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KUMASI

INSTITUTE OF DISTANCE LEARNING

SITE SELECTION FOR FIRE STATION IN BIRIM NORTH DISTRICT

BY:

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DECLARATION

I hereby declare that this submission is my own work toward the M Sc. And that to the best of my knowledge it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the University except due acknowledgement as been made in the text.

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DEDICATION

To the Glory of God

I dedicate this project to my wife, Mrs. Janet Manu, my lovely daughters Josephine, Anastasia and Wilhelmina Manu and my colleagues for their moral and mutual support.

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It is with great pleasure that I take this opportunity to recognize those who have played a major role in bringing this significant work to its full realization. It has been satisfying to see all the pieces come together, often in ways much better than I expected.

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ABSTRACT

This thesis seeks to identify the optimal location for sitting of a fire station in Birim North District using absolute centre heuristic method to ensure quick response to fire incidents and help fight fire outbreaks within the district thereby ensuring the future safety of people and property within the service coverage area. We focused on markets, hospitals, schools, residential facilities etc. and these were considered as the nodes or demand points. The Karis and Hakimi algorithm was used to compute the absolute centre from the distances obtained from the Floyd-Warshall's all pairs shortest path algorithm. The absolute centre was identified as (0) zero meters from node C. In other words the absolute centre was at node C and the facility has to be sited at C. The maximum distance to be travelled from the facility to a farthest node, node O (Akokoase) shall be 4745metres.

The fire station should be located at New Abirem to ensure minimum response time and travel distance in the service coverage area.

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INTRODUCTION

1.1 Background

Fires start when a flammable and or a combustible material with adequate supply of oxygen or another oxidizer is subjected to enough heat and is able to sustain a chain reaction. This is commonly called the 'Fire tetrahedron' Fire cannot exist without all of those elements being in place at the right conditions. Once ignited, a chain reaction must take place whereby the fire can sustain its own heat by further release of heat energy in the process of the combustion. It may propagate provided there is a continuous supply of an oxidizer and fuel. Propagation may be achieved through convection, conduction and radiation. Fire may be broadly grouped into two; structural fire and bush/wild/forest fire.

Since creation, fire has been and continues to be a useful tool in the very existence of human lives on this planet. Fire may be a blessing or a scourge depending on how it is managed. People have benefited from fire in the areas of transportation, energy generation, vegetation management, farming, heating cooking etc. However, others have had their properties and their livelihood destroyed by fire. Whole communities had been wiped out by fire. Offices such as the ministry of foreign affairs, markets such as Kumasi central market, Kantamanto market in Accra and many others have been engulfed by fire. Thus fire is indispensable on this planet but if it is not managed or controlled it will become a scourge and would threaten the very survival of human race. The destruction and damages left in the trail of fire outbreaks may translate into poverty, joblessness, homelessness and famine. The emotional stress victims of fire go through cannot be quantified.

It must be stated that all fires start very small but if not detected and acted upon quickly they escalate and become very destructive. The purpose of this study is to use mathematical method to identify a fire station in the Birim North District so that fires that break out can be quickly responded by fire fighters and suppress them before they escalate.

1.1.1 Causes of fire

As mentioned earlier, fires may be broadly grouped into two; Structural fires and wild/bush/forest fires reflecting the type of vegetation or fuel. Wild fire is an uncontrolled fire that occurs in the countryside or wilderness area. It differs from structural fire by its extensive size, the speed at which it can spread out from the original source and its ability to change direction unexpectedly and to jump gaps such as roads, rivers, streams and firebreaks. Wild fire may be caused either naturally or artificially through the activities of human beings. Four major natural causes are:

a. Ignition by lightning;
b. Ignition by volcanic eruption;
c. Ignition by sparks from rock falls
d. Spontaneous combustion.

Some of the man-made that causes fire outbreak are

i. Farming
ii. Honey extraction
iii. Group hunting

iv. Discarding live cigarette butts; and

v. Sparks from equipments

vi. Palm wine tapers

Wild fire behavior is often complex and variably dependent on factors such as fuel type, moisture content in the fuel, humidity, wind speed, topography, geographic location and the ambient temperature. A high moisture content usually prevent ignition and slows propagation because higher temperature are required to evaporate any water within the material and heat the material to its fire point. Dense forest usually provides more shade resulting in lower ambient temperature and greater humidity. Less dense material such as grasses and leaves are easier to ignite because they contain less water than denser material such as branches and trunks.

In societies which practice shifting cultivation where land is cleared quickly and farmed until the soil losses its fertility, slash and burn clearing is often considered the least expensive way to prepare land for future use. Forested area cleared by logging encourages the dominance of flammable grasses and abandoned logging roads overgrown by vegetation may act as fire corridors. People who indulge in palm wine taping also buy a sizable palm plantation for their businesses. In the cause of their activity they also live naked fire around which eventually ignite and cause bush fires.

Structural fire (commercial, Industrial, residential etc) may be caused by

i. Electricity;
ii. Naked flame/ naked fire
iii. Spontaneous combustion.

Most structural fires are attributed to misuse of electrical gadgets, illegal electrical connections, overloading of sockets, and unattended immiscible water heaters, keeping gas cylinders in our rooms etc. Careless handling and disposal of lighted matches, careless disposal of lives cigarette butts, the use of faulty electrical appliance/equipment etc may start fires in structures e.g. residential facility. The most significant factor in fire spread is the production of intense heat energy and the spread may be achieved through any of the following methods.

i. Conduction
ii. Convection
iii. Radiation.

The causes of fires mentioned above may be attributable to:

a. Accident;
b. Negligence;
c. Carelessness; and
d. Arson.

1.1.2 Firefighting History

Modern firefighting entails the application of an appropriate standard operating procedure (SOP) to control, confine and eventually extinguish a fire and also effect rescue of those trapped. It involves the use of fires equipment, personnel and appropriate firefighting methods to extinguish fires and limit the damage caused. Firefighting consist of removing one or more of the three elements essential for combustion, (fuel, heat and oxygen) or of interrupting the combustion chain reaction. However, the art of firefighting evolved differently in different part of the world.

The first Roman fire brigade had a group of slaves who were hired by an audible Marcus Egnatius Rufus. The Roman emperor Augustus took this idea from Rufus and built on it to form the (Vigils) in AD 6 to combat fires using 'bucket brigades' and pumps as well as poles, and hooks to tear down buildings in advance of the flames. The vigil patrolled the streets of Rome to look for fires and served as the police force [Paulison, David R. 2005]. Emperor Augustus is citied for instituting a vigil- firefighting corps (watchmen) in 24 BC and developed regulations for checking and preventing fire. In the pre- industrial era most cities had 'watchmen' who sounded an alarm when they saw signs of fire. The basic firefighting equipment in ancient Rome which was used till the early modern times was a bucket, passed from hand to hand to deliver water to a firescene. Ancient Rome is known to have had a fire department of approximately 7,000 paid firefighters. They are not only responded to and fought fires but also patrolled the street and had authority to impose corporal punishment on those who violated fire prevention codes. In 200 BC, Ctesibius of Alexandria devised the first- known fire pump but this idea was lost until the pump was reinvented in AD 1500.

In 1254 AD, a Royal decree of king saint Louis of France created the so- called guet bourgeois ('burgess watch') allowing the residents of Paris to establish their own night watches separate from the King's night watch to prevent and stop crime and fires. After the (100) Hundred Years' war, the population of Paris again expanded and the city which became the largest city in Europe in the 16th century experienced several great fires. Consequently, King Charles IX disbanded the resident's night watches and the King's watches were left as the only one responsible for checking crimes and fires (Paulison, 2005). The first modern fire brigades were created in France in the early 18th century.

The key breakthrough in firefighting was the invention of the first fire engines in the 17th century. They were simply tubs carried on runners, long poles or wheels and water was still supplied to the fire ground by "buckets brigade". The tub served as a reservoir rand sometimes housed a hand pump that forced the water through a pipe or nozzle to waiting buckets. Various nozzles were capable of projecting solid heavy streams of water in the form of steam spray or fog at flow rates between 57 litres to more than 380 litres per minute.

The invention of a hand stitched leather hose pipe in the Netherlands about 1672 enabled firefighters to work closer to the fire without endangering their engines and to increase the accuracy of water placement. At about the same time the development of pumping devices, made it possible to draw water from rivers and ponds. Manual pumps, rediscovered in Europe after 1500 AD (Allegedly used in Ausberg in 1518 and in Nuremberg in 1657 AD), were only force pumps and had a very short range due to lack of hoses. A German Inventor, Hans Hautsh improved the manual pump by creating the first suction and force pump and adding some flexible hoses to the pump. The fire engine was further developed by Richard Newshwam in 1725.

In U.K the great fire of London in 1666 set in motion changes which laid the foundations for organized future firefighting. The only equipment available to fight the fire in 1666 which burnt for five days was two- quart (2.28litres) hand syringes and a similar slightly larger syringe [Louisa et al., 2006]. In the wake of the fire, the city council established the first fire insurance company, "THE FIRE OFFICE" in 1677 which employed small teams of Thames watermen as firefighters and provided them with uniforms and arm badges showing the company to which they belonged. The first organized municipal fire brigade in the world however, was established in Edinburgh, Scotland, when the Edinburgh Fire Engine Establishment was formed in 1824. It was led by James Braidwood. In 1832, London Fire Engine Establishment was also formed.

On April, I 1853 the Cincinnati, ohio (USA) Fire Department became the first full- time paid professional fire department in the United States and the first in the world to use steam fire engines. The first horse- drawn steam engine for fighting fires was invented in 1829, but not accepted in structural firefighting until 1860, and ignored for another two years afterwards. Internal combustion engines arrived in 1907, built in the United States leading to the decline and disappearance of steam engines by 1925.

In the early 19th century copper rivets replaced the stitching on hoses and 15 metre-lengths coupled with brass fittings enabled firefighters to convey water through the narrow passages up the stairways and into buildings, while the pump operated in the street. In all the fire outbreaks, the fire companies or "bucket brigades" had to move from firehouses to the various fire scenes to fight the fires. The firehouses were a sort of social gathering places rather than a place where professionals would meet but it formed the basis for the establishment of fire stations. Water had been used in all circumstances as the extinguishing in the fire fighting.

1.1.3 Fire Hydrants

Globally, water is used by Fire Services as an extinguishant in firefighting since it:

i. is the least expensive firefighting agent;
ii. is plentiful;
iii. has a high specific heat capacity; and
iv. can be easily transported over long distances.

A Fire hydrant is an active fire protection measure and a source of water that firefighters tap from water mains to assist in extinguishing a fire. Every hydrant has one or more outlets to which a fire hose may be connected to provide a powerful source of water (about 3.5 bars) when the valve is opened. This hose can be further attached to a fire engine which can then use a powerful pump to boost the water pressure and possibly split it into multiple streams. The hose may be connected with a threaded, instantaneous or hermaphrodite coupling.

In order to provide sufficient water for firefighting, hydrants are sized to provide a minimum flow rate of 945 litres per minute (210 gallons/ min.) although most hydrants can provide much more. To prevent casual use or misuse, the hydrants requires special tools to be opened, usually large wrench with a pentagon- shaped sockets.

There are two types of pressurized fire hydrants; wet and dry- barrel. In a wet – barrel design, the hydrant is connected directly to the pressurized water source. The upper section, or barrel of the hydrant is always filled with water and each outlet has its own valve.

In the dry- barrel design, the hydrant is separated from the pressurized water source by a main valve in the lower section of the hydrant below ground. The upper section remains dry until the main valve is opened by means of key and bar. In colder climates where temperatures can fall below 0° C dry – barrel hydrants are usually used while wet- barrels are preferred in the temperate climates. Hydrants need to be readily recognizable and accessible and must not be obstructed.

In sparsely populated areas, hydrant must be spaced 150 meters apart and in densely populated areas they should be spaced 100 meters apart. Hydrants shall be placed a minimum of 15m from buildings being protected and 0.914 meters clear space shall be maintained around its circumference.

The Birim North District can boast of a few underground hydrants located at vantage points foreasy access but a good number of them have been sealed by stores and containers or are not functional. Most of the hydrants in the major markets especially the central market have structures built on them and therefore cannot be accessed during fire outbreaks. Any concrete or metal slab along the public roads with the inscription FH indicates a Fire hydrant.

1.1.4 Fire Management

Fire management has three specific objectives: protection, mitigation, and control and suppression. Decisions concerning the above vary with locations, events and policies. While it is the duty of firefighters to take all practicable measures to prevent and extinguish fires, one of the best fire control strategies is to reduce and manage, on a long term basis, vegetation and structural conditions that fuel the fire. Many preventive measures can help reduce the loss of lives and properties in the worst of fires.

1.1.5 Structural fire suppression in the Birim North District

Structural fires have been caused in the district mainly by electricity and naked flames and the methods or techniques applied in the extinguishment are not different from those that are applied elsewhere in the country by fire fighters. The methods used in the extinguishment are cooling, starvation and blanketing/ smothering depending on the medium of extinction being used. When a distress call is received at the control room at the Regional Headquarters, an appliance from the nearest fire station is dispatched to respond to the call. However, if the call is made at any of the stations closer to the incident scene, that station dispatches an appliance but informs the control room at the Headquarters. Currently the only fire station closer to Birim North District is at Nkawkaw about 40km away. The station has one appliance and a turntable ladder and water carrier in addition. The total strength of the fire personnel in the Kwahu West district (Nkawkaw) is 13 but not all of them are firefighters since the Service performs other functions apart from fighting fire.

The basic tactics of suppressing fires can be divided into the following categories;

i. Rescue operations
ii. Protection of structures exposed to the fire
iii. Confinement of the fire
iv. Salvage operations.

The Officer-in–Charge (crew leader), on arrival at the fire scene sizes up the situation and also estimates what additional assistance may be needed. Once the crew leader has appraised the situation, firefighters and equipment are deployed but rescue operations are always given priority. Depending on the size and intensity of the fire, firefighters may attack the fire either offensively or defensively. An early aggressive and offensive attack greatly reduces loss of life and property damage,

For effective fire suppression;

a. Quick response times are essential.
b. Adequate number of firefighters must be available for immediate attack.
c. Adequate water must be supplied for sustained firefighting
d. Firefighters must not perform multiple duties.

Sometimes the firefighters have to spend their effort and time to protect the surrounding structures that are exposed to the fire rather than concentrate on the structure on fire if it is realized that it has already been destroyed. Salvaging is carried out when there is the need for it to protect merchandise, household goods and the interior of buildings from smoke and water damage. Salvage sheets are usually used to cover such items.

The crew leader has a huge responsibility in ensuring:

i. The safety of the men and equipment he commits to the work.
ii. The safe arrival at the fire scene and back at the fire station.
iii. Effective and efficient utilization of all resources under his care

Additionally, he ensures the use of the appropriate and effective method to achieve total extinction of the fire as well as the management of the fire safety situation in and around the fire scene.

1.2 Problem Statement

Fire outbreak in Birim North District has been a major problem for some time now. In 2008 and 2009, Kwahu west municipal fire service responded to and fought 27 structure and wild fires within the Birim North District. The fires caused a lot of damaged and destructions to a number of residential, institutional, commercials etc. facilities and the estimated cost of damage to properties were GH¢340,596.00. There were 3 severe injuries recorded.

The earliest and latest arrival times from Nkawkaw to the incident scenes in the Birim North District were 18 minutes and 32 minutes respectively.

The problem, therefore, is to determine the optimal location where a fire station may be located or sighted in the Birim North District to ensure optimal response time for incident responders in the service coverage area. Such that the incident responders will respond to effect rescuer to those trapped by fire and suppress fire outbreaks quickly to save life and properties.

1.3 Objectives of the Study

Generally, the objective of this study seeks to identify a strategic site in the Birim North District using the absolute- centre heuristic method where a more central site may be found to locate the fire station.

1.4 Methodology

The data for this study will consist of secondary data collected from the Birim North District Assembly, Kwahu West Municipal and Regional fire service in Koforidua.

Floyd Warshall's algorithm will be used to obtain a matrix of the shortest paths among all the town pairs. Finally Absolute Centre Heuristic method which is the main tool of this study will be used to locate the fire station within the district.

1.5 Justification

Resources, both material and human are very essential for the development of any nation and these resources must be protected against fire. Many business and social service infrastructure have been destroyed by fire in the past and the huge capital outlay needed to reconstruct or rehabilitate these facilities could have been used to expand the economy in the areas of Health, Education, Housing etc. Many more of such facilities have been saved by the quick intervention of emergency responders.

It is therefore pertinent to ensure that more fire stations are opened but even more importantly, they are optimally located in communities so that emergency responders can always respond quickly and rapidly to save the situation. This would lead to investor confidence and invest more in the economy to create lob opportunities, etc and generally improve on Gross Domestic Product (GDP). As more job opportunities are created, there would be reduction or alleviation of poverty and consequently reduce poverty related social vices. This would also impact positively on security and people could go about their business without fear. Food and other crop production would increase and the citizens would get enough food to eat. The surplus could be stored for future use to ensure food security. Some of the food may even be exported to generate enough hard currency needed to purchase those essential materials that are not produced in the country.

In general the economy would grow and the lot of its citizens would improve.

1.6 Limitations of Study

It is noted here that:

i. Some of the documents were considered confidential and were not allowed to be taken out of the organization' premises, hence reference to some of the records were not detailed;

ii. Population density, nature and degree of hazards, road conditions, traffic density, road layout and their important factors could not be factored in the mathematical procedure; and

iii. Time and resources constraints limited the work to Birim North District.

1.7 Organization of the Study

The work is divided into five chapters;

Chapter 1- Provides the general introduction of the study.

Chapter 2- Reviews some of the available literature on location problem models.

Chapter 3- Looks at location problem models, strategies involved in fire station location, some location problem methods and network- based algorithms. It discusses methods and the basic models for the centre problem and its computation.

Chapter 4- Discusses data collection and their analyses.

Chapter 5- Gives conclusive statement from the study and recommendations for implementation by the Ghana National Fire Service.

1.8 Summary

Fire is an indispensable tool and without it life will be meaningless on this planet. Fire has been and continuous to be beneficial to mankind in the areas of cooking, hunting, Farming, transportation, energy production and vegetation management.

Nevertheless, fire has been the worst enemy of most nations and individuals as it has destroyed large arable farmlands, business entities, and properties worth billions of dollars and has even claimed precious lives.

The causes of fire, both structural and wild, their effects on the ecosystem and structural fire fighting in Birim North District have been discussed.

The history of fire fighting and the techniques that were adopted were not left out. The next chapter reviews some of the literature on the location problem under study and its variants.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Location policy is one of the most profitable areas of applied systems analysis. Location models seek the location of facilities (schools, hospitals, ware houses, fire stations etc) so as to optimize one of several objectives generally related to the efficiency of a system.

This chapter provides some of the methods or techniques that have been applied in solving location problems. All those techniques or methods were aimed at identifying the optimal location or 'a good location' for the sitting of a facility.

2.2 Review of Location Problem Models

Charikar et al., (2001), indicated that facility location problems were traditionally investigated with the assumption that all clients were to be provided services.

The authors explored a generalization of various facility location problems (K- center, K-median, uncapacitated facility location etc) to the case when only a specified fraction of the customers were to be served.

According to the authors what made the problems harder was that they had to also select the subset that should get service. They however, had to provide generalizations of various approximation algorithms to deal with this added constraints.

Lunday, (2005), considered that conditional covering problem on an undirected graph, where each node represents a site that must be covered by facility and facilities might only be established at these nodes. Each facility could cover all sites that lied within some common radius.

According to the authors, the difference between conditional covering problem and the more traditional set covering problem was that, in the conditional covering problem, facilities were incapable of covering the node at which they were located. Although this problem was difficult to solve on general graphs, there existed special structures on which the problem was easily solvable. The authors considered the special case in which the graph was a simple path. For the case in which facility location costs did not vary based on the site, the authors provided a regularly more complex dynamic programming algorithm to find the optimal solution.

Arasha Behzard Mohammed Modarres developed a method to obtain an optimal solution for multiple tour plant location problem (MTPLP), where the objective was to simultaneously locate facilities and to establish delivery houses from these facilities to a set of given customers. This approach in essence consisted of two prime transformations: the first converts MTPLP into limited number of generalized travelling salesman problems (GTSPs) while the second converts every resulting GTSP into a single travelling salesman problem (TSP).However, to reduce computational complexity involved in the above transformations, the author introduced an equivalent transformation that converted the underlying MTPLP. Consequently, the efficiency of the proposed algorithm relied on the method selected for solving the TSP.

Hongzhong J, (2005), first surveyed general facility location problems and identified models used to address common emergency situations, such as house fires and regular health needs. The author then analyzed the characteristics of large- scale emergencies and proposed a general facility location model that is site for large- scale emergencies. This

general facility location model could be cast as a covering model, a P-median model or a P-centre model, each suited for different needs in large- scale emergencies.
Illustrative examples where given to show how the proposed model could be used to optimize the location of facilities for medical supplies to address a large- scale emergencies.

Bhattacharya and Shi showed that a p (≥ 2) – centre location problem in general networks could be transformed to the well known Klee's measure problem. This resulted in an improved algorithm for the continuous case. The authors also showed that the discrete p-centre problem could be solved in O ($pn^p \log n$)time. The p- center problem is to locate p facilities on a network so as to minimize the largest distance from a demand point to its nearest facility. The p- median problem is to locate p- facilities on a network so as to minimize the average distance from one of the n demand points to one of the p facilities.

Charikar, Guha, et al., (2002), presented the first constant- factor approximation algorithm for the metric k- median problem. The k- median problem is one of the world-studied clustering problems that are; those problems in which the aim is to partition a given set of points into clusters so that the points within the cluster are relatively close with respect to some measure. For the metric k- median problem, and given n points in a metric space, the authors selected k of these to cluster sentence and then assigned each point to its closest selected centres. If point *j* was assigned to centres *i* the cost incurred was proportional to the distance between *i* and *j*. The goal was to select the k sentence that minimizes the sum of the assignment costs.

