KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY

INSTITUTE OF DISTANCE LEARNING

TIME SERIES ANALYSIS OF VALUE ADDED TAX REVENUE

COLLECTION IN GHANA.

A THESIS SUBMITTED IN PARTIAL FULFILMENT OF THE

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BY

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DECLARATION

I hereby declare that this submission is my own work towards the degree of M-Phil in Industrial Mathematics and that, to the best of my knowledge, it contains no material(s) previously published by another person(s) nor material(s), which have been accepted for the award of any other degree of the University, except where the acknowledgement has been made in the text.

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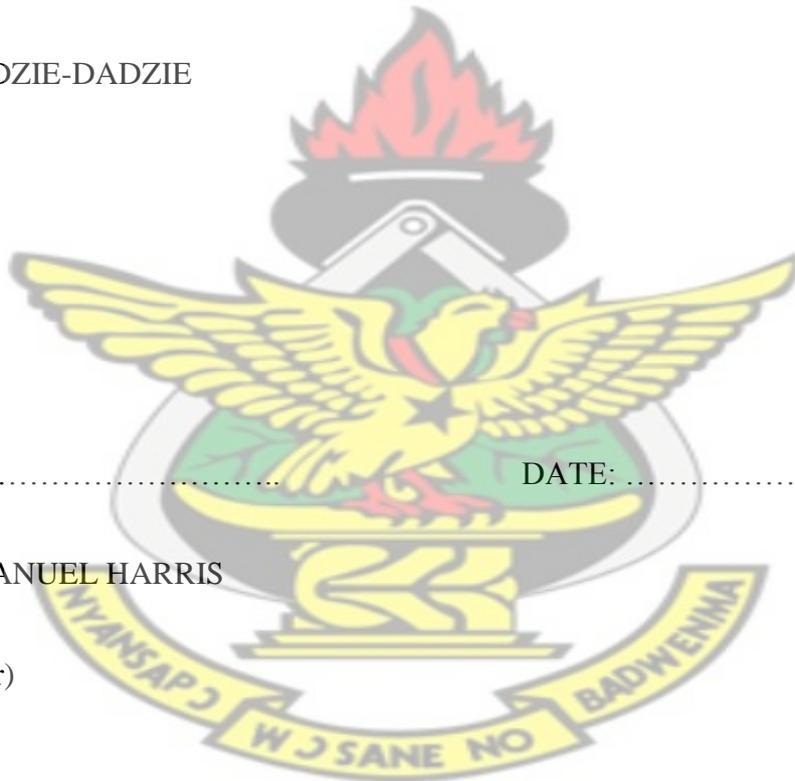
DEDICATION

This work is dedicated to my lovely wife Mrs. Sarah Edzie-Dadzie for the support and encouragement given to me and my daughter Nana Ama Sey.

ACKNOWLEDGEMENT
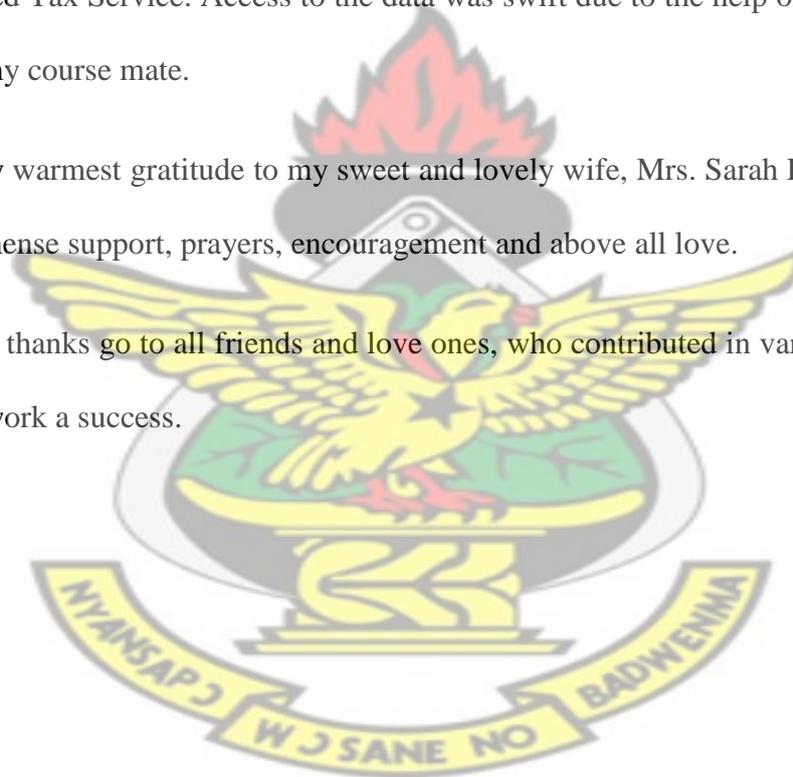
ABSTRACT

Economic growth is one of the major concerns of every government. In the quest to know how the Economy of Ghana is fairing through the various regimes of governance, increasing population and other factors that affects economic growth, this thesis has been purposed to study the behaviour pattern and trend in Ghana's Domestic and Import Value Added Tax Revenue in Ghana and to choose the best model among various ARIMA models which pose's high power of predictability for forecasting Ghana's Real Domestic and Import VAT Revenue. Finally, the data of yearly Domestic and Import VAT revenue in Ghana from 1999 to 2009 was examined.

The Box- Jenkins method of Time Series analysis was employed.

The model;

$$Y_t = 2.0145Y_{t-1} - 1.6275Y_{t-2} + 0.613Y_{t-3} + 1.6053e_{t-1} - 0.9999e_{t-2} + e_t + 0.3402$$

An ARIMA (2, 1, 2) was deduced to be the best model to forecast Domestic VAT revenue in Ghana,

Whilst the model;

$$Y_t = 0.1882Y_{t-1} + 0.4917Y_{t-2} + 0.3201Y_{t-3} - 0.3196e_{t-1} + e_t + 0.6093$$

An ARIMA (2, 1, 1) was deduced to be the best model to forecast Import VAT revenue in Ghana.

TABLE OF CONTENTS

LIST OF FIGURES

KNUST

LIST OF TABLE

LIST OF ABBREVIATIONS

VAT- Valued Added Tax

VATS- Valued Added Tax Service

GRA- Ghana Revenue Authority

IRS- Internal Revenue Service

CEPS- Custom, Excise and Preventive Service

RAGB- Revenue Agencies Governing Board

GET Fund- Ghana Education Trust Fund

NHIL- National Health Insurance Levy

LVO- Local VAT Office

VSO- VAT Sub Office

VSS- VAT Satellite Station

PoP- Point of Presence

LTU- Large Tax Payers Unit

VFRS- VAT Flat Rate Scheme

AR(p)-Autoregressive Process of order p

MA(q)- Moving Average Process of order q

ARMA (p, q) - Autoregressive Moving Average Process

ARIMA (p, d, q)- Autoregressive Integrated Moving Average of order (p, d, q)

FAUNRA- Federation Association and Union of National Revenue Agencies

GDP- Gross Domestic Product

OLS- Ordinary Least Squares

TARF- Tax Analysis and Revenue Forecasting

OECD- Organization for Economic Co-operation Development

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INTRODUCTION

1.1   Background of Study

Governments all over the world depend mainly on tax revenue for their developmental projects, (VAT News, 2003, Vol 1). There is therefore the need to mobilize more revenue to match fiscal expenditure in order to reduce budget deficit. This is more so, if the nation is to achieve its developmental objectives. It is in this light that institutions, organizations and agencies are established solely to mobilize revenue for the state (Daah, 2007).

Tax revenue generation in any nation constitutes an integral component of any government fiscal policy and many countries therefore depend mainly on taxation as a source of generating the required resources for funding government expenditure. The provision of social amenities such as good roads, schools, electricity, hospitals, portable water and sorting facilities are all funded by government through tax revenue. Moreover, tax revenue is used to maintain law and order as well as defences against external aggression by means of resourcing the security and enforcement agencies with the requisite modern security apparatus. In Ghana for instance, government use tax revenue collected to pay civil and public servants as well as provision of social welfare services such as National Health Scheme and National Youth Employment Programme.

Taxation as a means of generating the required revenue for government's developmental projects and provision of social services can be considered as essential

instrument for economic policy which has several benefits but however be a disincentive if not properly administered.

According to Bediako, (2007) cited Oteiku (1992), in Ghana; the tax system comprises all the tax laws on one hand and administrative machinery structures which implement these laws on the other hand. Tax laws, just like any other legal provisions, which impose duties and tariffs on people, however cannot in themselves guarantee the achievement of their underlying objectives. There is the need for certain machinery and structures to be set up in place for the achievement of those objectives. Tax administration is the aspect of the tax system which seeks for the structure and machinery to achieve the driving objectives of taxation. Thus, the laws usually outline all what to do while tax administration is mainly concerned with how to do.

Taxation is often defined as the levying of compulsory contributions by public authorities having tax jurisdiction, to defray the cost of their activities. It's also serve as a means by which government implements decisions to transfer resources from the private to the public sector which serves as a major instrument of social and economic policy (Ali-Nakyea, 2008, p.4). Tax also plays an important role in the redistribution of wealth as those in higher income brackets are encouraged to pay more as compared to those with lower incomes, what we called provisional assessment. The provisional assessment is like a letter that states categorically the amount of tax to be paid (usually in four instalments) in a calendar year and the tax payer has the right under the tax laws to object to the assessment by stating clear in a letter to the tax authorities why the tax burden is too much or stating precisely the grounds for the objection within nine (9) months of the commencement of the basis period to which the provisional assessment relates in each calendar year. On the basis of the reasons stated in the objection letter by the tax payer, the tax authorities will then decide among

other things to reduce, maintain or increase the tax. This system assists the tax authorities to have a rough estimate of amount of tax to be paid by each tax payer, however, most of the tax payers do not pay their taxes within the stipulated period and are always in arrears.

In Ghana, Ghana Revenue Authority (GRA) is in charge of tax collections. Under this authority are the following tax agencies, Internal Revenue Service (IRS), Custom, Excise and Preventive Service (CEPS) and Value Added Tax Service (VATS) have been established to mobilize tax revenue for the state. These tax revenue agencies represented by their various commissioners' report to the Commissioner, Ghana Revenue Authority. Formerly, the taxes collected were reported to the Minister of Finance and Economic Planning through the Revenue Agencies Governing Board (RAGB).

1.1.1 The Origin of Value Added Tax in Ghana

The Value added Tax Service (VATS), the youngest of the three agencies was first introduced in Ghana on 1st March,1995 by the VAT Act,1994 (Act 486) as part of the tax Reform Program which began in 1993. It was however, withdrawn by the government on 14th June, 1995. This withdrawal was in response to a general public outcry against a sudden increase in the prices of goods (including food items) which was blamed mainly on the introduction of VAT. Value Added tax was reintroduced on 16th March, 1998 under the Value Added Tax Act, 1998 (Act 546) and the Value Added Tax Regulations, 1998 (L.I. 1646). Its primary aim is the administration and collection of the Value Added Tax that replaced the Sales and Service Taxes previously administered by Custom, Excise and Preventive Service and Internal Revenue Service, respectively, Daah 2007.

The VAT Service is mandate to collect indirect taxes (that is, taxes on goods and services). The VAT Service is mandated to collect indirect taxes (that is, taxes on goods and services). The Value Added Tax rate from inception was ten percent. However, in 2000, an Act of Parliament, (Act 578) was passed to amend the rate to twelve and half percent, the two and half percent was to be given to the Ghana Education Trust Fund (GET Fund) Secretariat to support quality education delivery and infrastructural development. In 2004, another Act of Parliament, (Act 650) was promulgated to add a further two and half percent to support National Health Insurance Scheme. This additional rate was called the National Health Insurance Levy (NHIL). The Act gave the Finance

Minister the power to appoint a revenue collecting agency to collect this $2\frac{1}{2}$% levy on behalf of the National Health Insurance Scheme. In line with the functions defined within the Act, the service has the corporate vision of being an efficient, effective and modern tax administration that meets national and global standards. Its mission is to ensure the successful management and sustainable development of VAT in Ghana. It is committed to mobilizing revenue for national development by engendering public confidence in the administration of VAT through continuous education and fair application of the tax laws as a means of promoting voluntary compliance. This will be achieved through highly motivated staffs that are expected to act at all times with integrity, Daah, 2007.

In order to attain its goals, the organization has four departments, each having smaller units (VAT Service Management Report, 1999). These four departments are Operation, Enforcement and Debt Management, Research, Monitoring and Planning and Finance and Administration.

The Operations Department deals with the core activities of collecting taxes. To do this, it registers eligible businesses (referred to as "traders"). Upon their registration, a certificate is issued and VAT invoices are sold to these traders to issue to their clients or customers when services are rendered or sales are made.

The Enforcement and Management Unit ensures that eligible businesses registered, are complied with the dictates of the VAT Act (e.g. Insurance of tax invoice when Sales are made, submission of the tax returns, and paying the taxes due). Since the tax is self-accounting, periodic audits (otherwise known as Control and Verification) are performed by the Control and Verification Unit. In cases where fraud is suspected, such audits are performed by the Fraud and Special Control and Verification Unit.

The Research, Monitoring and Planning Department as the name suggest monitor's revenue performance and evaluate the implication of policies on the service as well as its traders. It researches in areas that affect the goals of the service. A case in Pont was a research carried out to see the impact of the load shedding exercise on the revenue collection by the service (Research Unit Quarterly Report, 2006).

The Finance and Administration Department collect the taxes from the traders and accounts for it at the Bank of Ghana. The Personnel Unit is in charge of employee welfare such as transfers and promotions amongst other duties. The Audit and Investigations unit is the Internal Audit section of the service.

There are about 16 outfield offices, sub-offices and point of presence covering at least all regions in Ghana. These offices are classified based on the concentration of registered trader population. The local VAT office (LVO) must have a trade population of not less than five thousand registered traders. It supervises the VAT sub-office (VSO), whose trader population must not be less than three thousand, and the VAT Satellite Station (VSS), which at least must be in all regional capitals

(irrespective of the member of registered traders there). The VATS Point of Presence (PoP), are offices within towns where it may not be prudent to open a VSS but for convenience an office needs to be located. These offices have clearly defined geographical boundaries. In terms of duties performed, the VSO and VSS performed similar duties (i.e. Control and Verification, Enforcement and Debt Management and Accounts). The only duties performed by the PoP's are the distribution of the tax returns to the registered traders, receiving of completed tax returns and the payment of taxes. The LVO, in addition to the duties performed by the VSO/VSS, performed duties as Information Systems Support, Research, etc which the VSO and VSS do not perform. The Large Tax Payers Unit (LTU) is a one-stop tax office. This office has representatives of all the three revenue collecting agencies and is for some selected businesses whose tax contribution together account for sixty percent of the total taxes collects in the country. The VAT collected by the LTU is accounted to the VATS. The service finances itself by retaining up to three percent of total revenue collected. This is permissible under the Revenue Agencies Retention Act, 2002.

The service in its quest to mobilize more revenue, from 1st September 2007 introduced a special method of collecting and accounting for VAT. It has been called the VAT Flat Rate Scheme (VFRS). It is designed for traders operating in the retail sector only. Under the scheme, registered retailers of taxable goods shall charge VAT/NHIL at a marginal rate of 3% on the value of each taxable item sold. Low level of literacy among retailers of taxable goods, especially those in the informal retail sectors, and the associated computational difficulties in calculating the VAT payable under the invoice credit method; have increased the burden of compliance for the retailer. This has necessitated the introduction of VFRS which is simpler to operate, comes with minimal record-keeping requirements and eliminates the difficulty of

apportioning input tax into deductible and non-deductible input taxes. VFRS has the potential to expand the tax net and create a level playing field for all retailers. The VAT service, since its establishment, has now become a permanent and important feature of the national revenue landscape. The service currently accounts for almost 27% of national tax revenue (VAT Service Dairy, 2007).

Value Added Tax is one of the principal means of indirect taxation in many countries including Ghana (VAT News 2003, Vol. 1). It is levied at both flat and variable percentage rates. Value Added Tax (VAT), defined as tax on consumption, is levied whenever the value of goods and services increases as they change hands in the curse of production, distribution and final sale to the consumer (Microsoft Encarta, 2006). In a public notice titled "why VAT in Ghana" (1998), Value Added Tax is defined as a tax applied on the value which is added to goods and services at each stage in the production and distribution chain.

On December 31, 2009 an assent was given to the Ghana Revenue Authority Act, 2009 (Act 791). The Act was passed to establish the Ghana Revenue Authority to replace the Value Added Tax Service, Internal Revenue Service and Custom, Excise and Preventive Service for the administration of taxes and to provide for related purposes in the country. Under this Act, Custom, Excise and Preventive Service has been called Customs Division whiles the Value Added Tax and Internal Revenue Service have been combined to become Domestic Tax Revenue Division of the Ghana Revenue Authority (Ghana Revenue Authority act, 2009, Act 791).

1.2.   Statement of the Problem

Almost all managerial decisions are based on past and present conditions to forecast future events. Every decision therefore, becomes operational at some point in the future (Hossein, 1994). A forecast is therefore needed and has become an

indispensable phenomenon throughout an organization. In Ghana, the Ghana Revenue Authority sets revenue targets to the revenue institutions. More often than not, these revenue institutions failed to achieve their given targets (VAT News, 2006, Vol 1). This could be attributed to the fact that the given targets are mostly unachievable, considering the unrealistically high targets and the economic conditions prevailing in the country.

It is against this background that this study seeks to ascertain the pattern of the Vat revenue collections in the country. It would also ascertain over the period of time the effect of any cyclical, seasonal and any other influences on the VAT revenue mobilization in Ghana. Forecast of expected revenue of VAT collections would be made to guide government to approximately budget its expenditure based on the projected revenue.

1.3   Objectives of the Study

The general objective of this study is to statistically analyze the Value Added Tax revenue collections in Ghana over the period of 1999 to 2009. The specific objectives are:

i.   To determine the trend pattern of VAT revenue collection for the period.

ii.   To fit a suitable model to the monthly indirect tax revenue data and

iii. Forecast the future VAT revenue expected for the next thirty six (36) months.

1.4   Methodology of the Study

Time series analysis is the main statistical tool that will be employed in the analyses of the monthly indirect tax revenue data in Ghana over the 11-year period (132

months) from 1999 to 2009. Graphical descriptions of the data will be used for easy understanding and interpretation at the various stages of the analysis.

The Box-Jenkins time series model theory will be considered in the modelling of the data. Models to be considered in the formulation includes Autoregressive process of order p (abbreviated to be an AR (p) process), Moving Average process of order q MA(q), mixed Autoregressive / Moving Average process (ARMA(p, q)) and Autoregressive Integrated Moving Average of order p. d. q (ARIMA (p, d, q)) depending on the nature of the sample autocorrelation function, sample partial autocorrelation function and other theoretical indicators to select the appropriate models or equations to identify the most appropriate one which best fit the indirect tax revenue data.

R software package will be used to sketch the graphs, estimate the model parameters, perform the diagnostic checks and also compute the forecasts.

1.4.1   Data Collection

The data throughout the study are secondary data and consist of monthly total indirect tax revenue collection. These data values were collected from the Research, Monitoring and Planning Department of the VAT Service and Monitoring and Evaluation Unit of CEPS, Headquarters Accra. The data covered the period from January 1999 to December 2009 (132 months).

Some of the indirect tax revenue data were recorded in the old currency (¢), that is from January, 1999 to June, 2007. However all the data values recorded in old currency were converted to the new currency (GH¢). The conversion from the old currency to the new currency has made the unit (currency) of measurement uniform and it would also help reduce errors in differencing.

9

1.5   Justification of the Study

The result of the study would among other things:

i.   Help the government and the outside world to have an idea of the pattern of tax revenue collected by the VAT Service over the period under consideration.

ii.   Be helpful to government to have a fair knowledge of the expected VAT for the subsequent year (3yrs).

iii.   Be made available for discussion at conference of tax revenue administrators, for example, federation Association and Union of National Revenue Agencies (FAUNRA).

iv.   Serve as a source of reference for further studies.

1.6   Scope and Limitation

The scope of the thesis was limited to the following:

➢ 11 years of monthly indirect tax (Domestic and Import vats) revenue collected in Ghana for the period January, 1999- December, 2009.

➢ Using box-Jenkins methodology to develop a time series model for both descriptive and forecasting purposes. In this case the explanatory capacity of the model was not addressed, as no additional independent variable was used for the modelling the time series data.

1.7   Organization of the Study

Every research must appropriately be organized to allow readers to follow the sequence of the study.

Chapter one of the study which is the introduction, dealt with the background to the study, statement of the problem, objectives of the study, methodology of the study, justification of the study, scope and limitation and organization of the study.

In chapter two, the researcher reviews the related literature regarding the VAT service in Ghana. The review also focuses on the approaches that have been adopted by previous researchers and limitations of their methods, as well as a discussion of the results from previous studies. Chapter three also discusses the methodology and procedures used in the analysis of the monthly Domestic and Import taxes revenue data.

Chapter four focuses on data analysis of the monthly indirect tax revenue data over the 11-year period from 1999 to 2009. The discussions and interpretations are also presented in this chapter. Finally, chapter five present the conclusion and recommendation and also summary of findings.
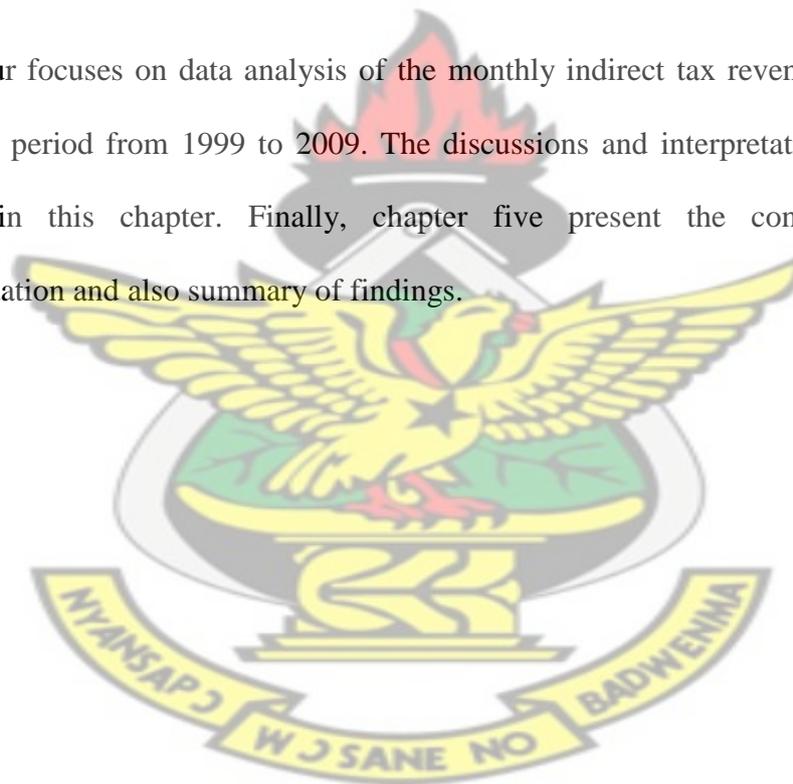
CHAPTER TWO

LITERATURE REVIEW

The introduction of Value Added Tax as a form of collecting indirect taxes from the populates of a nation for its socio-economic development dates back to the early 50's. According to Microsoft Encarta, 2006), France was the first country in the world to introduce Value Added Tax (VAT) in the year 1953. Since then many other countries have used this approach to collect most of its indirect taxes. According to Jordan-Bychkov (2005), the primary indirect tax used today throughout Europe is VAT. In more than 120 countries, the Value Added Tax as a major source of government revenue affects about four billion people, i.e. about 70% of the world's population (El- Ganainy 2006).

In view of the increasing number of countries using VAT as a way of collecting indirect taxes, it has become necessary for research work to be conducted on VAT revenue and its related issues. Many researchers have thus shown interest in conducting research on Value Added Tax revenue collections for countries over the period. Analysis of government tax with respect to Value Added Tax in France by Michel, 2003 concluded that the introduction of VAT in the country has led to a continuous increase in government total domestic revenue mobilization over the years. He said this has made France's gross domestic product (GDP) to rise to $1.76 trillion from $1.42 trillion, while per capita income increased to $29410 in 2003 from $29089 in 2002. He further observed that the government revenue target has always been achieved and VAT in particular has an upward trend over the period.

Alemayehu and Abebe (2004) on the other hand, after evaluating tax reforms in Ethiopia with particular reference to Value Added Tax performance, concluded that VAT revenue collection in Ethiopia has shown growing trends as compared with the replaced sales tax. They said that, that indicates a 50% growth in Value Added Tax collection as compared with those tax sales. They also contended that domestic VAT collection contributed 14.9% while that of import collection contributed 27.1% to the total revenue collections in the country. To this, Alemayehu and Abebe attributed the high contribution of import VAT to total VAT collection in the country to seemingly its well checked entry point. Moreover, the lower domestic VAT collection could be associated with administrative difficulty to collect the tax from the domestic economy and the existence of illegal practice on Value Added Tax which could be likened to the prevailing situation in Ghana and many other countries in Africa.

