

# **OPTIMAL WATER TREATMENT: A CASE STUDY OF KPONG HEADWORKS**

**By**

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## CERTIFICATION

I hereby declare that this submission is my own work towards the MSc. and that, to the best of my knowledge, it contains no material previously published by another person nor material, which has been accepted for the award of any other degree of the University, except where due acknowledgement has been made in the text.

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Date

## **DEDICATION**

This work is dedicated to my family for their love and support.

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## ACKNOWLEDGEMENT

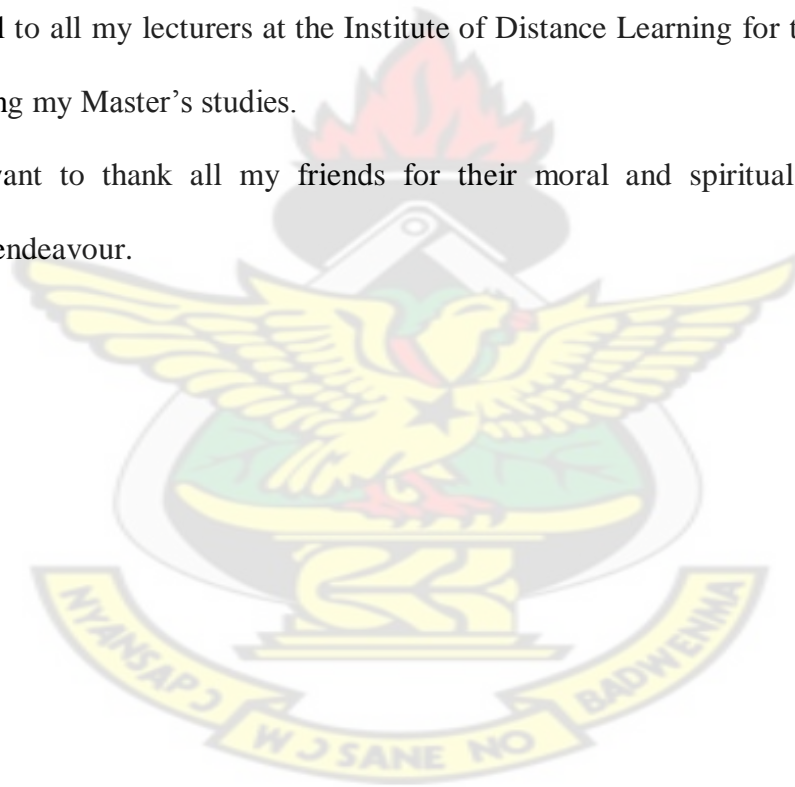
I would first of all like to give reverence to God who made all things possible.

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## ABSTRACT

Water is one of the most essential requirements to life and most human activities involve the use of water in one way or another. It is now universally accepted that providing a community with safe water, epidemics of water-borne diseases can be prevented. Untreated surface water has long been known to be the source of much human illness.

Over one billion people each year are exposed to unsafe drinking water due to poor source water quality and lack of adequate water treatment. The primary objective of any water supply scheme is to supply safe water in sufficient quantity.

The main objective of this study is to develop a water treatment cost model at Kpong Headworks using linear programming to minimize the cost of treating water.

The research showed that the factors which affect water treatment cost at Kpong Headworks includes; cost of personnel (labour), cost of electricity, cost of chemicals for treatment cost of fuel and lubricants, cost of repairs and maintenance, cost of raw water source, cost of other contractual, cost of civil structures of the treatment plant.

The most influential factors which make it very expensive to treat water at Kpong Headworks are cost of chemical, cost of electricity and the cost of fuel.

The study however revealed that, the average seasonal water treatment cost of Gh¢ 5576308.21 and Gh¢ 11633320.30 for dry and wet seasons respectively can be optimized to Gh¢ 1160000.00 in the dry season and Gh¢ 1200000.00 in the wet season to deliver the same quantity of water to consumers in each season.

Sensitivity analysis performed showed that the model developed will not minimize the total cost if the unit price of a bag of chemical, unit price of electricity and price of a litre of fuel in the objective function is increased by Gh¢ 1.00 in the dry season and Gh¢ 2.25 in the wet season.

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## CHAPTER ONE

### 1.0 INTRODUCTION

Water treatment at Kpong Headworks has been associated with very high cost over the years. The study looks at the most favourable options of treating water at the Headworks that will minimize cost.

This chapter discusses the background to the study, stating the problem at hand and specific objectives of the study. It will explain the reasons why the study is being carried out and the limitations. Finally, it will look at the organization of the study and the methodology.

### 1.1 BACKGROUND TO THE STUDY

Water is a chemical substance with the chemical formula  $H_2O$ . Water molecule contains one oxygen and two hydrogen atoms connected by covalent bonds. Water is a liquid at ambient conditions, but it often co-exists on Earth with its solid state, ice, and gaseous state (water vapor or steam). Water also exists in a liquid crystal state near hydrophilic surfaces. Under nomenclature used to name chemical compounds, dihydrogen monoxide is the scientific name for water, though it is almost never used. Water covers 70.9% of the earth's surface and is vital for all known forms of life.

On Earth, 96.5% of the planet's water is found mostly in oceans; 1.7% in groundwater; 1.7% in glaciers and the ice caps of Antarctica and Greenland; a small fraction in other large water bodies, and 0.001% in the air as vapor, clouds (formed of solid and liquid

water particles suspended in air), and precipitation. Only 2.5% of the Earth's water is freshwater, and 98.8% of that water is in ice and groundwater. Less than 0.3% of all freshwater is in rivers, lakes, and the atmosphere, and an even smaller amount of the Earth's freshwater (0.003%) is contained within biological bodies and manufactured products. Water on earth moves continually through the hydrological cycle of evaporation and transpiration (evapotranspiration), condensation, precipitation, and runoff, usually reaching the sea. Evaporation and transpiration contribute to the precipitation over land.

Access to safe drinking water has improved over the last decades in almost every part of the world, but approximately one billion people still lack access to safe water and over 2.5 billion lack access to adequate sanitation. However, some observers have estimated that by 2025 more than half of the world population will be facing water-based vulnerability. Water plays an important role in the world economy, as it functions as a solvent for a wide variety of chemical substances and facilitates industrial cooling and transportation. Approximately 70% of the fresh water used by humans goes to agriculture. (www.wikipedia.com)

According to the World Health Organization (2004), 1.1 billion people did not have access to an improved water supply in 2002, and 2.3 billion people suffered from diseases caused by contaminated water. Each year 1.8 million people die from diarrhoeal diseases, and 90% of these deaths are of children under 5 (WHO, 2004). Every human on our planet has a fundamental right to a reliable supply of clean water. Thus, there is a global need for clean water and every man, woman, and child has a fundamental right to a reliable supply.

The Ghana Water Company Limited (GWCL), formerly the Ghana Water and Sewerage Corporation, is responsible for planning, development and operation of water

supply systems in large towns and cities and some medium towns that are not under community management.

In Ghana, the average availability/access to safe drinking water in urban areas is 20hrs/day. In Accra, for example, it has been estimated that only approximately 25% of residents enjoy a 24-hour water supply. About 30% have an average of 12 hours service every day for five days a week. Another 35% have service for two days each week while the remaining residents on the outskirts of Accra are completely without access to piped water supplies. This pattern is more acute in other urban centres. The un-served areas depend on secondary supplies (i.e. vendors and mostly tanker service delivery or dedicated GWCL filling points).

Approximately 10.7 million people have access to improved water supplies in Ghana. Sixty one percent of the 8.4 million residents in the country's urban areas have improved water supply services provided by GWCL's networks. Hence, 3.3 million urban residents in Ghana depend on alternative water sources, (Ghana Water Sector Assessment, 2005).

The Kpong treatment plant is situated at latitude  $6.5^{\circ}$  North and an elevation of approximately 22 metres above sea level. The maximum and minimum shade temperatures are approximately  $45^{\circ}\text{C}$  and  $12^{\circ}\text{C}$  respectively and the relative humidity is approximately 95%. The Kpong treatment plant consists of two conventional sub systems, namely Kpong Old Works which was commissioned in 1954 and Kpong New Works, commissioned in 1967.

## **1.2 PROBLEM STATEMENT**

The Kpong Headworks which supplies water to Tema and parts of Accra spend an average of GH¢ 17,300,000.00 in the year for treating water, thus GH¢1,441,666.67 is used for treating water at Kpong monthly. Consumers have to pay more for the water they use because of the high cost associated with treating water. The problem at hand is to minimize the cost of treating water with respect to:

- (i) the cost of electricity use.
- (ii) the cost of fuel use.
- (iii) the cost of chemical use.

## **1.3 OBJECTIVES**

The objectives of the thesis are:

- (i) to develop water treatment cost model for Kpong Headworks.
- (ii) to use the model developed to determine the optimum cost of treating water at Kpong Headworks.

## **1.4 JUSTIFICATION**

The Kpong Treatment Plant supplies water to residents of Tema, Accra East, Somanya, Kpong, Akuse and its surrounding areas. As a result of the increasing demand for water by residents in these areas, optimizing the treatment plant will help the Ghana Water Company Limited (GWCL) reduce treatment cost associated with energy, chemicals and fuel. The results of this study would help GWCL to adopt workable strategies to effectively supply to the demand of consumers in the above mentioned areas.



## **1.5 METHODOLOGY**

This research is intended to use linear programming to develop a water treatment cost model to determine the optimum cost of treating water at the Kpong treatment plant.

The source of data will be from the Kpong treatment plant annual reports from 2008 to 2010. Data from the 2010 annual report will be used for the formulation of the model.

The computer implementation system called Six-Pap, will be used for the analysis of results.

## **1.6 SCOPE OF THE STUDY**

This research intends to analyse the cost involved in water treatment at Kpong Headworks with reference to the cost of chemical usage, electricity usage and fuel usage taking into consideration the two major seasons in the year.

## **1.7 LIMITATION OF THE STUDY**

The limitations of the study include:

- (i) Only the computer implementation system (sixpap) was used for the analysis of data, however, the simplex method of solving linear programming problems can also be used.
- (ii) The model was developed using production data for the year, 2010 because of time.

## **1.8 ORGANIZATION OF THE STUDY**

The study is organized into five chapters covering the literature review, methodology, results and discussion as well as conclusions and recommendations.

Chapter One gives an outline of the whole thesis which includes the background of the study. The problem statement portrays the problem faced and the needs of the current

research. Chapter two provides the literature review. Chapter three describes the methodology and data collected. Chapter four presents the results and discussion of the research which includes the analysis of the results. Finally, Chapter five includes the summary of results, conclusions and recommendations.

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## **CHAPTER TWO**

### **LITERATURE REVIEW**

#### **2.0 INTRODUCTION**

This chapter provides a brief review of the extensive literature that exists on the applications of linear programming, as relevant to the study.

#### **2.1 RELATED WORK ON LINEAR PROGRAMMING**

The problem of solving a system of linear inequalities dates back at least as far as Fourier, after whom the method of Fourier-Motzkin elimination is named. The three founders of the subject are considered to be Leonid Kantorovich, the Russian mathematician who developed the earliest linear programming problems in 1939, George Dantzig, who published the simplex method in 1947, and John von Neumann, who developed the theory of the duality in the same year. The earliest linear programming was first developed by Leonid Kantorovich, a Russian mathematician, in 1939. It was used during World War II to plan expenditures and returns in order to reduce costs to the army and increase losses to the enemy. The method was kept secret until 1947 when George B. Dantzig published the simplex method and John von Neumann developed the theory of duality as a linear optimization solution, and applied it in the field of game theory. Postwar, many industries found its use in their daily planning.

