# LOCATION OF NON-OBNOXIOUS FACILITY (HOSPITAL) IN KETU SOUTH DISTRICT

**BY**

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# DECLARATION

I hereby declare that this thesis is the result of my own original research with close supervisor by my supervisor and that no part of it has been presented to any institution or organization anywhere for the award of Mastersdegree. All inclusive for the work of others has been duly acknowledged.

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**ABSTRACT**

The main purpose of this research is to model the location of two emergency hospitals for Ketu South district due to the newness of the district. This is to help solve the immediate health needs of the people in the district. One essential way of doing this is to locate two hospitals which will be closer to all the towns and villages in order to reduce the cost of travelling and the distances people have to access the facilities (hospital).

In doing this, p-median and heuristics(RH1, RH2 and RRH) were employed to minimize the distances people have to travel to the demand point (hospitals) to access the facilities.

Floyd-warshall algorithm was also adopted to connect the ten (10) selected towns and villages together.

In conclusion, the best two sites for the location of the facilities in theKetuSouth districtwere Klikor-Agborume and Ehi.

## DEDICATION

I dedicate this thesis to GOD ALMIGHTY for His uncommon favour and Grace upon my life.

## ACKNOWLEDGEMENT

It has become a norm on academic ladder to acknowledge the assistance one receives from friends in writing a thesis like this. A number of people have contributed in diverse ways to the successful completion of this research. It is therefore, expedient for me to appreciate the useful contributions of such people.

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# CHAPTER ONE

# INTRODUCTION

## 1.1 HISTORICAL BACKGROUND OF THE HOSPITALS IN GHANA

A hospital can simply be defined as an institution where the sick or injured people are given medical or surgical care (Encyclopedia Britannica), The gold coast era has not seen any tremendous infrastructure development in the area of public health care system in Ghana by the former administrators for the initiation of the innate Ghanaians until the notable British Colonial Governor of Gold Coast, Sir Fredrick GordenGuggisberg's arrival in the Gold Coast (Ghana).

Governor Guggisberg's eight (8) years of administration (1919-1927) in the Gold Coast was the most advancing years in the development of the Gold Coast in the history of the colonial rule era in Ghana. Apart from railways and road infrastructures he is also noted to have established the Korle-Bu teaching hospital which was duly opened in 1923. The Korle-Bu teaching hospital is one of the best hospitals in the Sub-Saharan Africa especially West Africa, which offers health service to a lot of indigenous Ghanaians and neighouring countries, such as Togo, Burkina Faso and Ivory Coast as large.

According to Business Ghana (2011), before Governor Guggisberg came to the Gold coast (Ghana) only few hospitals were located in the populated coastal towns and cities such as Secondi-Takoradi and Accra, These few hospitals were established to cater for the Europeans who lived in those towns and cities but not for the innate Ghanaians. Because these hospitals were built for European population, they are called "European

Hospitals” For instance Ridge hospital in Accra and Takoradi hospital. Breakwaters were also constructed in Secondi and Accra between 1901 and 1908. Construction of an artificial deep-sea harbour was begun at Takoradi in 1926 and completed 1928. Until the construction of Temaharbour, in 1964, Takoradiharbour was the Ghana's sole deep sea harbour.

According to Acheampong (1993), as of 1950, Government hospitals all over the country were not more than fourteen (14). The rest of the hospitals were built and owned by European missionaries who added healing and education to their conversion of Africans to Christianity. Prominent among them were the Basel, Wesleyan, Bremen and Roman Catholic Missionaries. By 1939, the Gold Coast can only boast of thirty-eight (38) hospitals and dispensaries of which the majorities are owned by the missionaries. According to Business Ghana, (2011).

The attainment of independence in 1957 has brought about massive infrastructure development including schools, railways, roads and hospitals. Between 1957 and 1990, the establishment of hospitals by the Government has improved tremendously such that all the ten (10) regions in Ghana can now boast of well-equipped regional hospitals for their respective regions notable among them were, Ho, Sunyani, Cape Coast, Tamale, Bolgatanga and Wa Regional Hospital. The Government has also endeavour to establish a lot of district hospitals. To mention few are Aflao, Sogakope, Ada, Ejisu, Ketekrachi, Peki, Konoogo district hospitals.

In the 1980's the health infrastructure development in the country has dropped drastically due to the military (Caup-de'tal) interferences because of political and Economic instability.

In 1984, there was almost a total collapse of the health care system in Ghana because of the little attention which was payed to it. By the assistance of our donor partner and the economic growth in the Ghana's economy in the last 20 years has revamp the health sector in the country. This is because, out of this growth and help, more hospitals, Clinics, community health centers, health training schools were been built all over the country. It should however be noted that, health infrastructure in Ghana has improved to largely extent that almost all the districts in Ghana have hospitals or clinic and even some district have more than one (1) hospitals and clinics.