Brun, Chambas and Combes (2006), in their research titled, "Fiscal space in developing countries" evaluated VAT revenue within a context of tax transition and concluded as follows:

i. The relative contribution of internal indirect taxation has increased for most developing countries mainly due to Value Added Tax.

ii. Import VAT collected at the main borders has been an upward trend for the past ten (10) years.

However, they did not ascertain the variation existing between these two findings among the countries the research was conducted in. Thus, this work among other things, seeks to find what the case of Ghana is.

El- Ganainy, 2006 asserted that the pattern of Value Added Tax revenue collections in fourteen European Union countries has been an increasing one in her research on the topic "Essays on Value Added Tax". To El- Ganainy, among the reason for this

continuous increase is an increase in the VAT rate and consumption rate. She contended that one percent point increase in VAT rate led to a 0.23% point increase in growth rate.

The results she recorded were statistically significant at 10% significance level.

Similarly, in the Fiji economy, Narayan (2003) found out that a 25% increase in Fiji's VAT rate led to about 4% increases in government revenue. Thus, apart from the fact that Value Added Tax revenue collections have been increasing in most countries due to prudent measures put in place by the collecting agencies, an increase in the rate could also lead to a corresponding increase in revenue. El- Ganainy in her work also found that Value Added Tax revenue collections followed a cyclical fluctuation as well as seasonal effects especially in weather regulated economies in Central Europe.

 Similarly, Kearney (2003), evaluating Value Added Tax collections over the period 1993-2002 in South Africa observed that VAT is in important revenue source for the government. Kearney found out that it was the second highest revenue source next to income taxes and contributes 25% of total tax receipt, showing that the VAT base in South Africa is relatively broad and is steadily increasing. Kearney only evaluated actual collections made for the reviewing period and could not therefore make any comparative analysis with the target set.

Accurate revenue forecasts are important because they form the guidelines for budget development and set the tone for the budget process. In public- sector budgeting, the availability of resources circumscribes discussions about expenditures. As these discussions intensify in the face of mounting fiscal duress, reliable and informative revenue forecasts becomes critical elements of the budgeting process. However, government agencies rarely methodically forecast revenues for longer time periods than the next budget year.

14

Forecasting beyond one year can be very useful in identifying the direction and significance of financial and economic trends. This can also assist budget and finance officers, as well as senior management to anticipate future challenges and develop long- term plans.

Another important characteristic of a sound revenue forecasting system is that it forces budget and finance officers to identify assumptions related to future economic issues. It forces them to be more aware of economic conditions and relational impacts that could occur from national events.

The assumptions employed are more important determinants of reliability of the forecast, then are the specific techniques employed to produce the forecast.

Time series forecasting models such as moving averages, exponential smoothing and Box-Jenkins have not been extensively used in government agencies. However, empirical evidence indicates that these types of techniques can substantially improve accuracy in identifying the annually budgeted resource constraints and in updating current year forecast.

The limited use of these techniques has been linked to several derivative factors. Much of research shows that many budget and finance officers are not adequately exposed to these techniques in either their formal education or professional training. Any exposure they did have, was not sufficient for the officers to take the techniques to their own organization and apply them on a daily basis. Finance officers may also be reluctant to implement the new techniques due to an aversion to risk when it comes to experimenting with new methodologies.

In a research by Nikolov (2002) on "Tax Revenue Forecasting with Intervention Time Series Modelling" using monthly tax revenue collection from Republic of Macedonia

for the period January, 1998 to July, 2002; the modelling started with the Box-Jenkins model selection approach and continued with the effects of the intervention analyses. In conclusion, he recommended that the model could be used for forecasting but only for a few time units ahead because the variance of the forecasts in time series models becomes large in time.

Clower and Weinastein (2006) compared the time series using Box-Jenkins method with other time series methods when studying quarterly sales tax revenue from 1994 to 2005 in the city Arlington, Texas. In predicting future sales tax growth, three main statistical approaches were employed; Autoregressive Integrated Moving Average (ARIMA) model, Seasonal Exponential Smoothing and Ordinary Least Squares (OLS) Regression Analysis based on employment data. "Though not wildly different, each forecasting technique produced slightly different outcomes". The ARIMA model was considered as suggesting slower growth in total revenues and its forecasts were judged to be a lower bound for potential revenue growth compared to the other methods.

Braun (1988) conducted a research on "Measuring Tax Revenue Stability with Implications for Stabilization Policy: A Note". The main purpose of the study was to investigate whether aggregate state tax revenue data are characterized by a deterministic trend or a stochastic drift model using tax revenue data from the state of Georgia for the years 1950 to 1984, except for the general sales tax whose observations range from 1952 to 1984. The research revealed that forecasting tax revenue data using stochastic model was more robust and provided less errors compared to the deterministic trend model which provided a poor measure or forecast because it always assumed the tax data (observations) to be in the state of stationarity.

16

Fullerton (1989) also conducted a study on Forecasting state Government Revenues: A case study of Idaho State tax for the period 1967 to 1985. In the study, Econometric model, ARIMA model and Composite model (combination of Econometric and ARIMA models) were used and found that a composite model built with Econometric and univariate ARIMA projections of Idaho retail sales tax receipts provided better forecasts than either single model because the combined forecast variance was generally smaller than that of any of the single forecast methods considered.

Slobodnitsky and Drucker (2005) also studied on "VAT Revenue Forecasting in Israel" using monthly VAT revenues since 1987; the study compared the results of Co- integration estimation and ARIMA methods and decided over the best method suitable for VAT revenue forecasting. It was found out that the quarterly ARIMA specification performed the best with the least absolute deviation. " Monthly ARIMA came a close second and both budget forecast and co-integration were much less precise".

In addition, many government agencies may have little incentive to improve forecast accuracy if they have not experienced adverse consequences due to inaccurate forecasts.

Wildavsky (1986) noted that politicians generally accept revenue forecasts with little questioning or detail. Therefore, if there is no pressure to improve, many finance officers will continue with 'business as usual' and not take on the risk of introducing new and innovative techniques or strategies.

Budget and Finance officers also tend to have a conservative bias because they typically under forecast their entities revenue. This is mainly due to the requirement that they maintain as a balance budget. The preference towards judgemental

approaches to forecasting may be a reflection of this bias. Rubin, 1987 also suggests that decision makers may encourage under forecasting in order to make discretionary funds available during the fiscal year that can be re-allocated outside of the regular budget process. Although many reasons can be sighted explaining why budget officers desire to use mainly judgemental forecasting accuracy can be significantly improved with the use of a systematic approach.

Armah (2003) contend that despite the all too often celebrations by revenue collecting agencies that they have achieved their revenue targets, questions are being asked whether the revenue targets appropriately reflect the nation's macro-economic framework. He described as flawed the revenue setting mechanism is poor. While very limited research has been conducted with regards to revenue forecasting and its impact on national economy, there has been no research conducted related to the forecasting strategies adopted by the Value Added Tax Service. This study attempts to evaluate better revenue forecasting techniques for VATS.

According to Dadzie, 2006, a prediction is an invitation to introduce change into a system. There are several assumptions about forecasting:

1. There is no way to state what the future will be with complete certainty. Regardless of the methods that we use there will always be an element of uncertainty until the forecast horizon has come to pass.

2. There will always be blind spots in forecasts. We cannot for instant, forecast completely new technologies for which there are no existing paradigms.

3. Providing forecasts to policy- makers will help them formulate social policy. The new social policy, in turn, will affect the future, thus changing the accuracy of the forecast.

Effective taxation enhances sound economic growth and usually fosters a great protection of the poor and vulnerable in society. According to Bediako (2007), in the year 2000, E Attah Mills (prof.), former commissioner of IRS, who is the president of the Republic of Ghana, asserts: "Taxation is the most important tool available to governments for mobilizing financial resources". Over the years, several attempts have been made to capture more and more people from the informal sector into the tax net.

Examples of time series design are continuous monitoring of weekly (or monthly or yearly) revenue collection figures, monthly rainfall figures in Ghana and annual sales figures of fowl from poultry farmers in Ghana. Data collected on daily production for a manufacturing company, the data on annual sales for a corporation, the recorded end-of- month values of the prime interest rate and monthly recordings of people suffering from anaemia are all included in the statistical concept of time series.

In an annual program conducted by Duke Centre for internal Development at the Duke University in Durham, North Carolina, USA on Tax Analysis and Revenue Forecasting (TARF) for the past six years, the institute observed that statistical analysis of tax revenue and VAT for that matter for most of the participating countries has been an increasing one. It also asserted that various revenue data yielded different forecasting models. Thus, depending on the revenue pattern and the country, the model differed. In review of recent African countries experience of the fiscal impact of trade liberalization commissioned for the Organization for Economic Co-operation Development (OECD) developing centre, Fakasaku (2003) found that the overall impact of VAT implementation on trade liberalization has been enormous. Examining a database of about 22 African countries, he found that trade liberalization has increased VAT revenue to total government revenue of more than 20% in Mauritius,

more than 10% on La Cote D'Ivoire and Senegal and more than 5% in Cameroun, Tunisia and Mozambique. Ali (2005) worked on a similar topic of revenue and fiscal impact of trade liberalization: a case study of Niger. In Ali's analysis of revenue trend in Niger from 1990 to 2003, he observed that Niger witnessed a decline in revenue mobilization rather than an increase, resulting in a decline close to 2% of GDP and budgetary crisis. However, the introduction of VAT in the late 90's, Ali found that domestic revenue mobilization was increasing continuously throughout the period, though slowly and marginally, he concluded. Ali did not do any projection to ascertain the future trend in VAT collections. In a publication in Wednesday September 19, 2007 edition of the Daily Graphic written by Boahene Appiah, the chairman of the then Revenue Agencies Governing Board (RAGB), Mr. Kwabena Agyei speaking at a tax collectors Award Night said that for the last five years, revenue collections in Ghana had increased from ¢ 6.65 trillion to ¢ 23.7 trillion. He expressed hope that the trend in the increase of tax revenue in general and VAT in particular would be sustained. Expressing similar view, Mr. Harry Owusu, executive secretary of the RAGB reviewing the first three quarters of the revenue collections for 2006 in a Dairy Graphic publication of Tuesday, 24th October, 2006, declared that the country's domestic revenue mobilization was robust and on target for the third quarter of 2006. Mr. Owusu mentioned that total revenue from January to end of September, 2006 was on target as actual revenue was ¢ 20.96 trillion against a target of ¢ 21.02 trillion, a negative deviation of 0.26%. On Value Added Tax, he said the actual VAT collected was ¢ 3.15 trillion as against a target of ¢ 3.24 trillion leading to a marginal negative deviation of 0.03%. However, in most of the works above, and their corresponding findings, it was realized that there was no forecast of expected revenue collections and trend pattern in order to plan for the future.

CHAPTER THREE

METHODOLOGY OF THE STUDY

In this chapter the researcher looked at some theoretical background of time series and the method of time series which has been employed in modelling domestic and import vat revenue data in Ghana for the period under review. However, before we discuss the methodology we shall first look at some basic definitions.

3.0 THEORY OF TIME SERIES

3.1 Definition of time series

Any variable that is measured over a period of time in sequential order is called time series (Keller and Warrack, 2003). The period may be daily, weekly, monthly, quarterly or yearly. Examples of time series includes humidity, rainfall and temperature figures recorded over a period of time as well as heart beat of a person. Other examples include sales of product, production of goods, number of unemployed and unemployment rate over time.

On the other hand, time series is a set of observations measured sequentially through time. These measurements may be made continuously through time or be taken at a discrete set of time points. If a time series can be predicted exactly, it is said to be deterministic. But when the future is only partly determined by past values, so that exact predictions are impossible and must be replaced by the idea that future values have a probability distribution which is conditioned by knowledge of the past values it is called stochastic time series. The sequence of random variables $\{Y_t : t = 0, \pm 1, \pm 2, \pm 3, \ldots\}$ is called a stochastic process and serves as a model for an

observed time series. The systematic approach by which one goes about answering the mathematical and statistical questions posed by time correlation is commonly referred to as time series analysis. There are two types of time series, these are univariate and multivariate. Univariate time series are those where only one variable is measured over time, whereas multivariate time series are those, where more than one variable are measured simultaneously.

3.2 Applications of Time Series Analysis

Some applications of time series forecasting includes the following:

i. Economic Planning

ii. Sales forecasting

iii. Budgeting

iv. Financial risk management

v. Model evaluation

3.3 Objectives of Time Series Analysis.

The main objectives of time series analysis are:

3.3.1 Description of Data

When presented with a time series data, the first step in the analysis is usually to plot the data and obtain simple descriptive measures of the main properties of the series such as seasonal effect, trend etc. The description of the data can be done using summary statistics and or graphical methods. A time plot of the data is particularly valuable.

3.3.2   Interpretation of Data

When observations are taken on two or more variables, it may be possible to use the variation in one time series to explain the variation in another series. This may lead to a deeper understanding of the mechanism which generated a given time series. For example, sales are affected by price and economic conditions.

3.3.3   Forecasting of Data

Given an observed time series, one may want to predict the future values of the time series. This is an important task in sales forecasting and in the analysis of economic and industrial time series. Prediction is closely related to control problems in many situations, for examples if we can predict that manufacturing process is going to move off target, the appropriate corrective action can be taken.

3.3.4   Control

A good forecast enables the analyst to take action so as to control a given process, whether it is an industrial process, or an economy or whatever. When a time series is generated which measures the quality of a manufacturing process, the aim of the analysis may be to control the process. In statistical quality control, the observations are plotted on control charts and the controller takes action as a result of studying the charts. Box and Jenkins have described a more sophisticated control strategy which is based on fitting a stochastic model to the series from which future values of the series are predicted. The values of the process variables predicted by the model are taken as target values and the variables can form to the target values.

3.4   Components of a Time Series

Characteristic movement of time series may be classified into four main types often called components of time series. These four different components are trend, seasonal, cyclical and irregular or random variations.

3.4.1   Trend Variation

The Trend of a time series also known as a secular trend is a relatively smooth pattern, regular and long term movement exhibiting the tendency of growth or decline over a period of time. The trend is that part which the series would have exhibited, has there been no other factors affecting the values. The population growth together with advances in technology and methods of business organization are the main factors for the growth or upward trend in most of the economic and business data. The decline or downward trend may be due to the decreasing demand of the product, or a substitute taking its place, or difficulty in obtaining raw materials, etc. Many industries however initially show a steady growth until a saturation point is reached, and then the trend declines steadily. But sudden or frequent changes are incompatible with the idea of trend.

3.4.2   Seasonal Variation

Seasonal variation represents a type of periodic movement where the period is not longer than one year. Business activities are found to have brisk and slack periods at different parts of the year. The factors which cause this type of variation are the climatic changes of the different seasons, such as changes in rainfall, temperature, humidity, etc. and the customs and habits which people follow at different parts of the year. For short, a seasonal component is a pattern that is repeated throughout a time series and has a recurrence period of at most one year.

24

3.4.3 Cyclic Component

Cyclical fluctuation is another type of periodic movement where the period is more than a year. Such movements are fairly regular and oscillatory in nature. One complex period is called a cycle. Business cycles are caused by complex combination of forces affecting the equilibrium of demand and supply. Prosperity, decline, depression and recovery are usually considered to be the four phases of business cycles. The swing from prosperity to recovery and back again to different time series data are found to follow closely the same pattern of cyclical movement because of inter- relations between them.

3.4.4 Irregular or Random Variation

Irregular or Random variation movements are such variations which are caused by factors of an erratic nature. These are completely unpredictable or caused by such unforeseen events as wars, flood, earthquake, strike, lockout, etc. It may sometimes be the result of many small forces, each of which has a negligible effect, but their combination effect is not negligible. Random moment do not reveal any pattern of repetitive tendency and may be considered as a residual variation.

3.5   Terminologies/ Fundamental Concept

3.5.1 Stochastic Processes

A model that describes the probability structure of a sequence of observations is called a Stochastic Process.

3.5.2 Stationary Processes

A stochastic process (X_t) is said to be strictly stationary if the joint distribution of $X_{t_1}, X_{t_2}, \ldots, X_{t_n}$ is the same as the joint distribution of $X_{t_{1-k}}, X_{t_{2-k}}, \ldots, X_{t_{n-k}}$ for all $t_1, t_2, \ldots, t_n$ and lag k which is the time difference. In this case, $E(X_t) = E(X_{t-k})$, $\text{Var}(X_t) = \text{Var}(X_{t-k})$ for all t and k, and also $\text{Cov}(X_t, X_s) = \text{Cov}(X_{t-k}, X_{s-k})$ for all t, s and k.

In practice, it is rare to come across strict stationary process and therefore the need for a less restricted definition that is practicable. A process (X_t) is called Second order stationary (weekly stationary) if mean function is constant over time and its auto covariance function depends only on the lag, so that $E(X_t) = \mu$ and $\gamma_{t,t-k} = \gamma_{o.k}$.

3.5.3 White Noise Processes

A process (a_t) is called a White Noise if it is a sequence of uncorrelated random variables from a fixed distribution with constant mean $E(a_t) = \mu_a$, usually assumed to be zero, constant variance $\text{Var}(a_t) = \sigma^2{}_a$ and $Y_k = \text{Cov}(a_t, a_{t+k}) = 0$ for all k$\neq$ 0.

3.5.4 Differencing

It simply means relating the current values to the former values. Considering for instance, a series $(y_2, \ldots, y_n)$ which is formed from the original observed series $(x_2, \ldots, x_n)$.

First order differencing is given as $y_t = x_t - x_{t-1} = \nabla x_t$ (3.1.1)

For t = 2, 3... n.

Second order differencing is given as

$$\nabla^2 x_t = \nabla x_t - \nabla x_{t-1} = x_t - 2x_{t-1} + x_{t-2} \qquad (3.1.2)$$

At most, second order differencing is sufficient to attain stationarity.

3.5.5 Sample Autocorrelation Function

Consider an observed series $x_1, x_2, \ldots, x_N$. We can form N-1 pairs of observations, namely $(x_1, x_2), (x_2, x_3), \ldots, (x_{N-1}, x_N)$ where each pair of observation is separated by one time interval. Taking the observations in each as separate variables, we can compute the correlation coefficient between x_t and x_{t+1} as

$$r_k = \frac{\sum_{t=1}^{N-k}(x_t - \bar{x})(x_{t+k} - \bar{x})}{\sum_{t=1}^{N}(x_t - \bar{x})^2} \tag{3.1.3}$$

This is autocorrelation coefficient at lag k. A plot of r_k against lag k for k = 0, 1, 2 ... and M < N is called Correlogram or sample Autocorrelation Function (ACF).

3.5.6 Sample Partial Autocorrelation Function

The partial autocorrelation is the correlation between x_t and x_{t-1} after removing the effect of the intervening variables $x_{t-1}, x_{t-2}, \ldots, x_{t-k+1}$. It is usually called the Partial Autocorrelation Function (PACF) at lag k and denoted by $\emptyset_{kk}$ and is defined as

$$\emptyset_{kk} = \text{corr} (X_t, X_{t-k}|X_{t-1}, X_{t-2}, \ldots, X_{t-k+1}) \tag{3.1.4}$$

Thus, $\emptyset_{kk}$ measures the correlation between x_t and x_{t-k} given $x_{t-1}, x_{t-2}, \ldots, x_{t-k+1}$ or the correlation between x_t and x_{t-k} after adjusting for the effects of $x_{t-1}, x_{t-2}, \ldots, x_{t-k+1}$.

3.5.7 Moving Average (MA) Process

Consider a white noise process (a_t) with zero mean and variance σ_a^2, then a process (X_t) is called moving average process of order q (abbreviated to MA (q) process) if

$$X_t = a_1 - \beta_1 a_{t-1} - \beta_2 a_{t-2} - \cdots - \beta_q a_{t-q} \tag{3.1.5}$$

27

Where the β_i's are constants. The mean, E $(X_t) = 0$ and

$$\text{var}(X_t) = \sigma^2{}_a \left(1 + \sum_{i=1}^{q} \beta_i{}^2\right) = \gamma_0.$$

$$\gamma_k = \text{Cov}(X_t X_{t+k}) = \begin{cases} \sigma_a{}^2\left(-\beta_i + \sum_{i=1}^{q-k} \beta_i \beta_{i+k}\right) & k=1, 2, ..., q \\ 0, & \end{cases} \qquad (3.1.6)$$

$$k > q$$

and $\qquad k = 1, 2, ..., q$

$$\rho_k = \begin{cases} \dfrac{-\beta_k + \beta_1 \beta_{k+1} + \beta_2 \beta_{k+2} + \cdots + \beta_{q-k}\beta_q}{1 + \beta^2{}_1 + \beta^2{}_2 + \cdots + \beta_q{}^2} & k > q \\ 0, & \end{cases} \qquad (3.1.7)$$

Autoregressive (AR) Processes

Consider a white noise process (a_t) with zero mean and variance $\sigma_a{}^2$, then a process (X_t) is said to be an Autoregressive process of order p (abbreviated to an AR (p) process) if

$$X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \cdots + \alpha_p X_{t-p} + a_t \qquad (3.1.8)$$

This model resembles a multiple regression model but the dependent variable X_t is regressed on past values of X_t rather than separate independent or predictor variables.

We can derive that $\gamma_k = \alpha_1 \gamma_{k-1} + \alpha_2 \gamma_{k-2} + \cdots + \alpha_p \gamma_{k-p}$ \qquad (3.1.9)

and $\rho_k = \alpha_1 \rho_{k-1} + \alpha_2 \rho_{k-2} + \alpha_3 \rho_{k-3} + \cdots + \alpha_p \rho_{k-p}$ \qquad (3.1.10)

Thus $\gamma_0 = \alpha_1 \gamma_1 + \alpha_2 \gamma_2 + \cdots + \alpha_p \gamma_p + \sigma^2{}_a$ \qquad (3.1.11)

But we know that $\rho_{k=\frac{\gamma_k}{\gamma_0}}$, hence the variance may be written as

$$\gamma_{0=\dfrac{\sigma^2{}_a}{1 - \alpha_1 \rho_1 - \alpha_2 \rho_2 - \cdots - \alpha_p \rho_p}} \qquad (3.1.12)$$

3.5.9 Mixed Autoregressive Moving Average Process (ARMA)

The combination of AR and MA processes produced ARMA. That is partially autoregressive and partially moving average and it is denoted by

$$X_{t=}\alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \cdots + \alpha_p X_{t-p} + a_t - \beta_1 a_{t-1} - \beta_2 a_{t-2} - \cdots - \beta_q a_{t-q} \quad (3.1.13)$$

Where (X_t) is Mixed Autoregressive Moving Average process of orders p and q (abbreviated to ARMA (p, q)).

3.5.10 Autoregressive Integrated Moving Average Models (ARIMA)

If a non-stationary time series which has variation in the mean is differenced to remove the variation the resulting time series is known as integrated time series. It is called integrated because the stationary model which is fitted to the differenced data as to be summed or integrated to provide a model for the non-stationary data. All AR (p) models can be represented as ARIMA (p, 0, 0) that is no differencing and no MA (q) part, also MA (q) models can be represented as ARIMA (0, 0, q) meaning no differencing and no AR (p) part. The general model is ARIMA (p, d, q) where p is the order of the AR part, d is the degree of differencing and q is the order of the q MA part. Suppose a series (X_t) is non-stationary. If the dth difference of the series $W_{t=\nabla^d X_t}$ is a stationary ARMA (p, q) process, then (X_t) is said to be an ARIMA (p, d, q) process.

Considering an ARIMA (p. 1, q) process when the first order difference is

$W_t = X_t - X_{t-1}$, then we can express the new (differenced) series as

$$W_t = \alpha_1 W_{t-1} + \alpha_2 W_{t-2} + \cdots + \alpha_p W_{t-p} + \alpha_t - \beta_1 a_{t-1} - \beta_2 a_{t-2} - \cdots -$$

$$\beta_q a_{t-q} \quad (3.1.14)$$

or in the original form,

29

$$X_t - X_{t-1} = \alpha_1(X_{t-1} - X_{t-2}) + \alpha_2(X_{t-2} - X_{t-3}) + \cdots + \alpha_p(X_{t-p} - X_{t-p-1}) +$$

$$\alpha_{t-\beta_1 a_{t-1} - \beta_2 a_{t-2} - \cdots - \beta_q a_{t-q}} \hspace{3cm} (3.1.15)$$

Simplifying, we have

$$X_t = (1 + \alpha_1)X_{t-1} + (\alpha_2 - \alpha_1)X_{t-2} + (\alpha_3 - \alpha_2)X_{t-3} + \cdots$$

$$+(\alpha_p - \alpha_{p-1})X_{t-p} - \alpha_p X_{t-p-1} + a_t - \beta_1 a_{t-1} - \beta_2 a_{t-2} - \cdots - \beta_q a_{t-q} (3.1.16).$$

Equation (3.1.16) resembles non-stationary ARMA (p+1, q) process and it is called
the Difference Equation Form.