The linear-programming problem was first shown to be solvable in polynomial time by Leonid Khachiyan in 1979, but a larger theoretical and practical breakthrough in the field came in 1984 when Narendra Karmarkar introduced a new interior-point method for solving linear-programming problems. Dantzig's original example of finding the best

assignment of 70 people to 70 jobs exemplifies the usefulness of linear programming. The computing power required to test all the permutations to select the best assignment is vast; the number of possible configurations exceeds the number of particles in the universe. However, it takes only a moment to find the optimum solution by posing the problem as a linear program and applying the Simplex algorithm. The theory behind linear programming drastically reduces the number of possible optimal solutions that must be checked (wikipedia,2011).

One of the largest breakthroughs in solving linear programming problems was the introduction of the new interior point method by Narendra Karmarkar in 1984. Many ideas from linear programming have inspired central concepts of optimization theories. Examples are: Decomposition, duality and the importance of convexity and its generalizations. Nowadays, the applications of linear programming can be seen in most transportation, production and planning technologies. The use of LP can also be seen in company management and microeconomics, since companies try to minimize costs, maximise profits within their resources. (ezinemark.com)

According to Loucks et al., (1967), the first attempt to apply mathematical programming techniques to the river water quality problem was by Deininger (1965). In that work, a linear programming model was structured using various approximations of the differential equations that describe the dissolved oxygen profile of streams.

Loucks et al.,(1967) presented two linear programming models for determining the amount of wastewater treatment required to achieve, at minimum cost, a set of dissolved oxygen (DO) standards within a river basin. The two methods differ in the way they imposed the

minimum DO standards. One method was to set several DO constraints within each river reach which was assumed homogeneous. In this case, a single constraint is sufficient for the reaches for which the critical time is longer than the travel time from the beginning to the end of the reach. The second method was to ensure that the biochemical oxygen demand (BOD) concentration at the beginning of the reach is less than the critical BOD level. This critical BOD level is that which would result in a DO deficit equal to the maximum allowable deficit for the reach. It was emphasized that any comparison of quality standards should be based on an assessment of the resulting DO profile in each reach, and not on the changes in the minimum allowable DO concentrations. This is due to the following reason. There can be reaches of which a change in the minimum allowable DO level does not affect the DP profile of any other reaches, while such a change in some other reach may affect the DO profile in many reaches.

(Bundgaard-Nielsen et al., 1975) discussed the interaction between the level of effluent charge or taxation and the choice of treatment technology. A linear programming model was used to estimate the least-cost treatment alternatives. It is shown that there is a risk of overtaxation, i.e., simply increasing taxation may fail to improve water quality but only increase production costs and thus consumer prices. The possible inefficiency of a surcharge to abate pollution is similar to that of taxation, and it is also discussed by these authors.

Loucks et al., (1981) presented an example model, which can be solved by mixed integer programming. The purpose of the model is to determine the degrees of treatment for carbonaceous and nitrogenous BOD components at each waste outfall in a river basin. Effluent standards were expressed by maximum BOD levels, and, BOD and DO limits were

included as ambient standards. The objective of the analysis was to identify the treatment efficiencies that minimize the sum of the wastewater treatment costs. The possibility to start the solution procedure with a small number of constraint points (thereby reducing the computations) was pointed out. This leads to a trial-and-error solution procedure, as the solution must be checked to ensure that the concentrations within the entire river are acceptable. If the solution is unacceptable, the solution procedure should be repeated after setting constraints for the concentrations at the critical locations. This example was formulated using analytical equations to predict BOD and DO. Alternatively, a two-stage approach can be used to analyze the river system using finite-section models. In such a model, the river reach is divided into a number of reaches within which the quality parameters are constant.

Burn (1987) examined three model formulations for water quality management. The model formulations consisted of a linear cost minimization, a chance constrained optimization (considering the pollutant loadings as random variables) and, minimization of the variance of the quality response. These were applied to a problem involving five pollution sources at which treatment plants are located, and twelve receptor locations at which the water quality is of concern. The transfer coefficients describing the response at each receptor location for a unit release from each source have been prespecified. The results showed, as expected, that the greatest economic efficiency was obtained by the use of minimum cost formulation. The minimum variance model resulted in the poorest expected quality, but it guaranteed a small variability of the water quality than the other two models. Both the chance constrained formulation and the variance formulation indicated reductions of the mean water quality. The value of different model interpretations to the decision maker was emphasized, as the decision maker would then be better able to

choose a solution considering the implementation which incorporates the pertinent aspects of the problem.

Linear programming (LP) techniques can be utilised for solving groundwater quantity and quality management problems when the imposed physical and managerial constraints and the objective function are linear. The capability of LP techniques to solve large-scale problems and to guarantee global optimal solutions has attracted the widespread attention of many researchers in the groundwater management field.

Some of the aquifer management problems formulated and solved by using LP technique are discussed here. Lee and Aronofsky (1958) developed a linear programming management model to maximise profits from oil production. They used a response matrix that was developed using an analytical solution of the flow equation. They considered a transient management problem. Williams (1962) extended the work of Lee and Aronofsky (1958) to the scheduling of drilling operations. Wattenberger (1970) used a transient response matrix to develop a linear programming management model which sought to maximise well production.

Deninger (1970) presented a linear programming formulation to maximise water production from a well field. The author used the nonequilibrium formula of Theis (1935) to obtain the response matrix.

Aguado and Remson (1974) introduced linear programming (LP) based management models embedding the finite difference approximation of the governing differential equations as constraints in the formulation. They obtained solutions for example cases of confined and unconfined aquifers, one- and two-dimensional flow fields, and steady state



and transient flow conditions. In these examples, the objective was to maximise hydraulic heads at specified locations. The authors included as constraints some limits on the sum of production rates, and on the monotonicity of the nondecreasing heads in a specified direction. For a steady state one-dimensional unconfined flow case, the nonlinear formulation was converted to a linear formulation by taking the square of hydraulic heads as a linear variable. For a transient one-dimensional confined flow case the partial differential equations were approximated using the Crank–Nicolson scheme. Finite difference equations were written for all nodes for all time periods and then a single LP was solved to obtain the solution over time and space. The transient one-dimensional unconfined flow case is nonlinear. The authors used the predictor-corrector method of Douglas and Jones (1963) to approximate the nonlinear partial differential equation by a succession of system of linear difference equation. The predictor step did not result in an LP problem because, the number of equations are equal to the number of variables. It was tridiagonal and could be solved by using the Thomas algorithm. The corrector step resulted in an LP problem. The results of the corrector step were used for the predictor, and the results from the predictor were used for the corrector. Here management could be possible only for one time step.

Aguado et al., (1974) applied the LP formulation for dewatering of a large dry dock excavation to predict optimum number of wells, their locations, and rates of pumping needed to maintain ground water levels below specified elevations in a steady state. The objective was to minimise total pumping while maintaining steady state groundwater heads below some assigned value in the excavation area.

Remson et al., (1974) verified the results obtained from the LP management models against those obtained by using numerical and electrical analog ground-water models.

Morel-Seytoux (1975a,b) developed conjunctive surface water groundwater management models which were solved using linear programming. The author used the discrete kernel generator (Morel-Seytoux and Daly 1975) to develop the response matrix.

Alley et al., (1976) applied LP formulation to two-dimensional transient situations in a confined heterogeneous anisotropic aquifer. In the governing differential equation, the source/sink term was expressed as the sum of specified net source/sink and unknown source/sink terms. Their objective was to maximise the hydraulic heads for a portion of the management period such that a fixed total pumping was maintained with a certain minimum pumping from a specified location, while maintaining a certain minimum head during the specified period. For the remaining portion of the management period the objective was to maximise the pumping subject to maintaining of some lower limits for the heads at the interior nodes, with restrictions on pumping at some fixed nodes. This was a different objective and thus could not be formulated as a single problem with the previous one, though these two objectives were applicable to constituent parts in the total management period. The management period was divided into small periods of intervals and solved for each of these small periods separately by using LP.

The solution from the previous LP formulation was used as initial condition for the next period. Further they extended the methodology for steady state cases to study the feasibility of disposing of waste water by injection into an aquifer system.

The objective was to minimise total pumping from two lines of wells subject to:

- (i) a reversal of hydraulic gradient towards the pumping well,
- (ii) maintenance of monotonicity of head values to prevent the recharged waste product from reaching a particular area, and

(iii) to meet certain water demands for irrigation.

Futagami et al., (1976) presented a method to couple the finite element technique with linear programming for water pollution control. Here the objective was to maximise the pollutant issued from a waste outfall. Constraints of the model were the finite element form of the diffusion convection equation of pollutant movement and water quality requirements.

Molz and Bell (1977) used a procedure based on linear programming for the initial design of a well field that would create a zero gradient or a finite gradient in a given region. The objective was to maximise total pumping, while satisfying finite difference discretised flow equations for steady state conditions and specified head gradients.

Elango and Rave (1980) reported the performance of a finite element based linear programming model. Their study was limited to confined aquifers under steady state conditions. They presented two cases of problems. The first case related to efficient depressurisation of an aquifer. The aquifer considered in this problem was circular in shape in the plan view. The objective was to minimise pumping subject to flow equations, levels of depressurisations at various points, upper limits on pumping capacities and nonnegativity requirements. The flow constraints (equations) were obtained from (a) closed form solutions, and (b) finite element discretised equations. Their result showed the variability in the optimal solutions due to the differences in the chosen finite element configurations. Their second problem related to the maximisation of safe yield of the aquifer. They considered heterogeneity with respect to hydraulic conductivity. The constraints in this problem were finite element flow equations, restrictions on the piezometric head throughout the aquifer, and nonnegativity conditions.



Gorelick and Remson (1982) incorporated the steady state finite difference form of solute transport equation as embedded constraints. The authors maximised waste disposal at two locations while protecting water quality at supply wells and maintaining an existing waste disposal facility. Post optimality sensitivity analysis was performed using parametric programming. The objective of the second problem was to identify all sites suitable for waste disposal. They manipulated the linear programming management model so that the optimal value of the dual variable represented unit source impact indicators. It was possible to identify all feasible disposal sites by interpreting the solutions of two linear programming problems.

Gorelick (1982) presented a linear programming based model for maximising waste disposal at several facilities during several one-year planning periods. He used the response matrix approach. The concentration response matrix was obtained by using US Geological Survey method of characteristics, solute transport model (Konikow and Bredehoeft 1978).

The management model was applied to a hypothetical complex groundwater system. These large-field-scale management models were formulated as dual linear programming problems, which reduced the numerical difficulties and computation time for solution. The linear programming problems were also solved using MINOS (Murtagh and Saunders 1993) and MPS/III (Keltron Inc. 1979). The solution results indicated that waste disposal was enhanced by pulsing rather than maintaining constant disposal rates at various sites. Parametric linear programming was used for post optimality sensitivity analysis.

Heidari (1982) used linear programming in conjunction with response matrix approach for groundwater hydraulic management in the Pawnee Valley of south-central Kansas. He used MINOS to solve the linear programming problems.

Willis (1983) used linear programming to determine the optimal pumping scheme for three consecutive periods in order to meet agricultural water demands. The objectives were to maximise the sum of hydraulic heads and minimise the total deficit. The aquifer considered was unconfined, situated in the Yun Lin basin in Taiwan. The flow equation was quasilinearised using Taylor series expansion. This resulted in a linear approximation, which was solved using an iterative procedure. He used the response matrix approach.

