KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY

SELECTING THE BEST TRANSPORTATION MODEL FOR MINERAL WATER PRODUCING COMPANIES IN GHANA- A CASE STUDY OF GRATIS AQUA COMPANY(GAC)

BY

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DECLARATION

I hereby declare that this submission is my own work towards the award of the MSc. degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the university, except where due acknowledgement had been made in the text.

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DEDICATION

This study is dedicated to my son Alexander Owusu-Otchere, my wife Mavis Gyimah and my uncles Rev. Alexander Otchere Mainoo and Frederick Kwaku Agyeman Duah.

Abstract

Transporting goods from sources to destinations by road is very popular in Ghana and it can be seen in every business entity whose daily activities involve production. The production sites, storage facilities and final destinations are all link by road. It has always being a problem in finding ways to cut down transportation cost and it goes on to affect the decision making process and if proper strategic plans are not put in place can slow down the development of that establishment. In this study data of Gratis Aqua Company was considered and modeled as a transportation problem with unique quality method, minimum demand-supply method and best candidate method. It was observed that these models are capable of reducing the company's total transportation cost. This transportation model will be useful for making strategic decisions by managers of Gratis Aqua Company in making optimum allocation of the production from the three storage facilities (Juaso, Obogu and Konongo) to the various customers in the three districts of Asante Akim at a minimum transportation cost.

ACKNOWLEDGMENTS

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CHAPTER 1

Introduction

This chapter talks about the background of the study, problem statement, objectives of the thesis, methodology and justification. Organizational structure of the study is also discussed under this chapter.

1.1 Background of study

It is the dream of every company to effectively meet the demands of every customer at limited time by supplying products after the request is made in as much as increasing profit is at heart. The products are usually transported by sea, air or land. In Ghana the latter is most often used. It has been useful in a way but not all that efficient due to a number of factors that have led to transportation problem.

The transportation problem is concerned with finding an optimal distribution plan for a single commodity. A given supply of the commodity is available at a number of sources, there is a specified demand for the commodity at each of a number of destinations, and the transportation cost between each source-destination pair is known. In the simplest case, the unit transportation cost is constant. The problem is to find the optimal distribution plan for shipments from sources to destinations that minimizes the total transportation cost.To do this time becomes a major and important factor.

One sector of which transportation problem is considered to have effect on is water. Water plays an important role in the world economy, as it functions as a solvent for a wide variety of chemical substances and facilitates industrial cooling and transportation. Approximately 70 percent of the fresh water used by humans goes to agriculture.Water is life and that is why God in His own wisdom created water to cover about 70 percent of this earth.Water helps in so many activities of human life every second talk about washing, cooking, irrigation, construction and many more. It has been noticed that companies, factories, organizations and all the various institutions available use water in their outfits. There is no dispute about the fact that water is a need in human life because nobody can live without it. Man drinks a lot of water to be able to survive but that brings the question whether the water is clean and hygienic or it contains some particles of dirt. Safe drinking water is essential to humans and other lifeforms even though it provides no calories or organic nutrients. Access to safe drinking water has improved over the last decades in almost every part of the world, but approximately one billion people still lack access to safe water and over 2.5 billion lack accesses to adequate sanitation. There is a clear correlation between access to safe water and GDP per capita. However, some observers have estimated that by 2025 more than half of the world population will be facing water-based vulnerability. A recent report (November 2009) suggests that by 2030, in some developing regions of the world, water demand will exceed supply by 50 percent according to (www.wikipedia.org/wiki/water). Africa as a continent faces this challenge to provide clean and hygienic water to its people all the time. Ghana for one tried using the water companies established by the government but it has been observed their product cannot reach every home of the ordinary Ghanaian every time whereas the clean and hygienic aspect is nothing to write home about. The introduction of borehole water in the country has been a supportive mechanism to the country but yet comes with a whole lot of challenges such as being able to construct enough number to satisfy the needs of every citizen, repairs, pumping, accidents, etc. These problems brought the idea of mechanization of the drilled or borehole water but one may ask: how many people can afford?

Private mineral water producing companies have come to the rescue of everybody. Apart from the fact that the country has become dirty with the rubbers due to the bad attitude of most of the people in the country of not making good use of the dust bins provided by the central government and other non-governmental organizations, it has been observed that their work has been effectively done. There exist high competition between these numerous companies and that alone is gingering them to live up to expectation.

In transporting the sachet water and bottled water to the door step of the ordinary Ghanaian comes with a whole lot of challenges. Transportation cost which is a major challenge has always been so high due to a number of factors and this has actually being a burden on these companies as to how to minimize the cost. It is of this responsibility that the researcher has taken it upon himself to assist Gratis Aqua Company to minimize its total transportation cost.

1.1.1 Profile of Gratis Aqua Company

Historical Background

Every company has reasons for its establishment, location, target population, services and a lot more. Gratis Aqua for one was established on Friday 20th August, 1999 in Juaso, Ashanti region, Ghana. The idea was that most of the people living in the three districts of Asante Akim (South, North and Central) especially those in the south lacked frequent supply and access to safe and portable drinking water as at that time and any attempt to provide that need was going to be welcomed by the people. Although there were few setbacks like registration certificates, acquisition of land, initial capital to fund the whole project and acceptance of a new product. but through determination characterized by strategic planning and professionalism showed by all the workers, the company did not take time to gain grounds. Overall it started with six workers who acted as general manager, production and storage managers and distribution and sales managers. There was only one storage facility and one Kia truck available but was managed to do the supply. From time to time the company run water quality analysis test which has being the key to providing the people with the best purified drinking water at an affordable price. The supply started from the Juaso Township and extended to other towns of the Asante Akim south district. Right now the company supply all people within the three districts and has extended its reach to nearby towns in other districts in both the Ashanti and Eastern regions respectively. The company can now boast of huge staff, three actively used storage facilities, ten Kia trucks and five mini trucks which usually receive maintenance due to the bad nature of roads. High performance pumps have been installed to ensure the level of efficiency is not tempered with.

Location

Gratis Aqua Company(GAC) is located in the Asante Akim South District in Ashanti region, Ghana. The company is situated right at the centre of the district capital, Juaso to be precise. It can be sited behind the district authority cluster of schools (D/A primary and Junior High Schools).

Products/Brands

The company produces two main products namely:

- sachet water
- bottled mineral water

Storage and Warehousing

Finished products are temporarily stored in the storage facilities popularly referred to as store rooms by the company to ensure smooth and efficient distribution takes place afterwards when the need arises. There are three actively used store rooms and they are located at Juaso (at the production site),Konongo and Obogu. Each of the stores is capable of supplying products to each destination depending on availability, amount demanded and time.

Distribution

The distribution is done by vehicles which ply on both tiled and untilled roads to supply the products from sources (store rooms) to the various destinations (educational institutions, organizations, retailers, markets and even the individual at certain times) usually known as customers.

Vision Statement

Gratis Aqua Company is devoted to making both our planet and our customers healthier by serving water that is filtered and desalinated in such a way as to not pollute our environment nor further deplete our planets resources.

Mission Statement

To provide quality and purified water to our customers, using the best practices which are not harmful to the environment and people, but of the highest value that meets the standard of purified water specification with an acceptable customer service at heart and great delivery from our able staff.

1.2 Problem Statement

Management and shareholders would want to see increase in profit in the daily production and sales of the products but this demands critical thinking and strategic management. The cost of transporting products from sources to destinations has always been so high and it has been affecting managerial decision, Profit and assets of the company. This thesis therefore intend to assist the company(GAC) by selecting the best transportation model that will minimize the total transportation cost of transporting sachet water from sources to destinations.

1.3 Objectives of the study

This thesis seeks to:

- Use three models of transportation problem in minimizing total transportation cost of Gratis Aqua Company

- Recommend the best transportation model to Gratis Aqua Company

1.4 Methodology

This work will employ the use of unique quality method, minimum-demand supply method and best candidate method to minimize the total transportation cost of GAC

1.4.1 Method of data collection

The use of questionnaire and interview will be used as the two will be helpful tools in acquiring primary data. Secondary data will be much depended on since knowledge on various unit transportation cost, capacity of each storeroom and demands at various destinations are all helpful in the minimization of the total transportation cost.Information on the internet, Knust library, Lecture notes and other mathematical books that will be useful will be used.

