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DEPARTMENT OF MATHEMATICS

OPTIMAL LOAN PORTFOLIO, A CASE STUDY OF YAPRA RURAL BANK LIMITED, KWAME DANSO AGENCY

A Thesis submitted to the Department of Mathematics, in partial fulfillment

of the requirements for the award of degree of

Master of Science (Industrial Mathematics)

BY

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DECLARATION

I, hereby declare that this thesis entitled "OPTIMAL LOAN PORTFOLIO. A CASE STUDY OF YAPRA RURAL BANK LIMITED, KWAME DANSO AGENCY" represent my own work towards the award of MSc and that, to the best of my knowledge, except where due references is made has not been previously submitted to this or other institution for the award of degree, diploma or other qualification.

I also declare that I have wholly undertaken the study reported herein under supervision.

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ABSTRACT

Banking institutions in Ghana receive money on accounts which could be current or savings, pay and collect cheques drawn by or paid by customers, making of advances to customers. A bank does generate revenue in many ways including interest, transaction of fees and financial advice. The main method is through charging interest on the capital it lends out to customers. The main objectives of this study were; (i) to model the returns on loans given out by Yapra Rural Bank (ii) to determine the optimum loan portfolio for Yapra Rural bank using the revised simplex method. Both primary and secondary data were gathered from Yapra Rural Bank. The data included; type of loans, the interest rate and the probability of bad debt associated with each type of loan. The data was then modeled as a linear programming problem. The LP software also known as lips was used to solve the problem. It was observed that out of the two hundred and fifthty thousand Ghana cedis (GH¢250,000.00) made available for the disbursement to the various types of loans operated by the bank, it was revealed that (GH¢105,357.00) is to be disbursed as loan for agriculture, (GH¢26,785.70) should be given out to personal loans, (GH¢32,142.90) for transport loans, (GH¢12,500.00) for trading loans, and lastly (GH¢25,000.00) for salary loans. With these allocation the bank will make a maximum profit of sixty-two million, two hundred and twenty- three thousand Ghana cedis sixty pesewas (GH ¢62,223.60).

ABBREVIATIONS

ABBREVIATIONS

BOG .....Bank of Ghana

NBFI .....Non Banking financial Institutions

RCBs .....Rural and community Banks

SSNIT ..... Social security and national trust

NTHC ..... National trust holding company

IDA ..... International Development Association

HFC ..... Home finance company

GVCF ..... Ghana venture capital fund

NGO ..... Non Governmental organization

CU ..... Credit Unions

RMFI ..... Rural Micro finance Institutions

LPP ..... Linear programming problem

LP-Solver ..... Linear programming solver

RSM .... Revise simplex method

PD ..... Probability of Default

LGD ..... Loss given default

EAD ..... Exposure at default

IPM .... Interior-point method

LP ..... Linear programming

LTD .... Limited

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DEDICATION

To the Glory of the Almighty God, I dedicate this thesis to my dear late mother.

I truly love you and will miss you so much. May you rest in perfect peace until we meet

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CHAPTER 1

INTRODUCTION

1.0 BACKGROUND TO THE STUDY

Ghana's financial system is based on a number of banks and non-banking financial institutions, including the Bank of Ghana, which has the Central Bank, as the responsibility of advising the government on the implementation and control of monetary policies. Other institutions include commercial and merchant banks, discount houses, insurance companies, leasing companies, venture capital, a mortgage finance institution, and a stock exchange. Direct financing of projects in the country is provided by the commercial and other banking institutions. In an effort to ensure systematic development of the banking system, the Central Bank (Bank of Ghana), in addition to its traditional functions (for example formulation of monetary policies), also has the responsibility to ensure that banking is responsive to the needs of the public.

In attempt to encourage the establishment of new types of financial institutions, the Bank of Ghana pursues a liberal policy with regard to entry into the banking system, and is actively involved in the promotion of development and rural banking as well as in the establishment of discount houses. The minimum paid-up capital required for entry into the banking system is as follows; (www.bog.gov.gh 2008, October.)

1.1 TYPES OF BANKS

Commercial Banks: Commercial Banks are currently required to maintain a minimum of 57% of total deposits in liquid reserves. The Bank of Ghana fixes the Central Bank 'discount rate', which is used as the benchmark upon which commercial

banks base their interest rates. There are several Commercial and Development banks in Ghana.

The National Investment Bank: Is an industrial development bank providing financial assistance to manufacturing and processing industries, including agro-industrial projects. It maintains branches in all regions of the country.

The Agricultural Development Bank: Serves principally the agricultural sector - food production, livestock breeding, poultry farming and processing of agricultural produce. It has over Thirty one (31) branches throughout Ghana.

Leasing Companies: Though 'hire purchase' activities were conducted by the banks it was not until 1992 that a leasing law was enacted in Ghana. Since then, over three leasing companies have emerged and they are offering among others equipment leasing in Ghana. These include Ghana Leasing Company Limited, General Leasing Company Limited and LeaseAfric.

Venture Capital: Venture Capital provides capital for start-ups and high risk ventures. The Ghana Venture Capital Fund Limited (GVCF) - is managed by the Venture Fund Management Company. The Commonwealth Development Corporation is the lead investor and was joined by a few local banks and other foreign financial institutions. It has focused mainly on medium-sized, indigenous growth companies with expansion projects and shied away from start-ups because of the higher risks entailed.

Mortgage Financing: The Home Finance Company (HFC) is the leading secondary mortgage financing institution in Ghana. HFC was established in 1990 as the

implementing agency for a housing finance pilot scheme component for an Urban II Project provided to the Republic of Ghana by the International Development Association (World Bank). The IDA was joined by Social Security and National Insurance Trust (SSNIT), Merchant Bank and a number of insurance companies.

Discount Houses: In a bid to improve financial intermediation in the country, the non-bank financial institutions comprising the insurance and trust companies have joined forces with the banking institutions to establish discount houses in order to bring into single market institutions with cash balances for their intensive and effective use. These include the Consolidated Discount House and the Securities Discount Company, Gold Coast Securities Limited, and National Trust Holding Company (NTHC).

Non-Bank Financial Institutions: Ghana's non-bank financial institutions include the Social Security and National Insurance Trust (SSNIT), the Ghana Stock Exchange, Insurance companies, discount houses and other institutions.

Traditionally, rural development credit has been provided by two types of sources: institutional and non-institutional. In rural communities, non-institutional credit is provided by moneylenders, relatives, friends, traders, commission agents, cooperatives, consumers, distributors of farm inputs, and processors of agricultural products. Research has shown that the most common providers of loans in rural areas are friends and relatives who usually charge no interest or collateral (FAO 1994).

This credit market is small, however, and the total credit from these non-institutional sources is insufficient to implement rural development programs.

For rural development to proceed at a smooth pace, larger institutional sources of credit need to be created. In Ghana, institutional sources of credit are the commercial

banks, the Agricultural Development Bank, the National Investment Banks, and the Bank of Ghana Rural Banks. Until recently very few rural people, other than wealthy farmers and businessmen, had access to credit from these sources. The lack of interest in small rural credits by the National Investment Bank and the commercial banks is explained by the high cost of administering a large number of small credits spread over a wide area, coupled with the comparatively high level of default that has often accompanied small credits. The inability of rural borrowers to offer adequate security for loans, and the enormous risks associated with agricultural production, are the typical reasons given for the urban-based bias of commercial lending. The Agricultural Development Bank was created to service the rural sector in particular. It too, however, eventually began to concentrate on traditional urban-based banking activities.

To overcome many of these difficulties, the Ghanaian government, through the Bank of Ghana introduced the idea of rural banking into the country in 1976. According to the Association of Rural Banks (1992),

The aims of Rural Banks are:

(i) to stimulate banking habits among rural dwellers;

(ii) to mobilize resources locked up in the rural areas into the banking systems to facilitate development; and

(iii) to identify viable industries in their respective catchment [areas] for investment and development."

Due to these liberal policies of the Bank of Ghana many banks across Africa are opening branches in Ghana. This has also facilitated the opening of a lot of rural banks across the country. Currently there are over One Hundred and Twenty (120) Rural Banks in the country.

Rural Banks are unit banks established to provide facilities for the rural communities in which they are located. They are owned, managed and patronized by the local people. Some of these banks also operate agencies to cater for communities that are located far from the bank's facilities.

Savings mobilized through rural banks are invested in small-scale agricultural activities, cottage industries, transportation and trading. Rural banks also provide commercial banking services such as giving loans to people within the community in which they operate.

1.2 LOANS

A loan is a type of debt. Like all debt instruments, a loan entails the redistribution of financial assets over time, between the lender and the borrower. In a loan, the borrower initially receives or borrows an amount of money, called the principal, from the lender, and is obligated to pay back or repay an equal amount of money to the lender at a later time. Typically, the money is paid back in regular installments, or partial repayments in an annuity, each installment is the same amount. The loan is generally provided at a cost, referred to as interest on the debt, which provides an incentive for the lender to engage in the loan. Due to poor allocation of their loan disbursement they are not able to optimize their profits when they give out these loans, hence monies that could have been used to offer social services in the community in which they operate goes into “Bad Debts”.

A model is proposed to help Rural Banks allocate their funds available for loan disbursement optimally. The used of Yapra Rural Bank at Kwame Danso in the Sene West District of the Brong Ahafo Region as a case study is vital.

1.3 RURAL BANK OPERATIONS

The Bank of Ghana has streamlined Rural Bank lending operations to ensure that Bank credit actually benefits the small scale rural producer and the rural community. The Bank of Ghana has developed an Operational Manual for all Rural Banks. Applications are accepted from individuals, groups, associations, and companies. Recommendations to reject an application must be justified by specific and clearly stated reasons and cannot be based on vague suspicions.

Before granting a loan to a group, the Bank requires that there be mutual trust and respect among members. In the case of a group loan approval, members are held jointly and severally liable. The group cannot exceed 20 members, and the group leader must have a clean loan record. The Bank of Ghana has developed a mandatory sectorial allocation for Rural Bank loans. The Rural Banks try to reduce the cash element in the loans to the minimum possible to prevent the diversion of funds for purposes other than those for which they are granted. Loan repayment conditions are determined with reference to the borrower's capacity to repay. A "grace period" is allowed between the loan approval date and the time the borrower is expected to generate sufficient income to repay the loan.