Atwood and Gorelick (1985) presented a linear programming based design methodology for hydraulic gradient control aimed at containing and removing groundwater contaminants. They used the response matrix approach. Their design procedure used a two-stage procedure. In the first stage, solute transport simulation was used to predict the location of the shrinking plume boundary over time, assuming that the regional hydraulic gradient had been effectively flattened in the vicinity of the contaminant plume. The second stage determined the optimal well selection and pumping/recharge schedules by using the simulation-management model. Finally, a simulation model was used to verify the results.

Ahlfeld and Heidari (1994) presented an informative review of linear programming formulations for hydraulic control problems. For optimal management of a coastal aquifer in southern Turkey, Hallaji and Yazicigil (1996) used LP technique. The

authors proposed six LP models for steady state and transient state, and one quadratic optimisation model for steady state management of the aquifer system.

The general constraints were;

- (i) water demand constraints,
- (ii) drawdown limitations,
- (iii) maximum pumping rate constraints, and
- (iv) minimum pumping rate constraints.

The response matrix approach was used to obtain the drawdown limitations. However, the hydraulics of saltwater intrusion was not considered in the response matrix.

The objectives considered for the steady state management were:

- (i) maximisation of steady state water withdrawals from the existing wells,
- (ii) maximisation of withdrawals without any maximum limit on withdrawals from the wells,
- (iii) minimisation of the sum of drawdowns at pumping wells and saltwater-control nodes, and
- (iv) minimisation of the sum of the drawdowns at the saltwater-control nodes along the coast.

## **CHAPTER THREE**

### **METHODOLOGY**

#### **3.0 INTRODUCTION**

This chapter outlines the methodologies used for formulating the water treatment costmodel. It includes the definition of some terminologies, theoretical methods of solving linear programming problems which includes the graphical method and the simplex algorithm, duality, and SixPap, which is a software for solving linear programming models. The chapter also discusses some application areas of linear programming.

#### **3.1 LINEAR PROGRAMMING**

Linear Programming is that branch of mathematical programming which is designed to solve optimization problems where all the constraints as well as the objectives are expressed as linear function. It was developed by Denting in 1947. Its earlier application was solely related to the activities of the second' World War. However soon its importance was recognized and it came to occupy a prominent place in the industry and trade. Linear Programming is a technique for making decisions under certainty i.e.; when all the courses of options available to an organisation are known & the objective of the firm along with its constraints are quantified. That course of action is chosen out of all possible alternatives which yields the optimal results. Linear Programming can also be used as a verification and checking mechanism to ascertain the accuracy and the reliability of the decisions, which are taken solely on the basis of manager's experience-without the aid of a mathematical model. Linear programming (LP, or linear optimization) is a mathematical method for determining a way to achieve thebest outcome (such as maximum profit or lowest cost) in a given mathematical model for some list of requirements represented as

linear relationships. Linear programming is a specific case of mathematical programming (mathematical optimization). More formally, linear programming is a technique for the optimization of a linear objective function, subject to linearequality and linear inequality constraints. Its feasible region is a convex polyhedron, which is a set defined as the intersection of finitely many half spaces, each of which is defined by a linear inequality. Its objective function is a real-valued affine function defined on this polyhedron.

A linear programming algorithm finds a point in the polyhedron where this function has the smallest (or largest) value if such point exists.

Linear programs are problems that can be expressed in canonical form:

Maximize  $c^T x$

Subject to  $Ax \leq b$

And  $x \geq 0$

where  $x$  represents the vector of variables (to be determined),  $c$  and  $b$  are vectors of (known) coefficients and  $A$  is a (known) matrix of coefficients. The expression to be maximized or minimized is called the objective function ( $c^T x$  in this case). The equations  $Ax \leq b$  are the constraints which specify a convex polytope over which the objective function is to be optimized. (In this context, two vectors are comparable when every entry in one is less-than or equal-to the corresponding entry in the other. Otherwise, they are incomparable.)

Linear programming can be applied to various fields of study. It is used most extensively in business and economics, but can also be utilized for some engineering problems. Industries that use linear programming models include transportation, energy,

telecommunications, and manufacturing. It has proved useful in modelling diverse types of problems in planning, routing, scheduling, assignment, and design. (wikipedia, 2011)

Most business resource-allocation problems require the decision maker to take into account various types of constraints, such as capital, labor, legal, and behavioural restrictions. Linear-programming techniques can be used to provide relatively simple and realistic solutions to problems involving constrained resource allocation decisions. A wide variety of production, finance, marketing, and distribution problems have been formulated in the linear-programming framework. Consequently, managers should understand the linear-programming model so they may allocate the resources of the enterprise most efficiently, particularly in situations where important constraints are placed on the actions that may be taken.

Linear Programming is a method of planning and operation involved in the construction of a model of a real-life situation having the following elements:

(a) Variables, which denote the available choices and  
(b) the related mathematical expressions which relate the variables to the controlling conditions, reflect clearly the criteria to be employed for measuring the benefits flowing out of each course of action and providing an accurate measurement of the organization's objective. The method may be so devised as to ensure the selection of the best alternative out of a large number of alternatives available to the organization. Even though Linear Programming has wide and diverse applications, yet all linear programming problems have the following properties in common:

- (a) The objective is always the same (i.e.; profit maximization or cost minimization).
- (b) Presence of constraints which limit the extent to which the objective can be pursued/achieved.
- (c) Availability of alternatives i.e.; different courses of action to choose from, and



(d) The objectives and constraints can be expressed in the form of linear relation.

Regardless of the size or complexity, all LP problems take the same form i.e. allocating scarce resources among various competing alternatives. Irrespective of the manner in which one defines Linear Programming, a problem must have certain basic characteristics before this technique can be utilized to find the optimal values.

### 3.2 MATHEMATICAL MODEL OF LINEAR PROGRAMMING PROBLEM

Linear Programming is a mathematical technique for generating & selecting the optimal or the best solution for a given objective function.

The problem of Linear Programming may be stated as that of the optimization of linear objective function of the following form:

$$Z = C_1X_1 + C_2X_2 + \dots\dots\dots C_iX_i + C_nX_n$$

Subject to the linear constraints of the form:

$$a_{11}X_1 + a_{12}X_2 + a_{13}X_3 \dots\dots\dots + a_{1i}X_i + \dots\dots\dots + a_{1n}X_n \leq/\geq b_1$$

$$a_{j1}X_1 + a_{j2}X_2 + a_{j3}X_3 \dots\dots\dots + a_{ji}X_i + \dots\dots\dots + a_{jn}X_n \leq/\geq b_j$$

$$a_{m1}X_1 + a_{m2}X_2 + a_{m3}X_3 \dots\dots\dots + a_{mi}X_i + \dots\dots\dots + a_{mn}X_n \leq/\geq b_m$$

These are called the non-negative constraints. From the above, it is linear that a LP problem has:

- (i) linear objective function which is to be maximized or minimized.
- (ii) various linear constraints which are simply the algebraic statement of the limits of the resources or inputs at the disposal.
- (iii) Non-negativity constraints.

### 3.3 STEPS IN FORMULATING A LINEAR PROGRAMMING MODEL

Linear programming is one of the most useful techniques for effective decision making. It is an optimization approach with an emphasis on providing the optimal solution for resource allocation. How best to allocate the scarce organisational or national resources among different competing and conflicting needs (or uses) forms the core of its working. The scope for application of linear programming is very wide and it occupies a central place in many diversified decisional problems. The effective use and application of linear programming requires the formulation of a realistic model which represents accurately the objectives of the decision making subject to the constraints in which it is required to be made.

The basic steps in formulating a linear programming model are as follows:

- Step I. Identification of the decision variables.

The decision variables (parameters) having a bearing on the decision at hand shall first be identified, and then expressed or determined in the form of linear algebraic functions or in equations.

- Step II. Identification of the constraints.

All the constraints in the given problem which restrict the operation of a firm at a given point of time must be identified in this stage. Further these constraints should be broken down as linear functions in terms of the pre-defined decision variables.

- Step III. Identification of the objective.



### **3.4 GRAPHICAL SOLUTION OF THE LINEAR PROGRAMMING**

#### **PROBLEM**

In order to find a graphical solution of the linear programming problem the following steps must be employed:

- (i) Formulate the linear programming problem.
- (ii) Graph the constraints inequalities.
- (iii) Identify the feasible region which satisfies all the constraints simultaneously.

For 'less than or equal to' constraints the region is generally below the lines and 'for greater than or equal to' constraints, the region is above the lines.

- (iv) Locate the solution points on the feasible region.
- (v) These points always occur at the vertex of the feasible region.
- (vi) Evaluate the objective function at each of the vertex (corner point).
- (vii) Identify the optimum value of the objective function.

#### **3.4.1 FEASIBLE SOLUTION**

A set of values of the variables of a linear programming problem which satisfies the set of constraints and the non-negative restrictions is called a feasible solution.

#### **3.4.2 OPTIMAL SOLUTION**

A feasible solution of a linear programming problem which optimizes its objective function is called the optimal solution of the problem.

If none of the feasible solutions maximizes (or minimizes) the objective function, or if there are no feasible solutions, then the linear programming problem has no solutions.

If a linear programming problem has a solution, it is located at a vertex of the set of feasible solution.

If a linear programming problem has multiple solutions, at least one of them is located at a vertex of the set of feasible solutions. But in all the cases the value of the objective function remains the same.

### 3.5 DUALITY

Every linear programming problem where we seek to maximize the objective function gives rise to a related problem, called the dual problem, where we seek to minimize the objective function. The two problems interact in an interesting way: every feasible solution to one problem gives rise to a bound on the optimal solution in the other problem. If one problem has an optimal solution, so does the other problem and the two objective function values are the same. The equations below show a problem in standard form with  $n$  variables and  $m$  constraints on the left, and its corresponding dual problem on the right.

Max $c^T x$		min $b^T y$
s.t $Ax \leq b$		s.t $A^T y \geq c$
$x \geq 0$	$\Leftrightarrow$	$y \geq 0$
$n$ variables		$m$ variables
$m$ constraints		$n$ constraints

If the original or primal problem has the optimal solution  $x^*$ , its dual problem has an optimal solution  $y^*$  and  $c^T x^* = b^T y^*$ . If the primal problem is infeasible or unbounded, then the dual problem is infeasible or unbounded.

### 3.5.1 DUAL FORMATION

Following are the steps adopted to convert primal problem into its dual.

1. For each constraint in primal problem there is an associated variable in dual problem.
2. The elements of right hand side of the constraints will be taken as the co-efficient of the objective function in the dual problem.
3. If the primal problem is maximization, then its dual problem will be minimization and vice versa.
4. The inequalities of constraints should be interchanged from  $\geq$  to  $\leq$  and vice versa and the variables in both the problems and non-negative.
5. The rows of primal problem are changed to columns in the dual problem. In other words the matrix A of the primal problem will be changed to its transpose (A) for the dual problem.
6. The co-efficient of the objective function will be taken the right hand side of the constraints of the dual problem.

### 3.6 THE SIMPLEX METHOD

A procedure called the simplex method may be used to find the optimal solution to multivariable problems. The simplex method is actually an algorithm (or a set of instructions) with which we examine corner points in a methodical fashion until we arrive at the best solution—highest profit or lowest cost.

Objective function - a function that expresses the quantity to be maximised or minimised in terms of the other variables.  $C_j$  is the objective function coefficient.