1.5 Justification

According to management, the company has plans for expansion but it has been difficult to realize it due to high cost in transportation. Increase in cost of transporting products from sources to destinations is a major challenge to tackle and cannot be overlooked especially in this case that development is worked for. The ability of the company of being able to cut down or minimize its total transportation cost will be a plus in the sense that monies saved can be used to invest in the expansion works of the company and this study aims at achieving that. The transportation problem models to be employed in this study are tried and tested ones that have minimized the total transportation cost for industries, factories, companies and organizations that produce and supply their products to warehouses and consumers depending on the strategic plans of the business entity.Gratis Aqua Company for one cannot be exempted from benefiting from it as it will be useful in minimizing total transportation cost and most importantly it is easy to learn and use.

1.6 Organisation of the study

- The chapter one introduces the thesis in general, the background for Transportation Problem, the background of company (GAC), the problem statement, objective, methodology, justification, limitations of the study and the organization of the thesis

- Chapter two is concerned with the definition and the detailed literature review of the transportation problem/model

- Chapter Three discuss detailed methodology.This includes the formulation of the transportation problem, the transportation tableau, the solutions for the transportation problem, and methods for solving transportation problems to optimality

- Chapter Four provides an over view of the computational platforms for implementation and solution of the model and introduces the real-life data sets used in the solution process

- Chapter five which is the final chapter summarizes the conclusions with respect to overall aims of the project and proposed recommendation for future research/study. It reports the computational results and provides a comprehensive analysis of the outcome and performance of the proposed solution approaches

CHAPTER 2

Literature Review

2.1 Introduction

Transportation Problem (TP) is based on supply and demand of commodities transported from several sources to the different destinations. The sources from which we need to transport are referred to supply while the destinations where commodities arrive are referred to demand. It has been seen that on many occasions, the decision problem can also be formatting as TP. In general we try to minimize total transportation cost for the commodities transporting from source to destination. There is a type of linear programming problem that may be solved using a simplified version of the simplex technique called transportation method. Because of its major application in solving problems involving several product sources and several destinations of products, this type of problem is frequently called the transportation problem.

It gets its name from its application to problems involving transporting products from several sources to several destinations although the formation can be used to represent more general assignment and scheduling problems as well as transportation and distribution problems. The two common objectives of such problems are either,that is minimize the cost of shipping m units to n destinations or maximize the profit of shipping m units to n destinations. Source capacities, destinations requirements and costs of material shipping from each source to each destination are given constantly. The transportation problem can be described using linear programming mathematical model and usually it appears in a transportation tableau.

2.2 Background of transportation problem

The transportation problem and cycle cancelling methods are classical in optimization. The usual attributions are to the 1940's and later. However, Tolsto (1930) was a pioneer in operations research and hence wrote a book on transportation planning which was published by the National Commissariat of Transportation of the Soviet Union, an article called Methods of ending the minimal total kilometrage in cargo-transportation planning in space, in which he studied the transportation problem and described a number of solution approaches, including the, now well-known, idea that an optimum solution does not have any negative-cost cycle in its residual graph. He might have been the first to observe that the cycle condition is necessary for optimality. Moreover, he assumed, but did not explicitly state or prove, the fact that checking the cycle condition is also sufficient for optimality.

The transportation problem was formalized by the French mathematician (Monge, 1781). Major advances were made in the field during World War II by the So- viet/Russian mathematician and economist Leonid Kantorovich. Consequently, the problem as it is now stated is sometimes Known as the Monge-Kantorovich transportation problem. Kantorovich (1942), published a paper on continuous version of the problem and later with Gavurian, and applied study of the capacitated transportation problem (Kantorovich and Gavurin, 1949) many scientific disciplines have contributed toward analyzing problems associated with the transportation problem, including operation research, economics, engineering, Geographic Information Science and geography. It is explored extensively in the mathematical programming and engineering literatures. Sometimes referred to as the facility location and allocation problem, the transportation optimization problem can be modelled as a large-scale mixed integer linear programming problem.

The origin of transportation was first presented by Hitchcock (1941) also presented a study entitled (The Distribution of a Product from Several sources to numerous localities). This presentation is considered to be the first important contribution to the solution of transportation problems.

Koopman (1974) presented an independent study, not related to Hitchcock and called (Optimum Utilization of the Transportation System). These two contributions helped in the development of transportation methods which involve a number of shipping sources and a number of destinations. The transportation problem received this name because many of its applications involve determining how to optimally transport goods.

However it could be solved for optimally as an answer to complex business problem only in 1951, when George B. Dantzig applied the concept of Linear Programming in solving the Transportation models.

Dantzig (1963) then uses the simplex method on transportation problem as the primal simplex transportation method.

Stringer and Haley have developed a method of solution using a mechanical analogue. May be the first algorithm to find an optimal solution for the uncapacitated transportation problem was that of Efroymson and Ray. They assumed that each of the unit production cost functions has a fixed charge form. But they remark that their branch and bound method can be extended to the case in which each of these functions is concave and consists of several linear Segments. And each unit transportation cost function is linear.

Sharp.et.al developed an algorithm for reaching an optimal solution to the production of transportation problem for the convex case. The algorithm utilizes the decomposition approach it iterates between a linear programming transportation problem which allocates previously set plant production quantities to various markets and a routine which optimally sets plant production quantities to equate total marginal production costs, including a shadow price representing a relative location cost determined from the transportation problem. Williams applied the decomposition principle of Dantzing and Wolf to the solution of the Hitchcock transportation problem and to several generalizations of it. In this generalizations, the case in which the costs are piecewise linear convex functions is included. He decomposed the problem and reduced to a strictly linear program. In addition he argued that the two problems are the same by a theorem that he called the reduction theorem. The algorithm given by him, to solve the problem, is a variation of the simplex method with "generalized pricing operation". It ignores the integer solution property of the transportation problem so that some problems of not strictly transportation type, and for which the integer solution property may not hold be solved.

Shetty(1959) also formulated an algorithm to solve transportation problems taking nonlinear costs. He considered the case when a convex production cost is included at each supply center besides the linear transportation cost. Some of the approaches used to solve the concave transportation problem are presented as follows. The branch and bound algorithm approach is based on using a convex approximation to the concave cost functions. It is equivalent to the solution of a finite sequence of transportation problems. The algorithm was developed as a particular case of the simplified algorithm for minimizing separable concave functions over linear polyhedral as Falk and Soland.

Soland (1971) presented a branch and bound algorithm to solve concave separable transportation problem which he called it the "Simplified algorithm" in comparison with similar algorithm given by Falk and himself in 1969. The algo- rithm reduces the problem to a sequence of linear transportation problem with the same constraint set as the original problem.

A.C. Caputo. et. al. presented a methodology for optimally planning long-haul road transport activities through proper aggregation of customer orders in separate full truckload or less than truckload shipments in order to minimize total transportation costs. They have demonstrated that evolutionary computation techniques may be effective in tactical planning of transportation activities. The model shows that substantial savings on overall transportation cost may be achieved adopting the methodology in a real life scenario.

Roy and Gelders (1980) solved a real life distribution problem of a liquid bottled product through a 3-stage logistic system; the stages of the system are plant-depot, depot-distributor and distributor-dealer. They modelled the customer allocation, depot location and transportation problem as a 0-1 integer programming model with the objective function of minimization of the fleet operating costs, the depot setup costs, and delivery costs subject to supply constraints, demand constraints, truck load capacity constraints, and driver hours constraints. The problem was solved optimally by branch and bound, and Langrangian relaxation.

Tzeng et al. (1995) solved the problem of how to distribute and transport the imported Coal to each of the power plants on time in the required amounts and at the required 18 quality under conditions of stable and supply with least delay. They formulated a LP that Minimizes the cost of transportation subject to supply constraints, demand constraints, vessel constraints and handling constraints of the ports. The model was solved to yield optimum results, which is then used as input to a decision support system that help manage the coal allocation, voyage scheduling, and dynamic fleet assignment.

Equi et al.(1996) modelled a combined transportation and scheduling in one problem where a product such as sugar cane, timber or mineral ore is transported from multi origin supply points to multi destination demand points or transshipment points using carriers that can be ships, trains or trucks. They defined a trip as a full-loaded vehicle travel from one origin to one destination. They solved the model optimally using Langrangean Decomposition.

