming in solving the Transportation models.

Dantizig, (1963), then uses the simplex method on transportation problem as the primal simplex transportation method.

Stringer and Haley have developed a method of solution using a mechanical analogue.

May be the first algorithm to find an optimal solution for the uncapacitated transportation problem was that of Efroymson and Ray .

They assumed that each of the unit production cost functions has a fixed charge form.

But they remark that their branch-and - bound method can be extended to the case in which each of these functions is concave and consists of several linear Segments. And each unit transportation cost function is linear.

J. Frank Sharp.et.al developed an algorithm for reaching an optimal solution to the production-transportation problem for the convex case.

The algorithm utilizes the decomposition approach it iterates between a linear programming transportation problem which allocates previously set plant production quantities to various markets and a routine which optimally sets plant production quantities to equate total marginal production costs, including a shadow price representing a relative location cost determined from the transportation problem.

Williams applied the decomposition principle of Dantzing and Wolf to the solution of the Hitchcock transportation problem and to several generalizations of it.

In this generalizations, the case in which the costs are piecewise linear convex functions is included. He decomposed the problem and reduced to a strictly linear program. In addition he argued that the two problems are the same by a theorem that he called the reduction theorem.

The algorithm given by him, to solve the problem, is a variation of the simplex method with "generalized pricing operation". It ignores the integer solution property of the transportation problem so that some problems of not strictly transportation type, and for which the integer solution property may not hold be solved.

Shetty(1959) also formulated an algorithm to solve transportation problems taking nonlinear costs. He considered the case when a convex production cost is included at each supply center besides the linear transportation cost.

Some of the approaches used to solve the concave transportation problem are presented as follows.

The branch and bound algorithm approach is based on using a convex approximation to the concave cost functions. It is equivalent to the solution of a finite sequence of transportation problems. The algorithm was developed as a particular case of the simplified algorithm for minimizing separable concave functions over linear polyhedral as Falk and Soland.

Soland (1971) presented a branch and bound algorithm to solve concave separable transportation problem which he called it the "Simplified algorithm" in comparison with similar algorithm given by Falk and himself in 1969.

The algorithm reduces the problem to a sequence of linear transportation problem with the same constraint set as the original problem.

A.C. Caputo. et. al. presented a methodology for optimally planning long-haul road transport activities through proper aggregation of customer orders in separate full-truckload or less-than-truckload shipments in order to minimize total transportation costs. They have demonstrated that evolutionary computation techniques may be effective in tactical planning of transportation activities. The model shows that substantial savings on overall transportation cost may be achieved adopting the methodology in a real life scenario.

Roy and Gelders (1980) solved a real life distribution problem of a liquid bottled product through a 3-stage logistic system; the stages of the system are plant-depot, depot-distributor and distributor-dealer. They modelled the customer allocation, depot location and transportation problem as a 0-1 integer programming model with the objective function of minimization of the fleet operating costs, the depot setup costs, and delivery costs subject to supply constraints, demand constraints, truck load capacity constraints, and driver hours constraints.

The problem was solved optimally by branch and bound, and Langrangian relaxation.

Tzeng et al. (1995) solved the problem of how to distribute and transport the imported Coal to each of the power plants on time in the required amounts and at the required18 quality under conditions of stable and supply with least delay. They formulated a LP that Minimizes the cost of transportation subject to supply constraints, demand constraints, vessel constraints and handling constraints of the ports. The model was solved to yield optimum results, which is then used as input to a decision support system that help manage the coal allocation, voyage scheduling, and dynamic fleet assignment.

Equi et al.(1996) modelled a combined transportation and scheduling in one problem where a product such as sugar cane, timber or mineral ore is transported from multi origin supply points to multi destination demand points or transhipment points using carriers that can be ships, trains or trucks. They defined a trip as a full-loaded vehicle travel from one origin to one destination. They solved the model optimally using Langrangean Decomposition.

Saumis et al. (1991) considered a problem of preparing a minimum cost transportation plan by simultaneously solving following two sub-problem: first the assignment of units available at a series of origins to satisfy demand at a series of destinations and second, the design of vehicle tours to transport these units, when the vehicles have to be brought back to their departure point. The cost minimization mathematical model was constructed, which is converted into a relaxation total distance minimization, then finally decomposed to network problems, a full vehicle problem, and an empty vehicle problem. The problems were solved by tour construction and improvement procedures. This approach allows large problems to be solved quickly, and solutions to large problems to be solved quickly, and solutions to large test problems have been shown to be 1% 0r 2% from the optimum.

Equi et al. (1996) modelled a combined transportation and scheduling in one problem where a product such as sugar cane, timber or mineral ore is transported from multi origin supply points to multi destination demand points or transhipment points using carriers that can be ships, trains or trucks. They defined a trip as a full-loaded vehicle travel from one origin to one destination. They solved the model optimally using Langrangean Decomposition.

Goal Programming (GP) model and its variants have been applied to solve large-scale multi criteria decision-making problems. Charnes and Cooper (1960) first used the Goal Programming (GP) technique. This solution approach has been extended by Ijiri(965), Lee (1972), and others.

Lee and Moore (1973) used GP model for solving transportation problem with multiple conflicting objective. Arthur and Lawrence (1982) designed a GP model for production and shipping patterns in chemical and pharmaceutical industries.

Kwak and schniederjans (1985) applied GP to transportation problem with variable supply and demand requirements. Several other researchers Sharma et al. (1999) have also used the GP model for solving the transportation problem.

Veenan et al. proposed a heuristic method for solving transportation problem with mixed constraints which is based on the theory of shadow price. The solution obtained by heuristics method introduced by Veena et al is an initial solution of the transportation problems with constraints.

Klingman and Russell (1975) have developed an efficient procedure for solving transportation problems with additional linear constraints. Their method exploits the topological properties of basis trees within a generalized upper bound framework.

Swarup (1970) developed a technique, similar to transportation technique in linear programming to minimize a locally indefinite quadratic function, subject to Sharma and swarup, (1977b), have developed the same concepts for multi-dimensional transportation problem.

Further, et al. (1990) developed a heuristic, called TOM (Total Opportunity-cost Method), for obtaining an initial basic feasible solution for the transportation problem.

Gass (1990) detailed the practical issues for solving transportation problems and offered comments on various aspects of transportation problem methodologies along with discussions on the computational results, by the respective researchers. Sharma and Sharma (2000) proposed a new heuristic approach for getting good starting solutions for dual based approaches used for solving transportation problems

The transportation criterion is, however, hardly mentioned at all where the transportation problem is treated. Apparently, several researchers have discovered the criteria independently from each other. But most papers on the subject refer to the papers by Charnes and Klingman and Szwarc as the initial papers.

In Charnes and Klingman name it the more-for-less criteria (MFL), and they write: The criteria was first observed in the early days of linear programming history (by whom no one knows) and has been a part of the folklore known to some (e.g. A.Charnes and W.W.Cooper), but unknown to the great majority of workers in the field of linear programming.

The transportation criteria is known as Doigs criteria at the London School of Economics, named after Alison Doig who used it in exams etc. around 1959 (Doig did not publish any paper on it).

Since the transportation criteria seems not to be known to the majority of those who are working with the transportation problem, one may be tempted to believe that this phenomenon is only an academic curiosity, which will most probably not occur, in any practical situation. But that seems not to be true. Experiments done by Finke, with randomly generated instances of the transportation problem of size 100×100 and allowing additional shipments (post optimal) show that the transportation costs can be reduced considerably by exploiting the criteria properties. More precisely, the average cost reductions achieved are reported to be 18.6% with total additional shipments of 20.5%.

In a recent paper, Deineko & al. develop necessary and sufficient conditions for a cost matrix C to be protected against the transportation criteria. These conditions are rather restrictive, supporting the observations by Finke.

The existing literature has demonstrated the identifying cases where MFL paradoxical situation exists and also, has provided various methods for finding MFL solution for transportation problems. Gupta et al and Arsham obtained the more-for-less solution for the TPs with mixed constraints by relaxing the constraints and by introducing new slack variables. While yielding the best more-for-less solution, their method is tedious since it introduces more variables and requires solving sets of complex equations. The perturbed method was used for solving the TPs with constraints .

Adlakha et al. proposed a heuristic method for solving TPs with mixed constraints which is based on the theory of shadow price. In the heuristic algorithm for an MFL solution in Adlakha et al,Vogel Approximation Method (VAM) and MODI (Modified Distribution) method were used

Arsham developed an approach to post optimality analysis of the TPs through the use of perturbation analysis. Adlakha and Kowalski introduced a theory of absolute points for solving a TP and used these points for search opportunities to ship more for less in TP. Adlaka et al. developed an algorithm for finding an optimal MFL solution for TPs which builds upon any existing basic feasible solution.

The Since then, these problems have been studied extensively by many authors and have found applications in such diverse fields as geometry, fluid mechanics, Statistics, economics, shape recognition, inequalities and meteorology

CHAPTER THREE

METHODOLOGY

3.1 INTRODUCTION

This chapter reviews the proposed solution methodology and approach for handling transportation problem in Guinness Ghana Ltd.

The transportation problem seeks to minimize the total shipping costs of transporting goods from m origins (each with a supply s_i) to n destinations (each with a demand d_j), when the unit shipping cost from an origin, i, to a destination, j, is c_{ij}.

3.2 TRANSPORTATION PROBLEM

This is a type of linear programming problem that may be solved using a simplified version of the simplex technique called transportation method. Because of its major application in solving problems involving several product sources and several destinations of products, this type of problem is frequently called the transportation problem.

In a transportation problem, we have certain origins, which may represent factories where we produced items and supply a required quantity of the products to a certain number of destinations. This must be done in such a way as to maximize the profit or minimize the cost. Thus we have the places of production as origins and the places of supply as destinations. Sometimes the origins and destinations are also termed as sources and sinks.

Transportation model is used in the following:

- To decide the transportation of new materials from various centres to different manufacturing plants. In the case of multi-plant company this is highly useful.
- To decide the transportation of finished goods from different manufacturing plants to the different distribution centres. For a multi-plant-multi-market company this is useful.

These two are the uses of transportation model. The objective is minimizing transportation cost.

3.3 MATHEMATICAL FORMULATION

Supposed a company has m warehouses and n retail outlets. A single product is to be shipped from the warehouses to the outlets. Each warehouse has a given level of supply, and each outlet has a given level of demand. We are also given the transportation cost between every pair of warehouse and outlet, and these costs are assumed to be linear. More explicitly, the assumptions are:

- The total supply of the products from warehouse i = a_i ,where $i = 1,2,3\ldots m$
- The total Demand of the products at the outlet $j = b_j$, where $j = 1,2,3,\ldots n$.
- The cost of sending one unit of the product from warehouse i to outlet j is equal to C_{ij}, where $i = 1,2,3\ldots m$ and $j = 1,2,3,\ldots n$. The total cost of a shipment is linear in size of shipment.

