KWAME NKRUMAH UNIVERSITY

OF SCIENCE AND TECHNOLOGY KUMASI

MODELLING INVENTORY, DISTIBUTION ROUTE PROBLEM

A CASE STUDY OF LATEX FOAM PRODUCTS

BY

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DECLARATION

I SARBENG JOSEPH declare that except for reference to other people's work, which have dully been cited. This submission is my own work towards the Master of Science Degree and that it contains no material neither previously published by another person non presented elsewhere.

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ABSTRACT

Manufacturing firms generate and update production, inventory and distribution, which are plans that state when certain controllable activities for example processing of jobs by resources should take place. As production systems expand, there is the tendency for its inventory and distribution to become complex. The organizational perspective, which is the most complete, views production and distribution as a system of decision-making that transforms information about the manufacturing system into a plan. Distributors must perform a variety of tasks and use both formal and informal information to make scheduling decisions. Previous production distribution involves complicated iterative procedures. A new approach brings out the basic principle that leads to a simple solution. The objective of this study is to establish an efficient optimum distribution that will minimize the total production and inventory cost whilst satisfying customer demands for Latex Foam Ghana Limited. The production problem was modeled as a transportation problem. Optimality conditions were satisfied and solution was found using Excel Solver. Results from the analysis indicated that customer demands could be satisfied when overtime production is increased and that an efficient production distribution could ensure cost reduction, optimum utilization of capacity, proper inventory control and management, quality and ultimate optimality.

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DEDICATION

To my dear wife, Dora Asante and children, Helen Sarbeng, Irene Sarbeng, Bernard Sarbeng and Emmanuella Sarbeng.

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CHAPTER ONE

1.0 Introduction

The problem of distributing goods from depots to final consumers plays an important role in the management of many distribution systems, and its adequate programming may produce significant savings. In a typical distribution system, vehicles provide delivery, pick-up or repair and maintenance services to customers that are geographically dispersed in a given area. In its numerous applications, the common objective of distribution is to find a set of routes for the vehicles to satisfy a variety of constraints so as to minimize the total fleet operation cost.

Most of the manufacturing companies in Ghana utilize vehicles (trucks) to transport their products to the customers. The general problem in such a situation is how to take inventory of the products produced so as to minimize lost of goods at its destinations.

1.1 Background of the study

Latex foam rubber production limited is a manufacturer and exporter of polyurethane flexible foam, spring mattresses and pillows.

The company was incorporated on 8th March, 1969 to produce quality foam products for the bedding and furniture industry. The company entered the market using the Dunlop Technology under license from the Dunlop Company for production of its products. In 1972, three years after its inception, Latex foam started the production of Spring Interior Mattresses. Today the company has stood the test of time and is the oldest in the industry in Ghana. It is worth mentioning that both the interior production and assembly of the unit springs for the mattresses were done at the factory premises. It is also the leading manufacturer of quality foam products such as

(i) Foam Mattresses (e.g. Ultraflex, Ultrafirm and High Density Honeymoon mattresses)

(ii) Pillows (e.g. Orthopedic pillows, Dona pillows and Venus pillows),

(iii) Mattress Accessories (Divan Bed, Comforter and Protection Pad),

(iv) Sofa beds, Students mattresses, Upholstery and

(v) Therapeutic products, such as Reader's Pillow and Back Care Cushion, in Ghana and West Africa.

In 2007, because of the high quality of its products, the company was chosen to provide the mattresses for the houses that hosted the visiting Heads of States for the Ghana@50 celebrations. That year, the company became the first to produce high resilient foam for the Ghanaian market when they introduced the Ultra flexes Mattress which provides excellent relaxation and body support.

The company has branches in Ghana, Niger and Burkina Faso. Their products are also shipped to Togo, Mali, Benin Republic, Ivory Coast and Angola. Latex foam products are sought by furniture upholstery manufactures, assembly plants, department stores, hospitals, government institutions and the general public.

1.1.2 ***Production and Distribution***

The revolutions in transportation and communications technologies have increased the extent of the U.S. domestic markets over the last two centuries. Moreover, the expansion of markets is associated with major changes in the course of American economic history. The introduction of canals in the late eighteenth and the early nineteenth centuries is credited with increasing the levels of inventive activity and triggering industrialization (Sokoloff, 1988). Households became less self-sufficient and became specialized consumer-labourers; firm that specialized in the production of various goods emerged in great numbers. The division of labour within firms led to a re-organization of production and increased levels of productivity (Sokoloff, 1984a, 1984b). In the late 18th and he early 19th centuries, the expansion of the U.S domestic markets and industrialization caused a rapid decline in household production and a proliferation of specialized manufacturing firms in the American economy (Kim, 2000). In this period, the industrial structure was composed of single-unit firms who specialized in the production of manufacturing goods and wholesale merchants and retail store owners who distributed these goods. Since the manufacturing firms typically specialized in a narrow line of products. It was simply too costly for them to market their products directly to consumers. In this setting, the wholesale merchants, who bought and sold sufficient lines of products, were able to lower the costs of transactions more efficiently. The wholesale merchants were not only able to collect information on various manufactures by locating in major cities but were also able to collect information on rural consumer demand through the use of sales agents who traveled to rural country stores. In this period, most consumers were able to judge the quality of most products upon visual inspection. However, according to Kim, for some goods, they relied on the local producers' and retail merchants' reputation for honesty.

In the late nineteenth century, with advances in science and technology, it became increasingly difficult for consumers to discern the quality of products which they consumed. As incomes rose, consumers purchased a growing number of products for which they lacked basic knowledge to discern quality. Moreover, Kim indicated that, even the manufacturing processes of the most basic of products such as food became so sophisticated that consumers no longer had enough knowledge to discern whether a product was healthy or poisonous.

Finally, as regional domestic markets became increasingly integrated between the late 19th and the early 20th centuries, geographic specialization in economic activities increased (Kim, 1995).

1.1.3 Channels of Distribution in Ghana

Distribution could be broadly classified into Direct and Indirect distributions. There is direct distribution if the producer supplies the product directly to the consumer without the use of an intermediary or middle man. Indirect distribution involves the use of intermediaries or middlemen and retailers to make the product available to the consumer.

According to Jim (2012) there are three main channels of distribution of goods in Ghana. These are from the

(i) Producer to Consumer, where the producer sells directly to the consumer,

(ii) Producer to the Retailer and from the Retailer to the Consumer, where the wholesaler is by passed and the producer deals directly with the retailer, and

(iii) producer to the Wholesaler, from the Wholesaler to the Retailer and from the Retailer to the consumer.

1.1.4 The production planning and distribution interface

Production planning is the function of establishing an overall level of output, called the production plan. The process also includes any other activities needed to satisfy current planed levels of sales, while meeting the firm's general objectives regarding profit, productivity, lead times and customer satisfaction, as expressed in the overall business plan. A primary purpose of the production plan is to establish production rate that will achieve management objective of satisfying customer demands. Demand satisfaction could be accomplished through maintaining, raising or lowering of inventories or backlogs, while keeping the workforce relatively stable.

The production schedule is derived from the production plan; it is a plan that authorized the operations function to produce certain quantity of item within a specified time frame. Production schedule has three primary goals or objectives. The first involves due dates and avoiding late completion of jobs. The second goal involves throughput times; the system, from the opening of a job order until it is completed. The third goal concerns the utilization of work centers (Hurtubise et al., 2004).

According to Kriepl and Pinedo (2004), planning models differ from schedule models in a number of ways. First, planning models often cover multiple stages and optimize over medium term horizon, whereas schedule models are usually designed for a single stage (facility) and optimize over a short term horizon. Secondly, planning models uses more aggregate information, whereas scheduling models use more detailed information. Thirdly, the objective to be minimized in a planning model is typically a total cost objective and the unit in which this is measured is a monetary unit; the objective to be minimized in a schedule model is typical a function of the completion times of the jobs and the unit in which this measured is often or time unit. Nevertheless even though there are fundamental differences between these two types of models, they often have to be incorporated into a single frame work, share information, and interact extensively with one another.

1.1.5 Supply chain functions

The supply chain of an organization consists of different functions at each planning stage. According to (Ganeshan and Harrison, 1995), these functions can be broadly classified in the following four categories – location, production, inventory, and transportation. Each of these functions plays a major role in the overall performance of the supply chain. So it becomes essential to execute each one of them in an optimal manner to ensure an efficient supply chain performance. Organization has to make because it will affect the future performance of the organization for a long period of time. One of the factors that categorize the location decisions is capacity. Depending on whether capacity constraint is present or not, the problem is referred to as the capacitated or uncapacitated location problem. It is relatively difficult to obtain optimal solutions to capacitated location problems (CLP). Optimization algorithms as well as heuristic procedures have been proposed to solve these problems.

The location function deals with the decision about where to locate a facility keeping in mind the different constraints such as fixed costs, capacities, demands, labor costs, space constraints, distance from target markets, repulsion factors if any, local taxes, government regulations and so on. This is one of the strategic decisions that the

In a manufacturing facility, one of the most important functions is to prepare a production plan for a given time horizon. A master production schedule is developed based on the sales forecasts taking into account the resource constraints of the plant. It drives the detailed capacity and material planning process and presents a strategy that states the company's production goals. The master production schedule for a longer time period justifies purchase of equipment thus influencing the budgeting decisions as well. It thus forms a vital link between various supply chain functions making it important for inclusion in an integrated framework.

Managing the inventory process is an important task due to its costly nature. Stored inventory consumes space, incurs holding costs and handling costs, and most importantly locks up a huge amount of capital. Inventory may include raw materials, spare parts, factory supplies, work-in-process or finished products. Most often the amount of inventory that needs to be stored can be controlled by internal decisions processes such as production planning, materials requirement planning, distribution planning, etc. Moreover, this leads to an inter-relation and inter-dependence between the inventories function, production function, and distribution function. The internal decisions can also help in optimizing factors such as economic order quantity, safety stock, recorder level, active and reserve storage inventory, and so on. As a result, inventory management assumes an important role in supply chain optimization.

Transportation costs form one of the largest parts of the overall logistics costs in a company's functional structure. It represents the physical linking of the different phases and entities of the supply chain system. The various factors governing transportation costs include travel distance, number of shipments, hourly salaries, fixed costs, fuel charges, etc. Companies have to decide between different strategies, for example, maintaining an own fleet of trucks versus opting for a third party logistics provider, making transporter load (TL) or less than transporter load (LTL) shipments and so on. Routing and scheduling plans are also an important part of the overall transportation function. These decisions affect not just the costs but also customer satisfaction based on timely deliveries, reliability, and safety.

These functions have been mostly studied and optimized individually. However, in the recent years, there have been attempts to consider and analyze these functions collectively to study their interactions and provide a better solution for overall optimization. Integration refers to use of tools and techniques for a combined analysis of supply chain functions or entities to model their combinatorial behavior for determining the best policies for implementation. There are numerous ways to measure the supply chain performance. According to National Research Council (NRC), some of the most commonly used metrics are profitability, total revenues, costs, return on investments, response times, market share, quality, customer satisfaction, risk minimization, waste reduction, and so on. It also lists some detail metrics used by companies for internal operations such as inventory levels and capacity utilization, customer service, lead-times, accuracy of forecasts, logistics costs, obsolescence, turnovers, etc. Although these metrics are for internal measurement, they definitely measure the overall performance of the company with respect to the complete supply chain. In cases involving integration, it becomes important that the performance metrics for all participants are aligned to ensure a common focal point for all the efforts.

1.1.6 Inventory Costs.

In making any decision that affects inventory size, the following costs must be considered.

(i). Holding (or carrying) costs: This broad category includes the costs for storage facilities, handling, insurance, pilferage, breakage, obsolescence, depreciation, taxes, and the opportunity cost of capital. Obviously, high holding costs tend to favor low inventory levels and frequent replenishment.

(ii). Setup (or production change) costs. To make each different product involves obtaining the necessary materials, arranging specific equipment setups, filling out the required papers, appropriately charging time and materials, and moving out the previous stock of material.

If there were no costs loss of time in changing from one product to another, many small lots would be produced. This would reduce inventory levels, with a resulting savings in cost. One challenge today is to try to reduce these setup costs to permit smaller lot sizes.

(iii). Ordering costs. These costs refer to the managerial and clerical costs to prepare the purchase or production order. Ordering costs include all the details, such as counting items and calculating order quantities. The costs associated with maintaining the system needed to track orders are also included in ordering costs.

(iv). Shortage costs. When the stock of an item is depleted, an order for that item must either wait until the stock is replenished or be canceled. There is a trade-off between carrying stock to satisfy demand and the costs resulting from stock out. This balance is sometimes difficult to obtain, because it may not be possible to estimate lost profits, the effects of lost customers, or lateness penalties. Frequently, the assumed shortage cost is little more than a guess, although it is usually possible to specify a range of such cost.

Establishing the correct quantity to order from vendors or the size of lots submitted to the firm's productive facilities involves a search for the minimum total cost resulting from the combined effects of four individual cost: holding cost, setup cost, ordering cost, and shortage cost. Of course, the timing of these orders is a critical factor that may impact inventory cost.

1.1.7 Inventory Systems.

An inventory system provides the organizational structure and the operating policies for maintaining and controlling goods to be stocked. The system is responsible for ordering and receipt of goods: timing the order placement and keeping track of what has been ordered, how much, and from whom. The system must also follow up to answer such questions as: Has the supplier received the order? Has it been shipped? Are the dates correct? Are the procedures established for re ordering or returning undesirable merchandise?

1.2 Statement of the problem

Manufacturing firms produce and deliver products to their customers using a logistic distribution network. The planning and taking inventory in a company relies on mathematical techniques and holistic methods to allocate limited resources to the activities that have to be done. This allocation of resources has to be done in such a way that the company optimizes its objectives and achieves its goals Bradley et al., (1977). The problem facing Latex foam Ghana is how the company can take inventory and distributes its products in other to achieve its optimal results. This thesis wish to study the trend of production, inventory system and develop mathematical programming model to minimize labour, inventory cost and distribution of Latex foam products to meet demand of the numerous customers.