Constraint – a restriction that applies to the choice of values for the variables.

Standard maximization problem – a linear programming problem for which the objective function is to be maximized and all the constraints are “less-than-or-equal-to” inequalities.

Slack variable – a slack variable(S) is added variable wherever we have  $\leq$  constraints.

Artificial variable - An artificial variable(A) is a variable that has no physical meaning in terms of a real-world LP problem. It simply allows us to create a basic feasible solution to start the simplex algorithm. An artificial variable is not allowed to appear in the final solution to the problem. The presence of an artificial variable in the final tableau renders the system infeasible.

Surplus variable - A surplus variable(S) does have a physical meaning, it is the amount over and above a required minimum level set on the right-hand side of a greater-than-or-equal-to constraint. To handle  $\geq$  constraints, a “surplus” variable (S) is first subtracted and then an artificial variable (A) is added to form a new equation

Augmented matrix or tableau – matrix representing a system of linear equations.

Solution mix –the set of variables with non-zero values that form the solution to the problem.

Basic variable – a variable in the solution mix. A variable is basic if it has a coefficient of 1 and any number above or below it is 0.  $C_B$  is coefficient of basic variable.

Pivot column – the column of the tableau representing the variable to be entered into the solution mix.

Pivot row – the row of the tableau representing the variable to be replaced in the solution mix.

Pivot number – the element in both the pivot column and the pivot row.

Slack and surplus variables always have a coefficient of 0 in the objective function.

The coefficient of artificial variable is  $-M$  for maximization problems and  $M$  for minimization problems, where  $M$  is a large number, eg  $10^5$

A linear programming is in standard form if it seeks to maximize the objectives function

$Z = C_1X_1 + C_2X_2 + \dots + C_nX_n$  subject to the constraints.

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \leq b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \leq b_2$$

.

.

.

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \leq b_m$$

Where  $x_i \geq 0$  and  $b_i \geq 0$ . After adding slack variables, the corresponding system of constraint equations is

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n + S_1 = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n + S_2 = b_2$$

.

.

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n + S_m = b_m \text{ Where } S_i \geq 0.$$

The following steps are used to solve a linear programming problem in standard form

1. Convert each inequality in the set of constraints to an equation by adding slack variables.
2. Create the initial simplex tableau. The current value of objective function ( $Z_j$ ) for each column is found by multiplying the  $C_j$  of the row by the number in that row and  $j$ th column and summing. Compute the net evaluation row,  $C_j - Z_j$ .
3. Locate the most negative entry in the bottom row. The column for this entry is called the entering column. (If ties occur, any of the tied entries can be used to determine the entering column.)

4. Form the ratios of the entries in the “b-column” with their corresponding positive entries in the entering column. Choose the variable with the greatest positive  $C_j - Z_j$  to enter the solution for maximization problems and the least  $C_j - Z_j$  for minimization problems. The departing row corresponds to the smallest nonnegative ratio (If all entries in the entering column are 0 or negative, then there is no maximum solution. For ties, choose either entry.) The entry in the departing row and the entering column is called the pivot.
5. Use elementary row operations so that the pivot is 1, and all other entries in the entering column are 0. This process is called pivoting. The solution is optimal if all  $C_j - Z_j$  variables are all  $\leq 0$  for maximization problems and  $\geq 0$  for minimization problems. If not, go back to Step 3.
6. If you obtain a final tableau, then the linear programming problem has maximum solution, which is given by the entry in the lower-right corner of the tableau.

### 3.7 SENSITIVITY ANALYSIS

Sensitivity analysis is a systematic study of how sensitive optimal solutions are to (small) changes in the data. The basic idea is to be able to give answers to questions of the form:

1. If the objective function changes, how does the solution change?
2. If resources available change, how does the solution change?
3. If a constraint is added to the problem, how does the solution change?

### 3.8 DEGENERACY

An LP is degenerate if in a basic feasible solution, one of the basic variables takes on a zero value. Degeneracy is caused by redundant constraint(s) and could cost simplex method extra iterations.



### 3.9 COMPUTER IMPLEMENTATION SYSTEM (SIXPAP)

The computer implementation system called Six-Pap, which is developed to run one or both algorithms, Push-and-Pull and ordinary simplex, for solving different linear programming problems. The system also creates a comparison analysis between the two algorithms. The system was designed and built with MS Visual Studio 6.0 using MS Visual Basic. The system is object oriented, its functionality is based on the object structure, that consists of main functional objects, accompanied by useful supporting objects. The system is event-driven: the objects mostly communicate amongst themselves as well as the graphical user interface elements (user controls); thus by raising and watching for certain events. User can communicate with the system through a graphical user interface. It is a multi window interface consisting of three main windows with their own functional purpose and elements. Most of elements are User Controls: controls designed to fit the exact problem or deliver the exact functionality. That enhances the usability of the graphical user interface, and accompanied with appliance of error management using error handling routines, makes the system more stable and robust. Data is organized in three groups: definition data, result summary data and result detailed data. The definition and result summary data for each linear programming problem are stored in one plain text file. The first part of the file contains general data of the problem such as name, source and some comments. The second part contains data of the objective function, number of constraints, number of variables, equations and right hand side values. The third and/or fourth parts of the file are created when one or both algorithms were executed. They represent the result summary data. The data consists of the algorithm's name, the status of the result, optimal value of objective function (if exists), basic variable set BVS values, objective function variables  $x(j)$  values, degeneracy indicator, different efficiency counters (number of iterations, additions / subtractions, multiplications / divisions, loops and

decisions), time stamp and the result detailed file name. Detailed result shows a step-by-step algorithm execution by recording relevant data like basic variable sets, tableaux, outgoing, incoming variables and comments explaining the previous and/or next action for each iteration. All data files and data operations are handled by a subsystem called the Repository. User communicates with the Repository subsystem through the graphical user interface, especially through the Repository window.

The SixPap User Interface is a multi-document interface, using an MDIForm (the Main Window) as a container for three functional windows: the Repository Window, the Single Problem Window and the Analysis Window.

- **The Repository Window**

The Repository Window serves as an interface between the user and the Repository subsystem, which, as mentioned above, handles all data storage operations. It has a problem-level approach, meaning, that the main tasks concern a problem as a whole. Those tasks are selecting, opening, creating new, renaming, duplicating, adding to analysis, and deleting problems. The Repository Window, beside a standard window bar, consists of three main elements: the Repository Menu, the Trash Bin and the Problems Repository List. The Repository Menu is a MyActiveLabels user-control with menu options that execute the Repository tasks. The Trash Bin is a MyFileListView user control. It acts as a container for deleted problems. With some Repository Menu options they can become permanently deleted or can be restored to the Problem Repository. The Problem Repository is a MyFileListView user control, too. It is the main Repository display element, showing the collection of existing problems. With other Repository Window elements it takes care of the execution of the Repository sub-system tasks.



- **The Analysis Window**

The Analysis Window is created to work with multiple problems. It consists of two main elements (beside a standard window bar, stating the number of problems included in analysis): the Analysis Window Menu and the Analysis Window Problem List. The Analysis Menu consists of two User-Controls: a MyActiveLabels and a MyCheckLabels controls. The MyCheckLabels part enables user to select one or both algorithms to be executed (or results cleared). The MyActiveLabels part takes care of carrying out the Analysis Window tasks, such as clearing the Problem List, removing a single problem from the Problem List, exporting current Problem List as a Tab-delimited text file (it is suitable for further analyzing with e.g. MS Excel), resetting results and executing (selected) methods. The Problem List is based on MyGrid user-control. Its purpose is to display multiple problems with their definitions and result summaries for each linear programming algorithm, in a single table. That helps the user to find and understand some interesting behaviours of algorithms execution. The Problem List fields are explained in detail in the SixPap on-line help.

- **Single Problem Window**

The purpose of the Single Problem Window is to help the user to work with single (selected) problem and its details more easily. The Single Problem Window consists of four main elements (beside a standard window bar with the name of currently opened problem): the Single Problem Menu, the Definition, Result Summary and Result Details area. The Single Problem Menu helps executing tasks, such as saving and printing selected problem, and executing and resetting results for selected algorithms. It consists of a MyActiveLabels and MyCheckLabels usercontrols.

The problem general variables area is located immediately under the menu. It consists of six general variables, defining the linear programming problem: the objective function, the number of variables, the number of constraints, the problem name and two comment variables (the source of the problem and a general comment the user can give to the problem). All those variables can be edited, with exception of the problem name. The Definition tab contains the Problem Definition Tableau. It is based on a MyGrid user-control, and displays the problem definition in detail.

The Tableau can be easily edited. The Result Summary tab contains the Result Summary Table (it is also based on a MyGrid user-control) where the characteristic result values for one or both algorithms are displayed. There is also a column added, that displays a calculated difference between algorithm results.

### **3.10 APPLICATION AREAS OF LINEAR PROGRAMMING**

The important application areas of linear programming include:

- **Military Applications**

Paradoxically the most appropriate example of an organization is the military and worldwide, Second World War is considered to be one of the best managed or organized events in the history of the mankind. Linear Programming is extensively used in military operations. Such applications include the problem of selecting an air weapon system against the enemy so as to keep them pinned down and at the same time minimizes the amount of fuel used. Other examples are dropping of bombs on pre-identified targets from aircraft and military assaults against localized terrorist outfits.

- **Agriculture**

Agriculture applications fall into two broad categories, farm economics and farm management. The former deals with the agricultural economy of a nation or a region, while the latter is concerned with the problems of the individual farm. Linear Programming can be gainfully utilized for agricultural planning e.g. allocating scarce limited resources such as capital, factors of production like labour, raw material etc. in such a way 'so as to maximize the net revenue.

- **Environmental Protection**

Linear programming is used to evaluate the various possible alternative for handling wastes and hazardous materials so as to satisfy the stringent provisions laid down by the countries for environmental protection. This technique also finds applications in the analysis of alternative sources of energy, paper recycling and air cleaner designs.

- **Facilities Location**

Facilities location refers to the location non-public health care facilities (hospitals, primary health centres) and public recreational facilities (parks, community halls) and other important facilities pertaining to infrastructure such as telecommunication booths etc. The analysis of facilities location can easily be done with the help of Linear Programming.

- **Product-Mix**

The product-mix of a company is the existence of various products that the company can produce and sell. However, each product in the mix requires finite amount of limited resources. Hence it is vital to determine accurately the quantity of each product to be produced knowing their profit margins and the inputs required for producing them. The

primary objective is to maximize the profits of the firm subject to the limiting factors within which it has to operate.

- **Production**

A manufacturing company is quite often faced with the situation where it can manufacture several products (in different quantities) with the use of several different machines. The problem in such a situation is to decide which course of action will maximize output and minimize the costs.

Another application area of Linear Programming in production is the assembly by-line balancing - where a component or an item can be manufactured by assembling different parts. In such situations, the objective of a Linear Programming model is to set the assembly process in the optimal (best possible) sequence so that the total elapsed time could be minimized.

- **Transportation and Trans-Shipments**

Linear Programming models are employed to determine the optimal distribution system i.e.; the best possible channels of distribution available to an organisation for its finished products at minimum total cost of transportation or shipping from company's go down to the respective markets. Sometimes the products are not transported as finished products but are required to be manufactured at various sources. In such a situation, Linear Programming helps in ascertaining the minimum cost of producing or manufacturing at the source and shipping it from there.

- **Profit Planning and Contract**

Linear Programming is also quite useful in profit planning and control. The objective is to maximize the profit margin from investment in the plant facilities and machinery, cash on hand and stocking-hand.

