SEQUENTIAL ORDERING OF ROUTES FOR TRUCKS FOR EFFICIENT GARBAGE COLLECTION: CASE STUDY OF SEKONDI –TAKORADI METROPOLITAN ASSEMBLY (STMA).

By

Mathias Gyamfi BSc. (Mathematics)

A Thesis Submitted to Department of Mathematics, Kwame Nkrumah University of Science and Technology in Partial Fulfillment of the Requirements for the Degree of

Master of Science (Industrial Mathematics)

Institute of Distance Learning

KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI, GHANA

SEQUENTIAL ORDERING OF ROUTES FOR TRUCKS FOR EFFICIENT GARBAGE COLLECTION: CASE STUDY OF SEKONDI –TAKORADI METROPOLITAN ASSEMBLY (STMA).

PRESENTED BY

MATHIAS GYAMFI

(MSC. INDUSTRIAL MATHEMATICS, PG4066710)

THESIS SUPERVISOR

DR. S. K. AMPONSAH

APRIL , 2012

CERTIFICATION

I hereby declare that this submission is my own work towards the MSc. and that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the university, except where due acknowledgement has been made in the text.

MATHIAS GYAMFI ……………………… ……………………..

(PG 4066710) Signature Date

Students Name & ID

Certified by:

DR. SAMUEL K. AMPONSAH ……………………… ……………………..
Supervisor Signature Date

Certified by:

MR. KWAKU DARKWAH ……………………… ……………………..
Head of Department Signature Date

Certified by:

PROFESSOR I.K DONTWI ……………………… ……………………..
Dean, IDL Name Signature Date

DEDICATION

I dedicate this work to my mum, Elizabeth Adwoa Kaah and my siblings.

ACKNOWLEDGEMENT

To God be the glory, for great things He has done in my life and by His grace this work has come to a successful completion.

I wish to express my gratitude to my honorable supervisor, Dr. Samuel K. Amponsah, for the opportunity to work in an environment where I had all the necessary assistance and even more.

His patience and precision have been essential factors to the success of this research.

My gratitude also goes to Mr. John Gorkeh-Miah, head of STMAWMD and Mr. L. Ushack, Principal director for Operations Unit, STMAWMD, for given me the necessary assistance during data gathering process.

Special thanks to Mr. John Awuah Addor, for his constant encouragement, his great help in clarifying and synthesizing the results of this work.

I wish to thank Mr. Ekow Dadson, Surveying Department of STMA for helping in reading the Cartesian coordinates from the grid map.

I'm in debt with the people at the Department of Mathematics and Statistics, Takoradi Polytechnic and all the lecturers at KNUST Mathematics Department that have demonstrated sympathy and kindness to me.

Last but not least, special thanks to my lovely family and friends, for always encouraged and supported me in following my path.

May the Lord replenish whatever you have lost because of me.

ABSTRACT

Transportation of solid waste product from the cities in Ghana to the destinations for proper disposal has become a unique aim of most metropolitan assemblies in order to make the cities clean from dirt and thereby prevent the outbreak of some diseases and also make the environment smell good. Sekondi-Takoradi Metropolitan Assembly (STMA) has trucks to run to and from various garbage collection points for disposal. It is hard to believe that most of the garbage containers may get full but the trucks may not be available for collection due to the fact that the trucks may not operate effectively and efficiently. A careful study revealed that all the systems on which most of the officers assign truck to a specific route for garbage collection have no scientific basis. This resulted into inefficient ordering of trucks to routes. This work presents a case study of the Vehicle Routing Problem (VRP). The core objective is to minimize the total lengths taken by trucks of the Waste Management Department of Sekondi-Takoradi Metropolitan Assembly in transporting the waste from the Metropolis to the Dump Site. The problem was formulated as an Integer Programming Model and the Ant Colony Meta-heuristic for the Travelling Salesman Problem was used in obtaining optimal solution. Data on distances between potential garbage picking points were obtained. The Cartesian coordinates of the various garbage collection points were collected and used as the distance matrix table for each zone. The optimal solutions were obtained with the help of a Matlab implementation codes. The results revealed an outstanding performance of the Ant Colony Optimization Algorithm in terms of efficiency. In summary, there is reduction of total cost by GH$56000.35 which is 35% of the total cost.

TABLE OF CONTENT

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1

1.0 INTRODUCTION

Solid waste refers to wastes from households, municipal services, construction debris and the agricultural sector. This also includes non-hazardous, non-liquid wastes from institutions and industries. According to the World Bank (2001), its generation is greatly affected by a country's development. Generally, the more economically prosperous a country is, the more waste is generated per capita.

Solid waste management (SWM), on the other hand, pertains to the control of the generation, storage, collection, transfer and transport, processing and disposal of solid waste in a fashion that is in accordance to societal and economic needs while at the same time compliant to environmental standards and principles.

Solid waste is a telltale sign of how citizens' lifestyles change as a result of economic development. Furthermore, the distribution of waste generation in the different regions of a country is indicative of its degree of urbanization. In cities, where standard of living is higher, there is usually a higher waste output compared to rural areas. This is reflective of the case of a developing country like Ghana where its Western Regional capital (Sekondi-Takoradi) generates almost a quarter of the country's total waste generation (STMA, 2010).

1.1 BACKGROUND OF THE STUDY

Transportation of solid waste product from the cities in Ghana to the destinations for proper disposal has become a unique aim of most metropolitan assemblies in order to make the cities clean from dirt and thereby prevent the outbreak of some diseases and also make the environment smell good. Recent events in major urban centres in Africa have shown that the problem of waste management has become a monster that has aborted most efforts made by city authorities, state and federal governments, and professionals alike. A visit to any African city today will reveal aspects of the waste-management problem such as heaps of uncontrolled garbage, roadsides littered with refuse, streams blocked with junk, disposal sites constituting a health hazard to residential areas, and inappropriately disposed toxic wastes. The high rate of urbanization in African countries implies a rapid accumulation of refuse and Ghana is of no exception.

The high rate of urbanization in the country's regions mainly in the densely population areas (the cities) implies a rapid accumulation of refuse. These wastes, usually solid, consist of waste generated from human. Because these wastes are considered as useless they are disposed of. In the metropolis, Domestic, Industrial and commercial wastes are generated. Solid Industrial, Commercial and Domestic wastes are referred to as refuse, solid wastes generated in the households are referred to as domestic-type refuse.

In Ghana and for that matter Sekondi-Takoradi Metropolitan Assembly (STMA) has trucks to run to and from their various collection points where garbage are gathered to the destination for disposal. It is hard to believe that most of the garbage containers may get full but the trucks may not be available for collection due to the fact that the trucks may not run effectively and efficiently. A careful study revealed that all the systems on which most of the officers assign truck to a specific route for garbage collection have no scientific basis. This resulted into inefficient ordering of trucks to routes. The task of finding an efficient route is an important logistic problem (Bell and McMullen, 2004). When a metropolis is able to reduce the length of its delivery route then it would be able to provide a better service to the community and the nation at large. A typical truck routing problem involves determining the routes for several trucks from a given garbage points and returning to the disposal destination without exceeding its capacity. Some commercial streets in the cities in most cases are filled with garbage, producing bacteria and other harmful organism which sometimes results into the outbreak of contagious diseases and subsequently death of an individual. This phenomenon of individuals contracting these contagious diseases affects the production of the nation as the infected people may take some days, weeks or months to recover. Since the assigning of trucks to routes for garbage collection have no scientific basis, it affects the effectiveness and efficiency of garbage collection. Unnecessary delay in truck average time from garbage collection points to the disposal destination and therefore, a lot of garbage are left in the streets and also causing increase in cost of transportation, servicing the trucks and makes the streets smell bad. Sometimes the trucks find it difficult to get access to the garbage collection points due to the fact that some garbage collection points are located in the commercial streets which in most cases seem to be busy.

1.1.1 PROFILE OF THE SEKONDI-TAKORADI METROPOLITAN ASSEMBLY WASTE MANAGEMENT DEPARTMENT (STMAWMD)

The Sekondi-Takoradi Metropolitan Assembly (STMA) has its administrative Head Quarters at Sekondi. The Metropolis is made up of four (4) Sub Metros namely:

(i) Sekondi Sub Metro

(ii) Takoradi Sub Metro

(iii) Essikado and Ketan Sub Metro

(iv) Kwesimintsim and Effia Sub Metro

Out of these four (4) only two (2) are in operation and they are Sekondi Sub Metro and Takoradi Sub Metro. The non-operational Sub Metros are Essikado/Ketan and Effia Kwesimintsim. However, Essikado-Ketan and Effia Kwesimintsim Sub Metros are attached to Sekondi Sub Metros. The Metropolis is bordered on the West by Ahanta West District, to the North by Mpohor Wassa East, to the East by Shama District Assembly and to the South by the Gulf of Guinea. The Metropolis is at the West Coast and about 280km West of Accra and 151km East of La Cote D'Ivoire. The City is strategically located considering its closeness to the sea with a sea Port, Airport and accessibility to major cities either by road, by Air or by sea. The Local Government Act of 1993, Act 462 established and regulates the local government system in Ghana. Section 38 of this Act, provides for the establishment of 16 departments within a Metropolitan Assembly (MA) for the efficient discharge of its functions. The Waste Management Department (WMD) is one of the departments established by the MA to manage environmental sanitation services. STMA established its WMD in 1994. The Main Office of the WMD, is located at the Metropolitan Assembly, and consists of the Head of WMD, two Deputies – Deputy Head 1 (Planning, Research, Contract, Administration) Deputy Head 2 (Daily Technical Operations), six operational units namely the Solid Waste, Liquid Waste, Drains Maintenance unit, Plant & Equipment, Finance and Administration and Support services Unit. The numbers of skilled and unskilled employees are 57 and 444 respectively. The functions of STMAWMD is to collect, transport and disposal of solid and liquid waste, desilting, cleansing and maintenance of public drains, Street and kerb sweeping, Monitoring and supervision of private waste companies, supervision of the management of the final disposal site, and ensure efficient management and maintenance of public toilets and urinals. There are six trucks that are in good shape for the garbage collection: Two tractors, one compactor, one arm roll on and two tipper trucks. These trucks are used for street liter bins / door to door service. Other services rendered by STMAWMD are communal container lifting, supervising of activities at final disposal site, general clean up exercises and evacuation of refuse heaps

1.2 STATEMENT OF PROBLEM

The rapid rate of uncontrolled and unplanned urbanization in the developing nations of Africa has brought environmental degradation. Indeed, one of the most pressing concerns of urbanization in the developing world, especially in Africa, has been the problem of solid, liquid, and toxic-waste management. Recent events in major urban centres in Africa have shown that the problem of waste management has become a monster that has aborted most efforts made by city authorities, state and federal governments, and professionals alike. In recent years, several large foreign loans have been secured to help tackle the problems of environmental sanitation. A visit to any African city today will reveal aspects of the waste-management problem such as heaps of uncontrolled garbage, roadsides littered with refuse, streams blocked with junk, disposal sites, and inappropriately disposed toxic wastes which pose public health risks and asthetic burdens to the citizens they are meant to serve .

The aim of the research is to determine from the mathematical modeling point of view, how to minimize operational cost by determining the optimal route for the trucks, number and size of trucks to access and collect refuse at various garbage collection points taking into consideration various capacities and demand levels in order to keep the Metropolis clean and thereby provide a healthy life for her inhabitants.

1.3 OBJECTIVES

This research is focused on analyzing the transportation of garbage from STMA to the disposal destination. The objectives of the study are:

(i) to formulate a mathematical model that takes into account actual distances between various garbage picking points of the respective trucks.

(ii) to select truck stopping points from a set of potential truck stopping points.

(iii) to determine optimal routes for all trucks.

(iv) use results to recommend a cost effective way of managing transportation aspects of the waste collection operations in the metropolis.

1.4 METHODOLOGY

The data will be collected from the Sekondi-Takoradi Metropolitan Assembly which is the site for this research, and is purely primary and quantitative in nature. The research will also involve relevant literature from the library and the internet as well as other documented data made available by the waste management department of the STMA and other relevant information to be gathered from the field trips and interviews in respect of workers and management personnel

The model will be based on the transportation algorithm of Linear programming via meta-heuristic of ant colony optimization (ACO) as a traveling salesman problem which will be solved using Matlab implementation codes.

1.5 JUSTIFICATION

The truck routing systems used by almost every Garbage Collection Agent (GCA) in the Sekondi-Takoradi Metropolis are non-scientific. It is believed that when the research is successfully completed and recommendations well considered, Sekondi-Takoradi Metropolitan Assembly Waste Management Agents (STMAWMA) responsible for garbage collection will adopt a standard mathematical model (scientific-based transportation system) for their operations. Other Metropolis could also use the model to improve their waste transportation system and also if the model is adopted it will improve nationwide output in terms of efficient transportation of Solid Waste (SW) from the cities and the cost involved in transporting SW would be minimized.

It will also serve as source of literature for other researchers who are interested in the research area.

1.6 LIMITATIONS

This research should have covered all the garbage collection agents in the metropolis but it was limited to only STMAWMD garbage collection agents due to time and financial constraints. The entire research was to be completed within a given time period. However, time is needed sufficiently to review literature, understanding methodologies, testing alternative algorithms to select efficient ones. The limited time at the researcher's disposal led to few items being reviewed as literature. The quality of data depends not only on the amount of time one spends in gathering them but partially on how much money one is prepared to spend in gathering the data. The following difficulties were also encountered by the researcher in the course of data collection process. The nature of the study requires the researcher to collect data on distances from one garbage collection point to the other for all the trucks considered. Getting a reliable and efficient measuring instrument to take coordinates of the picking points did not come with ease, which could affects accuracy of the research and also cause a delay in the completion of the work.

1.7 ORGANIZATION

The research work is made up of five (5) chapters. Chapter one is made up of information on the background of the study, problem statement, objectives of the study, methodology, justification, limitations and organization of the study. Chapter two reviews related works of some authors about truck routing system (TRS) and also talks about ways of managing solid waste (MSW).

Chapter three presents the description and mathematical formulation of the problem with underlying assumptions. Chapter four talks about the experimental results of the data collected and display them in both figures and summary table. Chapter five, which is the last chapter presents the discussions of results, conclusions and recommendations.

CHAPTER 2

LITERATURE REVIEW

2.0 INTRODUCTION

The wish of every society is to grow in population, value, wealth and knowledge. However, a peak is always reached in the management of this growth, at which point additional development becomes counterproductive. It must also be said that values and production can diminish even before this peak is reached. This might be a result of poor management, poor programs, inadequate facilities, and so on. This is perhaps best illustrated by the positive and negative impacts of the urbanization process in Ghana.

Urbanization introduces society to a new, modern way of life, an improved level of awareness, new skills, a learning process, and so on. However, when the rate of urbanization gets out of control, it poses a big challenge to governance — optimizing forces become weakened, institutional capabilities become inadequate and ineffective, and, with these, the problems of urbanization are compounded.