In vertex k- center problem, the goal is to pick some vertices, called centres from a given undirected unweighted graph so as to minimize the maximum distance of any vertex from its closest centres. It is known that not only is k- centering an NP- complete problem, but approximating the k- centres problem with a factor better than 2 is still NP- complete. If the distance between any two vertices was given as a distance matrix (so distance computations take constant time), the 2-approximation k-centres problem could be solved in O (n log k) time. But to build such a distance matrix for a large- size road map was impractical since even storing the distance matrix was impractical, because it required n^2 space.

Chen R., (1993) suggested a method that enabled the solution of minisum and minimax location- allocation problems by using a differentiable approximation to the objective function and solving it by using nonlinear programming. This enabled the solution of relatively large problems but the result was not necessarily optimal since local minima might have been reached. Drezner presented heuristic and optimal algorithms for the p- centres problem in a plane. The heuristic method yielded results for problems with up to n= 30, p=5 or n=40 and p=4.

Caruso C., (2003), described an efficient exact method for p- centres problem. Their algorithm found the solution by updating, at each step, an upper and lower bound on the optimal solution. A tight lower bound to the optimal value was found in an initial phase of the algorithm, which consisted of solving linear programming sub- problems. Chen and Handler adapted a relaxation method to the p- centre problem in continuous Euclidean two- dimensional space. In the solution to the p- centre problem there was usually only one circle which was critical in the sense that two or three demand points were on its circumference. There was much freedom in the exact position of the other circles and therefore in the location of all but one of the centres. According to the authors, the value of the solution was determined by the radius of the critical circle, whereas the radii of the other circles might vary in size below this critical value. The authors proved that among all the optimal solutions to the minimax problem of serving n demand points in Euclidean space by p service point, there is at least one which all demand point are covered by critical circles, the largest of which is the value of the solution.

In order to locate a given number of emergency facilities along a road network. Garfinkle et al., (1977), examined the fundamental properties of p- centre problem. He modelled the p- centre problem using the integer programming and the problem was successfully solved by using a binary search technique and a combination of exact tests and heuristics.

Talwar (2002) utilized a p-centre model to locate and dispatch three emergency rescue helicopters to serve the growing EMS demands from accidents of tourist activities such as skiing- hiking and climbing at the north and south of Alpine mountain ranges. One of the model's aims was to minimize the maximum (worst) response times and the author used effective heuristics to solve the problem.

ReVelle C., (1989), formulated a model that sought to maximize a population which had a service available within a desired travel time with a stated reliability, given that only p servers were to be located. The authors computed the number pi of servers needed for reliable coverage of node i, and maximized the population in nodes, with pi or more servers.

In the fire protection area, ReVelle and Marianov, (1991), formulated a model, the probabilistic facility Location, Equipment, Emplacement, and Technique. This model considered the deployment of several types of vehicles, simultaneously covering each emergency as well as the sitting of depots or stations.

John C. Edwards developed a fire location computational model which could determined all possible path in a mine that smoke could travel from a fixed fire source to a smoke detector. The associated FORTRAN computer programmed could be utilized to determine the travel time from a source to a smoke detector. The difference in the travel time from an isolated fire source to two or more detectors could be used to isolate those airways in which the source of fire is located. The model also has application in mine emergency stage. To determine the optimum location of fire detectors, the mine network was divided into zones each of which was associated with a difference in calculated smoke arrival time between a pair of detectors.

Church and ReVelle (1974), and White and Case, (1974) developed a maximal covering location problem model that did not require full coverage to all demand points instead, the model sought the maximal coverage with a given number of facilities. The maximal covering location problem and different variant of it had been extensively used to solve various emergency service location problems a notable example was the work of Eaton et al., (1985) who used the maximal covering location problem to plan the emergency medical service in Austin, Texas. The solution gave a reduced average emergency responses time even with increased calls for service.

Sehilling et al., (2005), generalized the maximal covering location problem model to locate emergency fire fighting servers and depots in the city of Baltimore. In the authors' model, known as FLEET (facility location and Equipment Emplacement Technique) two different types of servers needed to be located simultaneously. A demand point was regarded as 'covered' only if both servers were located within a specified distance.

Daskin and Steere (1981) formulated hierarchical objective of location set covering problem for emergency medical service in order to find the minimum number of vehicles that were required to cover all demand areas while simultaneously maximizing the multiple coverage.

Benedict,(1983), Eaton et al., (1986), and Hogan and ReVelle (1986), developed maximal covering location problem models for emergency service that had a secondly "backup- coverage" objective. The models ensured that a second (backup) facility could be available to service a demand area in case that first facility was unavailable to provide services. Based on a hypercube queuing model, Javis, (1977), developed a descriptive model for operation characteristics of an EMS system with a given configuration of resources and a location model for determining the placement of ambulances to minimize average response time or other geographically based variables.

Marianov and ReVelle, (1996), created a realistic location model for emergency systems based on results from queuing theory. In their model the travel times or distance along arcs of the network were considered as random variables. The goal was to place limited numbers of emergency vehicles, such as ambulances, in a way as to maximize the cost of service.

Paluzzi (2004) discussed and tested a p- median based heuristic location model for placing emergency service facilities for the city of Carbondale. The goal of this model was to determine the optimal location for placing a new fire station. The results were compared with the results from other approaches and the comparison validated the usefulness and effectiveness of the p- median based location model.

Doeksen et at., (1976) used the general transportation model based on alternative objective functions to obtain optimal fire station locations for the rural fire system. The different objectives used to obtain the optimal sites include: minimizing response time to fire, minimizing total mileage for fighting rural or country fires and maximizing protection per dollar's wealth burnable property.

Plane and Hendrick, (1977), used the max covering distance concept to develop a hierarchical objective function permitted the simultaneous minimization of the number of fire stations and the maximization of the existing fire stations within the minimum total number of stations.

Hogg, (1968), used a set- covering technique, which minimizes the total number of fire appliance journey times to fires for any given number of fire stations, and applied this to the city of Bristol.

Badri et al.,(1998) underlined the need for a multi objective model in determining fire station location the authors used the multiple criteria modelling approach via integer goal programming to evaluate potential site in 31 sub- areas in the state of Dubai. Their model determined the location of fire stations and the areas they are suppose to serve. It considered eleven (11) strategic objectives that incorporated travel times and travel distance from stations to demand site, and also other cost-related objectives and criteria-technical and political in nature.

Tzeng and Chen, (1999) used a fuzzy multi objective approach to determine the optimal number and site of fire stations in Taipei's international airport. A genetic algorithm was then executed to weigh against the brute-force enumeration method. The results prove that the genetic algorithm was suitable for solving such location problems. Nevertheless, it efficiency still remained to be verified by means of large- scale problems.

Church, (2002) exhaustively reviewed the existing work linking GIS location science and asserted that GIS could support a wide range of spatial queries that aid location studies. He explored the integration of a heuristic algorithm into GIS for spatial optimization of fire station locations. This novel approach to solving optimization problems lead to a paradigm shift in solving analytical problems of a similar nature in the disciplines of transportation, networking and infrastructure design.

2.3 Summary

The different authors used different techniques/ methods or modified version of earlier methods to solve problems, all in an attempt to identify the optimal location of facility to provide services or and goods to optimize certain objectives.

A number of location models (p- centre, p- median, discrete etc) have been formulated and use to solve location problems. However, some of those techniques or methods produced better results than others. The next chapter discusses the methodology that will be used in determining the optimal location where a fire station may be located or sighted in the Birim North District to ensure optimal response time for incident responders in the service coverage area.

CHAPTER 3

METHODOLOGY

3.0 Introduction

This chapter reviews the methods, mathematical tools and algorithms that will be used to explain into details the basic model for the centre problem and then use an example to find the absolute centre and the edge on which it occurs on the given network.

3.1 Shortest Path Problem

Shortest path problems are the most fundamental and most commonly encountered problems in the study of transportation and communication networks [Salhi S, 1998]. There are many types of shortest path problems. For example, we may be interested in determining the shortest path from one specified node in the network to another specified node or we may need to find the shortest paths from a specified node to all other nodes. Shortest path between all pairs of nodes in a network are required in some problems while sometimes one wishes to find the shortest path from one given node to another given node that passes through certain specified intermediate nodes. In some applications, one requires not only the shortest path but also the second and the third shortest paths. There are instances when the actual shortest path is not required but only the shortest distance. We shall discuss two most important shortest path problems;

i. How to determine shortest path distance (a shortest path) from a specified node R to another specified node, T.

ii. How to determine shortest distances (and paths) from every node to every node in the network.

3.1.1 Single- Source Shortest Path Problem

In the single- source shortest path problem we have to find shortest path from source vertex k to all other nodes (vertices) in the network. Here an algorithm due to [Dijkstra E, 1959] shall be considered. This algorithm finds the shortest path from a source S to all other nodes in the network with non – negatives lengths.

Dijkstra's algorithm maintains a distance label d (*l*) with each node *i* which is an upper bound on the shortest path length from the source node to each node *j* at any intermediate step, the algorithm divides the nodes of the network under consideration into two groups those which it designates as permanently labelled (permanent) and those which it designates as temporal labelled (temporal). The distance label to any permanent node represents the shortest distance from the source node to that node. The basic idea of the algorithm is to find out from source node S and permanently labelled nodes in order of their distances from node S.

Initially, node S is assigned a permanent label of zero and each other node *j* a temporary equal to infinity. At each iteration, the label of a node *i* is the shortest distance from the source node along a path whose internal node (i.e. nodes other than S or the node i) itself are all permanently labelled. The algorithm selects a node *i* with the minimum temporary label (breaking ties arbitrary), makes it permanent and reaches out from that node- that is, it scans all the edges/arcs emanating from the node *i* to update the distance labels of adjacent nodes. The algorithm terminates when it has designated all nodes permanent. [Ahuja, R. K. et al., 1993]

3.1.2 All Pairs Shortest Path Problem

The shortest path between two nodes might not be a direct edge between them but instead involve a detour through other nodes. The all- pairs shortest path problem requires that we determine shortest path distances between every pair of nodes in a network.

The Floyd- Warshall's algorithm obtains a matrix of shortest path distance within O$\{n^3\}$computations. The algorithm is based on inductive arguments develops by an application of a dynamic programming technique. The Floyd- Warshall's algorithm remains of interest because it handles negative weighs edges correctly [Ahuja R.K et al., 1993] and [Boffery, 1983].

3.2 The Centre Problem Model

The centre problem model attempts to minimize the worst performance of a system and this addresses situation in which service inequity is more important than the average system performance. In location literature, the centre problem model is also referred to as the minimax model since it minimizes the maximum distance between any demand points and its nearest facility.

The centre problem model considers that a demand point is served by its nearest facility and therefore full coverage to all demand points is always achieved, however the full coverage requires only a limited number of facilities, say k. The vertex k- centre problem is a location problem where given cities and the distances between all pairs of cities are known and the aim is to choose k cities (centres) so that largest distance of a city to its nearest centre is minimal. The absolute centre problem on the other hand considers facility location problem on an edge or vertex of a network. [Mirchandi P. B, and Francis R.L, 1990].

A complete or weighted undirected network, G= (N,L) where N is the set of nodes/ vertices indicating population centres(markets, housing estates, hospitals, existing fire station etc) and L is the set of edges indicating the distances between the centres(nodes), with edge costs satisfying the triangle inequality ($|x+y|\leq|x|+|y|$) used in this application. Demands for service originate at the nodes of G. Each edge (p, q) has a real valued cost c(p,q) the Euclidean distance between nodes p and q which represents the cost of providing a service from node p to node q (or from node p to q). The goal is to locate one or more facilities to service the node demands such that a specified objective function is maximized or minimized. A facility can be located at any point on the network G, where the point x is either a node or a location on an edge (p, q). In the latter case the point is identified by d(x, p) and d(x, q) such that d (x, p) +d(x, q) =c (p, q) i.e., we add a new node "x" and new edges (x, p) and (x, q) with costs d(x, q),   d(x, q) such that d(x, p) + d(x, q) = c(p, q)

3.3 Basic Assumptions of the Centre Problem

In this section, we present an optimal or approximately optima location for an emergency semi – obnoxious services such as Fire Station. Locational decisions are based on many factors; some of which are physical, economical, social, environmental, or political.

However, the accessibility factor and its cost are more important to the customer using a spatial contest as accessibility factor.

The objective is to minimize the mazimum cost of serving one of several clients in the centre problem (minimax problem).That is the location which must be chosen from a finite number of potential sites selected in a preliminary stage.

3.4 The Centre Problem

The centre problem was first posed by Sylvester (1857) more than one hundred years ago. The problem asked for the centre of a circle that had the smallest radius to cover all desired destinations.

The k –centre model and its extensions had been applied in the context of locating facilities such as *EMS* centres, hospital, fire station and other public facilities.

For a point x on the network G, let m(x) denote max d(x, n_i) where d(x,n_i) is the cost or distance of the 'shortest' path between x and 'farthest' demand node $n_{i.}$ The general absolute center problem is formulated as min [m(x)] = min [max d(x, n)] subject to x∈ G.The above formulation is applied in finding the vertex and local centres.

The vertex centre (or node centre) $x_n \in N$ is a node such that for every node

y∈ N ,$m(x_n) \leq m(y)$,

The local centre of an edge (p, q) is a point x, on (p, q) such that for every point y on (p, q) m $(x_l) \leq m(y)$. The absolute centre x_a is a point on G such that for every point y on G, (y may be on an edge of G), m $(x_a) \ \leq m(y)$ [Mirchandi P.B, and Francis R.L, 1990].

To find a node center, we compute the matrix of the shortest paths costs (travel times, distances) for all pairs of nodes using the Floyd- Warshall's or Dijkstra'salgorithm, and then choose a node such that the maximum entry in its row in the matrix is smallest among the maximum entries of all rows.

For example figure 3.1 shows a network of an urbanized area with nodesn_1 , $n_2, n_3, n_4, and n_5$ respecting points where demand for services is generated.

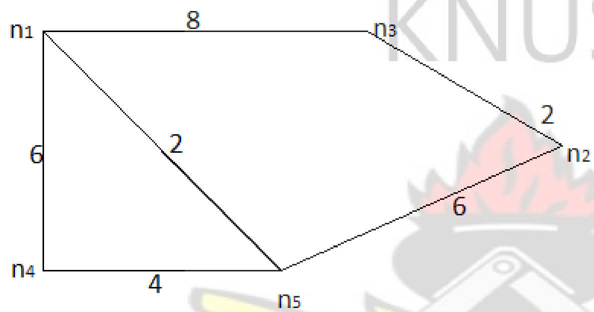


Figure 3.1

By using the Floyd- Warshall's algorithm we obtain a matrix of the shortest paths of the network of figure 3.1.

The algorithm computes d (p, q) for all node pairs p and q and are shown in table 3.1

Table 3.1

NODE	n_1	n_2	n_3	n_4	n_5	ROW(MAX)
n_1	0	8	6	6	2	8
n_2	8	0	10	10	6	10
n_3	8	2	0	12	8	12
n_4	6	10	12	0	4	12
n_5	2	6	8	4	0	8

From Table 3.1, the smallest among the entries in all rows occurs at either n_1 or $n_5$with

m (n_1) =m (n_5) = 8 and therefore n_5 or $n_1$may be taken as the node centre.

3.5 Finding the Absolute Centre

The absolute centre minimizes the cost (distances, travel time). We look for the path of minimum cost (Euclidean distance) by finding the shortest path among all pairs of vertices using the Floyd- Warshall's or Dijkstra algorithm. A vertex is a designated point in a network and an edge is a direct distance or arc between two vertices, p and q denoted by c(p,q) which is the edge cost or edge distance.

A shortest path is the total distance between two vertices which may not be direct but passing through other vertices. Thus a shortest path may not be a direct distance or cost between two vertices. This is denoted by d(p, q) and is described as the minimum path cost;

$$d(p, q) = \min \sum_{i=0}^{l-1} c(n_1, n_{i+1})$$

If x is a point on edge (p, q) then $d(x, q) = \min [d(x,n_1) + \sum_{i=1}^{l-1} c(n_i, n_{i+1})]$

Where n_1= p or q

Consider the edge (p, q) with a point x on it as shown in figure 3.2

Figure 3.2

$d(x, p) + d(x, q) = c(p, q)$

$\Rightarrow$, $d(x, p) = c(p, q) - d(x, q)$

For an undirected graph (a two – way road) with non negative weight (cost), put

$m(x) = \max_{n \in N} d(x, p)$

If x is on an edge or a node we require $m(x^1) \leq m(x)$, where $x_{pq} = x^1$, the distance (cost) of the point x on edge (p ,q) away from p.

To calculate $m(x^1)$

(i) Evaluate all vertices and find the vertex centre value and its cost.

(ii) Evaluate all edges to find the local centre with the minimum cost.

(iii) Compare the two costs, that is the minimum vertex centre cost and the minimum edge cost, the lowest of two costs is the solution, $m(x^1)$.

The local centre for each edge can be found as shown below. Consider an edge (p, q) with a point x on it.

Assuming we want to move from x to n_1 where n_1 is any node or vertex on the network G, we find the minimum cost by moving to n_1 through p and q.

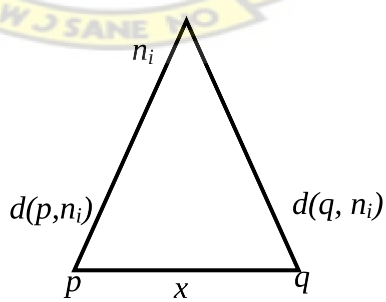


Figure 3.3

$x + d(p, n_i) = c(p, q) - x + d(q, n_i)$ is an edge and its cost is c(p, q).From the figure 3.3, d(p, x)=x and d(q, x)=c(p, x)- x hence d(p, x)+d(x, q)=c(p, q)

The movement from x to n_i (any of the nodes or vertices on the network G) can be done in two directions that is through p or q given rise to respectively the equations below;

$y_1 = x + d(p, n_1)$……………………………….(1)

$y_2 = c(p, q) - x + d(q, n_i)$…………………..(2)

Where y_1, is the distance from x to n_i through p and y_2 is the distance from x to n_i through q.

As x moves along the edge (p, q) there will be a point when the two distances or cost would be equal. At this point $y_1 = y_2$ and the kink/maximum/pareto point could be found. Solving for the path of equal cost we have

$x + d(p, n_i) = c(p, q) - x + d(q, n_i)$

$$\boldsymbol{X = \frac{c(p,q)\ + d(q,n_i) - d(p,n_i)}{2}}$$

Where x can be denoted by x_{ni} being the minimum cost. The equations (1) and (2) involving y_1 and y_2 are therefore used to draw graphs to draw edge (p, q) from which the local centre can be determined. As n_i assumes all the nodes on the network a number of equations will be generated under equations (1) and (2). These equations would then be sketch on the same axes in a given range obtained from solving the kink point for the paths of equal distance for each pair of equations. An upper envelope is then obtained by tracing all paths of lines beyond which there are no higher points for the x-value in the given on the graph. These graphs are indicated by thick lines. The local centre $x_{pq} = x_l$ is the point that minimizes the upper envelope. The absolute centre at termination of the process is the point x_a (node centre x_n or local centre $x_{pq} = x_l$) that assigned the least value to m(x) Using figure 3.1 we would evaluate in the given network to illustrate how the absolute centre can be found on a given network as follows;

Location on edge (n_1,n_3)

Consider $m(x)=y_1= x + d(p ,n)$……………………………………..(i)

$y_2 = c(p, q)- x + d(q , n)$…………………………………..(ii)

for i=1,2,3,4,5,…

Choosing n_3 as the origin, we let $p=n_3$ and $q= n_1$ such that $0 \leq x \leq c(p ,q)$

Putting $n_1 = n_1$, ie $i = 1$ then from table 3.1, we have $d(p, n_i) = d(n_3 ,n_1)=8$

Thus from (i) and (ii) $y_1 = x + 8$ and $y_2 = 8 –x$.

Solving for the path of equal distance or cost, we have $x + 8 =8 – x$, x=0 which is the kink point for two equations and being on the left endpoint of the interval. By sketching, the equation

$y_1=x + 8$ falls outside the range and hence rejected.

Putting $n_i = n_3$,i.e $i = 3$ then $d(p, n_i) = d(n_3, n_3) = 0$ and $d(q , n_i) = d(n_1, n_3) = 8$.

Thus $y_1 = x$, $y_2 = 16 – x$ and solving the path of equal distance or cost, we have $x = 16 – x$, $x = 8$ which is kink point. It is at the right end point of the interval. By sketching, the equation

$y_2 = 16 – x$ falls outside the range hence rejected. In both instances above, we accept and sketch the two equations below;

$y_1 = 8 – x$, $0 \leq x \leq 8$ …………………………………………..(i)

$y_2= x – x$, $0 \leq x \leq 8$ ………………………………………….(ii)

Putting $n_i = n_2$ i.e $i = 2$ then $d(p, n_i) = d(n_3, n_2) = 2$ and $d(q, n_i) = d(n_1, n_2) = 8$

The resulting equations $y_1 = x + 2$ and $y_2 = 16 - x$ when solved for the path of equal distance or cost, we have $x + 2 = 16 - x \Rightarrow x = 7$ which is the kink point. The following equations are then sketched in the given ranges

$y_1 = x + 2, 0 \leq x \leq 7$ …………………………………………(iii)

$y_2 = 16 - x, 7 \leq x \leq 8$ ………………………………………..(iv)

Putting $n_i = n_4$ i.e $i = 4$, then $d(p, n_i) = d(n_3, n_4) = 12$ and $d(q, ni) = d(n_1, n_4) = 6$.