1.3 Objectives of the study

The objectives of the study are:

(i) to study the trend of production and inventory activities in Latex foam Ghana

(ii) to ascertain whether overtime and regular time was necessary to improve productivity.

1.4 Justification of the study

The inventory system has diverse decision variables that can be considered as continuous like regular orders, demand on the stock, regular supply etc. On the other hand, there are discrete variables like special orders that come in at a particular time, theft or accidents that occur without any warning. As a result of these problems, the decision makers need efficient, production, inventory and distribution programming models that can enable them to minimize labour, inventory and distribution cost of latex foam products in attempt to meet demand of their numerous customers.

1.5 Methodology

A secondary data on production, inventory and distribution cost will be obtained from the General Manager of the Latex Foam Ghana Limited in Kumasi. Thus, a twelve month data on Latex Foam production capacities and expected demands (in units) for the period will be collected for the study. Regular and overtime production cost including inventory at the beginning of the year shall also be gathered for the study.

Mixed integer programming model, single commodity flow algorithm, MATLAB and Excel Solver software's will be used to analysis and solve the problem.

1.6 Significance of the study

This study focuses on production, inventory and distribution of Latex Foam Company. It will help to optimize the organizations product-mix and long-term resources allocation based on inventory levels, demand and forecast. In view of the benefits of production and distribution, this study will also help to take proper inventory before distributing to its depots. It will also provide real time information of the work in progress and capacity available.

The model will help to reduce the number of workforces in manufacturing firms and at the same time increase efficiency, production and offer superior value to the customer. In addition to profit maximization, the application of modern techniques of production will lead to major improvements in uniformity and quality. The model will serve as an efficient tool for providing decision support to the management.

1.7 The Manufacturing Environment and the means of Transportation in Ghana:

Compared to other Sub-Saharan African countries, Ghana is endowed with rich natural resources and skilled labour during independence. Immediately after independence, Ghana pursued a reward-oriented state-directed industrialization policy to modernize its economy. These policies were pursued because Ghana, at that time, lacked a strong domestic entrepreneurial know-how and did not want to depend on foreign investment for development (Adu, 1999). Inefficiencies in the management of the state-owned manufacturing enterprises has however, led to huge excess capacity. Today, most manufacturing firms in Ghana are privately owned.

There are four major manufacturing industries in Ghana, namely wood working, metal working, food processing, and textiles and garments and together they comprise 70% of manufacturing employers in Ghana. The sector has undergone several changes since the economic recovery programme of (CSAE and University of Ghana, 1994).

The characteristics of production today, are low costs, mass production, and considerable use of machinery and labour savings and the adoption of modern technological methods of production as well as the use of appropriate optimization skills with modern algorithmic features and the application of Just in Time (JIT) system of production. Manufacturing firms are becoming more customer and competitor-focused by improving quality, relationships with customers and suppliers, and distribution and delivery of their products. These strategic initiatives are being undertaken so as to reduce operating cost, increase demand, and to deal with heightened competition both on the domestic and foreign markets. Transportation in Ghana is accomplished by road, rail, air, and water.

Transport is essential so that products can be moved to places where they are required; to factories where they are manufactured or to markets where they are sold to the people. Ghana's transportation and communication networks are centered in the southern regions especially the areas in which cocoa and timber are produced. The northern and central areas are connected through a major road system; some areas however remain relatively isolated. The deterioration of the country's transportation and communications networks has been blamed for impeding the distribution of economic inputs and food as well as the transport of crucial exports.

1.7.1 Road Transport in Ghana

Road transport is by far the dominant carrier of freight and passengers in Ghana's land transport system. It carries over 95% of all passenger and freight traffic and reaches most communities, including the rural poor and is classified under three categories of trunk roads, urban roads, and feeder roads. The Ghana High Authority is tasked with developing and maintaining the country's trunk road network totaling thirteen thousand three hundred and sixty-seven (13,367) km, which makes up of 33% of Ghana's total road network of forty thousand one hundred and eighty-six (40,186) km (Ghana High Authority, 2011).

1.7.2 Rail Transport in Ghana

Rail transport facilitates long distance travel and the transport of bulky goods that cannot easily be transported by motor vehicles. Additionally, it is believed to be one of the safest forms of transport. The chances of accidents and breakdown are minimal as compared to other modes of transport. Moreover, it helps in the management of road traffic. The railway system in Ghana has historically been confined to plains south of the barrier range of mountains north of the city of Kumasi. However, a thousand and sixty-seven (1,067) mm, (3 ft 6 in) narrow gauge railway, totaling nine-hundred and thirty-five (935) km, is presently undergoing major rehabilitation and inroads to the interior are now being made. In Ghana, most of the lines are single tracked, and in 1997 it was estimated that thirty –two (32) km were double tracked. There are no rail links with adjoining countries (Ghana Railway Corporation, 2011).

1.7.3 Air Transport in Ghana

Domestic air transport in Ghana has in recent times received a little attention. This is normally patronized by the rich because it is relatively expensive. It is safe and very fast. Ghana has twelve landing fields, six with hard surfaced runaways. The most important ones are Kotoka International Airport in Accra and Sekondi-Takoradi, Kumasi and Tamale airports which serves as domestic air traffic (Clark, 1994).

1.7.4 Water Transport in Ghana

Domestic water transport in Ghana is as a result of non availability of road network connecting the source and destination in question, and its cost effectiveness as compared to other modes of transportation. This mode of transport is essential for passenger, liquid and dry cargo. Relatively short distance travel in the farming communities is normally done by canoes often powered by man. Long distance travel often involves passenger and cargo transport. The types of boats used under this mode are normally powered by internal combustion engines assisted by gear box and propellers. Passenger boat is normally used to transport passenger, charcoal and other food items like salt, fish and yams from the southern to the northern part of Ghana and vice versa. Also cargo boats aided by tug boats normally transport heavy industrial products like cement, and other minerals in the same direction. The Volta, Ankobra and Tano rivers provide one-hundred and sixty-eight (168) km of perennial navigation for launches and lighters; Lake Volta provides one thousand one hundred and twenty-five (1,125) km of arterial and feeder waterway. There are ferries on Lake Volta at Yeji. There are ports on the Atlantic Ocean at Takoradi and Tama for international transactions (Ghana Marine Transport, 2011).

1.8 Limitation of the study

Although Latex Foam Ghana has a lot of products on its products line, the scope of this study is limited to production, inventory and distribution of High Density Honeymoon Mattress produced for domestic purposes only. Different cost elements such as production, inventory, transportation, royalties, advertisement and taxes such as excise duty and Value Added Tax (VAT**)** and other practical constraints are normally considered in production planning models. The study takes into consideration the first three, i.e. production, inventory and transportation**.**

1.9 Organization of the Study

In this Chapter, we considered the background, problem statement and objectives of the study. The justification, methodology, scope and limitations of the study were also put forward. The remaining chapters of the study are organized as follows. In Chapter two we shall put forward pertinent literature in the area of production, inventory, distribution and transportation. Chapter three presents the research methodology of the study. Chapter four is devoted for the data collection and analysis of the study. Findings, summary, conclusions and recommendations will be presented in Chapter five.

CHAPTER 2

LITERATURE REVIEW

2.0 This Chapter reviews some of the research work that has been conducted so far in the field of Inventory and Distribution route problems.

2.1 Inventory Management

Inventory management deals with decisions regarding supply levels that correct amount of material and the correct time to reorder. There are many reasons for a company to hold excess inventory; variation in demand and production; poor quality and unreliable suppliers and shippers.

However, there are also good reasons to cut down the amount held in inventory; carrying cost, storage space and material handling. Thus an exchange has to be considered between the two situations.

Inventory management is defined as the direction and control of activities with the purpose of getting the right inventory in the right place at the right time in the right quantity in the right form and at the right cost (Cudjoe, 2010). Inventory is an important current asset with far reaching financial ramifications which deserves every organizations serious attention to ensure cost savings and optimum utilizations of scare resources.

(Cudjoe, 2010) explained that the terminology inventory was of American origin, which was synonymous with stock associated with British authors. Assets in the form of goods, property or services held for sale in the ordinary course of business, in the process of production for sale or to be consumed in the production of goods for sale or in the rendering of services. In order words, inventory may exist in three main forms namely; Finished goods Work in progress and raw materials.

(Cudjoe, 2010) said that Inventory was held in the following purposes;

(i) To enable the organization to achieve economies of scale.

(ii) To balance supply and demand.

(ii) To enable speculation activities.

(iv) To provide protection from uncertainties in demand and order cycle.

(v) To act as a buffer between critical and interfaces within the channel of distribution.

2.2 The Transportation Problem

Transportation networks are complex, large scale systems, and come in a variety of forms, such as road, rail, air and waterway networks. Transportation networks provide the foundation for the functioning of our economies and societies through the movement of people, goods and service (Nagurney, 2004).

(Koopman, 1947) based on the work done earlier by Hitchcock, led independent research on the tendencies of linear programs for the study of problems in Economics. Hence, referring to the classical case of the transportation problem as Hitchcock-Koopmans's transportation problem which aims at total transport cost minimization associated with moving a commodity to its final destination.

The transportation problem however, could be solved for optimally as an answer to complex business problem only in 1951, when Danzig applied the concept of linear programming in solving the transportation model (http;//businessmanagementcourse.org)

(Hammer, 1969) introduced a concept of time-minimizing algorithm for solving the transportation problem. In a related development, a school of thought is of the view that the objective of the transportation problem is to minimize total transportation cost plus expected penalty costs arising from stochastic transportation problems. This is seen as an iterative method for the solution of time-minimizing transportation problems. (Bhatia et al., 1974) discussed such a problem with stochastic demand and penalties for over supply and under supply demand. (Wilson, 1975) used a linear approximation method to solve the stochastic transportation problem as a capacitated problem.

2.3 Production and Distribution problem

In this section we present Computational study of Production and Distribution Planning, Production and Distribution model, Model of Vehicle Routing Assignment and Implementation of optimization model for scheduling of Production and Distribution.

2.3.1Computational study of Production and Distribution Planning

One of the few available approaches is presented by (Chandra and Fisher, 1994). The authors conducted a computational study to examine the value of coordinating production and distribution planning. Their paper is one of the first attempts to include the vehicle routing problem in the analysis, which analyzes, over a multiple period horizon, a multiple product production-distribution environment, solving both the lot sizing and the delivery routing problem. An integrated optimization model is proposed, based on a multi-stop routing problem formulation with the addition of setup constraints. Two different alternative solution approaches are evaluated to analyze the value of coordination. The first approach is to decouple the production and distribution decisions. The approach first determines a production schedule that minimizes the cost of setups and inventory holding subject to meeting total demand per period. This is done by solving a capacitated lot size problem which aggregates all customer demand in each period. The approach then schedules vehicle deliveries of products to customers subject to inventory availability as implied by the production schedule. This is done by solving standard vehicle routing problems using some well-known heuristics (Sweep, nearest neighbor rule, and a feasible insertion rule). The solution is later improved by combining the delivery to a customer at later period with delivery to the same customer at earlier period. These heuristics are not allowed to modify the production plan. In the second case, a coordinated approach, the procedure is the same as employed in first case except that now transportation decisions may entail changes in the production schedule. Computational results point out that the effectiveness of the integrated decision process, showing relevant cost savings, depends on structural parameters of the problem, such as production capacity, time horizon, number of products and customers.

The authors also emphasize the need for organizational changes in order to achieve effective coordination between production and distribution.

Van Buer et al., (1999) studied a problem from the newspaper industry where production and distribution are especially closely coupled since there can be no finished goods inventories.

They give a mathematical formulation for single-plant, multiple- product, single-period production scheduling and distribution routing problem. No production cost or inventory cost is presented in the model. To solve the problem, they use several heuristic search algorithms, such as tabu search, reactive tabu search, and simulated annealing with three different sets of cooling parameters. Using data from a particular newspaper and extensive computational experiments, they find no significant performance difference among these search algorithms. The authors concluded that re-using trucks that have completed earlier routes is the most important way to achieve low-cost solutions.

2.3.2 Production and Distribution model

Ozdamar and Yazgac (1999) considered a production-distribution model involving production and transportation decisions in a central factory and its warehouses. The model is based on the operating system of a multi-national company producing detergents in a central factory from which products are distributed to geographically distant warehouses. The overall system costs are optimized considering factory and warehouse inventory costs and transportation costs. Vehicles are of limit capacity. Although the number of vehicles is optimized, the authors do not consider vehicle routing issues since the warehouses are far apart. Constraints include production capacity, inventory balance and fleet size integrity. The authors adopt a hierarchical approach to make use of medium range aggregate information, as well as to satisfy weekly fluctuating demand with an optimal fleet size. Their approach first solves an aggregate model by aggregating the time periods and product families while omitting detailed capacity consumption by setup.

The aggregate model's optimal solution is then disaggregated for a single period on a rolling horizon basis in order to reduce problem size. Consistency between the aggregation and disaggregation models is obtained by imposing additional constraints on the disaggregation model. Infeasibilities in the disaggregated solution are resolved through an iterative constraint relaxation scheme, which is activated in response to infeasible solutions pertaining to different causes. The authors also investigated the robustness of the hierarchical model in terms of infeasibilities occurring due to the highly fluctuating nature of demand in the refined time periods and also due to the aggregation process itself.

2.3.3 Model for single plant production-distribution system:

Fumero and Vercellis (1999) developed a model for single plant production-distribution system, in which several products produced and delivered with limited available resources, for both production system and a homogenous distribution fleet. The tradeoff is among production setup cost, inventory cost and transportation cost. They use the same line of comparison as in Chandra and Fisher (1994). Two solution approaches are compared: coordinated and decoupled. The authors first present the integrated solution procedure. The utilize Lagrangean relaxation to produce four separate sub-problems, but at the same time preserve a global optimization perspective through the dual master problems. They then propose a decoupled approach, in which the production decision process is carried out independently from the transportation part of the system. Similar to Chandra and Fisher (1994), they first solve a multi-product capacitated lot-sizing problem, and then develop a delivery schedule based on the production schedule. The authors provide heuristics to solve both coordinated and decoupled cases, and thus they are only able to compare feasible solutions from these two approaches.