**1.2 Government Hospital Development and Goals/Objectives of the Ministry of Health**

The development of hospital infrastructure is a very essential thing which always appear on the top most list of the ministry of health (M.O.H) and the Ghana health service (G.H.S). The ultimate goal of the Ghana Health Service is to ensure a healthy and productive population that reproduces itself safely by providing the following health services: Promotive, Preventive, Curative and rehabilitative.

The health sector objectives include:

- To ensure the people live long, healthy, productive lives and produce without risk of injuries or death.

- To reduce the excess risk and burden of morbidity, mortality and disability especially in the poor and marginalized groups.
- To reduce inequalities in access to health population and nutrition service and health outcomes.

### 1.2.1 Health Sector Medium Term Development Plan (MTDP) Objectives.

To bridge the gaps in access to health and nutrition services and ensure sustainable financing arrangements that protect the poor, is as follows:

- Improve governance and strengthen efficiency and effectiveness in health services delivery.
- Improve access to quality maternal, neonatal, child and adolescent services
- Identify prevention and control of communicable and non-communicable diseases.
- Strengthen institutional care, including mental health service delivery.
- Development and promotion of proactive policies of the quality of clinical services.
- To enhance performance measurement, monitoring and use of information to improve productivity in the health sector.

According to Oppong (2002), The hospital development is very vital to the attainment of the Government goals/ objectives. To achieve quality health delivery, there should be well labeled infrastructures for the beneficiaries. In locating the facility, the geographical accessibility should be taken into consideration. Thus, the closeness of the facility to the consumers must be largely considered.

The total number of major hospitals in Ghana as at now (2011), is about 220.

**1.3 Historical Background of Ketu South District Hospital (Aflao)**

The Ketu South District is a district of Ghana in the Volta region. The district has the geographical area of 1, 130km2 and population of about 235, 852 (according to 2002 population census)



The Ketu South District Hospital was established to cater for the health needs of the staff and populace in the Ketu South district. The hospital provide health needs to the surrounding communities such as Denu, Agbozame, Klikor, Akame, Tokor, Ehi, Kpoglu and its environs. The hospital is located at Aflao in the Ketu South District. The hospital also offer health assistance to the business men and women who come to do business in the district, especially at Aflao in particular. People travel all over the country to Aflao to do business because of its business status.

People also travel from neighbouring countries such as Togo, to do business at Aflao in the Ketu South district because of its proximity to the capital city of Republic of Togo. These peoples (businessmen) sometimes visit the facility (Hospital) to meet the demand for their health needs when on business trips. The Ketu South District hospital offer its health services to about 235, 852 peoples in the districts, according to (2002) report by Wikipedia. The hospital can also boast of appreciable number of equipments and facilities owned by the hospital. These facilities consist of wards for male, female and children. Out-patient department (O.P.D), mortuary, operating theatres and so on. Despite its numerous services that the hospitals offer to its beneficiaries, the hospital is still faced with some challenges. According to hospital officials, the Ketu South district director of health, the hospital is unable to operate its mortuary and two operating theatres because of inadequate power supply. The hospital is also faced with frequent power outages and don't have a standby generator. This has made the hospital to operate under capacity.



The Ketu South District hospital was also faced with inadequate staff and lack of funding in the hospital. For instance, the district hospital has only one (1) medical superintendent in charge (Dr. AsareBediako). The Ketu South District hospital can be found at the south-eastern part of Volta Region.

The district is bordered to the North by the Akatsidistrict, to the South by the Gulf of Guinea. East by the republic of Togo and to the west by Keta district. By its strategic location, a common border with the republic of Togo, the district serves as the eastern Gate-way to Ghana where a continuous cross-border trading activity occurs. Due to all these activities in the district there is always so much pressure on the only district hospital despite the assistance offered by the private ones (hospitals) in the district.

### 1.4 Background of the study

In other to alleviate the unbearable pressure on the available facilities at Ketu South district hospitals, which was providing a health care services to over 235, 852 people and visitors simultaneously, there is an agent need to establish a new hospital in the District. This location of a new district hospital will go a long way to assist in providing a good quality health in the district. To locate this non-obnoxious facility in the Ketu South district, the researcher sort to look out for the health needs of the peoples in some of the major towns in the district and their population density in these towns were selected for the establishment of new district hospital. These towns were Denu, Klikor, Agbozume, Ehi, Amutinu, Weve, Adoteykope etc.