During the "operation period" of the loan, the Bank's Project Officer monitors the borrower to ensure proper use of funds and punctual repayment. Routine and emergency visits by the Project Officer are common during the operation period of the loan. Rescheduling may be allowed if there are circumstances which the loans committee or board of Directors accepts as "unforeseen developments." If there is default on the loan, the case is sent to the Bank's lawyer(s) for action. By 1990, the Rural Banks were experiencing negative profitability resulting in capital inadequacy

and, in some cases, the inability to meet depositors' withdrawal demands. The Bank of Ghana ordered a restructuring of the Rural Banks. By December of 1991 all of the Rural Banks had undergone diagnostic study conducted by outside consultants. The restructuring was designed to determine financial strength, organizational capability, and management status in line with existing statutory requirements. The Banks updated and standardized accounts and procedures. The Banks introduced internal control systems and management information systems. After the restructuring process, the number of Rural Banks meeting the capital adequacy requirement increased from 2 to 55

The Bank of Ghana has instituted measures to maintain public confidence in the remaining mediocre and distressed Rural Banks (Bank of Ghana 1995). Today, the Rural Banks are still given the opportunity to determine who should benefit from their credit resources. There is substantial evidence of misdirection and misapplication of rural credit by the Rural Banks and the rural people. Many rural banks appear to give credit to people who do not fall into the Bank of Ghana target groups. It is not uncommon to see many credit recipients spending borrowed credit on land litigation and funeral ceremonies instead of productive ventures. The Bank of Ghana initiated the Rural Bank system with the hope that small-scale rural producers and small towns would benefit from the new credit resources. It is uncertain whether or not the Rural Banks are fulfilling the basic functions for which they were created.

1.4 STRUCTURE AND PERFORMANCE OF RURAL AND MICRO FINANCE INDUSTRY

The financial system in Ghana falls into three main categories: formal, semi-formal, and informal:

• **Formal financial institutions:** are those incorporated under the Companies Code 1963 and licensed by the Bank of Ghana (BOG) under either the Banking Law 1989 or the Financial Institutions (Non-Banking) Law 1993 (NBFI Law) to provide financial services under Bank of Ghana regulation. Rural and Community Banks (RCBs) operate as commercial banks under the Banking Law, except that they cannot undertake foreign exchange operations. Their clientele is drawn from their local catchments area, and their minimum capital requirement is significantly lower. Among the nine specified categories of non-bank financial institutions (NBFIs), the Savings and Loans Companies (S&Ls), which are restricted to a limited range of services, are most active in micro and small-scale financial intermediation using microfinance methodologies.

• Non Governmental Organizations (NGOs) and Credit Unions (CUs) are considered to be **semi formal** – legally registered but not licensed by the Bank of Ghana. NGOs are incorporated as companies limited by guarantee (not for profit) under the Companies Code. Their poverty focus leads most of them to provide multiple services to poor clients, including micro credit, though mostly on a limited scale. They are not licensed to take deposits from the public and hence have to use external (usually donor) funds for micro credit. Credit Unions are registered by the Department of Cooperatives as cooperative thrift societies that can accept deposits from and give loans to their members. Although credit unions are nominally included in the NBFI Law, BOG has allowed the apex body Ghana Cooperative Credit Union Association to continue to regulate the societies pending the introduction of a new Credit Union Law.

• The **informal financial system** covers a range of activities known as *susu*, including individual savings collectors, rotating savings and credit associations, and savings and credit "clubs" run by an operator. It also includes moneylenders, trade creditors, self-help groups, and personal loans from friends and relatives. Moneylenders are supposed to be licensed by the police under the Moneylenders Ordinance 1957. The commercial banking system is dominated by a few major banks (among the 17 total) and reaches only about 5% of households, most of which are excluded by high minimum deposit requirements. With 60% of the money supply outside the commercial banking system, the RCBs, S&Ls, and the semi-formal and informal financial systems play a particularly important role in Ghana's private sector development and poverty reduction strategies. The assets of RCBs are nearly 4% of those of the commercial banking system, with S&Ls and CUs adding another 2%. While "RMFIs" is used to refer collectively to the full range of these institutions, they use different methodologies to reach different (albeit overlapping) clientele among farmers, rural households, the poor, and micro enterprises, and hence different regulatory and supervisory instruments may be appropriate.

1.5 LOANS TYPES IN GHANA

1.5.1 SECURED LOAN

A secured loan is that type of loan in which the borrower pledges some asset (e.g. a house or property) as collateral for the loan.

1.5.2 UNSECURED LOAN

Unsecured loans are monetary loans that are not secured against the borrower's assets.

These may be available from financial institutions under many different guises or

marketing packages-loan covenants. Acting as a provider of loans is one of the principal tasks for financial institutions.

1.5.3 MORTGAGE LOAN

A mortgage is a legal instrument that pledges a house or other real estate as security for repayment of a loan. By providing guarantee that the loan will be paid back, a mortgage enables a person to buy property without having the funds to pay for it outright.

1.5.4 CREDIT

A credit is transaction involving given out money or other property with the promise of repayment, usually at a fixed future date. The transferor thereby becomes a creditor, and the transferee, a debtor.

1.6 SOURCES OF LOANS

Banking is the business of providing financial services to consumers and businesses. The basic services a bank provides are checking accounts, which can be used like money to make payments and purchase goods and services; savings accounts and time deposits that can be used to save money for future use; loans that consumers and businesses can use to purchase goods and services and basic cash management services such as check cashing and foreign currency exchange. Four types of banks specialize in offering these basic banking services; these are commercial banks, savings and loan associations, savings banks, and credit unions.

1.6.1 COMMERCIAL BANKS

A bank is any financial institution that receives, collects, transfers, pays, exchanges, lends, invests, or safeguards money for its customers. This broader definition includes many other financial institutions that are not usually thought of as banks but which nevertheless provide one or more of these broadly defined banking services.

These institutions include finance companies, investment companies, investment banks, insurance companies, pension funds, security brokers and dealers, mortgage companies.

1.6.2 CREDIT UNIONS

These are financial cooperatives and credit associations that provide loans to its members at lower rates of interest than would otherwise be available. The capital funds of credit unions come from the purchase of shares by members, who receive yearly dividends on the basis of their investment. Credit unions are operated for the mutual benefit of their members and are usually formed by persons who share a common bond, such as membership in a church, lodge, trade union, or professional association. Many corporations have assisted their employees in establishing credit unions. The loans are usually for the acquisition of consumer goods rather than for the purchase of real estate.

1.7 ABUSES OF LOANS

One form of abuse in the granting of loans involves granting a loan in order to put the borrower in a position that one can gain advantage over.

Another form of abuse is where the lender charges excessive interest. In different time periods and cultures the acceptable interest rate has varied, from no interest at all to unlimited interest rates. Credit card companies in some countries have been accused by consumer organizations of lending at usurious interest rates and making money out of frivolous extra charges.

Abuses can also take place in the form of the customer abusing the lender by not repaying the loan or with intent to defraud the lender.

1.8 FUNDAMENTAL DUTIES

A Bank carries out money and credit policy in accordance with the needs of the economy and so as to maintain price stability. The Bank takes necessary measures to protect the domestic and international value of the national currency and regulates its volume and circulation. It also extends credits to banks and conducts open market operations in order to regulate money supply and liquidity in the economy.

Moreover, the Bank determines the terms and types of deposits, as well as their maturity dates and validity periods, and the parity of the national currency against gold and foreign currencies. It manages gold and foreign exchange reserves and trades in foreign exchange and precious metals on the stock exchange. The Bank, in particular, carries out the duties of financial and economic advisor, fiscal agent and treasurer to the Government.

1.8.1 FUNDAMENTAL POWERS

The Bank has the privilege of issuing bank notes and the authority to take decisions on money and credit issues and to submit proposals to the Government. The Bank

determines the rediscount, discount and interest rates applicable to its own transactions.

1.8.2 ADVISORY DUTIES

The Bank presents to the Government, when required, its views with regard to measures to be taken on money and credit, and submits advisory opinions on matters related to implementation of the Banking Law or on banking and credit issues in general, upon request of the Government.

The Bank can also be consulted prior to any decision granting permission for the establishment of banks and other financial institutions, as well as for the liquidation of such institutions for which the power to liquidate rests with the Government.

PROFILE OF YAPRA RURAL BANK

1.9.1 Organizational profile

Yapra rural bank was established as one of the numerous rural banks in Ghana. It was registered on the 24th October, 1984 with Certificate of Incorporation No.1983. However, Bank of Ghana licensed it as a Rural Bank on 12th March 1985 under licensed No.101.

Yapra Rural bank limited was established in a town called Kwame Danso in the Sene West District of the Brong-Ahafo region of Ghana. The bank has its head office at Prang, and can boast of seven other branches at Atebubu, Kwame Danso, Yeji, Abease, Kajaji, Zabrama and Parambo.

VISION

The vision of Yapra rural bank is to be the most preferred Rural Bank in Brong-Ahafo Region by 2012.

MISSION

Its mission is to ensure that its products and services meet the expectations of its varied and segmented customers. The management of the bank has stated that it will accomplish this mission by utilizing highly qualified, competent and well-motivated staff to provide quality and innovative products/services that responds to the dynamic market.

The hallmark of Yapra Rural Bank is to maintain its integrity in all its operations at all times.

CORE VALUES

The bank believes that with humility they will serve their customers and motivate their staff and retain them to keep the Bank as a going concern. The need to understand customer and staff needs and to respond to such needs at all times.

The hallmark of Yapra Rural Bank, Kwame Danso is to maintain its integrity in all its operations at all times. There is the need to succeed, this calls for total commitment from staff on continual basis on the grounds of fairness and integrity.

CORE PRODUCTS

The core products of the bank are lending and savings products.

Lending

The bank lends to the following Sectors:

Agriculture (cash crops, food crops, livestock, and fishing), Cottage Industry (agro-based and non-agro based), transport loans, trading and others.

Lending Products

The lending products for Yapra Rural Bank, Kwame Danso are overdraft and loans

The various types of loan provided by the bank are:

(i) Personal loans

(ii) Trade loans – Individuals, Enterprises, Corporate entities etc.

(iii) Salary loans

(iv) Agricultural Loans

(v) Transport Loans

Savings Products

The savings product of Yapra Rural Bank includes:

(i) Savings deposit accounts,

(ii) susu deposit accounts,

(iii) current/demand deposit accounts and

(iv) term/fixed deposit accounts.

1.10 STATEMENT OF THE PROBLEM

Rural banks in Ghana are grappling with huge challenges in managing their loan loss reserves due to bad loans and poor management systems applied by the banks. Due to poor allocation of funds some rural banks record marginal profits with some running at a lost. This situation is not different from that of the Yapra Rural bank, kwame Danso.

The main aim of this project is therefore to propose a linear model subject to some constraints for the established rural bank at Kwame Danso named Yapra Rural Bank to enable them disburse their funds allocated for loans optimally leading to maximization of profits.