The resulting equations, $y_1 = x + 12$ and $y_2 = 14 - x$ when solved for the path of equal distance or cost, we have $x + 12 = 14 - x$, $x = 1$ which is the kink point.

The following equations are then sketched within the given ranges.

$y_1 = x + 12 \quad , \quad 0 \leq x \leq 1$ ………………………………………(v)

$y_2 = 14 - x \quad , \quad 0 \leq x \leq 8$ ………………………………………(vi)

Putting $n_i = n_5$, i.e $i = 5$, then $d(p, n_i) = d(n_3, n_5) = 8$ and $d(q, n_i) = d(n_1, n_5) = 2$

The resulting equations, $y_1 = x + 8$ and $y_2 = 10 - x$ when solved for the path of equal distance or cost, we have $x + 8 = 10 - x \Rightarrow x = 1$

The following equations are then sketched within the given ranges.

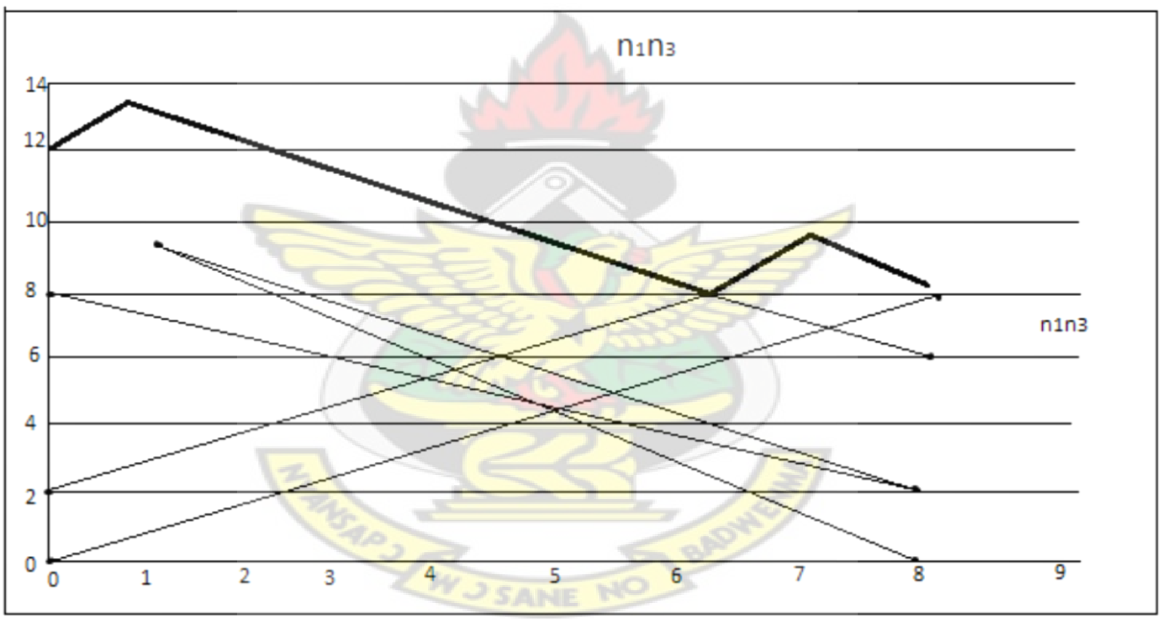
$y_1 = x + 8 \quad , \quad 0 \leq x \leq 1$ ………………………………………..(vii)

$y_2 = 10 - x \quad , \quad 0 \leq x \leq 8$ …………………………………………(viii)

The eight equations are then sketched on the axes as shown in fig 3.4.The minimum cost or distance of the path can be found from the graph using the upper envelope.

3.6 Construction of Upper Envelope

After sketching all the equations resulting from the location on edge (n_1n_3) on the same axes as shown in fig 3.4 there is the need to construct an 'upper envelope' which gives the minimum cost/distance of a shortest path from *x* to a farthest node on the given edge. To construct the upper envelope, we trace all paths of lines beyond which there are no higher points for the same *x*- value in the given range. This path is indicated by a thick line as shown in the figure.



$$Xn_1n_3 = 6, or\ 8\, f(x) = m(x_1) = 8$$

Figure 3.4

Thus the minimum cost on edge (n_1n_3) i.e. (Xn_1n_3), is selected by considering the point corresponding to the minimum cost for all nodes. In the example above, the minimum cost/distance for edge (n_1n_3) is given as (Xn_1n_3)= 6 or 8 and m (x)= 8 units.

The upper envelope is constructed for all graphs of the remaining edges and their corresponding minimum costs and the local centres taken on each edge. The minimum cost with the corresponding minimum value on the vertical axis or minimum local centre $\varkappa_1$ becomes the absolute centre.

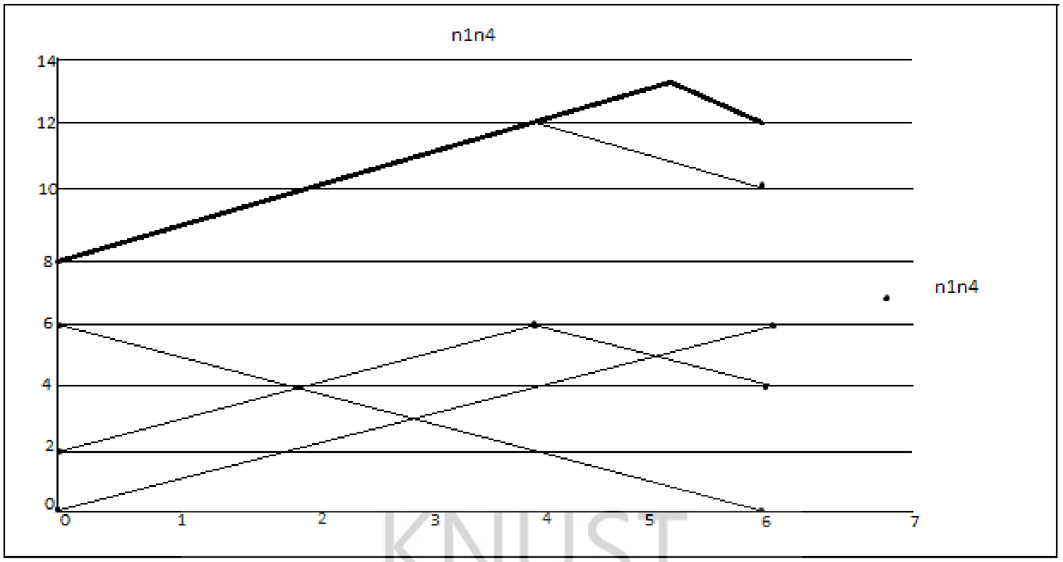
The diagrams in figures 3.5, 3.6, 3.7, 3.8, and 3.9 show the graphs of the remaining edges in the example with their corresponding minimum costs.

The computation of the equations for the locations on the other edges in the network is shown in Appendix I.

For each edge (p,q) , the local centre is found by plotting d $(\varkappa, n_1)$ for each node $n_1 \in N, where\ 0 \leq \varkappa \leq c(p,q)$.

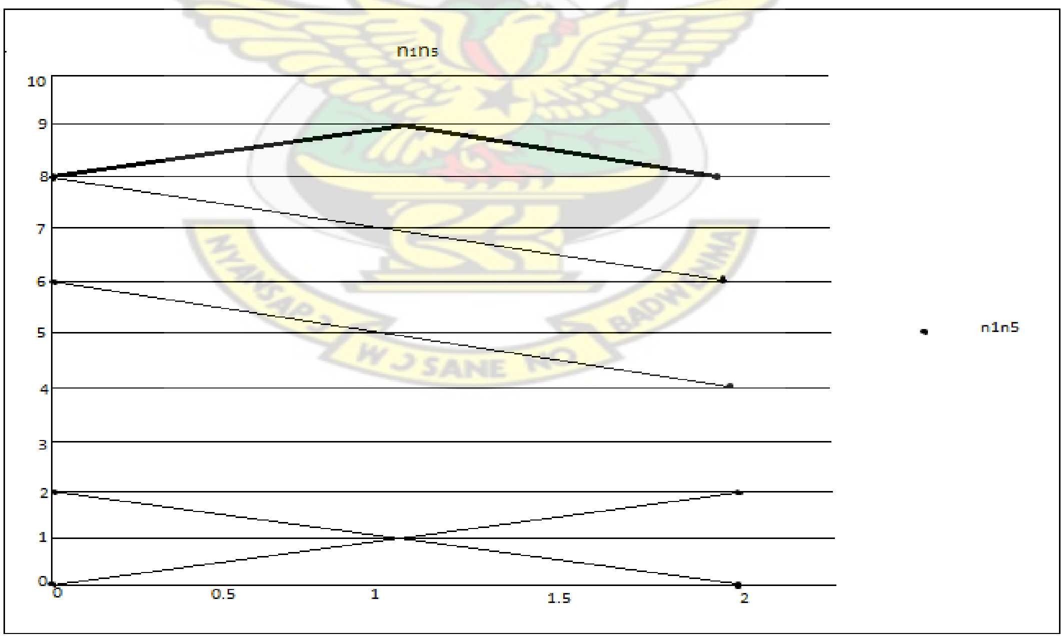
The local centre$\varkappa_{pq}$=$\varkappa_1$ is the point that minimizes the upper envelope. The absolute centre$\varkappa_a$ is the minimum point among the local centers. This occurs on the edge $(n_5 n_2)$ with $d(\varkappa_a, n_5) = 2\ and\ d(\varkappa_a, n_2) = 4, m(\varkappa_a) = 6$ which implies, the maximum distance from point$\varkappa$ to the farthest node is 6 units that is to both nodes $n_5\ and\ n_2$ hence the optimum location of the facility is on edge $(n_{2,} n_5)$ which is 2 units from nodes n_5 and 4 units from noden_2.

Finding a single absolute centre of a network is more involving. In practice, where a network has a large number of nodes, there would equally a large number of edges to be enumerated for their respective local centres.



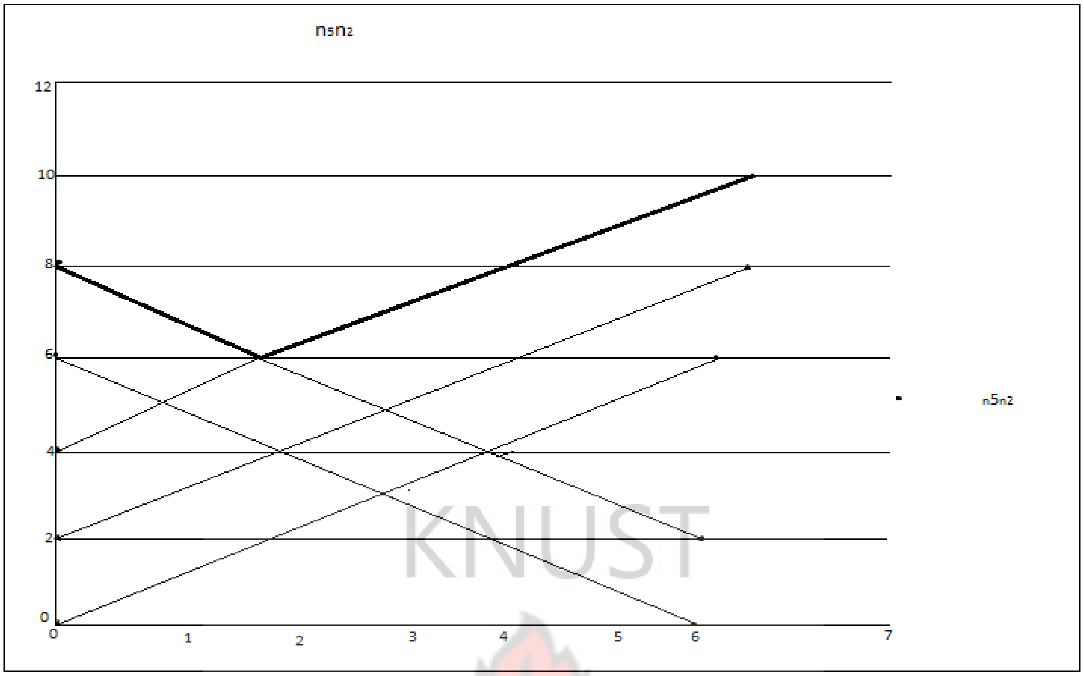
$xn_1n_4 = 0, m(\varkappa_l) = 8$

Figure 3.5



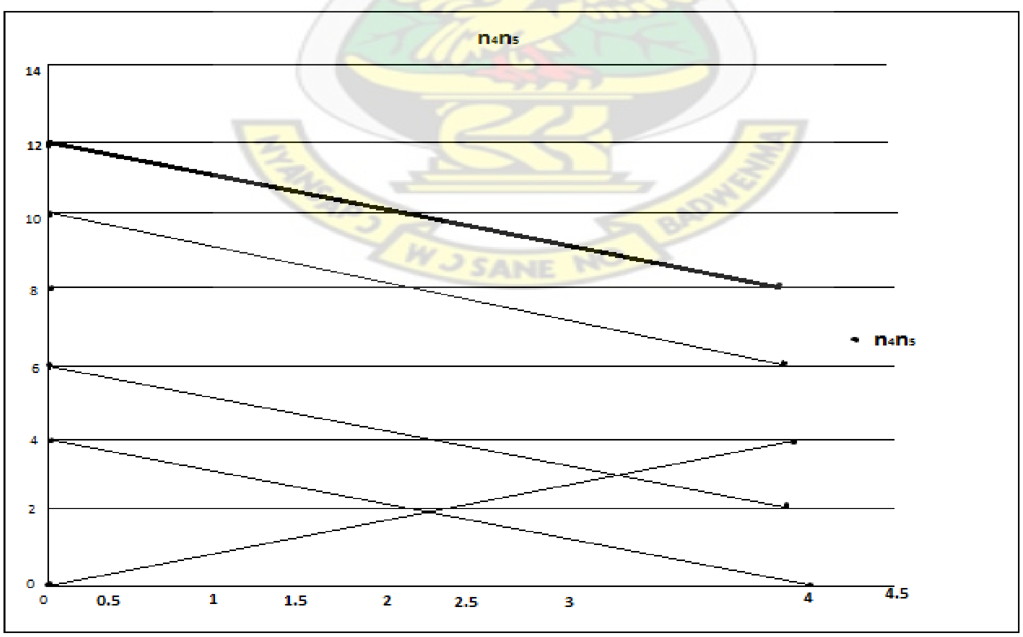
$xn_1n_5 = 0, or\ 2, m(\varkappa_l) = 8$

Figure 3.6



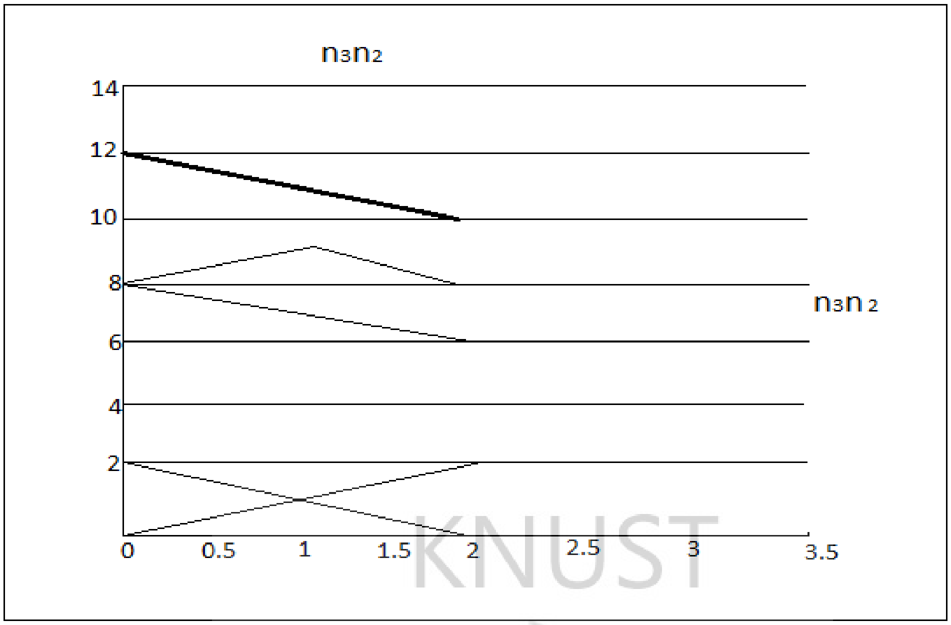
$xn_5n_2 = 2, m(\varkappa_l) = 6$

Figure 3.7



$xn_4n_5 = 2, m(\varkappa_l) = 8$

Figure3.8



$xn_3n_2 = 2, m(x_1) = 10$

Figure 3.9

$xn_3n_2 = 2, m(x_1) = 10$

Fortunately as indicated in the propositions (i) and (ii) below, many edges do not need to be explicitly enumerated for their respective local centres.

Propositions (i)

For the set of all points x on a fixed edge (p, q) of G, the maximum distance function m(x) is piecewise linear whose slope is always +1 or -1.

Preposition (ii)

For an edge (p, q) the local centre satisfies the equation, $m(X_l) \geq \frac{m(p) + m(q) - c(p, q)}{2}$

where *c(p, q)* denotes the cost of the edge *(p, q)*

Proof:

Consider any point on the edge *(p,q)*. Let $\varkappa: 0 \leq \varkappa \leq c(p,q)$ denotes the point p such that $\varkappa = 0$ *and the point* $\varkappa = c(p,q)$ denote q. We take d(*x*, p) to be *x* and d(*x*, q) to be c(p, q) $-\varkappa$.

The cost d(*x*, p) of a shortest path between *x* and the farthest demand node *p* is piecewise linear with a slope +1 or -1 at each point of *x*. Its value at *x*=0 is m(*p*) and its value at *x*= *c(p,q)* is m(*q*) where m(*p*)and m(*q*) are node centres for nodes *p and q.*

Hence, m (*x*)≥ $m(p) - \varkappa$ [For all $\varkappa: 0 \leq \varkappa \leq c(p,q)$]…………………….(i)

m (*x*)≥ $m(q) - (c(p,q) - \varkappa)$[For all $\varkappa: 0 \leq \varkappa \leq c(p,q)$]……….(ii)

By adding the two inequalities (i) and (ii), we obtain

m $(\varkappa_l) \geq \frac{m(p)+m(q)-c(p,q)}{2}$

Where $\varkappa_l$ simultaneously satisfies the two inequalities above.

From these propositions and from observation that, by definition, maximum distance associated with the node centre must be greater than or equal to the corresponding distance for the absolute centre i.e. m $(\varkappa_n) \geq m(\varkappa_a)$,we can derive the following test:

If for edge (p, q), m$(\varkappa_n) \leq \frac{m(p)+m(q)-c(p,q)}{2}$then the local center $\varkappa_l$ of (p, q) cannot improve on m($\varkappa_n$) and therefore need not to be found . This test which takes advantage of the fact that it is very simple to find the node centre ($\varkappa_n$), often leads to considerable reduction in the computation effort required to obtain the absolute centre. With respect to the five-node, six- edged network in fig. 3.1, we found easily that the node centre is at nodesn_1 and n_5 and that m $(\varkappa_n)$ =m $(n_1) = m(n_5)$=8.

On application of the test to the six edges of the network, we obtain:

Edges (n_1, n_3) : $\frac{m(n_1)+m(n_3)-c(n_1,n_3)}{2} = \frac{8+12-8}{2} = 6 < (8)$

Edges (n_2, n_3) : $\frac{m(n_2)+m(n_3)-c(n_2,n_3)}{2} = \frac{10+12-2}{2} = 10 > (8)$

Edges (n_2, n_5) : $\frac{m(n_2)+m(n_5)-c(n_2,n_5)}{2} = \frac{10+8-6}{2} = 6 < (8)$

Edges (n_1, n_5) : $\frac{m(n_1)+m(n_5)-c(n_1,n_5)}{2} = \frac{8+8-2}{2} = 7 < (8)$

Edges (n_1, n_4) : $\frac{m(n_1)+m(n_4)-c(n_1,n_4)}{2} = \frac{8+12-6}{2} = 7 < (8)$

Edges (n_4, n_5) : $\frac{m(n_4)+m(n_5)-c(n_4,n_5)}{2} = \frac{12+8-4}{2} = 8 = (8)$

The results of the test above clearly suggest that the local centre needs to be found for only edges.

((n_1, n_3), (n_2, n_5), (n_1, n_5), and (n_1, n_4).

This makes significant savings in the computational effort and time.

3.7 Invariance of local with respect to choice of any node as the origin.

It has being well established through research that any of the nodes of an edge can be chosen as the origin since each of the graphs that are obtained in both cases is a mirror reflection of the other.

The axis reflection may pass through any of the nodes. The edge (n_1, n_5) of the network in fig. 3.1 would be used to illustrate this fact.

Edge (n_1, n_5)

Choosing n_1 as the origin, we let p=n_1 and q= n_5 such that $\varkappa$:0$\leq \varkappa \leq c(p,q)$.

Putting $n_1 = n_1$ i.e. i = 1, then $d(p, n_1) = d(n_1, n_1) = d(n_1, n_1) = 0; d(q, n_i) = d(n_5, n_1) = 2$and $c(p,q) = c(n_1, n_5) = 2$. The resulting equation,$y_1 = \varkappa and y_2 = 4 - \varkappa$, when solved f\or the path of equal distance or cost, we have $\varkappa = 4 - \varkappa$,$\Rightarrow \varkappa = 2$ which is the kink point. putting

$n_i = n_5$, i.e. i=5, then $d(p, n_i) = d(\mathrm{n}_{1,}\mathrm{n}_5) = 2$ and d(q, $n_i) = d(\mathrm{n}_5, \mathrm{n}_5) = 0$.Theresulting equations,$y_1 = \varkappa + 2$ and$y_2 = 2 - \varkappa$when solved for the path equal distance, we have $\varkappa + 2 = 2 - \varkappa, \Rightarrow \varkappa = 0$whichis the kink point. The range of values of $\varkappa$ for the edge (n_1, n_5) shall be $0 \leq \varkappa \leq 2$.