3.6 Selecting From Competing Models

Sometimes, we can have a situation where one or more models best fit the series. In
such a case, a parsimonious model is chosen first. When two parsimonious models
best fit the data, we then go further and choose the model with minimum Akaike's
information criteria value and minimum residual variance.

3.6.1 Parsimony

Even though both diagnostic procedures describe above aid the analysis to arrive at a
forecasting model but neither procedure is considered the ultimate. We then introduce
the principle of parsimony. It requires that a forecaster uses few parameters in the
model. It also denotes that models should be simple and more accurate with the
prediction, everything else being equal. It is also easier to find estimators for
parameters in parsimonious models.

3.6.2 Akaike's Information Criteria (AIC)

AIC is a goodness of fit measure used to assess which of two parsimonious ARIMA
models are better when both have acceptable residuals. AIC has a function defined as

$$\text{AIC} = e^{\frac{2k}{T}} \sum_{i=1}^{T} \frac{e_t^2}{T} \qquad\qquad (3.1.17)$$

Where T is the sample size, k is the estimated number of parameters and e_t is the random error. The lower the AIC value, the better the model.

3.6.3 Residual Variance (RV)

Residual variance also called root mean square residual (RMS) or variance estimate as an alternative means of assessing the residuals. It is a measure of goodness of fit. The smaller the residual variance of the parsimonious models, the smaller the error and the better the model fits the time series data.

3.7 Methodology

Time series analysis is the main statistical tool employed in the analysis of the domestic and import vat revenue collection data. The data used for the analysis are secondary data and made up of monthly domestic and import vat revenue collection. These data were collected from Research, Monitoring and Evaluation Unit of Value Added Tax Service and Monitoring and Evaluation Unit of the Custom, Excise and Preventive Service, Headquarters, Accra. The data stretched from the January, 1999 to December, 2009(132 months). The software used for the analysis is R.

To begin with, the indirect tax revenue data values were vertically arranged from January, 1999 to December, 2009 in an Excel worksheet and then imported into R software using the R-Commander. As a requirement in R, the data were converted into time series data values using R-Commander 'dataset=ts(data, start=1999, frequency=12)'.

A descriptive analysis of the domestic and import vat revenue data (indirect tax revenue) was carried out by displaying a graphical representation of the monthly tax

31

revenue series and also computing the summary statistics such as the mean, standard deviation, variance, etc.

The indirect tax revenue was plotted against time (months) using the R-Commander 'plot (dataset, xlab="Time (Month)", ylab="Tax Revenue)")' in order to identify salient features such as trend, seasonality, outliers, discontinuities and stationarity in the dataset. The time plot depicted casual upward and downward behaviour with a general increasing linear trend which clearly indicated that the indirect tax series was non-stationary. In order to confirm this, the sample autocorrelation function of the indirect tax revenue data was computed and realised, as expected, that the autocorrelation coefficients at low lags were all 'large' and positive, and did not 'die out' or come down quickly to zero but declined gradually. The R-Commander used for computing the autocorrelation function was 'acf (dataset, 36)'. Because the tax revenue series was not stationary, differencing was therefore applied to transform the tax revenue data in order to attain the stationarity assumption, which is a prerequisite in Box-Jenkins modelling approach.

Furthermore, a first order differencing was carried with the R-Command 'difftax = diff (dataset)' to remove the trend component in the series. However, the number of observations was reduced from 132 to 131 due to the differencing.

Applying the rule of thumb that values exceeding $\pm \frac{2}{\sqrt{N}}$ are significantly different from zero, where N is the number of terms in the differenced series (in this case, N = 131), the sample autocorrelation function of the differenced tax revenue series (first-order differencing) revealed that the series had actually attained an appreciable stationarity. This means that apart from few autocorrelation coefficients at the low lags (1 or 2)

which were 'large' or exceeded the $\pm \frac{2}{\sqrt{N}}$ limits, the rest of the autocorrelation coefficients were close to zero or not significantly different from zero.

After achieving the stationarity assumption, the next task was to specify the order of the model. The behaviour of the autocorrelation function (ACF) and the sample partial autocorrelation function could be used to identify the model and the order that describes the stationary time series data. The order of an AR model could be assessed from the behaviour of the sample partial autocorrelation function (PACF). Theoretically, we expect 95% of the values of the partial autocorrelation coefficients, $\emptyset_{kk}$ to fall within the limits $\pm \frac{2}{\sqrt{N}}$ and values outside the range are significantly different from zero. The implication is that the sample partial autocorrelation function (PACF) of an AR (p) model 'cuts off' at lag p so that the values beyond p are not significantly different from zero. The sample partial autocorrelation function of the differenced tax revenue series was computed using the R –Command 'pacf (difftax, 36)'.

However, the order of a MA (q) model is usually clear from the sample autocorrelation function (ACF). The theoretical autocorrelation function of a MA(q) process 'cuts off' at lag q and values beyond q are not significantly different from zero. Similarly, the R command used for computing the autocorrelation function of the differenced tax revenue series was 'acf (difftax, 36)'. After specifying the orders for the AR and MA models using the sample partial autocorrelation function (PACF) and the sample autocorrelation function (ACF) respectively, three models were suggested.

Once we have specified the models, the next tax was to estimate the parameters of each of the three models suggested using the stationary series (differenced tax revenue

series) as well as to perform their corresponding diagnostic checks. The diagnostics actually helped us to evaluate the goodness-of –fit of the three models initially suggested.

On the basis of the diagnostic checks, the best-fit model was finally selected and used to make a 12-month forecast into the future of the Value Added Tax Service domestic tax revenue collection.

3.8   Box and Jenkins Method of Modeling

This method of forecasting by Box-Jenkins implements knowledge of autocorrelation analysis based on autoregressive integrated moving average models. This method is statistically sophisticated in analyzing and identifying a forecasting model that best fit the time series. The procedure is of four distinct stages namely; Identification, Estimation, Diagnostic checking, Forecasting.

The methodology has the following advantages;

1. It is logically and statistically accurate
2. Historical time series data are greatly utilized
3. Forecasting accuracy is increased

34

The schematic diagram below describes the four distinct phases of Box-Jenkins method;
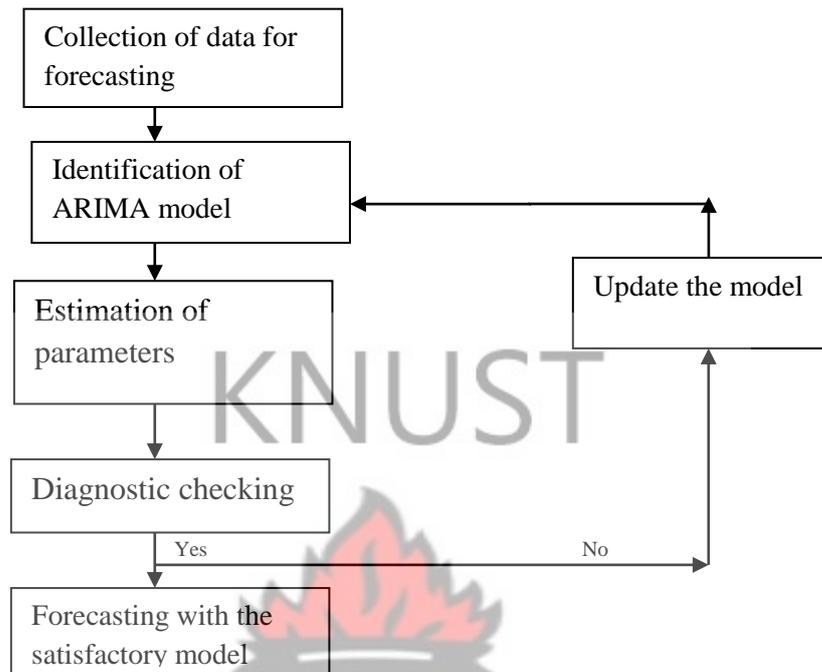
3.8.1 Identification of Models

Select the most appropriate ARMA model from the general class of ARMA (p, q) models denoted as

$$Y_t = \mu + \varphi_1 Y_{t-1} + \varphi_2 Y_{t-2} + \cdots + \varphi_p Y_{t-p} + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \cdots + \theta_p e_{t-p}\ldots$$

$$(3.1.18)$$

The p and q values are obtained by examining autocorrelation of the time series data.

3.8.2 Correlogram

A useful aid in interpreting a set of autocorrelation coefficient is a graph called a correlogram and it is a plot of autocorrelation against the lag (k). A correlogram can be used to get a general understanding of the following aspects of the series and effects of seasonal fluctuations.

35

When it is observed from a correlogram (a plot of autocorrelation coefficients) that the time series is non-stationary (have autocorrelation coefficients that fall outside the confident limit). It is differenced to achieve stationarity before we calculate partial auto-correlations and identify the ARIMA model properly. The order of p of the AR model is indicated by the number of partial autocorrelations that are statistically different from zero. Similarly, the order q of the moving average process is identified by the number of autocorrelation and partial autocorrelation trails off to zero.

SUMMARY OF HOW TO IDENTIFY MODELS

Model	Stationarity Condition	Invertibility Condition	Theoretical function	
			ACF coefficient	PACF coefficient
AR(p)	Yes	No	Dies down	Cut off after lag p
MA(q)	No	Yes	Cut off after lag q	Dies down
ARMA(p, q)	Yes	Yes	Dies down	Dies down

36

3.8.4   Estimation of Parameters

Once the preliminary model is chosen, the estimation stage begins. The purpose of the estimation is to find the parameter estimates that minimize the mean square error. An iterative non-linear least squares procedure is applied to the parameter estimates of an ARMA (p, q) model. The method minimizes the sum of squares of error Σe_t^2 given to form the model and data. The estimates usually converge on an optimal value for the parameters with a small number of iterations. However, some software programmes have been designed to estimate and test for parameter of parsimonious models. Among such programmes is the SPSS Analyze.

3.8.5   Diagnosis Checking and Q-Statistics

Residuals from the fitted model are examined to ensure that the model is adequate (random). Autocorrelation of the error term are estimated and plotted to determine whether they are statistically zero. Thus the observed value is test as a result of sampling error. This is the first test for adequacy. The second test for adequacy is the Q-test whose test statistic is given by

$$Q = n\,(n+2)\sum_{i}^{k} \frac{r_i^2}{n-i}.$$ (3.1.19)

The Q-statistics is approximately distributed as a Chi-square with (k-p-q) degrees of freedom where n is the length of the time series, r is the autocorrelation coefficient of the residual term, p and q are the order of the AR and MA process respectively and k is the first k autocorrelation being checked.

The hypothesis test is

   H_O: The model is adequate

H_1: The model is inadequate with (k-p-q) degrees of freedom

$H_0$is rejected if Q $>X^2$(k-p-q) and we conclude that the model is inadequate.

Under such circumstances, other ARMA model may be tried until a

satisfactory model is obtained.

3.8.6   Forecasting

When a satisfactory ARMA model has been found to be adequate, then it is

straightforward to forecast or predict for a period or several periods ahead. This is

done by using the estimates of the parameters to calculate new values of the series.

Large errors require that the parameters are recalculated for accuracy or an entirely

new forecasting model may be developed. For instance, if we have a model

ARIMA (1,1,0), the forecasting equation will be

$$Y_t = \mu + \varphi(Y_{t-1} - Y_{t-2}),$$ (3.1.20)

where Y_t= forecasted model, μ = constant mean,φ =ccoefficient of the

observed forecast and Y_{t-1}, Y_{t-2} =previous observed forecast

3.8.6.1   Basic Forecasting Methods

A forecasting method is a procedure for computing forecasts from present and past

values. As such it may simply be an algorithmic rule and need not depend on an

underlying probability model. Alternatively it may arise from identifying a particular

model for the given data and finding optimal forecasts conditional on that model.

In this section, we suppose that we are currently at time *t* and that we wish to use the

data up to this time, ieY_1, Y_2, ..., Y_{t-1}, Y_t, to make forecasts F$_{t+1}$, F$_{t+2}$, ..., F$_{t+m}$ of future

values of *Y*.

The methods to be considered are conventionally regarded as being divided in two groups:

(i) Averaging Methods.

(ii) Exponential Smoothing methods.

Though it is convenient to follow this convention, it is important to realise at the outset that this distinction is artificial in that all the methods in this section are methods based on averages. The averages are used for forecasting rather than for describing past data.

This point of potential confusion is made worse by the use of the name 'exponential smoothing' for the second group. These methods are also based on weighted averages, where the weights decay in an exponential way from the most recent to the most distant data point. The term smoothing is being used simply to indicate that this weighted average smoothes the data irregularities.

3.8. 6.2   Averaging Methods

The moving average forecast of order k, which we write as MA(k), is defined as

$$F_{t+1} = \frac{1}{k} \sum_{i=t-k+1}^{T} Y_i$$

(3.1.21)

This forecast is only useful if the data does not contain a trend-cycle or a seasonal component. In other words the data must be *stationary*. Data is said to be stationary if Y_t, which is a random variable, has a probability distribution that does not depend on t.

A convenient way of implementing this forecast is to note that

39

$$F_{t+1} = \frac{1}{k} \sum_{i=t-k+2}^{t+1} Y_i = F_{t+1} + \frac{1}{k}(Y_{t+1} - Y_{t-k+1}).$$

(3.1.22)

This is known as an updating formula as it allows a forecast value to be obtained from the previous forecast value by a simpler calculation than using the defining expression.

The only point of note is that moving average forecasts give a progressively smoother forecast as the order increases, but a moving average of large order will be slow to respond to real but rapid changes. Thus, in choosing k, a balance has to be drawn between smoothness and ensuring that this *lag* is not unacceptably large.

3.8.3 Single Exponential Smoothing

The single exponential forecast or single exponential smoothing (SES) is defined as

$$F_{t+1} = \alpha Y_t + (1 - \alpha)F_t$$

(3.1.23)

Where α is a given *weight* value to be selected subject to $0 < \alpha < 1$. Thus F_{t+1} is the weighted average of the *current* observation, Y_t, with the forecast, F_t, made at the previous time point $t-1$.

Repeated application of the formula yields

$$F_{t+1} = (1 - \alpha)^t F_1 + \alpha \sum_{j=0}^{t-1} (1 - \alpha)^j Y_{t-j}$$

(3.1.24)

Showing that the dependence of the current forecast on Y_t, Y_{t-1}, Y_{t-2} ... falls away in an exponential way. The rate at which this dependence falls away is controlled by α. The

40

larger the value of α, the quicker does the dependence on previous values fall away.

SES needs to be initialized. A simple choice is to use

$F_2 = Y_1$.

CHAPTER FOUR

RESULTS AND ANALYSIS

4.0   Introduction

This chapter entails an analysis of the monthly VAT performance (i.e. domestic and import vat) of the VAT SERVICE, with data available from the year 1999-2009. The statistical computing tool employed for this work is R software, and the Box Jenkins methodology of time series analysis is also used.

Because the data entails two (2) variables, namely domestic vat and import vat, an analysis on each is done and models fitted for each to be used for forecasting.

4.1   DOMESTIC VAT.

4.1.1   PRELIMNARY ANALYSIS
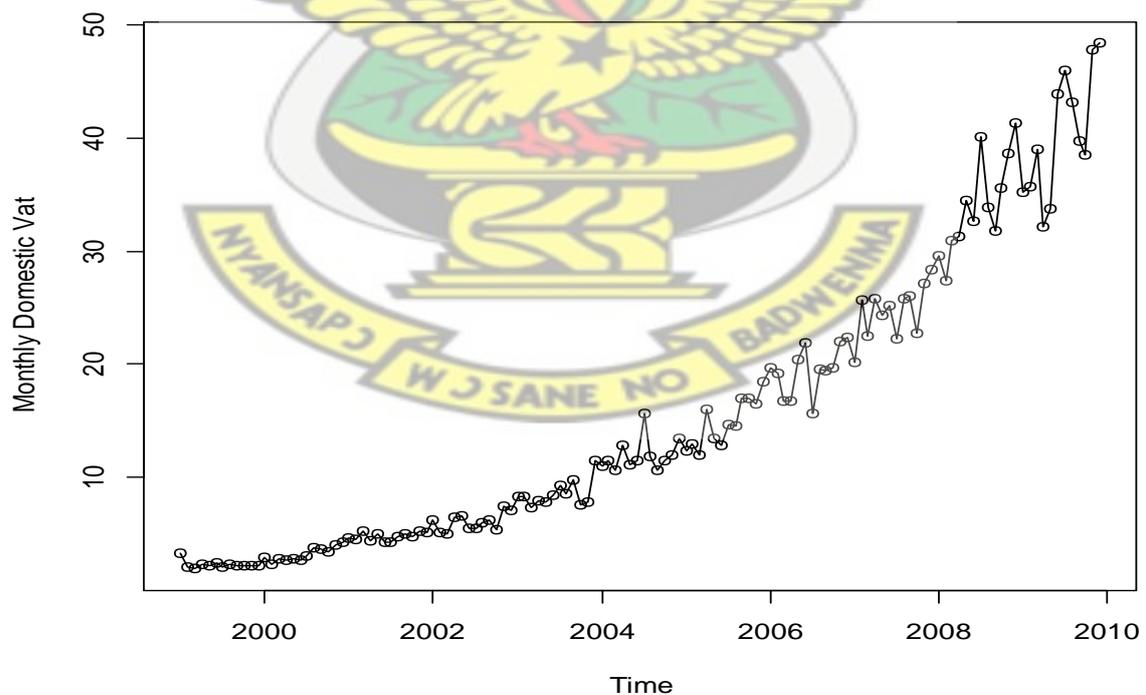
Figure 4.1.1 : Trend of Monthly Domestic VAT From JAN 1999 TO DEC 2009

Figure 4.1.1 above shows the pattern of monthly data of domestic vat (in millions of GH¢) obtained from the VAT service between January, 1999 and December, 2009.

We observe a gradually increasing underlying trend and a rather regular variation superimposed on the trend that seems to repeat over months, with maximum peak at 2009 (i.e. December), which recorded 48.36 million cedi in domestic vat. The minimum domestic vat amount occurred in March, 1999. The linear trend is made more evident after the decomposition of the observed data (in figure 4.1.1) as shown in figure 4.1.2 below.
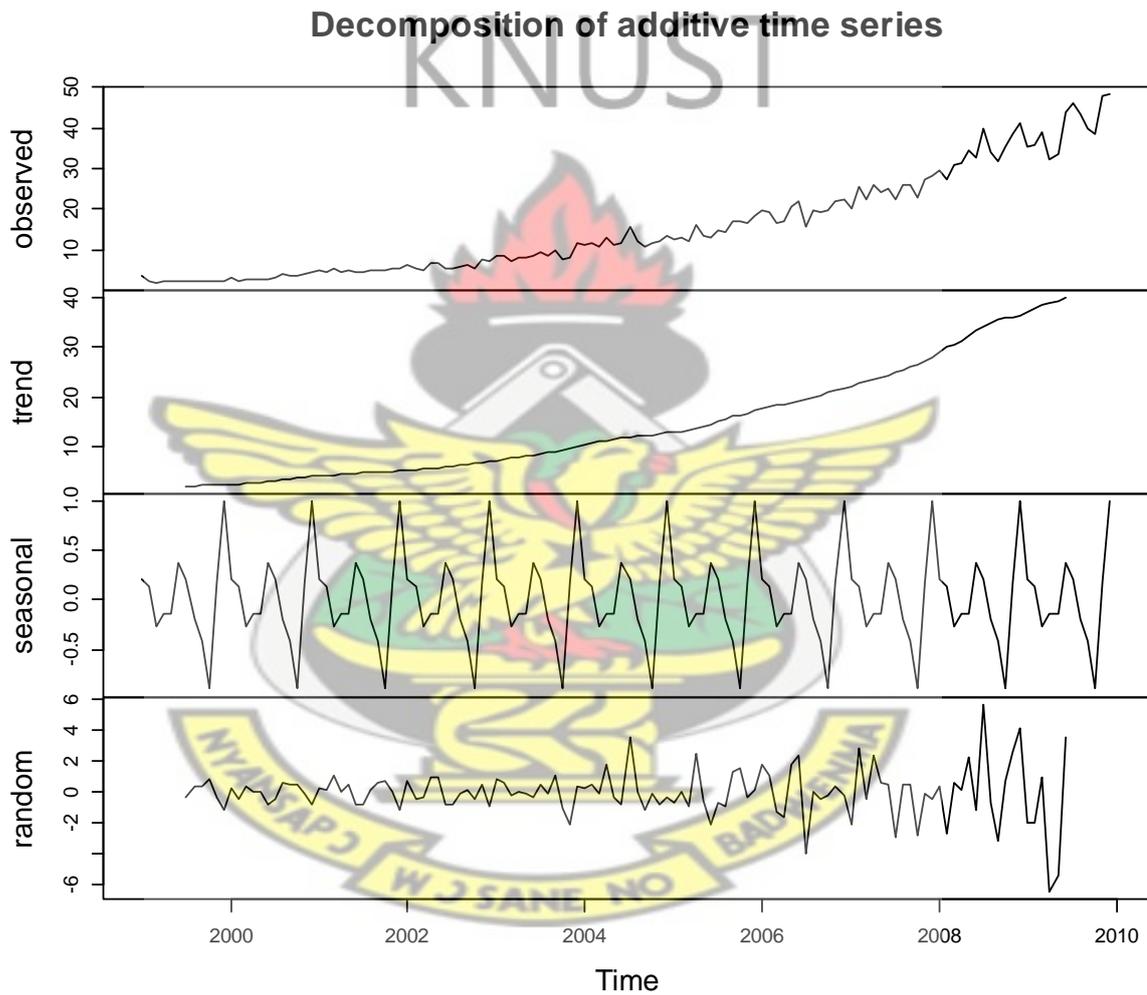

Figure 4.1.2: Decomposition of the domestic VAT series.

After decomposition, it is observed clearly that the data exhibits a systematic linear trend but the existence of seasonality is suggested. This is because the pattern

displayed in figure 4.1.2 could be as a result of the irregular component in the time series.

Table 4.1.1:   Summary statistics of domestic VAT data.

Minimum	1st Quartile	Median	Mean	3rd Quartile	Standard deviation	Maximum
1.900	5.038	11.490	15.450	22.530	12.50743	48.360

From Table 4.1.1 above, we observe that the minimum amount of domestic vat is 1.9 million, which occurred in March, 1999. The maximum amount of domestic vat is 48.36 million which also occurred in December, 2009.

The average domestic vat amount is nowhere equal to the median domestic vat amount. This indicates a non-symmetric behaviour of the domestic vat distribution. Also, from the observed pattern, it is seen clearly that the domestic vat data is not stationary.

3.5.1   ACHIEVING STATIONARITY

Due to the non-stationary nature of most business and economic time series, it is required that stationarity be achieved before building any model. To achieve stationarity on the time series that has trend in it, the series must be differentiated until it become stationary. This method is an integral part of the procedures advocated by Box and Jenkins (1970). For non-seasonal data, first order differencing is usually sufficient to attain apparent stationarity so that the new series $(y_1,\ldots,y_n{=}1)$ is formed from the original series $(x_1),\ldots,(x_n)$ by $Y_t{=}x_{t+1} - x_t = \nabla x_{t+1}$. Occasionally, second order differencing is required using the operator ∇^2 where $\nabla^2 x_{t+2} = \nabla x_{t+2} - $

$\nabla x_{t+1} \quad -2x_{t+1} + x_t$.Hence the number of times that the original series is differenced to achieve stationary is the Order of Homogeneity. Trend in variance is removed by taking logarithms.
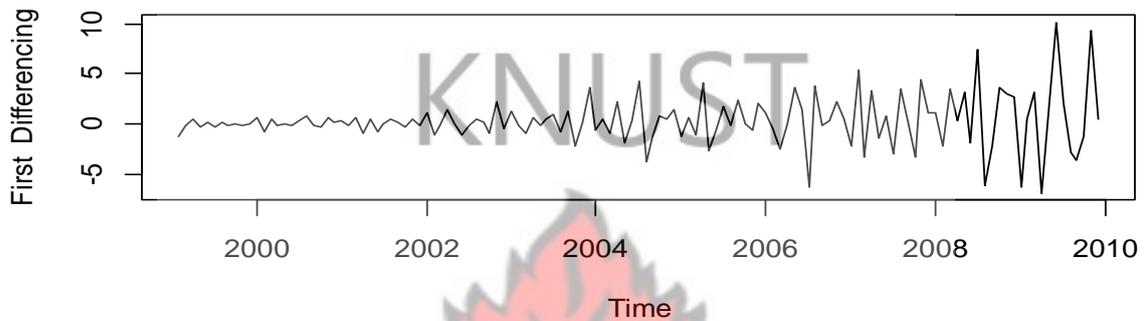
Figure 4.1.3(a) : Pattern of Domestic VAT Data After First Differencing
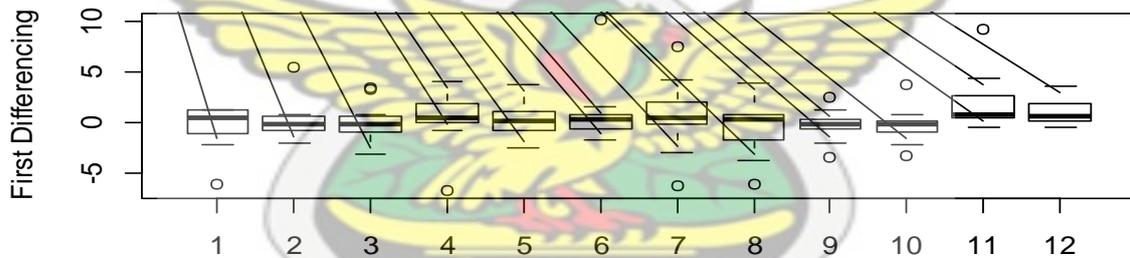
Figure 4.1.3(b): BoxPlot of Domestic VAT Data After First Differencing

From figure 4.1.3 (a) above, it can be seen that after the first differencing, the domestic VAT series becomes stationary. It can therefore be said that our data is non-seasonal (in reference to the methodological statement made in section 3.5.1), since for non-seasonal data, first order differencing is usually sufficient to attain apparent stationarity.