1.11 OBJECTIVES OF THE STUDY

Loan provides banks the highest profit hence the administration of loan portfolio really affect the banks profit. Thus some banks failure is due to poor loan management system. The research will be in line with the use of linear programming to determine the optimal loan portfolio to reduce the bank's operational risk during lending process.

The general objective of the research is to find out optimum loan portfolio for the bank, adherence to the regulations governing the activities of Yapra Rural Bank and their profitability levels and to make suggestions that would optimize the bank's loan portfolio and further enhance the operation of rural banks to create more wealth for their shareholders.

Specifically the research intends to:

(i) model the returns on loans given out by Yapra Rural Bank as linear programming problem (LPP)

(ii) determine the optimum loan portfolio for Yapra Rural Bank using the revised simplex method.

1.12 METHODOLOGY

Banks receive money on current or deposit account, pay and collect cheques drawn by or paid by customers, making advances to customers. A bank can generate revenue in different ways including interest, transaction of fees and financial advice. The main method is through changing interest on the capital it lends out to customers.

In order for the bank to maximize their profit, the LPP model will be used strictly based on the Bank's Loan Policy and its previous history on loan disbursement. The

model will be solved using the revised simplex method (RSM). The LP Solver also called lips which is a scientific tool or method is used for the analysis.

The data employed in this study are both primary and secondary. The primary data was collected through questionnaire administration which were answered by the credit officer of the bank and secondary data was obtained from publication and financial statement of the bank. This was used to determine the profitability of the bank and to find out its lending management practices. The analysis of the financial statement from the questionnaire will reveal the overall profit of the bank from loan and other financial activities. The sources of the materials included the internet, libraries, journals etc

1.13 JUSTIFICATION OF THE STUDY

The institution of Banks is one of the fastest growing institutions in Ghana which has a greater impact on the economy and the society. Among other things banks also give loans to prospective loan seekers.

This study would be a useful tool in the hands of the Ministry of Finance and Economic Planning. That is, the ministry of finance can issued series of policies to promote financial institutions which provide for a massive policy support for development of rural banks in Ghana.

Also, it is envisaged that the result or findings from this study would help to create an awareness of the functions of the rural banks in the rural areas to the people of Ghana and the policy makers.

The proposed model is going to help banks to efficiently distribute the funds they have available for loan in order to maximize their profit. The proposed model will

also help decision makers at the Bank to formulate prudent and effective loan policies. That is, it would help decision makers to measure the portfolio risk both for short term returns and hold long term strategy.

Again, it shall help mangers to minimize the funding of cost while lending against the market risk.

Finally, the findings from this study would form a data base for further research work into same or similar areas. This makes this study justifiable and worthwhile.

1.14 ORGANISATION OF THE STUDY

The first chapter of this thesis talks about the introduction to the topic. The second chapter 2 discusses literature review (work done by other researchers on the same or similar field). The third chapter deals with the methodology applied by the researcher in dealing with the problem or the topic (it is the actual work done by the researcher). The forth chapter is titled discussion; it is the one which explains the meaning of the result obtained from the work and its relevance and application. The final chapter which is the fifth is given the heading conclusion. It is the summary of the piece of work done by the researcher. It also gives recommendation to areas that can be researched in the near future by other researchers and some techniques that can help others to do good work in the same or similar area of research.

CHAPTER 2

LITERATURE REVIEW

2.0 INTRODUCTION

This section of the study seeks to find out what others have written about the subject matter. The section primarily deals with what other authors have done in regard to this subject matter.

Lending to firms is the basic business and risky activity for every commercial bank or rural bank. This means that loan portfolio management is the most important activity for getting the maximum return and minimum risk from banking loans. In this approach and over the recent years, there has been a large development in banking portfolio management methods. Whether due to weakness in the credit standard, loan portfolio problems have been the major cause of bank losses and failures.

Various Authors have proposed that banks should decide for the loans' portfolio, which should be determined by the needs of the firms, market competition and the capacity of the bank. Moreover, it should decide on the distribution of its capital among the various sorts of loans, which differ in duration and risk and are affected by the environment, the borrowers' deposits at the given bank as well as other factors.

2.2 Bank Loans

Borrowing money from the bank is called a bank loan. Loans constitute the most significant operation of rural banks.

According to Klaassen, (1998) a well planned loan policy is essential for the successful performance of the credit policy of a bank, for the acquisition of high return and the minimization of the risk that originates in credit extension. Loans on

one hand are the most important source of return for every rural bank, and on the other hand, the main item in banking risk area, so it is because of Modeling in this approach has developed over the past few years. There exists a wide variety of models for optimizing the portfolio that differ in their fundamental assumptions. Selection of the appropriate model is an important aspect of loan portfolio management.

2.3 LINEAR PROGRAMMING FOR BANK PORTFOLIO MANAGEMENT

Markowitz on Portfolio Theory changed the field more than any other single event. His doctoral thesis written in (1952) at the University of Chicago dealt with portfolio selection and in it he developed the basic portfolio model. Because of this work, Markowitz is often referred to as the "father of modern portfolio theory", and much subsequent research had been based on this effort (Sharpe, 1963, Fama, 1965 and Melnik, 1970).

Markowitz (1952, 1959) derived the expected rate of return for a portfolio of assets and an expected risk measure. Markowitz showed that the variance of the rate of return was a meaningful measure of risk under a reasonable set of assumptions and derived the formula for computing the variance of the portfolio. This portfolio variance formulation indicated the importance of diversification for reducing risk, and showed how to properly diversify. The Markowitz model is based on certain assumptions. Under these assumptions, a single asset or portfolio of assets is considered to be efficient if no other asset or portfolio of assets offers higher expected return with the same (or lower) risk, or lower risk with the same (or higher) expected return.

According to Robinson, (1961) Loans and investment are in fact complementary. Investment should be tailored to the strength, seasonality and character of loan demand. He reiterated that banks that experience sharp seasonal fluctuations in loan demand need to maintain more liquidity in their investment programmme. Moreover, during a boom when loan demand is high and credit-worthy customers are available, banks should allocate more funds to loans and fewer funds to investment, and vice versa during recession when loan demand is low. He said the crucial banking problem is to resolve the conflict between safety and profitability in the employment of bank funds. The conflict is essentially the problem between liquidity and the size of the earning assets. Robinson suggested that where there is a conflict between safety and profitability, it is better to err on the side of safety. The best practice is identifying procedures that can bring out the optimal mixture of management of banks funds.

He also proposed four priorities of the use of banks funds. These include primary reserves, (or protective investment), loans and advances (customer credit demand) and investment account (open market investment for income) in descending order of priority. His assessment has been fully supported in other works by Sheng-Yi andYong (1988).

A bank has to place primary reserves at the top of the priority in order to comply with the minimum legal requirement, to meet any immediate withdrawal demand by depositors and to provide a means of clearing cheques and credit obligations among banks.

Secondary reserves include cash items from banks, treasury bills and other short-term securities. Bank should have to satisfy customers' loan demand before allocating the balance of the funds in the investment market.

According to Tobin (1965) portfolio theory can be applied to bank portfolio management in that a bank would maximize the rates of return of its portfolio of assets, subject to the expected degree of risk and liquidity.

Chambers and Charnes (1961) applied linear programming analysis on the consolidated balance sheets of commercial banks in Singapore for the period 1978-1983. The results show that by large banks do not try to maximize the returns of their portfolios, subject to legal, policy, bounding and total assets constraints, which denote riskiness and liquidity of the portfolio of assets. In a direct way, banks conform to the portfolio choice theory; they have to balance yield and liquidity against security. They pointed out that although the computer cannot replace a manager, linear programming can serve as a useful guide.

Lakshmikantham et al. (1997), a linear programming problem in an inequality form having a bounded solution is solved error-free using an algorithm that sorts the inequalities, removes the redundant ones, and uses the p-adic arithmetic.

Yoshito (2004) considered the problem of finite dimensional approximation of the dual problem in abstract linear programming approach to control system design. A constraint qualification that guarantees the existence of a sequence of finite dimensional dual problems that computes the true optimal value. The result is based on the averaging integration by a probability measures.

Chambers and Chames (1961), as well as Cohen and Hammer (1967, 1972), developed a series of sophisticated linear programming models for managing the balance sheet of larger banks, while Waterman and Gee (1963) and Fortson and Dince (1977) proposed less elegant formulations which were better suited for the small to medium-sized bank. The use of linear and other types of mathematical programming

techniques has received extensive coverage in the banking literature. Several programming models have also been proposed for managing a bank's investment security portfolio, including those by Booth (1972).

Baldirer et al., (1981) used linear programming model to solve fundamental issues facing senior bank management of Central Carolina Bank and Trust Company in structuring the bank's balance sheet of approximately $360 million.

Greenberg et al., (1986) developed a framework for model formulation and analysis to support operations and management of large-scale linear programs from the combined capabilities of camps and analyze. Both the systems were reviewed briefly and the interface which integrates the two systems was then described. The model formulation, matrix generation, and model management capability of camps and the complementary model and solution analysis capability of analyze were presented within a unified framework. Relevant generic functions were highlighted, and an example was presented in detail to illustrate the level of integration achieved in the current prototype system. Some new results on discourse models and model management support were given in a framework designed to move toward an 'intelligent' system for linear programming modeling and analysis.

Jianq et al., (2004) proposed a novel linear programming based method to estimate arbitrary motion from two images. The proposed method always finds the global optimal solution of the linearized motion estimation energy function and thus is much more robust than traditional motion estimation schemes. As well, the method estimates the occlusion map and motion field at the same time. To further reduce the complexity of even a complexity-reduced pure linear programming method they presented a two-phase scheme for estimating the dense motion field. In the first step,

they estimated a relatively sparse motion field for the edge pixels using a non-regular sampling scheme, based on the proposed linear programming method. In the second step, they set out a detail-preserving variational method to upgrade the result into a dense motion field. The proposed scheme is much faster than a purely linear programming based dense motion estimation scheme. And, since they used a global optimization method linear programming in the first estimation step, the proposed two-phase scheme was also significantly more robust than a pure variational scheme.

2.4 PROBABILITY OF LOSS ON LOAN PORTFOLIO

A surveyed in Altman et al; (2003) agreed to Klaus Rheinberger and Martin Summer in their credit risk portfolio models, three parameters drives loan losses: The probability of default by individual obligors (PD), the loss given default (LGD) and the exposure at default (EAD). While the standard credit risk models focus on modeling the PD for a given LGD, a growing recent literature has looked closer into the issue of explaining LGD and of exploring the consequences of dependencies between PD and LGD.