(i) Number of production facilities (single vs. multiple).

(ii) Fleet characteristics (homogeneous vs. heterogeneous).

(iii) Transportation flexibility (single trip vs. multi-trip per transporter per period).

This research provide the first attempt so far to consider a multi-plant production planning and distribution problem, in which material transportation is carried out by heterogeneous fleets, each plant has an associated fleet, each transporter can make multiple trips in a time period, and customer demand may be met via deliveries by several transporters from different plants.

2.3.4 Model of Vehicle Routing Assignment

The "birth" of Lagrangian approach as it exists today occurred in 1970 when (Held and Karp, 1970, 1971) used a Lagrangian problem based on minimum spanning trees to devise a dramatically successful algorithm for the traveling salesman problem. Motivated by Held and Karp's success, Lagrange methods were applied in the early 1970s to scheduling problems (Fisher, 1973). Lagrangian methods had gained considerable currency by 1974 when (Geoffrion, 1974) coined the perfect name for this approach – "Lagrangian Relaxation".

(Ye and Xu, 2008) developed a fuzzy chance-constrained model of vehicle routing assignment model according to fuzzy theory. In the model, they considered the total costs which included preparing costs of each type of vehicle and the transportation costs as the objective function and the preparing costs and the commodity flow demand as fuzzy variables, and minimized the total costs at a predetermined confidence level, α. They converted the fuzzy constraints into their crisp equivalents by using fuzzy theory and used a priority-based genetic algorithm to solve the problem.

2.3.5 Implementation of optimization model for scheduling of Production and Distribution

Blumenfeld et al., (1987) reported on the successful implementation of an optimization model that integrated scheduling of production and distribution. A work done at Delco electronics division of General motors' resulted in about a 26% reduction in logistics cost.

Condotta, (2007) designed a Branch-and-Bound algorithm to optimally solve the general case of production transportation problem. In particular, a heuristic method was design to Agenerate feasible solutions and studies two possible lower bounds as well as important dominance properties. A second Branch-and-Bound algorithm based on a more efficient branching scheme and on a problem specific lower bound was also designed. Sahoo et al., (2011) developed an optimization based model and Decision Support System (DSS), for tactical supply chain planning for a large cement making firm in India. The DSS used Mixed Integer Linear Programming (MILP) model labeled as Manufacturing and Logistics Planner (MLP). The MLP gave a tactical plan (monthly or quarterly plan) for goods production and distribution which maximized the expected supply chain-wide contribution. The multi-time period model considered production cost, transportation cost, taxes such as excise duty, VAT and numerous practical constraints, and simultaneously calculated production mix at plants, primary transport allocation and mode selection from plants to warehouses and plants to markets, inventory stock taking at warehouses, and secondary transport allocation between warehouses and markets.

EKS, and GLU, (2002) proposed a class of optimization models that consider coordination of production, transportation and inventory decisions in a particular supply chain consisting of a number of facilities and retailers. The particular scenario presented considered a set of facilities where K different product types can be produced.

2.4 Planning Production and Distribution Problem

Chandra and Fisher in (Chandra and Fisher, 1994) are very close to our problem. The main differences in their paper are the production of several products, an unlimited fleet size and storage capacity at the plant, the absence of inventory holding cost at the plant, and split deliveries (multiple deliveries can be made by different vehicles to the same customer). Compared to our case, the multi-product option seems more complicated, but in fact the unlimited fleet and split deliveries make the problem easier because the hard bin-packing sub-problems consisting of assigning the demands to a limited number of vehicles are avoided. These authors proposed a first approach that computes separately one production plan and then a distribution plan, and a so-called coupled approach. In fact, the latter consists in searching cost-reducing changes in the two plans returned by the first approach. Savings between 3% and 20% are reported for the second method on instances with up to 10 products, 50 customers and 10 periods. It should be noted that the instances are weakly constrained: one third of them have a total demand per day limited to 85% of production capacity, one other third 60%, while the last third considers an unlimited production capacity.

Chandra. (1994) tackled a problem of preparation of orders in a regional warehouse to satisfy the demands of customers in the same region. If an order cannot be satisfied, the warehouse may transmit it to a higher echelon (e.g., a factory) but this induces a fixed cost. The tests conducted on different data sets show a cost reduction ranging from 5% to 14% when distribution and order preparation are coordinated.

Fumero and Vercellis, (1999) dealt with a problem closely related with the one studied by Chandra and Fisher. Their solution method, based on Lagrangean relaxation, is evaluated on smaller instances with up to 12 customers, 10 products and 8 periods. Here again, the algorithm is compared with an uncoupled approach and significant savings are obtained.

Erenguc et al., (1999) handled the same kind of problem. Like Chandra and Fisher, they start with a decomposition of the global problem into a production planning problem and a distribution problem. However, they relax some constraints in this first phase and reintroduce them progressively to ensure coordination and make the results of the first phase feasible for the global problem.

Metter's, (1996) investigated the coordination between a sorting centre and mail distribution. The objective includes the total cost, the reduction of routing delays. Bramel et al., (2000) solved a problem of cooperation among several factories that can make components or sub-assemblies for each other. However, the routing aspects are very simplified, since truckload transportation is assumed between factories. A review of the different problems raised by the coordination between production and distribution is presented in Sarmiento and Nagi, (1999).

2.5 Integrated models

One of the first steps to run an effective supply chain is the strategic positioning of the manufacturing facilities, warehouses and distribution centers. Canel and Khumawala, (1997) provided a review of the literature for the incapacitated multi-period international facilities location problem (IFLP). The authors formulated a Mixed-Integer Programming (MIP) model and solve it using the branch and bound method. The decision variables include the countries to locate manufacturing facilities, and their production and shipping levels. They provided a case study with an application of the solution procedure. The performance measurement criteria used here is the number of nodes needed to reach the optimal solution, and to verify the optimality. Computational time was also considered as another performance indicator. The research also identifies the scope for multi-stage problems wherein the location of manufacturing facilities would be accompanied by the optimal location of Distribution Centers (DC).

Canel et al., (2001) developed heuristic procedures for solving the MIP model of a similar IFLP. The heuristic procedures are tested for their computational efficiency. The profit maximization problem considered the other important factors for international location problems, such as, exchange rates, export incentives, tariffs, taxes, and so on. The multi-period model also takes into account the time-dependent variations in prices, costs, and demands. A number of quantitative models use Mixed-Integer Programming (MIP) to solve the supply chain optimization problems. One of the first attempts was done by Geoffrion and Graves, (1974), where a MIP model was formulated for the multi commodity location problem. This seminal research involved the determination of Distribution Center (DC) locations, their capacities, customer zones and transportation flow patterns for all commodities. A solution to the location portion of the problem was presented, based on Bender's Decomposition (BD). The transportation portion of the problem is decoupled into a separate classical transportation problem for each commodity. Their approach shows a high degree of effectiveness and advantage of using BD over branch-and-bound. The technique has been applied on a real problem to test its performance. However, the computational requirements and technical resources required for its implementation make it a difficult choice.

Cohen and Lee, (1988), developed an analytical model to establish a material requirements policy based on stochastic demand. They prepared four different sub-models with a minimum-cost objective. A mathematical algorithm at the end decides the optimal ordering policies to minimize the costs. The authors also developed a deterministic analytical MIP model to maximize the global after-tax profits through optimal policies for facility network design and material flows. The decision variables for the network design issues include location and capacities of all production facilities whereas those for material management issues include sourcing decisions, production and distribution planning. The model decides the optimal resource deployment for a particular policy option. It thus shows the robustness of the global manufacturing network which provides the company with increased flexibility in responding to changing scenarios by adjusting the sourcing, production, and distribution plans.

2.5.1 Model for production, transportation and distribution problem

A MIP model for a production, transportation, and distribution problem has been developed by Pirkul and Jayaraman, (1996) to represent a multi-product tri-echelon capacitated plant and warehouse location problem. The model minimizes the sum of fixed costs of operating the plants and warehouses, and the variable costs of transporting multiple products from the plants to the warehouses and finally to the customers. A solution procedure is provided based on Lagrangian Relaxation (LR) to find the lower bound, followed by a heuristic to solve the problem. The research shows computationally stable results for this combined approach. The heuristic, in particular, performs well with respect to approximations of optimality and solution times.

Fumero and Vercellis, (1991) used LR to solve a MIP model for integrated multi-period optimization of production and logistics operations. Two approaches are presented – integrated and decoupled – and results are compared. The objective in the integrated approach is to minimize the inventory, setup and logistics costs by employing LR which enables the separation of production and logistics functions although it facilitates global optimization of the objective function in the integrated model. In other words, the main model is decomposed into four sub-models: production, inventory, distribution, and routing. The approach separates the capacitated lot-sizing and vehicle routing decisions. In the decoupled approach, the production and logistics problems are analyzed and solved separately in two different models. The results from these two approaches are then compared to analyze their performance. Over a domain of problems of varying sizes, the overall performance ratio varied by around 10%. The final analysis shows that for problems of greater sizes (number of products, customers, time periods), the integrated approach gains relevance.

2.5.2 Strategic and operational planning

Sabri and Beamon.,(2000) present a multi-objective multi-product multi-echelon stochastic model that simultaneously address strategic and operational planning while taking into consideration the uncertainty in demand, and production and supply lead-times. The authors point to some gaps in the SC literature – most stochastic models presented so far consider only up to two echelons. Conversely, other larger models are mostly deterministic in nature. Moreover, these deterministic models consider only profitability and ignore other performance measures. Another observation is that strategic and operational levels are not considered simultaneously. The main model presented here consists of a mixed integer linear programming (MILP) sub-model for the strategic level, to determine the optimal number and location of manufacturing facilities and DCs, and assignment of service regions to DCs.

The stochastic operational level sub-model is an extension of Cohen et al., (1988), in that, it considers simultaneous optimization of non-linear production, distribution and transportation costs. A solution algorithm is then presented to integrate these two sub-models to achieve an overall supply chain performance vector. The approach presented here mainly emphasizes on integration of strategic and operational level decisions while considering demand uncertainty.

Ganeshan, (1999) presented an integrated model that synchronizes the inventory at

retailers and warehouse, and the demand at the warehouse.

One of the important contributions of this paper is the development of an inventory-logistics framework that includes inventory and transportation components in the total cost function. It also identifies the need for a model that caters to more than two echelons in the supply chain and a better way to model random delay at the warehouse.

Barbarosoglu et al., (1999) used Lagrangean Relaxation (LR) to solve a MIP model for an integrated production-distribution system. A heuristic was designed to aid in the problem solving thus further increasing the overall efficiency of the procedure. Due to the increased complexity of large scale integrated models, the paper highlights the importance of developing alternative solution techniques which are able to provide near optimal solutions.

CHAPTER THREE

METHODOLOGY

Introduction

The problem of balancing costs of regular and or overtime production and inventory storage to minimize the total cost of meeting given sales requirements can be set up as a transportation problem. The transportation problem received this name because many of its applications involve determining how to optimally transport goods. However, some of its important applications such as production scheduling problems actually have nothing to do with transportation. This chapter will focus on the development of an algorithm for solving distribution problems that has been modeled as a transportation problem.

The production problem involves the manufacturing of a single product, which can either be stored or shipped. The cost of production and the storage cost of each unit of the products are known. Total cost is made up total production cost plus total storage cost (as total shipping cost is presumed fixed or constant).

3.1Profile of Latex Foam Ghana Limited

Having consolidated its expansion program in Accra, Latex Foam on 12th September, 1996 established another factory in Kumasi in the premises of Ghana Industrial Holding Company (GIHOC) shoe factory at Atonsu- Agogo, with the aim of increasing its proximity to its numerous customers in the northern sector of Ghana.

The main objective of the company is to continue to be the leading manufacturer of quality foam

Products and also satisfy its numerous customers in the northern sector and parts of eastern and western regions of Ghana by providing them with quality and innovative foam products.

3.2 Labour and Machinery

The factory occupies an area of about 6000 square feet. At the moment, the company has about one hundred and fifty workers for the production and distribution of its cherished products. As a result of the numerical strength of the workers some of them capitalized on that and hence do not work as expected of them and therefore this thesis is about reducing labour and at the same time increase production.

The personnel, sales manager, manufactured products, and machines are all in a small building. Close to where the products are packed is a wooden structure which serves as a sales point for customers. The company therefore faces the problem of space at its premises. This does not help easily distribution of its finished products when ready for evacuation to its depots.

3.3 Inventory

Inventory is defined as the stock of any item or resource used in an organization. The inventory system is the set of policies and controls that monitor levels of inventory.

By convention, manufacturing inventories generally refers to items that contributes to or become part of a firms product output. Manufacturing inventory is typically classified into raw materials, finished products, component parts, supplies and work in process**.**

Observation of Latex foam company balance sheet reveals that a significant portion of its assets comprises inventories of raw materials such as gum, resins, tannins, components and subassemblies within the production process and finished goods. The company's manager dislikes taking inventories because he considers it as money placed in a drawer or assets tied up in investments that are not yielding return. The care of the stored materials is subjected to spoilage and obsolescence. As a result, the management of Latex foam limited are advocating for programming model that will help the company to reduce inventory activities, cost and the same time increase efficiency on the shop floor.

3.4 Transportation

The cost of distributing materials and finished products between different depots alone takes about sixty percent of the total production cost. How to minimum the cost of transporting material from the sources to the depots given constraints on the supply at each source and the demand at each depot is the concern of the management. Transportation therefore plays a major role in the distribution of the products and contributes to a greater portion of the total cost of production. It is therefore important to optimize the transportation system to minimize total cost and maximize total benefits.

The transportation problem is one of the subclasses of the linear programming problems for which simple and practical computational procedures have been developed that take advantage of the special structure of the problem

3.4.1 The production problem

The production problem is converted into transportation problem by considering the time periods during which production takes place at sources, and the time periods in which units will be shipped to destinations. The production capacities are taking to be suppliers.