Saumis et al. (1991) considered a problem of preparing a minimum cost transportation plan by simultaneously solving following two sub-problem: first the assignment of units available at a series of origins to satisfy demand at a series of destinations and second, the design of vehicle tours to transport these units, when the vehicles have to be brought back to their departure point. The cost minimization mathematical model was constructed, which is converted into a relaxation total distance minimization, then finally decomposed to network problems, a full vehicle problem, and an empty vehicle problem. The problems were solved by tour construction and improvement procedures. This approach allows large problems to be solved quickly, and solutions to large problems to be solved quickly, and solutions to large test problems have been shown to be 1% 0r 2% from the optimum.

Charnes and Cooper (1960) first used the Goal Programming (GP) technique. This solution approach has been extended by Ijiri(965), Lee (1972), and others. Lee and Moore (1973) used GP model for solving transportation problem with multiple objective.

Arthur and Lawrence (1982) designed a GP model for production and shipping patterns in chemical and pharmaceutical industries.

Kwak and schniederjans (1985) applied GP to transportation problem with variable supply and demand requirements. Several other researchers.

Sharma et al. (1999) have also used the GP model for solving the transportation problem.

Veenan et al. proposed a heuristic method for solving transportation problem with mixed constraints which is based on the theory of shadow price. The solution obtained by heuristics method introduced by Veenan is an initial solution of the transportation problems with constraints.

Klingman & Russel (1975) have developed an efficient procedure for solving transportation problems with additional linear constraints. Their method exploits the topological properties of basis trees within a generalized upper bound framework.

Sharma & Swarup (1977) developed a technique, similar to transportation technique in linear programming to minimize a locally indefinite quadratic function, subject to Sharma and swarup, (1977), have developed the same concepts for multi-dimensional transportation problem.

Gass (1990) detailed the practical issues for solving transportation problems and offered comments on various aspects of transportation problem methodologies along with discussions on the computational results, by the respective researchers. Sharma and Sharma (2000) proposed a new heuristic approach for getting good starting solutions for dual based approaches used for solving transportation problems.

The transportation criterion is, however, hardly mentioned at all where the transportation problem is treated. Apparently, several researchers have discovered the criteria independently from each other. But most papers on the subject refer to the papers by Charnes and Klingman and Szwarc as the initial papers. In Charnes and Klingman name it the more-for-less criteria (MFL), and they write: The criteria was first observed in the early days of linear programming history (by whom no one knows) and has been a part of the folklore known to some but unknown to the great majority of workers in the field of linear programming. The transportation criteria is known as Doigs criteria at the London School of Economics, named after Alison Doig who used it in exams etc. around 1959 (Doig did not publish any paper on it). Since the transportation criteria seems not to be known to the majority of those who are working with the transportation problem, one may be tempted to believe that this phenomenon is only an academic curiosity, which will most probably not occur, in any practical situation. But that seems not to be true. Experiments done by Finke, with randomly generated instances of the transportation problem and allowing additional shipments (post optimal) show that the transportation costs can be reduced considerably by exploiting the criteria properties. More precisely, the average cost reductions achieved are reported to be 18.6% with total additional shipments of 20.5%. In a recent paper, Deineko & al. develop necessary and sufficient conditions for a cost matrix C to be protected against the transportation criteria. These conditions are rather restrictive, supporting the observations by Finke. The existing literature has demonstrated the identifying cases where MFL paradoxical situation exists and also, has provided various methods for finding MFL solution for transportation problems. Gupta et al and Arsham obtained the more-for-less solution for the TPs with mixed constraints by relaxing the constraints and by introducing new slack variables. While yielding the best more-for-less solution, their method is tedious since it introduces more variables and requires solving sets of complex equations. The perturbed method was used for solving the TPs with constraints.

Adlakha et al. proposed a heuristic method for solving TPs with mixed constraints which is based on the theory of shadow price. In the heuristic algorithm for an MFL solution in Adlakha et al,Vogel Approximation Method (VAM) and MODI (Modified Distribution) method were used Arsham developed an approach to post optimality analysis of the TPs through the use of perturbation analysis. Adlakha and Kowalski introduced a theory of absolute points for solving a TP and used these points for search opportunities to ship more for less in TP. Adlaka et al. developed an algorithm for finding an optimal MFL solution for TPs which builds upon any existing basic feasible solution. Since then, these problems have been studied extensively by many authors and have found applications in such diverse fields as geometry, fluid mechanics, Statistics, economics, shape recognition, inequalities and meteorology.

Asare (2011) worked on the transportation problem of Guinness Ghana Limited (GGL) in October 2011. The GGL problem was solved with the linear programming module and the transportation module of The Management Scientist. The results from both the linear programming module and that of the transportation module of The Management Scientist yielded the same values, in terms of the optimal solution obtained.

Ablordepey (2012) also minimized the total transportation cost of a Beverage industry. The methods used were the vogels approximation method (VAM) and modified distribution method. She used Quantitative manager for windows (QMW) for the analysis. It was realized that there is much difference in the monthly transportation cost between the lean and festive season.

CHAPTER 3

Methodology

3.1 Introduction

This chapter deals with the methodology of the study and the approach for handling transportation problem in Gratis Aqua Company. The transportation problem seeks to minimize the total transportation costs of transporting goods from m origins (each with a supply s_i) to n destinations (each with a demand d_j), when the unit shipping cost from an origin, i, to a destination, j, is c_{ij}.

3.2 The transportation problem

One of the most important and successful applications of quantitative analysis to solving business problems has been in the physical distribution of products, commonly referred to as transportation problems. Basically, the purpose is to minimize the cost of shipping goods from one location to another so that the needs of each arrival area are met and every shipping location operates within its capacity. However, quantitative analysis has been used for many problems other than the physical distribution of goods. Consider a commodity which is produced at various centers called sources and is demanded at various other destinations. The production capacity of each source (availability) and the requirement of each destination are known and fixed. The cost of transporting one unit of the commodity from each source to each destination is also known. The commodity is to be transported from various sources to different destinations in such a way that the requirement of each destination is satisfied and at the same time the total cost of transportation is minimized. This optimum allocation of the commodity from various sources to different destinations is called Transportation Problem.

- Transportation models deals with the transportation of a product manufactured at different plants or factories (supply origins) to a number of different warehouses (demand destinations). The objective is to:
- satisfy the destination requirements within the plants capacity constraints at the minimum transportation cost.
- determine schedule of transportation to minimize total transportation cost.

A typical transportation problem contains

- Inputs:
- Sources with availability
- Destinations with requirements
- Unit cost of transportation from various sources to destinations

The linear programming problem representing the transportation problem is generally given as:
Minimize:

$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij}$$

subject to:

$$\sum_{i=1}^{m} C_{ij} \leq s_i \quad i = 1, 2, \cdots m$$

$$\sum_{i=1}^{n} X_{ij} \leq d_i \quad i = 1, 2, \cdots n$$

$$X_{ij} \geq U \text{ for all i and j}$$

3.3 The transportation model

In a transportation problem, we are focusing on the original points. These points may represent factories to produce items, and to supply a required quantity of the products to a certain number of destinations. This process must be done successfully in such a way as to maximize the profit or minimize the cost transportation. Therefore, the places of production and supply are collected as the original points and the destinations respectively. Sometimes the original and destinations points are also termed as sources and sinks. However, to illustrate a typical transportation model, suppose that m factories supply certain items to n warehouses. As well as, let factory i $(i = 1, 2, \cdots, m)$ produces a_i units, and the warehouse j $(j = 1, 2, \cdots, n)$ requires b_j units. Furthermore, suppose the cost of transportation from factory i to warehouse j is c_{ij}. The decision variables x_{ij} is being the transported amount from the factory i to the warehouse j. Typically, the main objective is to find the transportation pattern that will minimize the total of the transportation cost.