Also from the box plot in figure 4.1.3 (b) above, the means for each month are relatively close and show no obvious pattern of seasonality.

Also, if there is significant seasonality, the autocorrelation plot should show significant spikes at lags equal to the period of the series. For example, for monthly data, if there is a seasonality effect, we would expect to see significant peaks at lag 12, 24, 36, and so on (although the intensity may decrease the further out we go).

From figure 4.1.4 below, it can be seen from the sample ACF that lag 12 and the other lags following all lie within the significant bounds, hence showing no significant peaks.

Furthermore, the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test is computed to validate the null hypothesis that the differenced data is level or trend stationary.

KPSS Test for Level Stationarity

data: diff1

KPSS Level = 0.3461, Truncation lag parameter = 2, p-value = 0.1

Conclusion: At an α(alpha) 5% level of significance, we fail to reject the Null hypothesis that the domestic VAT series is trend or level stationary since the p-value (0.1) > 0.05, and hence conclude that the series is stationary.

Table 4.1.2: Summary statistics of differenced domestic VAT data.

Minimum	1st Quartile	Median	Mean	3rd Quartile	Standard deviation	Maximum
-6.810	-0.655	0.250	0.344	1.180	2.450147	10.150

The average of the differenced domestic VAT data is approximately equal to the median differenced domestic VAT data. This may indicate some symmetric behaviour of the differenced domestic VAT data distribution.

46

MODEL IDENTIFICATION

Sample ACF for Differenced Domestic VAT Data

Sample PACF for Differenced Domestic VAT Data
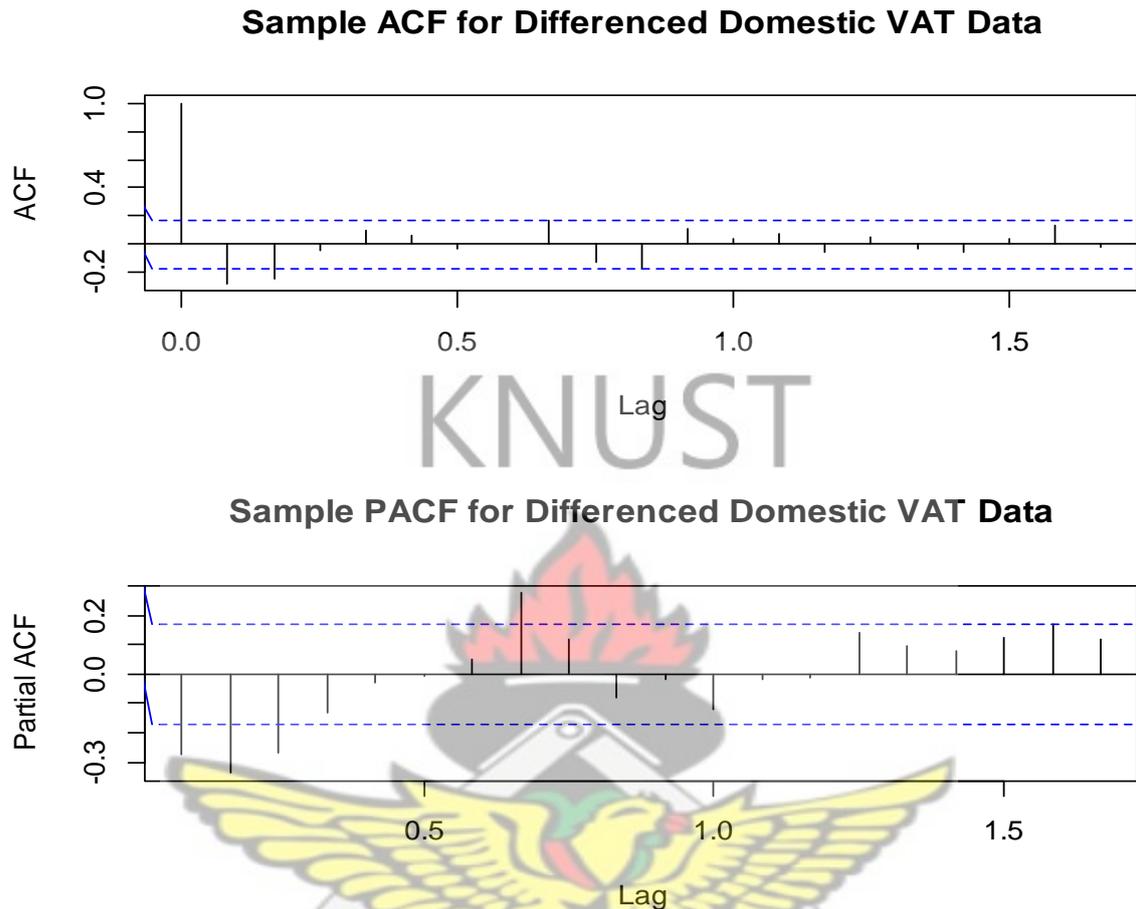
Figure 4.1.4 : Shows the Sample ACF (top) and Sample PACF (bottom) for the diff data

In order to select the appropriate model and also make more accurate forecasts, we fit several feasible ARIMA models in relation to the differenced data by making reference to its Sample ACF and Sample PACF (in Figure 4.1.4 above). Since the data is differenced once to attain stationarity (as shown by the KPSS Test), the fitted ARIMA models would be of order (p, d=1, q).

From the correlogram in figure 4.1.4, the sample ACF tails off to zero after the first two (2) significant lags, i.e. lag 1 and lag 2 exceed the significant bound and there after dies down exponentially. The lags that touches the bounds at lag 8 and 10 are however not significant.

Also the partial correlogram shows that the partial autocorrelations at lags 1, 2 and 3 exceed the significance bounds, and are negative as well. The partial autocorrelations tails off to zero after lag 3. Lag 8 which exceed the bound is however considered not significant since it may be due to chance.

From the foregoing analysis, the following ARIMA (Autoregressive integrated moving average) models are therefore possible for the data series:

- ARIMA(3,1,2)
- ARIMA(3,1,0)
- ARIMA(0,1,2)
- ARIMA(2,1,2)

4.1.2 ESTIMATION OF PARAMETERS AND DIAGNOSTIC CHECKING

At this point we proceed to estimate the parameters and investigate whether the residuals of the selected ARIMA models are normally distributed with mean zero and constant variance, and also whether there are no correlations between successive residuals (i.e. randomness of residuals).

To check for correlations between successive residuals, we make use of a correlogram and also the Ljung-Box test to further ascertain the adequacy (randomness) of the chosen model.

Also to check whether the residuals are normally distributed with mean zero and constant variance, we make use of a normality quantile-quantile plot (q-q plot) and a histogram.

If the residuals are normally distributed, the points on the normal quantile-quantile plot should approximately be linear, with residual mean as the intercept and residual

standard deviation as the slope whilst the shape of the histogram shows "a bell-like" shape.

- **ARIMA(3,1,2)**

Coefficients:

  ar1   ar2   ar3   ma1   ma2   drift

0.9795 -0.5575 -0.0568 -1.6039 1.0000 0.3446

s.e. 0.0881   0.1109   0.0883   0.0686   0.0845 0.1056

sigma^2 estimated as 3.778: log likelihood=-276.34

AIC=566.68   AICc=567.59   BIC=586.81

ME	RMSE	MAE	MPE	MAPE	MASE
-0.008512	1.93645	1.38023	-5.28502	11.65586	0.85977

ACF of ARIMA(3,1,2) Residuals
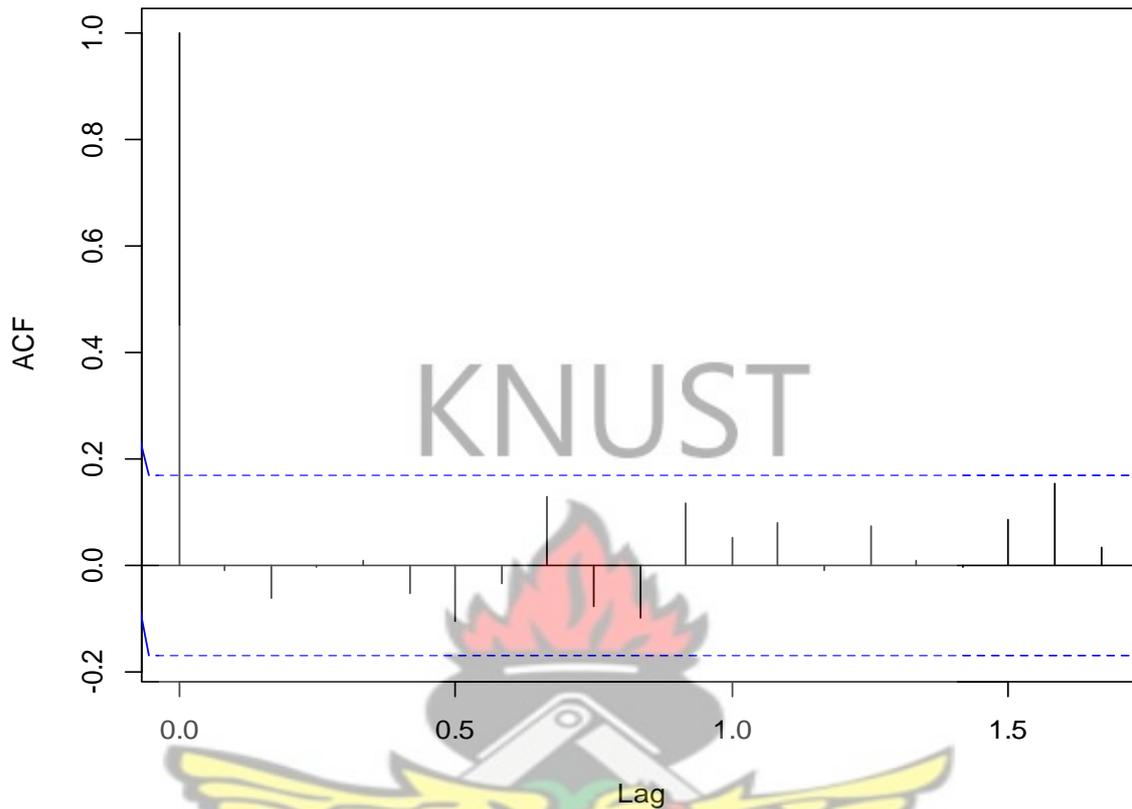
Figure 4.1.5: ACF of ARIMA(3,1,2) Residuals

Box-Ljung test:

data: model1$residuals

X-squared = 16.5638, df = 20, p-value = 0.6811

From figure 4.1.5 above, the ACF of residuals shows that none of the sample autocorrelations for the lags exceed the significant bounds. This gives an indication of a white noise process.

Also, from the Ljung-box test above, the computed p-value (i.e. 0.6811) is also greater than α (alpha) 5% level of significance.

Hence from these deductions, we fail to reject the null hypothesis that the series of residuals exhibits no autocorrelation and conclude that there is very little evidence for

non-zero autocorrelations in the residuals at all lags (i.e. the residuals are independently distributed).
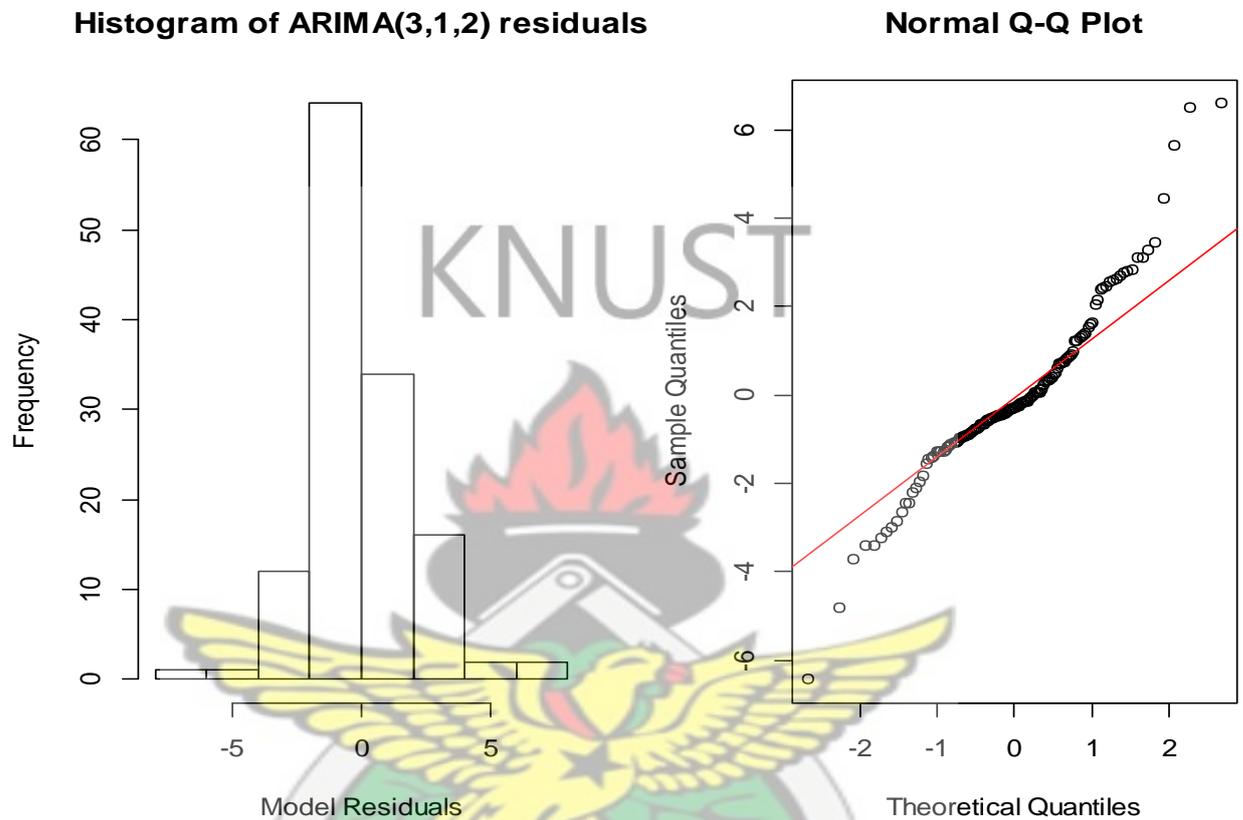
Figure 4.1.6: Shows the Histogram (left) and Normality plot (right) for the residuals of ARIMA (3, 1 ,2).

`From the plot in figure 4.1.6, the histogram of the residuals displayed above gives an indication of a symmetric distribution, thus it shape looks "bell-like" and certainly better for the fitted model. The QQ-normal plot for the residuals also throws more light on this since most of its residuals do not deviate that much from the line of best fit and it distribution looks approximately linear.

- **ARIMA(3,1,0)**

Coefficients:

      ar1   ar2   ar3   drift

  -0.4938 -0.5074 -0.3213 0.3294

s.e. 0.0839 0.0887   0.0874 0.0790

sigma^2 estimated as 4.331: log likelihood=-282.26

AIC=574.51   AICc=574.99   BIC=588.89

ME	RMSE	MAE	MPE	MAPE	MASE
-0.01347	2.07325	1.45469	-6.74412	12.9983	0.9062
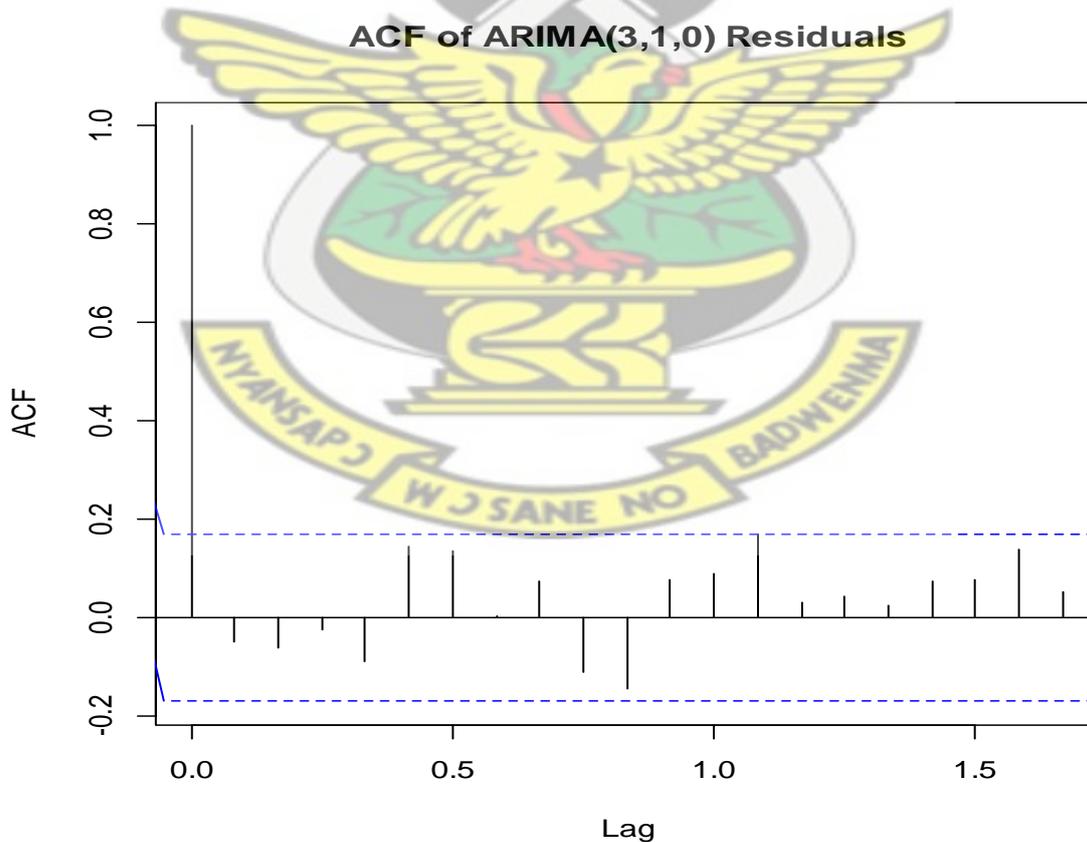
Figure 4.1.7: ACF of ARIMA(3,1,0) Residuals

Box-Ljung test:

data: model2$residuals

X-squared = 25.2007, df = 20, p-value = 0.1939

From figure 4.1.7, the ACF of residuals shows that none of the sample autocorrelations for the lags exceed the significant bounds.

Also, from the Ljung-box test above, the computed p-value (i.e. 0.1939) is also greater than α (alpha) 5% level of significance.

Hence from these deductions, we fail to reject the null hypothesis that the series of residuals exhibits no autocorrelation and conclude that there is very little evidence for non-zero autocorrelations in the residuals at all lags (i.e. the residuals are independently distributed).
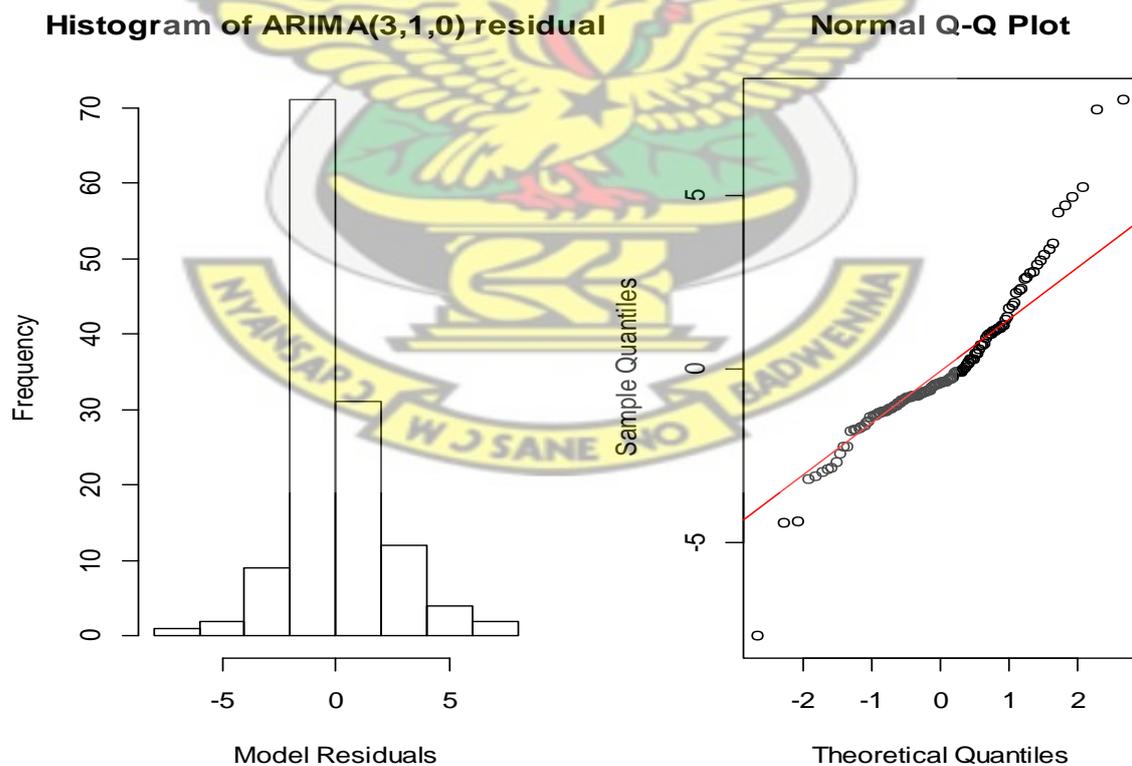
Figure 4.1.8: Shows the Histogram (left) and Normality plot (right) for the residuals of ARIMA (3, 1, 0).

From the plot in figure 4.1.8, the histogram of the residuals displayed above gives an indication of a symmetric distribution, thus it shape looks "bell-like" and certainly better for the fitted model. The QQ-normal plot for the residuals also throws more light on this since most of its residuals do not deviate that much from the line of best fit and it distribution looks approximately linear.

- **ARIMA(0,1,2)**

Coefficients:

      ma1   ma2 drift

   -0.467 -0.1893 0.3288

s.e. 0.093 0.0856 0.0653

sigma^2 estimated as 4.547: log likelihood=-285.32

AIC=578.64 AICc=578.96 BIC=590.14

ME	RMSE	MAE	MPE	MAPE	MASE
-0.01965	2.12435	1.50724	-7.86039	13.85057	0.93889

ACF of ARIMA(0,1,2) Residuals
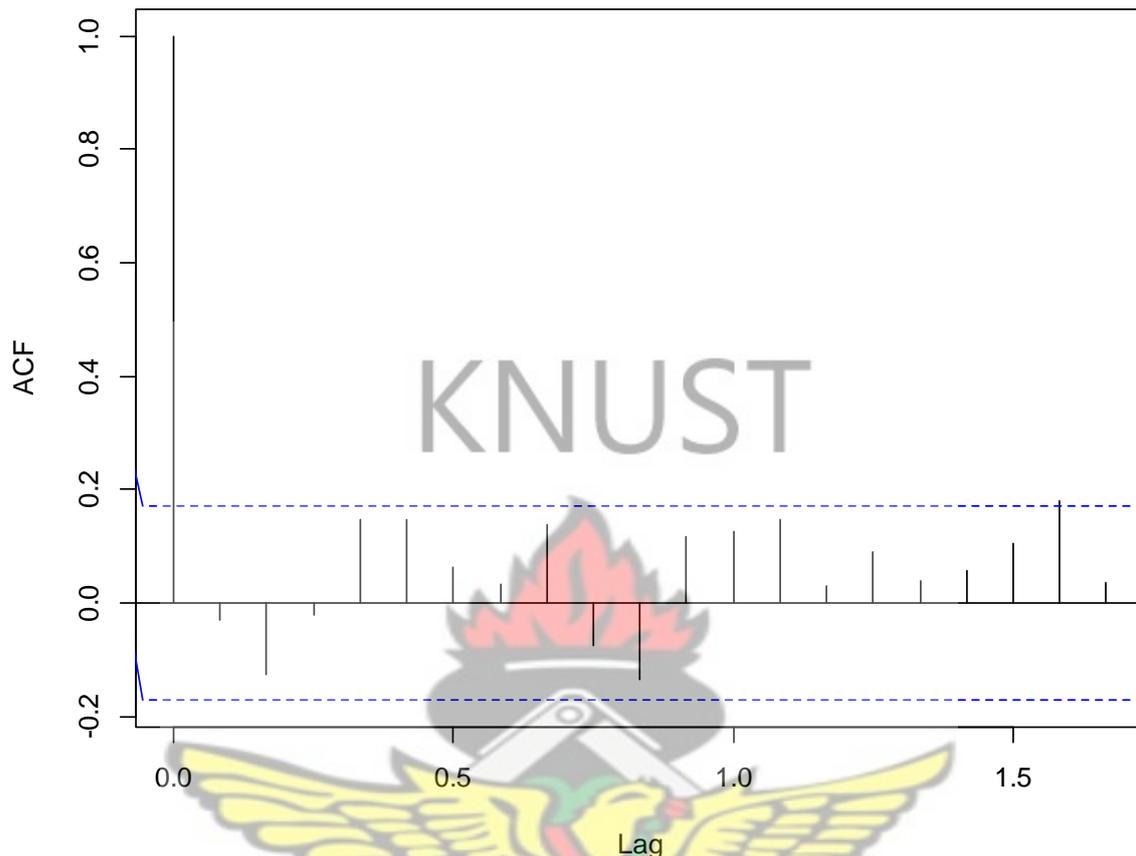
Figure 4.1.9: ACF of ARIMA(0,1,2) Residuals

Box-Ljung test:

data: model3$residuals

X-squared = 32.0433, df = 20, p-value = 0.04284

The correlogram in figure 4.1.9 shows the sample autocorrelation at lag 19 just exceeds the significance bounds. However, more of the lags seem to get closer to the significant bounds, hence we would expect at least one (1) out of 20 sample autocorrelations to exceed the 95% significance bounds.