By sketching, the equations $y_1 = \varkappa + 2$ and $y_2 = 4 - \varkappa$ fall outside the given range and hence rejected.

The following equations: $y_1 = \varkappa, 0 \leq \varkappa \leq 2$ … … ..(i)

$y_2 = 2 - \varkappa, 0 \leq \varkappa \leq 2$ … … (ii)

Are accepted and sketched within the given range.

Putting $n_1 = n_2$ i.e. $i = 2$, then $d(p, n_i) = d(n_i, n_2) = 8\ and\ d(q, n_i) = d(n_5, n_2) = 6$.

The resulting equations $y_1 = \varkappa + 8\ and\ y_2 = 8 - \varkappa$ when solved for the path of equal distance or cost, we have x+8=8$-\varkappa, \Rightarrow \varkappa = 0$ which is the kink point.

By sketching the equation $y_1 = \varkappa + 8$ falls outside the given range and hence rejected.

The equation $y_2 = 8 - \varkappa, 0 \leq \varkappa \leq 2$…….. (iii) is accepted and sketched.

Putting $n_1 = n_3$, i.e. i=3, then $d(p, n_\mathrm{i}) = d(\mathrm{n}_{1,}\mathrm{n}_3) = 8, \mathrm{d}(\mathrm{q}, n_\mathrm{i}) = d(\mathrm{n}_5, \mathrm{n}_3) = 8$. The resulting equation$y_1 = \varkappa + 8\ and y_2 = 10 - \varkappa$ when solved for the path of equal distance or cost, we have x+8=10$-\varkappa, \Rightarrow \varkappa = 1$ which is the kink point.

The following equations are then sketched within the given ranges:

$$y_1 = \varkappa + 8, 0 \leq \varkappa \leq 1\text{……… (iv)}$$

$$y_2 = 10 - \varkappa, 0 \leq \varkappa \leq 2\text{…….(v)}$$

Putting $n_1 = n_4$, i.e. i=4, then $d(p, n_\mathrm{i}) = d(\mathrm{n}_{1,}\mathrm{n}_4) = 6$, $\mathrm{d}(\mathrm{q}, n_\mathrm{i}) = d(\mathrm{n}_5, \mathrm{n}_4) = 4$.The resulting equations, $y_1 = \varkappa + 6\ and y_2 = 6 - \varkappa$ when solved for the path of equal distances or costs, we have $\varkappa - 6 = 6 - \varkappa, \Rightarrow \varkappa = 0$ which is the kink point.

By sketching the equation, $y_1 = \varkappa + 6$ falls outside the given range and hence rejected. The equation$y_2 = 6 - \varkappa, 0 \leq \varkappa \leq 2$.... (vi) is accepted within the given range.

Choosing n_5 as the origin we let $p = n_5 and q = n_1$and such that $\varkappa: 0 \leq \varkappa \leq c(p, q)$.

Putting $n_i = n_1$ i.e. i=1, then $d(p, n_\mathrm{i}) = d(\mathrm{n}_{5,}\mathrm{n}_1) = 2, \mathrm{d}(\mathrm{q}, n_\mathrm{i}) = d(\mathrm{n}_1, \mathrm{n}_1) = 0$and $c(p, q) = c(n_5, n_1) = 2$. The resulting equations, $y_1 = \varkappa + 2\ and y_2 = 2 - \varkappa$, when solvedfor the pathof equal distance or cost; we have $\varkappa + 2 = 2 - \varkappa, \Rightarrow \varkappa = 0$ which is the kink point.

Putting $n_i = n_5$ i.e.$i = 5$ then $d(p, n_i) = d(n_{5,} n_5) = 0$, $d(q, n_i) = d(n_1, n_5) = 2$.

The resulting equations $y_1 = x$ and $y_2 = 4 - \varkappa$, when solved for the path of equal distance or cost, we have x=4 - x, $\Rightarrow x$=2 which is the kink point. Thus the range of values of x shall be $0 \leq \varkappa \leq 2$.

By sketching, the equations, y_1 =x+2 and $y_2 = 4 - \varkappa$ falls outside the given range and hence rejected.

The equations $y_1 = \varkappa, \quad 0 \leq \varkappa \leq 2$……… (I)

$y_2 = 2 - \varkappa, 0 \leq \varkappa \leq 2$……. (ii)

are accepted and sketched within the given range.

Putting $n_i = n_2$ i.e. $i = 2$ then $d(p, n_i) = d(n_{5,} n_2) = 6$, $d(q, n_i) = d(n_1, n_2) = 8$.

The resulting equation, $y_1 = \varkappa + 6$ *and* $y_2 = 10 - \varkappa$, when solved for the paths of equal distance or costs, we have $\varkappa + 6$ =$10 - \varkappa$, $\Rightarrow \varkappa$=2 which is the kink point.

By sketching, the equation $y_2 = 10 - \varkappa$ falls outside the given range and hence rejected.

The equation, $y_1 = \varkappa + 6, 0 \leq \varkappa \leq 2$…………(iii) is accepted and sketched within the given range.

Putting $n_i = n_3$ i.e. $i = 3$ then $d(p, n_i) = d(n_{5,} n_3) = 8$, $d(q, n_i) = d(n_1, n_3) = 8$.

The resulting equations, $y_1 = \varkappa + 8$ and $y_2 = 10 - \varkappa$, when solved for the path of equal distance or cost, we have x+8=$10 - \varkappa$, $\Rightarrow x$=1which is the kink point.

The following equations are taken sketched within the given range.

$y_1 = \varkappa + 8, \quad 0 \leq \varkappa \leq 1$…………. (iv)

$y_2 = 10 - \varkappa$, 1$\leq \varkappa \leq 2$ … … ..(v)

Putting $n_i = n_4$ i.e. $i = 4$ then $d(p, n_i) = d(n_{5,} n_4) = 4$, $d(q, n_i) = d(n_1, n_4) = 6$.

The resulting equation $y_1 = \varkappa + 4$ and $y_2 = 8 - \varkappa$ when solved for the path of equal distance or cost, we have $\varkappa + 4 = 8 - \varkappa$, ⇒x=2 which is the kink point.

By sketching, the equation $y_2 = 8 - \varkappa$ falls outside the given range and hence rejected.

The equation, $y_1 = \varkappa + 4, 0 \leq \varkappa \leq 2$…………(vi) is accepted and sketched within the given range.

The graphs of edges $(n_{1,} n_5)$ and $(n_{5,} n_1)$ are then drawn the specified ranges. These graphs are shown respectively in fig 3.10 and fig 3.11.

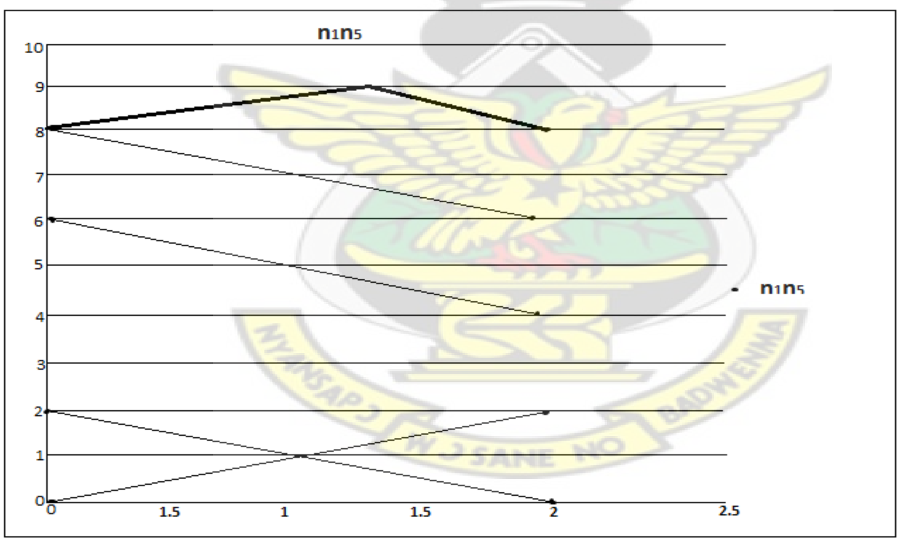


Figure 3.10

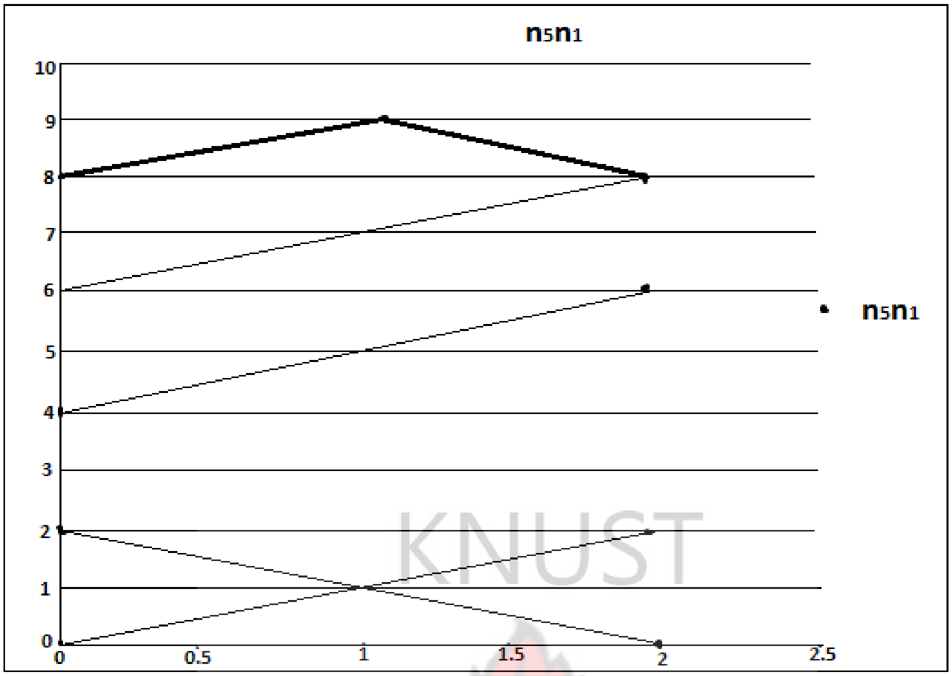


Figure 3.11

3.8 Summary

The decision of where to locate a facility depends on certain criteria unique to a given location problem. Optimizing some of the specific criteria raises the question of desirability of a facility by the community. Facilities are therefore classified into three categories as obnoxious, non- obnoxious and semi obnoxious.

The chapter also briefly discussed some network –based algorithms such as single-source shortest and all pair's shortest paths.

A detailed explanation of *p- centre* problem, a heuristic method which is the main tool in this project for the location of a fire station at the Birim North has been provided.

CHAPTER 4

DATA COLLECTION AND ANALYSES

4.1 Introduction

This chapter provides Birim North area map (Appendix III) which depicts the exact positions of some important facilities (school, markets, Hospitals etc.) specifying the Euclidean distances (along the road) between them. The data was obtained from Survey Division of the lands Commission, the district Education office, Town and country planning office, all in New Abirem and the Birim North District Assembly (BNDA) and the analysis would be done using the concept of the centre-problem to identify where a fire station has to be located in the district. The district distances along the road between some locations were provided by the Survey Division in New Abirem.

Locations Considered are:

A. Nkwateng
B. Ntronang
C. New Abirem
D. Mamaso
E. Old Abirem
F. Nyafoman
G. Noyem
H. Prasokuma
I. Afosu
J. Akoase
K. Kwea
L. Twapease
M. Labikrom
N. Hwekwae
O. Akokoase
P. Yayaso
Q. Akaikrom
R. Ofoasekuma
S. AmoanaPraso
T. Amenam
U. Bramkrom
V. Pankese

Table 4.1

NUMBER	EDGE CONSIDERED	DISTANCE (METRES)
1	(A, B)	1660
2	(A, C)	2590
3	(A, K)	1201
4	(A, N)	1330
5	(B, C)	1360
6	(B, R)	3400
7	(C, D)	1690
8	(C, F)	2320
9	(C, H)	2950
10	(C, O)	4745
11	(C, P)	1360
12	(C, I)	1150
13	(C, K)	4390
14	(D, E)	1270
15	(D, I)	2230
16	(D, M)	2080
17	(E, Q)	1210
18	(F, G)	1090
19	(F, H)	2890
20	(F, I)	1810
21	(F, S)	370
22	(F, T)	2800
23	(G, I)	990
24	(G, S)	670
25	(G, U)	3850
26	(H, T)	640
27	(I, I)	1090
28	(I, L)	2350
29	(I, M)	850
30	(I, U)	4030
31	(I, V)	3190

The data in Table 4.1 above was then developed into the network shown in figure 4.1

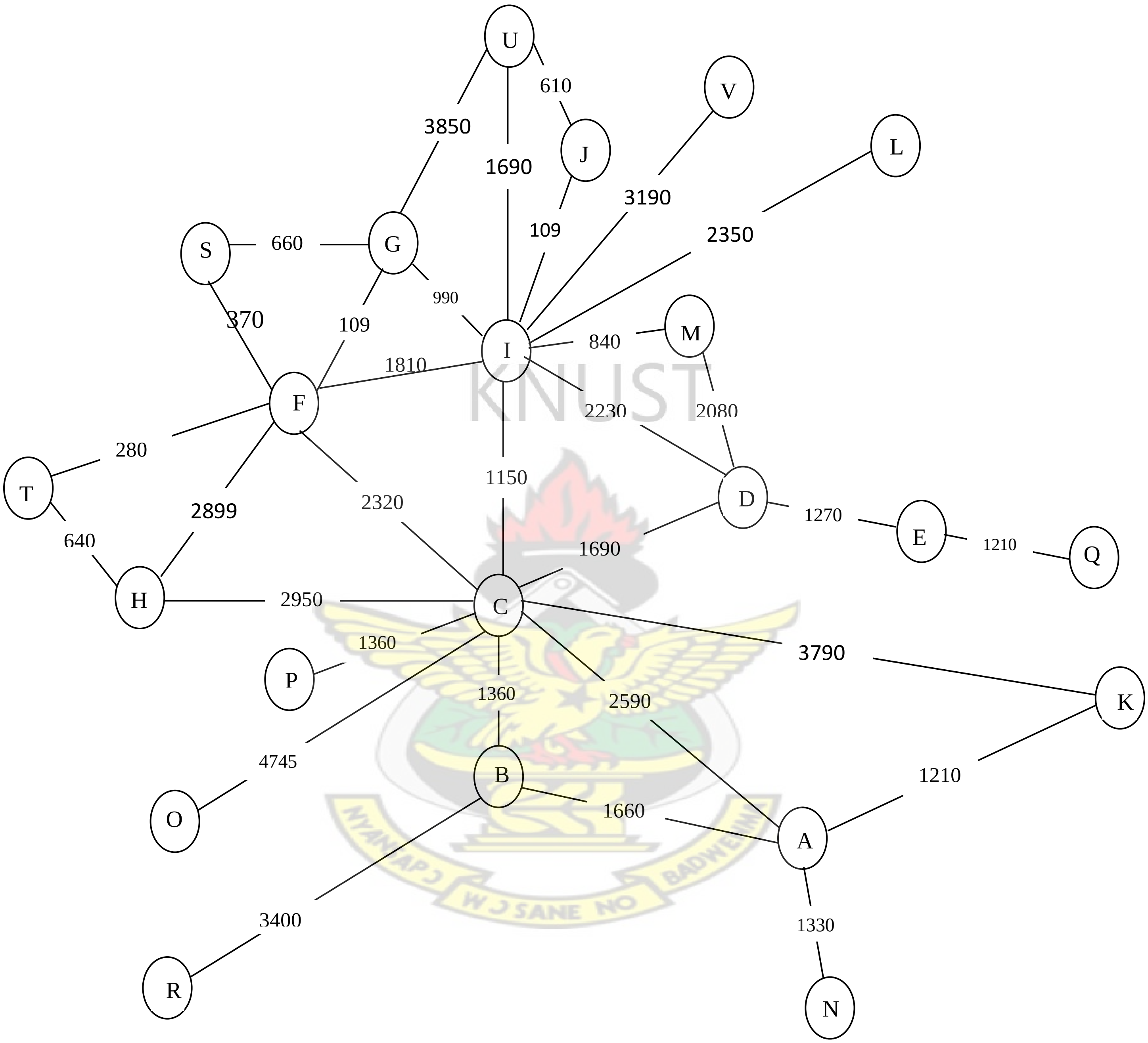

U
610
V
3850
1690
J
L
3190
109
2350
S
660
G
990
109
370
M
840
I
1810
F
2230
2080
280
T
1150
D
2899
2320
1270
E
1210
Q
640
1690
H
2950
C
1360
3790
P
1360
2590
K
4745
B
1210
1660
O
A
3400
1330
R
N

Figure 4.1

Table 4.2

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	ROW MAX
A	-	1660	2590	4270	5530	4900	4720	5530	3730	4810	1210	6070	4570	1330	7330	3940	6730	5050	5260	6160	5410	6910	7330
B	1660	-	1360	3040	4300	3670	3490	4300	2500	3580	2860	4840	3340	2980	6100	2710	5500	3400	4030	4930	4180	5680	6100
C	2590	1360	-	1690	2950	2320	2140	2950	1150	2230	3790	3490	1990	3910	4745	1360	4150	4745	2680	3580	2830	4330	4745
D	4270	3040	1690	-	1270	3400	3220	4630	2230	3310	5470	4570	2070	5590	6430	3040	2470	6430	3790	5260	3910	5410	6430
E	5530	4300	2950	1270	-	5260	4480	5890	3490	4570	6730	5830	3340	6850	7690	4300	1210	7690	5140	6520	5170	6670	7690
F	4900	3670	2320	3400	5260	-	1090	2890	1810	2890	6100	4150	2650	6220	7060	3670	6460	7060	370	2800	3490	4990	7060
G	4720	3490	2140	3220	4480	1090	-	3910	990	2080	5920	3340	1840	6040	6880	3490	5680	6880	660	3820	2680	4180	6880
H	5530	4300	2950	4630	5890	2890	3910	-	4090	5170	6730	6430	4930	6850	7690	4300	7090	7690	3250	640	5770	7270	7690
I	3730	2500	1150	2230	3490	1810	990	4090	-	1090	4930	2350	840	5050	5890	2500	4690	5890	1660	4600	1690	3190	5890
J	4810	3580	2230	3310	4570	2890	2080	5170	1090	-	6010	3430	1930	6130	6970	3580	5770	9670	2740	5680	610	4270	6970
K	1210	2860	3790	5470	6730	6100	5920	6730	4930	6010	-	7270	5770	2530	8530	5140	7930	6250	6460	7360	6610	8110	8530
L	6070	4840	3490	4570	5830	4150	3340	6420	2350	3430	7270	-	3190	7390	8230	4840	7030	8230	3400	6940	4030	5530	8230
M	4570	3340	1990	2080	3340	2650	1840	4930	840	1930	5770	3190	-	5890	6730	3340	4540	6730	2500	5440	2530	4030	6730
N	1330	2980	3910	5590	6850	6220	6040	6850	5050	6130	2530	7390	5890	-	8650	5260	8050	6370	6580	7480	6730	8230	8650
O	7330	6100	4745	6430	7690	7060	6880	7690	5890	6970	8530	8230	6730	8650	-	6100	8890	9490	7420	8320	7660	9070	9490
P	3940	2710	1360	3040	4300	3670	3490	4300	2500	3580	5140	4840	3340	5260	6100	-	5500	6100	4030	4930	4180	5780	6100
Q	6730	5500	4150	2470	1210	6560	5680	7090	4690	5770	7930	7030	4540	8050	8890	5500	-	8890	6340	7720	6370	7870	8890
R	5050	3400	4745	6430	7690	7060	6850	7690	5890	6970	6250	8230	6730	6370	9490	6100	8890	-	7420	8320	7570	9070	9490
S	5260	4030	2680	3880	5140	370	660	3250	1660	2740	6460	3400	2500	6580	7420	4030	6340	7420	-	3160	3340	4840	7420
T	6160	4930	3580	5250	6520	2800	3820	640	4600	5680	7360	6940	5440	7480	8320	4930	7720	8320	3160	-	6280	7780	8320
U	5410	4180	2830	3910	5170	3490	2680	5770	1690	610	6610	4030	2530	6730	7570	4180	6370	7570	3340	6280	-	4870	7670
V	6910	5680	4330	5410	6670	4990	4180	7270	3190	4270	8110	5530	4030	8230	9070	5680	7870	9070	4840	7780	4870	-	9070

4.1 All pairs shortest path for the data collected.

From the network in figure 4.1 the minimum distance matrix $d(i,j)$, that is the matrix of the shortest paths using the Floyd-Warshall's algorithm was obtained and is shown in Table 4.2.

4.1.1 Locating the Vertex/Node Centre

The node or vertex centre(xn) is chosen as the smallest among the maximum entries of all rows in the matrix. From table 4.2 the row with the minimum among the maximum entries occurs at node/vertex C with a maximum distance of 4745metres. Thus the node/vertex centre for the network in figure 4.1 is C, hence m(c) = 4745

4.1.2 Locating the Absolute Centre

An algorithm for determining the absolute centre of an undirected network/graph with thirty-one edges or links may be simply described as follows:

STEP I: For each edge L of G find the local centre(s) x_1

STEP II: Among the entire local centres(xi), choose the one with the smallest distance m(xi). The local centre(xi) with the minimum distance $m(xi)$is taken as the absolute of x_a of G.