3.3.1 The Decision Variables

The variables in the Linear Programming (LP) model of the TP will hold the values for the number of units shipped from one source to a destination.

The decision variables are:

X_{ij} = the size of shipment from warehouse i to outlet j,

Where $i = 1,2,3 \ldots m$ and $j = 1,2,3,\ldots n$.

This is a set of m.n variables.

3.3.2 The Objective Function

The objective function contains costs associated with each of the variables. It is a minimization problem.

Consider the shipment from warehouse i to outlet j. For any i and j, the transportation cost per unit C_{ij} and the size of the shipment is X_{ij}. Since we assume that the total cost function is linear, the total cost of this shipment is given by $c_{ij}\ xij$

Summing over all i and j now yields the overall transportation cost for all warehouse-outlet combinations. That is, our objective function is:

Minimize. $\sum_{i=1}^{m}\sum_{j=1}^{n} X_{ij}C_{ij}$

3.3.3 The Constraints

The constraints are the conditions that force supply and demand needs to be satisfied. In a Transportation Problem, there is one constraint for each node.

Let $\mathbf{a_1}$ denote a source capacity and $\mathbf{b_1}$ denote destination needs

i) The supply at each source must be used:

$$\sum_{j=1}^{n} X_{ij} = a_i, \ i = 1,2,3\ldots m$$

ii) The demand at each destination must be met:

$$\sum_{i=1}^{m} X_{ij} = b_j, \ j = 1,2,3\ldots n$$

and

(iii) Nonnegativity:

$$X_{ij} \geq 0, \ \forall \ \text{i and j}$$

The transportation model will then become:

Minimizing the transportation cost

$$\text{Minimize} \quad Z = \sum_{i=1}^{m}\sum_{j=1}^{n} X_{ij} C_{ij} \ \ldots\ldots\ldots \qquad \textbf{(1)}$$

$$\sum_{j=1}^{n} X_{ij} \leq a_i, \ (i = 1,2,3\ldots m) \ \ldots\ldots \ \textbf{(2)} \qquad \text{(Demand Constraint)}$$

$$\sum_{i=1}^{m} X_{ij} \geq b_j, \ (j = 1,2,3\ldots n) \ \ldots\ldots \ \textbf{(3)} \qquad \text{(Supply Constraint)}$$

$$\mathbf{X}_{ij} \geq 0, \ (i = 1,2,3\ldots m; \ j = 1,2,3\ldots n)$$

This is a linear program with $m.n$ decision variables, $m+n$ functional constraints, and $m.n$ nonnegative constraints.

$m =$ Number of sources

$n =$ Number of destinations

$a_i =$ Capacity of $i-th$ source (in tons, pounds, litres, etc)

$b_j =$ Demand of $j-th$ destination (in tons, pounds, litres, etc.)

$c_{ij} =$ cost coefficients of material shipping (unit shipping cost) between $i-th$ source and $j-th$ destination (in $ or as a distance in kilometers, miles, etc.)

$x_{ij} =$ amount of material shipped between $i-th$ source and $j-th$ destination (in tons, pounds, liters etc.)

A necessary and sufficient condition for the existence of a feasible solution to the transportation problem is that

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

Remark. The set of constraints

$$\sum_{i=1}^{m} X_{ij} = b_j \quad \text{and} \quad \sum_{j=1}^{n} X_{ij} = a_i$$

represents $m+n$ *equations* in $m.n$ non-negative variables. Each variable X_{ij} appears in exactly two constraints, one is associated with the origin and the other is associated with the destination.

UNBALANCED TRANSPORTATION PROBLEM

If $\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$

The transportation problem is known as an unbalanced transportation problem. There are two cases

Case(1)

$$\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j$$

Case (2) .

$$\sum_{i=1}^{m} a_i < \sum_{j=1}^{n} b_j$$

Introduce a dummy origin in the transportation table; the cost associated with this origin is set equal to zero. The availability at this origin is:

$$\sum_{j-1}^{n} b_j - \sum_{i=1}^{m} a_i = 0$$

3.4. TRANSPORTATION TABLEAU

The transportation problem can be described using linear programming mathematical model and usually it appears in a transportation tableau.

The model of a transportation problem can be represented in a concise tabular form with all the relevant parameters.

The transportation tableau (A typical TP is represented in standard matrix form), where supply availability (ai) at each source is shown in the far right column and the destination requirements (bi) are shown in the bottom row. Each cell represents one route. The unit shipping cost (Cij) is shown in the upper right corner of the cell, the amount of shipped material is shown in the centre of the cell. The transportation tableau implicitly expresses the supply and demand constraints and the shipping cost between each demand and supply point.

+ Table 1.0: THE TRANSPORTATION TABLEAU

Source supply

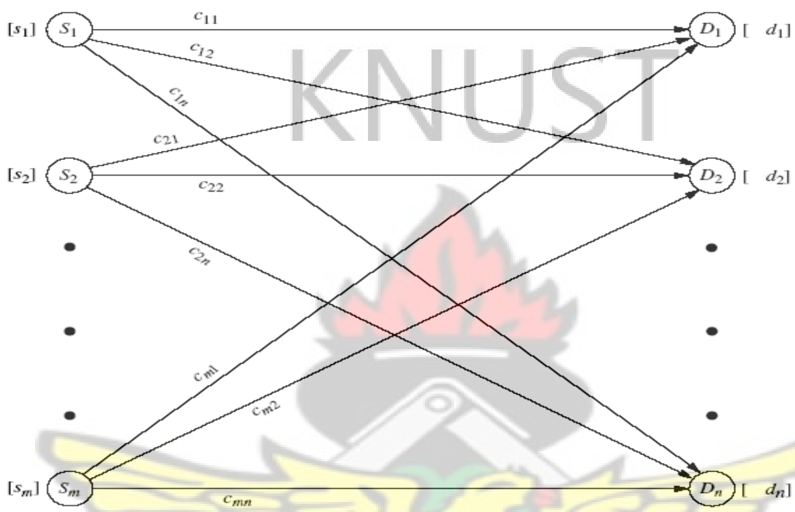
To Destination→ ↓From Source↓	D_1	D_2	…Dj…	D_n	Source Supply
S_1	c_{11} x_{11}	c_{12} x_{12}		c_{1n} x_{1n}	a_1
S_2	c_{21} x_{21}	c_{22} x_{22}		c_{2n} x_{2n}	a_2
$\ldots S_i \ldots$			c_{ij} x_{ij}		$\ldots a_i \ldots$
S_m	c_{m1} x_{m1}	c_{m2} x_{m2}		c_{mn} x_{mn}	a_m
Destination Requirements	b_1	b_2	$\ldots b_j \ldots$	b_m	$\sum a_i$ / $\sum b_j$

3.5 NETWORK REPRESENTATION OF TRANSPORTATION PROBLEM

Graphically, transportation problem is often visualized as a network with m source nodes, n sink nodes, and a set of m.n "directed arcs" This is depicted in Fig 1.

Figure .1 Network representation of the transportation problem

Unit of Supply(Si) Units of Demand(di)



In the diagram there are $S_1 \ldots S_n$ sources and $D_1 . \ldots D_n$ destination. The arrows show flows of output from source to destination .Each destination is linked to each source by an arrow. The number $C_1 \ldots C_n$ above each arrow represents the cost of transporting on that route.

Problems with the above structure arise in many applications. For example, the sources could represent warehouses and the sinks could represent retail.

DEGENCY IN TRANSPORTATION PROBLEM

Degeneracy exists in a transportation problem when the number of filled cells is less than the number of rows plus the number of columns minus one (m + n - 1). Degeneracy may be observed either during the initial allocation when the first entry in a row or column satisfies both the row and column requirements or during the Stepping stone method application, when the added and subtracted values are equal.

Transportation with ***m***-origins and ***n***-destinations can have **m+n-1** positive basic variables, otherwise the basic solution degenerates. So whenever the number of basic cells is less than ***m + n-1,*** the transportation problem is degenerate.

To resolve the degeneracy, the positive variables are augmented by as many zero-valued variables as is necessary to complete ***m +n –1*** basic variable.

The Initial Basic Feasible Solution (BFS)

Let us consider a T.P involving m origins and n destinations.

Since the sum of origin capacities equals the sum of destination requirements, a feasible solution always exists. Any feasible solution satisfying m + n – 1 of the m + n constraints is a redundant one and hence can be deleted. This also means that a feasible solution to a T.P can have at the most only

m + n – 1 strictly positive component, otherwise the solution will degenerate.

It is always possible to assign an initial feasible solution to a T.P. in such a manner that the rim requirements are satisfied. This can be achieved either by inspection or by following some simple

rules. We begin by imagining that the transportation table is blank i.e. initially all xij = 0. The simplest procedures for initial allocation discussed in the following section.

Feasible Solution (F.S.)

A set of non-negative allocations xij > 0 which satisfies the

row and column restrictions is known as feasible solution.

Basic Feasible Solution (B.F.S.)

A feasible solution to a m-origin and n-destination problem is

said to be basic feasible solution if the number of positive

Allocations are (m+n–1).

If the number of allocations in a basic feasible solutions are

less than (m+n–1), it is called degenerate basic feasible solution

(DBFS) (Otherwise non-degenerate).

Optimal Solution

A feasible solution (not necessarily basic) is said to be optimal

if it minimizes the total transportation cost.

Cell: It is a small compartment in the transportation tableau. *Circuit:* A circuit is a sequence of cells (in the balanced transportation tableau) such that

(i) It starts and ends with the same cell.

(ii) Each cell in the sequence can be connected to the next member by a horizontal or vertical line in the tableau.

Allocation: The number of units of items transported from a source to a destination which is recorded in a cell in the transportation tableau.