Handling of domestic-type refuse in the Sekondi-Takoradi metropolis has been a very challenging task from time immemorial, mainly due to the public's attitude and its multi-sectorial involvement (STMA-2003, end of year report on solid waste). Management of domestic waste involves a range of technologies associated with the control of the generation, storage, collection, transportation and disposal of all forms of solid wastes. To satisfy the World Health Organization (WHO) standards all of these processes need to be carried out within acceptable legal and social guidelines that will prevent solid waste hazards to the public, animals and the environment as a whole. These guidelines must be hygienically, aesthetically and economically acceptable. In this respect, waste management authorities must be responsive to public attitudes, collection, transportation, and treatment and disposal process (STMA, 2003). Between 1960 and 1984, the population of the Metropolis grew rapidly from 152,607 to 249,371, at a rapid growth rate of 3.5% per annum. The population now stands at about 360,000 (Ghana Statistical Service report, 2000). Thus the 2010 Population and Housing Census Report would give updated figure of the population of the Metropolis. The general sanitation of the metropolis has been so challenging and has led to environmental and health problems (Environmental Protection Agency, 1999). The domestic and commercial wastes are left in the various points of collection (STMA, 2001). This has led to an environmental and health nuisance for instance with regard to the destruction of the aesthetic nature of the environment, air pollution and outbreaks of waste related diseases like malaria, cholera and typhoid fever (Ministry of Health, Ghana, 2002).

In view of these the WHO in 1971 declared that ―solid waste management is an important aspect of environmental hygiene and needs to be integrated with total environmental planning. Aesthetic wise, indiscriminate waste or refuse dumping destroys the natural beauty of the environment and even the ―man created environment. Refuse left unattended to at the various collection points in the communities lead to disease out breaks. According to the Ministry of Health (MOH), Sekondi-Takoradi, the morbidity cases of some waste related diseases is as shown in Table 2.1.

Table2.1: Some Refuse related Diseases in Sekondi-Takoradi Metropolis

Year	Disease	Morbidity	Grand total	Percentage, %
2008	Malaria	86,293	158,086	54.59
	Cholera	5,517		3.49
	Typhoid Fever	6,276		3.97
2009	Malaria	89,447	196,762	45.46
	Cholera	7,200		3.66
	Typhoid Fever	9,115		4.63
2010	Malaria	65,814	176,005	37.39
	Cholera	1,879		1.07
	Typhoid Fever	8,312		4.72

Source: Metro Health Service, Sekondi

2.1 DEFINITION OF WASTE

Waste may be defined as a discarded material, which has no consumer value to the generator (Hagerty et al., 1973; Kreith et al, 1994). The term solid waste includes any garbage, refuse, sludge from a waste treatment plant, water supply treatment plant, or air pollution control facility, and other discarded material, including solid, liquid, semi-liquid, or contained gaseous materials resulting from industrial, commercial, mining, and agricultural operations and from community activities (Hagerty et al., 1973). This does not include solid or dissolved materials in domestic sewage or solid or dissolved materials in irrigation return flows or industrial discharges. Also excluded are agricultural wastes, including manures and crop residues, returned to the soil as fertilizers or solid conditioners (Hagerty et al., 1973).

2.2 CLASSIFICATION OF WASTE

There are varieties of solid waste which includes: food waste which may comprise of remnants of food stuff, Rubbish and these may be combustible (plastics, wood, textiles, leather, garden trimmings, papers etc) or non-combustible (tin cans, metal scraps, glass etc), Demolition and construction waste (construction waste and waste from razed building) Ashes and residues mainly from burnt woods, there are also Agricultural waste, Hospital waste etc.

For the purpose of this study, only domestic – type refuse generated in the metropolis will be considered.

2.3 SOLID WASTE TRANSPORTATION

Solid waste is been transported by three (3) main means, namely Motor vehicles, Railways and Water transport. Hydraulic and Pneumatic systems have also been used (Hagerty et al., 1973). In the Sekondi-Takoradi metropolis, refuse is been transported by motor vehicles and this process is known as direct haul. (Hardman et al., 1993)

2.3.1 Motor Vehicle Transport

Using of motor vehicles for the transportation of solid waste is possible where the collection point and the final disposal site are accessible by motor vehicles. Even though all vehicles can be used, usually the vehicles used include; semi-trailers, trailers, container trucks and compactors (Feachem et al., 1983). In the Sekondi-Takoradi metropolis, vehicles used must satisfy the following requirements:

(i) Vehicles must be designed for highway traffic.

(ii) Vehicle capacity must be such that the standard weight limits are not exceeded.

(iii) Waste must be covered during the transportation.

(iv) Methods of unloading must be simple and dependable.

(v) Transportation cost of the vehicle must be acceptably minimum.

2.3.2 Railway Transport

Transportation of solid waste via railways were common in the past, currently, in communities where road traffic is heavy, railways are used to transport wastes to remote areas where they are disposed. Areas where highway travel is difficult and railway lines exist, and where land for filling is available railway transport is used (Kreitht et al., 1994). Currently there is nowhere in the metropolis that railway transport is used.

2.3.3 Water Transport

Barges, scows and special boats have been used in the past to transport solid waste to processing locations and to sea side and ocean disposal sites. This was common in the United States and other advanced countries, but ocean disposal is no longer practiced by the United States, although some self-propelled vessels (such as the United States Navy garbage scows and other special boats) have been used. The most common practice is to use vessels towed by tugs or other special boats (Hagerty et al., 1973).

Major problem encountered when ocean vessels are used for the transportation of solid waste is that it is often impossible to sail the barges and boats during storm or times of heavy seas. In such cases, the waste must be stored and construction of expensive storage facilities may be necessary (Ivor and Seeley, 1992). Other means by which solid waste has been transported by water is the hydraulic and the pneumatic methods. The famous Walt Disney World amusement park in Orlando, Florida in the United States uses these methods. Other systems have been suggested for transportation of solid waste, these include conveyors, air-cushion and rubber-tyre trolleys, and underground conduits with magnetically transported gondolas. Unfortunately these systems have not been put into practice (Hagerty et al., 1973).

2.4 SOLID WASTE.

Solid wastes are waste materials that contain less than 70% water. This class includes such materials as household garbage, some industrial wastes, some mining wastes, and oilfield wastes such as drill cuttings. There are different types of solid waste depending on their moisture content. In the metropolis, household garbage forms about 80% of the total solid waste generated with industrial-type and the commercial type-solid waste refuse forming 20%.

2.5 SOLID WASTE DISPOSAL

There are so many ways of disposing solid waste but the most recommended ones are the Landfill, Composting and the Combustion/incineration methods. The Landfill method is the most common and probably accounts for more than 90% of the metropolis' solid waste (STMAWMD).

2.5.1 Landfill

Studies has shown that sanitary landfill is the cheapest satisfactory means of disposal (Kreith et al., 1994), but only if suitable land is within economic range of the source of the wastes; typically, collection and transportation account for 75% of the total cost of solid waste management. In a modern landfill, refuse is spread in thin layers, each of which is compacted by a bulldozer before the next is spread. When about 3 metre (about 10 ft) of refuse has been laid down, it is covered by a thin layer of clean earth, which also is compacted. Pollution of surface and groundwater is minimized by lining and contouring the fill, compacting and planting the cover, selecting proper soil, diverting upland drainage, and placing wastes in sites not subject to flooding or high groundwater levels. Gases are generated in landfills through anaerobic decomposition of organic solid waste. If a significant amount of methane is present, it may be explosive; proper venting eliminates this problem. Sites suitable for sanitary landfill are quarries, gravel pits, low- lying swamps, marshes etc. The best soil for a landfill is clay because clay is less permeable than other types of soil.

In choosing a place for a landfill site, the following must be considered.

(i) Proximity to residential areas.

(ii) Wind direction, and access by road, rail or water.

(iii) Areas close to airports must be avoided to prevent bird-strikes.

(iv) Site hydrogeology must be conducted to prevent pollution of water supplies and the underground water.

The essence of the thin layer of clean earth used and the compaction is to control the tip in the following ways:

(i) Limit the odour emission

(ii) Check the emergence of fly larvae

(iii) Check the breeding of flies and other insects

(iv) Allow easy rat control

(v) Prevent light refuse being blown away

(vi) Make tip less attractive to birds

(vii) Reduce the risk of fire

(viii) Provide good conditions for the biological degradation of organic matter in the tip

In the metropolis, the only landfill is the Kojokrom sanitary landfill, it covers $65km^2$ of land in the outskirt of the Kojokrom township.

2.5.2 Composting

Composting involves the biological stabilization of solid matter either under aerobic or anaerobic conditions (Purdom et al., 1971). This ultimately degrades susceptible organic matter to water, carbon dioxide and stabilized residue, principally humid substance called Compost. Even though composting is gaining grounds in the advanced countries it is mostly used in the developing countries this is as the result of the diminishing availability of landfill sites, the high cost and the relatively high degree of sophistication needed to operate incinerator safely and economically, also materials with carbon-nitrogen ratio greater than 50% are very slow to compost (Kreith et al, 1994). There are two main methods namely windrow composting which the Sekondi-Takoradi metropolis uses and the compost digesting. In composting, materials are spread out over a large land area so that microbes can decompose them, in the developed countries where there is enough resource for waste management the decomposable refuse is separated from the non-decomposable refuse and the decomposable (Ralph et al., 1986) one composted. In the developing countries the opposite is what is practiced, both the decomposable and the non-decomposable refuse are put together and composted. Maximum depth of 5-8 feet prevents compaction, but a depth of 4 feet is needed for insulation. The heap of compost is made by spreading the refuse for a marked duration, in open air windrows to allow biological activities to degrade the wastes (Purdom et al, 1971); the compost must be turned several times per week to supply sufficient oxygen time to keep the windrow aerobic and the biological activity constant.

The length of time required to produce compost varies with the number of times that the compost is been turned and also partially on the temperature of the compost. The time can vary from three or four weeks to several months, other factors are ambient temperatures and the chemical composition of the raw material. Where land limitations do not permit windrow composting to be practical, the compost digesting is used, in this process the refuse is placed in digesters and air is supplied from mechanical blowers, because air is forced through the piles of refuse there is no need to turn the compost to supply oxygen.

2.5.3 Incineration

In incinerators of conventional design, refuse is burned on moving grates in refractory-lined chambers; combustible gases and the solids they carry are burned in secondary chambers. Combustion is 85 to 90 percent complete for the combustible materials. In addition to heat, the products of incineration include the normal primary products of combustion—carbon dioxide and water—as well as oxides of sulfur and nitrogen and other gaseous pollutants; nongaseous products are fly ash and unburned solid residue. Emissions of fly ash and other particles are often controlled by wet scrubbers, electrostatic precipitators, and bag filters. It is very expensive to operate an incinerator due to the high power consumption nature of the incinerators to ensure complete combustion. Also the end products of the incinerators are always harmful to lives and a lot of resources are used to mitigate their harmful nature (Kreith et al., 1994).

2.5.4 Ocean or Sea Dumping

The modern-day sea disposal operation exists because of legislation, which spells out the operational details on the Dumping at Sea Act 1974 (Hagerty et al., 1973). It was taught in the past that Oceans and Seas were capable of receiving and making safe difficult toxic and hazardous waste, sewage sludge and radioactive waste deposited in them. One could suppose that this method is the ultimate ―dilute and disperse option.

This work, SORTEGC, solid waste transportation services (SWTS) is a real-life application of the standard vehicle routing problem (VRP) to solving contemporary routing problems in Sekondi-Takoradi, a Metropolis in the western region of Ghana. The general Vehicle Routing Problem calls for the determination of the optimal set of routes to be performed by a fleet of vehicles to serve a given set of customers.

Following the basis of this model, this section is devoted to review literature of related works by some researchers.

2.6 RELATED WORKS ON VEHICLE ROUTING PROBLEM

Bell and McMullen (2004) applied a meta-heuristic method of ant colony optimization (ACO) to establish a set of vehicle routing problems. The authors modified the ACO algorithm used to solve the traditional traveling salesman problem (TSP) in order to allow the search of multiple routes of the vehicle routing problem (VRP). Experimental results exhibited the success of the algorithm in finding solution within 1% of known optimal solution. The usage of multi ant colonies provides a comparatively competitive solution technique especially for larger problems. Also the size of the candidates list used within the algorithm became a significant factor in finding improved solution. The computational times for the algorithm compared favourably with other solution methods.

Dorigo and Gambardella (1997) introduced ant colony systems (ACS) to the TSP. In order to understand the operation of the ACS, experiments were conducted and the results showed that ACS outperforms other nature-inspired algorithms such as simulated annulling and evolutionary computations. The authors concluded by comparing ACS-3-opt, a version of ACS augmented with a local search procedure to some of the best performing algorithms for symmetric and asymmetric TSP's.

Lenstra et al., (1988) under took a survey of solution method for routing problems with time window constraints. The problem the authors considered include the travelling salesman problem, vehicle routing problem, pickup and delivery problem and the dial-a-ride problem(DARP). The authors implemented optimization algorithms that use brand and bound, dynamic programming and set partitioning and approximation algorithms based on construction, iterative improvement and incomplete optimization.

Ibraki et al., (2001) introduced a vehicle Routing problem with general time constraints. The problem was to minimize the sum of the distances travelled by a fixed number of vehicles, which visit every customer under capacity and time window constraints. The time window constraint was treated as a penalty function, which can be non- convex and discontinuous as long as it is piecewise linear function. First, the authors fixed the order of customers to visit and proceeded to determine the optimal start times to serve customers so that total time penalty of the vehicle is minimized. The authors proved how the problem could efficiently be solved using dynamic programming which was incorporated into the local search algorithms, In the local search, in addition to standard neighborhoods, the authors employed a new type of neighborhood called the cyclic exchange neighborhood, whose size generally growths exponentially with the input size. This difficulty was conquered by finding an efficient heuristic to find an improved solution in the cycle exchange neighborhood via the improvement graph. The computation results revealed good prospects for the proposed algorithms.

Sue (1994) studied the problem of permutation routing and sorting on several model of meshes with fixed and reconfigurable row and column buses. The author described two fast and fairly simple deterministic algorithms for permutation routing on two-dimensional networks and a more complicated algorithms for multi-dimensional networks. The algorithms were obtained by converting two known off-line routing schemes into deterministic routing algorithms which can be implemented on a variety of different models of meshes with buses. A deterministic algorithm for 1-1 sorting whose running time matches that for permutation routing, and another algorithm that matches the bisection lower bound on reconfigurable networks of arbitrary constant dimension, were introduced.