Step I of this simple two-step algorithm is however time consuming as described in chapter three. For each edge, one of the nodes on that edge has to be fixed as the origin and a number of equations have to be generated using the general

equations:

$$y1 = x + d(p, ni)$$
$$y2 = c(p, q) - x + d(q, ni)$$

where n_i = A, B, C …….V which are the individual nodes/ vertices on the network G, p is the origin and q is the other node or end note of that edge under consideration. A graph of those equations are then drawn on the same axis and $m(xi)$ is given by the minimum point on the upper envelope as described in chapter three.

From the prepositions 1 and 2 in chapter three the maximum distance associated with the vertex/node centre (x_n) must be greater than or equal to the corresponding distance for the absolute centre (x_a) that is $m(x_n) \geq m(x_a)$.

However, for edge (p, q) the local centre x_i satisfies

$$m(xi) \geq \frac{m(p) + \mathrm{m(q)} - \mathrm{c(p, q)}}{2}$$

where c (p,q) denotes the cost of edge(p, q).

Applying this test to the thirty-one edges of the network as shown in Appendix IV it was realised that the local centre needs to be found for only the edges (A C),(B C), (C P) (C H), (F C), (C I), (C D) and (K C)

A number of equations are also generated for each edge and these are shown in Appendix V.

4.2 Results

a) Vertex/Node Centre

Table 4.3

NODE	A	B	C	D	E	F	G	H	I
ROW MAX	7330	6100	4745	6430	7690	7060	6880	7690	5890

NODE	J	K	L	M	N	O	P	Q	R
ROW MAX	6970	8530	8230	6730	8650	9490	6100	8890	9490

NODE	S	T	U	V
ROW MAX	7420	8320	7570	9070

b) Local Center

Table 4.4

№	Edge	Local Centre (x_i)	Max Dis.[(m(x_i)]
1	(AC)	2590m from node A	6100
2	(BC)	1360m from node B	4745
3	(CP)	At C	4745
4	(CH)	At C	4745
5	(FC)	2340m from F	4745
6	(CI)	At C	4745

c) Absolute Centre

From the two propositions considered earlier on, the maximum distance associated with the node or vertex centre m(x_n) must be greater or equal to the corresponding distance for the absolute centre m(x_a) that is m(x_n) ≥ m(x_a).Also the maximum distance associated with the local centre m(x_i) must be greater or equal to the corresponding distance for the node centre that is m(x_i) ≥ m(x_n).

However, the absolute centre occurs on the edge where the minimum of all the maximum distances of the local centres occurs. From Table 4.3 and Table 4.4 and by observation the distance of the absolute centre m(x_a) is 4745 meters and occurs at node C from which all other nodes in the network is beyond the reached. No node in the network is beyond the maximum distance of the absolute centre.

4.3 Summary

The sector map of Birim North and the other data for the analysis were obtained from a number of institutions in the Birim North District. The data was processed into network and Floyd Warshall's algorithm used to compute the distance between all pairs of node /vertex in the matrix to determine the node/vertex centre. The determination of local centre son all the edges of the network were done by the use of the centre problem concept and the minimum of these was chosen as the absolute centre.

The absolute centre distance (cost) m(x_a) was found to be equal to the node centre distance cost m(x_n) hence the absolute centre for the network occurred at the Node C.

CHAPTER 5

CONCLUSION AND RECOMMENDATION

5.0 Introduction

This chapter presents the overall summary of the main concepts presented in this project; identify the strategic position for the location of a fire station as well as suggestions for future investigations.

5.1 Summary of Findings

In an effort to identify a strategic position or location for the establishment of a fire station at the Birim North District, using the absolute centre heuristic method, the following findings were made:

i. There is an existing fire station located at Nkawkaw but the best response time to fight fire outbreaks within the district is 18 minutes and the worst time is 32 minutes.

ii. Nkawkaw Fire Station responded to more than 89.3% of structure fires that broke out in the Birim North District between the years 2008 and 2009.

iii. In 2008 and 2009, 27 structural fires broke out in the district with the cost of damage estimated at GH¢340596.00. However, there were three (3) severe injuries recorded.

iv. The structural fires were attributed mainly to electrical faults, misuse of electrical equipment. The use of candles, leakage of gas cylinders and other naked flames without taken the necessary precautions by users.

v. The minimum of the local or edge centres is 4745 meters and it is located on the edges between B and C, A and C, C and P, C and H, F and C, C and I, C and D, K and C. Since the distance (cost) of the node centre is equal to the corresponding distance (cost) of the edge/local centre, it is chosen as the absolute centre of the network. This implies that the facility must be located at node C or in the neighborhood of node C and the maximum distance to be travelled from this facility to a farthest node is 4745 meters.

vi. The optimal service coverage area is found to be 4745 meters radius from the absolute centre.

5.2 Recommendations

It has been shown by research that, flashovers occur in residential structures if a fire burns for more than 10 minutes and that the fire will extend from the room of origin into bordering rooms [NFPA 1710].

A fire extension in residential structures' a study carried out by NFPA between 1994 and 1998 in USA showed that civilian deaths increased by 748 percent, injuries by 175 percent and dollar lost by 613 percent on the average when the fire is not contained to the fire origin [All rates one per 1000 fires].From the above it is obvious that Birim North District needs a fire station which should be strategically placed to respond to fire outbreaks immediately they are informed. Since Nkawkaw fire service during their worst situations has a responds time of 32 minutes, flashovers would have long occurred before arrival and would make search and rescue as well as bringing the fire under control very difficult if not impossible.It is therefore recommended to the management of Ghana

National Fire Service (GNFS) to strategically locate Fire Station at Birim North to ensure minimum responds time, minimum travel distance to all incidents in the service coverage area. This would result in the whole service coverage area benefiting from such a facility equally and no demand point is denied or unduly delayed of the services of the facility.

It is also recommended that a similar study is conducted in all districts where such facility is yet to be established and on all existing fire stations to identify whether or not they are optimally sited.

In future no fire station should be sited at a location without first using the appropriate methods/techniques to identify that the site is optimally located. It is however suggested that further investigations be conducted on this project taking into consideration traffic density, nature and degree of hazards and the nature and conditions of road networks in the service coverage area to identify an acceptable minimum response time.

REFERENCES

Amponsah, S.K, (2007). Optimization Technique. Vol. 1 pp 139-142

Amponsah, S.K, and Darkwa F .K. (2007).Operation Research pp 1-27, 38-62.

Badri, M.,(1998). A Multi-objective Model for Locating Fire Stations. European journal of Operational Research, Vol. 110 .pp 407-422.

Zhan B, and Noon C., (1998). Shortest path Algorithms: An evaluation using Real Road Network Transportation Science, Vol. 32, pp 65-73.

Brandeau, M. L., (1995). Objective in location problems. In: Drezner, Z.(ed.), facility location: A survey of applications and methods, Springer- Verleg, New York.

Calvo, A. and Marks, H. (1973). Location of Health care facilities: An analytical Approach. Socio-Economic Manning Sciences, Vol. 7, pp 407-422.

Carson Y. , (1990). Locating an ambulance on the Amherst campus of the State University of New York at Buffalo.

Caruso, C. , (2003). Dominant, an algorithm for the p-center problem in. Eur. J. Oper. Res., Vol.149, pp 53-64.

Charikar M, and samirk H., (2001) algorithm for facility location problem with outliers.

Chen, D.R. (2009). Computer and Operation Research Vol. 36 pp 1646-1655.

Chen, R. (1983). Solution of minisum and minimax location- allocation problems with Euclidean distances. Naval Res Log Quart., Vol. 30 pp 449-459.

Chen, R. and Handler, G. Y. (1993).The conditional P-Centre problem in the plane. Naval Research logistics, Vol. 40, pp 117-127.

Chen, R. and Handler, Y (1987). Relaxation method for the solution of the minimize location allocation problem in Euclidean space. Naval Res. Log.,Vol.34, pp 775-787.

Christo des, N, (1971). Graph Theory: An algorithm Approach. Academic Press, London.

Chung, C. H. (1986). Recent application of the maximal covering location planning (M.C.L.P) model. Journal of operational Research society, Vol. 37, pp .735-746.

Church, R. and Re Velle, C. (1974). The maximal covering problem, Papers of the Regional Service Association, Vol. 32,pp 101-118.

Church, R. L. (2002). Geographical information systems and location science. Computer Operation Research 29 (6) pp 541-562.

Matt C., (2007). An Evaluation of new Fire Station Location for Snoqualmine Pass Fire and Rescue pp 2-25.

Daskin, M. S. (1995) Network and Discrete Location: Models, Algorithms and application.

Daskin, M. S. (1995) Network and Discrete Location: Models, Algorithms and Application. John Wiley, New York.

Daskin, M. S. (2000). A new approach to solving the vertex P-center problem to optimality: Algorithm and computational results. Communication of the Japanese Operational Research Society, Vol. 9, pp 428-436.

Deterministic and probabilistic models. Annals of Operations Research, pp 303 322.

Dijkstra, E. W. (1959). A note on two problems in connection with graphs Numerische Mathematik, Vol.1 pp 269-271.

Doekson , G. and Oehrtman, R.(1976). Optimum locations for a Rural Fire System: A Study of a Major County in Oklahoma. Southern journal of Agricultural Economics, Vol. 12, pp 121-127.

Drezner Z. (1984). The p-centre problem-heuristic and optimal algorithms. J. Oper. Res. Vol. 8, pp 741-748.

Drezner, Z. and Hamacher, H. W. (2002).Facility Location Application and theory, Springer, Berlin.

Dyer, M. E. and Frieze, A. M. (1985).A simple heuristics for the p-centre problem. Operation Research. Lett., Vol. 3, pp 285-288.

Elloumi S., (2004). A new formulation and resolution method for the p-centre problem. INFORMS J. Comput Vol. 16, pp 84-94.

Aidoo E., (2007). Location of Semi- obnoxious facility on KNUST Campus.MSc. Thesis, Kwame Nkrumah University of Science and Technology Kumasi – Ghana.

Goldberg, D. E. (1989). Genetic Algorithms in Search, Optimization and Machine Learning. Addison – Wesley, Mass.

Goldman, A. (1991). Optimal centre location in Simple Networks. Transportation Science, Vol. 5, pp 212-221.

Gonzalez T. (1985). Clustering to minimize the maximum intercluster distance. In TheoretComput Science, Vol. 38, pp 293- 306.

Hakimi, S. L. (1964). Optimum location of switching centres and the absolute centres and medians of a graph. Operations Research, Vol. 12, pp 450- 459.

Hamacher, H. W. and Nickel, S. (1998). Classification of location models. Location Science, Vol. 6, pp 229-242.

Handler, G. (1973).Minimax location of facility in an undirected tree network. Transportation Science, Vol. 7, pp 287-293.

Her Majesty's (HM) Fire Service Inspectorate [1999]. Fire Service Manual Volume 2 (Fire Operation).

Hogg, J. (1973). The Sitting of Fire Stations. Operational Research Quarterly, Vol. 19, pp 275-287.

Hongzhong J., (2005). A modeling Framework for Facility Location of Medical Services for Large-scale Emergencies.

Iihan, T., (2002).An efficient exact algorithm for the vertex p-centre problem and computational experiments for different set covering sub- problems. Technical report.

Klose, A. and Drexl, A. (2004). Facility location models for distribution system design. European Journal of Operational Research, Vol. 162 pp 4-29.

Kuehn, A. A. and Hamburger, M. J. (1963).A heuristic program for locating warehouses. Management Science, Vol. 9, pp 643-666.

Lunday J., (2005). Algorithm for solving the conditional covering problems on paths.

Marianov, and V. ReVelle, C. (1992). The capacitated standard response fire protection sitting problem:

Marianov, V. and ReVelle, C. (1995). Siting Emergency Services in Facility Location: A survey of Applications and Methods (Z. Drezner, ed). Springer, N Y pp 199 223.

Michalewic Z. and Fogel, D. B. (2000a). How to solve it: Modern Heuristics, pp. 111-134. Springer-Verlag, Berlin Heidelberg, New York.

Mirchandani, P . B. and Francis, R.L. (1990). Discrete Location Theory .John Wiley and Sons, Inc., New York.

Monks J., (1996). Operations Management pp 82-103.

Nan Liu., (2006). Optimal sitting of Fire Stations using GIS and ANT Algorithm. Journal of computing in Civil Engineering, Vol. 20, pp 361- 369.

National Fire Prevention Association Standards 1710 and 1201.

Norman J., (1991). Fire Officers Handbook of Tactics.

Owen, S. H. and Daskin, M. S. (1998). Strategic facility location: A review. European Journal of operational Research. Vol. III, pp. 423-447.

Paluzzi, M. (2004).Testing a heuristic P- median location allocation model for sitting emergency service facilities. Paper presented at the Annual meeting of Association of American Geographers, Philadelphia, P. A.

Philip, M . A, (2010) Sitting of fire station (A Case study: Batama Sub-Metro).
MSc. Thesis, Kwame Nkrumah University of Science and Technology Kumasi-Ghana.

Plane, D. and Hendrick, T. (1977).Mathematical Programming and the Location of Fire Companies for the Denver Fire Department. Operations Research Vol. 25, pp 563-578.

Radke, J. (1995). Spatial Decision Support System for Urban/wild land Interface Fire Hazards. ES RI user conference papers and proceedings.

Repede, J. F. and Bernardo, J. J. (1994). Developing and validating a decision support system for locating emergency medical vehicles in Louisville, Kentucky. European Journal of Operational Research. Vol. 40, pp 58-69.

ReVelle, C. (1989). Review, extension and prediction in emergency service sitting models. European Journal of Operational Research Vol. 40, pp 58-69.

Schilling, D., (1979).The TEAM/FLEET Models for Simultaneous Facility and Equipment Siting. Transportation Science, Vol.167.

Suzuki, A. and Drezner, Z. (1996).The p-centre location problem in the area. Location Science Vol.4, pp 69-82.

Talwar (2002).A modeling framework for facility locating of medical service for large scale emergencies.

Toregas, C., (1971). The Location of Emergency Service Facilities. Operation Research, Vol. 19 pp 1363-1373.

Toregas, C. and ReVelle, C (1973). Binary Logic Solutions to a Class of Location Problems. Geographical Analysis, pp 145-155.

Tzeng, G. and Chen, Y.W (1999). The Optimal Location of Airport Fire Station: A fuzzy Multi-objective Programming and Revised Genetic Algorithm Approach. Transportation Planning and Technology, Vol. 23, pp 37-55.

Vijay, J. (1985). An algorithm for the p-centre problem in the plane. Trans. Sci, Vol. 19, pp 235-245.

Watson – Gandy, C.D.T. (1984). The multi-facility min-max Weber problem. Eur. J., Oper. Res., Vol. 18; pp 44-50.

APPENDIX I

Location on edge (n_1n_4)

Choosing n_1 as the origin, let p = n_1 and q = $n_4$such that $0 \leq x \geq c(p, q)$

Putting $n_i = n_1$ i.e. $i = 1$, then $d(p, n_i) = d(n_1, n_1) = 0$,

$d(q, n_1) = d(n_4, n_1) = 6$. and $d(p, q) = d(n_1, n_4) = 6$,

The resulting equation $y_1 = x$ and $y_2 = 12 - x$ when solved

$x = 12 - x$, $\Rightarrow x = 6$ (Kink point)

$y_1 = x$ $\quad 0 \leq x \leq 6$ …………….(i)

Putting $n_i = n_2$ i.e $i = 2$, then $d(p, n_i) = d(n_1, n_2) = 8$, $d(q, n_1) = d(n_4, n_1) = 10$

$y_1 = x + 8$ and $y_2 = 16 - x$ (when solved)

$x + 8 = 16 - x$, $\Rightarrow x = 4$(Kink point)

$y_1 = x + 8$ $\quad 0 \leq x \leq 4$……………(ii)

$y_2 = 16 - x$ $\quad 4 \leq x \leq 6$ …………(iii)

Putting $n_i = n_3$ i.e $i = 3$,then $d(p, n_i) = d(n_1, n_3) = 8$, $d(q, n_i) = d(n_4, n_3) = 12$.

$x + 8 = 18 - x$, $\Rightarrow x = 5$ (Kink point)

$y_1 = x + 8$ $\quad 0 \leq x \leq 5$ …………..(iv)

$y_2 = 18 - x$ $\quad 5 \leq x \leq 6$ …………(v)

Putting $n_i = n_4$ i.e $i = 4$, then $d(p, n_i) = d(n_1, n_4) = 6$, $d(q, n_i) = d(n_4, n_4) = 0$

$x + 6 = 6 - x$, $\Rightarrow x = 0$ (Kink point)

$y_1 = 6 - x$ $\quad 0 \leq x \leq 6$ ………(vi)

Putting $n_i = n_5$ i.e $i = 5$ then, $d(p, n_i) = d(n_1, n_5) = 8$, $d(q, n_i) = d(n_4, n_5) = 4$

$y_1 = x + 2$ and $y_2 = 10 - x$ when solved

$x + 2 = 10 - x$, $\Rightarrow x = 4$(Kink point)

$y_1 = x + 2 \qquad 0 \leq x \leq 4$ ………….(vii)

$y_2 = 10 - x \qquad 0 \leq x \leq 6$ ………..(viii)

Location on edge (n_1, n_5)

Choosing $n_i = p$ and $n_5 = q$ such that $c(p, q) = c(n_1, n_5) = 2$

Putting $n_i = n_1$ i.e $i = 1$, then $d(p, n_i) = d(n_1, n_1) = 0$, $d(q, n_i) = d(n_5, n_1) = 2$.

$y_1 = x$ and $y_2 = 4 - x$ when solved

$x = 4 - x$, $\Rightarrow x = 2$ (Kink point)

$y_1 = x \qquad 0 \leq x \leq 2$ ………..(i)

Putting $n_i = n_2$ i.e $i = 2$, then $d(p, n_i) = d(n_1, n_2) = 8$, $d(q, n_i) = d(n_5, n_2) = 6$

$y_1 = x + 8$ and $y_2 = 8 - x$ when solved

$x + 8 = 8 - x$, $\Rightarrow x = 0$ (Kink point)

$y_2 = 8 - x \qquad 0 \leq x \leq 2$ …………..(ii)

Putting $n_i = n_3$ i.e $i = 3$, then $d(p, n_i) = d(n_1, n_3) = 8$, $d(q, n_i) = d(n_5, n_3) = 8$

$y_1 = x + 8$ and $y_2 = 10 - x$ when solved

$x + 8 = 10 - x$, $\Rightarrow x = 1$ (Kink point)

$y_1 = x + 8 \qquad 0 \leq x \leq 1$ …….(iii)

$y_2 = 10 - x \qquad 0 \leq x \leq 2$ ……..(iv)

Putting $n_i = n_4$ i.e $i = 4$, then $d(p, n_i) = d(n_1, n_4) = 6$, $d(q, n_i) = d(n_5, n_4) = 4$.

$y_1 = x + 6$ and $y_2 = 6 - x$ when solved

$x + 6 = 6 - x$, $\Rightarrow x = 0$ (Kink point)

$y_2 = 6 - x \qquad 0 \leq x \leq 2$ ……..(v)

Putting $n_i = n_5$ i.e $i = 5$, then $d(p, n_i) = d(n_1, n_5) = 2$, $d(q, n_i) = d(n_5, n_5) = 0$

$y_1 = x + 2$ and $y_2 = 2 - x$ when solved

$x + 2 = 2 - x$, $\Rightarrow x = 0$ (Kink point)

$y_2 = 2 - x \qquad 0 \leq x \leq 2$ ……..(vi)

Location on edge (n_4,n_5)

Choosing $n_4 = p$ and $n_5 = q$ such that $c(p, q) = c(n_4, n_5) = 4$

Putting $n_i = n_1$ i.e $i = 1$, then $d(p, n_i) = d(n_4, n_1) = 6$,$d(q, n_i) = d(n_5, n_1) = 2$.

$y_1 = x + 6$ and $y_2 = 6 - x$ when solved

$x + 6 = 6 - x$, $\Rightarrow x = 0$ (Kink point)

$y_2 = 6 - x \qquad 0 \leq x \leq 4$ ………(i)

Putting $n_i = n_2$ i.e $i = 2$, then $d(p, n_i) = d(n_4, n_2) = 10$, $d(q, n_i) = d(n_5, n_2) = 6$

$y_1 = x + 10$ and $y_2 = 10 - x$ when solved

$x + 10 = 10 - x$,$\Rightarrow x = 0$ (Kink point)

$y_2 = 10 - \mathrm{x} \qquad 0 \leq x \leq 4$ …(ii)

Putting $n_i = n_3$ i.e $i = 3$, then $d(p, n_i) = d(n_4, n_3) = 12$, $d(q, n_i) = d(n_5, n_3) = 8$

$y_1 = x + 12$ and $y_2 = 12 - x$ when solved

$\mathrm{x} + 12 = 12 - \mathrm{x}$, $\Rightarrow \mathrm{x} = 0$ (Kink point)

$y_2 = x - 12 \qquad n_4 0 \leq x \leq 4$ …….(iii)

Putting $n_i = n_4$ i.e $i = 4$, then $d(p, n_i) = d(n_4, n_4) = 0$, $d(q, n_i) = d(n_5, n_4) = 4$

$y_1 = x$ and $y_2 = 8 - x$ when solved

$x = 8 - x$, $\Rightarrow x = 4$ (Kink point)

$y_1 = x \qquad 0 \leq x \leq 4$……(iv)

Putting $n_i = n_5$ i.e $i = 5$, then d(p, ni) = d(n_4, n_5) = 4, d(q, n_i) =d(n_5, n_5) =0

$y_1 = x + 4$ and $y_2 = 4 – x$ when solved

$x + 4 = 4 – x$, =>$x = 0$ (Kink point)

$y_2 = 4 – x \qquad 0 \leq x \leq 4$ …..(v)

Location on edge (n_5, n_2)

Choosing $p = n_5$ and $q = n_2$ such that c(p, q) = c(n_5, n_2) = 6

Putting $n_i = n_1$ i.e $_i$= 1, then d(p, n_i) = d(n_5, n_1) = 2, d(q, n_i) = d(n_2, n_1) = 8

$y_1 = x + 2$ and $y_2 = 14 – x$ when solved

$x + 2 = 2 – x$, => $x = 0$ (Kink point)

$y_2 = 2 – x \qquad 0 \leq x \leq 2$ ……(i)

Putting $n_i = n_3$ i.e i= 3, then d(p, n_i) = d(n_3, n_3) = 0, d(q, n_i) = d(n_2, n_3)=2.