Basic Variables: The variables in a basic solution whose values are obtained as the simultaneous solution of the system of equations that comprise the functional constraints

3.6 Solution for a transportation problem

3.6.1 Flow Chart Solution For the transportation Problem

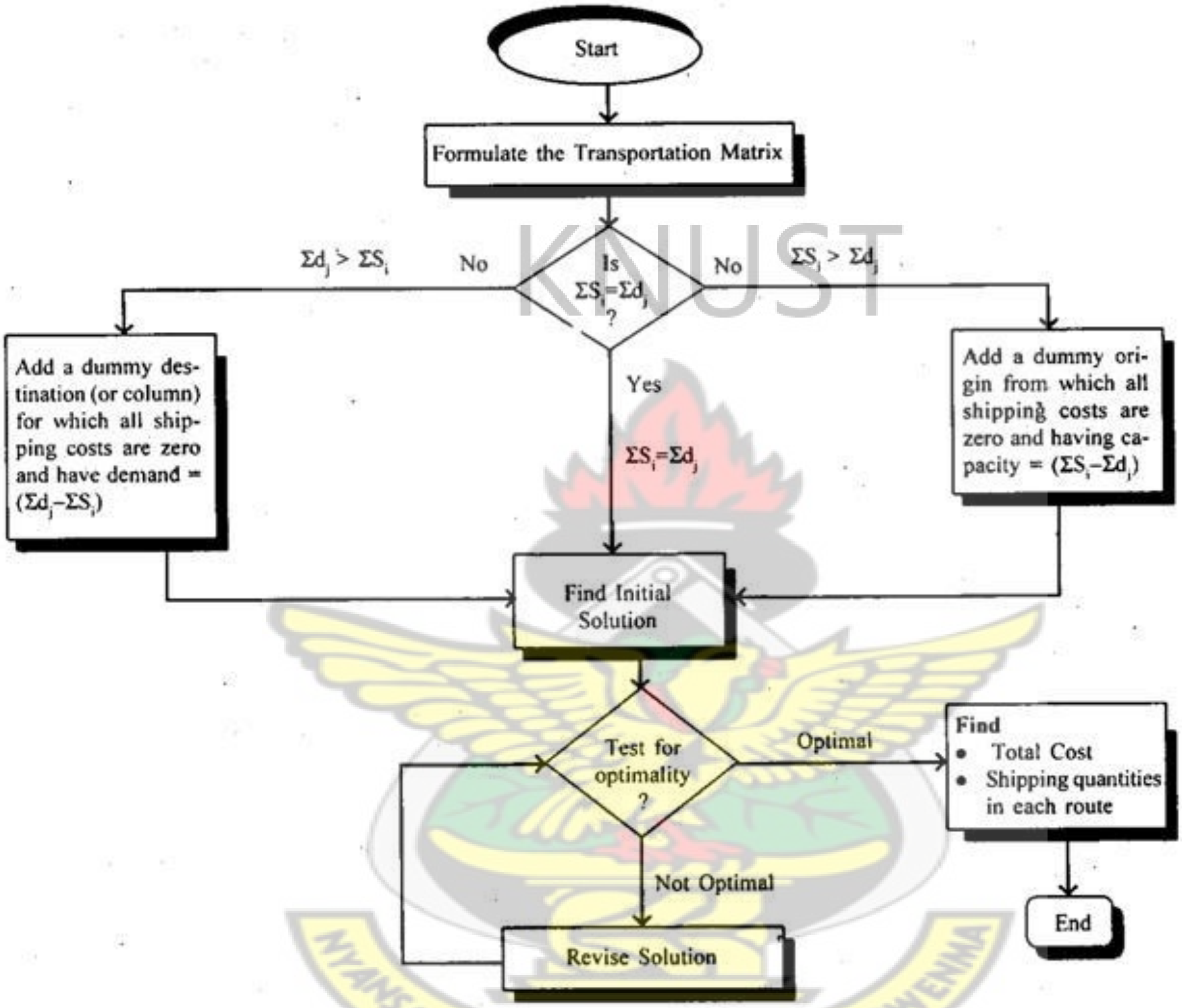


Figure 2: The flow chart showing the transportation problem approach

Summary description of the Flow chart

1. First the problem is formulated as transportation matrix.

2. Check weather is a balance transportation model?

3. If not balance add a dummy to either the supply or the demand to balance the transportation model**.**

4. Find the initial solution of the transportation problem.

5. Check whether the solution is optimized?

If the solution is not optimize Go to 4.

6. When optimal solution is obtained

8. We compute the total transportation cost and also shipped the respective quantity demand to its route.

3.6.2 Solution Algorithm For the transportation Problem

Transportation models do not start at the origin where all decision values are zero; they must instead be given an initial feasible solution

The solution algorithm to a transpiration problem can be summarized into following steps:

Step 1. Formulate the problem and set up in the matrix form.

The formulation of transportation problem is similar to LP problem formulation. Here the objective function is the total transportation cost and the constraints are the supply and demand available at each source and destination, respectively.

Step 2. Obtain an initial basic feasible solution.

This initial basic solution can be obtained by using any of the following methods:

i. North West Corner Rule
ii. Matrix Minimum(Least Cost) Method
iii. Vogel Approximation Method

The solution obtained by any of the above methods must fulfil the following conditions:

i. The solution must be feasible, i.e., it must satisfy all the supply and demand constraints. This is called RIM CONDITION**.**

ii. The number of positive allocation must be equal to m + n – 1, where, m is number of rows and n is number of columns

The solution that satisfies the above mentioned conditions are called a non-degenerate basic feasible solution.

Step 3. Test the initial solution for optimality.

Using any of the following methods can test the optimality of obtained initial basic solution:

i. Stepping Stone Method

ii. Modified Distribution Method (MODI)

If the solution is optimal then stop, otherwise, determine a new improved solution.

Step 4. Updating the solution

Repeat Step 3 until the optimal solution is arrived at.

3.6. 3 FINDING INITIAL BASIC FEASIBLE SOLUTION OF BALANCED TRANSPORTATION PROBLEMS

3.6.3.1 Northwest Corner Method (NWC)

The North West corner rule is a method for computing a basic feasible solution of a transportation problem where the basic variables are selected from the North – West corner (i.e., top left corner).

The method starts at the northwest-corner cell (route)

The major advantage of the north–west corner rule method is that it is very simple and easy to apply. Its major disadvantage, however, is that it is not sensitive to costs and consequently yields poor initial solutions

The Northwest Corner Method Summary of Steps

1. Allocate as much as possible to the cell in the upper left-hand corner, subject to the supply and demand conditions.

2. Allocate as much as possible to the next adjacent feasible cell.

3. Repeat step 2 until all rim requirements are met

ILLUSTRATIVE EXAMPLE 1 ON TRANSPORTATION PROBLEM

In this tableau the decision variable Xij , represent the number of tons of wheat transported from each grain elevator, i(where i=1,2,3) ,to each mill, j(where j= A,B,C). The objective function represents the total transportation cost for each route. Each term in the objective function reflects the cost of the tonnage transported for one route. The problem is to determine how many tons of wheat of transport from each grain elevator to each mill on monthly basis in order to minimize the total cost of transportation

ILLUSTRATIVE EXAMPLE1

<u>Table 2.0: A Balance transportation Problem</u>

From \ To	A	B	C	Supply
1	6	8	10	150
2	7	11	11	175
3	4	5	12	275
Demand	200	100	300	600

METHOD OF SOLUTIONS TO BALANCE PROBLEM USING NORTH WEST CORNER METHOD.

<u>Table2.1 .THE INTIAL NORTH WEST CORNER SOLUTION</u>

- In the northwest corner method the largest possible allocation is made to the cell in the upper left-hand corner of the tableau, followed by allocations to adjacent feasible cells.

From \ To	A	B	C	Supply
1	6 150	8	10	150
2	7 50	11 100	11 25	175
3	4	5	12 275	275
Demand	200	100	300	600

The Initial NW Corner Solution

This transportation tableau has:

The total supply= 200+100+300=600Units

The total supply=150+175+275= 600 units

Hence the tableau is balance

We first allocate as much as possible to cell 1A(northwest corner).this amount is 150 tons, since that is the maximum that can be supplied by grain 1 , even though 200 tons are demanded by mill A . This initial allocation, in this initial allocation is shown in Table 2. We next allocate to cell adjacent to cell 1A, in this case either cell 2A or cell 1B. However, cell 1B no longer represents a feasible allocation, because the total tonnage of wheat available at source 1 (i.e. 150tons) has been allocated. Thus, cell 2A represents the only feasible alternative, and as much as possible is allocated to this cell. The amount allocated at 2A can be either 175 tons, the supply available from source 2, or 50 tons, the amount now demanded at destination A. Because 50 tons is the most constrained amount, it is allocated to cell 2A. As shown in table 2. The third allocation is made in the same way as the second allocation. The only feasible cell adjacent to cell 2A is cell 2B. The most that can be allocated is either 100 tons(the amount demanded at mill B) or 125 tons(175 tons minus the 50 tons allocated to cell2A).the smaller(most constrained) amount, 100 tons, is allocated to cell 2B, as shown in Table 2.

The fourth allocation is 25 tons to cell 2C, and the fifth allocation is 275 tons to cell 3C, both of which are shown in Table 2.1

Testing for Optimality

The allocations made by the method is BFS since (m + n – 1) = 3 +3 – 1 = 5, which equals the number of allocations made.

Since the number of occupied cell 5 is equal (3+3-1),

The condition is satisfied

The initial solution is complete when all rim requirements are satisfied.

The starting solution (consisting of 4 basic variables) is

X 2A=50 tons,

X2B=100 tons,

X2C=25 tons

X3C=275 tons

Transportation cost is computed by evaluating the objective function:

$$Z = \$6x_{1A} + 8x_{1B} + 10x_{1C} + 7x_{2A} + 11x_{2B} + 11x_{2C} + 4x_{3A} + 5x_{3B} + 12x_{3C}$$

$$= 6(150) + 8(0) + 10(0) + 7(50) + 11(100) + 11(25) + 4(0) + 5(0) +!2(275)$$

$$= \$5,925$$

3.6.3.2 The Minimum Cell Cost (Least cost) Method

Matrix minimum method is a method for computing a basic feasible solution of a transportation problem where the basic variables are chosen according to the unit cost of transportation.

The minimum-cost method finds a better starting solution by concentrating on the cheapest routes. The method starts by assigning as much as possible to the cell with the smallest unit cost. Next, the satisfied row or column is crossed out and the amounts of supply and demand are adjusted accordingly. If both a row and a column are satisfied simultaneously, only one is crossed out, the same as in the northwest –corner method. Next, look for the uncrossed-out cell with the smallest unit cost and repeat the process until exactly one row or column is left uncrossed out.

Steps

1. Identify the box having minimum unit transportation cost (C_{ij}).

2. If there are two or more minimum costs, select the row and the column corresponding to the lower numbered row.

3. If they appear in the same row, select the lower numbered column.

4. Choose the value of the corresponding X_{ij} as much as possible subject to the capacity and requirement constraints.

5. If demand is satisfied, delete the column.

6. If supply is exhausted, delete the row.

7. Repeat steps 1-6 until all restrictions are satisfied.

In the minimum cell cost method as much as possible is allocated to the cell with the minimum cost followed by allocation to the feasible cell with minimum cost

APPLICATION OF LEAST COST METHODS TO ILLUSTRATIVE EXAMPLE 1 OF BALANCED TRANSPORTATION PROBLEM

Table 2.2 The starting solution using Minimum Cell Method

- In the minimum cell cost method as much as possible is allocated to the cell with the minimum cost followed by allocation to the feasible cell with minimum cost.