Therefore denote the number of units to be shipped during period i for shipment during time period j, and the unit production cost during time period i plus the cost of storing a unit of product from time period i until time period j.The problem is to find production, which will meet all demands at minimum total cost while satisfying all constraints of productive capacity and demands.

3.4.2 The transportation problem

The transportation problem arises frequently in planning for the distribution of the goods and services from several supply locations to several demand locations. Mathematically, a transportation problem is nothing but a special linear programming problem in which the objective function is to minimize the cost of transportation subject to the demand and supply constraints.

3.4.3 General formulation of a transportation problem;

Let Z be the total distribution cost and x_{ij} and the number of unites to be distributed from a source i to destination j. Let also s_i and d_j denote respectively the number of unites being supplied by source i and the number of unites being received by destination j and c_{ij} the unite cost of supplying s_i unites from source i to destination j.

The transportation problem is generally formulated as:

Minimize $Z = \sum_{i=1}^{m} \sum_{j=1}^{m} c_{ij} x$..(3.1)

Subject to $\sum_{j=1}^{n} x_{ij} = s_i \ (i = 1, 2....m)$ - (Supply constraints)……………(3.2)

$\sum_{j-1}^{n} x_{ij} = d_i \ (i = 1, 2....\text{n})$, (Demand constraints)…………………………………(3.3)

$x_{ij} \geq 0 \quad (i = 1, 2.....\text{m}, \text{j} = 1, 2, ...\text{n})$

The objective function (1) is the total cost of transportation.

Supply constraint (3.2) requires the total amount of commodity. $\sum_{j=1}^{n} x_{ij}$ leaving source s_i must not exceed the production capacity of source s_i.

Demand constraint (3) requires that the total amount of commodity. $\sum_{j=1}^{n} x_{ij}$ arriving at destination d_j must not be less than the demand at destination dj.

Table 3.0 shows the objective function $Z = \sum_{i=1}^{m} \sum_{j=1}^{m} c_{ij} x_{ij}$ as the total sum of elements in matrix table.

Table 3.1 matrix of objective function.

$C_{11}x_{11}$	$C_{12}x_{12}$	……………………	$C_{1n}x_{1n}$
$C_{21}x_{21}$	$C_{22}x_{22}$	……………………	$C_{2n}x_{2n}$
: : :	: : :		: :
$C_{m1}x_{m1}$	$C_{m2}x_{m2}$	……………………	$c_{mn}x_{mn}$

Table 3.1 shows the format of the transportation tableau. The row sum x_{ij}' s is less than or equal

to s_i for each row and the column sum of each x_{ij}'s is greater than or equal to d_j for each column. The table is called the transportation tableau.

The Table 3.1 Format of a transportation tableau

	Destination *(j)*				Supply(S_1)
	1	2		n	
1	c_{11} x_{11}	c_{12} x_{12}	……	X_{1n} X_{in}	s_1
Source *(i)* 2	c_{21} x_{21}	c_{22} x_{22}	……	c_{2n} x_{2n}	s_2
. .	. .	. .		. .	
M	c_{m1} x_{m1}	c_{m2} x_{m2}		c_{mn} x_{mn}	s_n
Demand(d_1)	d_1	D_2	……	d_n	

3.4.4 A Balanced Transportation Problem

In a" balanced transportation problem,'' the total supply is equal to the total demand at any instant.

$$\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$$

3.4.5 The Feasible Solutions Property of Transportation Problems

According to Hiller and Lieberman (2005), a transportation problem will have a feasible solution if and only if

$$\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$$

3.4.6 Integer Solution Property of Transportation Problems.

For transportation problems, where every s_i and d_j has an integer value, all the basic variables (allocations) in every basic feasible solution (BFS) (including an optimal one) also have integer values.

3.5 Methods of Solving Transportation Problems.

There are several methods for solving transportation problems. Two of such methods are the Stepping Stone Method and Lagrangian Relaxation based Methods. These methods are variants of the Simplex Method. The methods use an initial BFS computed from methods like the Northwest corner rule or Vogel's Approximation method, and improve upon the initial basic feasible solution to obtain an optimal solution.

Definitions

Cell: It is a small compartment in the transportation tableau.

Circuit: A circuit is a sequence of cells (in the balanced transportation tableau) such that

(i) It starts and ends with the same cell.

(ii) Each cell in the sequence can be connected to the next member by a horizontal or vertical line in the tableau.

Allocation: The number of units of items transported from a source to a destination which is recorded in a cell in the transportation tableau.

Basic Variables: The variables in a basic solution whose values are obtained as the simultaneous solution of the system of equations that comprise the functional constraints.

Basic Feasible Solution: A solution is called a basic feasible solution if;

(i) It involves $(m+n-1)$ cells with non-negative allocations.

(ii) There are no circuits among the cells in the solution.

3.5.1 Finding Initial Basic Solution of Balanced Transportation problems.

In this section we put forward the discussions of the methods of solving transportation problems.

3.5.2 The **Northwest corner rule**

The Northwest corner rule is a method for computing an initial basic feasible solution of a transportation problem where the basic variables are selected from the Northwest corner of the transportation tableau. Given a balanced transportation problem in a transportation tableau

1. (i) Begin in the upper left (or Northwest) corner of the transportation tableau

   (ii) Set x_{11} as large as possible. Clearly $x_{11} = \min\{s_1, d_1\}$

   (iii) If $x_{11} = S_1$ cross out row 1 of the transportation tableau; no more basic variables will come from row 1.

Also set $d_1 = d_1 - s_1$

(iv) If $x_{11} = d_1$ cross out column 1 of the transportation tableau; no more basic variables will come from column 1.

Also set $s_1 = s_1 - d_1$

(v) If $x_{11} = s_1 = d_1$ cross out either row 1 or column 1 (but not both)

- If you cross out row 1, set $d_1 = 0$

- If you cross out column 1, set $s_1 = 0$

(2) Continue applying this procedure to the most Northwest corner cell in the tableau that does not lie in the crossed-out row or column until you eventually reach a point where there is only one cell that can be assigned a value. Assign this cell a value equal to its row or column.

(3) A basic feasible solution has now been obtained.

Degeneracy

In cases of Degeneracy, the solution obtained by the Northwest corner method is not a basic feasible solution because it has fewer than (m+n-1) cells in the solution. This happens because at some point during the allocation, when a supply is used up, there is no cell with unfulfilled demand in the column.

To resolve degeneracy a zero allocation is assigned to one of the unused cell. Although there is a great deal of flexibility in choosing the unused cell for the zero allocation, the general procedure, when using the Northwest corner rule, is to assign it to a cell in such a way that it maintains an unbroken chain of allocated cells.

3.5.3 Vogel's Approximation Method

Vogel's Approximation method begins by computing for each row and column, a "penalty" equal to the difference between the smallest costs in the row and column and the second smallest. Row penalties are shown, along the right of each row and column penalties are shown below each column. Next we find the row or column with the largest penalty. The method is a variant of the least cost method and based on the idea that if for some reason, the allocation cannot be made to the least unit cost cell via row or column then, it is made to the next least cost cell in the row or column and the appropriate penalty paid for not being able to make the best allocation.

We choose the cell with the greatest row and column penalties. Allocate as much to this cell as the row supply or column demand will allow. This means either a supply is exhausted or a demand is satisfied. In either case, delete the row of the exhausted supply or the column of the satisfied demand. We re-compute new penalties (using only cells that do not lie in a crossed- out row or column), and repeat the procedure until only one uncrossed cell remains. We set this variable equal to the supply or demand associated with the variable, and cross out the variable's row and column, a basic feasible solution has now been obtained.

3.5.4 The Steppingstone Method

Suppose that we have a basic feasible solution, consisting of non-negative allocations in (m+n-1) cells, we call the cells which are not in the basic feasible solution unoccupied cells. Then for each unoccupied cell, a unique circuit begins and end the cell, consisting of that unoccupied cell and other cells all of which are occupied such that each row or column in the tableau either contains two or none of the cells of the circuit.

3.5.5 Test for optimality

To test the current basic feasible solution for optimality, we take each of the unoccupied cells in turns and place one unit allocation in it. This is indicated by just the sign + and -. Following the unique circuit containing this cell as described above place alternately the signs + and – until all the cells of the circuit are covered. Knowing the unit cost of each cell, we compute the total change in cost produced by allocation of one unit in the empty cell and the corresponding placements in the other cells of the circuit.

This change in cost is called improvement index of the unoccupied cell. If the improvement index of each unoccupied cell in the given basic feasible solution is non-negative then the current basic feasible solution is optimal since every reallocation increases the cost. If there is at least one unoccupied cell with a negative improvement index then a reallocation to produce a new basic feasible solution with a lower cost is possible and so the current basic feasible solution is not optimal. Thus the current basic feasible solution with a lower cost is possible and so the current basic feasible solution is not optimal. Thus the current basic feasible solution is optimal if and only if each unoccupied cell has a non-negative improvement index.

3.5.6 Improvement to optimality

If there exist at least one unoccupied cell in a given basic feasible solution which has a negative improvement index, then, the basic feasible solution is not optimal.

To improve on this solution, we find the unoccupied cell with the most negative improvement index say m using the circuit that was used in the calculation of its improvement index, find the smallest allocation in the cells of the circuit with the sign “–‘’.Call this smallest allocation k.

Subtract k from the allocations in the cells in the circuit with the sign "–'' and add it to all the allocations in the cells in the circuit with the sign'' +". This has the effect of satisfying the constraints on demand and supply in the transportation tableau. Since the cell which carried the allocation k now has a zero allocation, it is deleted from the solution and is replaced by the cell in the circuit which was originally unoccupied and now has an allocation k. The result of each reallocation is new basic feasible solution. The cost of this new basic feasible solution in N is less than the cost of the previous basic feasible solution. This new basic feasible solution is tested for optimality and the whole procedure repeated until an optimal solution is attained.

3.6 Lagrangian Relaxation Based Methods.

One of the most computationally useful ideas of the 1970's is the observation that many hard problems can be viewed as easy problem complicated by a relatively small set of side constraints. Making the side constraints dual produces a Lagrange problem that is easy to solve and whose optimal value is a lower bound (for minimization problems) on the optimal value of the original problem.

Lagrangian methods have gained considerable currency by 1974 when Geofrion (1974) coined the perfect name for this approach- Lagrangian Relaxation".

3.6.1 Equality Constraints for Lagrangian function.

Given the problem

$$P_1 : \text{minimize } f(\text{x})$$

Subject to g(x) = b, $x \in x$

The lagrangian function is defined to be

$$L(x,\lambda) = f(\mathrm{x}) + \lambda^T (b - g(\mathrm{x}))$$

The components $\lambda = (\lambda, ... \lambda_m)$ are known as the Lagrange multipliers.

3.6.2 Inequality constraints and complementary slackness.

When the functional constraints in the problem P_1 are in inequality form the problem becomes.

P2: Minimize *f(x)*

Subject to $g(x) \leq \mathrm{b},\ \mathrm{x} \in x$

It may be expressed in the previous form with equality constraints using slack variables as

P3: Minimize f(*x*)

Subject to g(x) + z = b, $x \in \times$ and $Z \geq 0$

The Lagrangian now becomes

$$L(\chi, \mathrm{z}, \lambda) = \mathrm{f(x)} + \lambda^T (b - g(x) - z\ )$$

and it must be minimized over $x \in x$ and $z \geq 0$.

Consider the term in the Lagrangian involving $-\lambda, z_i$, if $\lambda :> 0$ then letting z_i become arbitrarily large shows that this term can be made to approach $-\infty$ which implies that

$_{x \in X, x \to \geq 0}\inf l(x, z, \lambda) = \infty$. Thus, for a finite minimum of the Lagrangian we require that $\lambda_i \leq 0$ which case the minimum of the term $\lambda_1 Z_1$ is 0, since we could take $Z_i = 0$. Thus, with the inequality constraints in the problem, minimizing the Lagrangian always leads to sign conditions on the Lagrange multipliers, in this case $\lambda \leq 0$. There is also a joint condition on the Lagrange multiplier and the slack variables in that $\lambda_i Z_i = 0$ for each

i= 1…m or equivalently, $\lambda^T z = 0$

This condition is known as a complementary slackness condition; at least one of the variables λ_i and z_i must be zero (at the optimum solution) for each *i*.

3.6.3. Lagrange multipliers and the transportation problem.

A classical optimization problem is the transportation problem in which there are m sources of supply of a particular good $\{s_1......s_m\}$, with amounts $\{s_1......s_m\}$ available, and n destinations $\{D_1,......D_n\}$ at which there are demands $\{d_1......d_n\}$, respectively for the good.

For each pair $\{S_i, D_j\}$ there is a cost c_{ij} per unit for shipping from S_i to D_j.

Assumption:

$$\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$$, that is, total supply equals total demand.

The objective is to satisfy the demand from the supplies with the minimal transportation cost.

Let x_{ij} denote the flow from S_i to D_j .The transportation problem is the linear programming problem formulated as

Minimize $Z = \sum_{i=1}^{m} \sum_{j=1}^{m} c_{ij} x_{ij}$____(3.1)

Subject to $\sum_{j=1}^{n} x_{ij} = s_i \ (i = 1, 2....m)$ - (Supply constraints)……………….(3.2)

$\sum_{i=1}^{n} x_{ij} = d_i \ (i = 1, 2....\mathrm{n})$, (Demand constraints)……………………………..(3.3)

$x_{ij} \geq 0 \ (i = 1, 2.....\mathrm{m}, \mathrm{j} = 1, 2, ...\mathrm{n})$ ………………………………………(3.4)

Let λi and $\mathrm{V_j}$ be the Lagrange multipliers

The lagrangian for the balanced transportation problem is

$$L(x,\lambda,v) = \sum_{i=1}^{m}\sum_{i=1}^{n} c_{ij}x_{ij} + \sum_{i=1}^{m}\lambda_i\left(s_i - \sum_{i=1}^{n} x_{ij}\right) + \sum_{i=1}^{n} v_i\left(d_i - \sum_{i=1}^{m} x_{ij}\right)$$
$$= \sum_{i=1}^{m}\sum_{j=1}^{n}\left(c_{ij} - \lambda_i - v_i\right)x_{ij} + \sum_{i=1}^{m}\lambda_i s_i + \sum_{j=1}^{n} v_i d_j \ldots\ldots\ldots\ldots\ldots \quad (3.5)$$

The minimum of the Lagrangian over $x_{ij} \geq 0$ will be provided

$c_{ij} - \lambda_i - v_j \geq 0$ for each *i*,*j*(dual feasibility) and the optimum.