Bowerman, et al., (1995) proposed a new heuristic for urban school Bus Routing. The problem was formulated as a multi-objective model and a heuristic based on this formulation is developed. The study involves two interrelated problems. One has to do with the assignment of students to their respective bus stops and the second has to do with routing of buses to the bus stops. A problem of these characteristics is a location-routing problem. The nature of the formulation made it possible to organize their study into three layers, where layer one is the school, layer two is the bus stops and layer three the students. School buses routes cause interaction between layers one and two, while movements of student cause interaction between layers two and three. The heuristic approach to this problem involves two algorithms which catered for the multi-objective nature of the model. The first is a districting algorithm which groups students into clusters to be serviced by a unique school bus route. The second is a routing algorithm, which generates a specific school bus route that visits a sub set of potential bus stops sites.

Schowenaars et al., (2009) presented a new approach to fuel optimal path planning for multi vehicle using combination of LP and IP. The basic problem formulation was to have the vehicles moving from initials dynamic state to a final state without colliding with each other, while at the same time avoiding other stationary obstacles. The authors showed that the problem could be rewritten as a linear program with mixed integer program linear constraints that account for the collision avoidance. The problem was solved using the CPLEX optimizatice software with an AMPL|Matlab interface.

Marius (1985) designed and analyzed algorithms for vehicle routing and scheduling problems with time window constraints. The author described a variety of heuristics and conducted an extensive computational study of their performance. The problem set include routing and scheduling environment that differ in terms of the type of data used to generate the problem, the percentage of customers with time windows, their tightness and positioning and the scheduling horizon. The observation made was that several heuristics performed well in different problem environ; in particular, an insertion-type heuristics consistently gave very good results.

Desrochers, et al., (1991) proposed the development of a new optimization algorithm for the solution of VRPTW. The authors solved the LP relaxation of the set portioning formulation of the VRPTW by column generation. By this, feasible columns are added as required by solving a shortest path problem with time window and capacity constraint using dynamic programming. The LP solution obtained generally provides an excellent lower bound that is used in a branch-and-bound algorithm to solve integer set partitioning formulation. Their results indicate the success of the algorithm on a variety of practicalized benchmark VRPTW test problems. The algorithm was capable of optimally solving a 100 customer problems. This problem size is six times larger than any report presented by other published research prior to 1990.

Swersey and Ballard (1984) presented a work on scheduling of school buses. With the scheduling situation considered here, a set of routes each associated with a particular school is given. A single bus is assigned to each route to pick up students and arriving at their school within a specific time window. The problem includes finding the fewest buses needed to cover all routes whiles meeting the time window specifications. The authors presented two integer programming formulations of the scheduling problem and applied them to actual data from New Heaven, Connecticut for two different years as well as to 30 randomly generated problems. Linear programming relaxation of the integer programs was found to produce integer solutions more than 75% of the time. In the remaining cases, the authors observed the few functional values can be adjusted to integer values without increasing the number of buses needed. Their method reduces the number of buses needed by about 25% compared to the manual solutions developed by the New Heaven school bus scheduler.

Li and Fu (2002) presented a case study of the bus routing problem. It is formulated as a multi-objective combinatorial optimization problem. The objectives include minimizing the total number of buses required, the total travel time spent by all pupils at all pick-up points and the total bus travel time. The authors also aimed at balancing the loads and travel times between the buses. The authors proposed a heuristic algorithm, which was programmed and run efficiently on a PC. Numerical results were reported using test data from a kindergarten in Hong Kong. This proved to be effective as it save 29% of total travelling times when compared the system under practice.

Tillard et al., (1997) described a tabu search heuristic for the vehicle routing problem with soft time windows. This problem allows lateness at a customer location although a penalty is incurred and added to the objective value. In the tabu search, a neighbourhood of the current solution is created through an exchange procedure that swaps sequences of consecutive customers (segments) between routes. The tabu search also exploits an adaptive memory that contains the routes of the best previously visited solutions. New starting points for the tabu search are produced through combination of routes taken from different solution found in this memory. The authors reported many best known solutions on classical test problems.

A bounacer et al., (2009) proposed a two population meta-heuristics to the professional staff transportation problem (PSTP). PSTP has to do with building the vehicle routing for transporting staff of one or several companies in order to minimize total cost of transport, taking into account the level of service offered to its users. The meta-heuristics developed here in this paper include ACO and Genetic algorithm (GA). Experimental results proved both techniques to be efficient.

Goel and Gruhn (2006) studied a rich vehicle routing problem incorporating various complexities found in real-life applications. The real life requirements they considered include time window restrictions, heterogeneous vehicle fleets with different travel times, travel cost and capacity, multi-dimensional capacity constraints, vehicle compatibility constraints, orders with multiple pick up, delivery and service location, different start and end locations for vehicles and routes restrictions for vehicles. This problem known as General Vehicle Routing Problem (GVRP) is highly constrained and the search space is likely to contain many solutions such that it is impossible to go in for one solution to another using a single neighborhood structure. As a result, the authors proposed iterative improvement approaches based on the idea of changing the neighborhood structure during the search.

Palmgren, et al., (2001) proposed a column generation algorithm for the Log Truck Scheduling problem. Both pick-up and delivery are included in this problem. Its consist of finding one feasible route for each vehicle in order to satisfy demand of customers and in such way that total transport cost is minimized. The authors used a mathematical formulation of the log truck scheduling problem that is a generalized set partitioning problem. The column generation algorithm was applied to solving LP relaxed model and a branch and price algorithm for obtaining integer solutions.

Chen and Zhang (2005) introduced adaptive ant colony optimization. The authors' objective is to improve the critical factor influencing the performance of the parallel algorithm. In their work they proposed strategies for information exchange between processor selections based on sorting and on difference, which makes each processor choose another processor to communicate with and update the pheromone adaptively. In order to increase the ability of the search and avoid early convergence, the authors also introduced a method of adjusting the time interval of information exchange adaptively in accordance with the diversity of the solution. These techniques were applied to the travelling salesman problem on the massive parallel processors and experimental results revealed high convergence speed, high speed up and efficiency.

Ali, et al., (2009) proposed a solution to the minimum vertex cover problem using ant colony optimization. The authors introduced a pruning based ant colony algorithm to find approximate solution to the minimum vertex cover problem. The focus was on improving both time and convergence rate of the algorithm as such the authors introduced a visible set based on pruning paradigm for ant, where in each stop of their traversal, are not forced to consider all the renaming vertices to select the next one for continuing the traversal. The authors' technique was compared to two existing algorithms based on Genetic Algorithms and computational experiment evinced that ACO Algorithm demonstrates much effectiveness and consistency for solving the minimum vertex cover problem.

Ghiduk (2010) presented ant colony optimization based approach for generating a set of optimal paths to cover all definition –use associations (du-pairs) in the program under test. The objective is to use ant colony optimization to generate suit of test-data for satisfying the generated set paths. The authors introduced a case study to illustrate their approach and the algorithm proved to be very efficient.

Leeprechanon, et al., (2010) presented a paper which proposes the appreciation of ant colony optimization to solve a static transmission expansion planning (STEP) problem based on DC power flow model. The authors' major objective is to minimize investment cost of transmission lines added to existing network in order to supply forecasted load as economically as possible and subject to many system constraints-power balance, the generation requirement, line connections and thermal limits. In order to analyze and appraise the feasibility of the ACO, their proposed methodology was applied to the Gaver's six-bus system. Experimental result compared to other conventional approaches of Genetic Algorithm and Tabu search (TS) algorithm revealed the outperformance of ACO in convergence characteristics and computational efficiency.

Nazif and Lee (2010) proposed an "Optimal Crossover Genetic Algorithm for Vehicle Routing Problem with Time Windows". In this work, the authors considered a set of vehicles with limits on capacity and travel times available to service a set of customers with demands and earliest and latest time for serving. The authors' objective is to find routes for the vehicles to service all customers at minimal cost without violating capacity and travel time constraints of vehicles and time window constraints set by customers. Their proposed algorithm was tested with bench mark instances and also compared with other heuristics in the literature. Results proved the competitiveness of the proposed algorithm in terms of quality of the solution found.

Although this work addresses a contemporary routing problem using ACO, it also considers sensibility analysis on the number of ants needed for optimality. This is an important aspect of the work since it will assist in establishing a relationship between the number of ants and optimality.

Kontoravdis and Bard (1995) proposed a greedy randomized adaptive search procedure (GRASP) to VRP with time window. The objective was to address the problem of finding the minimum number of vehicles required to visit a set of node subject to time window constraints. The authors also considered a secondary object centered on minimizing the total distance travelled. Feasible solution obtained from GRASP for standard 100 modes data set as well as for a number of real –world problems with up to 417 customers. Experimental results revealed that their proposed procedure out performs techniques existing at the time and requires only a small fraction of time taken by exact method. The authors gauged the quality of solutions by applying three different lower bounding heuristics. The first considers the "bin parking" aspect of the problem with respect to vehicle capacity; the second is based on the maximum clique associated with customers' incompatibility; the third exploits the time window constraints.

Corberán et al., (2002) addressed the problem of routing school buses in rural areas. The authors approached this problem with a node routing model with multiple objectives that arise from conflicting viewpoints. From the point of view of cost, the number of buses used to transport students from their homes to school and back is minimized. From the service viewpoint, the authors minimized the time that a given student spend on route. The multi-objective employs a weighted function to combine individual objective functions into a single one. The authors developed a solution procedure that considered each objective separately and searched for a set of efficient solution instead of a single optimum. The authors' solution procedure is based on construction, improving and then combining solutions within the frame work of the evolutionary approach known as scatter search.

Schittekat, et al., (2006) formulated the school bus routing problem using a single objective integer programming model VRP by introducing several other interesting additional features. The authors considered a set of potential stops as well as a set of students who can walk to one or more of these potential stops. The goal of their routing problem is to select a subset of stops that will actually be visited by the buses; determine which stops each student should walk to; and develop a set of tours that minimize the total distance travelled by all buses. The problem was solved using a commercial integer programming solver and results on small instances were discussed.

Fisher (1981) implemented a Lagragian Relaxation Method for solving integer programming with many side constraints. The author considered the dual of the side constraints to obtain a Lagragian problem that is easy to solve and whose optimal value is a lower bound (for a minimization problem) on the optimal value of the original problem. The Lagragian has led to dramatically improved algorithms for a number of important problems in areas of routing, location, scheduling, and assignment and set covering.

CHAPTER 3

METHODOLOGY

3.0 INTRODUCTION

This chapter presents the problem formulation, Ant Colony Optimization metaheuristic and the linear programming model.

3.1 THE LINEAR PROGRAMMING MODEL

This work will be formulated into mathematical equations which will be solved using linear programming, specifically Integer Programming. Linear Programming is a mathematical programming model which deals with the allocation of resources to known activities with the objective of meeting a required goal such as maximizing profit or minimizing cost (mathworld.wolfram.com). The concept applied in linear programming is that all equations to be used in the analysis must be linear (straight line equations).Thus, there will be a formulation of linear equations that will determine (route length) of operation for each truck designated to a particular zone, these will be referred to as objective function and those linear equations which will describe the limitations under which the system must operate, called the constraints.

3.1.1 Standard Form

The simplex method for solving linear programming problems requires that the problem be expressed in standard form. A linear programming problem with all the constraints in equation form is said to be in its standard form. Since constraints of linear programming problem are often expressed as inequalities rather than equations, the inequalities are converted to equations by introducing variables to represent the slack or surplus between the left-hand side and right- hand side of each inequality.

The main features of the standard form are:

(i) The objective function is of the maximization or minimization type.

(ii) All constraints are expressed as equations.

(iii) All variables are restricted to be nonnegative.

(iv) The right-hand side constant of each constraint is nonnegative.

Basically the standard form reduces the linear program to a set of m equations in (m + n) unknowns which eventually lead to an infinite number of solutions. It should be noted that, it is computationally very difficult to determine every feasible point, it is therefore important to employ a method that locates the optimum solution after checking a finite number of solution points.

3.1.2 Simplex Method

The simplex method as developed by Dantzig (1947) is an iterative procedure for solving linear programming problems expressed in standard form. In addition the simplex method requires that the constraint equations be expressed as a canonical system from which a basic feasible solution can be readily obtained.

(i) Start with an initial basic feasible solution in canonical form.

(ii) Improve the initial solution if possible by finding another basic feasible solution with a better objective function value.

(iii) Step two is repeated until a particular basic feasible solution cannot be improved any further and this feasible solution eventually becomes the optimal solution and the simplex method terminates.

Definitions associated with the simplex method are;

(i) **Basis matrix**: This is an m x m non-singular matrix formed from m columns of the constraint matrix A. Since rank (A) = m, A contains at least one basis matrix.

(ii) **Basic variable**: A variable x_1 is said to be a basic variable in a given equation if it appears with a unit coefficient in that equation and zero in all other equations.

(iii) **Non-basic variable**: These are variables which are not basic, and therefore do not correspond to the columns of the basis matrix.

(iv) **Basic solution**: The solution obtained from a canonical system by setting the non-basic variables to zero and solving for the basic variable.

(v) **Basic Feasible Solution**: A basic feasible solution is a basic solution in which the values of the basic variables are nonnegative.

(vi) **Non-degenerate basic feasible solution**: This is a basic feasible solution with exactly m positive components.

(vii) **Optimal solution**: This is vector X such that it is feasible and its value of the objective function is larger than that of any other feasible solution. That is it is a feasible solution, which minimizes or maximizes the objective function.

(viii) **Pivot Operation**: A pivot operation is a sequence of elementary operations that reduces a given system to an equivalent system in which a specified variable has a unit coefficient in one equation and zero elsewhere.

By so doing, columns are introduced into and rows are eliminated from the basis matrix. The column that corresponds to the non-basic variable, which is about to be introduced into the basis, is called pivot column. The row that corresponds to the basic variable, which will leave the basis matrix as the algorithm iterates from one feasible solution to another, is called pivot row. The elements of the simplex tableau that is in both the pivot row and pivot column is called pivot elements.

3.2 The Branch-and-Bound Technique to Binary Integer Programming

Any bounded pure IP problem has only a finite number of feasible solutions; it is natural to consider using some kind of enumeration procedure for finding an optimal solution. Unfortunately, as discussed above, this finite number can be, and usually is, very large. Therefore, it is imperative that any enumeration procedure be cleverly structured so that only a tiny fraction of the feasible solutions actually need to be examined. For example, dynamic programming provides one such kind of procedure for many problems having a finite number of feasible solutions (although it is not particularly efficient for most IP problems). Another such approach is provided by the branch-and-bound technique. This technique and variations of it have been applied with some success to a variety of OR problems, but it is especially well known for its application to IP problems.

The basic concept underlying the branch-and-bound technique is to divide and conquer.

Since the original "large" problem is too difficult to be solved directly, it is divided into smaller and smaller sub problems until these sub problems can be conquered. The dividing (branching) is done by partitioning the entire set of feasible solutions into smaller and smaller subsets. The conquering (fathoming) is done partially by bounding how good the best solution in the subset can be and then discarding the subset if its bound indicates that it cannot possibly contain an optimal solution for the original problem.