- **Traveling Salesmen Problem**

Traveling salesmen problem refers to the problem of a salesman to find the shortest route originating from a particular city, visiting each of the specified cities and then returning back to the originating point of departure. The restriction being that no city must be visited more than once during a particular tour. Such types of problems can quite easily be solved with the help of Linear Programming.

- **Media Selection/Evaluation**

Media selection means the selection of the optimal media-mix so as to maximise the effective exposure. The various constraints in this case are: Budget limitation, different rates for different media (i.e.; print media, electronic media like radio and T.V. etc.) and the minimum number of repeated advertisements (runs) in the various media. The use of Linear Programming facilitates the decision making process.

- **Staffing**

Staffing or the man-power costs are substantial for a typical organisation which make its products or services very costly. Linear Programming techniques help in allocating the optimum employees (man-power or man-hours) to the job at hand. The overall objective is to minimize the total man-power or overtime costs.

- **Wages and Salary Administration**

Determination of equitable salaries and various incentives and perks becomes easier with the application of Linear Programming. LP tools” can also be utilized to provide optimal solutions in other areas of personnel management such as training and development and recruitment etc.

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## **CHAPTER FOUR**

### **DATA COLLECTION AND ANALYSIS**

#### **4.0 INTRODUCTION**

The first part of this chapter looks at the profile of Ghana Water Company Limited, the history and private sector participation in the urban water supply in Ghana. It also looks at the factors affecting water treatment cost and water quality and explains the water treatment process at Kpong Headworks.

The second part deals with the collection and analysis of data, model formulation and using the Six-Pap software for the analysis of results. Sensitivity analysis is also done on the model.

#### **4.1 PROFILE OF GHANA WATER COMPANY LIMITED**

Ghana Water Company Limited was established on 1<sup>st</sup> July 1999, following the conversion of Ghana Water and Sewerage Corporation into a state-owned limited liability company under the statutory Corporations (Conversion to Companies) Act 461 of 1993 as amended by LI 1648.

Presently the Company operates 83 urban water supply systems throughout the country. The installed capacity of all the systems is about 949,000m<sup>3</sup>/day. Present potable water demand in the urban areas is estimated at about 1,101,032m<sup>3</sup>/day whilst average daily production is about 691,690.90m<sup>3</sup>/day. Effective urban supply coverage is about 63%. Customer strength is currently 421,363 of which 42.8% are metered and 57.2% unmetered. (GWCL, 2012)

## **4.2 HISTORY OF WATER SUPPLY IN GHANA**

The first public water supply system in Ghana, then Gold Coast, was established in Accra just before World War I. Extensions were made exclusively to other urban areas among them the colonial capital of Cape Coast, Winneba and Kumasi in the 1920s.

During this period, the water supply systems were managed by the Hydraulic Division of Public Works Department. With time the responsibilities of the Hydraulic Division were widened to include the planning and development of water supply systems in other parts of the country.

In 1948, the Department of Rural Water Development was established to engage in the development and management of rural water supply through the drilling of bore holes and construction of wells for rural communities. After Ghana's independence in 1957, a Water Supply Division, with headquarters in Kumasi, was set up under the Ministry of Works and Housing with responsibilities for both urban and rural water supplies. During the dry season of 1959, there was severe water shortage in the country. Following this crisis, an agreement was signed between the Government of Ghana and the World Health Organisation (WHO) for a study to be conducted into the water sector development of the country. The study focused not only on technical engineering but also on the organisation of a national water and sewerage authority and methods of financing. Furthermore the study recommended the preparation of a Master Plan for water supply and sewerage services in Accra-Tema covering the twenty-year period 1960 to 1980.

In line with the recommendations of the WHO, the Ghana Water and Sewerage Corporation (GWSC), was established in 1965 under an Act of Parliament (Act 310) as a legal public utility entity. GWSC was to be responsible for:

- Water supply and sanitation in rural as well as urban areas.
- The conduct of research on water and sewerage as well as the making of engineering surveys and plans.
- The construction and operation of water and sewerage works,
- The setting of standards and prices and collection of revenues.

In the late 1970s and early 1980s, the operational efficiency of GWSC declined to very low levels mainly as a result of the deterioration in pipe connections and pumping systems. A World Bank report in 1998 states that: “The water supply systems in Ghana deteriorated rapidly during the economic crises of the 1970’s and early 1980’s when Government’s ability to adequately operate and maintain essential services was severely constrained.”

In 1957, there were 35 pipe-borne water supply systems in the country. The number of pipe-borne systems rose to 69 in 1961 and then to 194 in 1979. At this time, there were 2,500 hand pumped borehole systems in the country and by 1984, additional 3000 boreholes had been drilled and fitted with hand pumps. However by the late 1980s and early 1990s, 33% of the water supply systems had deteriorated greatly or completely broken down due to inadequate funding to carry out maintenance and rehabilitation.

To reverse the decline in water supply services, interventions in the area of sector reforms and project implementation were made in 1970, 1981 and 1988. These included interventions by the World Bank, IDA, donor countries and other external support agencies such as Austrian Government, Italian Government, Nordic Development Fund, the African Development Bank, Canadian International Development Agency, Department for International Development, KfW, GTZ, OECF, ECGD and CFD/ADF.

Though some gains were derived from these interventions, their general impact on service delivery was very disappointing. Due to the failure of these interventions to achieve the

needed results, several efforts were made to improve efficiency within the water supply sector in Ghana especially during the era of the Economic Recovery Programme from 1983 to 1993.

During this period, loans and grants were sought from the World Bank and other donors for the initiation of rehabilitation and expansion programmes, to train personnel and to buy transport and maintenance equipment. In addition, user fees for water supply were increased and subsidies on water tariffs were gradually removed for GWSC to achieve self-financing. Although subvention for both operational and developmental programmes was withdrawn in 1986, government funding for development programmes continued. The government at that time approved a formula for annual tariff adjustments to enable the corporation generate sufficient funds to cover all annual recurrent costs as well as attain some capacity to undertake development projects.

In 1987, a “Five-Year Rehabilitation and Development Plan” for the sector was prepared which resulted in the launching of the Water Sector Restructuring Project (WSRP). Multilateral and bilateral donors contributed \$140 million to support the implementation of the WSRP. The reforms were aimed at reducing unaccounted for water, introducing rationalisation through reduction of the workforce, hiring of professionals and training of the remaining staff. A strong focus in the WSRP was also on improved management and increased efficiency through organisational change of the water sector. Accordingly, a number of organisational reforms within the Ghanaian water sector were initiated in the early 1990s. As a first step, responsibilities for sanitation and small towns water supply were decentralized from Ghana Water and Sewerage Corporation to the District Assemblies in 1993.

The Environmental Protection Agency (EPA) was established in 1994 to ensure that water operations did not cause any harm to the environment. The Water Resources Commission (WRC) was founded in 1996 to be in charge of overall regulation and management of water resources utilization.

In 1997, the Public Utilities Regulatory Commission (PURC) came into being with the purpose of setting tariffs and quality standards for the operation of public utilities.

With the passage of Act 564 of 1998, Community Water and Sanitation Agency (CWSA) was established to be responsible for management of rural water supply systems, hygiene education and provision of sanitary facilities. After the establishment of CWSA, 120 water supply systems serving small towns and rural communities were transferred to the District Assemblies and Communities to manage under the community-ownership and management scheme.

Finally, pursuant to the Statutory Corporations (Conversion to Companies) Act 461 of 1993 as amended by LI 1648, on 1st July 1999, GWSC was converted into a 100% state owned limited liability, Ghana Water Company Limited, with the responsibility for urban water supply only.

In general, GWCL is responsible for:

- The planning and development of water supply systems in urban communities in the country.
- The provision and maintenance of acceptable levels of service to consumers in respect of quantity and quality of water supplied.



- Contracting for the design and construction, rehabilitation and expansion of existing as well as new works.
- The preparation of long term plans in consultation with appropriate coordinating authority established by the president
- The conduct of research and engineering surveys relative to water and related subjects.
- The conduct of other related or incidental activities. (GWCL, 2012)

### **4.3 PRIVATE SECTOR PARTICIPATION IN URBAN WATER DELIVERY IN GHANA**

On 22<sup>nd</sup> November 2005, GWCL signed a Management Contract with Vitens Rand Water Services BV of Netherlands, a consortium of Vitens International BV of the Royal Netherlands and Rand Water Services Pty of South Africa. Under the Management Contract which commenced on 6<sup>th</sup> June 2006, Vitens Rand Water Services BV, through its subsidiary, Aqua Vitens Rand Limited, operated the urban water systems for five years.

The management contract came to an end on 6<sup>th</sup> June 2011 and all the performance indicators showed that private involvement in the operations of GWCL did not bring about the expected positive improvement in urban water supply in Ghana.

Review of official documents and technical and audit reports of GWCL, AVRL and other independent institutions such as Fichtner/Hytas/Watertech and State Enterprises Commission proved that during the management contract period, the level of performance in almost all the systems was poor especially in respect of reduction in non revenue water, treatment plant operations, customer response plan, customer accounts receivable, customer collection, chemical usage, power consumption and public water consumption.



Presently a new company, Ghana Urban Water Limited, has been formed by government to take over the management of urban water systems in the country. According to the ministry of Water Resources Works and Housing, ‘the move is the most attractive short term option to allow government to take stock and seamlessly manage the operations of the urban water systems for a period of only 12 months.’ (GWCL, 2012)

**TABLE 4.1: RURAL-URBAN WATER COVERAGE BY REGION, 2010**

<b>Region</b>	<b>Estimated Rural Population</b>	<b>% Covered in 2010</b>	<b>Estimated Urban Population</b>	<b>% Covered in 2010</b>
<b>Ashanti</b>	3,265,624	72.64	2,617,060	39
<b>Brong Ahafo</b>	1,975,833	55.88	575,510	39
<b>Central</b>	1,559,278	56.77	1,097,440	51
<b>Eastern</b>	1,642,518	58.58	1,116,021	36
<b>Gt. Accra</b>	699,545	58.95	950,746	74
<b>Northern</b>	2,151,632	60.68	580,886	68
<b>Upper East</b>	1,187,524	59.22	185,529	41
<b>Upper West</b>	625,355	76.94	128,492	10
<b>Volta</b>	1,776,776	63.08	490,980	46
<b>Western</b>	1,692,083	52.45	665,764	60
<b>National</b>	<b>16,576,168</b>	<b>61.74</b>	<b>11,408,428</b>	<b>58</b>

Source: MWRWH/CWSA, 2010

#### **4.4 WATER QUALITY REQUIREMENTS FOR DRINKING WATER – GHANA STANDARDS**

The Ghana Standards for drinking water (GS 175-Part 1:1998) indicate the required physical, chemical, microbial and radiological properties of drinking water. The standards are adapted from the World Health Organizations Guidelines for Drinking Water Quality, Second Edition, Volume 1, 1993, but also incorporate national standards that are specific to the country's environment.

##### **4.4.1 PHYSICAL REQUIREMENTS**

The Ghana Standards set the maximum turbidity of drinking water at 5 NTU. Other physical requirements pertain to temperature, odor, taste and color. Temperature, odor and taste are generally not to be “objectionable”, while the maximum threshold values for color are given quantitatively as True Color Units (TCU) or Hazen units. The Ghana Standards specify 15 TCU or 15 Hazen units for color after filtration. The requirements for pH values set by the Ghana Standards for drinking water is 6.5 to 8.5 (GS 175-Part 1:1998).