Most of the papers on the issue of dependency between PD and LGD have been written for US data and usually find strong correlations between these two variables. The first papers investigating the consequences of these dependencies for credit portfolio risk analysis were Frye (2000a) and Frye (2000b) using a credit risk model suggested by Finger [1999] and Gordy (2000). The authors used a different credit risk model in the tradition of actuarial portfolio loss models and focus directly on two risk factors: an aggregate PD and an aggregate API as well as their dependence. The authors used this approach because their interest was to investigate the implications of

some stylized facts on asset prices and credit risk that have frequently been found in the macroeconomic literature for the risk of collateralized loan portfolios.

Jarrow (2001), Jokivuolle and Peura (2003), Carey and Gordy (2003), Hu and Perraudin (2002), Bakshi et al. (2001), G¨urtler and Heithecker (2005) and Altman et al., (2004) are various authors who believe that the credit risk model when use gives us maximal flexibility with assumptions about the distribution of systematic risk factors. There are a variety of models that try to capture the dependence between PD and LGD. These models look at bond data but some also cover loans. There is other literature that looks in some detail into the determinants of LGD.

Rasmussen et al., (2000) considered the dynamics of the Danish mortgage loan system several models are prepared to reject the choices of a mortgage,). The models were formulated as multi stage stochastic integer programs, which are difficult to solve for more than ten (10) stages. Linear Programming was used to obtain near optimal solutions for large problem instances.

According to Gendzio and Grothey (2005) recognized linear programming as a powerful tool to help decision making under uncertainty in financial planning. It shows how portfolio optimization problems with sizes measured in millions of constraints and decision variables featuring constraints on semi-variance, skewness or nonlinear activity functions in the objective can be solved.

Falkie et al., (1972) linear programming models have been used to solve many production planning problems for a multi-plant operations serving several customers. The main factors considered are availability of recourses, inventory restrictions and demand requirements. The objective function to be optimized is the total annual net returns from the operations.

Karmarkar (1984) proved that an Interior-Point Method (IPM) can solve LPs in polynomial time. The two decades that followed the publication of Karmarkar's paper have seen a very intense effort by the optimization research community to study theoretical and practical properties of IPMs. One of the early discoveries was that IPMs can be viewed as methods based on Newton's method but are modified to handle the inequality constraints. Some of the most important contributions were made by Nesterov and Ne-mirovski who showed that the IPM machinery can be applied to a much larger class of problems than just LPs.

Konno and Yamazaki (1991) propose a linear programming model instead of the classical quadratic model. Their approach is based on the observation that different measures of risk, such a volatility and $L1$-risk, are closely related, and that alternate measures of risk are also appropriate for portfolio optimization.

Amponsah et al., (2011) presented an optimal loan allocation mix policy from the steady state distribution of loan disbursement process. This was done by using monthly data on actual loan disbursement of four loan types for a period of twenty-four months, by using a transition matrix. From the estimated probability transition matrix, the steady state distribution indicated that in a long run, trade loan should constituted 77.3% of the total loan, 10.3% for service loan, 2.0% for production loan and 10.4% for susu loan.

Acharya et al., (2003) investigated defaulted bonds, Duellmann and Trapp (2004) look into recoveries of US corporate credit exposures, Grunert and Weber (2005) investigated recoveries of German bank loans and Schuermann (2004) summarizes existing knowledge about recoveries. While these papers show a nuanced picture of the determinants of recoveries that consists of many microeconomic and legal features

such as the industry sector in which exposures are held or the seniority of a claim all papers find that macroeconomic conditions play a key role.

2.5 BENEFITS OF LOAN PORTFOLIO TO MANAGEMENT AND DECISION MAKERS OF BANKS

Light et al., (2005) indicated that loan portfolio management is one of the responsibilities critical to the success of an institution. It is the dynamic process of managing an institution's primary earning assets to achieve the primary objectives of the board's strategic business and capital plans.

Loan portfolio encompasses all systems and processes used by management to adequately plan, direct, control and monitor the institutions lending operations. Loan portfolio ensures that all material aspect of lending operations are adequately controlled relative to the institutions risk bearing capacity.

Loan portfolio helps management and decision makers in the analysis of how business results are achieved, whether such results will continue and how the institution can optimize its opportunity and provide great benefit to its members.

Loan portfolio also helps decision makers to measure the portfolio risk both for short term returns and hold long term strategy. Finally, it helps mangers to minimize the funding of cost while lending against the market risk.

2.6 SUMMARY

In this chapter, other research works done by some scholars' in connection with rural bank operations, bank loans and Linear Programming Problems were reviewed. In the next chapter, we shall put forward guidelines used to model and solve Linear Programming Problems.

CHAPTER 3

METHODOLOGY

3.0 INTRODUCTION

Linear programming (LP) was developed during World War II, when a system with which to maximize the efficiency of resources was of utmost importance. New war-related projects demanded attention and spread resources thin. Programming was a military term that referred to activities such as planning schedules efficiently or deploying men optimally. George Dantzig, a member of the U.S. Air Force, developed the Simplex method of optimization in 1947 in order to provide an efficient algorithm for solving programming problems that had linear structures. Since then, experts from a variety of fields especially mathematics and economics, have developed the theory behind linear programming and explored its applications. This thesis will cover the main concepts in linear programming, including examples when appropriate. First, I explore the basic definition, basic assumptions or properties of linear programming problems (LPP), and method or theories of linear programs. In order to illustrate some applications of linear programming, I will present methods solution, concluding with the statement of the Revised Simplex method (RSM).

DEFINITION OF LPP

Linear Programming is a mathematical technique for optimum allocation of limited or scarce resources, such as labour, material, machine, money, energy and so on , to several competing activities such as products, services, jobs and so on, on the basis of a given criteria of optimality.

The term **'Linear'** is used to describe the proportionate relationship of two or more variables in a model. The given change in one variable will always cause a resulting proportional change in another variable.

The word, **'programming'** is used to specify a sort of planning that involves the economic allocation of limited resources by adopting a particular course of action or strategy among various alternatives strategies to achieve the desired objective.

Hence, **Linear Programming** is a mathematical technique for optimum allocation of limited or scarce resources, such as labour, material, machine, money, energy etc.

3.2 STRUCTURE OF LINEAR PROGRAMMING MODEL

The general structure of the Linear Programming model essentially consists of three components.

i) The activities (variables) and their relationships

ii) The objective function and

iii) The constraints

The activities are represented by $X_1, X_2, X_3 \ldots\ldots X_n$.

These are known as Decision variables.

The objective function of an LPP (Linear Programming Problem) is a mathematical representation of the objective in terms a measurable quantity such as profit, cost, revenue, etc.

Optimize (Maximize or Minimize) $Z=C_1X_1 +C_2X_2+ \ldots\ldots\ldots C_n X_n$

Where Z is the measure of performance variable

$X_1, X_2, X_3, X_4 \ldots\ldots X_n$ are the decision variables

And $C_1, C_2, \ldots C_n$ are the parameters that give contribution to decision variables.

The constraints; These are the set of linear inequalities and/or equalities which impose restriction of the limited resources.

3.3 ASSUMPTIONS

Several assumptions are implicit in linear programming problems. These assumptions are:

1. **Proportionality** The contribution of any variable to the objective function or constraints is proportional to that variable. This implies no dis-counts or economies to scale. For example, the value of $8x_1$ is twice the value of $4x_2$, no more or less.
2. **Additivity** The contribution of any variable to the objective function or constraints is independent of the values of the other variables.
3. **Divisibility** Decision variables can be fractions. However, by using a special technique called integer programming, we can bypass this condition. Unfortunately, integer programming is beyond the scope of this thesis.
4. **Certainty** This assumption is also called the deterministic assumption. This means that all parameters (all coefficients in the objective function and the constraints) are known with certainty. Realistically, however, coefficients and parameters are often the result of guess-work and approximation. The effect of changing these numbers can be determined with sensitivity analysis.
5. **Linearity** All relationships in the LP model (i.e. in both objective function and constraints) must be linear.

GENERAL MATHEMATICAL MODEL OF A LPP

3.4.1 Manipulating a Linear Programming Problem

Many linear problems do not initially match the canonical form, which is important when considering the Simplex algorithm. The constraints may be in the form of inequalities, variables may not have a nonnegativity constraint, or the problem may want to maximize z instead of minimize z. We now consider some ways to manipulate problems into the desired form.

Constraint Inequalities: We first consider the problem of making all constraints of a linear programming problem in the form of strict equalities. By introducing new variables to the problem that represent the difference between the left and the right-hand sides of the constraints we subtracting a slack variable from a "greater than or equal to" constraint or by adding an excess variable to a "less than or equal to" constraint, transforms inequalities into equalities.

Optimize (Maximize or Minimize) $Z=C_1X_1 + C_2X_2 +\ldots\ldots+C_nX_n$

Subject to constraints,

$$a_{11}x_1 + a_{12}x_2 + \ldots\ldots\ldots + a_{1n}x_n (\leq = \geq) b_1$$

$$a_{21}x_1 + a_{22}x_2 + \ldots\ldots\ldots + a_{2n}x_n (\leq = \geq) b_2$$

$$a_{21}x_1 + a_{22}x_2 + \ldots\ldots\ldots + a_{2n}x_n (\leq = \geq) b_3$$

$$a_{m1}x_1 + a_{m2}x_2 + \_\_\_ + a_{mn}x_n (\leq = \geq) b_m$$

$$x_1; \qquad x_2; \qquad : : : ; \qquad x_n \geq 0$$

In linear programming z, the expression being optimized is called the objective function. The variables x_1; x_2: : : x_n are called decision variables, and their values are subject to

m + 1 constraints (every line ending with a bi, plus the nonnegativity constraint). A set of x_1; x_2 : : : x_n satisfying all the constraints is called a feasible point and the set of all such points is called the feasible region. The solution of the linear program must be a point (x_1; x_2; : : : ; x_n) in the feasible region, or else not all the constraints would be satisfied. When there are only two variables, it is possible to solve linear programming problems graphically. Graphical solution is easy and illustrates most of the basic ideas about solutions of linear programming problems. The problem is that most problems involve more than two variables and graphical methods do not apply. Algorithms exist that can solve any linear programming problem. These algorithms are widely used in industry. The oldest and still most widely used algorithm is the simplex algorithm.

GUIDELINES FOR FORMULATING LINEAR PROGRAMMING MODEL

i) Identify and define the decision variable of the problem

ii) Define the objective function

iii) State the constraints to which the objective function should be optimized (i.e. Maximization or Minimization)

iv) Add the non-negative constraints from the consideration that the negative values of the decision variables do not have any valid physical interpretation.