To / From	A	B	C	Supply
1	6	8	10	150
2	7	11	11	175
3	4 200	5	12	275
Demand	200	100	300	600

Table 2.3. The Second Minimum Cell Cost Allocation

To / From	A	B	C	Supply
1	6	8	10	150
2	7	11	11	175
3	4 200	5 75	12	275
Demand	200	100	300	600

Table 2.4: The starting solution using Minimum Cell Method

The complete initial minimum cell cost solution; total cost = $4,550.

The minimum cell cost method will provide a solution with a lower cost than the northwest corner solution because it considers cost in the allocation process.

To / From	A	B	C	Supply
1	6	8 25	10 125	150
2	7	11	11 175	175
3	4 200	5 75	12	275
Demand	200	100	300	600

The Minimum Cell Cost Method Summary of Steps

1. Allocate as much as possible to the feasible cell with the minimum transportation cost, and adjust the rim requirements.

2. Repeat step 1 until all rim requirements have been met

3.6.3 .3 Vogel's Approximation Method (VAM)

VAM is an improved version of the least-cost method that generally, but not always, produces better starting solutions. VAM is based upon the concept of minimizing opportunity (or penalty) costs. The opportunity cost for a given supply row or demand column is defined as the difference between the lowest cost and the next lowest cost alternative. This method is preferred over the methods discussed above because it generally yields, an optimum, or close to optimum, starting solutions. Consequently, if we use the initial solution obtained by VAM and proceed to solve for the optimum solution, the amount of time required to arrive at the optimum solution is greatly reduced. The steps involved in determining an initial solution using VAM are as follows: The steps involved in determining an initial solution using VAM are as follows:

Step1. Write the given transportation problem in tabular form (if not given).

Step2. Compute the difference between the minimum cost and the next minimum cost corresponding to each row and each column which is known as penalty cost.

Step3. Choose the maximum difference or highest penalty cost. Suppose it corresponds to the i^{th} row. Choose the cell with minimum cost in the i^{th} row. Again if the maximum corresponds to a column, choose the cell with the minimum cost in this column.

Step4. Suppose it is the $(i, j)^{th}$ cell. Allocate min (a_i, b_j) to this cell. If the min $(a_i , b_j) = a_i$, then the availability of the i^{th} origin is exhausted and demand at the j^{th} destination remains as b_j-a_i and the i^{th} row is deleted from the table. Again if min $(a_i, b_j) = b_j$, then demand at the j^{th}

destination is fulfilled and the availability at the i^{th} origin remains to be a_i-b_j and the j^{th} column is deleted from the table.

Step5. Repeat steps 2, 3, 4 with the remaining table until all origins are exhausted and all demands are fulfilled.

- Method is based on the concept of *penalty cost* or *regret*.
- A penalty cost is the difference between the largest and the next largest cell cost in a row (or column).
- In VAM the first step is to develop a penalty cost for each source and destination.
- Penalty cost is calculated by subtracting the minimum cell cost from the next higher cell cost in each row and column.

Vogel's Approximation Method (VAM) Summary of Steps

1. Determine the penalty cost for each row and column.
2. Select the row or column with the highest penalty cost.
3. Allocate as much as possible to the feasible cell with the lowest transportation cost in the row or column with the highest penalty cost.
4. Repeat steps 1, 2, and 3 until all rim requirements have been met

APPLICATION OF VOLGEL'S APPROXIMATION METHOD TO ILLUSTRATIVE EXAMPLE 1 ON BALANCE TRANSFORMATION PROBLEM.

Table 2.5:The VAM Penalty Costs

From \ To	A	B	C	Supply	
1	6	8	10	150	2
2	7	11	11	175	4
3	4	5	12	275	1
Demand	200	100	300	600	
	2	3	1		

Table 2.6: The Initial VAM Allocation

- VAM allocates as much as possible to the minimum cost cell in the row or column with the largest penalty cost.

From \ To	A	B	C	Supply	
1	6	8	10	150	2
2	7 175	11	11	175	
3	4	5	12	275	1
Demand	200	100	300	600	
	2	3	2		

Table 2. 7:The Second VAM Allocation

After each VAM cell allocation, all row and column penalty costs are recomputed.

From \ To	A	B	C	Supply	
1	6	8	10	150	4
2	7 175	11	11	175	
3	4	5 100	12	275	8
Demand	200	100	300	600	
	2		2		

Table 2.8: The Third VAM Allocation

Recomputed penalty costs after the third allocation.

From \ To	A	B	C	Supply
1	6	8	10 150	150
2	7 175	11	11	175
3	4 25	5 100	12 150	275
Demand	200	100	300	600

<u>Table 2.9:</u> <u>The Initial VAM Solution</u>

- The initial VAM solution; total cost = $5,125
- VAM and minimum cell cost methods both provide better initial solutions than does the northwest corner method

To / From	A	B	C	Supply
1	[6]	[8]	[10] 150	150
2	[7] 175	[11]	[11]	175
3	[4] 25	[5] 100	[12] 150	275
Demand	200	100	300	600

3.5.4 METHODS FOR SOLVING TRANSPORTATION PROBLEMS TO OPTIMALITY

3.5.4 .1 AN OPTIMAL SOLUTION

To obtain an optimal solution by making successive improvements to initial basic feasible solution until no further decrease in the transportation cost is possible. An optimal solution is one where there is no other set of transportation routes that will further reduce the total transportation cost. Thus, we have to evaluate each unoccupied cell in the transportation table in terms of an opportunity of reducing total transportation cost. An unoccupied cell with the largest negative opportunity cost is selected to include in the new set of transportation routes (allocations). This value indicates the per unit cost reduction that can be achieved by raising the shipment allocation in the unoccupied cell from its present level of zero. This is also known as an incoming cell (or variable). The outgoing cell (or variable) in the current solution is the occupied cell (basic variable) in the unique closed path (loop) whose allocation will become zero first as more units are allocated to the unoccupied cell with largest negative opportunity cost. That is, the current solution cannot be improved further. This is the optimal solution.

The widely used methods for finding an optimal solution are:

- Stepping stone method (not to be done).
- Modified Distribution (MODI) method.

They differ in their mechanics, but will give exactly the same results and use the same testing strategy.

5. To develop the improved solution, if it is not optimal. Once the improved solution has been obtained, the next step is to go back to 3.

Note. Although the transportation problem can be solved using the regular simplex method, its special properties provide a more convenient method for solving this type of problems. This method is based on the same theory of simplex method. It makes use, however, of some shortcuts which provide a less burdensome computational scheme. There is one difference between the two methods. The simplex method performs the operations on a simplex table. The transportation method performs the same operations on a transportation table.

APPLICATION OF STEPPING STONE METHOD TO ILLUSTRATIVE EXAMPLE 1ON BALANCE TRANSFORMATION PROBLEM

3.5.3 .2 The Stepping-Stone Solution Method

Table 2.10: The Minimum Cell Cost Solution

- Once an initial solution is derived, the problem must be solved using either the stepping-stone method or the modified distribution method (MODI).

- The initial solution used as a starting point in this problem is the minimum cell cost method solution because it had the minimum total cost of the three methods used

From \ To	A	B	C	Supply
1	[6]	[8] 25	[10] 125	150
2	[7]	[11]	[11] 175	175
3	[4] 200	[5] 75	[12]	275
Demand	200	100	300	600

Table 2.11: The Allocation of One Ton from Cell 1A

The stepping-stone method determines if there is a cell with no allocation that would reduce cost if used

From \ To	A	B	C	Supply	
1	[6]	[8] 25	[10] 125	150	151
2	[7]	[11]	[11] 175	175	
3	[4] 200	[5] 75	[12]	275	
Demand	200	100	300	600	

Table 2.12: The Subtraction of One Ton from Cell 1B

- Must subtract one ton from another allocation along that row.

To / From	A	B	C	Supply
1	+1 6	−1 8 25	10 125	150
2	7	11	11 175	175
3	4 200	5 75	12	275
Demand	200	100	300	600
		99		

Table 2.13: The Addition of One Ton to Cell 3B and the Subtraction of One Ton from Cell 3A

- A requirement of this solution method is that units can only be added to and subtracted from cells that already have allocations, thus one ton must be added to a cell as shown.

To / From	A	B	C	Supply
1	+1 6	−1 8 25	10 125	150
2	7	11	11 175	175
3	−1 4 200	+1 5 75	12	275
Demand	200	100	300	600

Table 2.14: The Stepping-Stone Path for Cell 2A

An empty cell that will reduce cost is a potential entering variable.

- To evaluate the cost reduction potential of an empty cell, a closed path connecting used cells to the empty cells is identified.

From \ To	A	B	C	Supply
1	$\boxed{6}$	− $\boxed{8}$ 25	+ $\boxed{10}$ 125	150
2	+ $\boxed{7}$	$\boxed{11}$	− $\boxed{11}$ 175	175
3	− $\boxed{4}$ 200	+ $\boxed{5}$ 75	$\boxed{12}$	275
Demand	200	100	300	600

2A → 2C → 1C → 1B → 3B → 3A
+ \$7 – 11 + 10 – 8 + 5 – 4 = –\$1

Table 2.15: The Stepping-Stone Path for Cell 2B

The remaining stepping-stone paths and resulting computations for cells 2B and 3C

From \ To	A	B	C	Supply
1	$\boxed{6}$	+ $\boxed{8}$ 25	− $\boxed{10}$ 125	150
2	$\boxed{7}$	− $\boxed{11}$	+ $\boxed{11}$ 175	175
3	$\boxed{4}$ 200	$\boxed{5}$ 75	$\boxed{12}$	275
Demand	200	100	300	600

2B → 2C → 1C → 1B
+ \$11 – 11 + 10 – 8 = +\$2

Table 2.16:The Stepping-Stone Path for Cell 3C

From \ To	A	B	C	Supply
1	6	+ 8 25	− 10 125	150
2	7	11	11 175	175
3	4 200	− 5 75	+ 12	275
Demand	200	100	300	600

3C → 1C → 1B → 3B
+ $12 – 10 + 8 – 5 = +$5

Table 2.17: The Stepping-Stone Path for Cell 1A

- After all empty cells are evaluated, the one with the greatest cost reduction potential is the entering variable.
- A tie can be broken arbitrarily

From \ To	A	B	C	Supply
1	+ 6	− 8 25	10 125	150
2	7	11	11 175	175
3	− 4 200	+ 5 75	12	275
Demand	200	100	300	600

Table2.18: The Second Iteration of the Stepping-Stone Method

- When reallocating units to the entering variable (cell), the amount is the minimum amount subtracted on the stepping-stone path.

  - At each iteration one variable enters and one leaves (just as in the simplex method).

From \ To	A	B	C	Supply
1	6 25	8	10 125	150
2	7	11	11 175	175
3	4 175	5 100	12	275
Demand	200	100	300	600

Table 2.19: The Stepping-Stone Path for Cell 2A

Check to see if the solution is optimal.