Furthermore, the p-value for the Ljung-Box test computed above is 0.04284, indicating that there seem to be significant evidence for non-zero autocorrelations in the residuals for lags 1-20 (i.e. the residuals are dependently distributed).

To check whether the residuals are normally distributed with mean zero and constant
variance, we make a normality plot and a histogram of the residuals.

**Histogram of ARIMA(0,1,2) residuals**   **Normal Q-Q Plot**
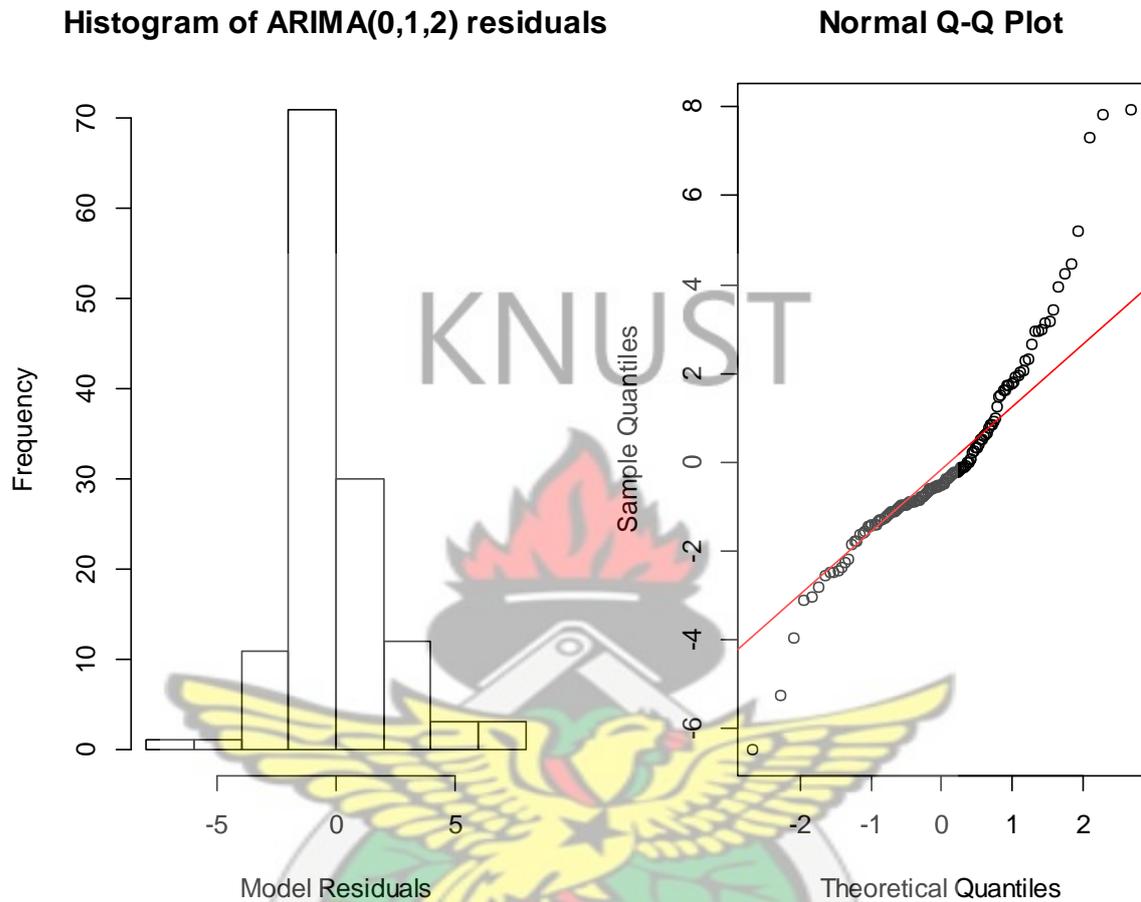
**Figure 4.1.10: Shows the Histogram (left) and Normality plot (right) for the
residuals of ARIMA (0, 1, 2).**

From the plot in figure 4.1.10, the histogram of the residuals displayed above gives an
indication of a symmetric distribution, thus it shape looks "bell-like" and certainly
better for the fitted model. The QQ-normal plot for the residuals also throws more
light on this since most of its residuals do not deviate that much from the line of best
fit and it distribution looks approximately linear.

- **ARIMA(2,1,2)**

Coefficients:

ar1   ar2   ma1   ma2 drift

1.0145 -0.613 -1.6053 0.9999 0.3402

s.e. 0.0697 0.070 0.0889 0.1100 0.1117

sigma^2 estimated as 3.788: log likelihood=-276.55

AIC=565.09   AICc=565.77   BIC=582.34

ME	RMSE	MAE	MPE	MAPE	MASE
-0.0032	1.93896	1.38327	-4.962244	11.46270	0.86167
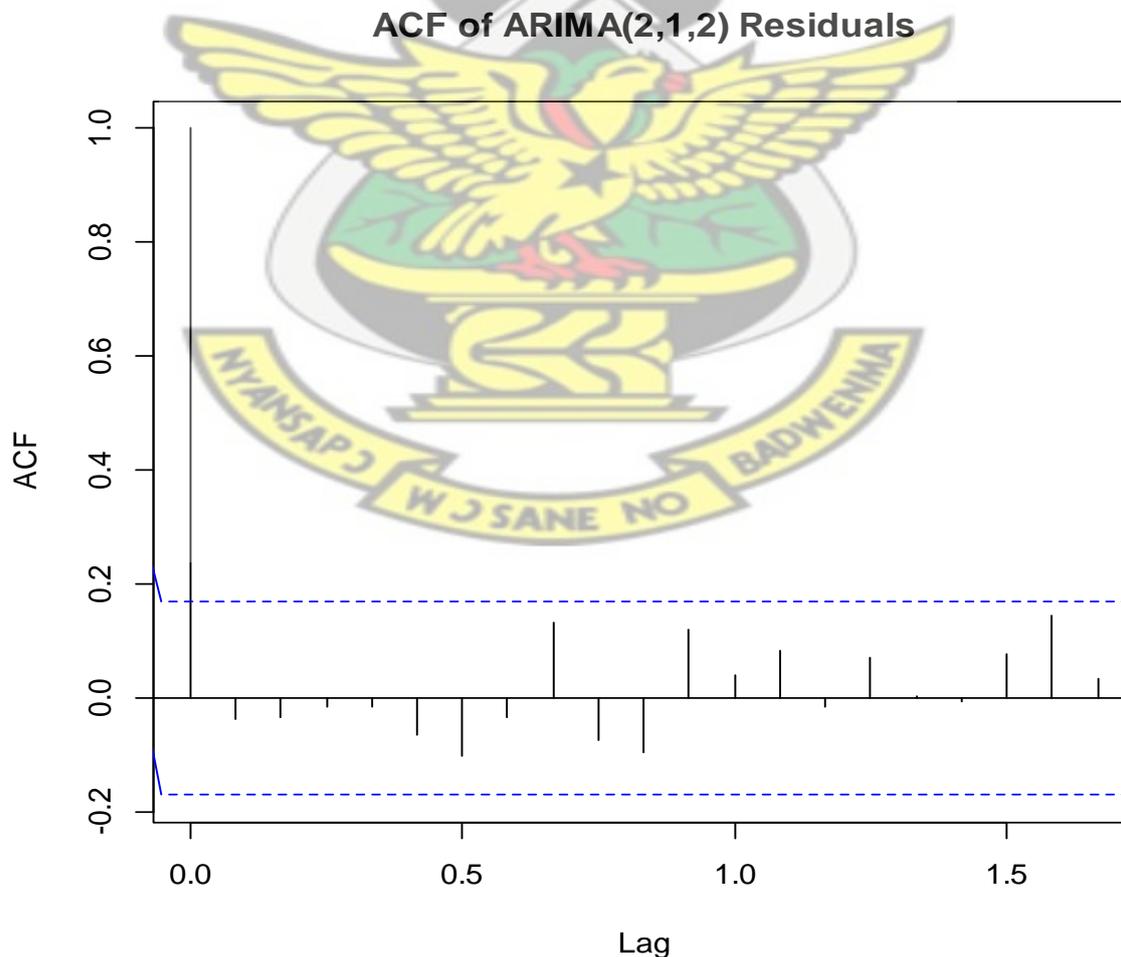
Figure 4.1.11: ACF of ARIMA(2,1,2) Residuals

Box-Ljung test:

data: model4$residuals

X-squared = 15.5578, df = 20, p-value = 0.7437

The correlogram in figure 4.1.11 shows that none of the sample autocorrelations at the lags exceeds the significance bounds. Also, more of the lags seem to get closer to zero. The ACF of residuals therefore gives an indication of a white noise process. Furthermore, the p-value for the Ljung-Box test computed above is 0.7437, indicating that there exist no significant evidence for non-zero autocorrelations in the residuals for lags 1-20 (i.e. the residuals are independently distributed).

To check whether the residuals are normally distributed with mean zero and constant variance, we make a normality plot and a histogram of the residuals.

Histogram of ARIMA(2,1,2) residuals   **Normal Q-Q Plot**
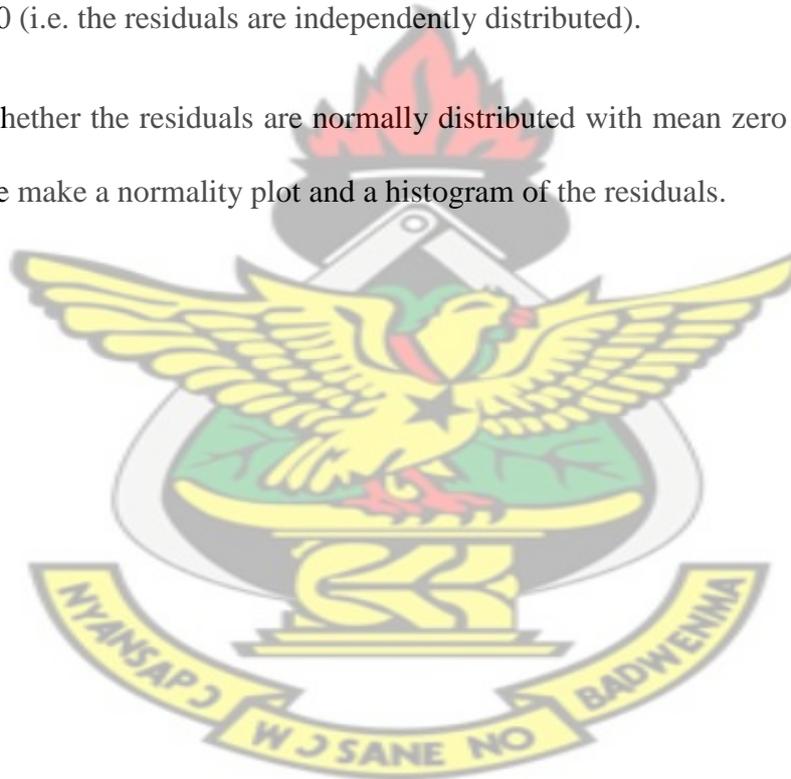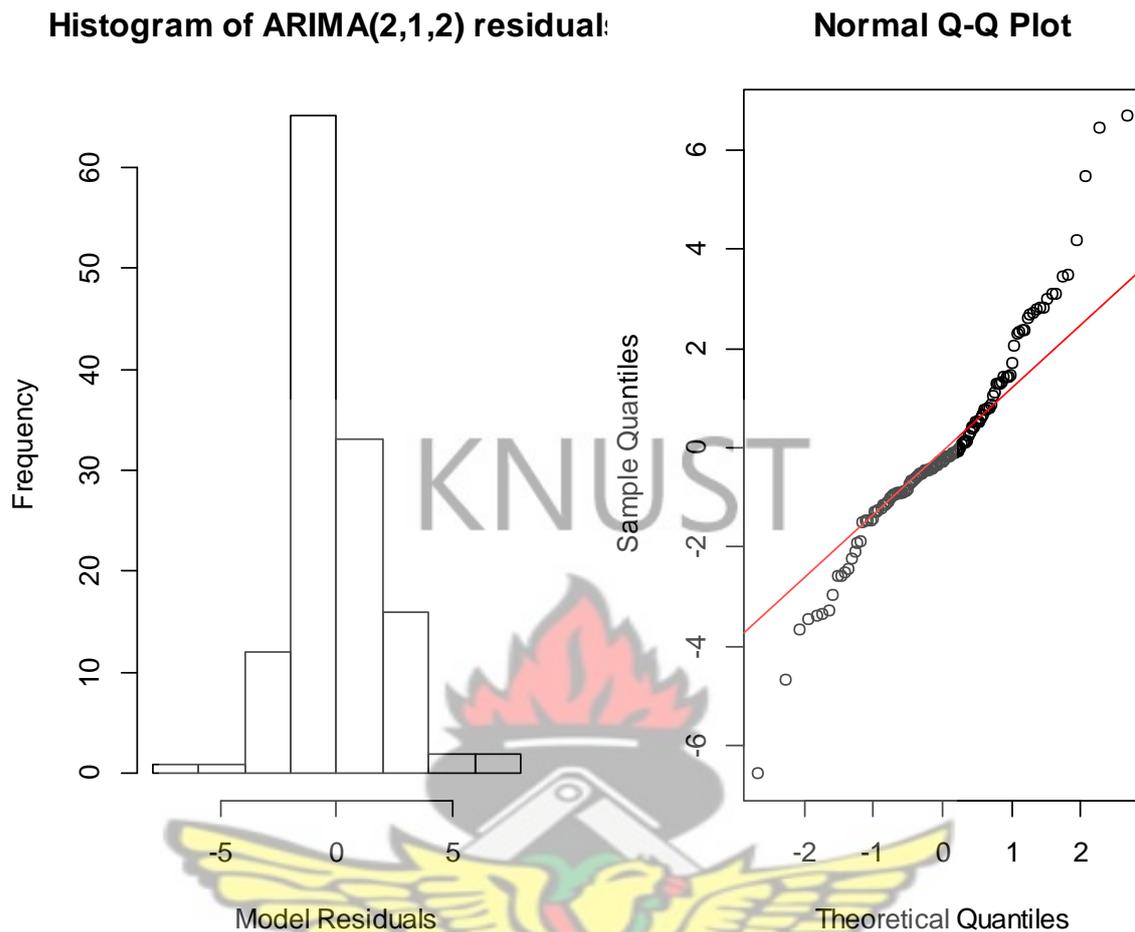
Figure 4.1.12: Shows the Histogram (left) and Normality plot (right) for the residuals of ARIMA (2, 1, 2).

From the plot in figure 4.1.12, the histogram of the residuals displayed above gives an indication of a plausible symmetric distribution, thus it shape looks "bell-like" and certainly better for the fitted model. The QQ-normal plot for the residuals also throws more light on this since most of its residuals do not deviate that much from the line of best fit and it distribution looks approximately linear.

4.1.3 Model Selection

In order to select the most appropriate model for our data, we compare all competing models and select the one with the minimum AIC (Akaike Information Criterion

value), Schwartz Bayesian Information Criterion (BIC) and Residual Variance. Other statistical tests like Root Mean Squared Error (RMSE), Mean Abs. Percent Error (MAPE), Bias Proportion or Mean Forecast Error (MFE) and Mean Absolute Scaled Error are also used in testing the forecast accuracy of the fitted models.

It should be noted that a model which fits the data well does not necessarily forecast well. The best model will be the one that achieves a compromise between the various information criterion values.

From the diagnostic checks above, since ARIMA (0, 1, 2) failed to satisfy the assumption of non-autocorrelation, it fails to stand as a possible competing model.

Table 4.1.3: AIC, BIC, Residual Variance and MASE for the possible Models

Model	Akaike Information Criterion (AIC)	Residual Variance	BIC	MASE
ARIMA(3,1,2)	566.68	3.778	586.81	0.85977
ARIMA(3,1,0)	574.51	4.331	588.89	0.90615274
ARIMA(2,1,2)	565.09	3.788	582.34	0.861664164

From table 4.1.3 above, it is clear that ARIMA(2,1,2) model is the best model for forecasting since its AIC and residual variance values are far lower than the other competing model.

Therefore, the chosen model for the domestic VAT series is of the form;

$$Y_t - Y_{t-1} = \alpha_1(Y_{t-1} - Y_{t-2}) + \alpha_2(Y_{t-2} - Y_{t-3}) - \theta_1 e_{t-1} - \theta_2 e_{t-2} + e_t + \mu$$

$$Y_t - Y_{t-1} = 1.0145 \quad (Y_{t-1} - Y_{t-2}) - 0.613(Y_{t-2} - Y_{t-3}) + 1.6053 \ e_{t-1}$$

$$- 0.9999e_{t-2} + e_t + 0.3402$$

OR

$$Y_t = (1 + \alpha_1)Y_{t-1} + (\alpha_2 - \alpha_1)Y_{t-2} - \alpha_2 Y_{t-3} - \theta_1 e_{t-1} - \theta_2 e_{t-2} + e_t + \mu$$

$$Y_t = 2.0145Y_{t-1} - 1.6275 \ Y_{t-2} + 0.613Y_{t-3} + 1.6053e_{t-1} - 0.9999e_{t-2} + e_t +$$

0.3402 $\hspace{10cm}$ (4.1)

This indicates that the fitted model is a linear combination of previous domestic VAT values, previous forecast errors and a constant.

4.1.4 Forecasting

We also make forecast using the best fitted model for the next three years. Below is the graph of the forecasts.


Forecasts From ARIMA(2,1,2)
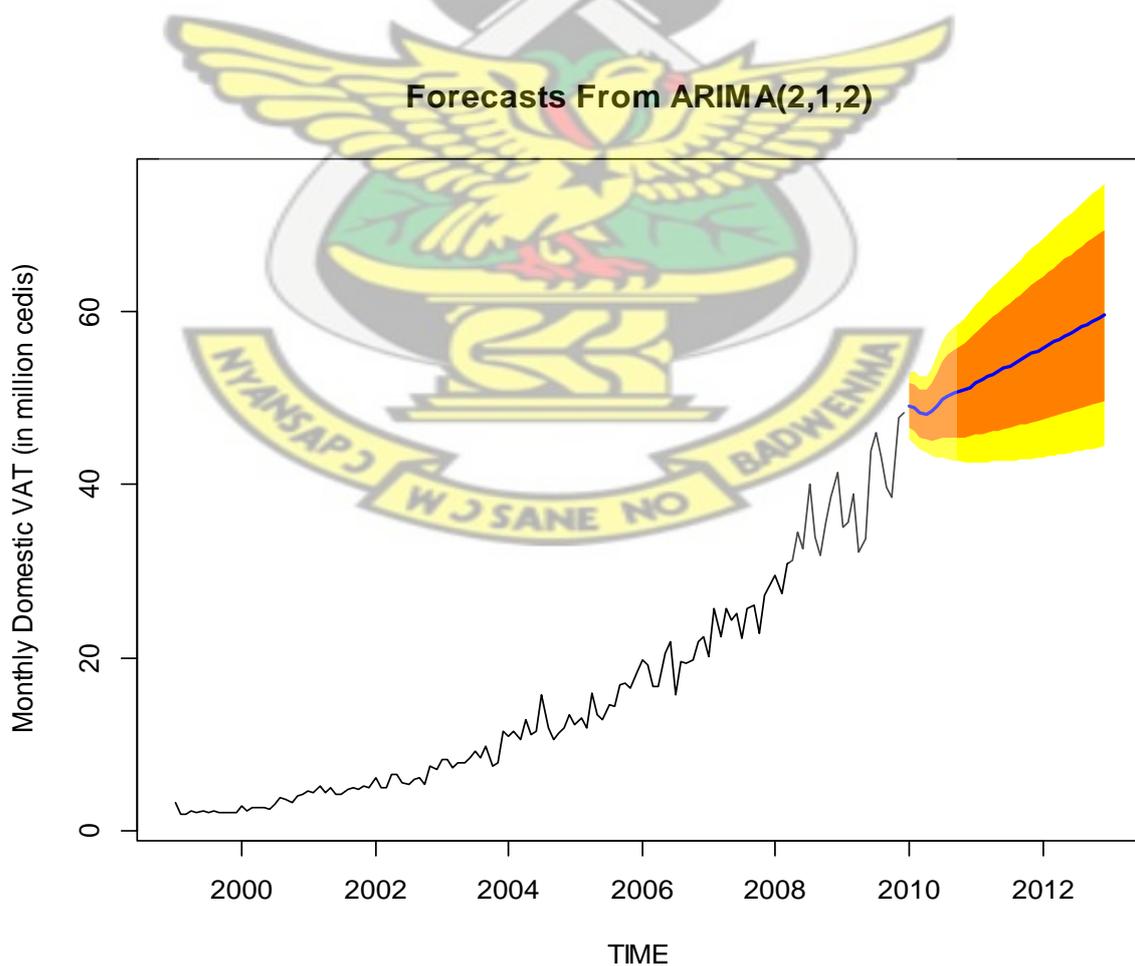
Figure 4.1.13: The forecasted domestic VAT (in millions) is shown by the blue line, whilst the orange and yellow shaded areas show 80% and 95% prediction intervals respectively.