$y_1 = x$ and $y_2 = 4 – x$ when solved

$x = 4 –x$, => $x = 2$ (Kink point)

$y_1 = x \qquad 0 \leq x \leq 2$ …….(ii)

Putting $n_i = n_4$ i.e i= 4, then d(p,n_i) =d(n_3, n_4) = 12, d(q, n_i) d(n_2, n_4) =10

$y_1 = x +12$ and $y_2 = 12 – x$ (when solved)

$x + 12 = 12 – x$, => $x = 0$ (Kink point)

$y_2 = 12 – x \qquad 0 \leq x \leq 2$……(iii)

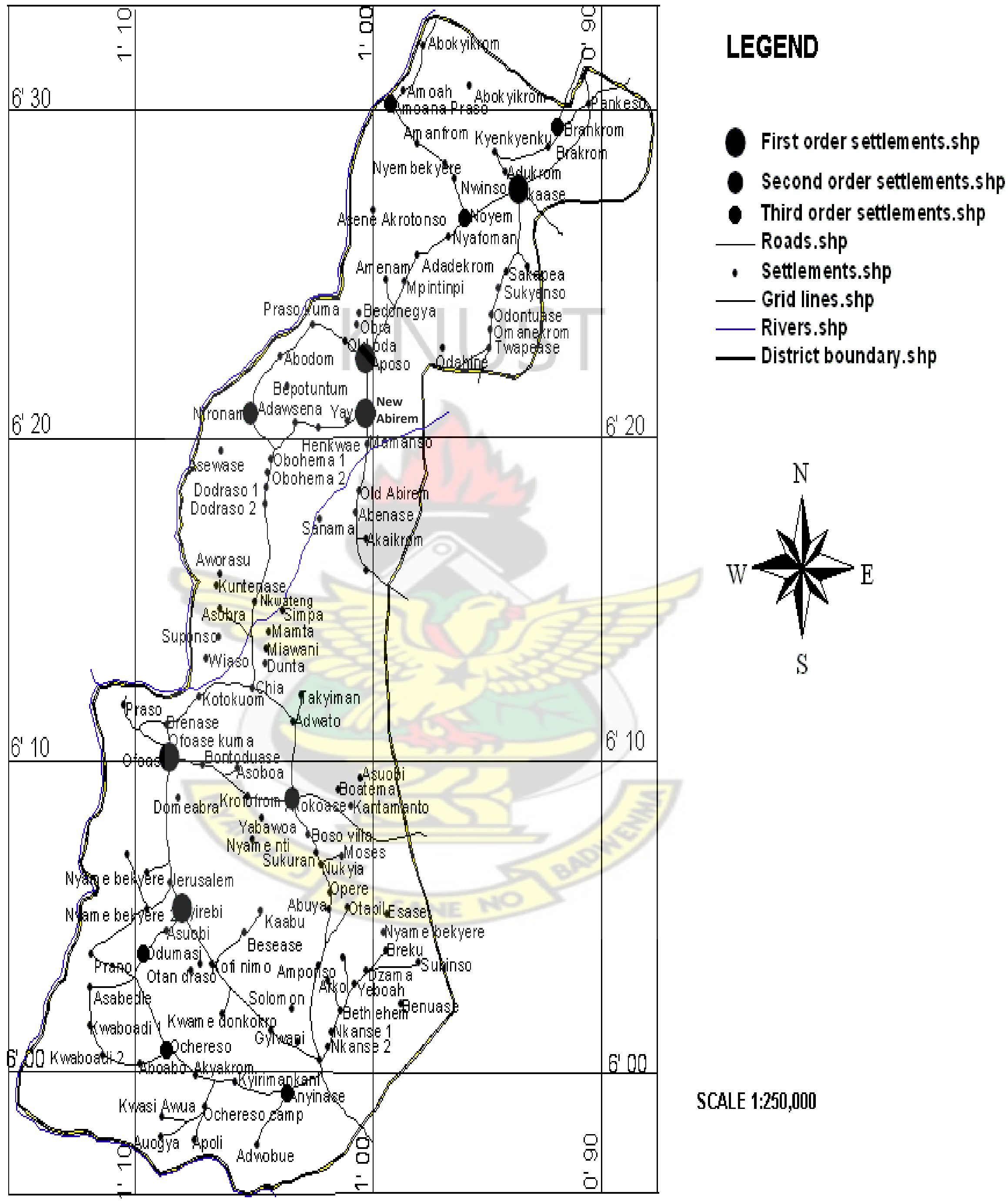
Putting $n_i = n_5$ i.e i= 5 , then d(p,n_i) = d(n_3, n_5) = 8, d(q, n_i) = d(n_2, n_5) = 6.

$y_1 = x+ 8$ and $y_2 = 8 – x$ when solved

$x + 8 = 8 – x$, => $x = 0$(Kink point)

$y_2 = 8 - x \qquad 0 \leq x \leq 2$ ……(iv)

APPENDIX II


LEGEND
First order settlements.shp
Second order settlements.shp
Third order settlements.shp
Roads.shp
Settlements.shp
Grid lines.shp
Rivers.shp
District boundary.shp
N
W
E
S
SCALE 1:250,000
6' 30
6' 20
6' 10
6' 00
1' 10
1' 00
0' 90
Abokyikrom
Amoah
Amoana Praso
Pankese
Amanfrom
Kyenkyenku
Brankrom
Brakrom
Nyembekyere
Adukrom
Nwinso
Akrotonso
Noyem
Nyafoman
Amenam
Adadekrom
Mpintinpi
Sukyenso
Praso Kuma
Beddnegya
Obra
Odontuase
Omanekrom
Twapease
Abodom
Aposo
Odahene
Bepotuntum
Adawsena
New Abirem
Henkwae
Obohema 1
Obohema 2
Asewase
Dodraso 1
Dodraso 2
Old Abirem
Abenase
Sanama
Akaikrom
Aworasu
Kuntenase
Nkwateng
Asubra
Simpa
Mamta
Miawani
Dunta
Wiaso
Chia
Kotokuom
Takyiman
Adwato
Praso
Brenase
Ofoase kuma
Bontoduase
Asoboa
Asuobi
Boatema
Domeabra
Krofofrom
Kantamanto
Yabawoa
Boso villa
Sukuran
Moses
Opere
Jerusalem
Abuya
Otapil
Esase
Kaabu
Asuobi
Besease
Nyame bekyere
Breku
Odumasi
Prano
Otan draso
Kofi nimo
Amponso
Dzama
Yeboah
Asabedie
Solomon
Bethlehem
Benuase
Kwaboadi 1
Kwame donkokro
Nkanse 1
Nkanse 2
Ochereso
Kwaboadi 2
Aboabo
Akyakrom
Kyirimankam
Anyinase
Kwasi Awua
Ochereso camp
Auogya
Apoli
Adwobue

APPENDIX III

A FLOWCHART FOR DETERMING ABSOLUTE CENTER OF A GIVEN NETWORK

Consider an edge from a given network

Use the Floyd Warshall's algorithm to obtain a matrix of the shortest paths between all pairs of nodes in the network

For an edge denote its vertices as p and q $where$ p is the origin and distance from x to q i.e. $d(q,x) = c(p,q) - x$

Choose a point x on the edge (p,q) such that the distance from p to x ie $d(p,x) = x$ and the distance x to q ie $d(q,x) = c(p,q) - x$

From the point x move through the vertices p and q to a vertex/node $n_{i=}1,2,3 \ldots k$ $where$ n_i is each of the vertices/nodes on the network under consideration and k is the number of vertices.
This will give rise to two equations: $y_1 = x + d(p,n_i) and$ $y_2 = c(p,q) - x + d(q,n_i)$ $where$ y_1 is the distance from x through p to n_i and y_2 is the distance from x through q to n_i
As the point x continues to move there will be a time that $y_1 = y_2$ and this will be the kink point for the equations above. The distances $c(p,q), d(p,n_1) and$ $d(q,n_1)$ shall be obtained from the matrix generated by the Floyd's warshall's algorithm from the network in step 1

If $n_i = p$ then the two equations shall be
$y_1 = x + d(p,p) and$ $y_2 = c(p,q) - x = d(q,p)$
but $d(p,q) = 0$ and $d(q,p) = c(p,q)$
Hence $y_1 = x$ and $y_2 = 2[c(p,q)] - x$
When $y_1 = y_2$, the Kink point shall be $x = c(p,q)$

I f $n_i = p$ then the two equations shall be
$y_1 = x + d(p,p) and$ $y_2 = c(p,q) - x = d(q,p)$
but $d(p,q) = 0$ and $d(q,p) = c(p,q)$
Hence $y_1 = x$ and $y_2 = 2[c(p,q)] - x$
When $y_1 = y_2$, the Kink point shall be $x = 0$

Draw the graph of $y_1 = x$ and $y_2 = c(p,q) - x$ in the interval $0 \leq x \leq c\ (p,q)$ on the same axes

Accept equation
$y_1 = x$

Accept the equation
$y_2 = c(p,q) - x$

Set n_i to each of the vertices/nodes on the network except p and q which are nodes of the edge under consideration to yield equation y_1 and y_2 as indicated in the step 4 above. These two equations for each n_i considered are then drawn on the same axes in the specified intervals obtained when y_1=y_2. The upper envelope is then constructed and $m(x_1)$, the minimum point on it identified.

Choose another edge which has not been considered and go through the whole process again

APPENDIX IV

$$\text{Edge } (K, C): \frac{m\,(K) + m(C) - c\,(K, C)}{2} = \frac{8530 + 4750 - 3790}{2} = 4745 = (4745)$$

$$\text{Edge } (K, A): \frac{m\,(K) + m\,(A) - c\,(K, A)}{2} = \frac{8530 + 7330 - 1210}{2} = 7325 > (4745)$$

$$\text{Edge } (A, N): \frac{m\,(A) + m\,(N) - c\,(A, N)}{2} = \frac{7330 + 8650 - 1330}{2} = 7325 > (4745)$$

$$\text{Edge } (A, B): \frac{m\,(A) + m\,(B) - c\,(A, B)}{2} = \frac{7330 + 6100 - 1660}{2} = 5885 > (4745)$$

$$\text{Edge } (A, C): \frac{m\,(A) + m(C) - c\,(A, C)}{2} = \frac{7330 + 4750 - 2590}{2} = 4745 = (4745)$$

$$\text{Edge } (B, R): \frac{m\,(B) + m(R) - c\,(B, R)}{2} = \frac{6100 + 9490 - 3400}{2} = 6095 > (4745)$$

$$\text{Edge } (B, C): \frac{m\,(B) + m(C) - c\,(B, C)}{2} = \frac{6100 + 4750 - 360}{2} = 4745 = (4745)$$

$$\text{Edge } (C, O): \frac{m(C) + m\,(O) - c(C, O)}{2} = \frac{4750 + 9490 - 4150}{2} = 5045 > (4745)$$

$$\text{Edge } (C, P): \frac{m(C) + m\,(P) - c(C, P)}{2} = \frac{4750 + 6100 - 1360}{2} = 4745 = (4745)$$

$$\text{Edge } (C, H): \frac{m(C) + m(H) - c(C, H)}{2} = \frac{4750 + 7690 - 2950}{2} = 4745 = (4745)$$

$$\text{Edge } (C, F): \frac{m(C) + m(F) - c(C\,, F)}{2} = \frac{4750 + 7060 - 2310}{2} = 4745 = (4745)$$

Edge (C, I : $\frac{m(C) + m(I) - c(C, I)}{2} = \frac{4750 + 5890 - 1150}{2} = 4745 = (4745)$

Edge (C, D): $\frac{m(C) + m(D) - c(C, D)}{2} = \frac{4750 + 6430 - 1690}{2} = 4745 = (4745)$

Edge (D, E): $\frac{m(D) + m(E) - c(D, E)}{2} = \frac{6430 + 7690 - 1270}{2} = 6425 > (4745)$

Edge (D, I): $\frac{m(D) + m(I) - c(D, I)}{2} = \frac{6430 + 5890 - 2230}{2} = 5045 > (4745)$

Edge (D, M): $\frac{m(D) + m(M) - c(D, M)}{2} = \frac{6430 + 6730 - 2080}{2} = 5545 > (4745)$

Edge (E, Q): $\frac{m(E) + m(Q) - c(E, Q)}{2} = \frac{6100 + 9490 - 3400}{2} = 6100 > (4745)$

Edge (B, C): $\frac{m(B) + m(C) - c(B, C)}{2} = \frac{7690 + 8890 - 1210}{2} = 7690 > (4745)$

Edge (H, F): $\frac{m(H) + m(F) - c(H, F)}{2} = \frac{7690 + 7060 - 2890}{2} = 5930 > (4745)$

Edge (F, S): $\frac{m(F) + m(S) - c(F, S)}{2} = \frac{7060 + 7420 - 370}{2} = 7055 > (4745)$

Edge (G, F): $\frac{m(G) + m(F) - c(G, F)}{2} = \frac{6880 + 7060 - 1090}{2} = 6425 > (4745)$

Edge (G, U): $\frac{m(G) + m(U) - (G, U)}{2} = \frac{6880 + 7570 - 2680}{2} = 5885 > (4745)$

Edge (G, I) : $\frac{m(G) + m(I) - c(G, I)}{2} = \frac{6880 + 5890 - 990}{2} = 5885 > (4745)$

Edge (U, J) : $\frac{m(U) + m(J) - (U, J)}{2} = \frac{7570 + 6970 - 610}{2} = 6965 > (4745)$

Edge (J, I) : $\frac{m(J) + m(I) - c(J, I)}{2} = \frac{6970 + 5890 - 1090}{2} = 5885 > (4745)$

Edge (I, M) : $\frac{m(I) + m(M) - c(I, M)}{2} = \frac{5890 + 6730 - 840}{2} = 5885 > (4745)$

Edge (S, G) : $\frac{m(S) + m(G) - c(S, G)}{2} = \frac{7420 + 6880 - 660}{2} = 6820 > (4745)$

Edge (I , L) : $\frac{m(I) + m(L) - c(I, L)}{2} = \frac{5890 + 8230 - 2350}{2} = 5885 > (4745)$

Edge (F, I) : $\frac{m(F) + m(I) - c(F, I)}{2} = \frac{7060 + 5890 - 1810}{2} = 5575 > (4745)$

Edge (I, V) : $\frac{m(I) + m(V) - c(I, V)}{2} = \frac{5890 + 9070 - 3190}{2} = 5885 > (4745)$

APPENDIX V

Location on edge (A, C)

Let A = p, C = q such that $0 \leq x \leq c(p, q)$ and c(p, q) = c(A, C) =2590

Putting n_i= A, then d(p, n_i) = d(A, A) = 0, d (q, n_i) =d(C, A)=2590,

$y_1 = x$ and $y_2 = 5180 - x$ when solved

$x = 5180 - x \Rightarrow x = 2580$ (Kink point)

$y_1 = x \qquad 0 \leq x \leq 2580$.. (i)

Putting n_i= B, then d(p, n_i)= d(A, B) = 1660, d (q, n_i) = d(q, n_i) = d(C, B) = 1360,

$y_1 = x+1660$ and $y_2 = 3950 - x$ when solved

$x_1+1660 = 3950 - x \Rightarrow x=1140$ (Kink point)

$y_1=x+1660 \qquad 0 \leq x \leq 1140$.. (ii)

$y_1= 3930 - x \qquad 1140 \leq x \leq 2590$... (iii)

Putting n_i = C, then d(p, n_i) = d(A, C)= 2590, d(q ,n_i) = d(C, C) = 0,

$y_1=x + 2590$ and $y_2 = 2590 - x$ when solved

$x + 2590 = 2590 - x, \Rightarrow x = 0$ (Kink point)

$y_2 = 2590 - x \qquad 0 \leq x \leq 2590$... (iv)

Putting n_i = D, then d(p, n_i) = d(A, D) = 4270, d(q, n_i) = d(C, D) = 1690,

$y_1 = x + 4270$ and $y_2 = 4270 - x$ when solved

$x + 4270 = 4270 - x \Rightarrow x = 0$ (Kink point)

$y_2 = 4270 - x \qquad 0 \leq x \leq 2590$.. (v)

Putting n_i = E, then d(p, n_i) = d(A, E)= 5530, d(q, n_i) = d(C, E) = 2950,

$y_1 = x + 5530$ and $y_2 = 5530 - x$ when solved

$x + 5530 = 5530 - x \Rightarrow x = 0$ (Kink point)

$y_2 = 5530 - x \qquad 0 \leq x \leq 2590$.. (vi)

Putting n_i= F , then d(p, n_i) = d(A, F) = 4900,d(q, n_i) = d(C, F) = 2320,

$y_1 = x + 4900$ and $y_2 = 4900 - x$ when solved

$x + 4900 = 4900 - x, \Rightarrow x = 0$ (Kink point)

$y_2 = 4900 - x \qquad 0 \leq x \leq 2590$... (vii)

Putting n_i= G, then $d(p, n_i)$ $d(A, G)$ =4720, $d(q, n_i) = d(C, G) = 2140$,

$y_1 = x + 4720$ and $y_2 = 4720 – x$ when solved

$x + 4720 = 4720 – x$, => $x = 0$ (Kink point)

$y_2 = 4720 – x$ $0 \leq x \leq 2590$.. (viii)

Putting n_i= H then $d(p, n_i) = d(A, H) = 5530$, $d(q, n_i) = (C, H) = 2950$,

$y_1 = x + 5530$ and y2 $5530 – x$ when solved

$x + 5530 = 5530 – x$, => $x = 0$ (Kink point)

$y_2 = 5530 – x$ $0 \leq x \leq 2590$.. (ix)

Putting n_i = I, then $d(p, n_i) = d(A, I) = 3730$,$d(q, n_i) = d(C, I) = 1150$,

$y_1 = x + 3730$ and $y_2 = 3730 – x$ when solved

$x + 3730 = 3730 – x$, => $x = 0$ (Kink point)

$y_2 = 3730 –x$ $0 \leq x \leq 2590$.. (x)

Putting n_i = J, then $d(p, n_i) = d(A, J) = 4810$, $d(q, n_i) = d(C, J) = 2230$,

$y_1 = x + 4810$ and $y_2 =4810 – x$ when solved

$x + 4810 = 4810 – x$, => $x = 0$ (Kink point)

$y_2 = 4810 – x$ $0 \leq x \leq 2590$.. (xi)

Putting n_i = K, then $d(p, n_i) = d(A, K) = 1210$, $d(q, n_i) = d(C, K) = 3790$,

$y_1 = x + 1210$ and $y_2 = 6370 – x$ when solved

$x + 1210 = 6370 – x$,=> $x = 2590$ (Kink point)

$y_1 = x + 1210$ $0 \leq x \leq 2590$.. (xii)

Putting n_i = L, then $d(p, n_i) = d(A, L) = 6070$, $d(q, n_i) = d(C, L) = 3490$,

$y_1 = x +6070$ and $y_2 = 6070 – x$ when solved

$x + 6070 = 6070 - x$, => $x = 0$ (Kink point)

$y_2 = 6070 – x$ $O \leq x \leq 2590$.. (xiii)

Putting n_i = M, then $d(p, n_i) = d(A, M) = 4570$, $d(q, n_i) = d(C, M) = 1990$,

$y_1 = x + 4570$ and $y_2 = 4570 - x$ when solved

$x + 4570 = 4570 - x$, $\Rightarrow x = 0$ (Kink point)

$y_2 = 4570 - x \qquad 0 \leq x \leq 2590$.. (xiv)

Putting n_i = N, then $d(p, n_i) = d(p, n_i) = 1330$, $d(q, n_i) = d(C,N) = 3910$,

$y_1 = x + 1330$ and $y_2 = 6490 - x$ *when* solved

$x + 1330 = 6490 - x$, $\Rightarrow x = 2590$ (Kink point)

$Y_1 = x + 1330 \quad 0 \leq x \leq 2590$.. (xv)

Putting n_i =0, then $d(p, n_i) = d(A, O) = 7330$, $d(q, n_i) = d(C, O) = 6100$,

$y_1 = x + 7330$ and $y_2 = 8680 - x$ when solved

$x + 7330 = 8670 - x$, $\Rightarrow x = 675$ (Kink point)

$y_1 = x + 7330 \qquad 0 \leq x \leq 67$..(xvi)

$y_2 = 8680 - x \qquad 675 \leq x \leq 2590$..(xvii)

Putting n_i = P, then $d(p, n_i) = d(A, P) = 3940$, $d(q, n_i) = d(C, P) = 1360$,

$y_1 = x + 3940$ and $y_2 = 3940 - x$ when solved

$x + 3940 = 3940 - x$, $\Rightarrow x = 0$ (Kink point)

$y_2 = 3940 - x \qquad 0 \leq x \leq 2590$ ....(xviii)

$y_2 = 8680 - x \qquad 675 \leq x \leq 2590$ ....(xix)

Putting n_i = Q, then $d(P, n_i) = d(A, Q) = 6730$, $d(q, n_i) = d(C, Q) = 4150$,