From \ To	A	B	C	Supply
1	− 6 25	8	+ 10 125	150
2	+ 7	11	− 11 175	175
3	4 175	5 100	12	275
Demand	200	100	300	600

2A → 2C → 1C → 1A
+ \$7 – 11 + 10 – 6 = \$0

Table 2.20: The Stepping-Stone Path for Cell 1B

To / From	A	B	C	Supply
1	− 6 / 25	+ 8	10 / 125	150
2	7	11	11 / 175	175
3	+ 4 / 175	− 5 / 100	12	275
Demand	200	100	300	600

1B → 3B → 3A → 1A
+ $8 – 5 + 4 – 6 = +$1

Table 2.21: The Stepping-Stone Path for Cell 2B

Continuing check for optimality

.

To / From	A	B	C	Supply
1	− 6 / 25	8	+ 10 / 125	150
2	7	+ 11	− 11 / 175	175
3	+ 4 / 175	− 5 / 100	12	275
Demand	200	100	300	600

2B → 3B → 3A → 1A → 1C → 2C
+ $11 – 5 + 4 – 6 + 10 – 11 = +$3

Table 2.22 : The Stepping-Stone Path for Cell 3C

From \ To	A	B	C	Supply
1	+ (6) 25	(8)	− (10) 125	150
2	(7)	(11)	(11) 175	175
3	− (4) 175	(5) 100	+ (12)	275
Demand	200	100	300	600

$3C \rightarrow 3A \rightarrow 1A \rightarrow 1C$
$+\$12 - 4 + 6 - 10 = +\$4$

- The stepping-stone process is repeated until none of the empty cells will reduce costs (i.e., an optimal solution).

- In example, evaluation of four paths indicates no cost reductions; therefore Table 16 solution is optimal.

- Solution and total minimum cost:

x_{1A} = 25 tons,

x_{2C} = 175 tons,

x_{3A} = 175 tons,

x_{1C} = 125 tons,

x_{3B} = 100 tons

$Z = \$6(25) + 8(0) + 10(125) + 7(0) + 11(0) + 11(175) + 4(175) + 5(100) + 12(0)$

$= \$4{,}525$

Table2.23: The Alternative Optimal Solution

- A multiple optimal solution occurs when an empty cell has a cost change of zero and all other empty cells are positive.

- An alternate optimal solution is determined by allocating to the empty cell with a zero cost change.

- Alternate optimal total minimum cost also equals $4,525

From \ To	A	B	C	Supply
1	[6]	[8]	[10] 150	150
2	[7] 25	[11]	[11] 150	175
3	[4] 175	[5] 100	[12]	275
Demand	200	100	300	600

The Stepping-Stone Solution Method Summary

1. Determine the stepping-stone paths and cost changes for each empty cell in the tableau.

2. Allocate as much as possible to the empty cell with the greatest net decrease in cost.

3. Repeat steps 1 and 2 until all empty cells have positive cost changes that indicate an optimal solution.

3.6.4.3 The Modified Distribution Method (MODI)

- MODI is a modified version of the stepping-stone method in which math equations replace the stepping-stone paths.

Step 1: Under this method we construct penalties for rows and columns by subtracting the least value of row / column from the next least value.

Step 2: We select the highest penalty constructed for both row and column. Enter that row / column and select the minimum cost and allocate min (a_i, b_j)

Step 3: Delete the row or column or both if the rim availability / requirements is met.

Step 4: We repeat steps 1 to 2 to till all allocations are over.

Step 5: For allocation all form equation $u_i + v_j = c_j$ set one of the dual variable u_i / v_j to zero and solve for others.

Step 6: Use this value to find $D_{ij} = c_{ij}-u_i-v_j$ of all D_{ij} ^{3}, then it is the optimal solution.

APPLICATION OF MODIFIED DISTRIBUTION METHOD TO ILLUSTRATIVE EXAMPLE 1 ON BALANCE TRANSFORMATION PROBLEM

Table 2.24: The Minimum Cell Cost Initial Solution

- In the table, the extra left-hand column with the u_i symbols and the extra top row with the v_j symbols represent values that must be computed.

- Computed for all cells with allocations:

$u_i + v_j = c_{ij}$ = unit transportation cost for cell $_{ij}$.

	v_j	$v_A =$	$v_B =$	$v_C =$	
u_i	From \ To	A	B	C	Supply
$u_1 =$	1	[6]	[8] 25	[10] 125	150
$u_2 =$	2	[7]	[11]	[11] 175	175
$u_3 =$	3	[4] 200	[5] 75	[12]	275
	Demand	200	100	300	600

Table 2.25: The Initial Solution with All ui and vj Values

Formulas for cells containing allocations:

x_{1B}: $u_1 + v_B = 8$

x_{1C}: $u_1 + v_C = 10$

x_{2C}: $u_2 + v_C = 11$

x_{3A}: $u_3 + v_A = 4$

x_{3B}: $u_3 + v_B = 5$

	v_j	$v_A = 7$	$v_B = 8$	$v_C = 10$	
u_i	From \ To	A	B	C	Supply
$u_1 = 0$	1	[6]	[8] 25	[10] 125	150
$u_2 = 1$	2	[7]	[11]	[11] 175	175
$u_3 = -3$	3	[4] 200	[5] 75	[12]	275
	Demand	200	100	300	600

Table 2.26: The Initial Solution with All ui and vj Values

- Five equations with 6 unknowns therefore let $u_1 = 0$ and solve to obtain:

$$v_B = 8,\ v_C = 10,\ u_2 = 1,\ u_3 = -3,\ v_A = 7$$

- Each MODI allocation replicates the stepping-stone allocation.

- Use following to evaluate all empty cells:

$$c_{ij} - u_i - v_j = k_{ij}$$

Where k_{ij} equals the cost increase or decrease that would occur by allocating to a cell.

For the empty cells in Table 26:

x_{1A}: $k_{1A} = c_{1A} - u_1 - v_A = 6 - 0 - 7 = -1$

x_{2A}: $k_{2A} = c_{2A} - u_2 - v_A = 7 - 1 - 7 = -1$

x_{2B}: $k_{2B} = c_{2B} - u_2 - v_B = 11 - 1 - 8 = +2$

x_{3C}: $k_{3C} = c_{3C} - u_3 - v_C = 12 - (-3) - 10 = +5$

Table 2.27: The Second Iteration of the MODI Solution Method

After each allocation to an empty cell, the u_i and v_j values must be recomputed

	v_j	$v_A =$	$v_B =$	$v_C =$	
u_i	From \ To	A	B	C	Supply
$u_1 =$	1	[6] 25	[8]	[10] 125	150
$u_2 =$	2	[7]	[11]	[11] 175	175
$u_3 =$	3	[4] 175	[5] 100	[12]	275
	Demand	200	100	300	600

The Second Iteration of the MODI Solution Method

Table 2.28: The New ui and vj Values for the Second Iteration

Recomputing u_i and v_j values:

x_{1A}: $u_1 + v_A = 6, v_A = 6$

x_{1C}: $u_1 + v_C = 10, v_C = 10$

x_{2C}: $u_2 + v_C = 11, u_2 = 1$

x_{3A}: $u_3 + v_A = 4, u_3 = -2$

x_{3B}: $u_3 + v_B = 5, v_B = 7$

	v_j	$v_A = 6$	$v_B = 7$	$v_C = 10$	
u_i	From \ To	A	B	C	Supply
$u_1 = 0$	1	[6] 25	[8]	[10] 125	150
$u_2 = 1$	2	[7]	[11]	[11] 175	175
$u_3 = -2$	3	[4] 175	[5] 100	[12]	275
	Demand	200	100	300	600

The New ui and vj Values for the Second Iteration

- Cost changes for the empty cells, $c_{ij} - u_i - v_j = k_{ij}$;

  x_{1B}: $k_{1B} = c_{1B} - u_1 - v_B = 8 - 0 - 7 = +1$

  x_{2A}: $k_{2A} = c_{2A} - u_2 - v_A = 7 - 1 - 6 = 0$

  x_{2B}: $k_{2B} = c_{2B} - u_2 - v_B = 11 - 1 - 7 = +3$

  x_{3C}: $k_{2B} = c_{2B} - u_3 - v_C = 12 - (-2) - 10 = +4$

- Since none of the values are negative, solution obtained is optimal.

- Cell 2A with a zero cost change indicates a multiple optimal solution.

The Modified Distribution Method (MODI) Summary of Steps

1. Develop an initial solution.
2. Compute the u_i and v_j values for each row and column.
3. Compute the cost change, k_{ij}, for each empty cell.
4. Allocate as much as possible to the empty cell that will result in the greatest net decrease in cost (most negative k_{ij})
5. Repeat steps 2 through 4 until all k_{ij} values are positive or zero.

3.6 Solving transportation problems with mixed constraints

A heuristic algorithm for solving transportation problems with mixed

Constraints and extend the algorithm to find a more-for-less (MFL) solution, if one exists. Though many transportation problems in real life have mixed constraints, these problems are not addressed in the literature because of the rigor required to solve these problems optimally. The proposed algorithm builds on the initial solution of the transportation problem.

Much effort has been concentrated on transportation problems (TP) with equality constraints. In real life, however, most problems have mixed constraints accommodating many applications that go beyond transportation related problems to include job scheduling, production inventory, production distribution, allocation problems, and investment analysis.

The more-for-less (MFL) paradox in a TP occurs when it is possible to ship more total goods for less (or equal) total cost, while shipping the same amount or more from each origin and to each destination and keeping all the shipping costs non-negative. The information of the occurrence of an MFL situation is useful to a manager in deciding which warehouse or plant capacities are to be increased, and which markets should be sought. It could also be a useful tool in analyzing and planning company acquisition, mergers, consolidations and downsizes. The so called MFL paradox in the transportation paradox has been covered from a theoretical stand point by Charnes and Klingman, and Charnes et al. Robb provides an intuitive explanation of the transportation occurrence. Adlakha and Kowalski, Adlakha et al. have given an algorithm for solving paradoxical situation in linear transportation problem.