$\left(c_{ij} - \lambda - v_{j)}\right)x_{ij} = 0$ for each $i.j$ (complementary slackness)

The steps for Lagrangian procedure for solving balanced transportation problems are then indicated as following:

1. **Initial assignment.** We start the algorithm by choosing an initial basic feasible solution (BBFS) by the Northwest method.
2. **Assign the Lagrangian multiplier.** Next, we choose the values for the Lagrange multipliers (λ_i, v_j) so that $c_{ij} - \lambda_i - v_j = 0$ for the basic cells; this ensures that the

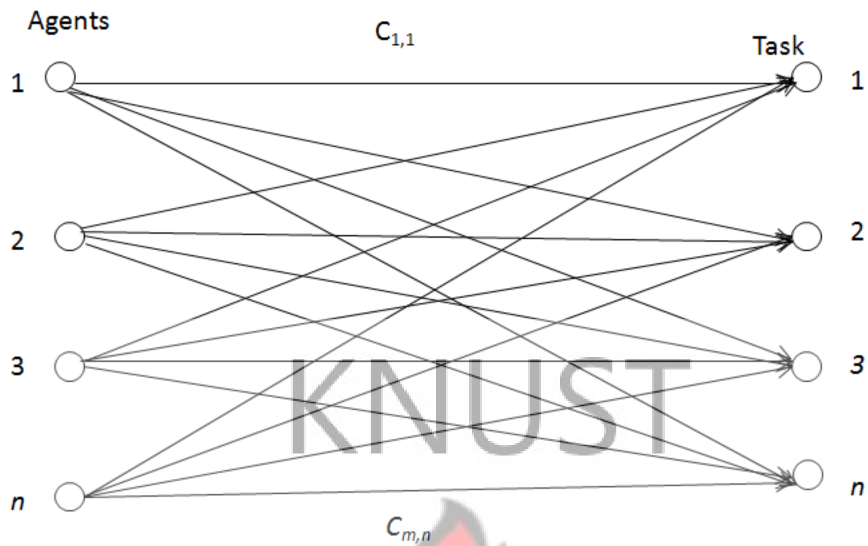
complementary slackness holds. Since only the sum $\lambda_i + v_j$ enter into all the calculations one of these multipliers may be chosen arbitrarily, $\lambda_1 = 0$

3. **Test for optimality.** We identify the non-basic cells for which $c_{ij} - \lambda_i - v_j < 0$; if all cells have $c_{ij} - \lambda_i - v_j \geq 0$ then the current solution is optimal. Otherwise go to step 4.
4. **Pivoting.** Choose the non-basic cell with the most negative value of $c_{ij} - \lambda_i - v_j$ (Pivot cell). Put an amount $\varepsilon > 0$ units of flow into the pivot cell. At the same time, add or subtract from the basic cells to maintain feasibility. Now choose the largest ε possible such that the flow is feasible.
5. The algorithm now returns to step 2 with this flow as the basic feasible flow.

3.7 The Assignment Problem.

The matching or assignment problem is one of the fundamental classes of combinatorial optimization problems. It is a special type of linear programming problem where agents are being assigned to perform tasks. The agents might be employees who need to be given work assignments. Assigning people to jobs is a common application of the assignment problem. However, the agents need not to be people. They would be machines, vehicles, plants, or even time slots to be assigned tasks.

In its most general form, the assignment problem can be stated as follows. A number of m agents and a number of n tasks are given, possibly with some restrictions on which agent can perform which particular task. A cost is incurred for each agent performing some task and the goal is to perform all tasks in such a way that the total cost of the assignment is minimized. The figure below shows the network representation of the assignment problem.



Network representation of assignment problem: C_{ij} denotes the cost of assigning agent I to task j.

The Linear Assignment Problem (LAP).

In the Linear Assignment Problem (LAP), the number of agents and tasks is the same and any agent can be assigned to perform any task. LAP is thus equivalent to the problem of finding an optimum weight vertex matching an nxn cost –weighted complete bipartite graph.

Formulation of Linear Assignment Problem.

Linear Assignment Problem can be formulated as follows. Given a set of agents A = {a_1, a_2 ……a_n} and a set with the same number of tasks.

T = {t_1, t_2 ….t_n} and the cost function C: A x T ----> R

Find a matching m: A ----> T such that the cost function

$\sum_{a \in \in A} c(a, m(a))$, is minimized

Usually, the weight function (i.e. the cost function) is viewed as a square real- valued matrix C with elements

$$C_{ij} = c(a_i, t_j).$$

This problem can be expressed as an integer linear program with the objective function.

Minimize $z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} x_{ij}$ ....(3.1)

Subject to the constraints $\sum_{i=1}^{n} x_{ij} = 1$ for all $j \in \{1, 2, ...n\}$ ....(3.2)

$$\sum_{j=1}^{n} x_{ij} = 1 \quad \text{for all } i \in \{1, 2, ...n\} \quad ....(3.3)$$

$$x_{ij} \in \{0,1\}, for,, all, i, j \in \{1, 2, ..., n\}$$

The variable x_{ij} represents the assignment of agent i to task j, taking value 1 if the assignment is done and 0 otherwise.

Constraints (5) require that every agent is assigned to exactly one task, and constraints (6) require that every task is assigned exactly one agent.

Except for the assumed integrality of the decision variable x_{ij}, the assignment problem is just a balanced transportation problem in which;

The number of supply and demand nodes is equal

Supply from every supply node (agent) is one

The demand at every demand node (task) is also one

Solution is required to be all integers.

The table below is a parameter table for the general assignment problem formulated as the transportation problem.

Table 3.17: Parameter table for assignment problem formulated as a transportation problem

		Task					Agent
		1	2		n-1	*n*	
Resource	1	c_{11}	c_{12}	………	$C_{1,n-1}$	$C_{1,n}$	1
	2	c_{21}	c_{22}	………	$C_{2,n-1}$	$C_{2,n}$	1
	.	. .	. .		. .	. .	. .
	.	$C_{m-1,1}$	$C_{m-1,2}$		$C_{m-1,n-1}$	$C_{m-1,n}$	1
	m	$C_{m,1}$	$C_{m,2}$		$C_{m,n-1}$	$C_{m,n}$	1
Activity		1	1	………	1	1	

For any linear assignment problem with n assignments to be made, the tableau shown in Table 3.17 has m=n, that is, both the number of agents (n) and the number of task (n) in this formulation equal the number of assignments (n).

Transportation problems in general have m+n-1 basic variables (allocations), so every basic feasible solution of linear assignment problems has 2n-1 basic variables, but exactly n of these x_{ij}

variables equals 1 (corresponding to the n assignments being made). Therefore, since all the variables are binary variables, there are always (n-1), degenerate variables $x_{ij}=0$.

3.7.2 Variants of Assignment Problem.

(i) Multiple optimum Solutions

This is a situation whereby more than one optimal solution is obtained and we therefore, have elasticity in decision making. Here, one can choose any of the solutions by experience or by using further considerations.

(ii) Maximization case in Assignment Problem

Some assignment problems entail maximizing the profit, effectiveness, or layoff of an assignment of agents to tasks or jobs to machines.

Unbalanced Assignment Problem

It is an assignment problem where the number of agents is not equal to the number of tasks. If the number of agents is less than the number of tasks then we introduce one or more dummy agents (rows) with zero cost values to make the assignment problem balanced. Likewise, if the number of tasks is less than the number of agents then we introduce one or more dummy tasks (columns) with zero cost values to make the assignment problem balanced.

Prohibited Assignment.

Sometimes it may happen that a particular resourced (say a man or machine) cannot be assigned to perform a particular activity. In such cases, the cost of performing that particular activity by a particular resource is considered to be very high, (written as M or ∞) so as to prohibit the entry of this pair of resource activity into the final solution

3.7.3 Methods for Solving Assignment Problems.

Methods such as the Stepping Stone method and the Lagrange multiplier for solving transportation problems can be used to solve the assignment problems. However, due to its special characteristics, the Hungarian Methods or Munkres Assignment Algorithm is usually used to solve such assignment problems.

Introduction to the Hungarian Method

A high degree of degeneracy in an assignment problem may cause the above mentioned methods to be inefficient in solving assignment problems. For this reason, and the fact that the algorithm is even much simpler than solution methods mentioned above, the Hungarian method is usually used to solve assignment problems.

The Hungarian Method was invented and published in 1955 by Harold Kuhn. The algorithm developed by Kuhn was largely based on the earlier works of two Hungarian mathematicians: DénesKönig and JenöEgerváry (Andras, 2004).

The main merit of Kuhn's Hungarian Method is that in the past half a century it has become the starting point of a fast developing area of efficient combinatorial algorithms. Its seminal ideas, developed originally for the weighted bipartite matching problem (that is, the assignment problem) have been applied by Ford and Fulkerson (1942) to the transportation problem and, more generally, to minimize cost flows, as well. The algorithm is used to solve an assignment problem of $n \times n$ cost matrix where each element represents the cost of assigning the *ith* agent to the *jth* task. By default, the algorithm performs a minimization on the element in the cost matrix.

The Hungarian Algorithm Due to Kuhn

Harold W. Kuhn, in his celebrated paper entitled *The Hungarian Method for the assignment problem,(Andras 2004)* described an algorithm for constructing a maximum weight perfect matching in a bipartite graph .Kuhn explained how the work of two Hungarian Mathematicians, DénesKöning and JenöEgerváry, had contribute to the invention of his algorithm, the reason why he named it the Hungarian Method.

Definitions

- *A graph* is an ordered pair G= (V, E) consisting of a finite set V and a subset E of element of the form(x, y) where x and y are in V. The elements in set V are called the vertices of the graph and those in set E are called the edges.
- *A bipartite graph is* a linear graph in which the nodes can be partitioned into two groups X and Y such that for every edge (i, j) node i is in X and node j is in Y. That is, a *graph* G= (V, E) is bipartite if there exists a partition $V=X\cup Y$ with $X \cap Y=\theta$ and $E \subseteq X \times Y$.
- ***The complete bipartite graph $K_{m:n}$*** is the graph with bipartition {X; Y} where /X/=*m* and /Y/=*n*, and each vertex of X is adjacent to every vertex of Y.
- **A matching M** of a general G= (V, E) is a subset of the edge with the property that no two of the edges of M share the same node. In other words, a subset $M \subseteq E$ such that $\forall v \epsilon V$ at most one edge in M is incident upon **v**.

A matching M is ***perfect*** if every vertex in G in incident with an edge in the matching.

The ***size*** of a matching is /M/ the number of edges in M.

- A ***path*** consists of a sequence of vertices from a starting vertex to an end vertex with edges linking successive vertices.

Alternating Paths

Let **M** be a matching of graph **G**. Vertex **v** is *matched* if it is an endpoint of edge in **M;** otherwise **v** is *free* of the matching. If (x, y) is a matched edge, then y is the mate of x. Nodes that are not incident upon any matched edges are called *exposed* (free) nodes. For example, in figure 3 below, the matched vertices are $x_2, x_3, x_4, x_6, y_2, y_4, y_5$, and y_6, the matched edges (deep black edges) are the set $(x_2,y_2),(x_3,y_5),(x_4,y_4),(x_6,y_6)$ and the exposed nodes are x_1, x_5, y_1 and y_3.

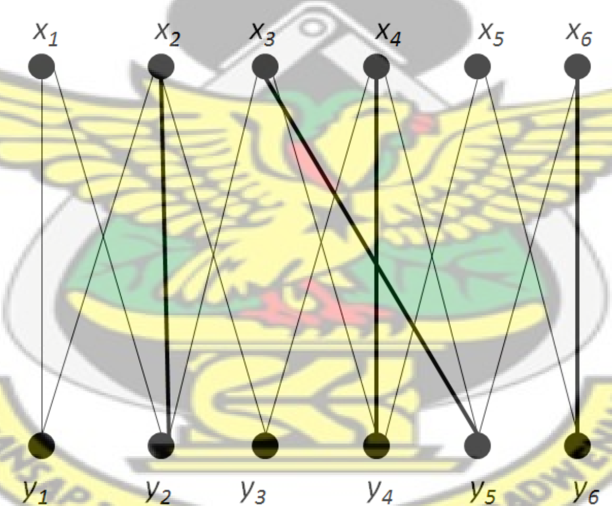


Figure 3: graph showing matched vertices, matched edges and alternating paths

A path is alternating if its edges alternate between **M** and **E-M.** In figure 3 above the alternating paths are

(i) $x_1 \rightarrow y_2 \rightarrow x_2 \rightarrow y_4 \rightarrow x_4 \rightarrow y_5 \rightarrow x_3 \rightarrow y_3$

(ii) $y_1 \rightarrow x_2 \rightarrow y_2 \rightarrow x_3 \rightarrow y_5 \rightarrow x_6 \rightarrow y_6 \rightarrow x_5$

(iii) $y_1 \rightarrow x_2 \rightarrow y_2 \rightarrow x_4 \rightarrow y_4 \rightarrow x_5$

Augmenting Path

An *alternating* path is *augmenting* if both endpoints are free or unsaturated. An augmenting path has one less edge in **M** than in **E-M.**

For example , in figure 4 below , vertices $x_1 y_2, x_2, y_4, x_4, y_5$ from an augmenting path.