The selections of these towns were done based on the population sizes and the health needs of the people. The proximity of these towns to the rest of the towns in the district were all considered so that the facility (Hospital) will be beneficial to all people in the Ketu South district.

### 1.5 Problem Statement

In locating this desirable facility (Hospital) which will provide the health needs of the people, selection of the site (town) which will benefit everybody in the district pose some challenges. This was because every major town in the district wants the facility to be located in their area, not a place (town) which will be closer to all towns and villages in the district.

Again, because of the geographical location of some towns and villages in the district, their proximity to the facility (Hospital) will be quite affected. Some villages were also dispersed along the length and breadth of the Ketu South district.

### 1.6 Objectives

- Tomodel the location oftwo hospitals as 2-median problem for Ketu South district.
- To optimally locate two sites for the location of two hospitals in Ketu South district using repeated reduction heuristics (RRH).

### 1.7 Thesis Organization

In chapter 1, we looked at a concise history of hospitals in Ghana and Ketu South district hospital.

In chapter 2, we shall review the literature.

In chapter 3, we shall discuss the methodology used in analyzing the data.

In chapter 4, we shall analyze the data and produce results.

In chapter 5, we shall also conclude and make recommendation for further studies.

### 1.8 The Methodology

The location problem was modeled as p-median problem. The following steps were used to solve it:

- Data on district map, population of various town and villages, distance between various towns sketch of the district maps were collected and used.
- Floyd-warshall algorithm was also used to connect the various towns and villages selected

- P-median algorithm was used to locate a common place to locate the facility in minimizing the weighted distance (wi) to the facility (hospital) and the cost of travelling to the facility
- The reduction heuristic ($RH_1$), $RH_2$ and repeated reduction heuristic (RRH) were used to select the best sites for the location of two emergency facilities (hospitals)
- The materials used in the data analysis were obtained from Ketu South district assembly and the town planning council.



**1.9 Justification of Study**

This study seek to find a common place or location site for the new district hospital, for Ketu South district which will be closer to all surrounding towns and villages in the district. This is to assist find the common place in the district which will minimize the distances people have to travel to access the facilities (Hospitals). The p-median and repeated reduction heuristics (RRH) will be used to select the best sites for location the facilities.

## CHAPTER TWO

## LITERATURE REVIEW

### 2.0 INTRODUCTION

According to Darkwah and Amponsah (2007), location is a position or site occupied or available for occupancy marked by some distinguishing features. The facility as a term also means something that is built, installed or established to serve a particular purpose. These include hospitals, supermarkets, churches, warehouses, science laboratories, schools, fire stations, airports, libraries and so on. According to Drezner et al (2002), location selection is a Science likely to have begun somewhere in the seventeen century. The initial premise was to find the spatial median problem. Essentially every location problems developed spawned by this initial mathematical model (problem).

The history is very convoluted and many authors attribute the origination of spatial median problem to different scholars during this time. For many years it appears that the problem was difficult to solve and its solution bore little fruit as a useful tool. The problem therefore remained in the realm of theory for scholarly debate until early twentieth century. The industrial revolution provided an outlet for application of the theories as businesses sought to locate their factories where they could minimize the sum of transportation costs. Such early solutions are based on what is popularly known as the Weber problem.

Drezner and Hamacher (2002) also stipulate that, since these early location the needs of the user. Most problems can be described as continues if there is an infinite or unknown number of possible location or discrete if only a predetermined number of possibilities exist. Continues location problem are normally designed as "site generating" because they are initially designed to find a limited number of possibilities (Drezner and Hamacher 2002).

According to Current et al (2002), location decisions are frequent made at all levels of human organizations such as firms, government agencies, international agencies etc. According to them such decision are strategic as possible. Thus, the involvement of huge capitals and resources are all long term decision plan.

Darkwah and Amponsah (2007), in public service oriented sitting problem, decision makers have to decide on the location of public services such as hospitals, schools, ICT centers, supermarkets etc while emphasizing on the accessibility of the people.

According to them, in locating a desirable (non-obnoxious) facility such as hospitals, certain vital decision must be taken based on the following factors. These factors include physical, economic, social, political and environmental factors. Location problem is concerned with the location of one or more facilities in some space so as to optimize some specified criteria. Often these criterions are linked with distribution costs of providing optimal access to the customers of the facility in question. This does not necessarily follow however when the facilities produce undesirable or an obnoxious effect. In this case, the risk to the local population far exceeds the benefit of close sitement of the facility. This therefore causes the location formulation to change to that of minimizing risk or equivalently maximizing some distance functions to the population centers (Darkwah and Amponsah 2007).