##### **4.4.2 MICROBIAL REQUIREMENTS**

The Ghana Standards specify that E.coli or thermotolerant bacteria and total coliform bacteria should not be detected in a 100ml sample of drinking water (0 CFU/100ml). The Ghana Standards also specify that drinking water should be free of human enteroviruses.

#### **4.5 FACTORS AFFECTING WATER QUALITY**

The factors which affects water quality includes:

- Natural contaminants

This includes contamination caused due to dried leaves, dead insects, bird droppings, animal faeces reaching the natural sources of water.

- Agricultural contaminants

These are the factors like agricultural runoffs, fertilizers, cleansers which reach the natural source of water and pollute it. Pesticides used on the crops also eventually seep down and contaminate the ground water.

- Industrial contaminants

There are various hazardous chemicals which also pollute the ground water by seeping in along with rain water.

- Microbial contaminants

These are the contaminants like bacteria, viruses, cysts which comfortably dwell in the old and rusty industrial pipes and when water travels through these pipes to reach your home, they get added to it. There are other contaminants like algae and traces of rust which also get added in similar manner.

#### **4.6 FACTORS AFFECTING WATER TREATMENT COST**

The factors which influence the treatment of water include:

- Cost of personnel (labour)
- Cost of electricity
- Cost of chemicals for treatment
- Cost of fuel and lubricants
- Cost of repairs and maintenance
- Cost of raw water source
- Cost of other contractual
- Cost of civil structures of the treatment plant. (Adombire M.A, 2007)

#### **4.7 WATER TREATMENT PROCESS AT KPONG HEADWORKS**

Water is extracted from a reservoir impounded between the Akosombo and Akuse dams on the River Volta. Screening is necessary at the intake to remove floating debris from the source water such as dead woods, leaves and rags. This water is pumped at a rate of to both Kpong Old Water treatment plant (with a capacity of about 33,750 m<sup>3</sup> per day) and Kpong New Water treatment plant (with a capacity of about 180,000 m<sup>3</sup> per day). The raw water quality is very good, on average having turbidity of less than 3NTU and colour below 5HU.

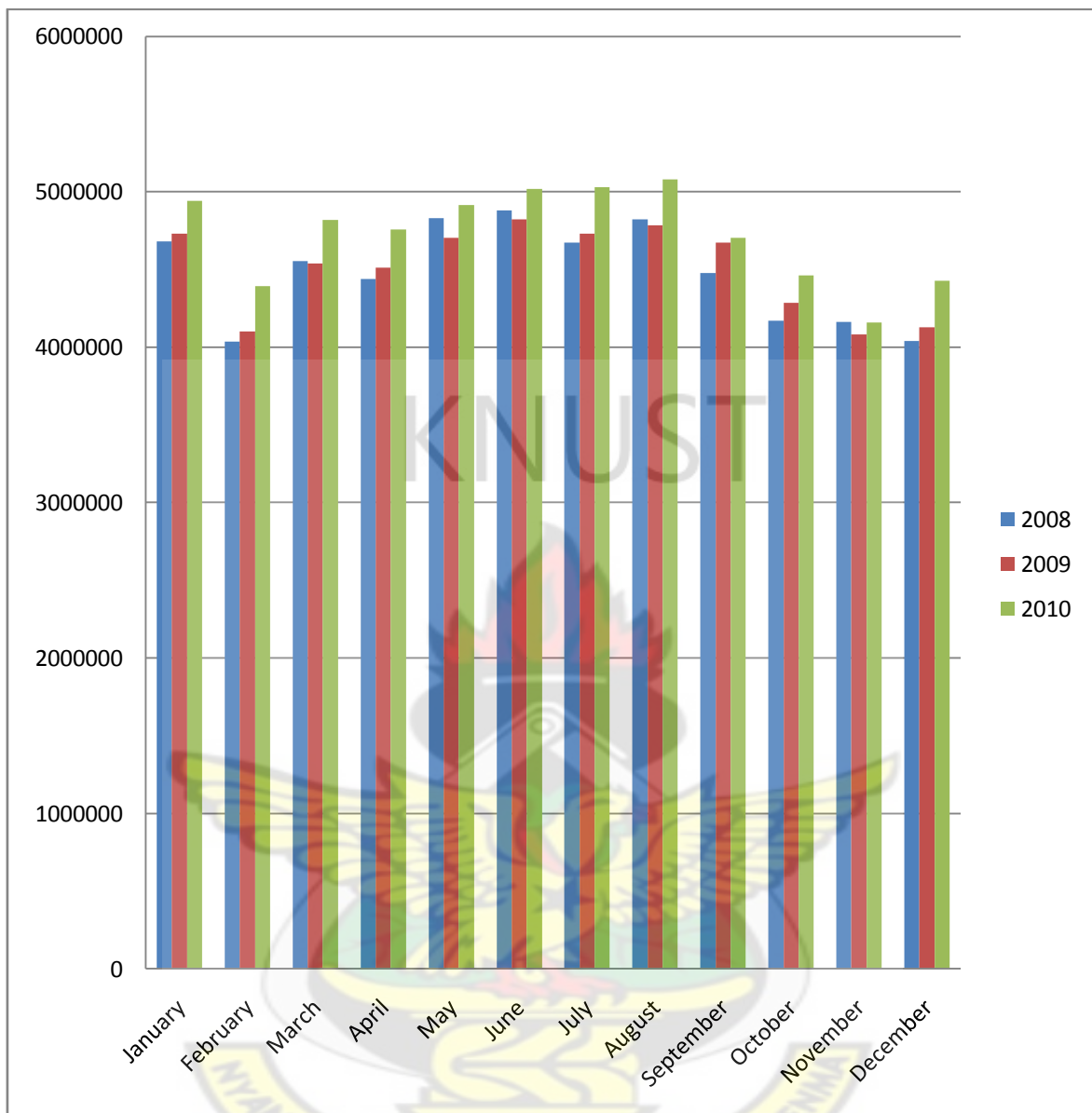
The raw water flows directly into the distribution chamber where the water is distributed to three clarifiers. The clarified water flows is run through a series of filters which trap and remove particles still remaining in the water column. Typically, beds of sand are used to accomplish this task. The filtered water is collected into the clear well where lime is added for P<sup>H</sup> adjustment. Chlorine or Calcium hypochlorite is also added for disinfection. Water coming out of a filter unit, may contain bacteria and other micro-organisms, some of which may be pathogenic. Disinfection kills the microbes, making the water safe to drink and preventing water-borne diseases. High lift pumps then transfers the water to consumers.

#### **4.8 DATA ANALYSIS**

The model has been divided into two parts. The first part is the dry season which is from November to March and the second is the wet season which is from April to October.

**TABLE 4.2: GWCL WATER PRODUCTION AT KPONG TREATMENT PLANT  
FROM 2008 TO 2010**

<b>MONTH/YR</b>	<b>2008 (m<sup>3</sup>)</b>	<b>2009 (m<sup>3</sup>)</b>	<b>2010(m<sup>3</sup>)</b>
<b>January</b>	4681072	4730712	4942043
<b>February</b>	4037401	4101847	4393236
<b>March</b>	4552710	4539160	4819182
<b>April</b>	4440250	4510284	4757565
<b>May</b>	4831028	4705183	4916207
<b>June</b>	4881202	4820822	5016880
<b>July</b>	4672235	4730173	5028864
<b>August</b>	4820713	4784024	5077559
<b>September</b>	4476146	4674269	4702978
<b>October</b>	4169173	4286107	4460300
<b>November</b>	4164252	4082635	4158815
<b>December</b>	4038205	4126304	4427637
<b>Total</b>	53764387	54091520	56701266
<b>Average</b>	<b>4480365.6</b>	<b>4507626.7</b>	<b>4725105.5</b>



**FIGURE 4.1: MONTHLY WATER PRODUCTION AT KPONG TREATMENT PLANT FROM 2008 TO 2010**



**TABLE 4.3: QUANTITY OF CHEMICALS, ELECTRICITY AND FUEL IN WATER TREATMENT FOR 2010**

<b>MONTH</b>	<b>Chemical(Drums)</b>	<b>Electricity (KWh)</b>	<b>Fuel (Litres)</b>
<b>January</b>	188	5501000	5627
<b>February</b>	174	4978000	5634
<b>March</b>	189	5343000	5696
<b>April</b>	183	5452000	5674
<b>May</b>	190	5899000	5684
<b>June</b>	185	6158000	5624
<b>July</b>	188	6321000	5583
<b>August</b>	191	6214000	5621
<b>September</b>	185	5752000	5643
<b>October</b>	189	5530000	5586
<b>November</b>	184	5233000	5636
<b>December</b>	187	5399424	5658
<b>Total</b>	2233	67780424	67666
<b>Average</b>	<b>186.08</b>	<b>5648368.67</b>	<b>5638.80</b>

**TABLE 4.4: COST OF CHEMICALS, ELECTRICITY AND FUEL IN WATER TREATMENT FOR 2010**

<b>MONTH</b>	<b>Chemical Cost (Gh¢)</b>	<b>Electricity Cost (Gh¢)</b>	<b>Fuel Cost (Gh¢)</b>	<b>Total Cost (Gh¢)</b>
<b>January</b>	19035	621086	6639.86	646760.86
<b>February</b>	17852.40	569260	6693.19	593805.59
<b>March</b>	19561.50	604854	6738.37	631153.87
<b>April</b>	19105.20	615659	6706.67	641470.87
<b>May</b>	20092.50	656919	6746.91	683758.41
<b>June</b>	19813.50	2084809	6658.82	2111281.32
<b>July</b>	20177.10	2134757	6587.94	2161522.04
<b>August</b>	20542.10	2107432	6660.89	2134634.99
<b>September</b>	19855.13	1958508	6743.39	19856.5210
<b>October</b>	20326.95	1888516	6703.2	1915546.15
<b>November</b>	19789.20	1796888	6706.84	1823384.04
<b>December</b>	20153.93	1854034	7015.92	1881203.85
<b>Total</b>	<b>236304.51</b>	<b>16892722</b>	<b>80602</b>	<b>17209628.51</b>
<b>Average</b>	<b>19692.04</b>	<b>1407726.83</b>	<b>6716.83</b>	<b>1434135.71</b>
<b>Ratio/Unit</b>	<b>105.83</b>	<b>0.25</b>	<b>1.19</b>	

#### 4.9 MODEL FORMULATION

The chemical cost, electricity cost and fuel cost(including other lubricants) which affect the total treatment cost are used to formulate the objective function.

Thus, Total treatment cost ( $C_T$ ) = C ( $C_C$ ,  $E_C$ ,  $F_C$ )

Where

$C_C$  is the chemical cost.

$E_C$  is the electricity cost.

$F_C$  is the fuel cost.