We shall now describe in turn these three basic steps—branching, bounding, and fathoming—and illustrate them by applying a branch-and-bound algorithm to the prototype example shown below:

(1) $6x_1 + 3x_2 + 5x_3 + 2x4 \leq 10$

(2) $x_3 + x4 \leq 1$

(3) $-x_1 + x_3 \leq 0$

(4) $-x_2 + x_4 \leq 0$ (3.1)

and

(5) x_j is binary, for $j = 1, 2, 3, 4$.

3.2.1 Branching

When you are dealing with binary variables, the most straightforward way to partition the set of feasible solutions into subsets is to fix the value of one of the variables (say, x_1) at $x_1 = 0$ for one subset and at $x_1 = 1$ for the other subset. Doing this for the prototype example divides the whole problem into the two smaller sub problems shown below.

Sub problem 1:

Fix $x_1 = 0$ so the resulting sub problem is

Maximize $Z = 5x_2 + 6x_3 + 4x_4$,

subject to

(1) $3x_2 + 5x_3 + 2x_4 \leq 10$

(2) $\quad x_3 + \quad x_4 \ \leq 1$

(3) $\quad x_3 \quad\quad \leq \ 0$

(4) $-x_2 \quad\quad + x_4 \ \leq \ 0$ (3.2)

(5) x_j is binary, for $j = 2, 3, 4.$

Sub problem 2:

Fix $\mathrm{x}_1 = 1$ so the resulting sub problem is

Maximize $\mathrm{Z} = 9 + \ 5\mathrm{x}2 + \ 6\mathrm{x}_3 + 4\mathrm{x}_4,$

subject to

(1) $3x_2 \ + \ 5x_3 \ + \ 2x_4 \leq 4$

(2) $\quad x_3 \ + \quad x_4 \ \leq 1$

(3) $\quad x_3 \quad\quad \leq 1$

(4) $-x_2 \quad\quad + \ x_4 \ \leq \ 0$ (3.3)

(5) $x_j \ is \ binary, \ for \ j = 1, 2, 3, 4.$

Figure 3.1 portrays this dividing (branching) into sub problems by a tree (with branches (arcs)) from the all node (corresponding to the whole problem having all feasible solutions) to the two nodes corresponding to the two sub problems. This tree, which will continue “growing branches” iteration by iteration, is referred to as the solution tree (or enumeration tree) for the algorithm. The variable used to do this branching at any iteration by assigning values to the variable (as with x_1 above) is called the branching variable. (Sophisticated methods for selecting branching variables are an important part of some branch-and-bound algorithms but, for simplicity, we always select them in their natural order—$x_1, x_2, \ . . . , x_n$ throughout this section.)

Later in the section you will see that one of these sub problems can be conquered (fathomed) immediately, whereas the other sub problem will need to be divided further into smaller sub-problems by setting $x_2 = 0$ or $x_2 = 1$.

For other IP problems where the integer variables have more than two possible values, the branching can still be done by setting the branching variable at its respective individual values, thereby creating more than two new sub problems. However, a good alternate approach is to specify a range of values (for example, $x_j = 0$ or $x_j = 3$) for the branching variable for each new sub problem.

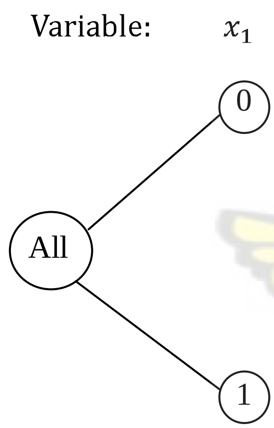

Figure 3.1: Figure 3.1 shows a solution tree created by the branching for the first iteration of the BIP branch and bound algorithm for (3.3)

3.2.2 Bounding

For each of these sub problems, we now need to obtain a bound on how good its best feasible solution can be. The standard way of doing this is to quickly solve a simpler relaxation of the sub problem. In most cases, a relaxation of a problem is obtained simply by deleting ("relaxing") one set of constraints that had made the problem difficult to solve.

For IP problems, the most troublesome constraints are those requiring the respective variables to be integer. Therefore, the most widely used relaxation is the LP relaxation that deletes this set of constraints.

To illustrate for the example, consider first the whole problem (3.3). Its LP relaxation is obtained by replacing the last line of the model (x_j is binary, for $j = 1, 2, 3, 4$) by the constraints that $x_j = 1$ and $x_j = 0$ for $j = 1, 2, 3, 4$. Using the simplex method to quickly solve this LP relaxation yields its optimal solution

$(x_1, x_2, x_3, x_4) = \left(\frac{5}{6}, 1, 0, 1\right)$, with $Z = 16\frac{1}{2}$

Therefore, $Z \leq 16\frac{1}{2}$, for all feasible solutions for the original BIP problem (since these solutions are a subset of the feasible solutions for the LP relaxation). In fact, as summarized below, this bound of $16\frac{1}{2}$ can be rounded down to 16, because all coefficients in the objective function are integer, so all integer solutions must have an integer value for Z. Bound for whole problem: $Z \leq 16$. Now let us obtain the bounds for the two sub problems in the same way. Their LP relaxations are obtained from the models in the preceding subsection by replacing the constraints that x_j is binary for $j = 2, 3, 4$ by the constraints $0 \leq x_j \leq 1\ for\ j = 2, 3, 4$. Applying the simplex method then yields their optimal solutions (plus the fixed value of x_1) shown below.

LP relaxation of sub problem 1: $(x_1, x_2, x_3, x_4) = (0, 1, 0, 1)$ with $Z = 9$.

LP relaxation of sub problem 2: $(x_1, x_2, x_3, x_4) = \left(1, \frac{4}{5}, 0, \frac{4}{5}\right)$, with $z = 16\frac{1}{5}$

The resulting bounds for the sub problems then are
Bound for sub problem 1: $Z \leq 9$,

Bound for sub problem 2: $Z \leq 16$.

Figure 3.2 summarizes these results, where the numbers given just below the nodes are the bounds and below each bound is the optimal solution obtained for the LP relaxation.

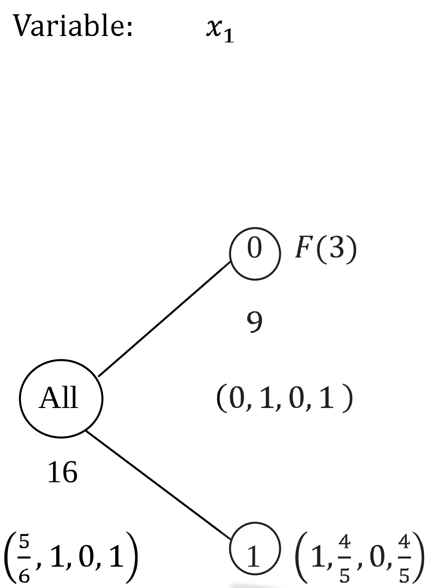

Figure 3.2: Figure 3.2 shows the results of bounding for the first iteration of the BIP branch and bound algorithm for (3.3)

3.2.3 Fathoming

A sub problem can be conquered (fathomed), and thereby dismissed from further consideration, in the three ways described below. One way is illustrated by the results for sub problem 1 given by the $x_1 = 0$ node in Figure 3.3. Note that the (unique) optimal solution for its LP relaxation, $(x_1, x_2, x_3, x_4) = (0, 1, 0, 1)$, is an integer solution. Therefore, this solution must also be the optimal solution for sub problem 1 itself. This solution should be stored as the first incumbent (the best feasible solution found so far) for the whole problem, along with its value of *Z*. This value is denoted by

$Z^* =$ Value of Z for current incumbent, so $Z^* = 9$ at this point. Since this solution has been stored, there is no reason to consider sub problem 1 any further by branching from the $x_1 = 0$ node, etc. Doing so could only lead to other feasible solutions that are inferior to the incumbent, and we have no interest in such solutions. Because it has been solved, we fathom (dismiss) sub problem 1 now.

The above results suggest a second key fathoming test. Since $Z^* = 9$, there is no reason to consider further any sub problem whose bound ≤ 9, since such a sub problem cannot have a feasible solution better than the incumbent. Stated more generally, a sub problem is fathomed whenever it's bound $\leq Z^*$.

This outcome does not occur in the current iteration of the example because sub problem 2 has a bound of 16 that is larger than 9. However, it might occur later for descendants of this sub problem (new smaller sub problems created by branching on this sub problem, and then perhaps branching further through subsequent "generations"). Furthermore, as new incumbents with larger values of $Z^*$are found, it will become easier to fathom in this way.

The third way of fathoming is quite straightforward. If the simplex method finds that a sub problem's LP relaxation has no feasible solutions, then the sub problem itself must have *no* feasible solutions, so it can be dismissed (fathomed).

In all three cases, we are conducting our search for an optimal solution by retaining for further investigation only those sub problems that could possibly have a feasible solution better than the current incumbent.

Summary of Fathoming Tests

A sub problem is fathomed (dismissed from further consideration) if

Test 1: It's bound $\leq Z^*$,

or

Test 2: Its LP relaxation has no feasible solutions, or

Test 3: The optimal solution for its LP relaxation is integer. (If this solution is better than the incumbent, it becomes the new incumbent, and test 1 is reapplied to all unfathomed Sub problems with the new larger Z^*.).

Figure 3.3 summarizes the results of applying these three tests to sub problems 1 and 2 by showing the current solution tree. Only sub problem 1 has been fathomed, by test 3, as indicated by $F(3)$ next to the $x_1 = 0$ node. The resulting incumbent also is identified below this node.

The subsequent iterations will illustrate successful applications of all three tests. However, before continuing the example, we summarize the algorithm being applied to this BIP problem. (This algorithm assumes that all coefficients in the objective function are integer and that the ordering of the variables for branching is $x_1, x_2, \dots, x_n$.)

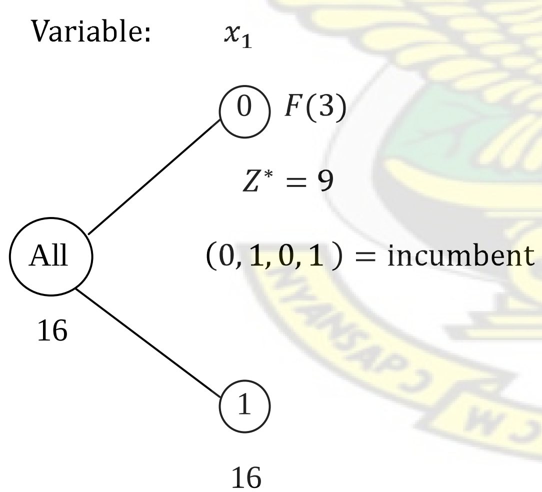

Figure 3.3: Figure 3.3 shows the solution tree after the first iteration of the BIP branch and bound algorithm for (3.3)

3.2.4 Summary of the BIP Branch and Bound Algorithm

Initialization: Set $Z^* = -\infty$. Apply the bounding step, fathoming step, and optimality test described below to the whole problem. If not fathomed, classify this problem as the one remaining "sub problem" for performing the first full iteration below.

Steps for each iteration

(i) Branching: Among the remaining (unfathomed) sub problems, select the one that was created most recently. (Break ties according to which the larger has bound.) Branch from the node for this sub problem to create two new sub problems by fixing the next variable (the branching variable) at either 0 or 1.

(ii) Bounding: For each new sub problem, obtain its bound by applying the simplex method to its LP relaxation and rounding down the value of Z for the resulting optimal solution.

(iii) Fathoming: For each new sub problem, apply the three fathoming tests summarized above, and discard those sub problems that are fathomed by any of the tests. [6]

Optimality test: Stop when there are no remaining sub problems; the current incumbent is optimal. Otherwise, return to perform iteration.

3.3 PROBLEM DESCRIPTION OF THE PROPOSED MODEL FOR SORTEGC

The problem is how to transport solid waste (garbage) from the city to the dump site for disposal. Residences of the customers are geographically dispersed around (or from) the depot and each truck is unique in terms of capacity.

The following assumptions are considered:

(i) Service is available to only customers whose residence is not within a walking distance from the dump site.

(ii) All customers to be serviced must walk to an allowed garbage picking point to dump the waste.

(iii) A truck must visits a given picking point only once.

(iv) Capacities of trucks must not be exceeded.

Parameters/Data

K_t = Capacity of truck t

T = Number of trucks

C_{ij} = Cost of traversing arc from i to j

S = Set of all potential picking points

G_{gi} = Binary variable that shows if a customer g can walk to picking point i or not

A = Set of all arcs between picking points.

O = Set representing the dump site or the depot

Decision Variables

$N_{(t)\ ij}$ = Number of times truck t traverses arcs from i to j

$$V_{(t)_i} = \begin{cases} 1, \text{if truck } t \text{ visit stop } i \\ 0, \text{otherwise} \end{cases}$$

$$P_{(t)_{ig}} = \begin{cases} 1, \text{if truck } t \text{ picks up garbage of customer } g \text{ at picking point } i \\ 0, \text{otherwise} \end{cases}$$

3.4 THE STMA SORTEGC MODEL

$$\text{Min } f = \sum_{(i;j)\in s} C_{ij} \sum_{t=1}^{4} N_{(t)\ ij} \tag{3.4}$$

$$s.t \quad \sum_{t=1}^{4} V_{(t)_o} \leq 4, \quad t = 1,2,3,4 \tag{3.5}$$

$$\sum_{j\in s} N_{(t)\ ij} = \sum_{j\in s} N_{(t)\ ij} = V_{(t)_i}\ ,\ \ \forall\, i \in S, \qquad t = 1,2,3,4 \tag{3.6}$$

$$\sum_{i\in G}\sum_{j\notin G} N_{(t)\ ji} \geq V_{(t)_h}, \quad \forall G \leq S\backslash\{0\}, h \in g,\ \ t = 1, \dots ,4 \tag{3.7}$$

$$\sum_{t=1}^{4} V_{(t)_i}\ \ \leq 1,\ \ \forall i \in S\backslash\{0\} \tag{3.8}$$

$$\sum_{t=1}^{4} P_{(t)_{ig}}\ \ \leq G_{gi}\ , \forall g\ \in G, i \in S \tag{3.9}$$

$$\sum_{i\in S}\sum_{g\in G} P_{(t)_{ig}} \leq K_t,\ \ t = 1,2,3,4 \tag{3.10}$$

$$P_{(t)_{ig}} \leq V_{(t)_i}, \forall i \in S, g \in G, t = 1,2,3,4 \tag{3.11}$$

$$\sum_{i\in s}\sum_{t=1}^{4} P_{(t)_{ig}}\ \ = 1\ ,\ \ \forall g \in G \tag{3.12}$$

$$V_{(t)_i} \in \{0, 1\}\ , \forall i \in S\ ,\ \ t = 1,2,3,4 \tag{3.13}$$

$$N_{(t)\ ij} \in \{0, 1\}, \forall i, j \in S\backslash i \neq j \tag{3.14}$$

$$P_{(t)_{ig}} \in \{0,1\}, \forall i, j \in S\backslash i \neq j \tag{3.15}$$

The objective function (3.4) minimizes the total route length covered by all trucks. Constraint (3.5) ensures that all trucks start from the depot (i.e. O). Constraint (3.6) ensures that if truck t visits picking point i then one arc is traversed by t entering and exiting i. Constrain (3.7) prevents the formation of sub-tours. This means that each cut defined by a customer set G is crossed by a number of arcs not less than the minimum number of trucks $n(T)$ required to serve set G. Constraint (3.8) ensures that a truck visits a particular picking point not more than one. Constraint (3.9) stipulates that every customer walks to his designated picking point only. Constraint (3.10) guarantees that respective capacities of trucks are not exceeded. Constraint

(3.11) ensures that garbage of costumer g designated to picking point i is picked up by truck t provided t visits stop i. Constraint (3.12) ensures that garbage of all costumers are picked up only once. Finally, (3.13), (3.14) and (3.15) represent the binary integrality constraints on all decision variables.