3.4 Network representation of the transportation problem

The transportation problem is concerned with finding an optimal distribution plan for a single commodity. A given supply of the commodity is available at a number of sources, there is a specified demand for the commodity at each of a number of destinations, and the transportation cost between each source-destination pair is known. In the simplest case, the unit transportation cost is constant. The problem is to find the optimal distribution plan for transporting the products from sources to destinations that minimizes the total transportation cost. This can be seen in Figure 3.1

Here sources indicate the place from where transportation will begin, destinations indicates the place where the product has to be arrived and c_{ij} indicates the transportation cost in transporting from source to destination and sink denotes the destination.

A transportation problem can be stated mathematically as follows:

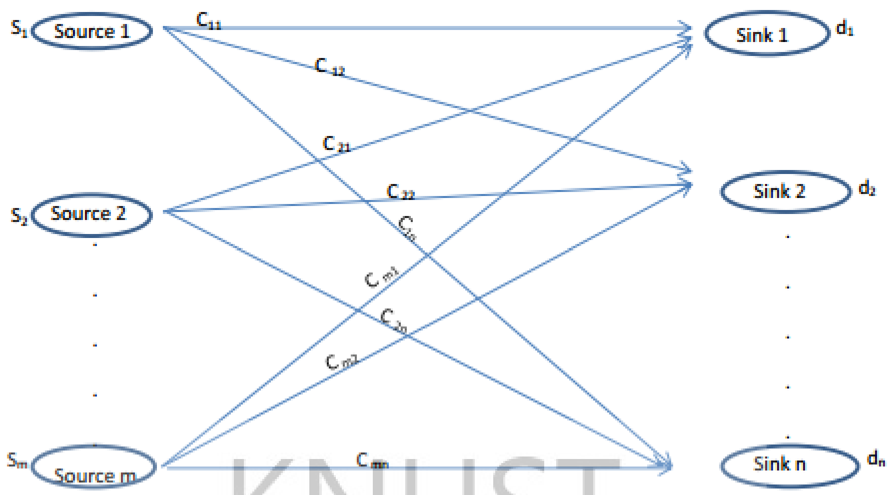

Figure 3.1: Network representation of the transportation problem

Let there be m sources and n destinations

Let s_i : the availability at the i_{th} source

d_j : the requirement of the j_{th} destination.

C_{ij} : the cost of transporting one unit of commodity from the i_{th} source to the j_{th} destination

3.5 Matrix model of a transportation problem

The Simplex tableau serves as a very compact format for representing and manipulating linear programs. In the same spirit, there is a need to introduce a tableau representation for transportation problems that are in the standard form as shown in Table 3.1 below

The transportation problem can be described using linear programming mathematical model and usually it appears in a transportation tableau. The model of a transportation problem can be represented in a concise tabular form with all the relevant parameters. The transportation tableau (A typical TP is represented in standard matrix form), where m represents number of sources, n stands for number of destinations and there is supply availability (S_i) at each source is shown in the far right column and the destination requirements (d_j)

Table 3.1: The Transportation Tableau

	Destination (j)					
Origin (i)	1	2	3	······	n	Supply (S_i)
1	X_{11} c_{11}	X_{12} c_{12}	X_{13} c_{13}	······	X_{1n} c_{1n}	s_1
2	X_{21} c_{21}	X_{22} c_{22}	X_{23} c_{23}	······	X_{2n} c_{2n}	s_2
3	X_{31} c_{31}	X_{32} c_{32}	X_{33} c_{33}	······	X_{3n} c_{3n}	s_3
$\vdots$				······		$\vdots$
m	X_{m1} c_{m1}	X_{m2} c_{m2}	X_{m3} c_{m3}	······	X_{mn} c_{mn}	s_m
Demand (d_j)	d_1	d_2	d_3	······	d_n	$\Sigma s_i = \Sigma d_j$

are shown in the bottom row. Each cell represents one route. The unit shipping cost (C_{ij}) is shown in the upper right corner of the cell, the amount of shipped material is shown in the centre of the cell. The transportation tableau implicitly expresses the supply and demand constraints and the shipping cost between each demand and supply point.

3.6 Definition of terms and conditions in the transportation problem:

Source

A point of origin where a supply of some commodity is available and that supply item(s) or commodities to various destinations are made possible

Destination

Destinations are points where the commodity is demanded

Supply

A supply schedule is a table which shows how much a source will be willing to supply a destination at particular prices under the existing circumstances. It usually refers to the capacity or how much a storage facility can hold and supply

Demand

The amount or quantity of a particular commodity that are needed by various destinations

Unit transportation cost

It refers to the transportation cost involved in shipping a commodity from a source to a destination. The costs of shipping from sources to destinations are indicated by the entries in the matrix

Total transportation cost

$$Z = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}X_{ij}$$

The constraints

Limitations on resource availability form what is known as a constraint set. Consider warehouse

i. The total outgoing shipment from this warehouse is the sum $X_{i1}+X_{i2}+\cdots+X_{in}$. In summation notation, this is written as $\sum_{i=1}^{n} X_{ij}$ since the total supply from warehouse i is a_i, the total outgoing shipment cannot exceed a_i. That is we must require.

$$\Sigma X_{ij} \leq a_i \quad \text{for } i = 1, 2, \cdots, m$$

Consider outlet j. The total incoming shipment at this outlet is the sum $X_{ij} +$

$X_{2j}+\cdots+X_{mj}$. In summation notation, this is written as $\sum_{i=1}^{m} X_{ij}$ since the demand at outlet j is b_j, the total incoming shipment should not be less than b_j. That is we must require

$$\sum_{i=1}^{m} X_{ij} \geq b_j \quad \text{for } j = 1, 2, \cdots, n$$

The results in a set of $m+n$ functional constraints and $X_{ij}'s$ should be nonnegative.

Basic variables

The variables in a basic solution whose values are obtained as the simultaneous solution of the system of equations that comprise the functional constraints

The decision variables

In the term linear programming, programming refers to mathematical programming. In this context, it refers to a planning process that allocates resources such as materials and resources in the best possible (optimal) way so that costs are minimized or profits are maximized. In LP, these resources are known as decision variables

A transportation scheme is a complete specification of how many units of the product should be shipped from each warehouse to each outlet. Therefore, the decision variables are: x_{ij} = the size of the shipment from warehouse i to outlet j, where $i = 1, 2, \cdots, m$ and $j = 1, 2, \cdots, n$. This is a set of $m \times n$ variables.

The objective function

The criterion for selecting the best values of the decision variables (e.g.,to maximize profits or minimize costs) is known as the objective function. Consider the shipment from warehouse i to outlet j. For any i and any j, the transportation cost per unit is c_{ij}; and the size of the shipment is x_{ij}. Since we assume that the cost function is linear, the total cost of this shipment is given by $c_{ij}x_{ij}$. Summing over all i and all j now yields the overall transportation cost for all warehouse outlet combinations. That is, our objective function is:

Minimize

$$\sum_{i=1}^{m}\sum_{i=1}^{n} C_{ij}X_{ij}$$

Cell

It refers to a small compartment in the transportation tableau

Allocation

The number of units of items transported from a source to a destination which is recorded in a cell in the transportation tableau.

Feasible solution

A set of non-negative values x_{ij}. $i = 1, 2, 3, \cdots, m$ and $j = 1, 2, 3, \cdots, n$ that satisfies the constraints is called a feasible solution to the transportation problem which implies that a set of positive individual allocations which simultaneously removes deficiencies is called a feasible solution

Basic feasible solution

A feasible solution to a m-origin, n-destination problem is said to be basic if the number of positive allocations are equal to $(m + n - 1)$

Optimal solution

A feasible solution (not basically basic) is said to be optimal if it minimizes the total transportation cost

3.7 Types of transportation problem

There are two basic types of transportation problem namely;

a. balanced transportation problem

b. unbalanced transportation problem

3.7.1 Balanced transportation problem

When the total availability is equal to the total requirement the problem ($i.e.\ \Sigma a_i = \Sigma b_j$) is said to be a balanced transportation problem

3.7.2 Unbalanced transportation problem

If the total availability at different sources is not equal to the total requirement at different destinations, ($i.e.\ \Sigma a_i \neq \Sigma b_j$), the problem is said to be an unbalanced transportation problem.

Steps to convert an unbalanced problem to a balanced one are

1) If $\Sigma a_i > \Sigma b_j$ i.e. the total availability is greater than the total requirement, a dummy destination is introduced in the transportation problem with requirement $= \Sigma a_i - \Sigma b_j$

2) The unit cost of transportation from each source to this destination is assumed to be zero.