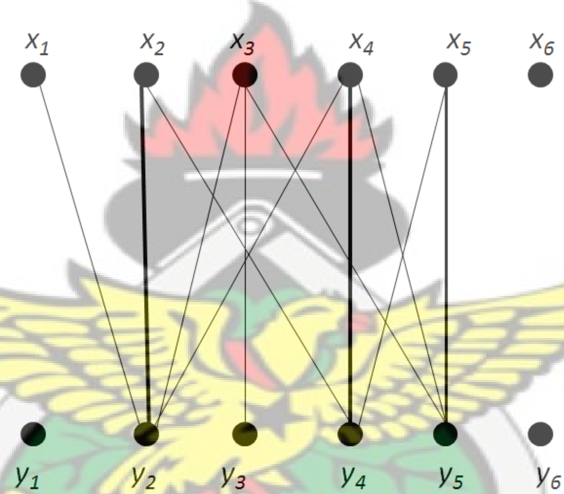


Figure 4: graph showing augmenting paths

Number of edges in **M** (i.e., number of deep black edges) in the augmenting path $x_1, y_2, x_2, y_2, y_4, x_4, y_5$ =2

Number of edges in E (i.e., the number of deep black edges plus the number of light black edges) in the augmenting path =5

Therefore, the number of edges in **E-M=5-2=3,** which is greater than the number of edges in **M** by 1.

Alternating Tree

An *Alternating tree* is two or more alternating paths all ending on some free vertex **v** as the root. Considering the matching **M** in figure 5(a), x_5 is the root because at x_5 three alternating paths

(i) x_5 ,y_6 ,x_6,

(ii) x_5 ,y_5 ,x_3 ,y_3 *and*

(iii) x_5 ,y_4 ,x_4 ,y_2 ,x_2 ,y_1 form alternating trees as indicated in figure 5(b).

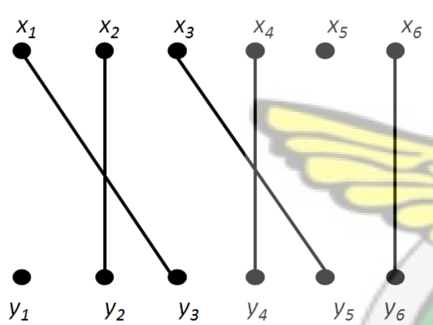


Fig. 5(a): A matching **M**

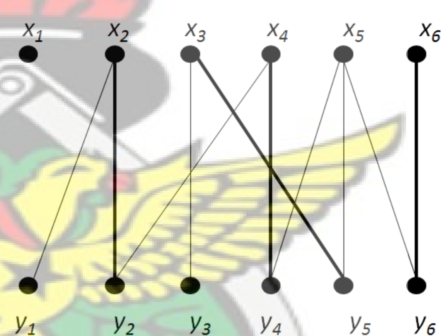


Fig. 5(b): An alternating tree

Weight Matching Bipartite Graphs

These are graphs in which each edge (i,j) has a weight or value *w*(i,j) .The weight of a matching **M** is the sum of the weights of the edges in **M,**

$$w(M) = \sum_{e \in M} w(e)$$

Each entry w_{ij} represents the weight of the edges between x_i and y_j.

Feasible Vertex Labeling

Let N be a network with each edge e giving an integer weight $w(e)$. A feasible vertex labeling for N is a function ℓ: V (N) →□ such that $\ell(x) + \ell(y) \geq w(x.y)$ for all $x \in X$ and $y \in Y$. $\ell(x)$ and $\ell(y)$ are the labeling of vertices x and y respectively and $w(x, y)$ is the maximum edge weight from vertex x to vertex y.

We define the size of ℓ by size $(\ell) = \sum_{v \in V(N).} \ell(v)$.

Lemma1: Let ℓ be a feasible vertex labeling for N and M be a perfect matching in N.

Then $w(M) \leq$ size (ℓ).

Proof: Let M= $\{xy_{1,\, x2}\, y_2 \ldots x_m y_m\}$.

Then $w(M) = \sum_{i=1}^{m} w(x_i y_i) \leq \sum_{i=1}^{m} \left[\ell(x_i) + \ell(y_i)\right] = \sum_{v \in V} \ell(v) = size(\ell)$

Since ℓ is a feasible vertex labeling.

Lemma 1 implies that the maximum weight of a perfect matching in N is less than or equal to the minimum size of a feasible vertex labeling of N.

Equality sub graph

Let ℓ be a feasible vertex labeling of N. The equality sub graph (with respect to ℓ) in N, G (ℓ) = (V, E) is the spanning sub graph of N containing all edges (x, y) for which

$E_\ell = \{(x, y) : \ell(x) + \ell(y) = w(x, y)\}$

Lemma 2: *Let ℓ be a feasible vertex labeling for N and M be a perfect matching in the equality sub graph G (ℓ). Then w (M) =size (ℓ) and hence M is maximum weight perfect matching in N and ℓ is minimum size feasible vertex labeling of N.*

Proof: Let M = $\{x_1y_1, x_2y_2, \ldots\ldots, x_my_m\}$. Since G (ℓ) is the equality sub graph of ℓ in N,

$\ell(x_i) + \ell(y_i) = w(x_iy_i)$, for all $1 \leq i \leq m$.

Thus

$$w(M) = \sum_{i=1}^{m} w(x_i, y_i) = \sum_{i=1}^{m} ((x_i) + \ell(y_i)) = \sum_{v \in (N)} \ell(v) = size(\ell)$$

The facts that M is a maximum weight, perfect matching in N and ℓ is a minimum size feasible vertex labeling of N now follows from Lemma 2.

Theorem 1 (Egerváry, 1931): *Let N be a weighted completed bipartite graph. Then the maximum weight of a perfect matching in N is equal to the minimum size of feasible vertex labeling of N.*

Proof: Let ℓ be a minimum size feasible vertex labeling of N and G=G (ℓ) be the equality sub graph for ℓ in N. By Lemma 2 it suffices to show that G has a perfect matching. We proceed by contradiction.

Suppose that G does not have a perfect matching. There exists a set

$s \subseteq x$ such that /T/</S/.

Let α = min {ℓ(x) + ℓ(y) – w(x y): x ∈ S, y ∉ T

Note that $\alpha > 0$ since there are no edges in the equality sub graph from S to Y-T an d hence we have

$\ell(x)+\ell(y)>w(x,y),\ forall, x\in S, y\notin T$ .We may now define a feasible vertex labeling ℓ' of N as follows

For each $v \in V(N)$, let $\ell'(v)=\begin{cases} \ell(v)-\alpha,\ for, v\in S \\ \ell(v)+\alpha,\ for, v\in T \\ \ell(v)........otherwise \end{cases}$

Suppose $\ell'(v)$ is not feasible vertex labeling of N.

Then we have $\ell'(x)+\ell'(y)<w(x,y)$ for some $x\in X$ and $y\in Y$

Since ℓ is a feasible vertex labeling of N, we must have $x\in S, y\in Y-T$

But then the definition of ∝ implies that

$$\begin{aligned} &\ell(x)+\ell(y)-w(x,y)\geq\alpha, \\ &\ell'(x)+\ell'(y)-w(x,y)\geq 0 \end{aligned} \quad \text{and hence,}$$

Thus ℓ' is a feasible vertex labeling of N. Since ∝ >0 and S > T we have

Size ℓ'=size (ℓ) - ∝ (S – T) < size (ℓ). This contradicts the fact that ℓ is a minimum size feasible vertex labeling of N. Thus G has a perfect matching.

3.7.4 The Kuhn-Munkres Algorithm (Hungarian Method)

Suppose N is a network obtained from Km: m by giving each edge *e* an integer weight

w(*e*). The algorithm iteratively constructs a sequence of feasible vertex labeling $\ell_1, \ell_2, \ldots$ for N such that size (ℓ_{i+1}) < size (ℓ_i), and a sequence of matching M_i such that M_i is a maximum matching in the equality sub graph G (ℓ_i), for all $i \geq 1$. It stops when it finds a feasible vertex labeling ℓ_i for which M_i is perfect matching in G (ℓ_i).

Initial Step

Construct a feasible vertex labeling $\ell_1(x)=\max\{w(x\,y): y \in Y\}$ for each $x \in X$, and $\ell_1(y)=0$ for all $y \in Y$. for each $x \in X$, and $\ell_1(y)=0$ for all $y \in Y$.

Construct a maximum matching M_1 in $G(\ell_1)$

Iterative step:

Suppose we have constructed a feasible vertex labeling ℓ_i of N and, and a maximum matching M_i in $G=G(\ell_i)$, for some $i \geq 1$.

(i) If M_i is complete for G (ℓ_i), then M_i is optimal. Stop. Otherwise, there is some unmatched $x \in X$. Set $S=\{x\}$ and $T=\emptyset$.

(ii) Let $N_{G(\ell i)}(S)$ be the neighbour of set S in the equality sub graph G (ℓ_i), where $S \subseteq X$. If $N_{G(\ell i)}(S) \neq T$, go to step (iii). Otherwise, $N_{G(\ell i)}(S) = T$.

Compute $\alpha=\min\{\ell_i(x)+\ell_i(y)-w(xy); x \epsilon S, y \epsilon T^c\}$, where T^c denotes the complement of T in Y and construct a new labeling ℓ_{i+1} by

$$\ell_{i+1}(v) = \begin{cases} \ell_i(v)-\alpha, if, v \in S \\ \ell_i(v)+\alpha, if, v \in T \\ \ell_i(v)..........otherwise, \end{cases} \quad \text{for each } v \in V(N).$$

(iii) Choose a vertex y in $N_{G(\ell i)}$ (S), not in T. If y is matched in M_i, say with z ϵX, replace S by S $\cup${z} and T by T $\cup$ {y}, and go to step (ii).Otherwise, there will be an M-alternating path from x to y , and we may use this path to find a larger matching M_{i+1} in G (ℓ_i). Replace M_i by M_{i+1} and go to step (i).

Correctness of the Method

(i) We can always take the trivial labeling ℓ and empty matching M=□ to start the algorithm.

(ii) If the labeling ℓ in the neighborhood of S is equal to T, we saw that we could always update labels to create a new feasible matching ℓ'.

(iii) If the labeling ℓ in the neighborhood of S is not equal to T, we can by definition, always augment the alternating tree by choosing some $x \in S$ and $y \in Y\text{-}T$ such that $(x,y) \in E_\ell$. Note that at some point, the y chosen must be free, and in which case we augment M. So, the algorithm always terminates and when it does terminate M is a perfect matching in E_ℓ so by Kuhn-Manures theorem, it is optimal.

3.7.5 Matrix Reduction from the Hungarian Method.

One way of looking at the assignment problem and the Hungarian method is in terms of a matrix. Given *n* agent and *n* task, and non negative edges *e(i,j)*, i =1,2…,n ,j = 1,2,…,n represented by the cost c_{ij} of assigning agent i to task j, the problem is to find the cost minimizing assignment.

The method operates directly on the cost table for the problem. More precisely, it converts the original cost table into a series of equivalent cost tables until it reaches one where an optimal solution is obtained.

The steps of the method as outlined by Hiller and Lieberman (2005) are as follows:

1. Subtract the smallest number in each row from every number in the row. Enter the result in a new table.
2. Subtract the smallest number in each column of the new table from every number in the column .Enter the result in another table.
3. Test whether an optimal assignment can be made. We do this by counting the minimum number of lines needed to cover (i.e., cross out) all zeros. If the number of lines equals the number of rows, then an optimal set of assignments is possible. In that case, go to step 6.Otherwise, go to step 4.
4. If the number of lines is less than the of rows, modify the table in the table in the following way:

   a. Subtract the smallest uncovered number from every uncovered number in the table.

   b. Add the smallest uncovered number in 4(a) to the intersections of covering lines.

   c. Numbers cross out but not at the intersections of cross-out lines carry over unchanged to the next table.

5. Repeat steps 3 and 4 until an optimal set of assignments is possible.
6. Make the assignments one at a time in positions that have zero elements. Begin with rows or columns that have only one zero. Since each row and each column needs to receive exactly one assignment, cross out both the rows and the columns that are not yet crossed out

to select the next assignment, with reference again given to any such row or column that has only one zero that is not crossed out. Continue until every row and column has exactly one assignment and so has been crossed out.

3.7.6 The Munkres Assignment Algorithm (Modified Hungarian).

The Hungarian Method was later revised by James Munkres in 1957 and has since been known as the Munkres assignment algorithm or the Kuhn-Munkres algorithm. His contribution to Kuhn's algorithm was that he introduced the procedure for finding

(i) A minimal set of lines which contain all zeros

(ii) A maximal set of independent zeros,

(iii) "starred zeros" and 'primed zeros" and

(iv) Alternating sequence between "starred zeros" and "primed zeros"

By default, the algorithm perform a minimization on the elements in the cost matrix

The modified Hungarian method on the cost matrix of an assignment problem involves the following steps:

1. Subtract the row minimum from each row.
2. Find a zero (Z) in the resulting matrix. If there are no starred zeros in its column or row, star the zero. Repeat for each zero.
3. Cover each column that has a starred zero.
   (i) If all the columns are covered, then the assignment is optimal.
   (ii) Otherwise, go to step 4.
4. (a) (i) Find a non-covered zero and prime it
   (iii) If there is no starred zero in the tow containing this prime zero, go to step 5

(iv) Otherwise, cover this row and uncover the column containing the starred zero.

(v) Continue in his manner until there are no uncovered zeros left.

(b) Save the smallest uncovered value in the cost matrix and go to step 6

5. Construct a series of alternating primed and starred zeros as follows. Let Z_o represent the uncovered zero found in Step 4. Let Z_1denote the starred zero in the column of Z_o (if any) .Let Z_2 denote the primed zero in the row of Z_1 (there will always be one). Continue until the series terminates a primed zero that has no starred zero in its column. Un-star each starred zero of the series, prime each starred zero of the series, erase all primes and uncover every line in the matrix. Return to step 3.

Add the value found in step 4 to every element of each covered row, and subtract it from every element of each covered row, and subtract it from every element of each uncovered column. Return to Step 4(b) without altering any stars, primes, or covered lines. Return to Step 4 without altering any stars, primes or covered lines.

CHAPTER 4

4.0 DATA COLLECTION, ANALYSIS AND DISCUSSIONS

Data was collected from the factory site of Latex Foam Company Limited, Kumasi.