SORTEGC is a hard combinatorial problem, and also the number of customers and that of potential garbage picking points, as exists in the STMAWMD area of operation are large. As a result to this situation, the IP model described above cannot be solved efficiently using the branch- and-bound or any exact polynomial algorithm. A heuristic approach must be used and the researcher's choice is the ACO heuristics, which has been explained below.

3.5 ANT COLONY OPTIMIZATION

Ant Colony Optimization is one of the newest metaheuristic for the application to CO problems. The basic ideas of ACO were introduced in Dorigo (1992) and successively extended in Dorigo et al. (1999, 1996). St¨utzle and Dorigo (2002), Dorigo and St¨utzle (2002). In this section we present the description of ACO given in Dorigo and Di Caro (1999). ACO was inspired by the foraging behavior of real ants. This behavior—as described by Deneubourg et al. (1990)—enables ants to find shortest paths between food sources and their nest. Initially, ants explore the area surrounding their nest in a random manner. As soon as an ant finds a source of food, it evaluates quantity and quality of the food and carries some of this food to the nest. During the return trip, the ant deposits a pheromone trail on the ground. The quantity of pheromone deposited, which may depend on the quantity and quality of the food, will guide other ants to the food source. The indirect communication between the ants via the pheromone trails allows them to find the shortest path between their nest and food sources. This functionality of real ant colonies is exploited in artificial ant colonies in order to solve CO problems. In ACO algorithms the pheromone trails are simulated via a parameterized probabilistic model that is called the pheromone model. The pheromone model consists of a set of model parameters whose values are called the pheromone values. The basic ingredient of ACO algorithm is a constructive heuristic that is used for probabilistically constructing solutions using the pheromone values.

In general, the ACO approach attempts to solve a CO problem by iterating the following two steps:

(i) Solutions are constructed using a pheromone model, that is, a parameterized probability distribution over the solution space.

(ii) The solutions that were constructed in earlier iterations are used to modify the pheromone values in a way that is deemed to bias the search toward high quality solutions.

The ACO metaheuristic framework is shown in Algorithm 1. It consists of three algorithmic components that are gathered in the Schedule activities construct. The Schedule activities construct does not specify how these three activities are scheduled and synchronized. This is up to the algorithm designer. In the following we explain these three algorithm components in more detail Ant Based Solution Construction: As mentioned above, the basic ingredient of ACO algorithm is a constructive heuristic for probabilistically constructing solutions. A constructive heuristic assembles solutions as sequences of solution components taken from a finite set of solution components $C = \{c_1 \cdot \cdot \cdot , c_n\}$. A solution construction starts with an empty partial solution S^P =<>. Then, at each construction step the current partial solution S^P is extended by adding a feasible solution component from the set $N(S^P) \subseteq C \backslash S^P$.

Algorithm 1: Ant Colony Optimization

input: an instance *x* of a CO problem

while termination conditions not met **do**

Schedule Activities

Ant Based Solution Construction()

Pheromone Update()

Daemon Actions()

end Schedule Activities

sbest ← best solution in the population of solutions

end while

output: *sbest*, "candidate" to optimal solution for *x* solution construction mechanism.

The process of constructing solutions can be regarded as a walk (or a path) on the so-called construction graph GC = (C, L) whose vertices are the solution components *C* and the set *L* are the connections. The allowed walks on *GC* are hereby implicitly defined by the solution construction mechanism that defines the set $N(S^P)$ with respect to a partial solution S^P. The choice of a solution component from $N(S^P)$ is at each construction step done probabilistically with respect to the pheromone model, which consists of pheromone trail parameters T_i that are associated to components $c_i \in C$. The set of all pheromone trail parameters is denoted by *T*. The values of these parameters—the pheromone values—are denoted by τi. In most ACO algorithms the probabilities for choosing the next solution component also called the transition probabilities are defined as follows:

$$P(C_i / S^P) = \frac{T_i^{\alpha}.n(C_i)^{\beta}}{\sum_{C_j \in N(S^P)} T_j^{\alpha}.n(C_j)^{\beta}} , \forall\, C_i \in N(S^P) \qquad (3.16)$$

where n is a weighting function, which is a function that, sometimes depending on the current partial solution, assigns at each construction step a heuristic value $n(C_i)$ to each feasible solution component $C_j \in N(S^P)$. The values that are given by the weighting function are commonly called the heuristic information. Furthermore, α and β are positive parameters whose values determine the relation between pheromone information and heuristic information.

3.5.1 Pheromone Update

In ACO algorithms we can find different types of pheromone updates. First, we outline a pheromone update that is used by basically every ACO algorithm. This pheromone update consists of two parts. First, a pheromone evaporation, which uniformly decreases all the pheromone values, is performed. From a practical point of view, pheromone evaporation is needed to avoid a too rapid convergence of the algorithm toward a sub-optimal region. It implements a useful form of forgetting, favoring the exploration of new areas in the search space. Then, one or more solutions from the current or from earlier iterations are used to increase the values of pheromone trail parameters on solution components that are part of these solutions. As a prominent example, we outline in the following the pheromone update rule that was used in Ant System (AS) Dorigo (1992), which was the first ACO algorithm proposed. This update rule, which we henceforth call AS-update, is defined by

$$T_i \leftarrow (1-P).T_i + P.\sum_{\{S \in G_{iter} | C_i \in S\}} F(S) \qquad (3.17)$$

3.6 ITERATED LOCAL SEARCH

Local Search starts from a solution s, often randomly generated, and explores the neighborhood $N(s)$. The pseudo-code is shown in Algorithm 2. There are two major ways of implementing the function Choose Improving Neighbour. The first way is called first-improvement. A first-improvement function scans the neighborhood $N(s)$ and returns the first solution that is better than s. In contrast, a best-improvement function exhaustively explores the neighborhood and returns one of the solutions with the lowest objective function value. An iterative improvement procedure that uses a first-improvement function is called first-improvement local search, respectively best-improvement local search (or steepest descent local search) in the case of a best-improvement function. Both methods stop at local minima. Therefore, their performance strongly depends on the definition of a neighborhood structure N. A Local Search algorithm partitions the search space S into so-called basins of attraction of local minima. The basin of attraction of a local minimum $s* \in S$ is the set of all solutions s for which a deterministic iterative improvement local search terminates in $s*$ when started from the initial solution s.

In practice, LS define a correspondence between the set S and the subset $S*$ of globally minimal solutions. One of the most evolved methods of LS is the metaheuristic ILS. In general, an explorative local search method is effective if it is able to find high quality local minima, i.e., if it can find the basins of attraction of high quality local minima. However, when the search space is huge or when the

Algorithm 2 Local Search

input: an instance x of a CO problem

$s \leftarrow$ Generate Initial Solution()

repeat

$s* \leftarrow$ Choose Improving Neighbor ($N(s)$)

$s \leftarrow s*$

until no improvement is possible

$sbest \leftarrow s$

output: *sbest*, "candidate" to optimal solution for x

Algorithm 3 Iterated Local Search

input: an instance x of a CO problem

$s \leftarrow$ Generate Initial Solution()

$s* \leftarrow$ LS(s)

repeat

$s\_ \leftarrow$ Perturbation ($s*$, *history*)

$s\_* \leftarrow$ LS ($s\_$)

$s* \leftarrow$ Apply Acceptance Criterion ($s*$, s_*, *history*)

until termination conditions not met

$sbest \leftarrow s*$

output: *sbest*, "candidate" to optimal solution for x

basins of attraction of high quality local minimum are small, this goal is difficult to reach. In these cases an effective local search method might be defined only on the set of local minima $S*$, instead of on the whole set S. Unfortunately, in most cases there is no feasible way of introducing a neighborhood structure for $S*$. Instead, ILS algorithms perform a trajectory along local minima $s_1*, s_2*, \ldots, s_t*$ without explicitly introducing a neighborhood structure on $S*$ by applying the scheme that is shown in Algorithm 3. At each iteration, the current solution (which is a local minimum) is perturbed and a local search method is applied to the perturbed solution. Then, the local minimum that is obtained by applying the local search method is either accepted as the new current solution, or not. The importance of the perturbation is obvious: too small a perturbation might not enable the system to escape from the basin of attraction of the local minimum just found. On the other side, too strong a perturbation would make the algorithm similar to a random restart local search. Therefore, the requirement on the perturbation method is to produce a starting point for local search such that a local minimum different from the current solution is reached. However, this new local minimum should be closer to the current solution than a local minimum produced by the application of the local search to a randomly generated solution. The acceptance criterion acts as a counterbalance, as it filters and gives feedback to the perturbation action, depending on the characteristics of the new local minimum. The design of ILS algorithms has several degrees of freedom in the generation of the initial solution, the choice of the perturbation method and the acceptance criterion. Furthermore, the history of the search process can be exploited both in form of short and long term memory. In the following we describe the three main algorithmic components of ILS.

Generate Initial Solution: The construction of initial solutions should be fast (computationally not expensive), and initial solutions should be a good starting point for local search. The fastest way of producing an initial solution is to generate it at random. Another possibility is to use constructive heuristics. It is worth underlining that an initial solution is considered a good starting point depending on the particular local search method applied and on the structure of the problem instance under consideration, thus the algorithm designer's goal is to find a good trade-off between speed and quality of solutions. Perturbation (*s*∗, *history*): The perturbation is usually non-deterministic in order to avoid cycling. Its most important characteristic is the strength.

3.7 SIMULATED ANNEALING

The concept of Simulated Annealing is derived from Statistical mechanics in the area of natural Sciences. A piece of regular metal in its natural state has the magnetic directions of its molecules aligned in a uniform direction. In the preparation of alloys the metals are heated to a very high temperature where the molecules acquire higher energy state. The basic structure of the metallic bonds break down and magnetic directions of the molecules are oriented randomly. Annealing is the slow cooling of the metallic material so that at the natural temperature conditions the metal will achieve regularity of alignment of the magnetic direction so as to make the metal stable for use. Hasty cooling of solids result in defective crystal. In 1953 Metropolis and others recognized the use of Boltzmain Law to simulate the efficient equilibrium condition of a collection of molecules at a given temperature and thus facilitated annealing. When the metal is heated to higher temperature with higher energy state and it is being cooled slowly it is assumed that for a finite drop in temperature the system state change in the sense that the molecules assume new configuration of arrangement. The configuration depends on parameters like temperature, the energy of the system and others. Combining the parameters we obtain an energy function from which the configuration can be obtained. In 1983 Kirk Patrick showed how Simulated Annealing of Metropolis could be adapted to solve problems in Combinational Optimization. The following analogy was made:

(i) (a) Annealing looks for system state at a given temperature and energy.

(b) Optimization looks for feasible solution of the combinatorial problems

(ii) (a) Cooling of the metal is to move from one system state to another

(b) Search procedure (algorithm scheme) tries one solution after another in order to find the optimal solution.

(iii) (a) Energy function is used to determine the system state and energy

(b) Objective (cost) function is used to determine a solution and the objective functions value.

(iv) (a) Energy results in evaluation of energy function and the lowest energy state corresponds to stable state.

(b) Cost results in evaluation of objective function and the lowest objective function value corresponds to the optimal solution.

(v) (a) Temperature controls the system state and the energy

(b) A control parameter is used to control the solution generation and the objective function value.

Given an optimization problem, we put it in the form min $f\,(\,x)$ such that $x \in S$, S = feasible solution. The basic SA algorithm is detailed below with the following parameter identification

$X^{(i)}$ =Solution (system state); $F(x)$ =Objective function (Energy function); k = Iteration number (time check in cooling process); $\delta = f(x^1) - f(x^0)$ (Energy change between states x^1 and x^0); T = Control parameter (Temperature of system); $g(T)$ =control parameter function (Temperature function); $e^{-(\delta|T)}$ = Choice probability function (Boltzmann probability function);

It provides the condition under which a non-improvement solution is not discarded.

Step 1

(i) Select an initial solution $X^{(0)}$ assign $X^{(b)} = X^{(0)}$

(ii) Set $k = 0$, and select an initial temperature (control parameter) $T_k = T_o$

For K= 0 assign $T_b = T_o$

(iii) Select a temperature function $g(T_k)$

Step 2

Choose a solution $X^{(1)}$ in $N(X^{(0)})$ *and compute* $\delta = f(X^{(1)}) - f(X^{(0)})$

Step 3

(i) If $\delta = 0$ or [$\delta > 0$ and $e^{-(\delta|T)} \geq \theta : \theta \leftarrow U = (0, 1)$], accept the new

Solution $X^{(1)}$. Assign $X^{(0)} \leftarrow X^{(1)}$ and keep the new $X^{(0)}$ such that $X^{(0)} = X^{(b)}$ set $T_b = T_k$

Step 4

If some stopping criteria are satisfied; stop.