3.6.1 MATHEMATICAL MODEL FOR THE TRANSPORTATION PROBLEM WITH MIXED CONSTRAINTS:

Minimize Z= $\sum_{i-1}^{m}\sum_{j=1}^{n} C_{ij} X_{ij}$

Subject to

$$\sum_{j=1}^{n} X_{ij} \geq a_i \qquad i \in U \qquad (1)$$

$$\sum_{j=1}^{n} X_{ij} \leq a_i \qquad i \in V \qquad (2)$$

$$\sum_{j=1}^{n} X_{ij} = a_i \qquad i \in W \qquad (3)$$

$$\sum_{i=1}^{m} X_{ij} \geq b_j \qquad j \in Q \qquad (4)$$

$$\sum_{i=1}^{m} X_{ij} \leq b_j \qquad j \in T \qquad (5)$$

$$\sum_{i=1}^{m} X_{ij} = b_j \qquad j \in S \qquad (6)$$

$$x_{ij} \geq 0 \qquad (i, j) \in I * j$$

Where $a_i > 0, \forall_i \in I$; $b_j > 0, \forall_j \in J$

I= the index set of supply points= $\{1,2,3...,m\}$

J= the index set of destination= $\{1,2,3...,n\}$

c_{ij} = the unit cost of transportation from the ith supply point to jth destination

where U, V and W are pairwise disjoint subsets of { 1,2,3,...,m } such that U $\cup$ V $\cup$ W = {1, 2, 3... m}; Q, T and S are pairwise disjoint subsets of { 1,2,3,...,n } such that $Q \cup T \cup S$ = { 1,2,3,...,n }; c_{ij} is the cost of shipping one unit from supply point i to the demand point j ; a_i is the supply at supply point i ; b_i is the demand at demand point j and X_{ij} is the number of units shipped from supply point i to demand point j.

Now, the LBP (least bound problem) for the problem (P) is given below:

Minimize Z= $\sum_{i-1}^{m}\sum_{j=1}^{n} C_{ij} X_{ij}$

$$\sum_{j=1}^{n} X_{ij} = a_i \quad , \ i \in U \qquad (1)$$

$$\sum_{j=1}^{n} X_{ij} = 0 \quad , \ i \in V \qquad (2)$$

$$\sum_{j=1}^{n} X_{ij} = a_i \quad , \ i \in W \qquad (3)$$

$$\sum_{i=1}^{m} X_{ij} = b_j \qquad j \in Q \qquad (4)$$

$$\sum_{i=1}^{m} X_{ij} = 0 \qquad j \in T \qquad (5)$$

$$\sum_{i=1}^{m} X_{ij} = b_j \qquad j \in S \qquad (6)$$

$X_{ij} x \geq 0$, $i = 1,2,...,m$ and $j= 1,2,...,n$ and integer (7)

REMARKS: Asharm proved that the existence of a MFL situation in a regular TP requires only one condition namely, the existence of a location with negative plant to-market shipping shadow price. The shadow prices are easily calculated from the solution of the TP with mixed constraints.

The MFL solution is obtained from the optimal solution distribution by increasing and decreasing the shipping quantities while maintaining the minimum requirements for both supply and demand.

The plant-to-market shipping shadow price (also called Modi index) at a cell (i, j) is ui + vj where ui and vj are shadow prices corresponding to the cell (i, j) .

The negative Modi index at a cell (i, j) indicates that we can increase the ith plant capacity / the demand of the jth market at the maximum possible level.

3.6.2 Zero point method

We, now introduce a new method called the zero point method for finding an optimal solution to a transportation problem with mixed constraints in a single stage.

The zero point method proceeds as follows.

Step 1. Construct the transportation table for the given TP with mixed constraints.

Step 2. Subtract each row entries of the transportation table from the row minimum and then subtract each column entries of the resulting transportation table after using the Step 1 from the column minimum.

Step 3. Check if each column demand can be *accomplished* from the *joint* of row supplies whose reduced costs in that column are zero. Also, check if each row supply can be *accomplished* from the *joint* of column demands whose reduced costs in that row are zero. If so, go to Step 6. (Such reduced transportation table is called the allotment table). If not, go to Step 4.

Step 4. Draw the minimum number of horizontal lines and vertical lines to cover all the zeros of the reduced transportation table such that some entries of row(s) or / and column(s) which do not satisfy the condition of the Step3. are not covered.

Step 5: Develop the new revised reduced transportation table as follows:

(i) Find the smallest entry of the reduced cost matrix not covered by any lines.

(ii) Subtract this entry from all the uncovered entries and add the same to all entries lying at the intersection of any two lines.

and then, go to Step 3.

Step 6: Select a cell in the reduced transportation table whose reduced cost is the maximum cost. Say (α ,β) . If there is more than one, then select any one.

Step 7: Select a cell in the α -row or/ and β − column of the reduced transportation table which is the only cell whose reduced cost is zero and then, allot the maximum possible to that cell such that its row or its column condition is satisfied. If such cell does not occur for the maximum value,

Theorem 1. Any optimal solution to the problem (P1) where

(P1) Minimize $\overset{*}{Z} = \sum_{i-1}^{m}\sum_{j=1}^{n}(C_{ij} - U_i - V_j)X_{ij}$

subject to (1) to (7) are satisfied ,

where U_i and V_j are some real values, is an optimal solution to the problem (P)

where

(P) Minimize Z= $\sum_{i-1}^{m}\sum_{j=1}^{n} C_{ij}X_{ij}$

subject to (1) to (7) are satisfied.

Theorem 2. If { $\overset{0}{X}_{ij}$, i= 1,2,…,m and j =1,2,…,n } is a feasible solution to the problem (P) and c_{ij} - u_{ij} − v_{ij} ≥ 0 , for all i and j where u_{ij} and v_{ij} are some real values, such that the minimum of Minimize Z= $\sum_{i-1}^{m}\sum_{j=1}^{n}(C_{ij} - U_i - V_j)X_{ij}$
Subject to (1) to (7) are
Satisfied, is zero, then { $\overset{0}{X}_{ij}$, i= 1,2,…,m and j =1,2,…,n } is an optimum solution to the problem (P).

Theorem 3. The solution obtained by the zero point method for a TP with mixed constraints (P) is an optimal solution for the problem (P).

Theorem 4. The optimal MFL solution of a TP with mixed constraints is an optimal solution of a TP with mixed constraints which is obtained from the given TP with mixed constraints by changing the sign of columns and rows having negative Modi indices from $\leq$ to = and = to $\geq$.

3.6.3 Optimal MORE-FOR-LESS (MFL) procedure:

We use the following procedure for finding an optimal MFL solution to a TP with mixed constraints.

Step 1. Form the LBP which is obtained from TP with mixed constraints by changing all inequalities to equalities with the lowest possible feasible right-hand side values.

Step 2. Balance LBP and find an optimal solution of the balanced LBP using the transportation algorithm.

Step 3. Place the load(s) of the dummy row(s)/ column(s) of the balanced LBP at the lowest cost feasible cells of the given TP to obtain a solution for the TP with mixed constraints.

Step 4. Create the Modi index matrix using the solution of the given TP obtained in the Step 3.

Step 5. Identify negative Modi indices and related columns and rows. If none exist, this is an optimal solution to TP with mixed constraints (no MFL paradox is present). STOP.

Step 6. Form a new TP with mixed constraints by changing the sign of columns and rows having negative Modi indices from $\leq$ to = and = to $\geq$ in the given problem.

Step 7. Obtain a solution of the new TP with mixed constraints using the Step 1 to the Step 3.

Step 8. The optimal solution for the new TP with mixed constraints obtained from the Step 7 is an optimal MFL solution of the given TP with mixed constraints (by the Theorem 1.).

ILLUSTRATIVE EXAMPLE 2 ON MIXED CONSTRAINT TRANSPORTATION PROBLEM

The proposed method for finding an optimal MFL solution to a TP with mixed constraints is illustrated by the following example.

Table3.1 TP with a mixed constraint

	1	2	3	SUPPLY
1	2	5	4	=5
2	6	3	1	≥ 6
3	8	9	2	≤ 9
DEMAND	=8	≥ 10	≤ 5	

Now, LBP for the given TP with mixed constraints is given below.

Table 3.2: LBP for TP with mixed constraints

	1	2	3	SUPPLY
1	2	5	4	=5
2	6	3	1	= 6
3	8	9	2	=0
DEMAND	=8	= 10	0	

Now, the optimal solution of LBP is given below by the transportation algorithm

Table 3.3: Optimal solution of LBP

	1	2	3	SUPPLY
1	2	5	4	=5
2	6	3	1	= 6

3	8	9	2	=0
4	3	4	0	=7
DEMAND	=8	= 10	0	

Using the step 3, we obtain the following solution for the given problem.

Table 3.4 :optimal solution

	1	2	3	SUPPLY
1	**5**			=5
2	3	10	**0**	≥ 6
3			**0**	≤ 9
DEMAND	=8	= 10	0	

Therefore, the solution for the given problem is

X_{11}=5, X_{12}=3, X_{22}=10, X_{23}=0, X_{33}=0 for a flow of 18 units with the total transportation cost is $58

Now the Modi index for the optimal solution of given problem is shown below.

Table3.5 :The modi index

	V1	V2	V3	Ui
Ui	**2**	-1	-3	-4
U2	6	3	**1**	0
U3	7	4	**2**	1
Vj	6	3	1	

Since the first row and the second and third columns have negative Modi indices, we

Consider the following new TP with mixed constraints

<u>Table3.6: TP with Mixed Constraints</u>

	1	2	3	SUPPLY
1	2	5	4	=5
2	6	3	1	≥ 6
3	8	9	2	≤ 9
DEMAND	=8	≥10	≤ 5	

Now LBP for the new TP with mixed constraints is given below.

<u>Table3.7: LBS for TP with mixed constraints</u>

	1	2	3	SUPPLY
1	2	5	4	=5
2	6	3	1	= 6
3	8	9	2	=0
4	3	4	0	=12
DEMAND	=8	= 10	=5	

Using the step 3, we obtain the following solution for the new TP with Mixed constraints given problem.

<u>Table3.8 :Solution for the TP with mixed constraints</u>

	1	2	3	SUPPLY
1	**8**	0		≥ 5
2		10	**5**	≥ 6
3			**0**	≤ 9
DEMAND	≥ 8	≥10	=5	

Now the Modi index for the solution of the new TP is given below.of given problem is shown below.

Table3.9 :Modi index of the TP

	V1	V2	V3	Ui
Ui	**2**	5	3	2
U2	0	3	**1**	0
U3	1	4	**2**	1
Vj	0	3	1	

Since all the Modi indices are positive, the current solution is an optimal solution of the new TP with mixed constraints . Thus, by the Theorem 1. the optimal MFL Solution for the given TP with mixed constraints is X_{11} = 8, X_{12}= 0, X_{22} = 10 X_{23} =5 and X_{33} = 0 for a flow of 23 units with the total transportation cost is $51. The solution is better than the solution obtained earlier because the shipping rate per unit is now 2.22.

Note 1: For calculating Modi indices, we need $n + m - 1$ loading cells. So, we keep the cells that would be loaded using the zero point method even with a load of zero

3.7 Sensitivity Analysis of TP

This involves the development of understanding how the information in the final tableau can be given managerial interpretations. This will be done by examining the application of sensitivity analysis to the linear programming problems. To analyze sensitivity in linear programming, after obtaining the optimal solution, one of the right-hand-side values or coefficients of objective function are changed, then, the changes in optimal solution and optimal value are examined.