But Cost = Quantity x Price

Therefore, Total Treatment Cost ( $C_T$ ) =  $\sum_i^n W_i X_i = W_1 X_1 + W_2 X_2 + W_3 X_3$

Where  $W_1$  = Unit Chemical Price

$W_2$  = Unit Electricity Price

$W_3$  = Unit Fuel Price

$$W_1 = \frac{\text{Average price of chemical}}{\text{Average drum of chemical}} = \frac{19692.04}{186.08} = 105.83$$

$$W_2 = \frac{\text{Average price of electricity}}{\text{Average unit of electricity}} = \frac{1407726.83}{5648368.67} = 0.2492 = 0.25$$

$$W_3 = \frac{\text{Average price of fuel}}{\text{Average litre of fuel}} = \frac{6716.83}{5638.80} = 1.19$$

The Objective function is

Minimize Treatment Cost ( $C_T$ ) =  $\sum_i^n W_i X_i = W_1 X_1 + W_2 X_2 + W_3 X_3$

Minimize  $C_T = 105.83X_1 + 0.25X_2 + 1.19X_3$

In order to determine the constraints to the objective function the following must be calculated for the two major seasons in the year.

$$\text{Average Seasonal Quantity/Cost} = \frac{\text{Total Monthly Quantity/Cost}}{\text{Number of Months}}$$

$$\text{Usage Ratio} = \frac{\text{Seasonal Cost}}{\text{Seasonal Quantity}}$$

**TABLE 4.5: AVERAGE SEASONAL QUANTITIES OF CHEMICAL, ELECTRICITY AND FUEL FOR 2010**

Season	Chemical (Drums)	Electricity (KWh)	Fuel (Litres)
Dry	184.4	5290884.8	5650.2
Wet	187.29	5903714.29	5630.7

**TABLE 4.6: AVERAGE SEASONAL COST OF CHEMICAL, ELECTRICITY, AND FUEL FOR 2010**

Season	Chemical (Gh¢)	Electricity (Gh¢)	Fuel (Gh¢)
Dry	19278.41	1089224.4	6758.84
Wet	19987.50	1635228.57	6686.83

**TABLE 4.7: USAGE RATIOS OF CHEMICAL, ELECTRICITY AND FUEL FOR 2010**

Season	Chemical	Electricity	Fuel
Dry	104.5467	0.21	1.196213
Wet	106.7195	0.27698	1.187566

**TABLE 4.8: DRY SEASONAL ALLOCATION OF CHEMICALS, ELECTRICITY AND FUEL FOR 2010**

	<b>Chemical House</b>	<b>Pump House</b>	<b>Transport</b>	<b>Total</b>
<b>Chemical Cost</b>	17134.97	0	0	17134.97
<b>Electricity Cost</b>	0	961734	0	961734
<b>Fuel Cost</b>	1982.73	2399.74	3354.12	7736.59
<b>Total</b>	<b>19117.7</b>	<b>964133.74</b>	<b>3354.12</b>	

**TABLE 4.9: WET SEASONAL ALLOCATION OF CHEMICAL, ELECTRICITY AND FUEL FOR 2010**

	<b>Chemical House</b>	<b>Pump House</b>	<b>Transport</b>	<b>Total</b>
<b>Chemical Cost</b>	17134.97	0	0	17134.97
<b>Electricity Cost</b>	0	1305488.59	0	1305488.59
<b>Fuel Cost</b>	1982.73	2399.74	3354.12	7736.59
<b>Total</b>	<b>19117.7</b>	<b>1307888.3</b>	<b>3354.12</b>	

**TABLE 4.10: COST/QUANTITY (QTY) OF FUEL IN BOTH DRY AND WET SEASON FOR 2010**

	Chemical House		Pump House		Transport		Total
	Cost Gh¢	Qty	Cost Gh¢	Qty	Cost Gh¢	Qty	Cost Gh¢
<b>Dry</b>	8508.16	6805.53	9967.29	8637.17	15318.73	12808.30	33794.2
<b>Wet</b>	12746.34	10078.32	13952.91	11774.61	20108.57	17562.07	46807.8

**TABLE 4.11: USAGE RATIO OF FUEL IN CHEMICAL HOUSE, PUMP HOUSE AND TRANSPORTATION IN BOTH DRY AND WET SEASON FOR 2010**

	Chemical House	Pump House	Transport
<b>Dry</b>	1.2502	1.1540	1.1960
<b>Wet</b>	1.2647	1.1850	1.1450

The constraints to the objective function are expressed in the form of inequalities which includes:

- **Chemical House**

Product of usage ratio of chemical by the number of drums used +

Product of usage ratio of fuel used in chemical house by the number of litres used  $\geq$

Least seasonal cost for chemical house.

- **Pump House**

Product of usage ratio of electricity by the number of electrical units used +



Product of usage ratio of fuel used in pump house by the number of litres used  $\geq$   
Least seasonal cost for pump house.

- **Transport**

Product of usage ratio of fuel by the number of litres used  $\leq$   
Seasonal cost for transport.

- Nonnegativity constraints on the variables.

Therefore the models for the two seasons are:

Model for dry season

$$\text{Minimize } C_T = 105.83X_1 + 0.25X_2 + 1.19X_3$$

Subject to:-

$$104.5467X_1 + 1.2502X_3 \geq 19117.7$$

$$0.21X_2 + 1.1540X_3 \geq 964133.74$$

$$1.1960X_3 \leq 3354.12$$

$$X_1, X_2, X_3 \geq 0$$

Model for wet season

$$\text{Minimize } C_T = 105.83X_1 + 0.25X_2 + 1.19X_3$$

Subject to:-

$$106.7195X_1 + 1.2647X_3 \geq 19117.7$$

$$0.27698X_2 + 1.1850X_3 \geq 1307888.3$$

$$1.1450X_3 \leq 3354.12$$

$$X_1, X_2, X_3 \geq 0$$

#### 4.9.1 WATER TREATMENT COST FOR DRY SEASON

Analysis of results by the use of Computer Implementation System (SixPap)

##### S T D . S I M P L E X

Number of variables : 3

Number of constraints : 3

Problem Definition:

$$\text{MIN: } 105.83X_1 + 0.25X_2 + 1.19X_3$$

$$\text{Constr.: } 104.5467X_1 + 0 X_2 + 1.2502X_3 \geq 19117.7$$

$$0 X_1 + 0.21X_2 + 1.1540X_3 \geq 964133.74$$

$$0 X_1 + 0 X_2 + 1.1960X_3 \leq 3354.12$$

$$X_1, X_2, X_3 \geq 0$$

Initial tableau

i	BVS	1	2	3	4S	5S	6S	7A	8A	RHS
1	7	104.547	0	1.2502	-1	0	0	1	0	19,117.7
2	8	0	0.21	1.154	0	-1	0	0	1	9.64e+05
3	6	0	0	1.196	0	0	1	0	0	3,354.12
Zj		1.11e+05	221.99	2.54e+03	-1058.3	-1058.3	0	0	0	1.04e+09

Zj positive does exist.

Iteration No.: 1

k= 1, r= 1

i	BVS	1	2	3	4S	5S	6S	7A	8A	RHS
1	1	1	0	0.012	-0.0096	0	0	0.0096	0	182.8628
2	8	0	0.21	1.154	0	-1	0	0	1	9.64e+05
3	6	0	0	1.196	0	0	1	0	0	3,354.12
Zj		0	221.993	1.22e+03	-1.0123	-1.058.3	0	-1.06e+03	0	1.02e+09

Zj positive does exist.

Iteration No.: 2

k= 3, r= 3

i	BVS	1	2	3	4S	5S	6S	7A	8A	RHS
1	1	1	0	0	-0.0096	0	-0.01	0.0096	0	149.3264
2	8	0	0.21	0	0	-1	-0.9649	0	1	9.61e+05
3	3	0	0	1	0	0	0.8361	0	0	2.80e+03
Zj		0	221.993	0	-1.0123	-1.058.3	-1.20e+03	-1.06e+03	0	1.02e+09

Zj positive does exist.

Iteration No.: 3

k= 2, r= 2

i	BVS	1	2	3	4S	5S	6S	7A	8A	RHS
1	1	1	0	0	-0.0096	0	-0.01	0.0096	0	149.3264
2	2	0	1	0	0	-4.7619	-4.5947	0	4.7619	4.58e+06
3	3	0	0	1	0	0	0.8361	0	0	2.80e+03
Zj		0	0	0	-1.0123	-1.1905	-1.2118	-1.06e+03	-1.06e+03	1.16e+06

Zj positive does NOT exist.

## RESULT TESTING

constraint no. 1 TRUE :  $19,117.7 \geq 19,117.7$

constraint no. 2 TRUE :  $9.64e+05 \geq 9.64e+05$

constraint no. 3 TRUE :  $3,354.12 \leq 3,354.12$

Test result: OK

## SUMMARY

Status: OPTIMAL

Degeneracy: NO

Test to constraints: OK

MIN:  $1.16e+06$

BVS: [1, 2, 3 ]

$x(1) = 149.3264$

$x(2) = 4.58e+06$

$x(3) = 2.80e+03$

Number of iterations: 3

Number of additions/subtractions: 202

Number of multiplications/divisions: 247

Number of loops: 246

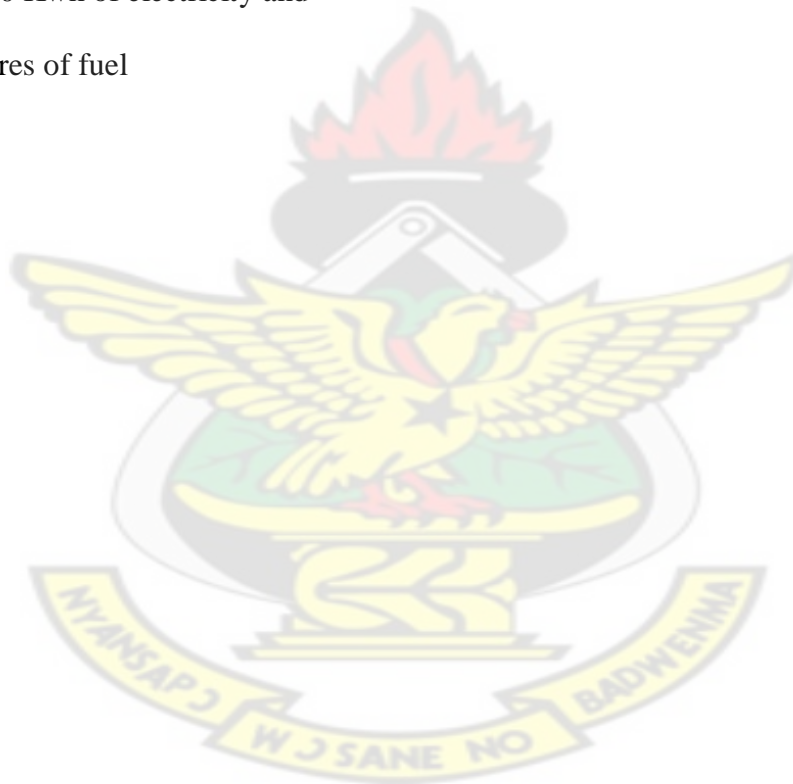
Number of decisions: 232

Therefore the total cost for water treatment in dry season is Gh¢1160000.00, with

$X_1 = 149.3264$  drums of chemicals

$X_2 = 4580000$  Kwh of electricity and

$X_3 = 2800$  litres of fuel



#### 4.9.2 WATER TREATMENT COST FOR WET SEASON

##### S T D . S I M P L E X

Number of variables : 3

Number of constraints : 3

Problem Definition:

$$\text{MIN: } 105.83X_1 + 0.25X_2 + 1.19X_3$$

$$\text{Constr.: } 106.7195X_1 + 0X_2 + 1.2647X_3 \geq 19117.7$$

$$0X_1 + 0.27698X_2 + 1.1850X_3 \geq 1307888.3$$

$$0X_1 + 0X_2 + 1.1450X_3 \leq 3354.12$$

$$X_1, X_2, X_3 \geq 0$$

Initial tableau

i	BVS	1	2	3	4S	5S	6S	7A	8A	RHS
1	7	106.7195	0	1.2647	-1	0	0	1	0	19,117.7
2	8	0	0.277	1.185	0	-1	0	0	1	1.31e+06
3	6	0	0	1.145	0	0	1	0	0	3,354.12
Zj		1.13e+05	292.878	2.59e+03	-1,058.3	-1,058.3	0	0	0	1.40e+09

Zj positive does exist.