Step 5

Update the temperature $T_{(k+1)} = g(T_k)$ and set K= K+1

3.8 TABU SEARCH

Tabu Search is one of the most successful metaheuristic for the application to CO problems. The basic ideas of TS were introduced by Glover in 1986, based on his earlier ideas. A description of the method and its concepts can be found in Glover and Laguna (1997). The basic idea of TS is the explicit use of search history, both to escape from local minima and to implement an explorative strategy. We first describe a simple version of TS in order to introduce the basic concepts; then, we explain a more applicable algorithm. A simple TS algorithm (see Algorithm 4) is based on a best-improvement local search and uses a short term memory to escape from local minima and to avoid cycles.3. The short term memory is implemented as a tabu list that keeps track of the most recently visited solutions and excludes them from the neighborhood of the current solution. In the following we will refer to the restricted neighborhood of a solution *s* as the allowed set, which we will denote by *Na*(*s*). At each iteration the best solution from the allowed set is chosen as the new current solution. Furthermore, in procedure Update (*TabuList*, *s*, *s*_) this solution is added to the tabu list and if the tabu list has reached its maximum capacity, one of the solutions that were already in the tabu list is removed. Tabu lists are usually handled in a FIFO manner. The algorithm stops when a termination criterion is met. It might also terminate if the allowed set is empty. The use of a tabu list prevents from returning to recently visited solutions, therefore it prevents from endless cycling4 and forces the search to accept even uphill-moves. The length *l* of the tabu list—known in the literature as the tabu tenure—controls the memory of the search process. With small tabu tenures the search will concentrate on small areas of the search space. On the opposite, large tabu tenure forces the search process to explore larger regions, because it forbids revisiting a higher number of solutions. The tabu tenure can be varied during the search, leading to more robust algorithms. The implementation of short term memory in terms of a list that contains complete solutions is not practical, because managing a list of complete solutions is highly inefficient. Therefore, instead of the solutions themselves, the solution components that are involved in moves are stored in the tabu list. Since different types of moves that work on different types of solutions components can be considered, a tabu list is usually introduced for each type of solution component. The different types of solution components and the corresponding tabu lists define the tabu conditions which are used to filter the neighborhood of a solution and generate the allowed set. Storing solution components instead of complete solutions is much more efficient, but it introduces a loss of information, as forbidding for example the introduction of a certain solution component in a solution means assigning the tabu status to probably more than one solution. Thus, it is possible that unvisited solutions of high quality are excluded from the allowed set. To overcome this problem, aspiration criteria are defined which allow to include a solution in the allowed set even if it is forbidden by tabu conditions. The most commonly used aspiration criterion applies to solutions which are better than the best solution found so far. This more applicable tabu search algorithm is shown in Algorithm 5. Reference of successful applications of TS can be found in Glover and Laguna (1997).

Algorithm 4 Simple Tabu Search

input: an instance *x* of a CO problem

s ← Generate Initial Solution()

Tabu List ← ∅

while termination conditions not met **do**

Na(*s*) ← *N*(*s*) \ *TabuList*

*s*_ ← argmin{*f*(*s*")|*s*" ∈ *Na*(*s*)}

Update (*Tabu List, s, s*_)

$s \leftarrow s\_$

end while

sbest $\leftarrow s$

output: *sbest*, "candidate" to optimal solution for *x*

Algorithm 5 Tabu Search

input: an instance *x* of a CO problem

$s \leftarrow$ Generate Initial Solution()

Initialize Tabu List (*TabuList*1, · · · , *Tabu Listr*)

while termination conditions not met **do**

$Na(s) \leftarrow \{s\_ \in N(s) | s\_$ does not violate a tabu condition, or satisfies at least one aspiration criterion

$s\_ \leftarrow \text{argmin}\{f(s'') | s'' \in Na(s)\}$

Update Tabu List (*TabuList*1, · · · , *Tabu Listr, s, s_*)

$s \leftarrow s\_$

end while

sbest $\leftarrow s$

output: *sbest*, "candidate" to optimal solution for *x*

CHAPTER 4

DATA COLLECTION AND ANALYSIS

4.0 INTRODUCTION

This chapter presents ant colony results of the data taken on the various zones for the respective trucks. The chapter is divided into four sections, since the area of the study was divided into four zones, where each section provides ACO results for each truck for each zone. The results are obtained by an ant colony programme written in Matlab implementation codes. Each figure comprises two panels namely, lower and upper. The upper pannel shows the distance covered by various ants at every iterative point whilst the lower displays the complete optimal tour of the best ant.

The data involves length of distances between various picking points for each zone. The distances between picking points were measured and a grid map was used to find the cartesian coordinates which gave rise to the distance matrix as shown in the Tables 4.1 to 4.4. The distances were recorded with the aid of a car that reads distances digitally. One can also use a software called the eye-calculator to convert GPS information into a coordinate system.

Table 4.1 Distance Matrix for Zone I

S/NO	Picking Points	X Coordinates	Y Coordinates
1	Pentecost junction	625	90
2	Housing	624	92
3	Polyede	622	89
4	Acalima junction	625	88
5	Poly Gate	626	88
6	Rok Fm	627	85
7	Doctor's Flat	626	86
8	Unicorm Hostel	626	87

Source: Field Survey Dec. 2011.

Table 4.1 shows the Cartesian coordinates of the various garbage picking points in zone I. The original total route length for zone I tour for collecting the garbage is 30.22km.

Table 4.2 Distance Matrix for Zone II

S/NO	Picking Points	X Coordinates	Y Coordinates
1	Anaji Fie	618	96
2	C. K. Man junction	621	95
3	Snaps junction	622	96
4	Namibia	617	96
5	Top ridge School junction	620	91
6	St Francis	620	93
7	Alhaji Bawah	622	93
8	Effiakuma bus stop	621	91
9	Number 9	623	94
10	Skyy	625	97
11	Summer Lodge	623	96

Source: Field Survey Dec. 2011.

Table 4.2 shows the Cartesian coordinates of the various garbage picking points in zone II. The original total route length for zone II tour for collecting the garbage is 38.18km.

Table 4.3 Distance Matrix for Zone III

S/NO	Picking Points	X Coordinates	Y Coordinates
1	Last hour	626	75
2	Planters Lodge	635	78
3	African beach	622	75
4	MTN Office	624	79
5	Police Reserve	625	82
6	Takoradi Int. School	625	81
7	IPMC	626	79
8	European Hospital	627	78
9	Good News Fm	627	82

Source: Field Survey Dec. 2011.

Table 4.3 shows the Cartesian coordinates of the various garbage picking points in zone III. The original total route length for zone III tour for collecting the garbage is 46.15km.

Table 4.4 Distance Matrix for Zone IV

S/NO	Picking Points	X Coordinates	Y Coordinates
1	Pipe Ano	621	89
2	School junction	621	88
3	Ahamadiya	622	88
4	Tadisco junction	619	90
5	TTI junction	622	90
6	Time Enterprise	623	89
7	Mrs. Cudjoe	623	91
8	Air force Flat	617	87
	Sawmill	616	88
10	Kwasimintim bus stop	614	89

Source: Field Survey Dec. 2011.

Table 4.4 shows the Cartesian coordinates of the various garbage picking points in zone IV. The original total route length for zone IV tour for collecting the garbage is 38.23km.

4.1 ACO OUTPUT FOR TRUCK OF ZONE I

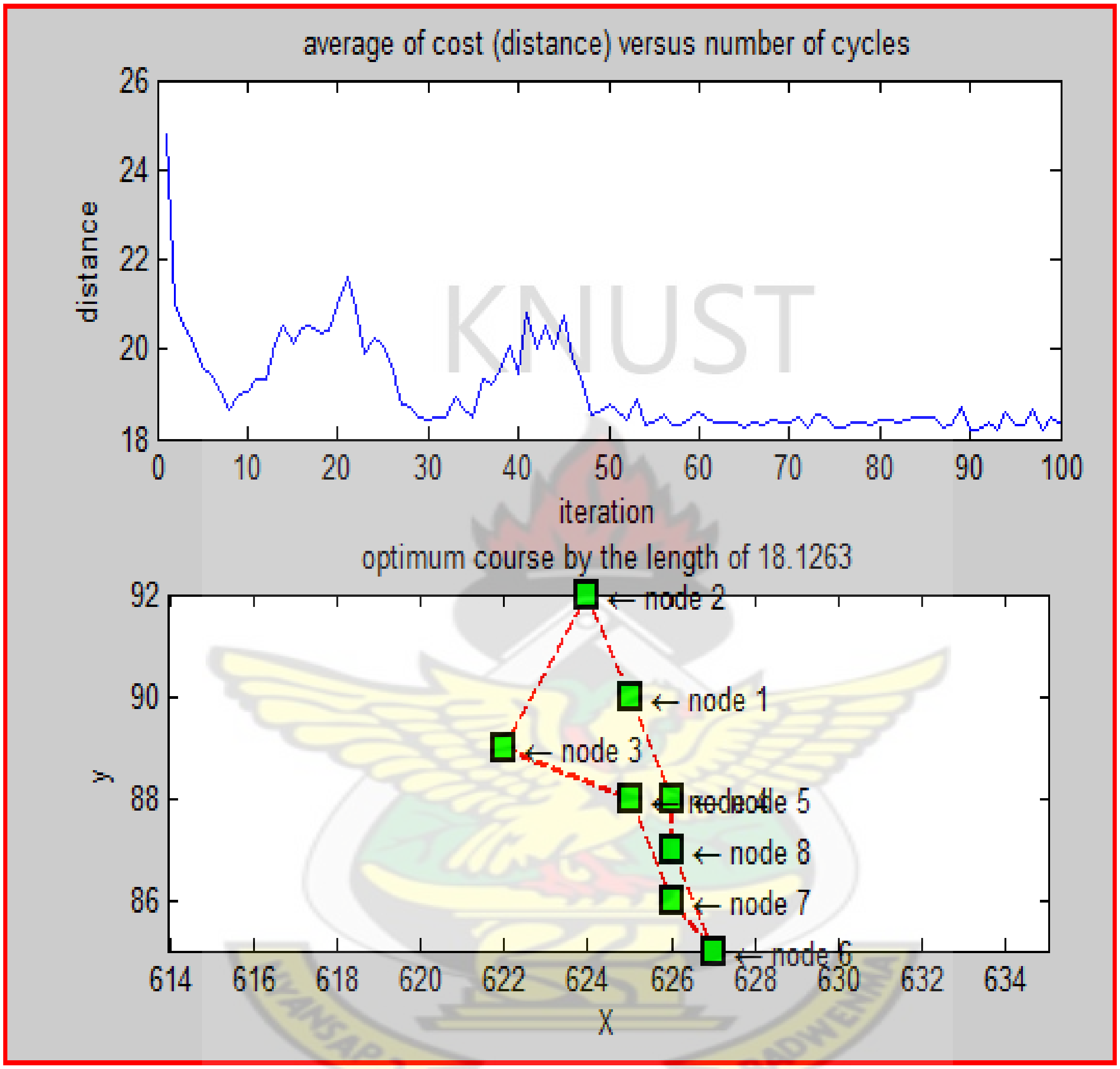

Figure 4.1: Ant colony result for 200 ants.

Figure 4.1 depicts that for 200 ants the best ant will cover an optimum course by the length of about 18.126km, which is better than distance covered by truck of zone I. This means that the optimal route length displaced by truck of zone I using ant colony is 18.126km.

4.2 ACO OUTPUT FOR TRUCK OF ZONE II

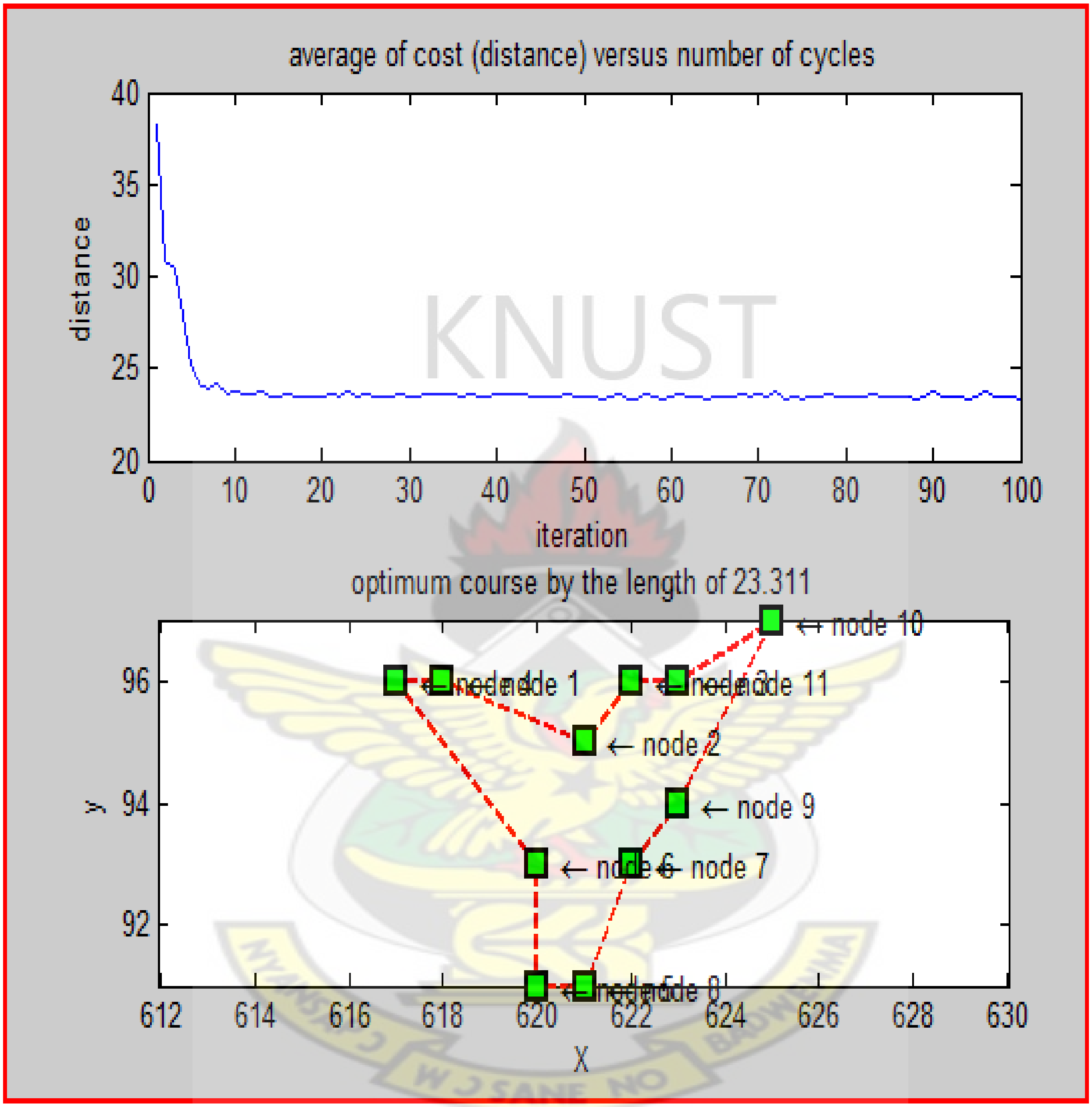

Figure 4.2: Ant colony result for 200 ants.

Figure 4.2 depicts that for 200 ants, the optimum length completed by the best out of 200 ants is approximately 23.311km. This shows a reduction of 39.5% of the original route length. Therefore, the optimum course for truck of zone II using ant colony is approximately by the length of 23.311km.