3) If $\Sigma a_i < \Sigma b_j$ i.e. the total availability is less than the total requirement, a dummy source is introduced in the transportation problem with requirement $= \Sigma b_j - \Sigma a_i$. The unit cost of transportation from each destination to this source is assumed to be zero.

After making the necessary modifications in the given problem to convert it to a balanced problem, it can be solved using any of the methods.

- Include a dummy source or a dummy destination having a supply d or demand d to convert it to a balanced transportation problem.

- Where $d = \Sigma b_j - \Sigma a_i$ or $\Sigma a_i - \Sigma b_j$ respectively

3.8 Formulation of transportation problem

Suppose there are m warehouses $(w_1, w_2, w_3, \cdots, w_m)$, where the commodity is stocked and n markets where it is needed.
Let the supply available in warehouses be $a_1, a_2, a_3, \cdots, a_m$.
The demands at the markets $(m_1, m_2, m_3, \cdots, m_n)$ be $b_1, b_2, b_3, \cdots, b_n$.
The unit cost of shipping from warehouse i to a market j is C_{ij} $(C_{11}, C_{12}, \cdots, C_n)$,
Let $X_{11}, X_{12}, X_{13}, \cdots, X_{mn}$ be the distances from warehouse to the markets, we want to find an optimum shipping schedule which minimises the total cost of transportation from all warehouses to all the markets.

3.8.1 Transportation algorithm

This algorithm minimizes the cost of transporting goods from m origins to n destinations along $m * n$ direct routes from origin to destination. If the sum of the supplies at the m sources equals the sum of the demands at the destinations, then the problem is called balanced and the algorithm proceeds as described below. If the problem is not balanced to begin with, it can be balanced by adding a fictitious supply node (when supplies are short) or a fictitious demand node (when excess supplies are available) to balance the problem before beginning the algorithm. Origins are customarily listed along the left side of the table with supply amounts listed along the right side of the table, and Demands are customarily listed along the top of the table with demand amounts listed along the bottom side of the table. Per unit transportation costs are given in small boxes at the top of each cell in the rectangular matrix, where a zero unit cost is used for an unshipped units column (excess supply), and either a zero unit cost or a penalty unit cost is used for a shortage row (deficient supplies).
Like the more general Simplex Algorithm, the procedure works in two phases. The first Phase I algorithm allocates supplies to demands using a greedy minimal unit cost approach to generate a feasible solution, which however is not necessarily

optimal. Then an optimizing Phase II procedure follows which checks for optimality conditions, and makes cost reducing improvements to the solution in case optimality conditions are violated. The Phase II iterations stop when the optimality conditions are finally met, at which time no further cost reductions are possible.

3.8.2 Mathematical statement of the transportation problem

The classical transportation problem can be stated mathematically as follows: Let a_i denotes quantity of product available at origin i, b_j denotes quantity of product required at destination j, C_{ij} denotes the cost of transporting one unit of product from source/origin i to destination j and x_{ij} denotes the quantity transported from origin i to destination j.

Assumptions: $\Sigma a_i = \Sigma b_j$

This means that the total quantity available at the origins is precisely equal to the total amount required at the destinations. This type of problem is known as balanced transportation problem. When they are not equal, the problem is called unbalanced transportation problem. Unbalanced transportation problems are then converted into balanced transportation problem using the dummy variables.

3.8.3 Degeneracy

The number of constraints in transportation table is $(m+n)$, where m denotes the number of rows and n denotes the number of columns. The number of variables required for forming a basis is one less, i.e. $(m + n - 1)$. This is so, because there are only $(m + n - 1)$ independent variables in the solution basis.

In other words, with values of any $(m+n-1)$ independent variables being given, the remaining would automatically be determined on the basis of those values. Also, considering the conditions of feasibility and non-negativity, the numbers of basic variables representing transportation routes that are utilized are equal to $(m + n - 1)$ where all other variables are non-basic, or zero, representing the unutilized routes. It means that a basic feasible solution of a transportation problem has exactly $(m+n-1)$ positive components in comparison to the $(m+n)$ positive components required for a basic feasible solution in respect of a general linear programming problem in which there are $(m+n)$ structural constraints to satisfy.

Degeneracy exists in a transportation problem when the number of filled cells is less than the number of rows plus the number of columns minus one $(m+n-1)$. Degeneracy may be observed either during the initial allocation when the first entry in a row or column satisfies both the row and column requirements or during the Stepping stone method application, when the added and subtracted values are equal. Degeneracy requires some adjustment in the matrix to evaluate the solution achieved. The form of this adjustment involves inserting some value in an empty cell so a closed path can be developed to evaluate other empty cells. This value may be thought of as an infinitely small amount, having no direct bearing on the cost of the solution. Procedurally, the value (often denoted by the Greek letter epsilon, -) is used in exactly the same manner as a real number except that it may initially be placed in any empty cell, even though row and column requirements have been met by real numbers. A degenerate transportation problem is where we can see that if there are boxes which were not assigned to the matrix, it would be impossible to evaluate several cells.

Once a box has been inserted into the solution, it remains there until it is removed by subtraction or until a final solution is reached. While the choice of where to put an allocation is arbitrary, it saves time if it is placed where it may be used to evaluate as many cells as possible without being shifted. In the phase II discussion, we refer to Basic cells and Non-basic cells to distinguish between cells which may have positive flow and those which are currently set to zero flow. The transportation problem theorem tells us that the basic cells will always lie in cells which correspond to a spanning tree in the network model for the problem, so there will always be exactly $m + n - 1$ Basic cells

3.9 Solution for a transportation problem

3.9.1 Solution algorithm for the transportation problem

The solution algorithm to a transpiration problem can be summarized into following steps:

Step 1 . Formulate the problem and set up in the matrix form. The formulation of transportation problem is similar to LP problem formulation. Here the objective function is the total transportation cost and the constraints are the supply and demand available at each source and destination, respectively.

Step 2. Obtain an initial basic feasible solution and optimal solution. This solution can be obtained by using any of the following methods:

i. Unique quality method

ii. Minimum-demand supply Method

iii. Best candidate Method

The solution obtained by any of the above methods must fulfil the following conditions:

i. The solution must be feasible, i.e., it must satisfy all the supply and demand constraints (popularly known as RIM CONDITION)

ii. The number of positive allocation must be equal to $m + n - 1$, where, m is number of rows and n is number of columns

The solution that satisfies the above mentioned conditions are called a non-degenerate basic feasible solution.

Repeat Step 3 until the optimal solution is obtained.

3.9.2 Unique Quality Method

The major advantage this method has over the commonly used methods is complexity. This is less complex as compared to these existing methods in the sense that this method gives the initial basic feasible solution as well as the optimal solution whereas the other methods do not work the same way. The following are the steps for solving the Transportation Problem using the unique quality method.

Summary of Steps of the unique quality method

Step 1 Select the first row (source) and verify which column (destination) has minimum unit cost. Write that source under column 1 and corresponding destination under column 2. Continue this process for each source. However if any source has more than one same minimum value in different destination then write all these destination under column 2.

Step 2 Select those rows under column-1 which have unique destinations. However, if destinations are not unique then follow step 3. Next delete that row/column where supply/demand exhausted.

Step 3 If destination under column-2 is not unique then select those sources where destinations are identical. Next find the difference between minimum and next minimum unit cost for all those sources where destinations are identical.

Step 4 Check the source which has maximum difference. Select that source and allocate a minimum of supply and demand to the corresponding destination.

Delete that row/column where supply/demand exhausted.

Remark 1 For two or more than two sources, if the maximum difference happens to be same then in that case, find the difference between minimum and next minimum unit cost for those sources and select the source having maximum difference. Allocate a minimum of supply and demand to that cell.

Next delete that row/column where supply/demand exhausted.

Step 5 Repeat steps 3 and 4 for remaining sources and destinations till $(m+n-1)$ cells are allocated.