Example 1

A manufacturer produces two types of models M_1 and M_2. Each model of the type M_1 requires 4 hours of grinding and 2 hours of polishing; whereas each model of M_2

requires 2 hours of grinding and 5 hours of polishing. The manufacturer has 2 grinders and 3 polishers. Each grinder works for 40 hours a week and each polisher works 60 hours a week. Profit on M_1 model is Rs.3.00 and on model M_2 is Rs.4.00.Whatever produced in a week is sold in the market. How should the manufacturer allocate his production capacity to the two types of models, so that he makes maximum profit in a week?

i. Identify and define the decision variable of the problem

Let X_1 and X_2 be the number of units of M_1 and M_2 model.

ii. Define the objective function

Since the profits on both the models are given, the objective function is to maximize the profit.

Max $Z = 3X_1 + 4X_2$

iii. State the constraints to which the objective function should be optimized (i.e. Maximization or Minimization) There are two constraints one for grinding and the other for polishing. The grinding constraint is given by $4X_1 + 2X_2 < 80$.

Number of hours available on grinding machine per week is 40 hrs. There are two grinders.

Hence the total grinding hour available is $40X2 = 80$ hours.

The polishing constraint is given by

$2X1 + 5X2 < 180$

Number of hours available on polishing machine per week is 60 hrs. There are three grinders.

Hence the total grinding hour available is $60X_3 = 180$ hours.

Finally we have,

Max $Z = 3X_1 + 4X_2$

Subject to constraints,

$4X_1 + 2X_2 < 80$

$2X_1 + 5X_2 < 180$

$X_1, X_2 > 0$

Example 2

A firm can produce 3 types of cloth, A, B and C and 3 kinds of wool are required Red, Green and Blue.1 unit of length of type A cloth needs 2 meters of red wool and 3 meters of blue wool.1 unit of length of type B cloth needs 3 meters of red wool, 2 meters of green wool and 2 meters of blue wool.1 unit type of C cloth needs 5 meters of green wool and 4 meters of blue wool. The firm has a stock of 8 meters of red, 10 meters of green and 15 meters of blue. It is assumed that the income obtained from 1 unit of type A is Rs.3, from B is Rs.5 and from C is Rs.4. Formulate this as an LPP.

i) Identify and define the decision variable of the problem

Let X_1, X_2 and X_3 are the quantity produced of cloth type A, B and C respectively.

ii) Define the objective function

The incomes obtained for all the three types of cloths are given; the objective function is to maximize the income.

$\text{Max } Z = 3X_1 + 5X_2 + 4X_3$

iii) State the constraints to which the objective function should be optimized.

The above objective function is subjected to following three constraints.

$2X_1 + 3X_2 \leq 8$

$2X_2 + 5X_3 \leq 10$

$3X_1 + 2X_2 + 4X_3 \leq 15$

$X_1, X_2, X_3 \geq 0$

Finally we have,

Max $Z = 3X_1 + 5X_2 + 4X_3$ is subjected to three constraints

$2X_1 + 3X_2 \leq 8$

$2X_2 + 5X_3 \leq 10$

$3X_1 + 2X_2 + 4X_3 \leq 15$

$X_1, X_2, X_3 \geq 0$

3.6 MINIMIZATION PROBLEMS

Example 3

A person requires 10, 12, and 12 units chemicals A, B and C respectively for his garden. A liquid product contains 5, 2 and 1 units of A, B and C respectively per jar. A dry product contains 1, 2 and 4 units of A, B and C per carton. If the liquid product sells for Rs.3 per jar and the dry product sells for Rs.2 per carton, how many of each should be purchased, in order to minimize the cost and meet the requirements?

i) Identify and define the decision variable of the problem

Let X_1 and X_2 be the number of units of liquid and dry products.

ii) Define the objective function

The cost of Liquid and Dry products are given; the objective function is to minimize the cost.

Min. $Z = 3X_1 + 2X_2$

iii) State the constraints to which the objective function should be optimized.

The above objective function is subjected to following three constraints.

$5X_1 + X_2 > 10$

$2X_1 + 2X_2 > 12$

$X_1 + 4X_2 > 12$

$X_1, X_2 > 0$

Finally we have,

Min. $Z = 3X_1 + 2X_2$ is subjected to two constraints

$5X_1 + X_2 > 10$

$2X_1 + 2X_2 > 12$

$X_1 + 4X_2 > 12$

$X_1, X_2 > 0$

LINEAR PROGRAMMING: GRAPHICAL SOLUTION

Example 1

Solve the following LPP by graphical method

Maximize $Z = 5X_1 + 3X_2$

Subject to constraints

$2X_1 + X_2 \leq 1000$

$X_1 \leq 400$

$X_2 \leq 700$

$X_1, X_2 \geq 0$

Solution:

The first constraint $2X_1 + X_2 \leq 1000$ can be represented as follows.

We set $2X_1 + X_2 = 1000$

When $X_1 = 0$ in the above constraint, we get,

$2 \times 0 + X_2 = 1000$

$X_2 = 1000$

Similarly when X2 = 0 in the above constraint, we get,

$2X_1 + 0 = 1000$

$X_1 = 1000/2 = 500$

The second constraint $X_1 \leq 400$ can be represented as follows,

We set $X_1 = 400$

The third constraint $X_2 \leq 700$ can be represented as follows,

We set $X_2 = 700$

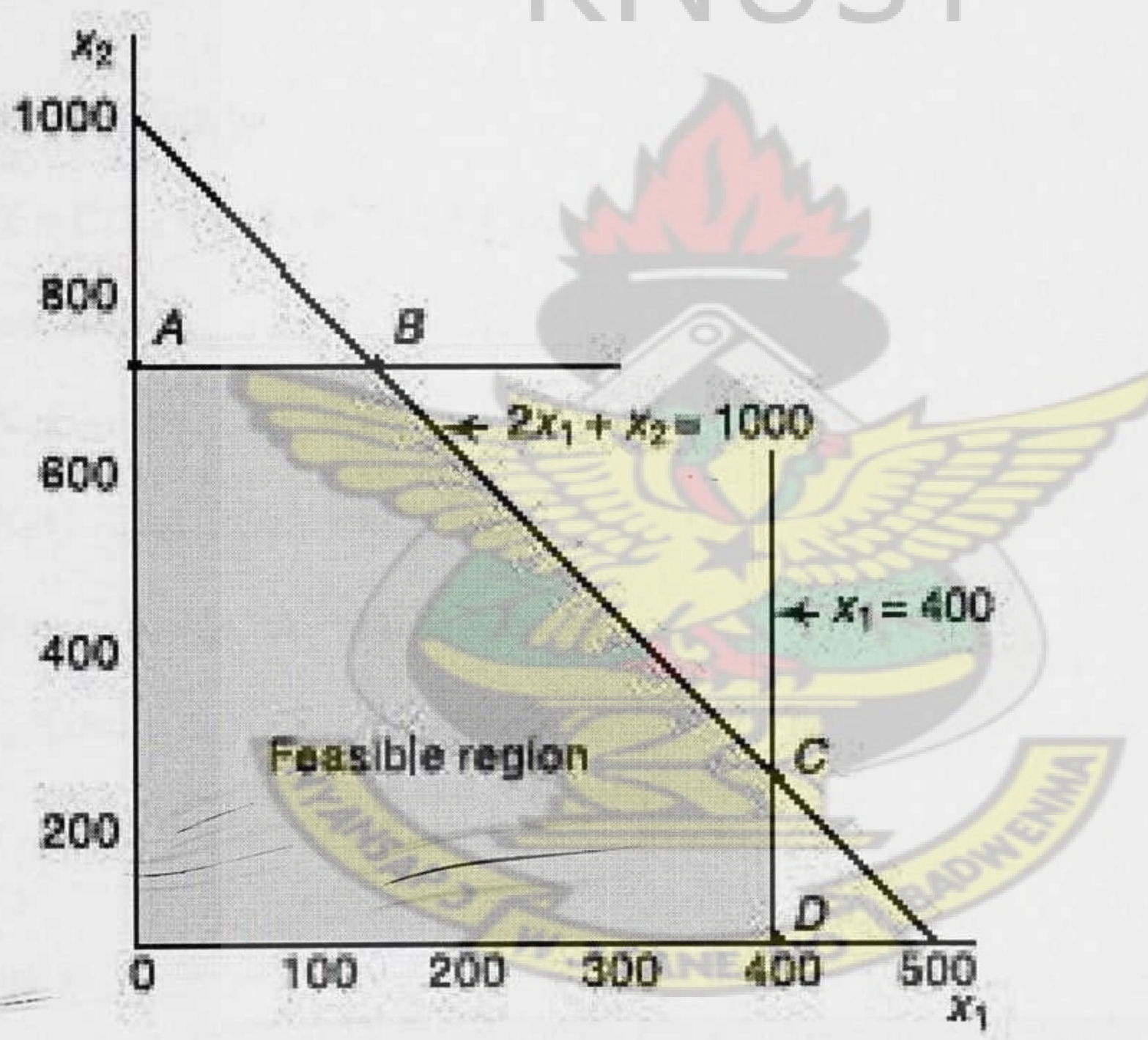

Figure1

The constraints are shown plotted in the above figure

Point	X_1	X_2	$Z = 5X_1 + 3X_2$
0	0	0	0
A	0	700	Z = 5 x 0 + 3 x 700 = 2,100
B	150	700	Z = 5 x 150 + 3 x 700 = **2,850*** Maximum
C	400	200	Z = 5 x 400 + 3 x 200 = 2,600
D	400	0	Z = 5 x 400 + 3 x 0 = 2,000

The Maximum profit is at point B

When $X_1 = 150$ and $X_2 = 700$

$Z =$ **2850**

3.8 DUALITY IN LINEAR PROGRAMMING

Duality in Linear Programming, for every LPP there is a unique LPP associated with it involving the same data and closely related optimal solution. The original problem is then called primal problem while the other is called its Dual problem.