4.3 ACO OUTPUT FOR TRUCK OF ZONE III

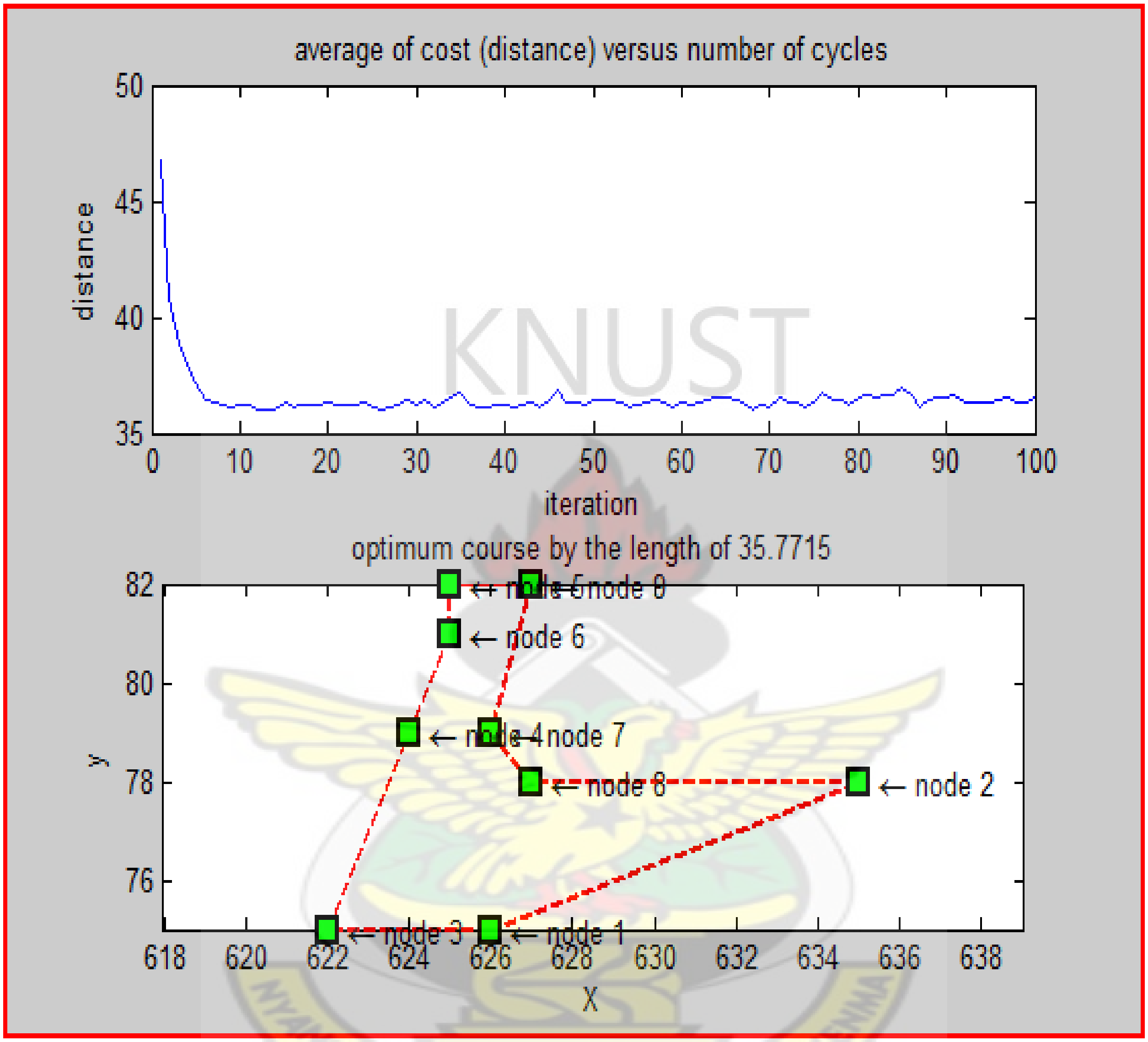

Figure 4.3: Ant colony result for 200 ants.

Figure 4.3 depicts that for 200 ants, the optimum length completed by the best out of 200 ants is approximately 35.772km. This shows a reduction of 21.7% of the original route length. Therefore, the optimum course for truck of zone III using ant colony is approximately by the length of 35.772km.

4.4 ACO OUTPUT FOR TRUCK OF ZONE IV

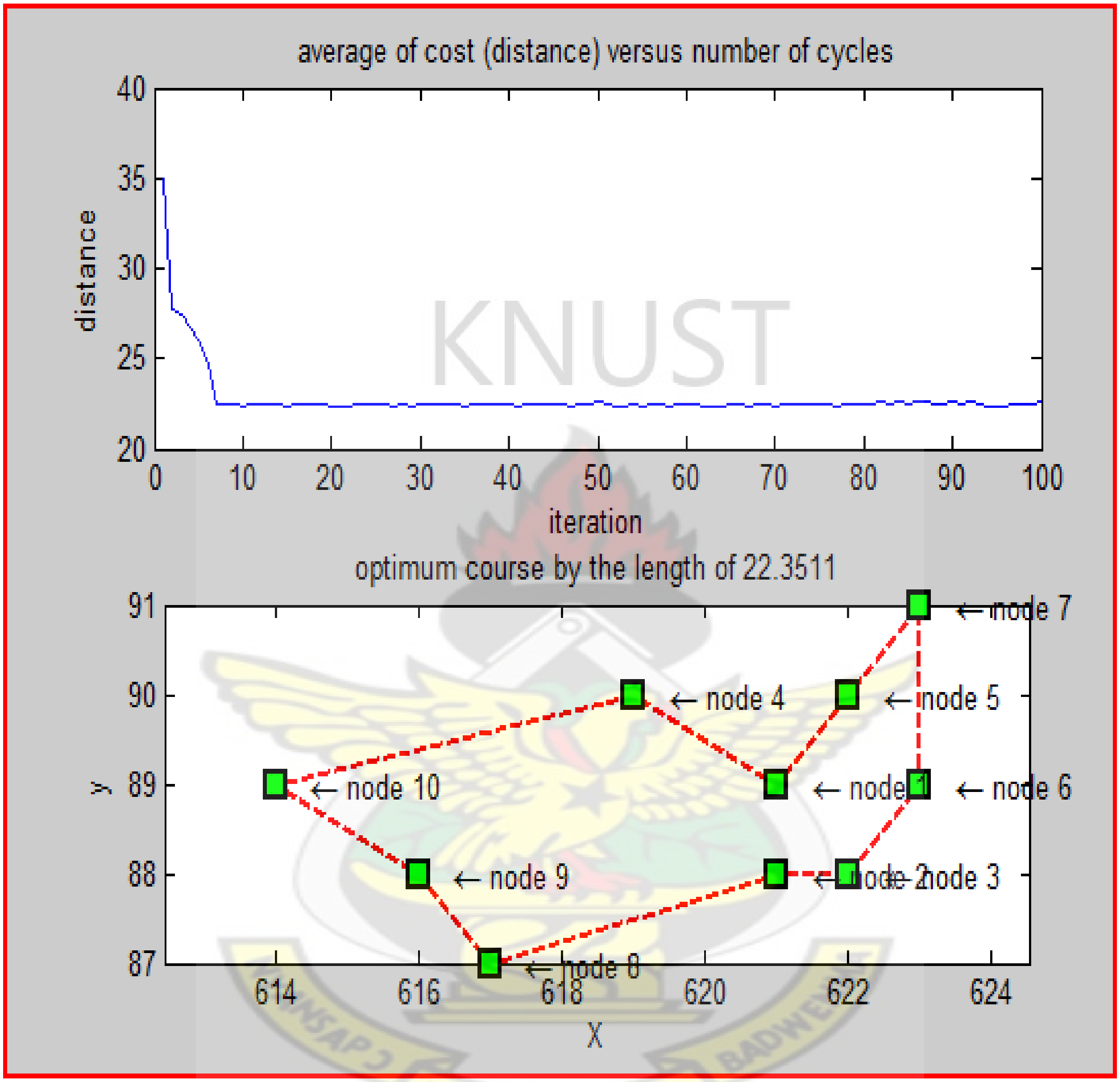

Figure 4.4: Ant colony result for 200 ants.

Figure 4.2 depicts that for 200 ants, the optimum length completed by the best out of 200 ants is approximately 22.351km. This shows a reduction of 42.1% of the original route length. Therefore, the optimum course for truck of zone II using ant colony is approximately by the length of 22.351km.

CHAPTER 5

DISCUSSIONS, CONCLUSIONS AND RECOMMENDATIONS

5.0 INTRODUCTION

This chapter presents the discussion of the experimented results, conclusions and recommendations based on the results of the experiment.

5.1 SUMMARY OF FINDINGS

The research work is on the construction of routes for vehicles using the transportation system of solid waste of Sekondi-Takoradi Metropolitan Assembly Waste Management Department (STMAWMD) as a case study. In this study, the routes were constructed by simulating the behaviour of artificial ants (naturally know as ants). Since ants naturally move in “colonies” the methodology was based on the idea that a group of ants are allowed to explore various routes that link a given number of garbage picking points in a given zone. As these ants are put to work different ants will take route that link a given numbers of garbage picking points and the result of the best ant will be considered as the optimal route length. The results shown in the graphs are based on the tour of the best ants. The STMAWMD has four trucks that are in good condition to operate on different routes. However, it is important to note that the researcher performed several experiments on more than those presented, but considering the objectives of the work only the cases that gave the optimal routes for each truck is presented.

The result in Figure 4.1 shows the optimal route lengths that will yield the optimal cost for truck of zone I. For 200 ants the optimal course for truck of zone I is by the length of approximately

18.126km. It can therefore be concluded that the optimal course for truck of zone I is about 18.126km, which represent a reduction in cost by about 40%.

The optimal course for truck of zone II is shown in Figure 4.2. For 200 ants the optimal course for truck of zone II is by the length of approximately 23.311km. It can therefore be concluded that the optimal course for truck of zone II is about 23.311km, which represent a reduction in cost by about 22%.

The optimal course for truck of zone III is shown in Figure 4.3. For 200 ants the optimal course for truck of zone III is by the length of approximately 35.772km. It can therefore be concluded that the optimal course for truck of zone III is about 35.772km, which represent a reduction in cost by about 39.5%.

The result in Figure 4.4 depicts the optimal route lengths that will yield the optimal cost for truck of zone IV. For 200 ants the optimal course for truck of zone IV is by the length of approximately 22.351km. It can therefore be concluded that the optimal course for truck of zone IV is about 22.351km, which represent a reduction in cost by about 42%.

Also, high numbers of ants are required for optimal route length to begin to improve than are required for large number of nodes. The potential picking points for trucks of zone I, II, III and IV are respectively 8, 11, 9 and 10

In general, the ant colony optimization has performed very well by reducing the total original route length by about 53.22km, which is a reduction from 152.78km to 99.56km and represents about 35% reduction in the total cost. The total cost for transporting the waste per month for 2010 is GH$160,001.00, which means if STMAWMD is going to adopt the ant colony optimization approach used by the researcher the cost would be reduced to GH$104000.65.

5.2 CONCLUSIONS

The ant colony optimization algorithm has shown that it is one of the most powerful tools for solving hard and complex combinatorial optimization problems like the VRP. Comparing the results of the ant colony to the existing transportation system at STMAWMD, the existing system is inefficient since it is not scientific- based.

The results show the possibility of the ant colony optimization heuristic to converge the solution to optimality.

The following conclusions can be made based on the results.

(i) Optimal route length for trucks of zone I, II, III and IV are respectively 18.126km, 23.311km, 35.772km and 22.351km approximately.

(ii) Cost of services rendered by trucks of zone I, II, III and IV are reduced by 40%, 39.5%, 21.7% and 42.1% , respectively.

(iii) In general, the total cost of transporting the waste is reduced by approximately 35% which means the cost is reduced to GH$104000.65 instead of spending GH$160001.00 per month.

(iv) The number of potential garbage picking points that will be selected for trucks of zone I, II, III and IV are 8, 11, 9 and 10 respectively.

5.3 RECOMMENDATIONS

(i) It is suggested that Sekondi-Takoradi Metropolitan Assembly Waste Management Department implements or adopt a system which is scientific based in assigning their trucks.

(ii) Back-tour to the already visited routes must be avoided.

(iii) Bad routes that join shortest paths between two potential garbage points must be developed.

(iv) In addition, the researcher recommends that any further research based on this study should consider first, the assignment of trucks to various tours before constructing their respective routes and this must also be included in the ant colony algorithm. It should also consider assignment of customers to various garbage collection points.

REFFERENCES

1. Aarts, E. H. L., Korst, J. H. M. and Van laarhoven, P. J. M. (1997). Simulated annealing. In local search in combinatorial optimization, John Wiley & Sons, Chichester, UK. Pages 91–120.
2. Abounacer, R., Bendrielch, G., Boukachour, J., Dkhissi, B. and Alaoui, A. E. (2009). Population Metaheuristic to solve the Professional Staff Transportation Problem. International Journal of Computer Science and Network Security (iicsns-06), 9(7):22-34.
3. Addor, J. A. (2011). Efficiency of the school bus transportation service: a case study of the system at Woodbridge School Complex (WBSC) MSc thesis; Kwame Nkrumah University of Science and Technology, Kumasi, Ghana.
4. Agyen, K. J. (2011). Transportation model for waste collection in the Kumasi Metropolis. MSc thesis; Kwame Nkrumah University of Science and Technology, Kumasi, Ghana.
5. Amponsah, S. K. And Darkwah, F. K. (2007). Operations Research, Kwame Nkrumah University Of Science and Technology, Kumasi, Ghana, (P.9-23; 47-62).
6. Amponsah, S. K. (2008). Operations Research, Kwame Nkrumah University of Science and Technology, Kumasi, Ghana, (P.48-116; 139-144).
7. Arunya, B., Nanth, S. and Rawinkhan, S. (2010). Strategic planning for newspaper delivery problem using vehicle routing algorithm with time window. Advance Online Publication, pp. 13-16.
8. Baker, E. K. and Schaffer, J. R. (1998). Solution improvement heuristics for the vehicle routing scheduling problem with time window constraints, American Journal of Mathematics and Management Sciences, 6:261-300

9. Banson, R. and Naadimuthu, G. (1997). Schaum's Outline of theory and problems of operations research, (2nd edition), Mcgraw-Hill Companies, New York, USA, (p.124-132).

10. Barnhart, C., Johnson, E. L. (1998). Branch-and-price: *Column* generation for solving huge integer programs, Operation Research, 46(3).

11. Baver, A., Bullnbeimer, B., Hart, R. F. and Straues, C. (1999). Ant colony optimization approach for the single machine tool tardiness problem proceedings of the 1999 congress on evolutionary computation, p.1445-1450.

12. Bell, J. E. and McMullen, P. R. (2004). Ant Colony Optimization Techniques for the Vehicle Routing Problem. Advanced Engineering Informatics, 18:41-48.

13. Bellman, R. (1957). Dynamic Programming. Princeton University Press, Princeton, NJ.

14. Bennet, B and Gazis, D. (1975). School Bus Routing By Computer. Transportation Research, 6:317-326.

15. Bertsimas, D. J. and Simchi-Levi, D. (1996). A new generation of vehicle routing research: robust algorithms, addressing uncertainty. Operations Research, 44(2):216–304.