The forecasted values and standard errors are given in table 4.1.4 and 4.1.5 below respectively:

Table 4.1.4: Forecasted Values for ARIMA (2, 1, 2)

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug
2010	49.13626	48.83630	48.25981	48.06246	48.41928	49.10587	49.78731	50.26137
2011	51.64938	52.05225	52.42540	52.76062	53.07560	53.39329	53.72612	54.07267
2012	55.78461	56.12287	56.46370	56.80575	57.14746	57.48808	57.82779	58.16727

	Sep	Oct	Nov	Dec
2010	50.52822	50.71197	50.93843	51.25918
2011	54.42385	54.77132	55.11218	55.44861
2012	58.50705	58.84729	59.18781	59.52832

Table 4.1.5: Standard Errors for ARIMA (2, 1, 2)

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug
2010	1.960538	2.126989	2.164893	2.266678	2.565703	3.011069	3.423988	3.720609
2011	4.619818	4.811534	4.989860	5.152685	5.304847	5.452459	5.599154	5.745258
2012	6.424733	6.551205	6.675771	6.798333	6.918711	7.036853	7.152874	7.266979

	Sep	Oct	Nov	Dec
2010	**3.923630**	**4.085775**	**4.247421**	**4.426694**
2011	**5.889117**	**6.029054**	**6.164525**	**6.296090**
2012	**7.379357**	**7.490124**	**7.599330**	**7.706996**

## 4.2 Import Vat

4.2.1 Preliminary Analysis
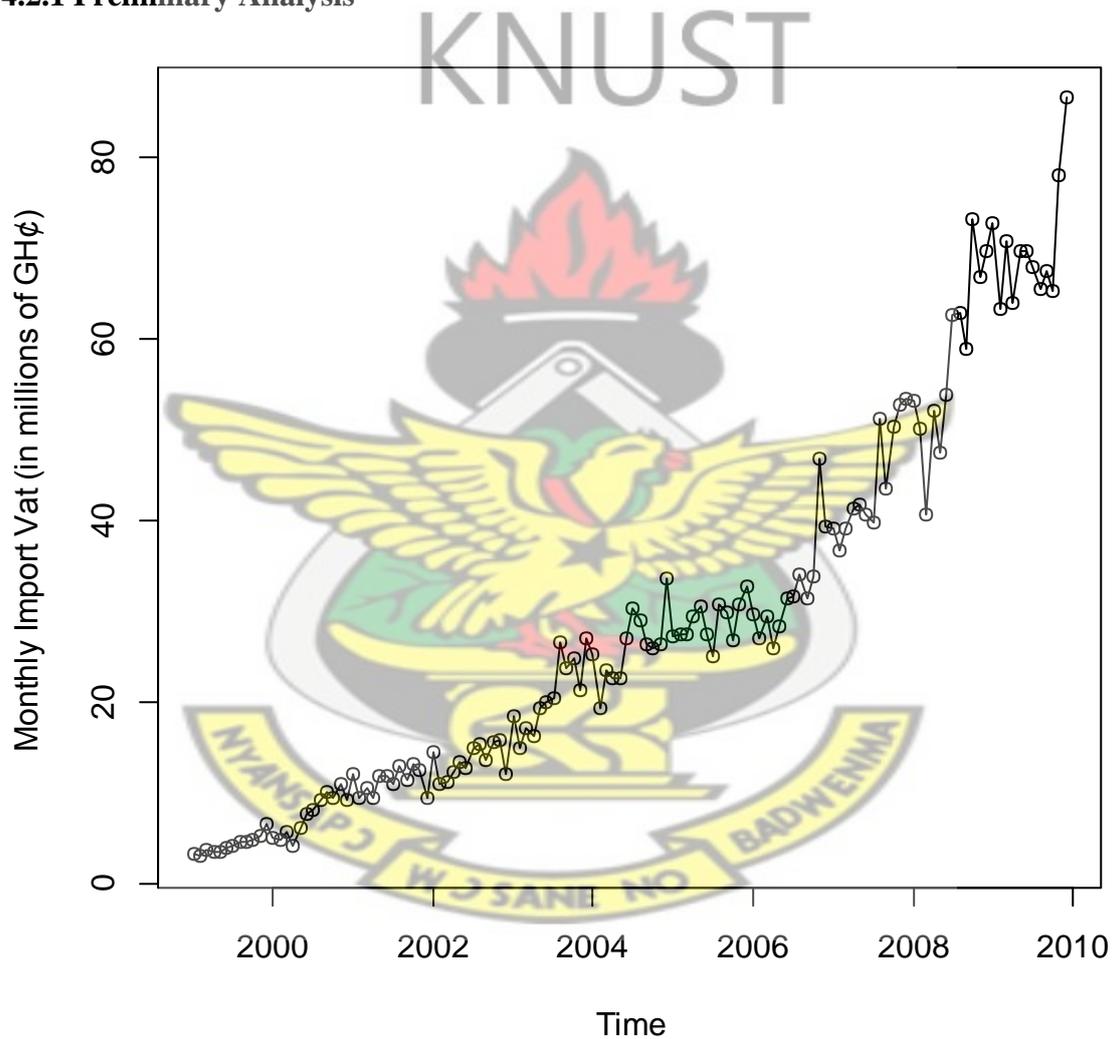
Figure 4.2.1 : Trend of Monthly Import VAT From JAN 1999 TO DEC 2009

Figure 4.2.1 above shows the pattern of monthly data of import vat (in millions of

GH¢) obtained from the CEPS service between January, 1999 and December, 2009.

We observe a gradually increasing underlying trend and a rather regular variation superimposed on the trend that seems to repeat over months, with maximum peak at 2009 (i.e. December), which recorded 86.58 million cedis in import vat. The minimum import vat amount occurred in February, 1999. The linear trend is made more evident after the decomposition of the observed data (in figure 4.2.1) as shown in figure 4.2.2 below.
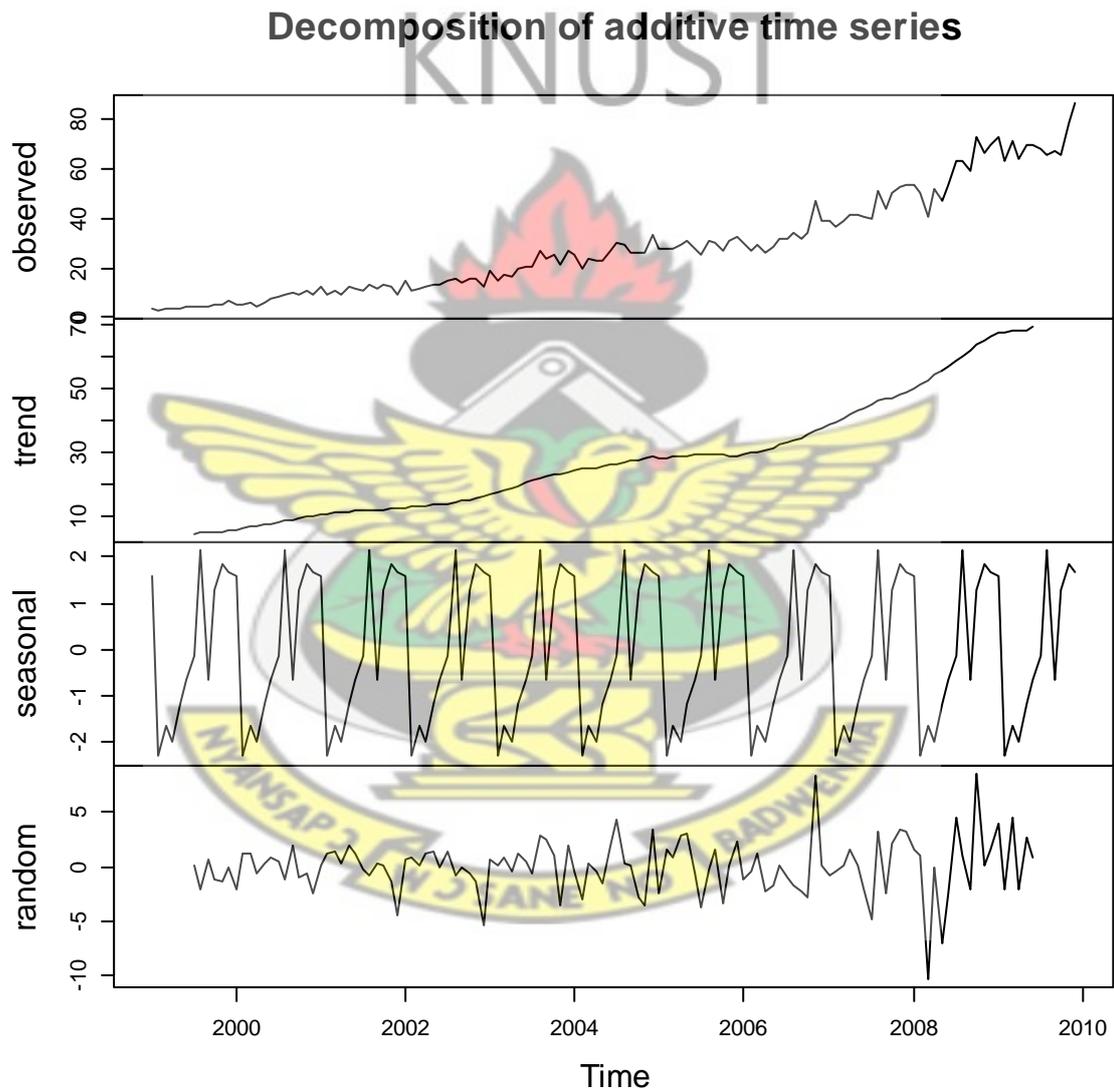

**Figure 4.2.2: Decomposition of the Import VAT series.**

After decomposition, it is observed clearly that the data exhibits a linear trend but the existence of seasonality is suggested. This is because the pattern displayed in figure 4.2.2 could be as a result of the irregular component in the data series.

Table 4.2.1: Summary statistics of import VAT data.

Minimum	1st Quartile	Median	Mean	3rd Quartile	Standard deviation	Maximum
3.02	11.97	26.21	28.82	39.96	20.70458	86.58

From Table 4.2.1 above, we observe that the minimum amount of import vat is 3.02 million cedis, which occurred in February, 1999. The maximum amount of import vat is 86.58 million cedis which also occurred in December, 2009.

The average import vat amount is a little higher than the median import vat amount. This may indicate a non-symmetric behaviour of the import vat distribution. Also, from the observed pattern, it is seen clearly that the import vat data is not stationary.
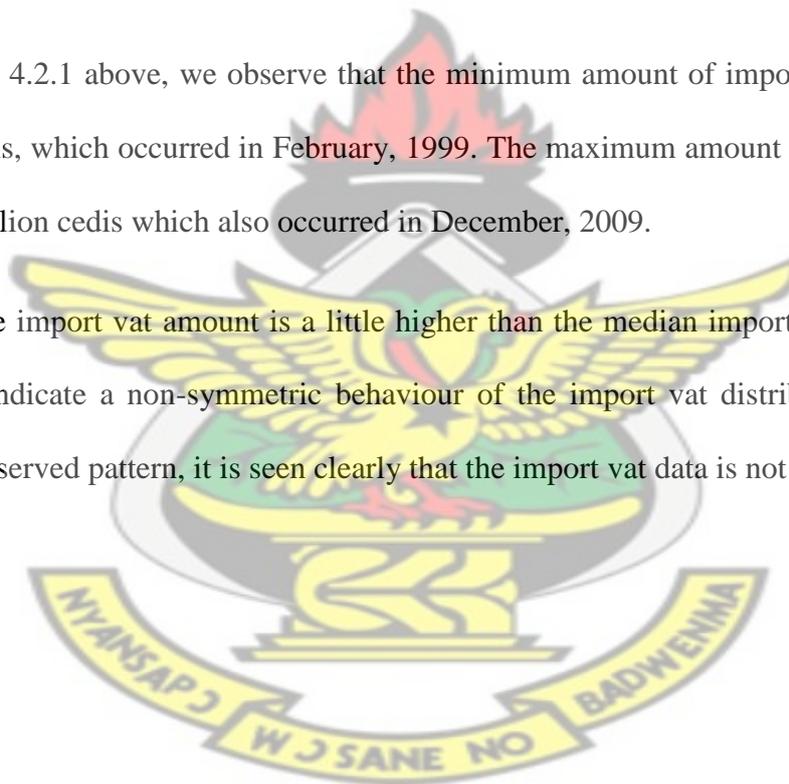
4.2.2 Achieving Stationarity:

The observed data is first differenced and tested for stationarity.
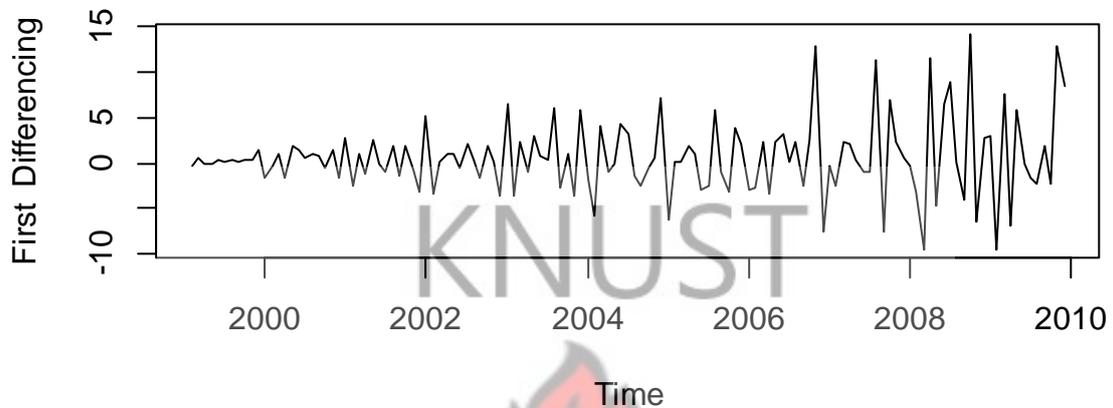
Figure 4.2.3(a) : Pattern of Import VAT Data After First Differencing
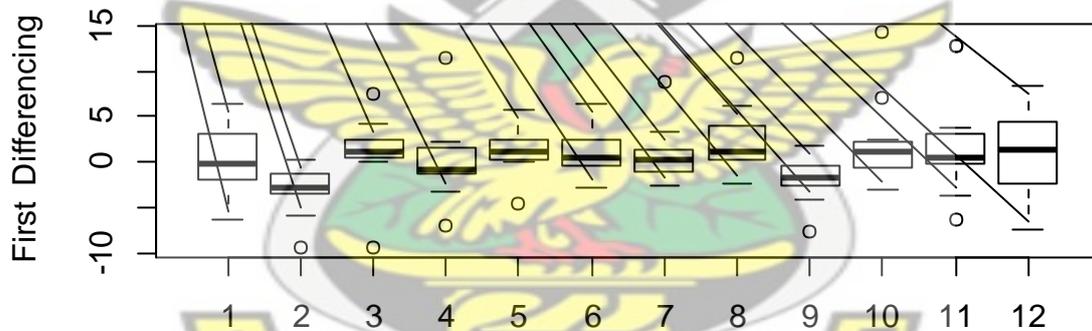
Figure 4.2.3(b) : Boxplot of Import VAT Data After First Differencing

From figure 4.2.3 (a) above, it can be seen that after the first differencing, the import VAT series becomes stationary. It can therefore be said that our data is non-seasonal (in reference to the methodology in chapter 3), since for non-seasonal data, first order differencing is usually sufficient to attain apparent stationarity.

Also from the boxplot in figure 4.2.3 (b) above, the means for each month are relatively close and show no obvious pattern of seasonality.

Furthermore, if there is significant seasonality, the autocorrelation plot should show significant spikes at lags equal to the period of the series. For example, for monthly data, if there is a seasonality effect, we would expect to see significant peaks at lag 12, 24, 36, and so on (although the intensity may decrease the further out we go).

From figure 4.2.4 (a) below, it can be seen from the sample ACF that lag 12 dies down completely, hence showing no significant peak.

Furthermore, the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test is computed to validate the null hypothesis that the differenced data is level or trend stationary.

4.2.3   KPSS Test for Level Stationarity

data: diff1

KPSS Level = 0.3031, Truncation lag parameter = 2, p-value = 0.1

Conclusion: At an α(alpha) 5% level of significance, we fail to reject the Null hypothesis that the import VAT series is trend or level stationary since the p-value (0.1) > 0.05, and hence conclude that the series is stationary.

4.2.4 Model Identification

Sample ACF for Differenced Import VAT Data   **Sample PACF for Differenced Import VAT Data**
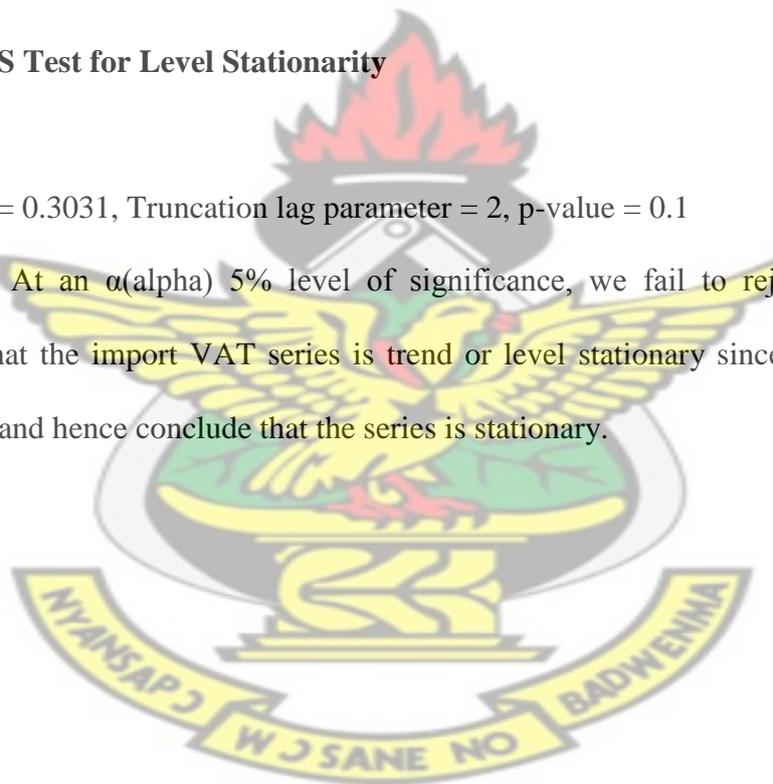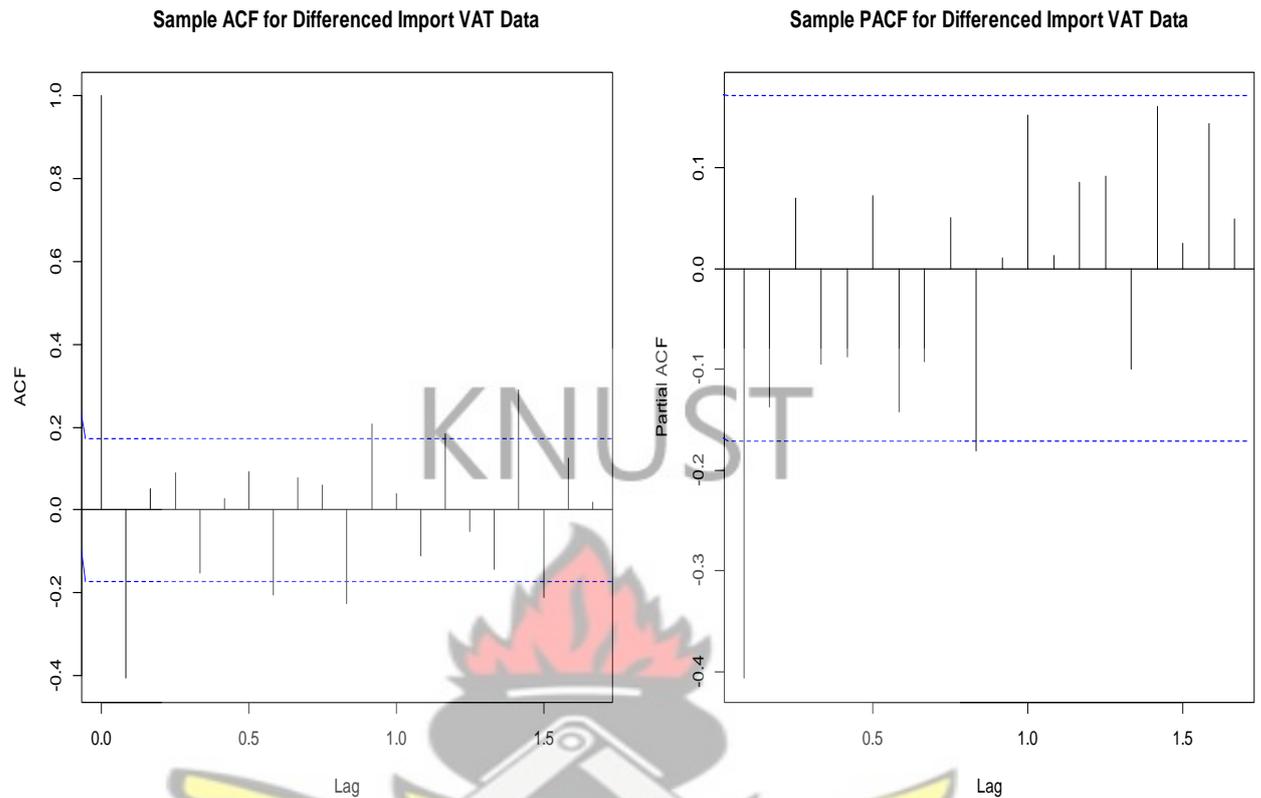
Figure 4.2.4(a) : Shows the Sample ACF for the diff data

Figure 4.2.4(b) : Shows the Sample PACF for the diff data

In order to select the appropriate model and also make more accurate forecasts, we fit several feasible ARIMA models in relation to the differenced data by making reference to its Sample ACF and Sample PACF (in Figure 4.2.4 above). Since the data is differenced once to attain stationarity (as shown by the KPSS Test), the fitted ARIMA models would be of order (p, d=1, q).

From the correlogram in figure 4.2.4(a), the sample ACF after the first significant cut at lag 1, tails off to zero and then cuts again at lags 7, 10, 11, 14, 17 and 18. But at lags 10, 11,14 and 17 respectively, the lags cut is consistent. The autocorrelations for these latter lags exceed the significance bounds too, but it may be likely that this is due to chance.

Also the partial correlogram (b) shows that the partial autocorrelations at lag 1 exceed the significance bounds, and is negative as well. The partial autocorrelations tails off to zero afterwards and cut again at lag 10. However, this lag that exceeds the bounds is not significant since it may be due to chance.

From the foregoing analysis, the following ARIMA (Autoregressive integrated moving average) models are therefore possible for the import data series:

- ARIMA(1,1,1)
- ARIMA(1,1,0)
- ARIMA(0,1,1)
- ARIMA(1,1,2)
- ARIMA(2,1,1)

4.2.5 ESTIMATION OF PARAMETERS AND DIAGNOSTIC CHECKING

At this point we proceed to estimate the parameters and investigate whether the residuals of the selected ARIMA models are normally distributed with mean zero and constant variance, and also whether there are no correlations between successive residuals (i.e. randomness of residuals).

To check for correlations between successive residuals, we make use of a correlogram and also the Ljung-Box test to further ascertain the adequacy (randomness) of the chosen model.

Also to check whether the residuals are normally distributed with mean zero and constant variance, we make use of a normality quantile-quantile plot (q-q plot) and a histogram.

If the residuals are normally distributed, the points on the normal quantile-quantile plot should approximately be linear, with residual mean as the intercept and residual standard deviation as the slope whilst the shape of the histogram shows "a bell-like" shape.

- **ARIMA(1,1,1)**

Coefficients:

	ar1	ma1	drift
	-0.0634	-0.4478	0.5942
s.e.	0.2446	0.2444	0.1692

sigma^2 estimated as 13.54: log likelihood=-356.71

AIC=721.43   AICc=721.75   BIC=732.93

ME	RMSE	MAE	MPE	MAPE	MASE
-0.008024	3.6663	2.46274	-3.97206	10.56486	0.852971
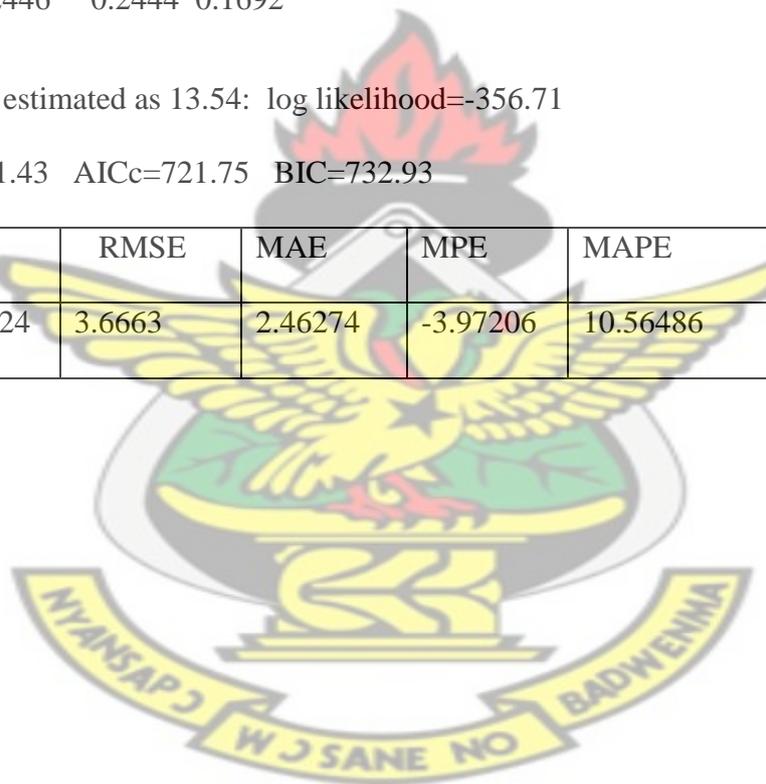
ACF of ARIMA(1,1,1) Residuals
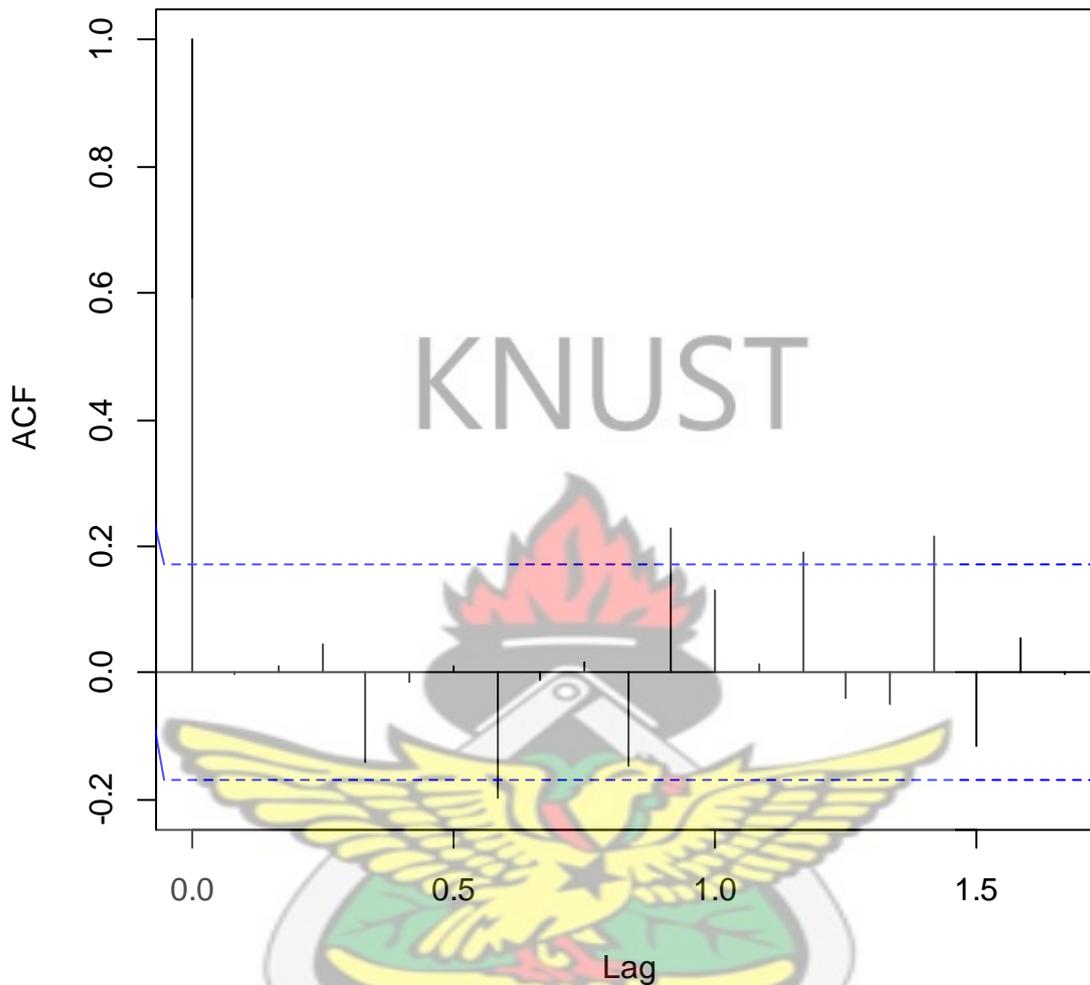

Figure 4.2.5: ACF of ARIMA(1,1,1) Residuals

Box-Ljung test:

data: model1$residuals

X-squared = 37.9014, df = 20, p-value = 0.009105

From figure 4.2.5 above, the ACF of residuals shows that the sample autocorrelations at lags 7, 11, 14 and 17 exceed the significance bounds, with four (4) other lags getting close to the bounds. This might probably have some level of significance,

since we would expect at least four (4) out of 20 sample autocorrelations to exceed the 95% significance bounds.