Let the primal problem be

Maximize $Z = C_1X_1 + C_2X_2 + \ldots\ldots + C_nX_n$

Subject to constraints,

$a_{11}X_1 + a_{12}X_2 + \ldots\ldots\ldots\ldots\ldots + a_{1n}X_n \leq, b_1$

$a_{21}X_1 + a_{22}X_2 + \ldots\ldots\ldots\ldots\ldots + a_{2n}X_n \leq, b_2$

$a_{31}X_1 + a_{32}X_2 + \ldots\ldots\ldots\ldots\ldots + a_{3n}X_n \leq, b_3$

$a_{m1}X_1 + a_{m2}X_2 + \ldots\ldots\ldots\ldots\ldots + a_{mn}X_n \leq, b_m$

and $X_1, X_2 \ldots . X_n \geq 0$

Then its Dual is

Minimize $G = b_1W_1 + b_2W_2 + b_3W_3 + \ldots\ldots + b_mW_m$

Subject to constraints,

$a_{11}W_1 + a_{21}W_2 + a_{31}W_3 + \ldots\ldots\ldots\ldots + a_{m1}W_m \geq C_1$

$a_{12}W_1 + a_{22}W_2 + a_{32}W_3 + \ldots\ldots\ldots\ldots + a_{m2}W_m \geq C_2$

$a_{13}W_1 + a_{23}W_2 + a_{33}W_3 + \ldots\ldots\ldots\ldots + a_{m3}W_m \geq C_3$

$a_{1n}W_1 + a_{2n}W_2 + a_{3n}W_3 + \ldots\ldots\ldots\ldots + a_{mn}W_m \geq C_n$

$W_1, W_2, W_3 \ldots\ldots W_m \geq 0$

Example 1

Write the Dual of the following LPP

Min $Z = 2X_2 + 5X_3$

$X_1 + X_2 > 2$

$2X_1 + X_2 + 6X_3 < 6$

$X_1 - X_2 + 3X_3 = 4$

and $X_1, X_2, X_3 > 0$

Rearrange the constraints into a standard form, we get

Min $Z = 0X_1 + 2X_2 + 5X_3$

Subject to constraints,

$X_1 + X_2 + 0X_3 > 2$

$-2X_1 - X_2 - 6X_3 > -6$

$X_1 - X_2 + 3X_3 > 4$

$-X_1 + X_2 - 3X_3 > -4$

and $X_1, X_2, X_3 > 0$

The Dual of the above primal is as follows

Max.$G = 2W_1 - 6W_2 + 4W_3 - 4W_4$

Subject to constraints,

$W_1 - 2W_2 + W_3 - W_4 < 0$

$W_1 - W_2 - W_3 + W_4 < 2$

$0W_1 - 6W_2 + 3W_3 - 3W_4 < 5$

$W_1, W_2, W_3, W_4 > 0$

Max $G = 2W_1 - 6W_2 + 4(W_3 - W_4)$

Subject to constraints,

$W_1 - 2W_2 + (W_3 - W_4) < 0$

$W_1 - W_2 - W_3 + W_4 < 2$

$0W_1 - 6W_2 - 3(W_3 - W_4) < 5$

$W_1, W_2, W_3, W_4 > 0$

$\text{Max.}G = 2W_1 - 6W_2 + 4W_5$

Subject to constraints,

$W_1 - 2W_2 + W_5 < 0$

$W_1 - W_2 - W_5 < 2$

$0W_1 - 6W_2 - 3W_5 < 5$

$W_1, W_2, > 0$, W_5 is unrestricted in sign

3.9 DESCRIPTION OF THE REVISED METHOD

The Revised Simplex Method is commonly used for solving linear programs. This method operates on a data structure that is roughly of size *m* by *m* instead of the whole tableau. This is a computational gain over the full tableau method, especially in sparse systems (where the matrix has many zero entries) and/or in problems with many more columns than rows. On the other hand, the revised method requires extra computation to generate necessary elements of the tableau: The revised simplex method is especially efficient for linear programs that are sparse and have high aspect ratio (n/m). A linear program is sparse if most of the elements of the dictionary are 0, and it has a high aspect ratio if n/m is large. Updating any of the representations used by the revised method is usually, at worst, of order m_2. On the other hand, pivoting on the explicit representation of the dictionary takes order *mn*. Thus for high aspect ratios the standard method takes much more work. Fortunately, in our distributed method, this work is done in parallel with linear speedup in a straightforward way. This will be

made clearer as we derive performance models for the revised and full tableau methods later in this section.

3.10 The Revised Simplex Method

While solving linear programming problem on a digital computer by regular simplex method, it requires storing the entire simplex table in the memory of the computer table, which may not be feasible for very large problem. But it is necessary to calculate each table during each iteration.

The revised simplex method which is a modification of the original method is more economical on the computer, as it computes and stores only the relevant information needed currently for testing and / or improving the current solution. i.e. it needs only

- The net evaluation row Δj to determine the non-basic variable that enters the basis.
- The pivot column
- The current basis variables and their values (X_B column) to determine the minimum positive ratio and then identify the basis variable to leave the basis.

The above information is directly obtained from the original equations by making use of the inverse of the current basis matrix at any iteration.

There are two standard forms for revised simplex method

- **Standard form-I** – In this form, it is assumed that an identity matrix is obtained after introducing slack variables only.
- **Standard form-II** – If artificial variables are needed for an identity matrix, then two phase method of ordinary simplex method is used in a slightly different way to handle artificial variables.

3.9.1 Steps for solving Revised Simplex Method in Standard Form

Solve by Revised simplex method

Max $Z = 2x_1 + x_2$

Subject to

$3x_1 + 4x_2 \leq 6$

$6x_1 + x_2 \leq 3$

and $x_1, x_2 \geq 0$

SLPP

Max $Z = 2x_1 + x_2 + 0s_1 + 0s_2$

Subject to

$3x_1 + 4x_2 + s_1 = 6$

$6x_1 + x_2 + s_2 = 3$

and $x_1, x_2, s_1, s_2 \geq 0$

Step 1 – Express the given problem in standard form

- Ensure all $b_i \geq 0$
- The objective function should be of maximization
- Use of non-negative slack variables to convert inequalities to equations

The objective function is also treated as first constraint equation

$Z - 2x_1 - x_2 + 0s_1 + 0s_2 = 0$

$3x_1 + 4x_2 + s_1 + 0s_2 = 6$ -- (1)

$6x_1 + x_2 + 0s_1 + s_2 = 3$

And $x_1, x_2, s_1, s_2 \geq 0$

Step 2 – Construct the starting table in the revised simplex form

Express (1) in the matrix form with suitable notation

$\beta_0^{(1)}$ $\beta_1^{(1)}$ $\beta_2^{(1)}$

e_1 $a_1^{(1)}$ $a_2^{(1}$ $a_3^{(1)}$ $a_4^{(1)}$

X_B

$$\begin{bmatrix} 1 & -2 & -1 & 0 & 0 \\ 0 & 3 & 4 & 1 & 0 \\ 0 & 6 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} Z \\ X_1 \\ X_2 \\ S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ 3 \end{bmatrix}$$

Column vector corresponding to Z is usually denoted by e1 matrix B_1, which is usually denoted as B1 = $[\beta_0^{(1)}, \beta_1^{(1)} \ldots \beta_n^{(1)}]$

Hence the column $\beta_0^{(1)}$,$\beta 1^{(1)}$, $\beta_2^{(1)}$ constitutes the basis matrix B_1 (whose inverse B_1^{-1} is also B_1)

Basic Variables	B_1^{-1}			X_B	X_k	X_B / X_k
	e1 (Z)	$\beta_1^{(1)}$	$\beta_2^{(1)}$			
Z	1	0	0	0		
S_1	0	1	0	6		
S_2	0	0	1	3		

$a_1^{(1)}$	$a_2^{(1)}$
-2	-1
3	4
6	1

Step 3 – Computation of Δj for $a_1^{(1)}$ and $a_2^{(1)}$

Δ_1 = first row of $B_1^{-1} * a_1^{(1)} = 1 * -2 + 0 * 3 + 0 * 6 = -2$

Δ_2 = first row of $B_1^{-1} * a_2^{(1)} = 1 * -1 + 0 * 4 + 0 * 1 = -1$

Step 4 – Apply the test of optimality

Both Δ_1 and Δ_2 are negative. So find the most negative value and determine the incoming vector.

Therefore most negative value is $\Delta 1 = -2$. This indicates $a_1(^1)$ (x_1) is incoming vector.

Step 5 – Compute the column vector X_k

$X_k = B_1^{-1} * a_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} -2 \\ 3 \\ 6 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 6 \end{bmatrix}$$

Step 6 – Determine the outgoing vector. We are not supposed to calculate for Z row.

Basic Variables	B_1^{-1}			X_B	X_k	X_B / X_k
	e1 (Z)	$\beta_1^{(1)}$	$\beta_2^{(1)}$			
Z	1	0	0	0	-2	-
S_1	0	1	0	6	3	2
S_2	0	0	1	3	6 ↑ incoming	1/2→outgoing

Step 7 – Determination of improved solution

Column e1 will never change, x_1 is incoming so place it outside the rectangular boundary

	$\beta_1^{(1)}$	$\beta_1^{(1)}$	X_B	
R_1	0	1/3	1	X_1 -2
R_2	1	-1/2	9/2	3
R_3	0	1/6	1/2	6

Make the pivot element as 1 and the respective column elements to zero.

	$\beta_1^{(1)}$	$\beta_1^{(1)}$	X_B	X_1
R_1	0	1/3	1	0
R_2	1	-1/2	9/2	0
R_3	0	1/6	1/2	1

Construct the table to start with second iteration

Basic Variables	B_1^{-1}			X_B	X_k	X_B / X_k
	e1 (Z)	$\beta_1^{(1)}$	$\beta_2^{(1)}$			
Z	1	0	1/3	1		
S_1	0	1	-1/2	9/2		
S_2	0	0	1/2	½		

$a_4^{(1)}$	$a_2^{(1)}$
0	-1
0	4
1	1

$\Delta_4 = 1 * 0 + 0 * 0 + 1/3 * 1 = 1/3$

$\Delta_2 = 1 * -1 + 0 * 4 + 1/3 * 1 = -2/3$

Δ_2 is most negative. Therefore $a_2^{(1)}$ is incoming vector.