Step 6 Total cost is calculated as sum of the product of cost and corresponding allocated value of supply/ demand. That is,

$$\text{Total cost} = \Sigma\Sigma C_{ij} X_{ij}$$

ILLUSTRATIVE EXAMPLE 1 ON TRANSPORTATION PROBLEM

A firm owns facilities at six places. It has manufacturing plants at places A, B and C with daily production of 50, 40 and 60 units respectively. At point D, E and F it has three warehouses with daily demands of 20, 95 and 35 units respectively. Per unit shipping costs are given in the following table. If the firm wants to minimize its total transportation cost, how should it route its products?

Table 3.2: A Balance Transportation Problem

	To / From	Warehouse			Supply
		D	E	F	
Plant	A	6	4	1	50
	B	3	8	7	40
	C	4	4	2	60
	Demand	20	95	35	150

Solution

Step 1 The minimum cost value for the corresponding sources A,B,C are 1, 3 and 2 which represents the destination F, D and F respectively which is shown in Table 3.2

Column 1	Column 2
A	F
B	D
C	F

Step 2 Here the destination D is unique for source B and allocate the cell (B, D) min (20, 40) =20. This is shown in Table 3.3

Table 3.3:

From \ To	D	E	F	Supply
A	6	4	1	50
B	$\boxed{20}$ 3	8	7	40 (20)
C	4	4	2	60
Demand	20	95	35	150

Step 3 Delete column D as because this destination demand is exhausted and adjust supply as (40-20) =20. Next the minimum cost value for the corresponding sources A,B,C are 1, 7 and 2 which represents the destination F, F and F respectively which is shown in Table 3.4

Column 1	Column 2
A	F
B	F
C	F

Here the destinations are not unique because sources A, B, C have identical destination F. so we find the difference between minimum and next minimum unit cost for the sources A, B and C. The differences are 3, 1 and 2 respectively for the sources A, B and C. That is:

Column 1	Difference in minimum and next minimum
A	3
B	1
C	2

Step 4: Here the maximum difference is 3 which represents source A. Now allocate the cell (A, F), min (50, 35) = 35 which is shown Table 2.2

Table 3.4:

From \ To	E	F	Supply
A	4	35 1	50 (15)
B	8	7	20
C	4	2	60
Demand	95	35	150

Step 5: Delete column F as demand is exhausted. Next adjust supply as (50-35) =15. Next the minimum unit cost for the corresponding sources A, B and C are 4, 8 and 4 which represents the destination E, E and E respectively which is shown in Table 2.2

Column 1	Column 2
A	E
B	E
C	E

Here the source A, B, C have identical destination E, so we must find minimum difference. However only one column remain and hence minimum difference can not be obtained. So allocate the remaining supply 15, 20 and 60 to cells (A, E) (B, E) and (C, E) which is shown in Table 3.5

Table 3.5:

To / From	E	Supply
A	15 4	15
B	20 8	20
C	60 4	60
Demand	95	150

Step 6: Here (3+3-1) =5 cells are allocated and hence we got our feasible solution. Next we calculate total cost as some of the product of cost and its corresponding allocated value of supply/demand which is shown in Table 3.6

Table 3.6: Final Table

To / From	D	E	F	Supply
A	6	15 4	35 1	50
B	20 3	20 8	7	40
C	4	60 4	2	60
Demand	20	95	35	150

Testing for Optimality

The allocations made by this method is feasible since

$$(m + n - 1) = 3 + 3 - 1 = 5, which equals$$

the number of allocations made. Since the number of occupied cell 5 is equal $(3 + 3 - 1)$, The condition is satisfied The solution is complete when all rim requirements are satisfied.

Total Transportation cost

Next step is to calculate the total transportation cost: Total cost: $(15*4)+(35*1)+(20*3)+(20*8)+(60*4) = 555$

3.9.3 Minimum Demand-Supply Method

Algorithm for solving Transportation Problem using alternate method: The following are the steps for solving Transportation Problem

Step 1 Formulate the problem and set up in the matrix form. The formulation of TP is similar to that of LPP. So objective function is the total transportation cost and constraints are the supply and demand available at each source and destination respectively.

Step 2 Select that row/column where supply/demand is minimum. Find the minimum cost value in that respective row/column. Allocate minimum of supply demand to that cell.

Step 3 Adjust the supply/demand accordingly.

Step 4 Delete that row/column where supply/demand is exhausted.

Step 5 Continue steps 1 to step 3 till $(m + n - 1)$ cells are allocate.

Step 6 Total cost is calculated as sum of the product of cost and corresponding assigned value of supply/demand. That is, Total cost $= \Sigma\Sigma C_{ij}X_{ij}$

APPLICATION OF MINIMUM DEMAND-SUPPLY METHOD TO ILLUSTRATIVE

EXAMPLE 1 OF BALANCED TRANSPORTATION PROBLEM

Step1: General transportation matrix is shown in Table 3.7

Table 3.7:

	To / From	Warehouse D	E	F	Supply
Plant	A	6	4	1	50
	B	3	8	7	40
	C	4	4	2	60
	Demand	20	95	35	150

Step 2: In table 2.5, among supply and demand, minimum is demand which represents column D. In column D, the minimum unit cost is in cell (B, D). Corresponding to this cell demand is 20 and supply is 40. So allocate min (20, 40) = 20 to cell (B, D).

Step 3: For row B is adjusted as 40-20=20, which is shown in Table 3.8

Table 3.8:

To / From	D	E	F	Supply
A	6	4	1	50
B	20 3	8	7	40 (20)
C	4	4	2	60
Demand	20	95	35	150

Step 4 Since demand in column D is exhausted and hence delete column D.

Step 5 Next among supply and demand, minimum is supply which represents row B. In row B, the minimum unit cost is in cell (B, F). Corresponding to this cell demand is 35 and supply is 20. So allocate min (20, 35) =20 to cell (B, F). For column F is adjusted as 35-20=15, which is shown in Table 3.9

Table 3.9:

From \ To	E	F	Supply
A	4	1	50
B	8	$\boxed{20}$ 7	~~20~~
C	4	2	60
Demand	95	35-20 = 15	150

Step 6 Since supply in row B is exhausted and hence delete row B. Next among supply and demand, minimum is demand which represents column F. In column F, the minimum cost value is in cell (A, F). Corresponding to this cell demand is 15 and supply is 50. So allocate min (15, 50) =15 to cell (A, F). Now row A is adjusted as 50-15=35, which is shown in Table 3.10

Table 3.10:

From \ To	E	F	Supply
A	4	$\boxed{15}$ 1	50-15 =35
C	4	2	60
Demand	95	~~15~~	150

Step 5 Since supply in column F is exhausted and hence delete column F. Next among supply and demand, minimum is supply which represents row A. In row A, the minimum cost value is in cell (A, E). Corresponding to this cell demand is 95 and supply is 35. So allocate min (95, 35) =35 to cell (A, E). Now column E is adjusted as 95-35=60, which is shown in Table 3.11

Table 3.11:

To / From	E	Supply
A 4	35 1	~~35~~
C	60 4	~~60~~
Demand	~~95~~	150

Step 5 Here only one cell C is remains so allocate min (60, 60) =60 to cell (C, E). The final allocated supply and demand is shown in Table 3.12

Table 3.12: Final Table

To / From	D	E	F	Supply
A	6	35 4	15 1	50
B	20 3	8	20 7	40
C	4	60 4	2	60
Demand	20	95	35	150

In Table 3.12, $(3 + 3 - 1) = 5$ cells are allocate and hence we got our feasible **solution.**

Total cost:

$$(35 * 4) + (15 * 1) + (20 * 3) + (20 * 7) + (60 * 4) = 595$$

3.9.4 Best Candidate Method

Algorithm for solving Transportation Problem using the Best Candidate Method:

Step 1:Check the matrix balance, If the total supply is equal to the total demand, then the matrix is balanced and also apply if the total supply is not equal to the total demand, then we add a dummy row or column as needed to make supply is equal to the demand. So the transportation costs in this row or column will be assigned to zero.

Step 2: Appling BCM to determine the best combination that is to produce the lowest total weight of the costs, where is one candidate for each row and column.

Step 3: Identify the row with the smallest cost candidate from the choosen combination. Then allocate the demand and the supply as much as possible to the variable with the least unit cost in the selected row or column. Also, we should adjust the supply and demand by crossing out the row/column to be then assigned to zero. If the row or column is not assigned to zero, then we check the selected row if it has an elemnt with lowest cost comparing to the determined element in the chossen combination, then we elect it.