According to Darkwah and Amponsah (2007), the whole essence of a sitting problem is to locate several facilities within an environment so as to optimize their location. This optimization may vary depending on the particular objective function chosen. This could be any of the following.

- Minimize average travel time or cost.
- Minimize average response time.
- Minimize maximum average time or cost.

### 2.1. Types of Facilities

In this modern day's society, the number of facilities available to people in their communities often defines the quality of life of the people "Any facility that provides some services to group of people in a community where it is sited is called physical Entity." These serviceable facilities are grouped into three categories. These are desirable (non-obnoxious), semi-obnoxious and obnoxious facilities.

#### 2.1.1 Non-obnoxious (desirable) facilities

These are facilities which are located or sited close to the consumer so that the consumer can optimally access the facility to the fullest. These facilities include hospitals, churches, schools, libraries, supermarkets, ICT centers, garages, shops, warehouses etc. Since the consumer's accessibility to the facility is very important, the facility ought to be located close to the consumer. This implies that the proximity of the facility must be at its best so that the consumer can access the facility at any time as often as possible.

### 2.1.2 Semi-obnoxious Facilities

In most cases, some facilities need a very high of accessibility which provides negative or unpleasant effects on people who live around the facility. For instance, location of waste disposal site is a need for depositing of the waste produced by the local population. The disposal site may be offensive to look at and emit unpleasant smell. These two contradicting points cause the facility to be described as semi-obnoxious (Darkwah and Amponsah(2007).

Brimberg and Juel (1998), introduced the term semi-desirable facility. They argued that the facilities cannot be classified as being purely desirable or purely obnoxious. Garbage dump sites, airports and power plants are typical semi-desirable facilities.

### 2.1.3 Obnoxious Facilities

According to Darkwah and Amponsah (2007), it is a type of facility which is valuable but harmful to the populace. These facilities include military installations, nuclear power stations and pollution produced by the industrial plants. They argued that the presence of these facilities is dangerous to the surrounding communities and so therefore lowering the prices of houses because nobody wants to live around them. The undesirable effects of obnoxious facilities outweigh their accessibility requirement.

Erkut and Neuman (1998) defined an obnoxious facility as one that generates a disservice to the people nearby while producing an intended product or service. They argue that if minimization of the undesirable effect is the only concern, transportation cost to/from the facility to be located may be prohibitive.

**2.2 Location Models**



There are many location models that have been developed to optimally solve location problems. These models are developed to assist site facilities at a particular node which will benefit everyone equally. This will reduce a transportation costs, placing desirable facilities to ensure satisfaction, prevent situation of obnoxious facilities which will be closer to human settlement. According to current et al (2002), the basic location models are set covering, maximum covering, p-center, p-median, p-dispersion, fixed charge, hub, maximum etc. The main objective is to optimally locate a new facility which will be accessible to all the consumers at a demand node. These models will assist in reducing distances and as a result will reduce the transportation cost and increase the consumers' patronage. According to Klose and Drexl Andreas (2003), indicated, that the objectives of facility location, that the objectives of facility location may be either of the minimum, maximum or minimize. The minimum models are developed to minimize average distances. The maximum models are designed to increase or maximize short distances. The model is normally applied when locating an obnoxious facility such as nuclear power and military installation. Finally maximum models are developed to minimize long distances. This is normally applied to the desirable facilities or non-obnoxious facilities.

In the actual fact, minimum and maximum models are mostly used by companies and the public sector as a whole.

### 2.2.1 Break-Even Analysis

According to Darkwah and Amponsah (2007), the location break even analysis is the use of cost-volume analysis to make an economic comparison of location. Thus to determine which one provides the least cost. Location model break even analysis can be done mathematically or graphically. The graphic approach has advantage of providing range of volumes over which each location is preferable.

Break-even analysis method employs four steps, these are:

- For each location, determine the fixed and variable costs
- Plot the total costs for each location on one graph
- Identify the ranges of output for which each location has the lowest total cost.
- Solve algebraically the break-even analysis point over the identified ranges.

The location break - even analysis is determined by the equation

$y = ax + b$, where

$a$ = variable cost

$b$ = fixed cost

$x$ = volume of business

$y$ = cost of business

### 2.2.2 Factor Rating method

The factor rating approach is one of the numerous approaches which were used to locate facilities, taken into accounts the number of factors. According to Darkwah and Amponsah (2007), these factors include: labour, cost (wages) unionization productivity, labour availability, proximity to raw materials and suppliers, proximity to markets, state and government, fiscal policies, environmental regulation, utilities, site cost, transportation etc.

To them, when applying the factor rating method, the following method must be vigorously followed.