Compute the column vector

$$\begin{bmatrix} 1 & 0 & 1/3 \\ 0 & 1 & -1/2 \\ 0 & 0 & 1/6 \end{bmatrix} * \begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} -2/3 \\ 7/2 \\ 1/6 \end{bmatrix}$$

Determine the outgoing vector

Basic Variables	B_1^{-1}			X_B	X_k	X_B / X_k
	e1 (Z)	$\beta_1^{(1)}$	$\beta_2^{(1)}$			
Z	1	0	1/3	1	-2/3	-
S_1	0	1	-1/2	9/2	7/2	1/2→outgoing
S_2	0	0	1/6	½	1/6 ↑ incoming	3

Determination of improved solution

	$\beta_1^{(1)}$	$\beta_1^{(1)}$	X_B	X_2
R_1	0	1/3	1	-2/3
R_2	1	-1/2	9/2	7/2
R_3	0	1/6	1/2	1/6

	$\beta_1^{(1)}$	$\beta_1^{(1)}$	X_B	X_2
R_1	4/21	5/21	13/7	0
R_2	2/7	-1/7	9/7	1
R_3	-1/21	8/42	2/7	0

Basic Variables	B_1^{-1}			X_B	X_k	X_B / X_k
	e1 (Z)	$\beta_1^{(1)}$	$\beta_2^{(1)}$			
Z	1	4/21	5/21	13/7		
S_1	0	2/7	-1/7	9/7		
S_2	0	-1/21	8/42	2/7		

$a_1^{(1)}$	$a_2^{(1)}$
0	0
0	1
1	0

$\Delta_4 = 1 * 0 + 4/21 * 0 + 5/21 * 1 = 5/21$

$\Delta_3 = 1 * 0 + 4/21 * 1 + 5/21 * 0 = 4/21$

Δ_4 and Δ_3 are positive. Therefore optimal solution is Max Z = 13/7, x1= 2/7, x2 = 9/7

11.3 Worked Example s

Example 1

Max $Z = x_1 + 2x_2$

Subject to

$x_1 + x_2 \leq 3$

$x_1 + 2x_2 \leq 5$

$3x_1 + x_2 \leq 6$

and $x_1, x_2 \geq 0$

Solution

SLPP

Max $Z = x_1 + 2x_2 + 0s_1 + 0s_2 + 0s_3$

Subject to

$x_1 + x_2 + s_1 = 3$

$x_1 + 2x_2 + s_2 = 5$

$3x_1 + x_2 + s_3 = 6$

and $x_1, x_2, s_1, s_2, s_3 \geq 0$

Standard Form

$Z - x_1 - 2x_2 - 0s_1 - 0s_2 - 0s_3 = 0$

$x_1 + x_2 + s_1 + 0s_2 + 0s_3 = 3$

$x_1 + 2x_2 + 0s_1 + s_2 + 0s_3 = 5$

$3x_1 + x_2 + 0s_1 + 0s_2 + s_3 = 6$

and $x_1, x_2, s_1, s_2, s_3 \geq 0$

Matrix form

$\beta_0^{(1)}$ $\beta_1^{(1)}$ $\beta_2^{(1)}$ $\beta_3^{(1)}$

e_1 $a_1^{(1)}$ $a_2^{(1)}$ $a_3^{(1)}$ $a_4^{(1)}$ $a_5^{(1)}$

$$\begin{bmatrix} 1 & -1 & -2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 3 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Z \\ X_1 \\ X_2 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 5 \\ 6 \end{bmatrix}$$

Revised simplex table

Basic	B_1^{-1}			X_B	X_k	X_B / X_k
	e1	$\beta_1^{(1)}$	$\square_2^{(1)}$			
Variables	(Z)					
Z	1	0	0	0		
S_1	0	1	0	3		
S_2	0	0	0	5		
S_3	0	0	1	6		

Additional table

$a_1^{(1)}$	$a_2^{(1)}$
-1	-2
1	1
1	2
3	1

Computation of Δj for a1

Δ_1 = first row of $B_1^{-1} * a_1^{(1)} = 1 * -1 + 0 * 1 + 0 * 1 + 0 * 3 = -1$

Δ_2 = first row of $B_1^{-1} * a_2^{(1)} = 1 * -2 + 0 * 1 + 0 * 2 + 0 * 1 = -2$

$\Delta_2 = -2$ is most negative. So $a_2^{(1)}$ (x_2) is incoming vector.

ompute the column vector X_k

$X_k = B_1^{-1} * a_2^{(1)}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} -2 \\ 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 2 \\ 1 \end{bmatrix}$$

Basic variables	B_1^{-1}				X_B	X_k	X_B / X_k
	e1 (Z)	$\square_1^{(1)}$	$\square_2^{(1)}$	$\square_3^{(1)}$			
Z	1	0	0	0	0	-2	-
S_1	0	1	0	0	3	1	3
S_2	0	0	1	0	5	2	5/2→
S_3	0	0	0	1	6	1	6
						↑	

Improved Solution

	$\square_1^{(I)}$	$\square_1^{(I)}$	$\square_3^{(I)}$	X_B	X_k
R_1	0	0	0	0	-2
R_2	0	0	0	3	1
R_3	1	1	0	5	2
R_4	0	0	1	6	1

R_1	$\square_1^{(I)}$	$\square_2^{(I)}$	$\square_3^{(I)}$	X_B	X_k
R_2	0	1	0	5	0
R_3	1	-12	0	1/2	0
R_4	0	1/2	0	5/2	1
	0	-1/2	1	7/2	0

Revised simplex table for II iteration

Basic variables	B_1^{-1}				X_B	X_k	X_B / X_k
	e1 (Z)	$\square_1^{(I)}$	$\square_2^{(I)}$	$\square_3^{(I)}$			
Z	1	0	0	0	0	-2	-
S_1	0	1	0	0	3	1	3
S_2	0	0	1	0	5	2	5/2→
S_3	0	0	0	1	6	1	6
						↑	

$\Delta 1 = 1 * -1 + 0 * 1 + 1 * 1 + 0 * 3 = 0$

$\Delta 4 = 1 * 0 + 0 * 0 + 1 * 1 + 0 * 0 = 1$

$\Delta 1$ and $\Delta 4$ are positive. Therefore optimal solution is Max Z = 5, x1= 0, x2 = 5/2

CHAPTER 4

DATA ANALYSIS AND PRESENTATION

4.0 Introduction

In this chapter, we shall formulate and analyze the data collected from the bank. Solutions and findings from the model shall be presented on tables.

4.1 Sources and data collection

The data used was both primary and secondary data. The primary data was gathered through the administration of questionnaires for the credit officer of the bank to fill. The secondary data on the other hand was extracted from the bank published annual reports and financial statements. This category of data was mainly in quantitative form. It is believed that secondary data are likely to be of higher-quality than could be obtained by collecting empirical data..

A model was proposed and solved to help maximize its net profit. The bank is in the process of formulating a loan policy involving a total fund of GH¢250,000.00 Being a full service facility, the bank is obligated to grant loan to different clients.

The table 4.1 provides the types of loans (column one), the interest rate charge (column two) and the probability of bad debts (column three) on each associated loan type of Yapra Rural bank as estimated for 2013 financial year.

Table 4.1 Loans available to the Yapra Rural Bank

Types of loan	Interest rate (r_i)	Probability of bad debts (K_i)
X_1: agriculture	0.34	0.04
X_2: personal	0.34	0.02
X_3: transport	0.28	0.05
X_4: trading	0.28	0.02
X_5: salary	0.28	0.01

Bad debts are assumed unrecoverable and hence produce no interest revenue. For policy reasons, there are limits on how the bank allocates the fund. The bank requires that the disbursement of fund for loan should be done as follows:

1. Allocates at most 55% of the total funds to agriculture and Salary loan.
2. Transport loan should be at least 50% of the personal, trading and salary loans (to ensure optimality).
3. The sum of trading and salary loans should be at most 15% of the total funds.
4. The sum of personal and transport loans should be at least 50% of agriculture, trading and salary loans.
5. The sum of agriculture and trading loans should be at most 30% of the total funds.
6. Trading loans should not exceed 5% of the total funds
7. The total ratio of bad debt on all loans may not exceed 0.5

4.2 Proposed Loan model for Yapra Rural bank, Kwame Danso Agency

4.2.1 Formulation

Base on the empirical data, we formulate the proposed model for Yapra Rural Bank LTD.

Decision variables

The variables of the model are defined as follows;

Let

X_1 = Amount for agriculture loan

X_2 = Amount for personal loan

X_3 = Amount for transport loan

X_4 = Amount for trading loan

X_5 = Amount for salary loan

4.2.2 Objective Function

The objective of the bank is to maximize its net return, Z which comprise of the difference between revenue from interest and lost fund due to bad debts for each amount of loan disburses.

Table 4.2 depicts the loan amount (column one), the amount of bad debts to each loan type (column two) and finally the amount contributing to profit associated with the various types of loans operated by the bank.

Table 4.2 The amount and percentage impose on the loan allocated to the various loan items

Loan Amount	Amount of bad debts (K_i x_i)	Amount Contributing to profit $(1-K_i)\,x_i$
X_1	$0.04x_1$	$0.96x_1$
X_2	$0.02x_2$	$0.98x_2$
X_3	$0.05x_3$	$0.95x_3$
X_4	$0.02x_4$	$0.98x_4$
X_5	$0.01x_5$	$0.99x_5$

Profit on loan is given by

$Z = A_1 (1-K_1) x_1 + A_2 (1-K_2) x_2 + A_3 (1-K_3) x_3 + \ldots\ldots$ where $K_i > 0$

The above expression can be summarized as below

$$\text{Maximize } Z = \sum_{i=1}^{5} A_i (1-K_i) x_i$$

Z is the optimal solution,

A_i is the coefficients of objective function, and

X_i is the various loan items

$(1 - K_i)$ xi is the amount contributing to profit

The Objective function is Maximize

$Z = 0.34\,(0.96x_1) + 0.34\,(0.98x_2) + 0.28\,(0.95x_3) + 0.28(0.98x_4) + 0.28(0.99x_5)$

$$Z = 0.3264x_1 + 0.3332x_2 + 0.266x_3 + 0.2744x_4 + 0.2772x_5$$

4.2.3 Constraints

The problem has eight (8) constraints

1. Limit on total funds available (x_1, x_2, x_3, x_4, x_5)

   The total funds available for disbursement is GH¢ 250000

   $x_1 + x_2 + x_3 + x_4 + x_5 \leq 250000$

2. Limit on agriculture loans (x_1) and personal loan (x_2)

   Allocates at most 55% of the total funds to agriculture and transport loan

   $x_1 + x_3 \leq 0.55\ (250000)$

   $x_1 + X_3 \leq 137500$

3. Limit on transport (x_3), personal (x_2), trading (x_4), and salary (x_5) loans

   transport loan should be at least 50% of the personal, trading and salary loans to ensure optimality.

   $x_3 \geq 0.5\ (x_2 + x_4 + x_5)$

   $0.5x_2 - x_3 + 0.5x_4 + 0.5x_5 \leq 0$

4. Limit on trading (x_4) and salary loans (x_5)

   The sum of trading and salary loans should be of most 15% of the total funds.