The company manufactures and sells innovative, high quality industrial and consumer related products throughout the world. It produces foam materials based on orders from its registered customers. Orders that are released into production have to be translated into jobs with associated due dates. These jobs often have to be processed on the machines in a work center in a given order or sequence. The processing of jobs may sometimes be delayed if certain machines are busy.

Pre-emptions may occur when high priority jobs are released which have to be processed at once. Unexpected events on the shop floor, such as machine breakdowns or longer-than-expected processing times, also have to be taken into account, since they may have a major impact on the distributions. Developing, in such an environment, a detailed schedule of the tasks to be performed helps maintain efficiency and control of operations.

This chapter will focus on computational procedure, data analysis and finding of an optimal inventory and distribution of Latex Foam Products.

4.1 Computational Procedure and Data Analysis

Table 4.1 shows the company's production capacities (regular and overtime) and expected demands for Latex Products from January- December 2011.

Table 4.1 Expected demand and capacity data for Latex Foam Ghana-2011.

Month	Foam Demand	Regular Capacity	Overtime Capacity
January	14956	16831	5049
February	12309	12080	3624
March	13456	10876	3262
April	10681	11606	3481
May	12549	12494	3748
June	6589	6300	1890
July	14622	15558	9157
August	13412	15226	4567
September	13164	15608	4682
October	14766	12161	3648
November	14831	15334	4600
December	12095	13421	4026

Inventory/Work In Progress (WIP) at the beginning of January 2011=155439 Mattresses.

The production manager decides on how much should be produced based on the demands taking into consideration the plant capacities. Production is carried out throughout the day (24 hours) in three (3) shifts made up of eight (8) hours per shift. Goods produced cannot be allocated prior to being produced and also, goods produced in a particular month are allocated to the demands in the month ahead. Regular production cost per a foam mattress is GH¢2.1 and the overtime cost per mattress is GH¢2.1.

Units produced on regular shifts are not available for shipments during production; they are generally sold during the next month. Unit produced during overtime shifts must be used to meet demands in the same month as produced.

Production takes place at both regular and overtime shifts for each of the twelve months. Each of these months is a source. A thirteenth source is added, i.e. the WIP, since it can also be used to satisfy demand.

Any unused capacity will be shipped to the dummy demand point. Dummy demands are only created to balance the production problem and so all their allocations do not count. To ensure that no goods are used to meet demand during a month prior to their production, a prohibitively large cost (say GH¢10,000) is assigned to any cell that corresponds to using a regular production to meet demand for a current or an earlier (or previous) month. In the same way, since units produced during overtime must be used to meet demands in the same month as produced, a prohibitively large cost is also assigned to a cell that corresponds to using overtime production to meet next month's demand. It was noticed that the company incurred a regular production cost of GH¢311500 and an overtime cost of GH¢133500 giving a total production cost of GH¢445000 for producing 153430 mattresses for the period.

In view of the huge cost incurred by the company, the production tableau was modeled as a transportation problem in order to minimize the total cost of production whilst satisfying demand.

Formulation of the problem

The formulation takes into account the unit cost of production, $C_{ij,}$ the supply at a_i at source s_j and the demand d_j at destination for $i \in (1, 2, ...25)$ and $j \in (1, 2, ..., 12)$. The problem is:

Minimize $\sum_{i=1}^{25} \sum_{j=1}^{12} c_{ij} x_{ij}$

Subject to $\sum_{j=1}^{12} x_{ij} \leq a_i, ...i = 1, 2, ...25$ (Supply constraints) $\sum_{i=1}^{25} x_{ij} \leq d_j, ...j = 1, 2, ...12$

(Demand constraints)

The objective is to determine the amount of x_{ij} allocated from source I to a destination j such that the total production cost

$$\sum_{i=1}^{25} \sum_{j=1}^{12} c_{ij} x_{ij} \quad \text{is minimized.}$$

Thus, we minimize:

$$\sum_{i=1}^{25} \sum_{j=1}^{12} c_{ij} x_{ij}$$

Subject to the following supply constraints (regular and overtime capacity):

$$x_{1,1}+x_{1,2}+x_{1,3}+x_{1,4}+x_{1,5}+x_{1,6}+x_{1,7}+x_{1,8}+x_{1,9}+x_{1,10}+x_{1,11}+x_{1,12}\le 155439$$
$$x_{2,1}+x_{2,2}+x_{2,3}+x_{2,4}+x_{2,5}+x_{2,6}+x_{2,7}+x_{2,8}+x_{2,9}+x_{2,10}+x_{2,11}+x_{2,12}\le 16831$$
$$x_{3,1}+x_{3,2}+x_{3,3}+x_{3,4}+x_{3,5}+x_{3,6}+x_{3,7}+x_{3,8}+x_{3,9}+x_{3,10}+x_{3,11}+x_{3,12}\le 12080$$
$$x_{4,1}+x_{4,2}+x_{4,3}+x_{4,4}+x_{4,5}+x_{4,6}+x_{4,7}+x_{4,8}+x_{4,9}+x_{4,10}+x_{4,11}+x_{4,12}\le 10876$$
$$x_{5,1}+x_{5,2}+x_{5,3}+x_{5,4}+x_{5,5}+x_{5,6}+x_{5,7}+x_{5,8}+x_{5,9}+x_{5,10}+x_{5,11}+x_{5,12}\le 11606$$
$$x_{6,1}+x_{6,2}+x_{6,3}+x_{6,4}+x_{6,5}+x_{6,6}+x_{6,7}+x_{6,8}+x_{6,9}+x_{6,10}+x_{6,11}+x_{6,12}\le 12494$$
$$x_{7,1}+x_{7,2}+x_{7,3}+x_{7,4}+x_{7,5}+x_{7,6}+x_{7,7}+x_{7,8}+x_{7,9}+x_{7,10}+x_{7,11}+x_{7,12}\le 6300$$
$$x_{8,1}+x_{8,2}+x_{8,3}+x_{8,4}+x_{8,5}+x_{8,6}+x_{8,7}+x_{8,8}+x_{8,9}+x_{8,10}+x_{8,11}+x_{8,12}\le 15558$$
$$x_{9,1}+x_{9,2}+x_{9,3}+x_{9,4}+x_{9,5}+x_{9,6}+x_{9,7}+x_{9,8}+x_{9,9}+x_{9,10}+x_{9,11}+x_{9,12}\le 15226$$
$$x_{10,1}+x_{10,2}+x_{10,3}+x_{10,4}+x_{10,5}+x_{10,6}+x_{10,7}+x_{10,8}+x_{10,9}+x_{10,10}+x_{10,11}+x_{10,12}\le 15608$$
$$x_{11,1}+x_{11,2}+x_{11,3}+x_{11,4}+x_{11,5}+x_{11,6}+x_{11,7}+x_{11,8}+x_{11,9}+x_{11,10}+x_{11,11}+x_{11,12}\le 12161$$
$$x_{12,1}+x_{12,2}+x_{12,3}+x_{12,4}+x_{12,5}+x_{12,6}+x_{12,7}+x_{12,8}+x_{12,9}+x_{12,10}+x_{12,11}+x_{12,12}\le 15334$$
$$x_{13,1}+x_{13,2}+x_{13,3}+x_{13,4}+x_{13,5}+x_{13,6}+x_{13,7}+x_{13,8}+x_{13,9}+x_{13,10}+x_{13,11}+x_{13,12}\le 13421$$
$$x_{14,1}+x_{14,2}+x_{14,3}+x_{14,4}+x_{14,5}+x_{14,6}+x_{14,7}+x_{14,8}+x_{14,9}+x_{14,10}+x_{14,11}+x_{14,12}\le 5049$$
$$x_{15,1}+x_{15,2}+x_{15,3}+x_{15,4}+x_{15,5}+x_{15,6}+x_{15,7}+x_{15,8}+x_{15,9}+x_{15,10}+x_{15,11}+x_{15,12}\le 3624$$
$$x_{16,1}+x_{16,2}+x_{16,3}+x_{16,4}+x_{16,5}+x_{16,6}+x_{16,7}+x_{16,8}+x_{16,9}+x_{16,10}+x_{16,11}+x_{16,12}\le 3262$$
$$x_{17,1}+x_{17,2}+x_{17,3}+x_{17,4}+x_{17,5}+x_{17,6}+x_{17,7}+x_{17,8}+x_{17,9}+x_{17,10}+x_{17,11}+x_{17,12}\le 3481$$
$$x_{18,1}+x_{18,2}+x_{18,3}+x_{18,4}+x_{18,5}+x_{18,6}+x_{18,7}+x_{18,8}+x_{18,9}+x_{18,10}+x_{18,11}+x_{18,12}\le 3748$$
$$x_{19,1}+x_{19,2}+x_{19,3}+x_{19,4}+x_{19,5}+x_{19,6}+x_{19,7}+x_{19,8}+x_{19,9}+x_{19,10}+x_{19,11}+x_{19,12}\le 1890$$
$$x_{20,1}+x_{20,2}+x_{20,3}+x_{20,4}+x_{20,5}+x_{20,6}+x_{20,7}+x_{20,8}+x_{20,9}+x_{20,10}+x_{20,11}+x_{20,12}\le 9157$$
$$x_{21,1}+x_{21,2}+x_{21,3}+x_{21,4}+x_{21,5}+x_{21,6}+x_{21,7}+x_{21,8}+x_{21,9}+x_{21,10}+x_{21,11}+x_{21,12}\le 4567$$
$$x_{22,1}+x_{22,2}+x_{22,3}+x_{22,4}+x_{22,5}+x_{22,6}+x_{22,7}+x_{22,8}+x_{22,9}+x_{22,10}+x_{22,11}+x_{22,12}\le 4682$$
$$x_{23,1}+x_{23,2}+x_{23,3}+x_{23,4}+x_{23,5}+x_{23,6}+x_{23,7}+x_{23,8}+x_{23,9}+x_{23,10}+x_{23,11}+x_{23,12}\le 3648$$
$$x_{24,1}+x_{24,2}+x_{24,3}+x_{24,4}+x_{24,5}+x_{24,6}+x_{24,7}+x_{24,8}+x_{24,9}+x_{24,10}+x_{24,11}+x_{24,12}\le 4600$$
$$x_{25,1}+x_{25,2}+x_{25,3}+x_{25,4}+x_{25,5}+x_{25,6}+x_{25,7}+x_{25,8}+x_{25,9}+x_{25,10}+x_{25,11}+x_{25,12}\le 4026$$

And the following demand constraints:

$$x_{1,1}+x_{2,1}+x_{3,1}+x_{4,1}+x_{5,1}+x_{6,1}+x_{7,1}+x_{8,1}+x_{9,1}+x_{10,1}+x_{11,1}+x_{12,1}\le 14956$$
$$x_{1,2}+x_{2,2}+x_{3,2}+x_{4,2}+x_{5,2}+x_{6,2}+x_{7,2}+x_{8,2}+x_{9,2}+x_{10,2}+x_{11,2}+x_{12,2}\le 12309$$
$$x_{1,3}+x_{2,3}+x_{3,3}+x_{4,3}+x_{5,3}+x_{6,3}+x_{7,3}+x_{8,3}+x_{9,3}+x_{10,3}+x_{11,3}+x_{12,3}\le 13456$$
$$x_{1,4}+x_{2,4}+x_{3,4}+x_{4,4}+x_{5,4}+x_{6,4}+x_{7,4}+x_{8,4}+x_{9,4}+x_{10,4}+x_{11,4}+x_{12,4}\le 10681$$
$$x_{1,5}+x_{2,5}+x_{3,5}+x_{4,5}+x_{5,5}+x_{6,5}+x_{7,5}+x_{8,5}+x_{9,5}+x_{10,5}+x_{11,5}+x_{12,5}\le 12549$$
$$x_{1,6}+x_{2,6}+x_{3,6}+x_{4,6}+x_{5,6}+x_{6,6}+x_{7,6}+x_{8,6}+x_{9,6}+x_{10,6}+x_{11,6}+x_{12,6}\le 6589$$
$$x_{1,7}+x_{2,7}+x_{3,7}+x_{4,7}+x_{5,7}+x_{6,7}+x_{7,7}+x_{8,7}+x_{9,7}+x_{10,7}+x_{11,7}+x_{12,7}\le 14622$$
$$x_{1,8}+x_{2,8}+x_{3,8}+x_{4,8}+x_{5,8}+x_{6,8}+x_{7,8}+x_{8,8}+x_{9,8}+x_{10,8}+x_{11,8}+x_{12,8}\le 13412$$
$$x_{1,9}+x_{2,9}+x_{3,9}+x_{4,9}+x_{5,9}+x_{6,9}+x_{7,9}+x_{8,9}+x_{9,9}+x_{10,9}+x_{11,9}+x_{12,9}\le 13164$$
$$x_{1,10}+x_{2,10}+x_{3,10}+x_{4,10}+x_{5,10}+x_{6,10}+x_{7,10}+x_{8,10}+x_{9,10}+x_{10,10}+x_{11,10}+x_{12,10}\le 14766$$
$$x_{1,11}+x_{2,11}+x_{3,11}+x_{4,11}+x_{5,11}+x_{6,11}+x_{7,11}+x_{8,11}+x_{9,11}+x_{10,11}+x_{11,11}+x_{12,11}\le 14831$$
$$x_{1,12}+x_{2,12}+x_{3,12}+x_{4,12}+x_{5,12}+x_{6,12}+x_{7,12}+x_{8,12}+x_{9,12}+x_{10,12}+x_{11,12}+x_{12,12}\le 12095$$

4.2 Using Excel Solution to obtain the BFS and the optimal solution:

Excel solver shall be used to find the solution to the distribution formulation. Excel solver is a windows package which can be used to obtain the optimal solution to a production and distribution problem. Before using the Excel solver, an initial table is created. This is shown in Table 4.2.

Each cell in Table 4.2 contains the cost per unit of the product plus the storage cost but in this study the storage cost is zero since production is strictly based on order from the customer. For example, in the cell C_{11} the cost is 2.1. A high cost of 10000 is put in cells where production is not feasible. For example, in the cell C_{21}, the cost is 10000. This is because one cannot produce in the month of February to meet a demand in January and so a high cost is allocated to that effect.