The balanced relation between supply and demand in transportation problem makes it difficult to use traditional sensitivity analysis methods.

Therefore, in the process of changing supply or demand resources, at leastone more resource needs to be changed to make the balanced relation possible.

In this study, utilizing the concept of complete differential of changes for sensitivity analysis of right-hand-side parameter in transportation problem, a method is set forth. This method examines simultaneous and related changes of supply and demand without making any change in the basis. The mentioned method utilizes Arasham and Kahn's simplex algorithm to obtain basic inverse matrix

.

CHAPTERS FOUR

DATA COLLECTION AND ANALYSIS

4.0 Introduction

Guinness Ghana LTD is one of the top five worldwide brewery companies. Transportation cost represents about 25% of the total production cost. The company has outsourced its transportation to external logistics services Providers. Guinness Ghana Ltd has registered about 20 transporters who operate with 97 trucks. Each of the plants at the various sites namely Achimota and Kaasi has its own constraint with respect to plant and warehouse capacity. Thus, there is a limit capacity at each plant. The total plant capacity for Kaasi and Achimota per day is 1800 and 1600 crates dependants on the plant efficiency.

This project is intended to minimize the total transportation cost from two production site namely Kaasi (Kumasi), and Achimota (Accra) to its numerous key distributors geographically scattered all over Ghana which are numbered about 52.. Guinness Ghana Ltd faces challenges on how to optimally distribute its products among the 52 Key distributors with a minimum transportation cost. As each site has its limit that supply and each customer a certain demand at a time.

4.1 Data Collection

For the purpose o this study, data was collected from Guinness Ghana Ltd, in the brewery the volume of the liquid is quantified in millilitres. Also creates of drinks are packed on pallets. The required data includes: A list of all products, sources, demand for each product by customer, the full truck transportation cost,.

The study concerned the supply of Malta Guinness from two production sites Kaasi and Achimota to 9 key distributors geographically scattered in the regions of Ghana. The study covered data gathered on the periods July07-June08, and Sept08-June09.

The transportation cost for full truckload of 1512 cases was known as were as production capacities. The demand for each destination was also known in advance. Demand and production capacity were expressed in cases while the cost of transportation were express in Ghana cedis.

4.2 Data source

The data used for the analysis was collected from the logistics manager of Guinness Ghana Breweries Ltd. The data included the transport cost per full truck of 1512 cases of malt from production plant to the various key distributors, quantity demanded of Malta Guinness by the various distributors and capacities for the two plants ACH and KAS sited at Achimota in Accra and Kaasi in Kumasi respectively

4.3 JULY07-JUNE08 TRANSPORTATION MATRIX FOR GGL PROBLEM

The collected data for JULY07-JUNE08 (thousand) on transportation cost is shown in the table below. This data indicates the transportation matrix showing the supply (capacity), demand, and the unit cost per full truck.

<u>Table 4.1the matrix representation of the problem (10^3)</u>

PLANT	FTA	RICKY	OBIBAJK	KADOM	NAATO	LESK	DCEE	JOEMAN	KBOA	CAPACITY
ACH	39.99	126.27	102.70	81.68	38.81	71.99	31.21	22.28	321.04	1298
KAS	145.36	33.82	154.05	64.19	87.90	107.98	65.45	39.08	167.38	1948
DEMAND	465	605	451	338	260	183	282	127	535	

4.3.1 Formulation Problem

Let Y1=plant site at ACH

Y_2=plant site at KAS

X_{ij} = the units shipped in crates from plant *i* to distribution centre *j*

i =1, 2, 3... 9. and j=1, 2, 3..., 9.

Using the shipping cost data of Table 4.1 the annual transportation cost in thousand of Cedis is written as.

Minimize

$39.99x_{11} + 126.27x_{12} + 102.70x_{13} + 81.68x_{14} + 38.81x_{15} + 71.99x_{16} + 31.21x_{17} + 22.28x_{18}$

$+321.044x_{19} + 145.36x_{21} + 33.82x_{22} + 154.05x_{23} + 64.19x_{24} + 87.90x_{25} +$

$+ 107.98x_{26} + 64.45x_{27} + 39.08x_{28} + 167.38x_{29}$

Consider capacity constraint

$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} \leq 1298$

$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} + x_{27} + x_{28} + x_{29} \leq 1948$

Demand constraint

$x_{11} + x_{21} = 465$

$x_{12} + x_{22} = 605$

$x_{13} + x_{23} = 451$

$x_{14} + x_{24} = 338$

$x_{15} + x_{25} = 260$

$x_{16} + x_{26} = 183$

$x_{17} + x_{27} = 282$

$x_{18} + x_{28} = 127$

$x_{19} + x_{29} = 535$

$x_{ij} \geq o$ for all i and j

4.3.2 OPTIMAL SOLUTION-July2007-June2008

Using the management scientist 5.0 for Linear programming module the optimal solution obtained is displayed below:

OPTIMAL SOLUTION

Objective Function Value = **245,498**

Table 4.2 Optimal Solution

Variable	Value	Reduced cost
X_{11}	465.000	0.000
X_{12}	0.000	128.000
X_{13}	451.000	0.000
X_{14}	0.000	53.480
X_{15}	260.000	0.000
X_{16}	122.000	0.000
X_{17}	0.000	2.750
X_{18}	0.000	19.190
X_{19}	0.000	189.650
X_{21}	0.000	69 .380
X_{22}	605.000	0.000
X_{23}	0.000	15.360
X_{24}	338.000	0.000
X_{25}	0.000	13.100
X_{26}	61.000	0.000
X_{27}	282.000	0.000
X_{28}	127.000	0.000
X_{29}	535.000	0.000

SENSITIVITY REPORT

Table 4.2.1 sensitivity Report 1

Constraints	Slack/Surplus	Dual Prices
1	0.000	34.990
2	0.000	-1.000
3	0.000	-74.980
4	0.000	-32.820
5	0.000	-137.690
6	0.000	-63.190
7	0.000	-73.800
8	0.000	-106.980
9	0.000	-63.450
10	0.000	-38.080
11	0.000	-166.380

OBJECTIVE COEFFICIENT RANGES

Table 4.2.2Sensitivity Report 2

Variable	Lower Limit	Current Value	Upper Limit
X_{11}	-	-39.990	88.450
X_{12}	-254.710	-126.270	-
X_{13}	-	-102.700	-
X_{14}	135.160	-81.680	-
X_{15}	-	-38.810	30.570
X_{16}	-	-71.990	-18.510
X_{17}	-33.960	-31.210	-
X_{18}	-41.470	-22.280	-
X_{19}	-510.690	-321.040	-
X_{21}	-214.740	-145.360	-
X_{22}	-	-33.820	-20.720
X_{23}	-169.410	-154.050	-
X_{24}	-	-64.190	-48.830
X_{25}	-101.000	-87.900	-
X_{26}	-	-107.980	81.6700
X_{27}	-	-64.450	-61.7000
X_{28}	-	-39.080	-25.980
X_{29}	-	-167.380	-148.190

RIGHT HAND SIDE RANGES

Table4.2.3 Sensitivity Report3

Constraints	Lower Limit	Current Value	Upper Limit
1	-	1298.000	-
2	-	1948.000	-
3	-	465.000	-
4	-	605.000	-
5	-	451.000	-
6	-	338.000	-
7	-	260.000	-
8	-	183.000	-
9	-	282.000	-
10	-	127.000	-
11	-	535.000	-

4.3.3COMPUTATIONAL PROCEDURE

The management scientist software is windows-based software designed for use with many of the techniques represented in Operations management theory book.

The management scientist 6.0 software packaged was employed to solve this transportation problem. The management scientist software is mathematical tool solver for optimization and mathematical programming in operations research. The Management Science module used is based on simplified version of the simplex technique called The Transportation Simplex Method.

The transportation simplex method is a special version of Simplex Method used to solve Transportation Problems.

It was run on Intel(R) Core(TM) Duo CPU machine with 4.0GB of RAM.

Based on the data gathered (Table4.1 and Table 5.1) that were used in running the management scientist program, produced the same output for the ten trials

4.3.5 RESULTS AND DISCUSSION

The above transportation problem was solved with linear programming module and transportation module of the Management Scientist, and the optimal solution obtained was the same for each results.

The computer solution (see fig 4.05) shows that the minimum total transportation cost is GH¢245,497,537 Ghana cedis

The values for the decision variables show the optimal amounts to ship over each route. The logistics manager should follow the following distribution list if want to optimize the distribution:

Ship 465000 case of malt Guinness from Plant ACH to distributor FTA.

Ship 451000 case of malt Guinness from Plant ACH to distributor OBIBA JK

Ship 260000 case of malt Guinness from Plant ACH to distributor NAATO

Ship 122000 case of malt Guinness from Plant ACH to distributor LESK

Ship 605000 case of malt Guinness from Plant KAS to distributor RICKY

Ship 338000 case of malt Guinness from Plant KAS to distributor KADOM

Ship 61000 case of malt Guinness from Plant KAS to distributor LESK

Ship 282000 case of malt Guinness from Plant KAS to DCEE

Ship 127000 case of Malt Guinness from Plant KAS to distributor JOEMA and

Ship 535000 case of malt Guinness from Plant KAS to distributor KBOA

Table4.2.4 :Transportation output

TRANSPORTATION OUTPUT TABLE				
PLANT SITE(SOURCE)	DISTRIBUTOR(DESTINATION)	FULLTRUCK PER CASE(000)	COST PER FULL TRUCK LOAD(¢)	TOTAL COST(¢)
ACH	FTA	465	39.99	18595.35
ACH	RICKY	0	126.27	0
ACH	OBIBA JK	451	102.70	46317.7
ACH	KADOM	0	81.68	0
ACH	NAATO	260	38.81	10090.6
ACH	LESK	122	71.99	8782.78
ACH	DCEE	0	31.21	0
ACH	JOEMA	0	22.28	0
ACH	KBOA	0	321.04	0
KAS	FTA	0	145.36	0
KAS	RICKY	605	33.82	20461.1
KAS	OBIBA JK	0	154.05	0
KAS	KADOM	338	64.19	21696.22
KAS	NAATO	0	87.90	0
KAS	LESK	61	107.98	6586.78
KAS	DCEE	282	65.45	18456.9
KAS	JOEMA	127	39.08	4963.16
KAS	KBOA	535	167.38	89548.3
The total transportation cost is ,:				245,498

4.3.6 The sensitivity Analysis

. Using the Arsham and Kahn Algorithm, this analysed the sensitivity of right-hand-side values of the transportation problem.

The values of supply and demand's changes in this problem are consequently shown as Δs_1=10,000, Δs_2=15,000, Δd_1=5000, Δd_2=-5000, Δd_6=3000,d Δd_7=2000 and Δd_9=10000.