   $x_4 + x_5 \leq 0.15\ (250{,}000)$

   $x_4 + x_5 \leq 37500$

5. Limit on personal (x_2), transport (x_3) and agriculture (x_1), trading (x_4) and salary loans (x_5).

The sum of personal and transport loans should be at most 50% of agriculture, trading and salary loans.

$$x_2 + x_3 \leq 0.5\ (x_1 + x_4 + x_5)$$

$$0.5x_1 - x_2 - x_3 + 0.5x_4 + 0.5x_5 \geq 0$$

6. Limit on agriculture (x_1) and trading loan (x_4)

   The Sum of agriculture and trading loans should be at least 30% of the total funds.

   $$x_1 + x_4 \geq 0.3\ (250000)$$

   $$x_1 + x_4 \geq 75000$$

7. Limit on trading loan (x4) trading loans should exceed 5% of the total funds

   $$x_4 \geq 0.05\ (250000)$$

   $$x_4 \geq 12500$$

8. Limit on bad debts

   The total ratio of bad debt on all loans may not exceed 0.05

   $$\frac{0.04x_1 + 0.02x_2 + 0.05x_3 + 0.02x_4 + 0.01x_5}{x_1 + x_2 + x_3 + x_4 + x_5} \leq 0.05$$

   $$-0.01x_1 - 0.03x_2 + 0x_3 - 0.03x_4 - 0.04x_5 \leq 0$$

9. Non – negativity

   $$x_1, x_2, x_3, x_4, x_5 \geq 0$$

4.3 Resulting Linear Programming Problem

Maximize

$$Z = 0.3264x_1 + 0.3332x_2 + 0.266x_3 + 0.2744x_4 + 0.2772x_5$$

Subject to

$$x_1 + x_2 + x_3 + x_4 + x_5 \leq 250000$$

$$x_1 + x_3 \leq 137500$$

$$0.5x_2 - x_3 + 0.5x_4 + 0.5x_5 \leq 0$$

$x_4 + x_5 \leq 37500$

$0.5x_1 - x_2 - x_3 + 0.5x_4 + 0.5x_5 \leq 0$

$x_1 + x_4 \leq 75000$

$x_4 \leq 12500$

$-0.01x_1 + 0x_3 - 0.0x_4 - 0.04x_5 \leq 0$

$x_1, x_2, x_3, x_4, x_5, \geq 0$

Thus,

$x_1 + x_2 + x_3 + x_4 + x_5 \leq 250000$.....(1)

$x_1 + 0x_2 + x_3 + 0x_4 + 0x_5 \leq 137500$.....(2)

$0x_1 + x0.5x_2 - x_3 + 0.5x_4 + 0.5x_5 \leq 0$.....(3)

$0x_1 + 0x_2 + 0x_3 + x_4 + x_5 \leq 37500$.....(4)

$0.5x_1 - x_2 - x_3 - 0.5x_4 + 0.5x_5 \leq 0$.....(5)

$x_1 + 0x_2 + 0x_3 + x_4 + 0x_5 \leq 75000$.....(6)

$0x_1 + 0x_2 + 0x_3 + x_4 + 0x_5 \leq 12500$.....(7)

$-0.01x_1 - 0.03x_2 + 0x_3 - 0.03x_4 - 0.04x_5 \leq 0$.....(8)

4.4 Solution of the LP model

The LP solver (lips) was used to solve the linear systems above on a Toshiba laptop with the following information;

Model: satellite C655

Rating: 3.3 windows Experience Index

Processor: Intel(R) Celeron(R) CPU 925@2.30 GHz 2.29 GHz

Installed memory (RAM): 2.00 GB (1.87 GB usable)

System type: 64-bite operating system

In other to check accuracy of results from the laptop, number of trial was (3) which gave same solution each with (6) iterations. Lipsy uses revised simplex as described in chapter three.

4.4.1 Input data

Writing the linear system in matrix form, we have the input data as;

$$Z = [0.3264 \quad 0.3332 \quad 0.266 \quad 0.2744 \quad 0.2772]$$

$$\mathbf{A}=\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0.5 & -1 & 0.5 & 0.5 \\ 0 & 0 & 0 & 1 & 1 \\ 0.5 & -1 & -1 & -0.5 & 0.5 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -0.01 & -0.03 & 0 & -0.03 & -0.04 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 250000 \\ 137500 \\ 0 \\ 37500 \\ 0 \\ 75000 \\ 12500 \\ 0 \end{bmatrix}$$

That is, AX = B

4.4.2 Optimal Solution

Table 4.3 Shows the decision variables (column one), the optimal value of the variables (column two) and the status of the variables (column three).

Table 4.3 Results - variable

Decision variable	**Value**	**Status**
X_1: Agriculture loans	105357	Basic
X_2: Personal loans	26785.7	Basic
X_3: Transport loans	32142.9	Basic
X_4: Trading loans	12500	Basic
X_5: Salary loans	25000	Basic

Source: field data

The table 4.3 revealed that funds for loans disbursement should be allocated to all the various loans types with the various amounts indicated. As such, the statuses of the variables are all basic since none of them is zero for that matter.

Table 4.4 Optimal Value (Z) = GH¢ 62,223.60

Variable	**Objective coefficient**	**Total contribution objective to function**
X_1: Agriculture loans	0.3264	GH¢ 34388.50
X_2: Personal loans	0.3332	GH¢ 8925
X_3: Transport loans	0.2660	GH¢ 8550.01
X_4: Trading loans	0.2744	GH¢ 3430
X_5: Salary loans	0.2772	GH¢ 6930
Total value of the objective function (z)		GH¢ **62,223.60**

Source: field data

Table 4.4 depicts the variables (column one) and objective coefficient (column two) and the objective value contribution (column three). Optimal value of the various contributions to the objective function is GH¢ 62,223.60 This implies that the bank would be making a net profit annually of six-two thousand two hundred and twenty-three Ghana cedis sixty pesewa.

Table 4.5 shows the constraint (column one) the current right hand side (column two) and the Dual price (column three).

Table 4.5 Results-Constraints

Constraints	Value	Current RHS	Dual Price
1	201786	250000	0
2	137500	137500	0.412971
3	0	0	0.320114
4	37500	37500	713/3500
5	0	0	-0.173143
6	117857	7500	0
7	12500	12500	-0.175943
8	-3232.14	0	0

Source: field data

The dual price for constraint (2) is 0.412971, constraint (3) is 0.320114, constraint (4) is 0.20371 and constraint (5) is -0.173143.These are non zero because they correspond to the active constraints at the optimum, hence their slack variables are non basic (0), so the dual cannot be zero.

The table shows the dual values for the constraints. The dual value is the change in optimal value or profit per unit change in the resource or right hand side of the various constraints. The Table 4.5 shows that the dual value corresponding to the second constraint is 0.412971. This indicates that the optimal profits will increase by 0.412971 for each increase in the total amount to be disbursed. Thus it is advisable to increase the amount available for allocation if extra funds are available. Similarly, constraint three will also cause for the profit to be increase by 0.320114 for each increase in the third constraint. This indicates that, if the bad debt is reduced, it will increase the bad debt ratio with the total loan facility thereby decreasing the risk involved and increasing the profit.

However, a value of -0.173143, and -0.175943 shows that there is a decrease of -0.173143 for constraint 5, and -0.175943 for constraint 7 in the value of the objective function per unit increase in the amount allocated to satisfy these constraints.

Moreover, the studies have shown in Table 4.5 that, the model will yield an optimum amount of sixty-two thousand two hundred and twenty-three Ghana cedis sixty pesewas (GHS 62,223.60) as a profit from the loan disbursement.

In short, after solving the loan portfolio using Lips, it was realized that, in order for the bank to achieve the maximum returns on the total loans given, it should allocate GHS 105357.00 for agricultural loans, GHS 26785.70 for personal loans, GHS 32142.90 for transport Loans, GHS 12500.00 for trading and lastly GHS25000.00 for salary loans.

In so doing the bank will be able to make a net return profit of GHS 62,223.60 on the total loans given per annum.

4.5 Discussions

The results were found after six (6) iterations. The optimal solution or value was found to be GH¢ 62,223.60 Table 4.1 depicts decision variables (column 1), the optimal value of the variables (column 2), objective cost or the objective function coefficients. The variables show that funds for the loans should be allocated for all the loans types the bank operates. That is, agriculture loans, personal loans, transport loans, trading loans and salary loans with the amounts indicated to each type of loan.``

The analysis revealed that, the agriculture loan contributed GH¢ 105357.00, personal loan contributed GH¢ 26785.70, transport loans GH¢ 32142.90, trading loans GH¢ 12500.00 and finally salary loan contributed GH¢ 25,000.00 The objective function value is Z = GH¢ 62,223.60. As shown in Table 4.4 Yapra Rural Bank, Kwame Danso Agency if allocate the funds available for loans in 2013, will be making a net profit of GH¢ 62,223.60 compared with the annual profit of GH¢ 44,501.20 in the year 2012.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

5.0 CONCLUSION

Most rural banking institution in the country do not have any scientific method for allocating of funds for loans. As a result the objective for this research was to model a linear programming model for Yapra Rural Bank has to help Yapra Rural into allocations of their funds for loans. This model shows that if Yapra Rural Bank Ltd agrees to use the model they can make an annual profit of GH¢ 62,223.60 on loans as compared to GH¢ 44,501.20 profits made on loans in 2012. As such, the conclusion is that the scientific method used to develop the proposed model can increase Yapra Rural Bank Ltd net profit.

5.1 RECOMMENDATIONS

1. I recommend that using mathematical and scientific methods to give out loans can help rural banking institutions and all financial institutions in the country to increase their net profits.

2. I also recommend that Yapra Rural Bank Limited should stick to this model in their allocation of funds for loans.

3. Again, it is recommended that all microfinance and commercial banking institutions should use Mathematical methods and scientific methods in most of their businesses. –

4. The limitations encountered include unpreparedness of the institution to give out data, it should be noted that these models cannot be used to take decision outside.

5. In future, other scientific and mathematical methods could be used for study of the same topic or other related areas.

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APPENDIX I

Kwame Nkrumah University of Science and Technology

Institute of Distance Learning

Research Questionnaire

This questionnaire seeks to collect data for the study "optimum loan portfolio selection". A case study of Yapra Rural Bank limited B/A.

The data is solely needed for academic purpose and would be conducted in a highly confidential manner. Thank you

Please tick the appropriate one.

1. Which of the following areas is the bank investing in 2013

a. Agriculture { }

b. Personal loan { }

c. Transport { }

d. Trading { }

e. Salary { }

f. Funeral { }

g. Others (specify).....

2. How much is being proposed for investment in loan for 2013?

.....

3. What are the respective interest rates?

a. Agriculture .....

b. Personal loan .....

c. Transport .....

d. Trading .....

e. Salary .....

f. Funeral .....

g. Others (specify) .....

4. Do you provide for bad debt?

a. Yes { }

b. No { }

5. What is the probability of bad debt on each portfolio

a. Agriculture .....

b. Personal loan .....

c. Transport .....

d. Trading .....

e. Salary .....

f. Funeral .....

g. Others (specify).....

Organizational profile

1. Year of establishment .....
2. Year of license .....
3. Certificate of incorporation No: .....
4. License No: .....
5. Number of branches .....
6. Main vision.....
.....
7. Main mission.....
.....
8. Core values .....
9. Core products.....
10. Lending products
    a. .....
    b. .....
    c. .....
    d. .....
    e. .....
11. Saving products
    a. .....
    b. .....
    c. .....
    d. .....
    e. .....

Thank very much.