For a solution to the production problem to exist, the total demand should be equal to the total supply. The total supply according to Table 4.2 is 312934 and the total demand is 153430. Since the total supply is greater than the total demand, a dummy or fictitious demand of 159504 i.e. (312934 – 153430) is created to balance the production problem with a cost per unit of zero.

The IBFS and the optimal solution to the problem are shown in Table 4.3. The IBFS gives the initial allocations of production resources necessary to meet a demand. Each cell contains the respective allocations for each of the periods during the financial year. A cell with no allocation is called an unoccupied cell or an empty cell. The optimal solution gives the allocations which will minimize the total cost of production.

Findings

All constraints and optimality conditions were satisfied and a solution was found. The following findings were observed after the analysis:

The regular shift production of 5171 mattresses in January was fully used to supply the demand for February without any overtime.

The demand for March was satisfied by the regular shift production of 9631 mattresses and overtime production of 3262 mattresses.

Demand for April was met by the regular shift production of 7200 mattresses and overtime production of 3481 mattresses.

In May a total of 8801 mattresses were produced in regular shift and 3748 mattresses produced in overtime to clear the demand for that month.

There was a regular shift production of 4699 in June as compared to an overtime production of 1890.

Again a regular shift production of July was 5465 and its corresponding production with respect to its overtime went up to 9157 mattresses.

The demand for August regular shift production was 8845 mattresses whereas its overtime production came to 4567 mattresses.

Also the demand for September was met by 8482 mattresses production in normal shift and 4682 mattresses overtime production.

From the analysis so far it could be seen that overtime productions was necessary throughout the year.

The optimal solution gave the final total cost of production Gh¢296,803.50.

The company could have reduced total production cost by (Gh¢ 148196.50) 33.30 %.

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

The application of the model showed how the monthly allocations should be done in order to reduce the cost of production. It also showed which months the stocks available should be allocated to so that they do not pile up unnecessarily.

From the finding it's evident that efficient distribution could reduce production and inventory cost whilst satisfying customer demands. The demand and supply at each level were determined using the Excel solver. The company was able to reduce total production cost by 33.30 % (Gh¢ 148196.50) resulting in the reduction of labour and raw- material cost. The due date and through put times were achieved as overtime become very necessary and the time orders spent in the system was minimized.

Computer-based scheduling could help manufacturers to easily attend to customers' orders, improve on time delivery and create realistic schedules. This confirms the fact that computerized scheduling tools outperform older manual scheduling tools.

The analysis also suggests that production scheduling and control can facilitate the production processes in a number of ways. Production inventory and distribution can result in optimum utilization of capacity.

Companies, with the help of production scheduling could distribute their production in a way to ensure that production capacities such as employees and machinery do not remain idle.

Overtime production was very necessary. The company could not have achieved demand without overtime production. This suggests that the company should necessarily maintain the working or labour force for its production activities. Overtime should not only be carried out as urgent order but throughout the production activities. To a large extent a certain level of inventory is necessary for production since that could be used to supplement some demand.

A good production scheduling ensures quality in terms of processes, products and packaging. It can be ascertained that product ion scheduling and control is of immense importance to every production firm in terms of capacity utilization, inventory control and more importantly, improving the company's response to time and quality. As such, effective production scheduling and control contributes to time, quality and cost parameters of a company's success. Companies would reap a lot of savings if they could incorporate this type of scheduling in their production activities.

It is therefore recommended that companies especially, production firms should employ the usage of the transportation model to achieve optimum level of production at a minimum cost.

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APPENDIX

Initial Table to be used by Excel Solver

	January	February	March	April	May	June	July	August	September	October	November	December	Dummy	Supply
Source 1	2.1	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000	0	0	155439
Source 2	10000	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	0	16831
Source 3	10000	10000	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	0	12080
Source 4	10000	10000	10000	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	0	10876
Source 5	10000	10000	10000	10000	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	0	11606
Source 6	10000	10000	10000	10000	10000	2.1	2.1	2.1	2.1	2.1	2.1	2.1	0	12494
Source 7	10000	10000	10000	10000	10000	10000	2.1	2.1	2.1	2.1	2.1	2.1	0	6300
Source 8	10000	10000	10000	10000	10000	10000	10000	2.1	2.1	2.1	2.1	2.1	0	15558
Source 9	10000	10000	10000	10000	10000	10000	10000	10000	2.1	2.1	2.1	2.1	0	15226
Source 10	10000	10000	10000	10000	10000	10000	10000	10000	10000	2.1	2.1	2.1	0	15608
Source 11	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000	2.1	2.1	0	12161
Source 12	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000	2.1	0	15334
Source 13	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	10000	0	13421
Source 14	2.1	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000	0	5049
Source 15	10000	2.1	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000	0	3624
Source 16	10000	10000	2.1	10000	10000	10000	10000	10000	10000	10000	10000	10000	0	3262
Source 17	10000	10000	10000	2.1	10000	10000	10000	10000	10000	10000	10000	10000	0	3481
Source 18	10000	10000	10000	10000	2.1	10000	10000	10000	10000	10000	10000	10000	0	3748
Source 19	10000	10000	10000	10000	10000	2.1	10000	10000	10000	10000	10000	10000	0	1890
Source 20	10000	10000	10000	10000	10000	10000	2.1	10000	10000	10000	10000	10000	0	9157
Source 21	10000	10000	10000	10000	10000	10000	10000	2.1	10000	10000	10000	10000	0	4567
Source 22	10000	10000	10000	10000	10000	10000	10000	10000	2.1	10000	10000	10000	0	4682
Source 23	10000	10000	10000	10000	10000	10000	10000	10000	10000	2.1	10000	10000	0	3648
Source 24	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000	2.1	10000	0	4600
Source 25	10000	10000	1000	10000	10000	10000	10000	10000	10000	10000	10000	2.1	0	4026
DEMAND	14956	12309	13456	10681	12549	6589	14622	13412	13164	14766	14831	12095	159504	

Optimal Solution Generated by Excel Solver

Optimal cost = $296,803.50	January	February	March	April	May	June	July	August	September	October	November	December	Dummy	Supply
Source 1	5171											12095	130997	7176
Source 2			9631	7200										
Source 3			563											11517
Source 4														10876
Source 5					8801									2805
Source 6						4699							7795	
Source 7							5465						835	
Source 8								8845					6713	
Source 9									8482				6744	
Source 10										11118			4490	
Source 11											10231		1930	
Source 12														15334
Source 13	4736	8685												
Source 14	5049													
Source 15		3624												
Source 16			3262											
Source 17				3481										
Source 18					3748									
Source 19						1890								
Source 20							9157							
Source 21								4567						
Source 22									4682					
Source 23										3648				
Source 24											4600			
Source 25														4026

Marginal Costs

	January	February	March	April	May	June	July	August	September	October	November	December	Dummy	Supply
Source 1		9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9			
Source 2	9997.9	0			0	0	0	0	0	0	0	2.1	0	0
Source 3	9997.9	9997.9		0	0	0	0	0	0	0	0	2.1	0	
Source 4	9997.9	9997.9	9997.9	0	0	0	0	0	0	0	0	2.1	0	
Source 5	9997.9	9997.9	9997.9	9997.9		0	0	0	0	0	0	2.1	0	
Source 6	9997.9	9997.9	9997.9	9997.9	9997.9		0	0	0	0	0	2.1		0
Source 7	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9		0	0	0	0	2.1		0
Source 8	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9		0	0	0	2.1		0
Source 9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9		0	0	2.1		0
Source 10	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9		0	2.1		0
Source 11	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9		2.1		0
Source 12	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	2.1	0	
Source 13			0	0	0	0	0	0	0	0	0	10000	0	0
Source 14		9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	10000	0	0
Source 15	9997.9		9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	10000	0	0
Source 16	9997.9	9997.9		9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	10000	0	0
Source 17	9997.9	9997.9	9997.9		9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	10000	0	0
Source 18	9997.9	9997.9	9997.9	9997.9		9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	10000	0	0
Source 19	9997.9	9997.9	9997.9	9997.9	9997.9		9997.9	9997.9	9997.9	9997.9	9997.9	10000	0	0
Source 20	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9		9997.9	9997.9	9997.9	9997.9	10000	0	0
Source 21	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9		9997.9	9997.9	9997.9	10000	0	0
Source 22	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9		9997.9	9997.9	10000	0	0
Source 23	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9		9997.9	10000	0	0
Source 24	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9		10000	0	0
Source 25	9997.9	9997.9	997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	9997.9	2.1	0	

Final Solution Generated by Excel Solver

	January	February	March	April	May	June	July	August	September	October	November	December	Dummy	Supply
Source 1	5171	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	12095	130997	7176
Source 2	[9997.9]	[0]	9631	7200	[0]	[0]	[0]	[0]	[0]	[0]	[0]	[2.1]	[0]	[0]
Source 3	[9997.9]	[9997.9]	363	[0]	[0]	[0]	[0]	[0]	[0]	[0]	[0]	[2.1]	[0]	11517
Source 4	[9997.9]	[9997.9]	[9997.9]	[0]	[0]	[0]	[0]	[0]	[0]	[0]	[0]	[2.1]	[0]	10876
Source 5	[9997.9]	[9997.9]	[9997.9]	[9997.9]	8801	[0]	[0]	[0]	[0]	[0]	[0]	[2.1]	[0]	2805
Source 6	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	4699	[0]	[0]	[0]	[0]	[0]	[2.1]	7795	[0]
Source 7	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	5465	[0]	[0]	[0]	[0]	[2.1]	835	[0]
Source 8	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	8845	[0]	[0]	[0]	[2.1]	6713	[0]
Source 9	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	8482	[0]	[0]	[2.1]	6744	[0]
Source 10	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	11118	[0]	[2.1]	4490	[0]
Source 11	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	10231	[2.1]	1930	[0]
Source 12	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[2.1]	[0]	15334
Source 13	4736	8685	[0]	[0]	[0]	[0]	[0]	[0]	[0]	[0]	[0]	[10000]	[0]	[0]
Source 14	5049	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[10000]	[0]	[0]
Source 15	[9997.9]	3624	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[10000]	[0]	[0]
Source 16	[9997.9]	[9997.9]	3262	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[10000]	[0]	[0]
Source 17	[9997.9]	[9997.9]	[9997.9]	3481	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[10000]	[0]	[0]
Source 18	[9997.9]	[9997.9]	[9997.9]	[9997.9]	3748	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[10000]	[0]	[0]
Source 19	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	1890	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[10000]	[0]	[0]
Source 20	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	9157	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[10000]	[0]	[0]
Source 21	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	4367	[9997.9]	[9997.9]	[9997.9]	[10000]	[0]	[0]
Source 22	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	4682	[9997.9]	[9997.9]	[10000]	[0]	[0]
Source 23	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	3648	[9997.9]	[10000]	[0]	[0]
Source 24	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	4600	[10000]	[0]	[0]
Source 25	[9997.9]	[9997.9]	[997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[9997.9]	[2.1]	[0]	4026

Summary of Shipment Costs Generated by Excel Solver

	January	February	March	April	May	June	July	August	September	October	November	December	Dummy	Supply
Source 1	5171/$10859.1											12065/$0	130997/$0	7176/$0
Source 2			9631/$20225.1	7200/$15120										
Source 3			568/$1192.8											11517/$0
Source 4														10876/$0
Source 5					8801/$18482.1									2805/$0
Source 6						4169/$8627.199							7795/$0	
Source 7							5465/$11476.5						835/$0	
Source 8								8845/$18574.5					6714/$0	
Source 9									8482/$17812.2				6744/$0	
Source 10										11118/$23347.8			4490/$0	
Source 11											10231/$21485.1		1940/$0	
Source 12														15834/$0
Source 13	4736/$9945.6	8685/$18238.5												
Source 14	5049/$10602.9													
Source 15		3624/$7610.399												
Source 16			3262/$6850.2											
Source 17				3481/$7310.1										
Source 18					3748/$7870.8									
Source 19						1860/$3906								
Source 20							9157/$19229.7							
Source 21								4567/$9590.699						
Source 22									4482/$9412.199					
Source 23										3648/$7660.8				
Source 24											4600/$9660			
Source 25														4026/$0

Summary Results From Excel Solver

From	To	Shipment	Cost per unit	Shipment cost
Source 1	January	5171	2.1	10859.1
Source 1	December	12095	0	0
Source 1	Dummy	130997	0	0
Source 1	Dummy	7176	0	0
Source 2	March	9631	2.1	20225.1
Source 2	April	7200	2.1	15120
Source 3	March	563	2.1	1182.3
Source 3	Dummy	11517	0	0
Source 4	Dummy	10876	0	0
Source 5	May	8801	2.1	18482.1
Source 5	Dummy	2805	0	0
Source 6	June	4699	2.1	9867.899
Source 6	Dummy	7795	0	0
Source 7	July	5465	2.1	11476.5
Source 7	Dummy	835	0	0
Source 8	August	8845	2.1	18574.5
Source 8	Dummy	6713	0	0
Source 9	September	8482	2.1	17812.2
Source 9	Dummy	6744	0	0
Source 10	October	11118	2.1	23347.8
Source 10	Dummy	4490	0	0
Source 11	November	10231	2.1	21485.1
Source 11	Dummy	1930	0	0
Source 12	Dummy	15334	0	0
Source 13	January	4736	2.1	9945.6
Source 13	February	8685	2.1	18238.5
Source 14	January	5049	2.1	10602.9
Source 15	February	3624	2.1	7610.399
Source 16	March	3262	2.1	6850.2
Source 17	April	3481	2.1	7310.1
Source 18	May	3748	2.1	7870.8
Source 19	June	1890	2.1	3969
Source 20	July	9157	2.1	19229.7
Source 21	August	4567	2.1	9590.699
Source 22	September	4682	2.1	9832.199
Source 23	October	3648	2.1	7660.8
Source 24	November	4600	2.1	9660
Source 25	Dummy	4026	0	0
Optimal cost = Gh¢296,803.50				