$y_1 = x + 6730$ and $y_2 = 6730 - x$ when solved

$x + 6730 = 6730 - x \qquad \Rightarrow x = 0$ (Kink point)

$y_2 = 6720 - x \qquad 0 \leq x \leq 2590$.. (xix)

Putting n_i = R, then $d(p, n_i) = d(A, R) = 5050$, $d(q, n_i) = d(C, R) = 4745$,

$y_1 = x + 5050$ and $y_2 = 7330 - x$ when solved

$x + 5050 = 7330 - x \Rightarrow x = 1440$ (Kink point)

$y_1 = x + 5050 \qquad 0 \leq x \leq 2590$.. (xx)

$y_2 = 7330 - x \qquad 0 \leq x \leq 2590$.. (xxi)

Putting n_i =S, then d(p, n_i,) = d(A, S) = 5260, d(q, n_i) = d(C, S) = 2680,

$y_1 = x + 5260$ and $y_2 = 5260 - x$ when solved

$x + 5260 = 5260 - x$, $\Rightarrow x = 0$ (Kink point)

$y_2 = 5260 - x \quad 0 \leq x \leq 2590$.. (xxii)

Putting n_i = T, then d(p, n_i) =d(A, T) = 6160, d(q, n_i) = d(C, T) =3580,

$y_1 = x + 6160$ and $y_2 = 6160 - x$ when solved

$x + 6160 = 6160 - x$, $\Rightarrow x = 0$ (Kink point)

$y_2 = 6160 - x \quad 0 \leq x \leq 2590$.. (xxiii)

Putting n_i,=U then d(p, n_i) = d(A, U) = 5410,d(q, n_i) = d(C, U) = 2830,

$y_1 = x + 5410$ and $y_2 = 5410 - x$ when solved

$x + 5410 = 5410 - x$, $\Rightarrow x = 0$ (Kink point)

$y_2 = 5410 - x \quad 0 \leq x \leq 2590$.. (xxiv)

Putting n_i = V, then d(p, n_i) =d(A, V) = 6910, d (q, n_i) = d(C, V) =4330,

$y_1 = x + 6910$ and $y_2 = 6910 - x$ when solved

$x + 6910 = 6910 - x$, $\Rightarrow x = 0$ (Kink point)

$y_2 = 6910 - x \quad 0 \leq x \leq 2590$.. (xxv)

<u>Location on edge (B C)</u>

Let B = p, C = q such that $0 \leq x \leq c(p, q)$ and c(B, C)=1350

Putting n_i = A, then d (p, n_i) =d(B, A) = 1660, d(q, n_i) = d(C, A) =2590,

$y_1 = x + 1660$ and $y_2 = 3940 - x$ when solved

$x + 1660 = 3940 - x$ $\Rightarrow x = 1140$ (Kink point)

$y_1 = x + 1660 \quad 0 \leq x \leq 1140$.. (i)

$y_2 = 3940 - x \quad 1140 \leq x \leq 1350$.. (ii)

Putting n_i = B then $d(p, n_i) = d(B, B) = 0$, $d(q, n_i) = d(C, B) = 1350$,

$y_1 = x$ and $y_2 = 2710 - x$ when solved

$x = 2710 - x$ $\Rightarrow x = 1360$ (Kink point)

$y_1 = x$ $0 \leq x \leq 1350$.. (iii)

Putting n_i = C, then $d(p, n_i) = d(B, C) = 1350$, $d(q, n_i) = d(C, C) = 0$,

$y_1 = x + 1350$ and $y_2 = 1350 - x$ when solved

$x + 1350 = 1350 - x \Rightarrow x = 0$ (Kink point)

$y_2 = 1350 - x$ $0 \leq x \leq 1350$.. (iv)

Putting n_i = D, then $d(p, n_i) = d(B, D) = 3040$, $d(q, n_i) = d(C, D) = 1690$,

$y_1 = x + 3040$ and $y_2 = 3040 - x$ when solved

$x + 3040 = 3040 - x$, $\Rightarrow x = 0$ (Kink point)

$y_2 = 3040 - x$ $0 \leq x \leq 1350$ ....(v)

Putting n_i = E, then $d(p, n_i) = d(B, E) = 4300$, $d(q, n_i) = d(C, E) = 2950$,

$y_1 = x + 4300$ and $y_2 = 4300 - x$ when solved

$x + 4300 = 4300 - x$, $\Rightarrow x = 0$ (kink point)

$y_2 = 4300 - x$ $0 \leq x \leq 1350$ ....(vi)

Putting n_i =F, then $d(p, n_i) = d(B, F) = 3670$, $d(q, n_i) = d(C, F) = 2320$,

$y_1 = x + 3670$ and $y_2 = 3670 - x$ when solved

$x + 3670 = 3670 - x$, $\Rightarrow x = 0$ (Kink point)

$x_2 = 3670 - x$ $0 \leq x \leq 1350$ ....(vii)

Putting n_i = G, then $d(p, n_i) = d(B, G) = 3490$, $d(p, n_i) = d(C, G) = 2140$,

$y_1 = x + 3490$ and $y_2 = 3490 - x$ when solved

$x + 3490 = 3490 - x$, $\Rightarrow x = 0$ (Kink point)

$y_1 = 3490 - x$ $0 \leq x \leq 1350$.. (viii)

Putting n_i = H, then d (p, n_i) = d(B, H) =4300, d (q, n_i) = d(C, D) = 2950,

$y_1 = x + 4300$ and $y_2 = 4300 - x$ when solved

$x + 4300 = 4300 - x, \Rightarrow x = 0$ (Kink point)

$y_2 = 4300 - x \quad 0 \leq x \leq 1350$ .... (ix)

Putting n_i = I, then d(p, n_i) = d (B, I) = 2500, d(q, n_i) =d(C, I) = 1150,

$y_1 = x + 2500$ and $y_2 = 2500 - x$ when solved

$x + 2500 = 2500 - x, \Rightarrow x = 0$ (Kink point)

$y_2 = 2500 - x \quad 0 \leq x \leq 1350$ .... (x)

Putting n_i = J, then d (p, n_i) = d (B, J) = 3580, d (q, n_i) = d(C, J) = 2230,

$y_1 = x + 3580$ and $y_2 = 3580 - x$ when solved

$x + 3580 = 3580 - x, \Rightarrow x = 0$ (Kink point)

$y_2 = 3580 - x \quad 0 \leq x \leq 1350$ .... (xi)

Putting n_i = Q, then d(p, n_i) = d(B, Q) = 5500,d (q, n_i) = d(C, Q) = 4150,

$y_1 = x + 5500$ and $y_2 = 5500 - x$ when solved

$x + 5500 = 5500 - x, \Rightarrow x = 0$ (Kink point)

$y_2 = 5500 - x \quad 0 \leq x \leq 1350$ .... (xii)

Putting n_i = R, then d(p, n_i) = d(B, R) = 3400, d (q, n_i) = d(C, R) = 4745,

$y_1 = x + 3400$ and $y_2 = 6100 - x$ when solved

$x + 3400 = 6100 - x, \Rightarrow x = 0$ (Kink point)

$y_1 = x + 3400 \quad 0 \leq x \leq 1350$ .... (xiii)

Putting n_i = S, then d (p, n_i) = d (B, S) = 4030, d (q, n_i) = d (C, S) =2680,

$y_1 = x + 4030$ and $y_2 = 4030 - x$ when solved.

$x + 4030 = 4030 - x, \Rightarrow x = 0$ (Kink point)

$y_2 = x + 4030 \quad 0 \leq x \leq 1350$.. (xiv).

Putting n_i = T, then d (p, n_i) = d (B, T) = 4930, d (q, n_i) = d (C, T) =3580,

$y_1 = x + 4930$ and $y_2 = 4930 - x$ when solved.

$x + 4930 = 4930 - x$, => $x = 0$ (Kink point)

$y_2 = x + 4930 \quad 0 \leq x \leq 1350$.. (xv).

Putting n_i = U, then d (p, n_i) = d (B, U) = 4180, d (q, n_i) = d (C, U) =2830,

$y_1 = x + 41880$ and $y_2 = 4180 - x$ when solved.

$x + 4180 = 4180 - x$, => $x = 0$ (Kink point)

$y_2 = x + 4180 \quad 0 \leq x \leq 1350$.. (xvi).

Putting n_i = V, then d (p, n_i) = d (B, V) = 5680, d (q, n_i) = d (C, V) =4330,

$y_1 = x + 5680$ and $y_2 = 4330 - x$ when solved.

$x + 5680 = 5680 - x$, => $x = 0$ (Kink point)

$y_2 = x + 5680 \quad 0 \leq x \leq 1350$.. (xvii).

<u>Location of edge on (C, P)</u>

Let C = p, P = q such that $0 \leq x \leq c$ (p, q) and c (p, q) = c (C, P) = 1350.

Putting n_i = A, then d (p, n_i) = d (C, A) = 2590, d (q, n_i) = d (P, A) = 3940,

$y_1 = x + 2590$ and $y_2 = 5290 - x$ when solved.

$x + 2590 = 5290 - x$, => $x = 1350$ (Kink point)

$y_1 = x + 2590 \quad 0 \leq x \leq 1350$.. (i)

Putting n_i = B, then d (p, n_i) = d (C, B) = 1360, d (q, n_i) = d (P, B) = 2710,

$y_1 = x + 1360$ and $y_2 = 4060 - x$ when solved.

$x + 1360 = 4060 - x$, => $x = 1350$ (Kink point)

$y_1 = x + 1360 \quad 0 \leq x \leq 1350$.. (ii).

Putting n_i = C, then d (p, n_i) = d (C, C) = 0, d (q, n_i) = d (P, C) = 1360,

$y_1 = x$ and $y_2 = 2710 - x$ when solved.

$x + 2710 - x$, => $x = 1350$ (Kink point)

$y_1 = x \quad 0 \leq x \leq 1350$... (iii).

Putting $n_i = D$, then $d(p, n_i) = d(C, D) = 1690$, $d(q, n_i) = d(P, D) = 3040$,

$y_1 = x + 1690$ and $y_2 = 4390 – x$ when solved.

$x + 1690 = 4390 – x$, => $x = 1350$ (Kink point)

$y_1 = x + 1690 \quad 0 \leq x \leq 1350$.. (iv).

Putting $n_i = E$, then $d(p, n_i) = d(C, E) = 2950$, $d(q, n_i) = d(P, E) = 4300$,

$y_1 = x + 2950$ and $y_2 = 5650 – x$ when solved.

$x + 2950 = 5650 – x$, => $x = 1350$ (Kink point)

$y_1 = x + 2950 \quad 0 \leq x \leq 1350$.. (v).

Putting $n_i = F$, then $d(p, n_i) = d(C, F) = 2320$, $d(q, n_i) = d(P, F) = 3670$,

$y_1 = x + 2320$ and $y_2 = 5020 – x$ when solved.

$x + 2320 = 5020 – x$, => $x = 1350$ (Kink point)

$y_1 = x + 2320 \quad 0 \leq x \leq 1350$.. (vi).

Putting $n_i = G$, then $d(p, n_i) = d(C, G) = 2140$, $d(q, n_i) = d(P, G) = 3490$,

$y_1 = x + 2320$ and $y_2 = 4840 – x$ when solved.

$x + 2320 = 4840 – x$, => $x = 1350$ (Kink point)

$y_1 = x + 2320 \quad 0 \leq x \leq 1350$.. (vii).

Putting $n_i = H$, then $d(p, n_i) = d(C, H) = 2950$, $d(q, n_i) = d(P, H) = 3490$,

$y_1 = x + 2950$ and $y_2 = 4840 – x$ when solved.

$x + 2950 = 4840 – x$, => $x = 945$ (Kink point)

$y_1 = x + 2950 \quad 0 \leq x \leq 945$... (viii).

$y_2 = 4840 – x \quad 945 \leq x \leq 1350$.. (ix)

Putting $n_i = I$, then $d(p, n_i) = d(C, I) = 1150$, $d(q, n_i) = d(P, I) = 2500$,

$y_1 = x + 1150$ and $y_2 = 3850 – x$ when solved.

$x + 1150 = 3850 – x$, => $x = 1350$ (Kink point)

$y_1 = x + 1150 \quad 0 \leq x \leq 1350$.. (x).

Putting $n_i = J$, then $d(p, n_i) = d(C, J) = 2230$, $d(q, n_i) = d(P, J) = 3580$,

$y_1 = x + 2230$ and $y_2 = 4930 – x$ when solved.

$x + 2230 = 4930 – x$, => $x = 1350$ (Kink point)

$y_1 = x + 2230 \quad 0 \leq x \leq 1350$.. (xi).

Putting $n_i = K$, then $d(p, n_i) = d(C, K) = 3790$, $d(q, n_i) = d(P, K) = 5140$,

$y_1 = x + 3790$ and $y_2 = 6490 – x$ when solved.

$x + 3790 = 6480 – x$, => $x = 1350$ (Kink point)

$y_1 = x + 3790 \quad 0 \leq x \leq 1350$.. (xii).

Putting $n_i = L$, then $d(p, n_i) = d(C, L) = 3490$, $d(q, n_i) = d(P, L) = 4840$,

$y_1 = x + 3490$ and $y_2 = 6190 – x$ when solved.

$x + 3490 = 6190 – x$, => $x = 1350$ (Kink point)

$y_1 = x + 3490 \quad 0 \leq x \leq 1350$.. (xiii).

Putting $n_i = M$, then $d(p, n_i) = d(C, M) = 1990$, $d(q, n_i) = d(P, M) = 3340$,

$y_1 = x + 1990$ and $y_2 = 4690 – x$ when solved.

$x + 1990 = 4690 – x$, => $x = 1350$ (Kink point)

$y_1 = x + 1990 \quad 0 \leq x \leq 1350$.. (xiv).

Putting $n_i = N$, then $d(p, n_i) = d(C, N) = 3910$, $d(q, n_i) = d(P, N) = 5260$,

$y_1 = x + 3910$ and $y_2 = 3610 – x$ when solved.

$x + 3910 = 6610 – x$, => $x = 1350$ (Kink point)

$y_1 = x + 3910 \quad 0 \leq x \leq 1350$.. (xv).

Putting $n_i = O$, then $d(p, n_i) = d(C, O) = 4745$, $d(q, n_i) = d(P, O) = 6100$,

$y_1 = x + 4745$ and $y_2 = 7450 – x$ when solved.

$x + 4745 = 7450 – x$, => $x = 1352.5$ (Kink point)

$y_1 = x + 4745 \quad 0 \leq x \leq 1352.5$.. (xvi).

Putting n_i = P, then $d(p, n_i) = d(C, P) = 1350$, $d(q, n_i) = d(P, P) = 0$,

$y_1 = x + 1350$ and $y_2 = 1350 – x$ when solved.

$x + 1350 = 1350 – x$, => $x = 0$ (Kink point)

$y_1 = x + 1350 \quad 0 \le x \le 1350$.. (xvii).

Putting n_i = Q, then $d(p, n_i) = d(C, Q) = 4150$, $d(q, n_i) = d(P, Q) = 5500$,

$y_1 = x + 4150$ and $y_2 = 6850 – x$ when solved.

$x + 4150 = 6850 – x$, => $x = 1350$ (Kink point)

$y_1 = x + 4150 \quad 0 \le x \le 1350$.. (xviii).

Putting n_i = R, then $d(p, n_i) = d(C, R) = 4745$, $d(q, n_i) = d(P, R) = 6100$,

$y_1 = x + 4745$ and $y_2 = 7450 – x$ when solved.

$x + 4745 = 7450 – x$, => $x = 1352.5$ (Kink point)

$y_1 = x + 4745 \quad 0 \le x \le 1352.5$.. (xix).

Putting n_i = S, then $d(p, n_i) = d(C, S) = 2680$, $d(q, n_i) = d(P, S) = 4030$,

$y_1 = x + 2680$ and $y_2 = 5350 – x$ when solved.

$x + 2680 = 5350 – x$, => $x = 1350$ (Kink point)

$y_1 = x + 2680 \quad 0 \le x \le 1350$.. (xx).

Putting n_i = T, then $d(p, n_i) = d(C, T) = 3580$, $d(q, n_i) = d(P, T) = 4930$,

$y_1 = x + 3580$ and $y_2 = 6280 – x$ when solved.

$x + 3580 = 6280 – x$, => $x = 1350$ (Kink point)

$y_1 = x + 3580 \quad 0 \le x \le 1350$.. (xxi)

Putting n_i = U, then $d(p, n_i) = d(C, U) = 2830$, $d(q, n_i) = d(P, U) = 4180$,

$y_1 = x + 2830$ and $y_2 = 5530 – x$ when solved.

$x + 2830 = 5530 – x$, => $x = 1350$ (Kink point)

$y_1 = x + 2830 \quad 0 \le x \le 1350$.. (xxii)

Putting n_i = V, the d (p, n_i) = d(C, V) = 4330, d(q, n_i) = d(P, V) = 5680,

$y_1 = x + 4330$ and $y_2 = 7030 – x$ when solved.

$x + 4330 = 7030 – x$, => x = 1350 (Kink point)

$y_1 = x + 4330 \quad 0 \leq x \leq 1350$.. (xxiii)

Location on edge (C, H)

Let C = p, H = q such that $0 \leq x \leq c$ (p, q) and c (p, q) = c (C, H) = 2940.

Putting n_i = A, then d(p, n_i) = d(c, A) = 2590, d(q, n_i) = d(H, A) =5530,

$y_1 = x + 2590$ and $y_2 = 8470 – x$ when solved.

x + 2590 and 8470 – x => x = 2940 (Kink point)

$y_1 = x + 1350 \quad 0 \leq x \leq 2940$.. (i).

Putting n_i = B, then d(p, n_i) = d(C, B) = 1350, d(q, n_i) = d(H, B) = 4300,

$y_1 = x + 1350$ and $y_2 = 7240 –x$ when solved.

$x + 1350 = 7230 – x$ => x = 2940 (Kink point)

$y_1 = x + 1350 \quad 0 \leq x \leq 2940$.

Putting n_i = C, then d(p, n_i) = d(C, C) = 0, d(q, n_i) = d(H, C) = 2940,

$y_1 = x$ and $y_2 =5890 – x$ when solved

$x = 5890 – x$, => x = 2940 (Kink point)

$y_1 = x \quad 0 \leq x \leq 2940$.

Putting n_i = D, then d(p, n_i) = d(C, D) = 1690, d(q, n_i) = d(H, D) = 4630,

$y_1 = x + 1690$ and $y_2 = 7570 – x$ when solved.

$x + 1690 = 7570 – x$, => x = 2940 (Kink point)

$y_1 = x + 1690 \quad 0 \leq x \leq 2940$.. (iv)

Putting n_i = E, then d(p, n_i) = d(C, E) = 2940, d(q, n_i) = d(H, E) = 5890,

$y_1 = x + 2940$ and $y_2 = 8820 – x$ when solved.

$x + 2940 = 8820 – x$, => x = 2940 (Kink point)

$y_1 + x = 2940 \quad 0 \leq x \leq 2940$.. (v)

Putting n_i = F, then $d(p, n_i) = d(C, F) = 2320$, $d(q, n_i) = d(H, F) = 5890$,

$y_1 = x + 2320$ and $y_2 = 5830 – x$ when solved.

$x + 2320 = 5830 – x$, => $x = 1755$ (Kink point)

$y_1 = x + 2320 \quad 0 \le x \le 1755$.. (vi)

$y_2 = 5830 – x \quad 1755 \le x \le 2940$.. (vii)

Putting n_i = G, then $d (p, n_i) = d(C, G) = 2140$, $d(q, n_i) = d(H, G)$,

$y_1 = x + 2140$ and y_2 $6850 – x$ when solved.

$x + 2140 = 6850 – x$, => $x = 2355$ (Kink point)

$y_1 = x + 2140 \quad 0 \le x \le 2355$.. (viii)

$y_2 = 6850 – x \quad 2355 \le x \le 2940$.. (ix)

Putting n_i = H, then $d(p, n_i) = d(C, H) = 2940$, $d(q, n_i) = d(H, H) = 0$,

$y_1 = x + 2940$ and $y_2 = 2940 – x$ when solved.

$x + 2940 = 2940 – x$, => $x = 0$ (Kink point)

$y_2 = 2940 – x \quad 0 \le x \le 2940$.. (x)

Putting n_i = I, then $d(p, n_i) = d(C, I) = 1150$, $d(q, n_i) = d(H, I) =4090$,

$y_1 = x + 1150$ and $y_2 = 7030 – x$ when solved.

$x + 1150 = 7030 – x$ => $x = 2940$ (Kink point)

$y_2 = x + 1150 \quad 0 \le x \le 2940$.. (xi)

Putting n_i = J, then $d(p, n_i) = d(C, J) = 2230$, $d(q, n_i) = d(H, J) = 5170$,

$y_1 = x + 2230$ and $y_2 = 8110 – x$, when solved.