Step 4: Elect the next least cost from the chosen combination and repeat Step 3 until all columns and rows is exhusted.

APPLICATION OF BEST CANDIDATE METHOD TO ILLUSTRATIVE EXAMPLE 1 OF BALANCED TRANSPORTATION PROBLEM

General transportation matrix is shown in Table 3.13

Step 1: In this problem, the matrix is balanced, where the total supply is equal to the total Demand = 150

Step 2: By using BCM, we determine the best combination that will produce the lowest total weight of the costs, whereby one candidate should be selected for each row and the same for each column. The result from applying BCM is shown

Table 3.13:

	To / From	Warehouse			Supply
		D	E	F	
Plant	A	6	4	1	50
	B	3	8	7	40
	C	4	4	2	60
	Demand	20	95	35	150

in Table 3.14

Table 3.14:

To / From	D	E	F	Supply
A	6	4	1	50
B	3	8	7	40
C	4	4	2	60
Demand	20	95	35	150

The best candidates that give minimum cost have been selected as 1, 3 and 4 representing (row 1 and column 3), (row 2 and column 1) and (row 3 and column 2) respectively as shown in table 3.14 above.

Step3: Identify the row with the smallest cost candidate from the chosen combination. Then allocate the supply and the demand to the variable with the least unit cost in the selected row and column. Also adjust the supply and the demand by crossing out the row/column to be then assigned to zero. If the column is assigned to zero, then elect the cost in the cell because it has a lower cost than the determined cell from the chosen combination

Column 3 is exhausted as seen in table 3.3 above. Supply in row 1 is left with 15 and that should be allocated to the cell with the next minimum unit cost (that

Table 3.15:

From \ To	D	E	F	Supply
A	6	4	$\boxed{35}$ 1	50 (15)
B	3	8	7	40
C	4	4	2	60
Demand	20	95	~~35~~	150

is 4) in the same row 1 in the sense that demand and supply requirements are met. This can be shown in table 3.16

Table 3.16:

From \ To	D	E	Supply
A	6	$\boxed{15}$ 4	~~15~~
B	3	8	40
C	4	4	60
Demand	20	95(80)	150

Row 1 is exhausted and demand in column 2 is scaled down to 80 as seen in table 3.16. Now there should be allocation in the next minimum/selected unit cost (thus 3) and that can be shown in table 3.17.

As seen in table 3.17 Column 1 is exhausted.

Step 4: Elect the next least cost from the choosen combination and repeat Step 3 untill all columns and rows are exhusted shown in table 3.18

In Table 3.19, $(3 + 3 - 1) = 5$ cells are allocate and hence feasible solution is

Table 3.17:

To / From	D	E	Supply
B	$\boxed{20}$ 3	8	40(20)
C	4	4	60
Demand	~~20~~	80	150

Table 3.18:

To / From	E	Supply
B	$\boxed{20}$ 8	~~20~~
C	$\boxed{60}$ 4	~~60~~
Demand	~~80~~	150

Table 3.19: Final Table

To / From	D	E	F	Supply
A	6	$\boxed{15}$ 4	$\boxed{35}$ 1	50
B	$\boxed{20}$ 3	$\boxed{20}$ 8	7	40
C	4	$\boxed{60}$ 4	2	60
Demand	20	95	35	150

obtained. Total cost:

$$(35 * 1) + (15 * 4) + (20 * 3) + (20 * 8) + (60 * 4) = 555$$

3.10 Sensitivity Analysis

This involves the development of understanding how the information in the final tableau can be given managerial interpretations. This will be done by examining the application of sensitivity analysis to the linear programming problems. To analyze sensitivity in linear programming, after obtaining the optimal solution, one of the right-hand-side values or coefficients of objective function are changed, then, the changes in optimal solution and optimal value are examined. The balanced relation between supply and demand in transportation problem makes it difficult to use traditional sensitivity analysis methods. Therefore, in the process of changing supply or demand resources, at least one more resource needs to be changed to make the balanced relation possible. In this study, utilizing the concept of complete differential of changes for sensitivity analysis of right-hand-side parameter in transportation problem, a method is set forth. This method examines simultaneous and related changes of supply and demand without making any change in the basis. The mentioned method utilizes Arasham and Kahn's simplex algorithm to obtain basic inverse matrix

CHAPTER 4

DATA COLLECTION AND ANALYSIS

4.1 Introduction

This chapter discusses data collection and analysis of the study. Gratis Aqua Company has a production point which feeds three different storage facilities located in Juaso, Obogu and Konongo respectively. These three storage facilities then supply mineral water to various destinations within the three districts of Asante Akim namely; Asante Akim North District (AAN), Asante Akim South District (AAS) and Asante Akim Central (AAC).

The packaged mineral water which is temporary stored in the storage facilities are transported in Kia trucks by road to all the destinations in the three districts. The average monthly supply of mineral water from each source and the monthly demand from each district were recorded with the corresponding unit transportation cost from each source to each district.

The data was then modeled as a transportation problem with unique quality method, minimum demand-supply method and best candidate method to ensure the total transportation cost is improved to optimality.

4.2 Factors Affecting Transportation Cost in Gratis Aqua Company

The company known to be one of the best mineral water producing companies in the Asante Akim districts do face some challenges in transportation. With the interview and questionnaire organized by the researcher it was realized that the actual transportation problems are attributed to;

1. Lack of communication
2. Fuel cost
3. Bad (rough) roads
4. Maintenance cost

4.3 Data Collection

Data was collected from Gratis Aqua Company and in the in the mineral water producing companies the volume of the liquid is quantified in millilitres (ml). The volume of one sachet of water is 500ml. Also sachet water is packed in transparent polythene bags. One bag of sachet water contains a total of 30 pieces (sachet water). The required data includes: A list of all products, sources, demand for each product by customer, the full truck transportation cost. The study concerned the supply of sachet water from three storage facilities that is Juaso, Obogu and Konongo to consumers and various point of sales in the three districts of Asante Akim. The study covered data gathered on the periods September 2013-February 2014. The transportation cost for full truckload was known as were production capacities. The demand for each destination was also known in advance. Demand and production capacity were expressed in quantity while the cost of transportation were expressed in Ghana cedis.

4.4 Data source

The data used for the analysis was collected from the transport office of Gratis Aqua Company.

4.5 Transportation matrix for Gratis Aqua Company problem

The collected data on transportation cost is shown in the table below. This data indicates the transportation matrix showing the supply (capacity), demand, and the unit cost per full truck.

Table 4.1: The matrix representation of the problem

To / From	AAC	AAN	AAS	SUPPLY
Juaso	2	4	1	3500
Obogu	3	5	1	2500
Konongo	1	2	2	4000
DEMAND	5000	2000	3000	10000

4.6 UNIQUE QUALITY METHOD

Juaso ········· AAS

Obogu ········· AAS

Konongo ········· AAC

AAC is unique as such Konongo-AAC has to be allocated

Juaso ········· AAS

Obogu ········· AAS

Since there is no unique destination, difference between minimum and next minimum needs to be found in each row.

Juaso ········· 1

Obogu ········· 2

The highest is in row 2 (ie. Obogu) and as such the cell with the minimum unit transportation cost in that row has to be allocated.