Also, from the Ljung-box test above, the computed p-value (i.e. 0.009105) is also far lower than α (alpha) 5% level of significance.

Hence from these deductions, we reject the null hypothesis that the series of residuals exhibits no autocorrelation and conclude that there is some evidence of autocorrelations in the residuals at certain lags (i.e. the residuals are dependently distributed).
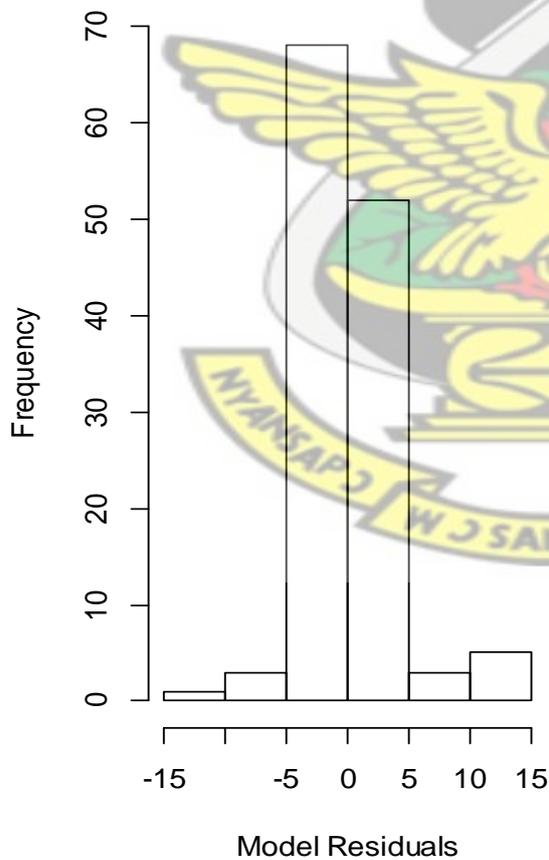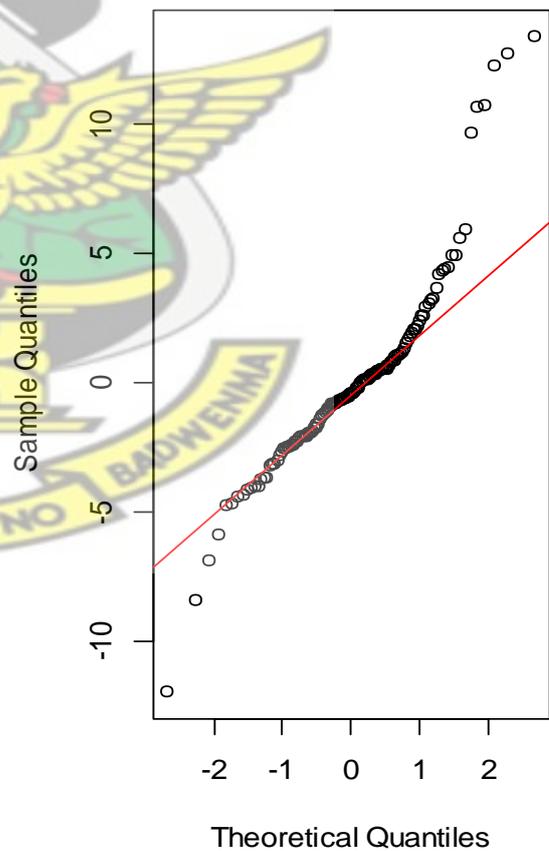
Figure 4.2.6: Shows the Histogram (left) and Normality plot (right) for the residuals of ARIMA (1, 1, 1).

From the plot in figure 4.2.6, the histogram of the residuals displayed above gives an indication of a plausible symmetric distribution, thus it shape looks "bell-like" and certainly better for the fitted model. The QQ-normal plot for the residuals also throws more light on this since most of its residuals do not deviate that much from the line of best fit and it distribution looks approximately linear.

Since the residuals of this fitted model fails to satisfy the assumption of independence (i.e. zero autocorrelation), it is therefore considered as an inadequate model.

- **ARIMA(1,1,0)**

Coefficients:

ar1   drift

  -0.4128 0.5499

s.e.0.0803 0.2319

sigma^2 estimated as 13.97: log likelihood=-358.69

AIC=723.38   AICc=723.57   BIC=732.01

ME	RMSE	MAE	MPE	MAPE	MASE
0.096	3.7234	2.5007	-2.7675	10.2574	0.86612

ACF of ARIMA(1,1,0) Residuals
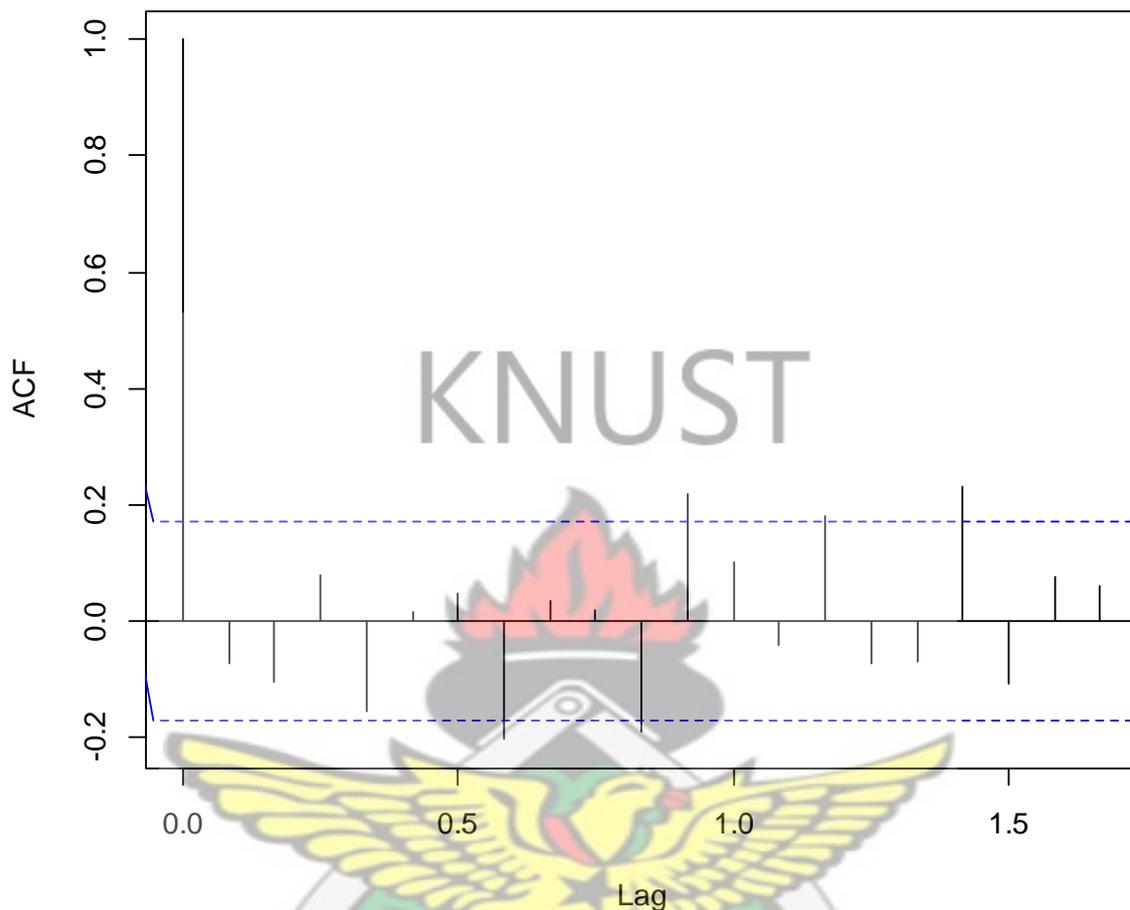
Figure 4.2.7: ACF of ARIMA(1,1,0) Residuals

Box-Ljung test:

data: model2$residuals

X-squared = 44.8301, df = 20, p-value = 0.001164

From figure 4.2.7 above, the ACF of residuals shows that the sample autocorrelations at lags 7, 10, 11, 14 and 17 exceeds the significance bounds and this might probably have some level of significance, since we would five out of 20 sample autocorrelations to exceed the 95% significance bounds.

Also, from the Ljung-box test above, the computed p-value (i.e. 0.001164) is also less than α (alpha) 5% level of significance.

74

Hence from these deductions, we reject the null hypothesis that the series of residuals exhibits no autocorrelations and conclude that there is some evidence of autocorrelations in the residuals at certain lags (i.e. the residuals are not independently distributed).
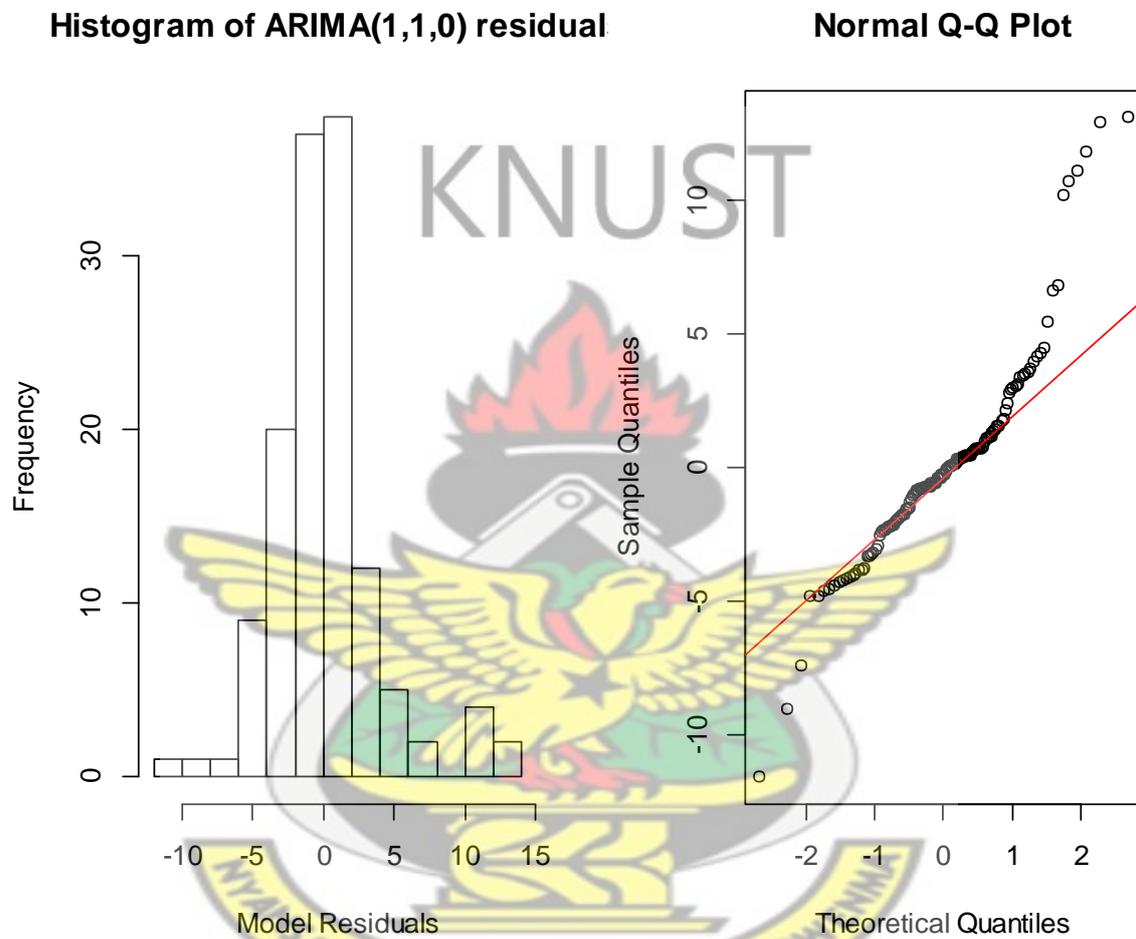

Figure 4.2.8: Shows the Histogram (left) and Normality plot (right) for the residuals of ARIMA (1, 1, 0).

From the plot in figure 4.2.8, the histogram of the residuals displayed above gives an indication of a plausible symmetric distribution, thus it shape looks "bell-like" and certainly better for the fitted model. The QQ-normal plot for the residuals also throws

more light on this since most of its residuals do not deviate that much from the line of best fit and it distribution looks approximately linear.

Since the residuals of this fitted model fails to satisfy the assumption of independence (i.e. zero autocorrelation), it is therefore considered as an inadequate model.

- **ARIMA(0,1,1)**

Coefficients:

ma1   drift

   -0.5060 0.5841

s.e.0.0882 0.1604

sigma^2 estimated as 13.55: log likelihood=-356.75

AIC=719.49   AICc=719.68   BIC=728.12

ME	RMSE	MAE	MPE	MAPE	MASE
0.00313	3.66702	2.47054	-3.9850	10.6050	0.8557

ACF of ARIMA(0,1,1) Residuals
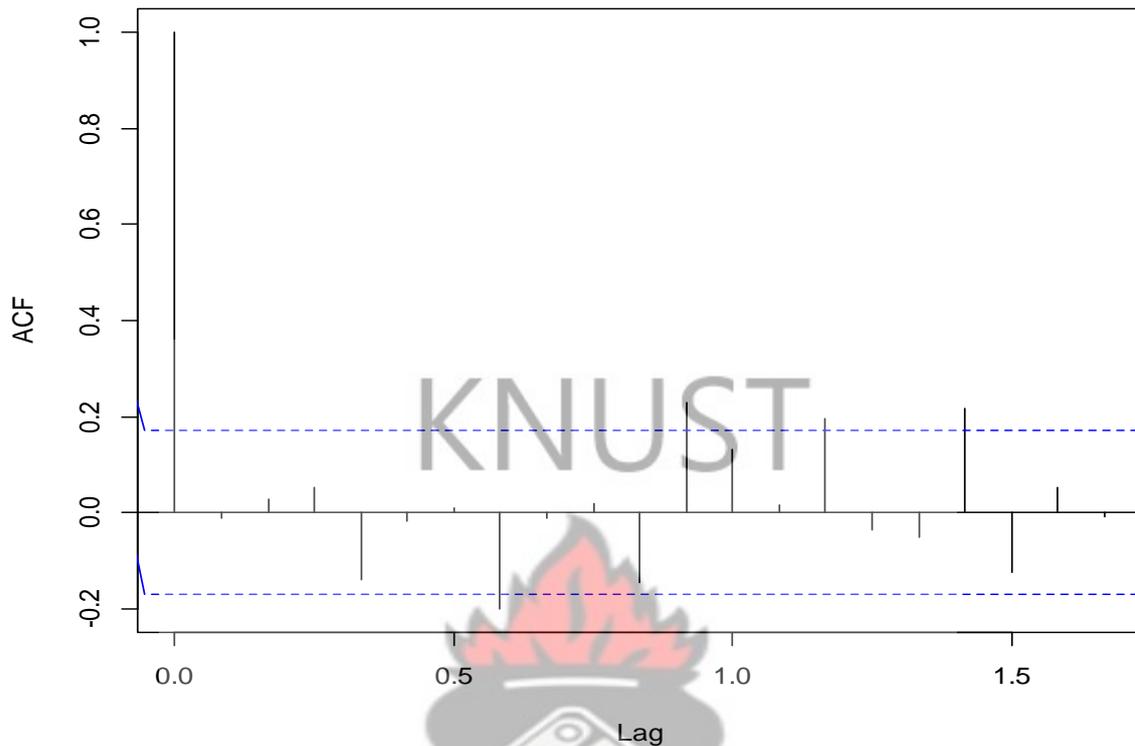
Figure 4.2.9: ACF of ARIMA(0,1,1) Residuals

Box-Ljung test:

data: model3$residuals

X-squared = 38.8105, df = 20, p-value = 0.007038

From figure 4.2.9 above, the ACF of residuals shows that the sample autocorrelations at lags 7, 11, 14 and 17 exceeds the significance bounds, with four other lags getting close enough to the bounds. This might probably have some level of significance, since we would expect at least four out of 20 sample autocorrelations to exceed the 95% significance bounds.

Also, from the Ljung-box test above, the computed p-value (i.e. 0.007038) is also less than α (alpha) 5% level of significance.

Hence from these deductions, we reject the null hypothesis that the series of residuals exhibits no autocorrelations and conclude that there is some evidence of autocorrelations in the residuals at certain lags (i.e. the residuals are not independently distributed).
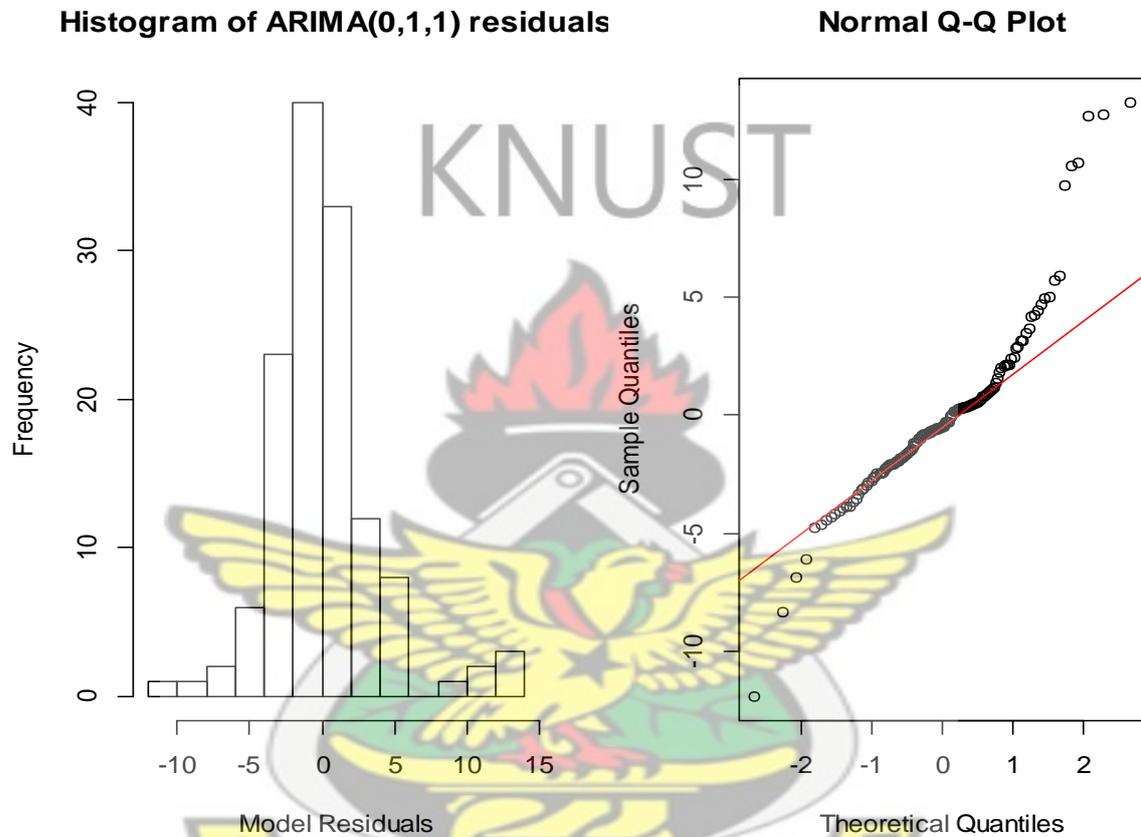
Figure 4.2.10: Shows the Histogram (left) and Normality plot (right) for the residuals of ARIMA (0, 1, 1).

From the plot in figure 4.2.10, the histogram of the residuals displayed above gives an indication of a plausible symmetric distribution, thus it shape looks "bell-like" and certainly better for the fitted model. The QQ-normal plot for the residuals also throws more light on this since most of its residuals do not deviate that much from the line of best fit and it distribution looks approximately linear.

Since the residuals of this fitted model fails to satisfy the assumption of independence (i.e. zero autocorrelation), it is therefore considered as an inadequate model.