$x + 2230 = 8110 – x$, => $x = 2940$ (Kink point)

$y_1 = x + 2230 \quad 0 \le x \le 2940$.. (xii)

Putting n_i = K, then $d(p, n_i) = d(C, K) = 3790$, $d(q, n_i) = d(H, K) = 6730$,

$y_1 = x +3490 =$ and $y_2 = 9670 – x$ when solved

$x + 3790 = 9670 – x$, => $x = 2940$ (Kink point)

$y_1 = x + 3790 \quad 0 \le x \le 2940$.. (xiii)

Putting n_i = L, then $d(p, n_i) = d(C, L) = 3480$, $d(q, n_i) = d(H, L) = 6430$,

$y_1 = x + 3490$ and $y_2 = 9370 – x$ when solved

$x + 3490 = 9370 – x, \Rightarrow x = 2940$ (Kink point)

$y_1 = x + 3490 \quad 0 \leq x \leq 2940$.. (xiv)

Putting n_i= M, then $d(p, n_i) = d(C, M) = 1990$, $d(q, n_i) = d(H, M) = 4930$,

$y_1 = x + 1990$ and $y_2 = 7870 – x$ when solved

$x + 1990 = 7870 – x, \Rightarrow x = 2940$ (Kink point)

$y_1 = x + 1990 \quad 0 \leq x \leq 2940$.. (xv)

Putting n_i = N, then $d(p, n_i) = d(C, N) = 3910$, $d(q, n_i) = d(H, N) = 6850$,

$y_1 = x + 3910$ and $y_2 = 9790 – x$ when solved

$x + 3910 = 9790 – x, \Rightarrow x = 2940$ (Kink point)

$y_1 = x + 3910 \quad 0 \leq x \leq 2940$.. (xvi)

Putting n_i = O, then $d(p, n_i) = d(C, O) = 4745$, $d(q, n_i) = d(H, O) = 7690$,

$y_1 = x + 4745$ and $y_2 = 9790 – x$ when solved

$x + 4745 = 10630 – x, \Rightarrow x = 2942.5$ (Kink point)

$y_1 = x + 4745 \quad 0 \leq x \leq 2942.5$.. (xvii)

Putting n_i = P, then $d(p, n_i) = d(C, P) = 1350$, $d(q, n_i) = d(H, P) = 4300$,

$y_1 = x + 1350$ and $y_2 = 7230 – x$ when solved

$x + 1350 = 7230 – x, \Rightarrow x = 2940$ (Kink point)

$y_1 = x + 1350 \quad 0 \leq x \leq 2940$.. (xviii)

Putting n_i = Q, then $d(p, n_i) = d(C, Q) = 4150$, $d(q, n_i) = d(H, Q) = 7090$,

$y_1 = x + 4150$ and $y_2 = 10030 – x$ when solved

$x + 4130 = 10030 – x, \Rightarrow x = 2940$ (Kink point)

$y_1 = x + 4150 \quad 0 \leq x \leq 2940$.. (xix)

Putting n_i = R, then $d(p, n_i) = d(C, R) = 4745$, $d(q, n_i) = d(H, R) = 7690$,

$y_1 = x + 4745$ and $y_2 = 10630 - x$ when solved

$x + 4745 = 10630 - x$, => $x = 2942.5$ (Kink point)

$y_1 = x + 4745 \quad 0 \le x \le 2942.5$.. (xx)

Putting n_i = S, then $d(p, n_i) = d(C, S) = 2680$, $d(q, n_i) = d(H, S) = 3250$,

$y_1 = x + 2680$ and $y_2 = 6190 - x$ when solved

$x + 2680 = 6190 - x$, => $x = 2940$ (Kink point)

$y_1 = x + 2680 \quad 0 \le x \le 1755$... (xxi)

$y_2 = 6190 - x \quad 1755 \le x \le 1755$.. (xxii)

Putting n_i = T, then $d(p, n_i) = d(C, T) = 3580$, $d(q, n_i) = d(H, T) = 640$,

$y_1 = x + 3580$ and $y_2 = 3580 - x$ when solved

$x + 3580 = 3580 - x$, => $x = 0$ (Kink point)

$y_2 = x + 3570 \quad 0 \le x \le 2940$.. (xxiii)

Putting n_i = U, then $d(p, n_i) = d(C, U) = 2830$, $d(q, n_i) = d(H, U) = 5770$,

$y_1 = x + 2830$ and $y_2 = 8710 - x$ when solved

$x + 2830 = 8710 - x$, => $x = 2940$ (Kink point)

$y_1 = x + 2830 \quad 0 \le x \le 2940$.. (xxiv)

Putting n_i = V, then $d(p, n_i) = d(C, V) = 4330$, $d(q, n_i) = d(H, V) = 7270$,

$y_1 = x + 4330$ and $y_2 = 10210 - x$ when solved

$x + 4330 = 10210 - x$, => $x = 2940$ (Kink point)

$y_1 = x + 4330 \quad 0 \le x \le 2940$.. (xxv)

Location on edge (F, C)

Let F=p, C=q such that $0 \le x \le c(p,q)$ and $c(p,q) = c(F, C) = 5530$

Putting n_i = A, then $d(p,n_i) = d(F, A) = 4900$, $d(q, n_i) = d(C, A) = 2590$

$y_1 = x + 4900 - x$ and $y_2 = 4900 - x$ when solved

$x + 4900 = 4900 - x \Rightarrow x = 0$ (Kink point)

$y_2 = 4900 - x \qquad 0 \le x \le 2320$.. (i)

Putting n_i = B, then $d(p,n_i) = d(F, B) = 3670$, $d(q, n_i) = d(C, B) = 1360$

$y_1 = x+3670$ and $y_2 = 3670-x$ when solved

$x + 3670 = 3670 - x, \Rightarrow x=0$ (Kink Point)

$y_2 = 3670 - x \qquad 0 \le x \le 2320$.. (ii)

Putting n_i= c, then $d(p, n_i) = d(F, C) = 2320$, $d(q, n_i) = d(C, C) = 0$

$y_1 = x + 2320$ and $y_2 = 2320 - x$ when solved

$x + 2320 = 2320 - x, \Rightarrow x=0$ (Kink Point)

$y_2 = 2320 - x \qquad 0 \le x \le 2320$.. (iii)

Putting n_i = D, then $d(p,n_i) = d(F, D) = 3990$, $d(q, n_i) = d(C, D) = 1690$

$y_1 = x + 3990$ and $y_2 = 3990 - x$ when solved

$x + 3990 = 3990 - x, \Rightarrow x=0$ (Kink Point)

$y_2 = 3990 - x \qquad 0 \le x \le 2320$.. (iv)

Putting n_i = E, then $d(p,n_i) = d(F,E) = 5260$, $d(q, n_i) = d(C,E) = 2950$,

$y_1 = x + 5260$ and $y_2 = 5260 - x$ when solved

$x + 5260 = 5260 - x, \Rightarrow x = 0$ (Kink Point)

$y_2 = 5260 - x \qquad 0 \le x \le 2320$.. (v)

Putting n_i = F, then $d(p, n_i) = d(F, F) =0$, $d(q, n_i) = d(C,F) = 2320$,

$y_1 = x$ and $y_2 = 4630 – x$ when solved

$x = 4360 - x, \Rightarrow x = 2320$ (Kink point)

$y_i = x \qquad 0 \le x \le 2320$.. (vi)

Putting n_i =G, then $d(p, n_i)$ =d(F, G) = 1090 $d(q, n_i)$ = d(C, G) = 2320

$y_i = x + 1090$ and $y_2 = 4450 – x$ when solved

$x + 1090 = 4450 - x$, => x=1680 (Kink point)

$y_i = x+1090$ $0 \leq x \leq 1680$.. (vii)

$Y_2 = 4450 - x$ $1680 \leq X \leq 2320$................................... (viii)

Putting n_i=H then(p, n_i)=d(F, H)=2890, $d(q, n_i)$=.d(C,H)=2950

$y_1=x+2890$ and $y_2=5260 - x$ when solved

$x + 2890=5260 - x$, => x =1185 (Kink point)

$y_1=x+2890$ $0 \leq x \leq 1185$.. (ix)

$y_2=5260 - x$ $1185 \leq x \leq 2320$.. (x)

Putting n_i=I then $d(p, n_i)$= d(F, I)=1810 $d(q, n_i)$= d(C,I)= 1150,

$y_1=x+1810$ and $y_2=3460 – x$ when solved

$x + 1810=3460 - x$, => x = 825 (Kink point)

$y_1 = x+1810$ $0 \leq x \leq 825$.. (xi)

$y_2 = 3460-x$ $825 \leq x \leq 2320$.. (xii)

Putting n_i=J, then $d(p,n_i)$= d(F,J)=2890 $d(q,n_i)$= d(C,J)=2230

$y_1= x +2890$ and $y_2 = 4540 – x$ when solved

$x + 2890 = 4540 - x$, => x=825 (Kink point)

$y_1=x+2890$ $0 \leq x \leq 825$……… (xiii)

$y_2= 4540 - x$ $825 \leq x \leq 2320$.. (xiv)

Putting n_i = K, then d(p, ni) = d(F, K) = 6100, $d(q, n_i)$ = d(C, K) = 3790

$y_i = x + 6100$ and $y_2 = 6100 – x$ when solved

$x + 6100 = 6100 - x$, => x=0 (Kink point)

$y_2 = 6100 - x$ $0 \leq x \leq 2320$.. (xv)

Putting n_i = L, then $d(p, n_i) = d(F, L) = 4150$, $d(q,n_i) = d(C, K) =3490$,

$y_i = x+4150$ and $y_2 = 5800 - x$ when solved

$x + 4150 = 5800 - x$, => $x = 825$ (Kink point)

$y_i = x+4150 \quad 0 \le x \le 825$.. (xvi)

$y_2 = x + 4150 \quad 825 \le x \le 2320$... (xvii)

Putting n_i= M, then $d(p,n_i)= d(F,M)=2650$, $d(q, n_i)= d(C, M)=1990$

$y_1=x + 2650$ and $y_2 = 5800 - x$ when solved

$x+2650 = 4300 - x$, => $x=825$ (Kink point)

$y_1=x+2650 \quad 0 \le x \le 825$.. (xviii)

$y_2 = 4300 - x \quad 825 \le x \le 2320$.. (xix)

Putting n_i=N, then $d(p, n_i)= d(F, N)=6220$, $d(q, n_i)= d(C, N)=3910$

$y_1=x + 6220$ and $y_2 = 6220 - x$ when solved

$x + 6220 = 6220 - x$, => $x=0$ (Kink point)

$y_2 = 6220 - x \quad 0 \le x \le 2320$... (xx)

Putting n_i=O, then $d(p, n_i)= d(F, O)=7060$, $d(q, n_i)= d(C, O)=4750$

$y_1= x + 7060$ and $y_2 = 7060 - x$ when solved

$x+7060 = 7060 - x$, =>$x=0$ (Kink point)

$y_2 = 7060 - x \quad 0 \le x \le 2320$... (xxi)

Putting n_i=P, then $d(p,n_i)= d(F,P)=3670$, $d(q, n_i)= d(C, P)=1350$,

$y_1 = x+3670$ and $y_2 = 3670 - x$ when solved

$x + 3670 = 3670 - x$, => $x=0$ (Kink point)

$y_2 = 3670 - x \quad 0 \le x \le 2320$... (xxii)

Putting n_i = Q, then $d(p, n_i) = d(F, Q) =6460$,

$y_i =x + 6460$ and $y_2 = 6460 - x$ when solved

$x + 6460 = 6460 - x$, => $x=0$ (Kink point)

$y_2 = 6460 - x \quad 0 \le x \le 2320$.. (xxiii)

Putting n_i = R, then $d(p, n_i) = d(F, R) = 7060$, $d(q, n_i) = d(C, R) = 4750$,

$y_i = x + 7060$ and $y_2 = 7060 - x$ when solved

$x + 7060 = 7060 - x, \Rightarrow x = 0$ (Kink point)

$y_2 = 7060 - x \qquad 0 \leq x \leq 2320$.. (xxiv)

Putting n_i = S, then $d(p, n_i) = d(F, S) = 360$, $d(q, n_i) = d(C, S) = 2680$,

$y_i = x + 360$ and $y_2 = 4990 - x$ when solved

$y_i = x + 360 = 4990 - x, \Rightarrow x = 2320$.. (Kink point)

$y_i = x + 360 \qquad 0 \leq x \leq 2320$.. (xxv)

Putting n_i = T, then $d(p, n_i) = d(F, T) = 2800$, $d(q, n_i) = d(C, T) = 3580$

$y_i = x + 2800$ and $y_2 = 5890 - x$ when solved

$x + 2800 = 5890 - x, \Rightarrow x = 1545$ (Kink point)

$y_i = x + 2800 \qquad 0 \leq x \leq 1545$.. (xxvi)

$y_2 = 5890 - x \qquad 1545 \leq x \leq 2320$.. (xxvii)

Putting n_i = U, then $d(p, n_i) = d(F, U) = 3490$, $d(q, n_i) = d(C, U) = 2830$,

$y_i = x + 3490$ and $y_2 = x + 5140$ when solved

$x + 3490 = 5140 - x, \Rightarrow x = 825$ (Kink point)

$y_i = x + 3490 \qquad 0 \leq x \leq 825$.. (xxviii)

$y_2 = x + 5140 \qquad 825 \leq x \leq 2320$.. (xxix)

Putting n_i = V, then $d(p, n_i) = d(F, V) = 4190$ $d(q, n_i) = d(C, V) = 4330$,

$y_i = x + 4190$ and $y_2 = x + 6640$ when solved

$x + 4190 = 6640 - x, \Rightarrow x = 1225$ (Kink point)

$y_i = x + 4190 \qquad 0 \leq x \leq 1225$.. (xxx)

$y_2 = x + 6640 \qquad 1225 \leq x \leq 2320$.. (xxxi)

Location on edge (C, I)

Let C = p, I = q such that $0 \le x \le C(p, q)$ and $c(p, q) = c(C, I) = 1140$

Putting n_i =A, then $d(p, n_i) = d(C, A) = 2590$, then $d(q, n_i) = d(I, A) = 3730$

$y_i = x+2590$ and $y_2 = 4870 – x$ when solved

$x + 2590 = 4870 - x, \Rightarrow x = 1140$ (Kink point)

$y_i = x + 2590$ $\quad 0 \le x \le 1140$.. (i)

Putting n_i = B, then $d(p, n_i) = d(C, B) = 2590$, $d(q, n_i) = d(I, B) = 2500$,

$y_i = x + 1350$ and $y_2 = 3630 - x$ when solved

$x + 1350 = 3630 - x, \Rightarrow x = 1140$ (Kink point)

$y_i = x + 1350$ $\quad 0 \le x \le 1140$.. (ii)

Putting n_i = C, then $d(p, n_i) = d(C, C) = 0$ $d(q, n_i) = d(I, C) = 1140$

$y_i = x$ and $y_2 = 2290 – x$ when solved

$x = 2290 - x, \Rightarrow x = 1140$ (Kink point)

$y_i = x$ $\quad 0 \le x \le 1140$.. (iii)

Putting n_i = D, then $d(p, n_i) = d(C, D) = 1690$ $d(q, n_i) = d(I, C) = 2230$,

$y_i = x + 1690$ and $y_2 = 3370 – x$ when solved

$x + 1690 = 3360 - x, \Rightarrow x = 840$ (Kink point)

$y_i = x + 1690$ $\quad 0 \le x \le 840$.. (iv)

$y_2 = 3370 - x$ $\quad 840 \le x \le 1140$... (v)

Putting n_i = E, then $d(p, n_i) = d(C, E) = 2950$, $d(q, n_i) = d(I, C) = 3490$,

$y_i = x + 2950$ and $y_2 = 4630 – x$ when solved

$x + 2950 = 4630 - x, \Rightarrow x = 840$ (Kink point)

$y_i = x + 2950$ $\quad 0 \le x \le 840$.. (vi)

$y_2 = 4630 - x$ $\quad 840 \le x \le 1140$.. (vii)

Putting n_i = F, then $d(p, n_i) = d(C, F) = 2320$, $d(q, n_i) = d(I, F) = 1810$,

$y_i = x + 2320$ and $y_2 = 2950 - x$ when solved

$x + 2320 = 2920 - x$, => $x = 315$ (Kink point)

$y_i = x + 2320 \qquad 0 \le x \le 315$.. (viii)

$y_2 = 2950 - x \qquad 315 \le x \le 1140$.. (ix)

Putting n_i = G, then $d(p, n_i) = d(C, A) = 2140$, $d(q, n_i) = d(I, G) = 990$,

$y_i = x + 2140$ and $y_2 = 2140 – x$ when solved

$x + 2140 = 2140 - x$, => $x = 0$ (Kink point)

$y_2 = 2140 - x \qquad 0 \le X \le 1140$.. (x)

Putting n_i = H, then $d(p, n_i) = d(C, H) = 2950$,$d(q, n_i) = d(I, H) = 4090$,

$y_i = x + 2950$ and $y_2 = 5230 - x$ when solved

$x + 2950 = 5230 - x$, => $x = 1140$ (Kink point)

$y_i = x + 2950 \qquad 0 \le x \le 1140$.. (xi)

Putting n_i = I, then $d(p, n_i) = d(C, I) = 1140$, $d(q, n_i) = d(I, I) = 0$

$y_i = x + 1140$ and $y_2 = 1140 – x$ when solved

$x + 1140 = 1140 - x$, => $x = 0$ (Kink point)

$y_2 = 1140 - x \qquad 0 \le x \le 1140$.. (xii)

Putting n_i = J, then $d(p, n_i) = d(C, J) = 2230$, $d(q, n_i) = d(I, J) = 1090$,

$y_i = x + 2230$ and $y_2 = 2230 – x$ when solved

$x + 2230 = 2230 - x$, => $x = 0$ (Kink point)

$y_2 = 2230 - x \qquad 0 \le x \le 1140$.. (xiii)

Putting n_i = K, then $d(p, n_i) = d(C, K) = 3790$, $d(q, n_i) = d(I, K) = 4930$,

$y_i = x + 3790$ and $y_2 = 6070 – x$ when solved

$x + 3790 = 6070 - x$, => $x = 1140$ (Kink point)

$y_2 = x + 3790 \qquad 0 \le x \le 1140$.. (xiv)

Putting n_i = L, then d(p, n_i) = d(C, L) = 3490, d(q, n_i) = d(I, L) = 2350,

$y_i = x + 3490$ and $y_2 = 3490 – x$ when solved

$x + 3490 = 3490 - x$, => $x = 0$ (Kink point)

$y_2 = 3490 - x$ $\quad 0 \le x \le 1140$.. (xv)

Putting n_i = M, then d(p, n_i) d(C, M) = 1990,d(q, n_i) = d(I, M) = 840,

$y_i = x + 1990$ and $y_2 = 1990 - x$ when solved

$x + 1990 = 1990 - x$, => $x = 0$ (Kink point)

$y_2 = 1990 - x$ $\quad 0 \le x \le 1140$.. (xvi)

Putting n_i = N, then d(p, n_i) = d(C, N) = 3910, d(q, n_i) = d(I, N) = 5050,

$y_i = x + 3910$ and $y_2 = 6190 – x$ when solved

$x + 3910 = 6190 - x$, => $x = 1140$ (Kink point)

$y_i = x + 3910$ $\quad 0 \le x \le 1140$.. (xvii)

Putting n_i = 0, then d(p, n_i) = d(C, P) = 1350, d(q, n_i) = d(I, O) = 5890,

$y_i = x + 4750$ and $y_2 = 7030 - x$, when solved

$x + 4750 = 7030 - x$, => $x = 1140$ (Kink point)

$y_i = x + 4750$ $\quad 0 \le x \le 1140$.. (xviii)

Putting n_i = P, then d(p, n_i) = d(C, P) = 1350, d(q, n_i) = d(I, P) = 2500,

$y_i = x + 1350$ and $y_2 = 3640 – x$ when solved

$x + 1350 = 3640 - x$, => $x = 1140$ (Kink point)

$y_i = x + 1350$ $\quad 0 \le x \le 1140$.. (xix)

Putting n_i = Q, then d(p, n_i) = d(C, Q) = 4150, d(q, n_i) = d(I, Q) = 4690,

$y_i = x + 4150$ and $y_2 = 5830 – x$ when solved

$x + 4150 = 5830 - x$, => $x = 840$ (Kink point)

$y_i = x + 4150$ $\quad 0 \le x \le 840$.. (xx)

$y_2 = 5830 - x$ $\quad 840 \le x \le 1140$.. (xxi)

Putting $n_i = R$, then $d(p, n_i) = d(C, R) = 4750$, $d(q, n_i) = d(I, R) = 5890$,

$y_i = x + 4750$ and $y_2 = 7030 – x$ when solved

$x + 4750 = 7030 - x$, => $x = 1140$ (Kink point)

$y_i = x + 4750$ $0 \leq x \leq 1140$... (xxii)

Putting $n_i = S$, then $d(p, n_i) = d(C, S) = 2680$, $d(q, n_i) = d(I, S) = 1660$,

$y_i = x + 2680$ and $y_2 = 2800 – x$ when solved

$x + 2680 = 2800 - x$, => $x = 60$ (Kink point)

$y_i = x + 2680$ $0 \leq x \leq 60$... (xxiii)

$y_2 = 2800 - x$ $60 \leq x \leq 1140$... (xxiv)

Putting $n_i = T$, then $d(p, n_i) = d(C, T) = 3580$, $d(q, n_i) = d(I, T) = 4600$

$y_i = x + 3580$ and $y_2 = 5740 – x$ when solved

$x + 3580 = 5740 - x$, => $x = 1090$ (Kink point)

$y_i = x + 3580$ $0 \leq x \leq 1090$.. (xxv)

$y_2 = 5740 - x$ $1090 \leq x \leq 1140$... (xxvi)

Putting $n_i = U$, then $d(p, n_i) = d(C, U) = 2830$, $d(q, n_i) = d(I, U) = 1690$,

$y_i = x+2830$ and $y_2 = 2830 – x$ when solved

$x + 2830 = 2830 - x$, => $x = 0$ (Kink point)

$y_2 = 2830 - x$ $0 \leq x \leq 1140$...(xxvii)

Putting $n_i = V$, then $d(p, n_i) = d(C, V) = 4330$, $d(q, n_i) = d(I, V) = 3190$,

$y_i = x + 4330$ and $y_2 = 4330 – x$ when solved

$x + 4330 = 4330 - x$, => $x = 0$ (Kink point)

$y_2 = 4330 - x$ $0 \leq x \leq 1140$... (xxviii)