16. Bertsimas, D. J. (1992). A vehicle routing problem with stochastic demand. Operations Research, 40(3):574–585.

17. Bertsimas, D. J., Chervi, P. and Peterson, M. (1995). Computational approaches to stochastic vehicle routing problems. Transportation Science, 29(4):342–352.

18. Bianchi, L., Birattari, M., Chiarandini, M., Manfrin, M., Mastrolilli, M., Paquete, L., Rossi-Doria, O. and Schiavinotto, T. (2004). Metaheuristic for the vehicle routing problem with stochastic demands. Technical Report IDSIA-06-04, IDSIA, Manno, Switzerland.

19. Bianchi, L., Mastrolilli, M., Birattari, M., Manfrin, M., Chiarandini, M., Paquete, L. and Rossi-Doria, O. (2004). Report for task 5: Research on the vehicle routing problem with stocastic demand. Internal document of the Metaheuristics Network.

20. Bianchi, L., Gambardella, L. M. and Dorigo, M. (2002). An ant colony optimization approach to the probabilistic traveling salesman problem. In Parallel Problem Solving from Nature - PPSN VII, volume 2439 of Lecture Notes in Computer Science, pages 883–892, Berlin, Germany, Springer Verlag.

21. Bianchi, L., Gambardella, L. M. and Dorigo, M. (2002). Solving the homogeneous probabilistic traveling salesman problem by the ACO metaheuristic. In Proceedings of ANTS 2002 – Third International Workshop on Ant Algorithms, volume 2463 of Lecture Notes in Computer Science, pages 176–187, Berlin, Germany, Springer Verlag.

22. Bland, J. A. (1999). Space Planning by Ant Colony Optimization. International Journal Computer Applied Technology, 6:320-328.

23. Blanton, J. L. and Wainwright, R.L. (1993). Multiple Vehicle Routing with Time and Capacity Constraint Using Genetic Algorithms, in S Forrest Eds, Proceedings of the Fifth International Conference on Genetic Algorithm, Morgan Kaufmann, San Mateo, part A: p. 452-459.

24. Blum, C. and Roli, A. (2003). Metaheuristics in combinatorial optimization: Overview and conceptual comparison. ACM Computing Surveys, 35(3):268–308.

25. Bonding, L. and Berman, L. (1979). Routing and Scheduling of School Buses by Computer Transportation Science, 13: 113-129.

26. Boryczka, U., Boryczka, M. (2001). Multi-cast Ant colony System for Bus Routing Problem, Proceedings of the 4th Metaheuristics International conference, 3:521-523.

27. Bowerman, R., Hall, B., and Calamai, P. (1995). A Multi-objective Optimization Approach to Urban School Bus Routing: Formulation and Solution Method. Transportation Research part A; Policy and Practice, 29A:107-123

28. Bylander, T. (1997). A linear programming Heuristic for Optimal Planning. Proceedings of AAAI-97.

29. Cairncrose, S. and Feachem, R., (1988). Environmental Health Engineering in the Tropics, John Wiley & Son's. Baffin‘s lane Chichester England, pp 196-202.

30. Ceder, A., Golany, B. and Tal, O. (2001). Creating Bus Timetable with Maximal Synchronization. Transportation Research, part A 35:913-928.

31. Ceril, L., Smith, W., Pike, Paul W., Murrill (1970). Formulation and Optimization of Mathematical Models Copyright by International Textbook Company Louisiana State University, pp554-575.

32. Cern´y, V. (1985). A thermodynamical approach to the traveling salesman problem: an efficient simulation algorithm. Journal of Optimization Theory and Applications, 45(1):41-51.

33. Chen, L. and Zhang, C. (2005). Adaptive Ant Colony Algorithm. International conference on Network Computing, 3611:1239-1249.

34. Chibuzor, J. E. (2005). A MATLAB Toolkit for Linear Programming. Master of Science Thesis. Department of Electrical and Electronic Engineering, University of London, (p. 7-31).

35. Deneubourg, J. L., Aron, S., Goss, S. and Pasteels, J. M. (1990). The self-organizing exploratory pattern of the argentine ant. Journal of Insect Behaviour, 3:159–168.

36. Desrochers, M., Desrosiers, J. and Marius, S. (1992). A New Optimization Algorithm for the Vehicle Routing Problem With time Windows. Operations Research, 40(2):342-354.

37. Dorigo, M. (1992). Ant Colony Optimization for vehicle routing problem. PhD thesis, Politecnico di Milano, Milan, Italy.

38. Dorigo, M. and Gambardella, L. M. (April 1997). Ant Colony System: A cooperative learning approach to the travelling salesman problem. IEEE Transactions on Evolutionary Computation, 1(1):53–66.

39. Dorigo, M. and Di Caro, G. (1999). Ant colony Algorithm for Discrete Optimization, Artificial Life, 5:137-172.

40. Dorigo, M. and Di Caro, G. (1999). The Ant Colony Optimization meta-heuristic. In D. Corne, M. Dorigo, and F. Glover, editors, New Ideas in Optimization, McGraw-Hill, London, UK, pages 11–32.

41. Dorigo, M. and St¨utzle, T. (2002). The ant colony optimization metaheuristic: Algorithms, applications and advances. In F. Glover and G. Kochenberger, editors, Handbook of Metaheuristics, volume 57 of International Series in Operations Research & Management Science, pages 251–285. Kluwer Academic Publishers, Norwell, MA.

42. Dorigo, M. and St¨utzle, T. (2004). Ant Colony Optimization. MIT Press, Cambridge, MA.

43. Dorigo, M., Di Caro, G. and Gambardella, L. M. (1999). Ant algorithms for discrete optimization. Artificial Life, 5(2):137–172.

44. Dorigo, M., Gamberdella, L. M. (1997). Ant colony for the Travelling Salesman Problem. Biosystems, 43(1):73-81.

45. Dorigo, M., Maniezzo, V. and Colorni, A. (1996). Ant System: Optimization by a colony of cooperating agents. IEEE Transactions on Systems, Man, and Cybernetics – Part B, 26(1):29–41.

46. Dumas, Y., Desrosiers, J. and Soumis, F. (1991). The Pickup and Delivery Problem with Time Window. European Journal of Operational Research, 54:7-22.

47. Feachem, R., Mara, D., McGarry, M., (1983). Water, Waste and Health in Hot Climate. John Wiley and Sons New York, Reprinted pp 320 – 327.

48. Fisher, M. L. (1981). The Lagragian Relaxation Method for Solving Integer programming Problems. Management Science, 27:1-18.

49. Foster, B.A. and Ryan, D.M. (1976). An Integer programming Approach for the Vehicle Scheduling Problem. Operations Research, 27: 367-384.

50. Gass and Saul I., (1958). Linear Programming Methods and Application McGraw- Hill Book Company, Inc., New York pp 74 – 76.

51. Gendreau, M., Laporte, G. and S´eguin R. (1995). An exact algorithm for the vehicle routing problem with stochastic demands and customers. Transportation Sciences, 29(2):143–155.

52. Gendreau, M., Laporte, G. and S´eguin R. (1996). A tabu search heuristic for the vehicle routing problem with stochastic demands and customers. Operations Research, 44(3).

53. Ghiduk, A. S. (2010). A new software Data-flow Jesting Approach via Ant Colony Algorithm. Universal Journal of Computer science and engineering Technology, 1(1)64-72.

54. Gillet, B. C. and Miller, L. R. (1974). A Heuristic Algorithm for the Vehicle Dispatch Problem. Operation Research, 22(2):340-349.

55. Glover, F. (1977). Heuristics for integer programming using surrogate constraints. Decision Sciences, 8:156–166.

56. Glover, F. (1986). Future paths for integer programming and links to artificial intelligence. Computers & Operations Research, 13:533–549.

57. Glover, F. and Laguna, M. (1997). Tabu Search. Kluwer Academic Publishers, Norwell, MA, second edition.

58. Goel, A., Gruhn, V., (2006). A General Vehicle Routing Problem. Elsevier Science Publishers North-Holland, (p.1-16).

59. Green, D. W., Perry, H. R.., (1997). Perry's Chemical Engineering Handbook McGraw-Hill Companies Inc. U.S.A.

60. Gupta R., Singh, B. and Pandey, D. (2010). Multi-objective Fuzzy Vehicle Routing Problem: A case study. International Journal Contemp. Maths. Science, 5(29) 1439-1454.

61. Hagerty, D. J., Pavani, J. L., and Heer, Jr. J. E., (1973). Solid Waste Management Littan Educational Publishing Inc. New York pp 44-47.

62. Hillier, F.S. and Liebermann, G. J., (2001). Introduction to Operations Research (7th edition), McGraw-Hill, an imprint of McGraw-Hill companies, New York, USA, (p.580-585).

63. Ibaraki, T., (2001). Effective Local Search Algorithm for the Vehicle Routing Problem with General Time Window Constraints. Proceedings of the 4th Metaheuristics International conference, p. 293-297.

64. Jaillet, P. (1985). Probabilistic Traveling Salesman Problems. PhD thesis, MIT, Cambridge, MA.

65. Johnson, D.S. and McGeoch, L. A. (1997). The traveling salesman problem: A case study in local optimization. In E.H.L. Aarts and J.K. Lenstra, editors, Local Search in Combinatorial Optimization, John Wiley & Sons, New York, NY, pages 215–310.

66. Ketabi, A. and Fevillet, R., (2009). Ant Colony Search Algorithm for Optimal General Start up during Power system Restoration. Hindawi Publishing Corporation, (p.1-11).

67. Kirkpatrick, S., Gelatt, C. D. and Vecchi, M. P.(1983). Optimization by simulated annealing. Science, 220(4598):671–680.

68. Kontoravdis, G. and Bard, J.F. (1995). A GRASP for the Vehicle Routing Problem with Time Windows. ORSA Journal on Computing, 7(1):11-23

69. Leepre Chanon, N., Limsakul, P. and Pothiya, S. (2010). Optimal Transmission Expansion Planning using Ant Colony Optimization. Journal of Sustainable Energy and Environment, 1:71-76.

70. Lenstra, J. K. Desroches, M., Savelbergh, M. W. P. and Soumis, F., (1988). Vehicle Routing with Time Windows: Optimization and Approximation. Elsevier Science Publishers, North-Holland, (pp.65-83).

71. Li, L. and Fu, E. (2002). The School Bus Routing Problem: a Case Study. Journal of the operation Research Society, 53:552-558.

72. Louren¸co, H. R., Martin, O. and St¨utzle, T. (2001). A beginner’s introduction to iterated local search. In Proceedings of MIC’2001 - Meta-heuristics International Conference, volume 1, Porto, Portugal,

73. Louren¸co, H. R., Martin, O. and St¨utzle, T. (2002). Iterated local search. In F. Glover and G. Kochenberger, editors, Handbook of Metaheuristics, volume 57 of International

Series in Operations Research & Management Science, Kluwer Academic Publishers, Norwell, MA, pages 321–353.

74. Marius, M. S. (1987). Algorithms for the Vehicle Routing and Scheduling Problem with Time Window Constraint. Operations Research, 35:254-265.

75. Martin, W., Otto, S. W. and Felten, E. W. (1991). Large-step markov chains for the traveling salesman problem. Complex Systems, 5(3):299–326.

76. McMullen, P. R. (2001). Ant colony Optimization Approach for addressing a JIT Sequencing Problem with Multiple Objectives. International Journal of Artificial Intelligence Engineering, 15:309-317.

77. Moscato, P. (1989). On evolution, search, optimization, genetic algorithms and martial arts: Towards memetic algorithms. Technical Report Caltech Concurrent Computation Program, Report. 826, California Institute of Technology, Pasadena, CA,

78. Nada, M. A. A. S. (2009). Ant Colony optimization algorithm. Ubicc Journal 4:825 -825.

79. Nazif, H. and Lee, L.S. (2010). Optimization Crossover Genetic Algorithm for Vehicle Routing Problem with Time Window, 7(1):95-101.

80. Nemhauser, G. I. and Wolsey, A. L. (1988). Integer and Combinatorial Optimization. John Wiley & Sons, New York, NY.

81. Palmgren, M. (2001). Optimization Method for Log Truck Scheduling. Thesis No. 880, Linkoping’s University, Sweden.

82. Ryan, D. M. and Foster, B. A., (1981). A Integer programming Approach to Scheduling. North-Holland Publishing Company, Harwell, UK, (p.269-280).

83. Savas, E. (1978). On equity in providing Public services. Journal of Management Science, (24): pp 800-808.

84. Schittekat, P., Sevaux M. and Sörensen K. (2009). A Mathematical Formulation for a School Bus Routing Problem. IEEE, p.1552-1556.

85. Schouwennaars, T., (2001). Mixed Integer Programming for Multi-vehicle Path Planning. Proceedings of the European Control Conference, p.2603-2608.

86. Secomandi, N. (1998). Exact and heuristic dynamic programming algorithms for the vehicle routing problem with stochastic demands. PhD thesis, University of Houston, Houston, TX.

87. Secomandi, N. (2001). A rollout policy for the vehicle routing problem with stochastic demands. Operations Research, 49(5):796–802.

88. St¨utzle, T. and Dorigo, M. (2002). A short convergence proof for a class of ACO algorithms. IEEE Transactions on Evolutionary Computation, 6(4):358–365.

89. St¨utzle, T. (1998). Local Search Algorithms for Combinatorial Problems - Analysis, Algorithms and New Applications. PhD thesis, Technische Universit¨at Darmstadt, Darmstadt, Germany. Published in 1999 - Infix, Sankt Augustin, Germany - volume 220 of DISKI.

90. Taillard, E. (1993). Parallel Iterative Search Method for Vehicle Routing Problems. International Journal of Networks, 23:661-673.

91. Taillard, E.P., Badeau, P., Gendreau, M., Guertin, F. and Potvin, J. Y., (1997). A Tabu Search Heuristic for the Vehicle Routing Problem with Soft Time Windows. Journal of Transportation Science, 31:170-186.

92. Tarasewich, P. and McMullen, P.R. (2002). Swan Intelligence: Power in Numbers. Commune ACM, 48(8):62-67.

93. Thangiah, S.R., Nygards, K. and Juell, P. (1991). A G.A. System for Vehicle Routing with Time Windows. Proceedings of the 7th IEEE conference on AI Applications, IEEE Press Miam, FL pp.422-25.

94. Vossen, I., Ball, M., Lotem, A. and Mau, D., (1999). On the use of Integer Programming Models in AI Planning (revised edition). Proceedings of International Joint Conference Artificial Intelligence (IJCAI-99), Nagoya, Japan.

95. Yang, Z., Yu, B. and Chang, C. (2007). A Parallel Ant Colony Algorithm for Bus Network Optimization. Computer-aided Civil and infrastructure Engineering, 22:44-55.