Thus $\Sigma\Delta s = \Sigma\Delta d$. Implementing the above changes in the transportation problem using the Arsham algorithm the basic solution is change as follows:

The computer solution shows that the minimum total transportation cost is GH¢244,129,447 Ghana cedis. Which is clearly shows that, if Guinness Ghana Ltd management is to implement such changes in supply and demand, it will help in decreases transportation cost to GH¢1,368,090 Ghana cedis.

The logistics manager should follow the following distribution pattern if want to optimize the distribution:

Ship 47, 000 case of malt Guinness from Plant ACH to distributor FTA.

Ship 451,000 case of malt Guinness from Plant ACH to distributor OBIBA JK

Ship 260,000 case of malt Guinness from Plant ACH to distributor NAATO

Ship 127,000 case of malt Guinness from Plant ACH to distributor LESK

Ship 600,000 case of malt Guinness from Plant KAS to distributor RICKY

Ship 338,000 case of malt Guinness from Plant KAS to distributor KADOM

Ship 59,000 case of malt Guinness from Plant KAS to distributor LESK

Ship 282,000 case of malt Guinness from Plant KAS to DCEE

Ship 127,000 case of Malt Guinness from Plant KAS to distributor JOEMA and

Ship 525,000 case of malt Guinness from Plant KAS to distributor KBOA

4.4 JULY08-JULY09 Transportation matrix For GGBL Problem

The collected data for JULY08-JULY09 (thousand) on transportation cost is shown in the table below. This data indicates the transportation matrix showing the supply (capacity), demand, and the unit cost per full truck

Table 4.3.1 the matrix representation of the problem (10^3)

PLANT	FTA	RICKY	OBIBA JK	KADOM	NAATO	LESK	DCEE	JOEMA	KBOA	CAPACITY
ACH	90.79	88.21	82.08	68.99	30.59	424.91	30.60	13.87	70.85	1736
KAS	228.74	37.60	176.41	72.95	114.32	173.09	73.37	38.61	239.20	2419
DEMAND	907	576	445	335	272	431	304	128	757	

4.4.1 Formulation Problem

Let Y_1 =plant site at ACH

Y_2 =plant site at KAS

X_{IJ} = the units shipped in crates from plant i to distribution centre j

i =1, 2, 3... 9. and j =1, 2, 3..., 9.

Using the shipping cost data in Table 4.4.5 the annual transportation cost in thousand of Cedis is written as.

Minimize

$90.79x_{11} + 88.21x_{12} + 82.08x_{13} + 68.99x_{14} + 30.59x_{15} + 424.91x_{16} + 30.60x_{17} + 13.87x_{18} + 70.85x_{19}$

$+ 228.74x_{21} + 37.60x_{22} + 176.41x_{23} + 72.955x_{24} + 114.32x_{25}$

$+ 173.09x_{26} + 73.37x_{27} + 38.61x_{28} + 239.2020x_{29}$

Consider capacity constraint

$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} \leq 1736$

$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} + x_{27} + x_{28} + x_{29} \leq 22419$

Demand constraint

$x_{11} + x_{21} = 907$

$x_{12} + x_{22} = 576$

$x_{13} + x_{23} = 445$

$x_{14} + x_{24} = 335$

$x_{15} + x_{25} = 272$

$x_{16} + x_{26} = 431$

$x_{17} + x_{27} = 304$

$x_{18} + x_{28} = 128$

$x_{19} + x_{29} = 757$

$x_{ij} \geq o$ for all i and j

4.4.2 The management Scientist Solution (July 2008- July 2009)

(i) Using the management scientist 5.0 for Linear programming module.

Objective Function Value = 386729.91

<u>Table.4.3.2 the management Science solution (July08-July09)</u>

Variable	Value	Reduced cost
X_{11}	907.000	0.000
X_{12}	0.000	0.000
X_{13}	72.000	0.000
X_{14}	0.000	90.370
X_{15}	0.000	10.600
X_{16}	0.000	346.180
X_{17}	0.000	51.200
X_{18}	0.000	69.590
X_{19}	757.000	0.000
X_{21}	0.000	188 .560
X_{22}	576.000	0.000
X_{23}	373.000	0.000
X_{24}	335.000	0.000
X_{25}	272.000	0.000
X_{26}	431.000	0.000
X_{27}	304.000	0.000
X_{28}	128.000	0.000
X_{29}	0.000	74.020

THE SENSITIVE SECTION OF THE OUTPUT

Table4.3.3: Sensitivity Report1

Constraints	Slack/Surplus	Dual Prices
1	0.000	54.150
2	0.000	-40.180
3	0.000	-144.940
4	0.000	2.580
5	0.000	-136.230
6	0.000	-32.770
7	0.000	-74.140
8	0.000	-132.880
9	0.000	-33.550
10	0.000	1.570
11	0.000	-125.00

OBJECTIVE COEFFICIENT RANGES

Table4.3.4 Sensitivity Report 2

Variable	Lower Limit	Current Value	Upper Limit
X_{11}	-	90.790	279.350
X_{12}	-100.350	88.210	-
X_{13}	8.060	82.080	92.680
X_{14}	-21.380	68.990	-
X_{15}	19.990	30.590	-
X_{16}	78.730	424.910	-
X_{17}	-20.600	30.600	-
X_{18}	-55.720	-22.280	-
X_{19}	-	70.850	144.870
X_{21}	40.180	228.740	-
X_{22}	-	37.600	226.160
X_{23}	165.810	176.410	250.430
X_{24}	-	72.950	163.320
X_{25}	-	114.320	124.920
X_{26}	-	173.060	519.240
X_{27}	-	73.730	124.930
X_{28}	-	38.610	108.200
X_{29}	165.180	239.200	-

RIGHT HAND SIDE RANGES

Table4.3.5: Sensitivity Report 3

Constraints	Lower Limit	Current Value	Upper Limit
1	1664.000	1736.000	1736.000
2	1843.000	2419.000	2419.000
3	907.000	907.000	907.000
4	576.000	576.000	576.00
5	445.00	445.000	1021.000
6	335.000	335.000	911.00
7	272.000	272.00	848.000
8	431.00	431.00	1007.000
9	304.000	304.00	880.000
10	128.00	128.00	704.00
11	757.00	757.000	829.000

4.4.3 RESULTS AND DISCUSSION

The GGL problem (Table5.1) was solved with the linear programming module and the transportation module of The Management Scientist. The results from both the linear programming module and that of the transportation module of The Management Scientist yielded the same values, in terms of the optimal solution obtained. The computer solution (Table 5.4) shows the minimum total transportation cost is GH ¢386,729.91. The value for the decision variables shows the optimal amount of drinks to be ship over each route.

For variable X13=907,907 cases of drinks should be transported from site ACH to distributor FTA. To minimize the transportation cost the management of Guinness Ghana Ltd should make the following shipments:

Ship 907,000 cases of Malta Guinness from Plant ACH to Distributor FTA

Ship 72,000 cases of Malta Guinness from Plant ACH to distributor OBIBA JK

Ship 757,000 cases of Malta Guinness from Plant ACH to distributor KBOA

Ship 576,000 cases of Malta Guinness from Plant KAS to distributor RICKY

Ship 373,000 cases of Malta Guinness from Plant KAS to distributor OBIBA JK

Ship 335,000 cases of Malta Guinness from Plant KAS to distributor KADOM

Ship 272,000 cases of Malta Guinness from Plant KAS to distributor NAATO

Ship 431,000 cases of Malta Guinness from Plant KAS to distributor LESK

Ship 304,000 cases of Malta Guinness from Plant KAS to distributor DCEE

Ship 128,000 cases of Malta Guinness from Plant KAS to distributor JOEMAN

TABLE 4.3.6 THE OPTIMAL SOLUTION (JULY08-JULY09)

TRANSPORTATION OUTPUT TABLE				
PLANT SITE(SOURCE)	DISTRIBUTOR(DESTINATION)	FULLTRUCK PER CASE(0000)	COST PER FULL TRUCK LOAD	TOTAL COST(GH 000)
ACH	FTA	907	90.79	82346.53
ACH	RICKY	0	88.21	0
ACH	OBIBA JK	72	82.08	5909.76
ACH	KADOM	0	68.99	0
ACH	NAATO	0	30.59	0
ACH	LESK	0	424.91	0
ACH	DCEE	0	30.60	0
ACH	JOEMA	0	13.87	0
ACH	KBOA	757	70.85	53633.45
KAS	FTA	0	228.74	0
KAS	RICKY	576	37.60	21657.6
KAS	OBIBA JK	373	176.41	65800.93
KAS	KADOM	335	72.95	24438.25
KAS	NAATO	272	114.32	31095.04
KAS	LESK	431	173.09	74601.79
KAS	DCEE	304	73.37	22304.48
KAS	JOEMA	128	38.61	4942.08
KAS	KBOA	0	239.20	0
The total transportation cost is ,:				386,729.91

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusion

The transportation cost is an important element of the total cost structure for any business

The transportation problem was formulated as a Linear Programming and solved with the standard LP solvers such as the Management scientist module to obtain the optimal solution.

The computational results provided the minimal total transportation cost and the values for the decision variables for optimality. Upon solving the LP problems by the computer package, the optimum solutions provided the valuable information such as sensitivity analysis for Guinness Ghana Ltd to make optimal decisions

Through the use of this mathematical model (Transportation Model) the business (GGBL) can identify easily and efficiently plan out its transportation, so that it can not only minimize the cost of transporting goods and services but also create time utility by reaching the goods ad services at the right place ad right time. This intend will enable them to meet the corporative objective such as education fund, entertainment and other support they offered to people of Ghana

The study recorded total minimization of transportation cost during the periods of June2007-June 2008 and July08-July09 financial period.The value for the decision variable produced the optimal amounts to be ship to each distributor of Guinness Ghana Ltd.

5.2 Recommendations

Based on the results and findings of this study, I recommend to the management of Guinness Ghana Breweries Group to seek to the application of mathematical theories into their operations as a necessary tool when it comes to decision making, not only in the area logistics(the transportation Problem), but in production as well as administration.

This study employed mathematical technique to solve management problems and make timely optimal decisions. If the GGL managers are to employed the proposed transportation model it will assist them to efficiently plan out its transportation scheduled at a minimum cost.

There are number of programs that can assist in construction of TP and LP problems. Probably the best known is GAMS-General Algebraic modelling system. This provides a high level language for easy representation of complex problems. In Future I recommend the solution of large-scale transportation problems through aggregation. This proposed method is applicable to any transportation problem.

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APENDIX

Conversions.

Full truck load of Malta Guinness =1512 cases

24 bottles =1 case

84 cases =1 pallet

Acronyms and Symbols

LP= Linear Programming

GGBL= Guinness Ghana LTD

TP= Transportation Problem

ACH= Achimota Plant

KAS= Kaasi Plant

KD=Key Distributor