Table 4.2:

To / From	AAC	AAN	AAS	SUPPLY
Juaso	2	4	1	3500
Obogu	3	5	1	2500
Konongo	$\boxed{4000}$ 1	2	2	~~4000~~
DEMAND	5000 (1000)	2000	3000	10000

Table 4.3:

To / From	AAC	AAN	AAS	SUPPLY
Juaso	2	4	1	3500
Obogu	3	5	$\boxed{2500}$ 1	~~2500~~
DEMAND	1000	2000	3000(500)	10000

Table 4.4:

To / From	AAC	AAN	AAS	SUPPLY
Juaso	$\boxed{1000}$ 2	$\boxed{2000}$ 4	$\boxed{500}$ 1	3500
DEMAND	1000	2000	500	10000

Table 4.5: Unique quality method final table

From \ To	AAC	AAN	AAS	SUPPLY
Juaso	1000 2	2000 4	500 1	3500
Obogu	3	5	2500 1	2500
Konongo	4000 1	2	2	4000
DEMAND	5000	2000	3000	10000

Total Transportation cost

$= (2 * 1000) + (4 * 2000) + (1 * 500) + (1 * 2500) + (1 * 4000)$

$= 2000 + 8000 + 500 + 2500 + 4000$

$= 17000$

4.7 MINIMUM DEMAND-SUPPLY METHOD

With reference to table 3.8 the minimum among both supply and demand is 2000 representing the demand for AAN (column 2), as such the cell with the minimum unit transportation cost in that column has to be allocated as shown in table 4.6 below

The next minimum among supply and demand is 2000 which represents the supply for Konongo (ie. Row 3). Therefore the cell with the minimum unit transportation cost has to be allocated as shown in table 4.7 below

Table 4.6:

From \ To	AAC	AAN	AAS	SUPPLY
Juaso	2	4	1	3500
Obogu	3	5	1	2500
Konongo	1	$\boxed{2000}$ 2	2	4000(2000)
DEMAND	5000	~~2000~~	3000	10000

Table 4.7:

From \ To	AAC	AAS	SUPPLY
Juaso	2	1	3500
Obogu	3	1	2500
Konongo	$\boxed{2000}$ 1	2	~~2000~~
DEMAND	5000(3000)	3000	10000

The next minimum falls on supply for Obogu and as such the cell with the minimum unit transportation cost on that row (ie. Row 2) has to be allocated as shown in table 4.8 below

Table 4.8:

From \ To	AAC	AAS	SUPPLY
Juaso	2	1	3500
Obogu	3	$\boxed{2500}$ 1	~~2500~~
DEMAND	3000	3000(500)	10000

Table 4.9:

To / From	AAC	AAS	SUPPLY
Juaso	3000 2	500 1	~~3500~~
DEMAND	~~3000~~	~~500~~	10000

Table 4.10: Minimum demand-supply method final table

To / From	AAC	AAN	AAS	SUPPLY
Juaso	3000 2	4	500 1	3500
Obogu	3	5	2500 1	2500
Konongo	2000 1	2000 2	2	4000
DEMAND	5000	2000	3000	10000

Total Transportation cost

$= (2 * 3000) + (1 * 500) + (1 * 2500) + (1 * 2000) + (2 * 2000)$

$= 6000 + 500 + 2500 + 2000 + 4000$

$= 15000$

4.8 BEST CANDIDATE METHOD

With reference to table 4.1 the best candidates are 2, 1 and 2 representing (row 1, column 1), (row 2, column 3) and (row 3, column 2) respectively as shown in table 4.11 below

Table 4.11:

To / From	AAC	AAN	AAS	SUPPLY
Juaso	2	4	1	3500
Obogu	3	5	1	2500
Konongo	1	2	2	4000
DEMAND	5000	2000	3000	10000

The minimum out of the selected ones is 1 representing row 2 and column 3 as such the demand and supply constraint for the mentioned row and column must be fulfilled as shown in table 4.12 below

Table 4.12:

To / From	AAC	AAN	AAS	SUPPLY
Juaso	2	4	1	3500
Obogu	3	5	2500 1	2500
Konongo	1	2	2	4000
DEMAND	5000	2000	3000(500)	10000

The demand constraint has still not exhausted therefore the 500 left in the third column has to be supplied to the next minimum in that same column which is one as shown in table 4.13 below

The next selected minimum is 2 representing row 1 column 1 and as such there should be allocation in that cell as shown in table 4.14 below

Table 4.13:

To / From	AAC	AAN	AAS	SUPPLY
Juaso	2	4	500 1	3500(500)
Obogu	3	5	2500 1	~~2500~~
Konongo	1	2	2	4000
DEMAND	5000	2000	~~3000~~	10000

Table 4.14:

To / From	AAC	AAN	SUPPLY
Juaso	3000 2	4	~~3000~~
Konongo	1	2	4000
DEMAND	5000(2000)	2000	10000

Table 4.15:

To / From	AAC	AAN	SUPPLY
Konongo	2000 1	2000 2	~~4000~~
DEMAND	~~2000~~	~~2000~~	10000

Total transportation cost

$= (2 * 3000) + (1 * 500) + (1 * 2500) + (1 * 2000) + (2 * 2000)$

$= 6000 + 500 + 2500 + 2000 + 4000$

$= 15000$

Table 4.16: Best Candidate method final table

To / From	AAC	AAN	AAS	SUPPLY
Juaso	3000 2	4	500 1	3500
Obogu	3	5	2500 1	2500
Konongo	2000 1	2000 2	2	4000
DEMAND	5000	2000	3000	10000

Table 4.17: The optimal cost solution

To / From	AAC	AAN	AAS
Juaso	3000		500
Obogu			2500
Konongo	2000	2000	

4.9 Interpretation for optimal cost solution

The optimal solution results show that the optimal transportation cost was GH ¢15,000. The Juaso storage facility should supply two districts thus AAC and AAS with a capacity of 3000 and 500 respectively.

The Obogu storage facility should also supply only the AAS district with a monthly capacity of 500 whiles the Konongo storage facility should supply only two districts namely: AAC and AAN with capacities of 2000 and 2000 respectively.

This shows that the transportation of products from Juaso to AAN district should be cancelled. Also transportation of products from the Obogu storage facility to both AAC and AAN districts should be cancelled as well as transporting products from the Konongo storage facility to the AAS district should be stopped. This will help do away with unnecessary transportation cost

Table 4.18:

From	To	Shipment	Cost per unit	Shipment cost
Juaso	AAC	3000	2	6000
Juaso	AAS	500	1	500
Obogu	AAS	2500	1	2500
Konongo	AAC	2000	1	2000
Konongo	AAN	2000	2	4000

The optimal solution depicts that a total of 3000 bags of sachet water should be transported from Juaso to AAC district at a cost of GH ¢6000, a total of 500bags should be transported from Juaso to AAS district at a cost of GH¢500. Also 2500bags should be transported from Obogu to the AAS district at a cost of GH¢2500, 2000bags should be transported from Konongo to the AAC district at a cost of GH¢2000. Lastly, a total of 2000bags should be transported from the Konongo storage facility to the AAN district at a cost of GH¢4000.

4.10 Summary

In this chapter, the researcher considered the data collected from Gratis Aqua mineral water producing company and modeled it. The data was modeled as a transportation problem and the total transportation cost was optimised using unique quality method, minimum demand-supply method and best candidate method

CHAPTER 5

SUMMARY, CONCLUSION AND RECOMMENDATION

5.1 Introduction

This chapter talks about summary of the results and findings, drawing conclusion on what the result says and finally recommending managerial skills and strategies that will help improve the transportation cost for Gratis Aqua mineral water producing company.

5.2 Summary

This study was specifically conducted to minimize the total transportation cost of Gratis Aqua Company. Both primary and secondary data was collected from the company. It was identified that apart from the production site the company has three storage facilities which redistribute the products across the three districts of Asante Akim. The data therefore gives information concerning the number of sources, number of destinations, capacities at each source, demand requirement at each destination and unit transportation cost from each source to each destination. The data was modeled as a transportation problem by three different transportation models namely: unique quality method, minimum demand supply method and best candidate method.

5.3 Conclusion

After going through the analysis, the result produced shows an improvement upon the transportation cost that the company incur. The unique quality method produced a total transportation cost of GH ¢17,000.00, whilst both minimum demand-supply method and best candidate method gave a reduced total transportation cost of GH ¢15,000.00 This signifies that the minimum demand-supply method and best candidate method are transportation models that can be useful in reducing the transportation cost of mineral water producing companies in Ghana.

5.4 Recommendations

I recommend best candidate method as a transportation model and mathematical tool that can be used to minimize the total transportation cost for Gratis Aqua Company and any other mineral water producing company in Ghana. This is in the sense that this model has a lesser computational time, it is not complex and has shown consistency. Also I recommend that the Konongo storage facility should supply only customers in the Asante Akim Central and Asante Akim North districts whereas the Juaso storage facility should supply Asante Akim South and Asante Akim Central and finally the Obogu storage facility should supply customers within the Asante Akim South district only. Monies saved can therefore be used to increase the production points and also either expand the capacities of the storage facilities or construct new storage facilities so that the supply of products can go beyond the three districts.

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