Iteration No.: 1

k= 1, r= 1

i	BVS	1	2	3	4S	5S	6S	7A	8A	RHS
1	1	1	0	0.0119	-0.0094	0	0	0.0094	0	179.1397
2	8	0	0.277	1.185	0	-1	0	0	1	1.31e+06
3	6	0	0	1.145	0	0	1	0	0	3,354.12
Zj		0	292.8779	1.25e+03	-0.9917	-1,058.3	0	-1.06e+03	0	1.38e+09

Zj positive does exist.

Iteration No.: 2

k= 3, r= 3

i	BVS	1	2	3	4S	5S	6S	7A	8A	RHS
1	1	1	0	0	-0.0094	0	-0.0103	0.0094	0	144.4247
2	8	0	0.277	0	0	-1	-1.0349	0	1	1.30e+06
3	3	0	0	1	0	0	0.8734	0	0	2.93e+03
Zj		0	292.8779	0	-0.9917	-1,058.3	-1.10e+03	-1.06e+03	0	1.38e+09

Zj positive does exist.

Iteration No.: 3k= 2, r= 2

i	BVS	1	2	3	4S	5S	6S	7A	8A	RHS
1	1	1	0	0	-0.0094	0	-0.0103	0.0094	0	144.4247
2	2	0	1	0	0	-3.6104	-3.7365	0	3.6104	4.71e+06
3	3	0	0	1	0	0	0.8734	0	0	2.93e+03
Zj		0	0	0	-0.9917	-0.9026	-0.9902	-1.06e+03	-1.06e+03	1.20e+06

Zj positive does NOT exist.

## RESULT TESTING

constraint no. 1 TRUE :  $19,117.7 \geq 19,117.7$

constraint no. 2 TRUE :  $1.31e+06 \geq 1.31e+06$

constraint no. 3 TRUE :  $3,354.12 \leq 3,354.12$

Test result: OK

## SUMMARY

Status: OPTIMAL

Degeneracy: NO

Test to constraints: OK

MIN:  $1.20e+06$

BVS: [1, 2, 3 ]

$x(1) = 144.4247$

$x(2) = 4.71e+06$

$x(3) = 2.93e+03$

Number of iterations: 3

Number of additions/subtractions: 202

Number of multiplications/divisions: 247

Number of loops: 246

Number of decisions: 232

Therefore the total cost for water treatment in wet season is Gh¢1200000.00, with

$X_1 = 144.4247$  drums of chemicals

$X_2 = 4710000$  Kwh of electricity and

$X_3 = 2930$  litres of fuel

Optimal cost for dry season : Gh¢ 1160000.00

Optimal cost for wet season : Gh¢ 1200000.00

Old cost for dry season : Gh¢ 5576308.21

Old cost for wet season : Gh¢ 11633320.3



#### 4.9.3 SENSITIVITY ANALYSIS OF THE MODEL

Below is a sensitivity analysis on the model for the two seasons.

Old cost for dry season : Gh¢ 5576308.21

Old cost for wet season : Gh¢ 11633320.3

**TABLE 4.12: SENSITIVITY ANALYSIS OF THE MODEL**

CHANGE IN UNIT	UNIT CHEMICAL COST	UNIT ELECTRICITY COST	UNIT FUEL COST	OPTIMAL COST (DRY SEASON)	OPTIMAL COST (WET SEASON)
0	105.83	0.25	1.19	1160000.00	1200000.00
0.01	105.84	0.26	1.20	1210000.00	1240000.00
0.03	105.86	0.28	1.22	1300000.00	1340000.00
0.08	105.91	0.33	1.27	1530000.00	1570000.00
0.40	106.23	0.65	1.59	2990000.00	3080000.00
0.80	106.63	1.05	1.99	4830000.00	4970000.00
0.90	106.73	1.15	2.09	5280000.00	5440000.00
1.00	106.83	1.25	2.19	5740000.00	5910000.00
1.20	107.03	1.45	2.39	6660000.00	6850000.00
1.80	107.63	2.05	2.99	9400000.00	9680000.00
2.00	107.83	2.25	3.19	10300000.00	10600000.00
2.20	108.03	2.45	3.39	11200000.00	11600000.00
2.25	108.08	2.50	3.44	11500000.00	11800000.00
2.50	108.33	2.75	3.69	12600000.00	13000000.00
3.00	108.83	3.25	4.19	14900000.00	15300000.00

From table 4.12 above, it can be observed that the model for the dry season will not minimize the total cost for treating water in dry season if the unit price of a drum of chemical, unit price of electricity and price of a litre of fuel in the objective function is increased by Gh¢ 1.00.

Similarly, the model for wet season will not also minimize the total cost for treating water if factors such as unit price of a drum of chemical, unit price of electricity and the price of a litre of fuel are increased by Gh¢ 2.25.



## **CHAPTER FIVE**

### **CONCLUSIONS AND RECOMMENDATIONS**

#### **5.0 INTRODUCTION**

This chapter looks at the summary of the research findings, conclusions and recommendations.

#### **5.1 SUMMARY OF FINDINGS**

It can be observed from the analysis carried out in chapter four that the average seasonal water treatment cost of Gh¢ 5576308.21 for dry season can be optimized to Gh¢ 1160000.00.

Similarly, the average seasonal water treatment cost of Gh¢ 11633320.30 for wet season can also be optimized to Gh¢ 1200000.00.

#### **5.2 CONCLUSIONS**

The study has shown that the cost of treating water at Kpong Headworks can be reduced with respect to the most influential factors which affect treatment cost, namely cost of chemical, cost of electricity and the cost of fuel.

#### **5.3 RECOMMENDATIONS**

It is recommended that a treatment cost of Gh¢ 1160000.00 in the dry season and Gh¢ 1200000.00 in the wet season can pump the same volume of water to consumers at the reduce cost of Gh¢ 4416308.21 and Gh¢ 10433320.3 less in the dry and wet seasons respectively.



To reduce costs associated with chemicals, it is recommended that chlorine residual in the final treated water be monitored to keep it at an appreciable value. Chlorine residual in the final treated water at the Headworks should be in the range of 0.5-2mg/L.

Also, capacitor banks should be installed on all electrical motors to improve the power factor to avoid paying power factor surcharges, thus reducing costs associated with electricity usage.

#### **5.4 AREAS FOR FURTHER STUDIES**

The following areas can be researched into:

- (i) Improving operation of drinking water treatment through modelling.
- (ii) Optimal design of water distribution system
- (iii) Water distribution system operation



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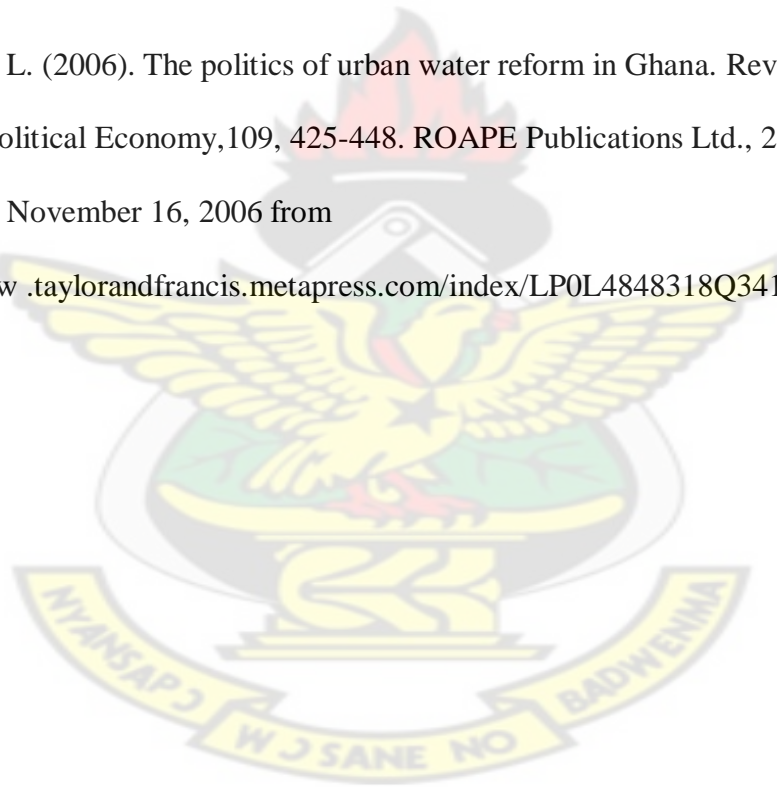
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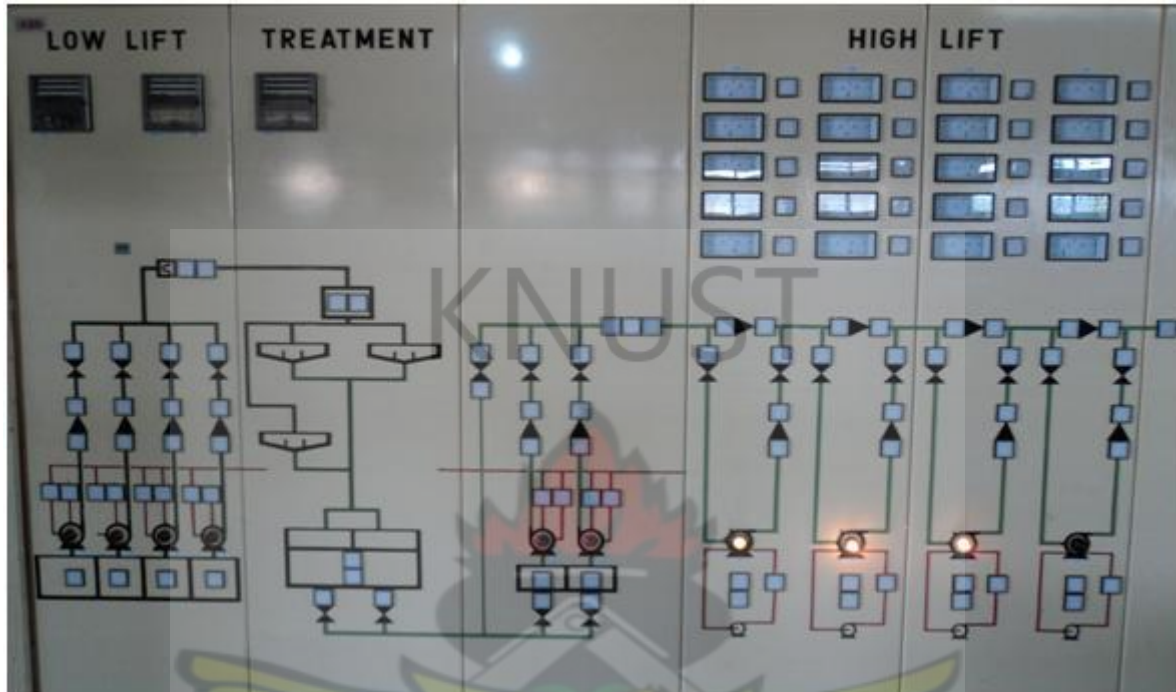
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## APPENDIX

### A: THE DESIGN OF KPONG WATER TREATMENT PLANT



### B: HIGH LIFT PUMPS AT KPONG HEADWORKS



# KNUST