- **ARIMA(2,1,1)**

  Coefficients:

	ar1	ar2	ma1	drift
	-0.8118	-0.3201	0.3196	0.6093
s.e.	0.3162	0.1390	0.3279	0.1991

  sigma^2 estimated as 13.45: log likelihood=-356.24

  AIC=722.49 AICc=722.97 BIC=736.86

ME	RMSE	MAE	MPE	MAPE	MASE
-0.0056	3.65316	2.48659	-3.6412	10.5277	0.8612
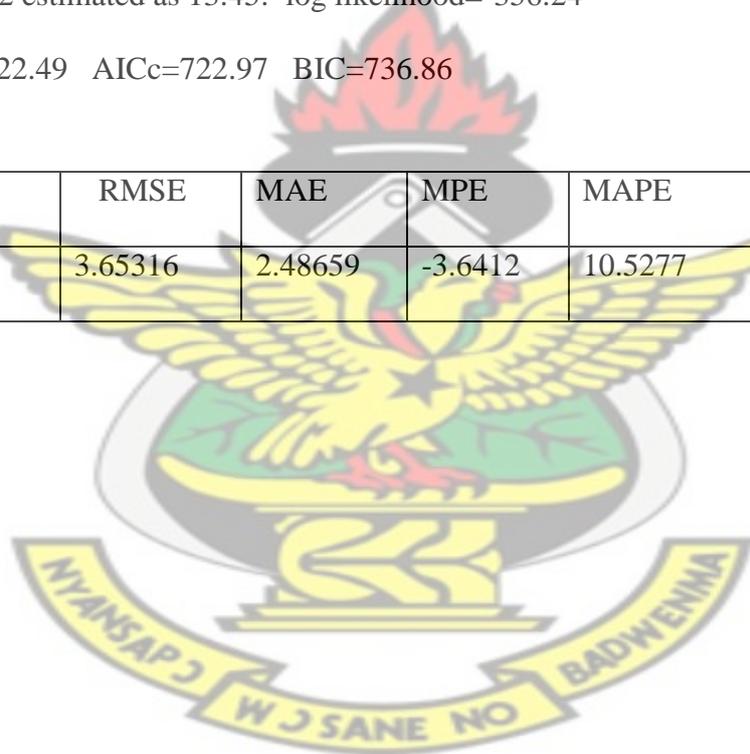
ACF of ARIMA(2,1,1) Residuals
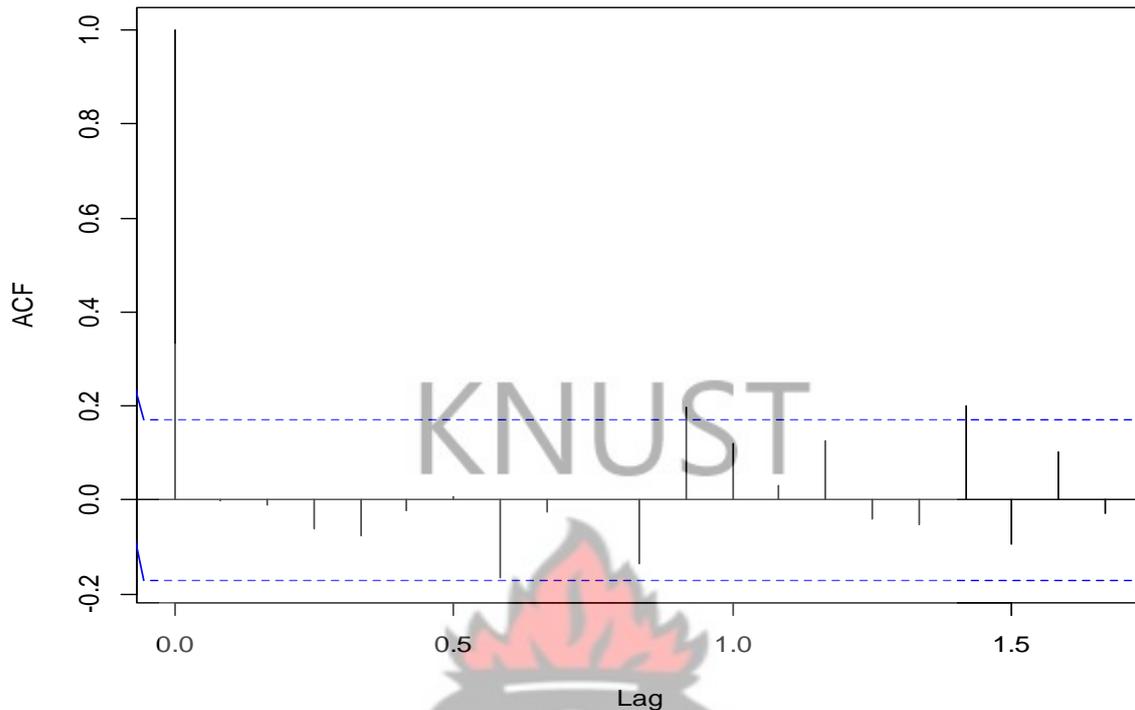
Figure 4.2.11: ACF of ARIMA(2,1,1) Residuals

Box-Ljung test:

data: model5$residuals

X-squared = 28.4808, df = 20, p-value = 0.0985

From figure 4.2.11 above, the ACF of residuals shows that the sample autocorrelations at lags 11 and 17 only exceed the significance bounds and this might probably be due to chance, since we would expect at most two out of 20 sample autocorrelations to exceed the 95% significance bounds. Also, most of the lags die down.

Also, from the Ljung-box test above, the computed p-value (i.e. 0.0985) is greater than α (alpha) 5% level of significance.

Hence from these deductions, we fail to reject the null hypothesis that the series of residuals exhibits no autocorrelations and conclude that there is little evidence of autocorrelations in the residuals for all lags (i.e. the residuals are independently distributed).
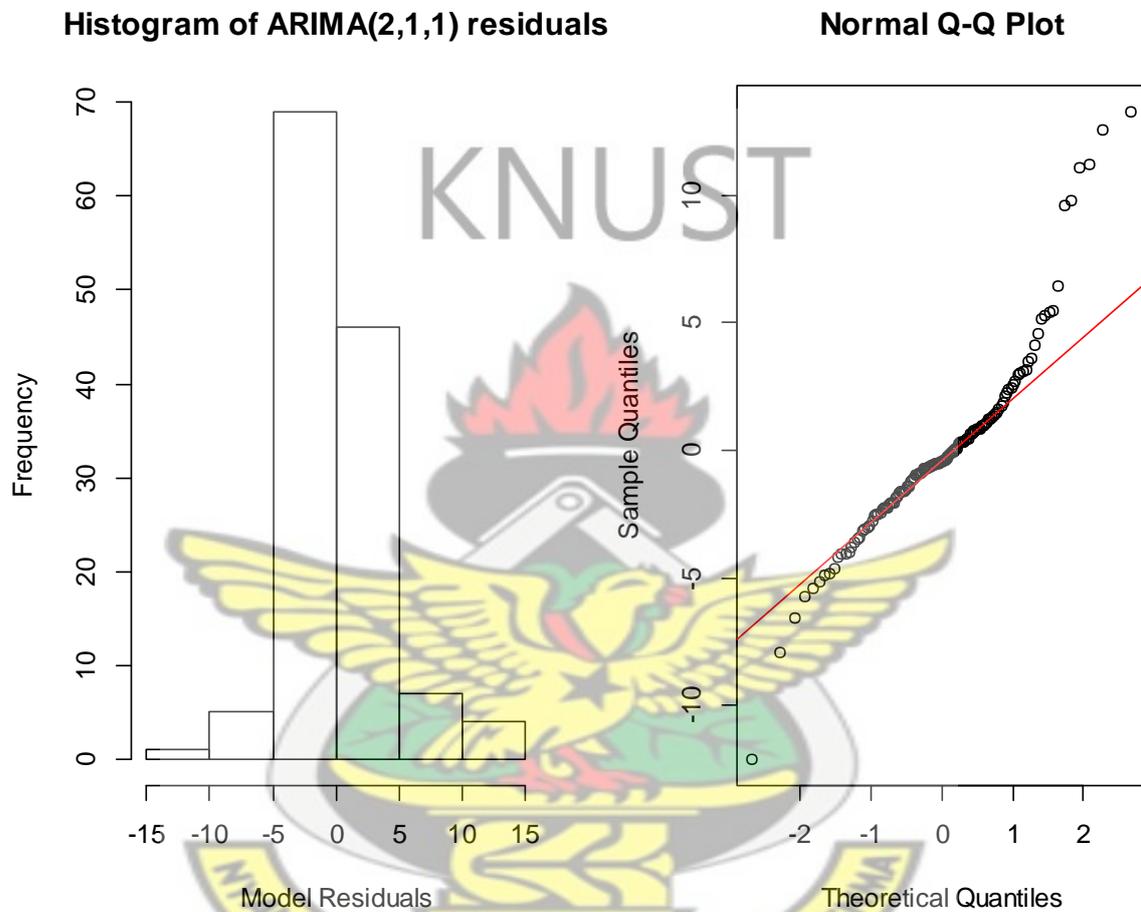
Figure 4.2.12: Shows the Histogram (left) and Normality plot (right) for the residuals of ARIMA (2, 1, 1).

From the plot in figure 4.2.12, the histogram of the residuals displayed above gives an indication of a plausible symmetric distribution, thus it shape looks "bell-like" and certainly better for the fitted model. The QQ-normal plot for the residuals also throws

more light on this since most of its residuals do not deviate that much from the line of best fit and it distribution looks approximately linear.

- **ARIMA(1,1,2)**

  Coefficients:

        ar1   ma1   ma2   drift

     -0.1145   -0.3971   -0.0268   0.5943

  s.e.   0.8580   0.8548   0.4444   0.1684

  sigma^2 estimated as 13.54: log likelihood=-356.71

  AIC=723.42   AICc=723.9   BIC=737.8

ME	RMSE	MAE	MPE	MAPE	MASE
-0.008813	3.66622	2.46248	-3.98513	10.5679	0.85288

ACF of ARIMA(1,1,2) Residuals
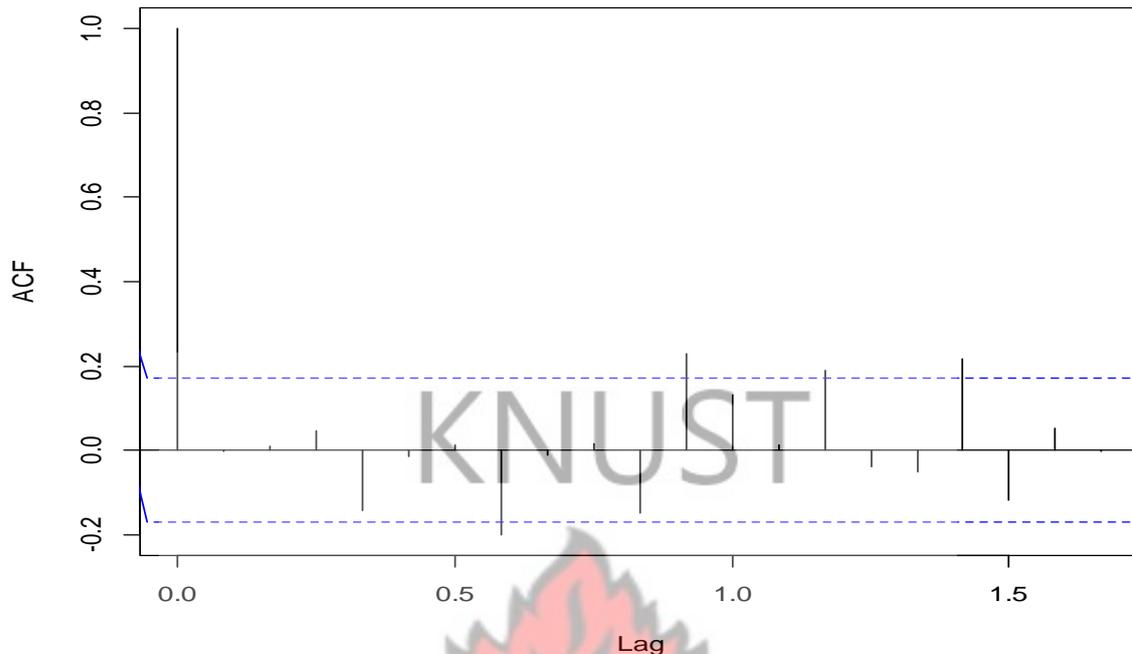
Figure 4.2.13: ACF of ARIMA(1,1,2) Residuals

Box-Ljung test:

data: model4$residuals

X-squared = 37.998, df = 20, p-value = 0.008861

From figure 4.2.13 above, the ACF of residuals shows that the sample autocorrelations at lags 7, 11, 14 and 17 exceeds the significance bounds and this might probably have some level of significance, since we would expect at least four (4) out of 20 sample autocorrelations to exceed the 95% significance bounds.

Also, from the Ljung-box test above, the computed p-value (i.e. 0.008861) is also less than α (alpha) 5% level of significance.

Hence from these deductions, we reject the null hypothesis that the series of residuals exhibits no autocorrelations and conclude that there is some evidence of autocorrelations in the residuals at certain lags (i.e. the residuals are not independently distributed).
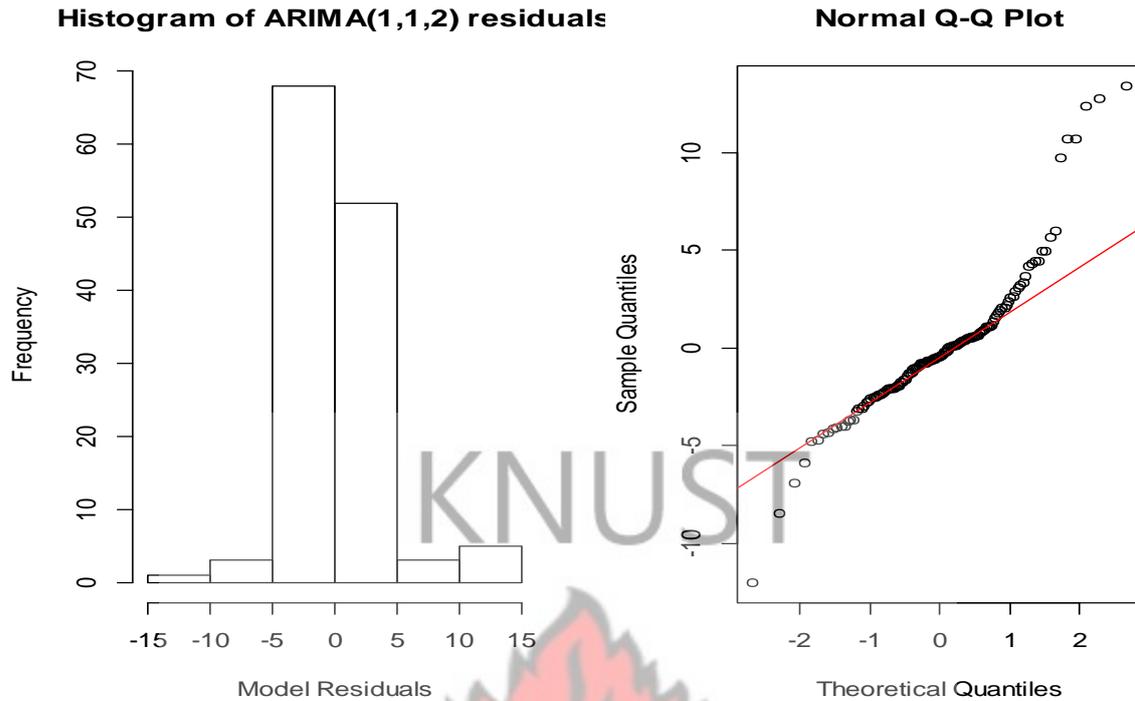
**Figure 4.2.14: Shows the Histogram (left) and Normality plot (right) for the residuals of ARIMA (1, 1, 2).**

From the plot in figure 4.2.14, the histogram of the residuals displayed above gives an indication of a plausible symmetric distribution, thus it shape looks "bell-like" and certainly better for the fitted model. The QQ-normal plot for the residuals also throws more light on this since most of its residuals do not deviate that much from the line of best fit and it distribution looks approximately linear.

Since the residuals of this fitted model fails to satisfy the assumption of independence (i.e. zero autocorrelation), it is therefore considered as an inadequate model.

4.2.6   Model Selection

In order to select the most appropriate model for our data, we compare all competing models and select the one with the minimum AIC (Akaike Information Criterion value), Schwartz Bayesian Information Criterion (BIC) and Residual Variance. Other

statistical tests like Root Mean Squared Error (RMSE), Mean Abs. Percent Error (MAPE), Bias Proportion or Mean Forecast Error (MFE) and Mean Absolute Scaled Error are also used in testing the forecast accuracy of the fitted models.

It should be noted that a model which fits the data well does not necessarily forecast well. The best model will be the one that achieves a compromise between the various information criterion values.

Since four(4) of the models, namely ARIMA(1,1,1), ARIMA(1,1,0), ARIMA(0,1,1) and ARIMA(1,1,2) failed the diagnostic test, we fail to accept them as possible competing models. Therefore, the only adequate model left is ARIMA (2, 1, 1).

Table 4.2.2: Summary of AIC, BIC, Residual Variance and MASE for the possible Model

Model	Akaike Information Criterion (AIC)	Residual Variance	BIC	MASE
ARIMA(2,1,1)	722.49	13.45	736.86	0.86123042

Therefore, the chosen model for the import VAT series is of the form;

$$Y_t - Y_{t-1} = \alpha_1(Y_{t-1} - Y_{t-2}) + \alpha_2(Y_{t-2} - Y_{t-3}) - \theta_1 e_{t-1} + e_t + \mu$$

$$Y_t - Y_{t-1} = -0.8118\ (Y_{t-1} - Y_{t-2}) - 0.3201(Y_{t-2} - Y_{t-3}) - 0.3196e_{t-1} + e_t + 0.6093$$

OR

$$Y_t = (1 + \alpha_1)Y_{t-1} + (\alpha_2 - \alpha_1)Y_{t-2} - \alpha_2 Y_{t-3} - \theta_1 e_{t-1} + e_t + \mu$$

$$Y_t = 0.1882Y_{t-1} + 0.4917\ Y_{t-2} + 0.3201Y_{t-3} - 0.3196e_{t-1} + e_t + 0.6093 \quad (4.2)$$

This indicates that the fitted model is a linear combination of previous import VAT values, a previous forecast error and a constant.

4.3 Forecasting

We also make forecast using the best fitted model for the next three years. Below is the graph of the forecasts.
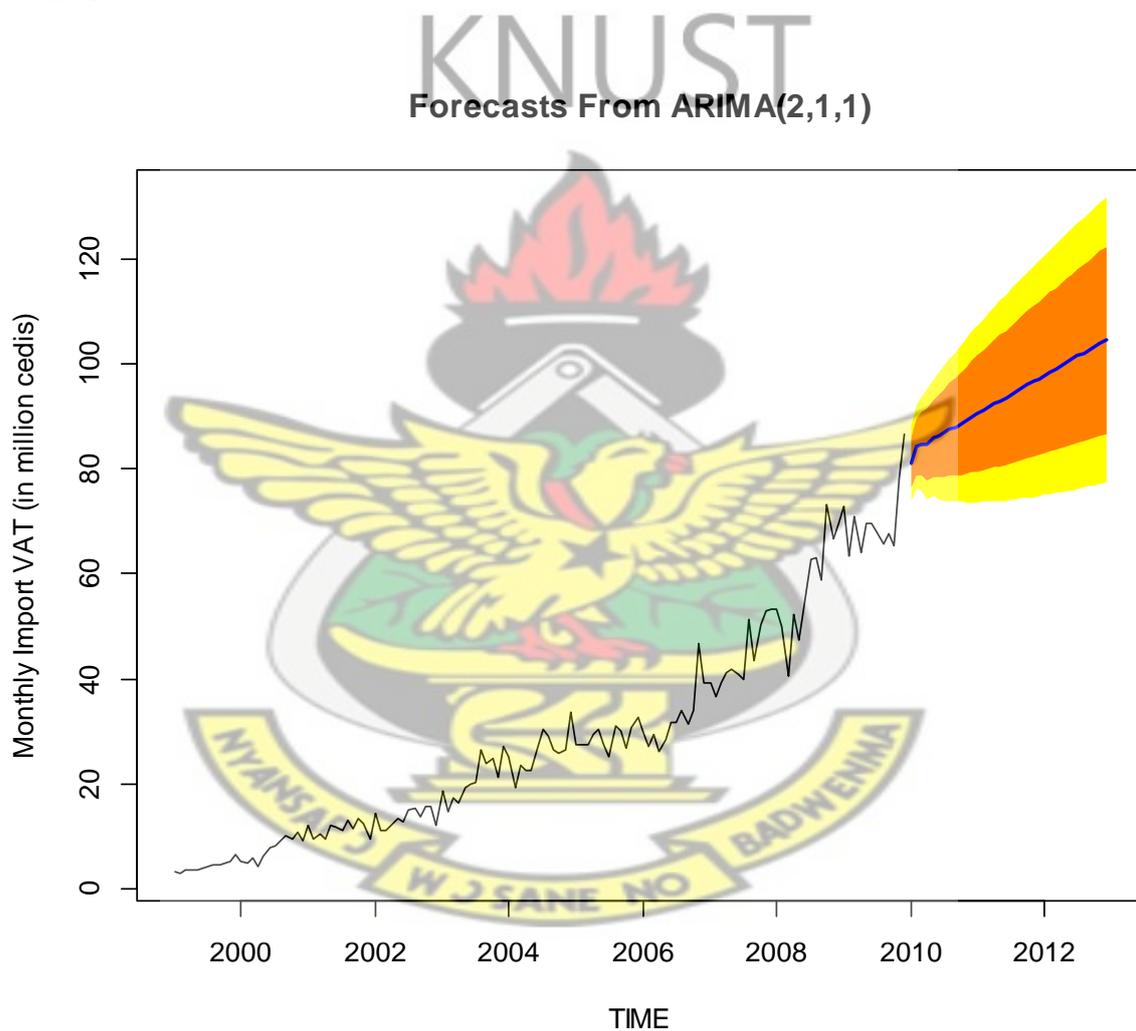

Figure 4.2.15: The forecasted import VAT (in millions) is shown by the blue line, whilst the orange and yellow shaded areas show 80% and 95% prediction intervals respectively.

The forecasted values and standard errors are given in table 4.2.3 and 4.2.4 below respectively:

Table 4.2.3: Forecasted Values for ARIMA (2, 1, 1)

	Jan	Feb	Mar	Apr	May	Jun	Jul
2010	81.1797	84.1704	84.7702	84.6248	85.8498	86.2008	86.8227
2011	90.5128	91.1191	91.7288	92.3387	92.9473	93.5569	94.1661
2012	97.8218	98.4311	99.0403	99.6496	100.2589	100.8682	101.4775

	Aug	Sep	Oct	Nov	Dec
2010	87.5044	88.05084	88.68795	89.29476	89.89713
2011	94.7754	95.38469	95.99394	96.60322	97.21251
2012	102.0867	102.69601	103.30528	103.91456	104.52384

Table 4.2.4: Standard Errors for ARIMA (2, 1, 1)

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug
2010	3.6671	4.1127	4.6424	5.2702	5.6822	6.1360	6.54551	6.9206
2011	8.5848	8.8799	9.1654	9.4422	9.71120	9.9729	10.2279	10.477
2012	11.6413	11.861	12.076	12.2872	12.4951	12.6995	12.9008	13.0989

	Sep	Oct	Nov	Dec
2010	7.28803	7.6318	7.9619	8.2797
2011	10.7198	10.9574	11.1900	11.4179
2012	13.2941	13.4865	13.6761	13.8632

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1.0   Conclusions

The general objective of this study was to use Time Series to analyse the Value Added Tax revenue collections in Ghana over the period under review. The specific objectives were;

1. To determine the trend pattern of VAT revenue collection for the period.

2. To fit a suitable ARIMA model to the monthly indirect tax revenue data.

3. To forecast the future revenue expected for the next 36 months ahead.

Using the Box-Jenkins methodology and having satisfied all the model assumptions, the results revealed that ARIMA (2, 1, 2) was the best model and fits the observed VAT revenue pattern well in the domestic front whilst ARIMA (2, 1, 1) was also selected as the best fitted model for import VAT values.

The third objective of the research work was to use the two (2) models to forecast the values for both Domestic and Import Vats, for the next three years. The forecast values for ARIMA (2, 1, 2) model which was fitted to the domestic vat data is shown in table 4.1.4 whilst that of ARIMA (2, 1, 1) which was fitted to the import vat data shown in table 4.2 was seen to be increased. Therefore using the models suggested in forecasting for the period January, 2010 to December, 2012 was appreciable.

5.2.0   Recommendations

The ARIMA (2, 1, 2) and ARIMA (2, 1, 1) models are therefore recommended for forecasting the Domestic and Import vat respectively in Ghana. Since the ARIMA

models chosen showed up an increasing revenue pattern, it is recommended for Ghana Revenue Authority to adopt it in future predictions.

The following precautionary measures should be taken into consideration in order to prevent any decline of the revenue collections as forecasted by the models:

- The Ghana Revenue Authority should not set target depending on the government expenditure.

- Economic conditions prevailing in the country should not force GRA to set unrealistic target to meet the demands.

- Unnecessary tax exemptions, reliefs, remittances evasions must be avoided.
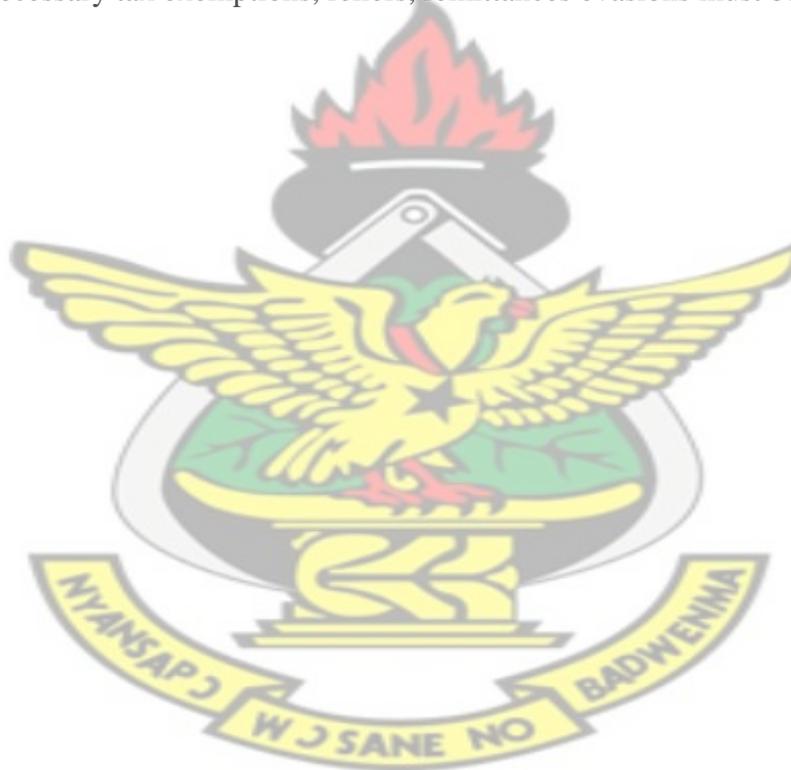
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