These are:

- Develop a list of relevant factors
- Assign a weight to each factor to reflect its relative importance in the company's objectives.
- Develop a scale for each factor E.g. I to 10 or 10 to 100
- Have management or related people to score each relevant factor using the scale developed above.
- Multiply the score by the weight assigned to each factor and total it for each location.
- Make a recommendation based on the maximum point score considering the result of quantitative approaches as well.

### 2.2.3: Center of Gravity Method

Darkwah and Amponsah (2007), stipulated in their academic course book that, the center of gravity method is a mathematical techniques used for finding the location of a distribution center that swill minimize distribution cost. According to them the approach takes into account the location of market, the volume of goods transported to these markets and transportation cost in finding the best location center.

## 2.3 P - Center Location Problems Model

To Hakimi (1965), the p-center problem minimizes the maximum distance between a demand node and it closest servicing facility, given the already predetermined number of facilities to locate. It can be either a vertex p-center problem, where the set of facility site are restricted to the node of network are restricted to the node of the network, or an absolute p- center problem, which permits facilities to be located anywhere along the arcs of the network. Both versions can be either weighted or unweighted in the problem, all demand nodes are treated equally. In the weighted model, the distances between demand nodes and facilities are multiplied by weight associated with the demand node. For example, this weight might represent a node's importance or, more commonly, the level of its demand. The p-center model is used when the goal is to minimize the maximum distance and the objective is to minimize the maximum distance, thus minimizing the response time, between a school and its closest resource center

According to Handler and mirchandani, (1979) and Handler (1990), its integer value distances can be assumed, the unweighted vertex or absolute p-center problem is most often solved using a binary search over a range of coverage distances. For each coverage distances, a set covering problem is solved. When the solution to the set problem equals p, the minimum associated coverage distance is the solution to the p-center problem.

### 2.4 P - Median Location Model

Another important way to measure the effectiveness of facility location is by evaluating the average (total) distance between the demand points and the facilities. When the average (total) distance decreases, the accessibility and effectiveness of the facilities increases. According to Current, et al. (2002),the p-median problem finds the locations of p facilities to minimize the demand weighted total distance between demand nodes and facilities to which they are assigned. It is one of the most popular models used in facility location decision making. The p-median problem, introduced by Hakimi(1964), takes this measure into account and is defined as determine the location of p facilities so as to minimize the average (total) distance between demands and facilities.

Later, Revelleand Swain (1970) formulated the p-median problem as a linear integer problem and used a branch and bound algorithm to solve the problem. Since its formulation the p-median model has been enhanced and applied to a wide range of emergency facility location problems.

Carbone (1974), formulated a deterministic p-median model with the objective of minimizing the distance travelled by a number of users to fixed public facilities such as medical or day-care centers. Recognizing the number of users at each demand node is uncertain, the author further extended the deterministic p-median model to a chance constrained model. Calvo and marks (1973) constructed a p-median model to locate multi-level heath care facilities including central hospitals, community hospitals and local reception centers. The model seeks to minimize distance and user costs, and maximize demand and utilization.

Later, the hierarchical p-model was improved by Tien et al. (1983) and mirchandani (1987) by introducing new features and allowing various allocation schemes to overcome the deficient organization problem across hierarchies.

Paluzzi (2004), discussed and tested a p-median based heuristic location model for placing emergency service facilities for the city of Carbondale ii. The goal of this model was to determine the optimal locations for placing a new fire station by minimizing the total aggregate distances from the demand sites to the fire station. The results were compared with the results from other approaches and the comparison validated the usefulness and effectiveness of the p-median based location model.

Madel (1988) develop a p-median model and used priority dispatching to optimally locate emergency units for a tiered Emergency Medical Support (EMS) system that consist of advanced life-support (ALS) units and basic life-support (BLS) units. The model can also be used to examine other system parameters including the balance between ALS and BLS units, and different dispatch rules. Uncertainties have also been considered in many p-median models. Mirchandani (1980) examined a p-median problem to locate fire-fighting emergency units with consideration of stochastic travel characteristics and demand patterns. The author took into accounts the situations that a facility may not be available to serve a demand and used a markov process to create a system in which the state were specified according to the demand distributions, services and travel time, and sever availability.

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According to Milind, S. et al. (2010), robust airline scheduling under block-time uncertainty airline schedule development continues to remain one of the most challenging planning activities for any airline. An airline schedule comprises a list of flights and specifies the origin, destination, scheduled departure, and arrival time of each flight in the airline's network. A critical component of the schedule development activity is the choice of flight block-times, which depend on several factors. Many airlines decide schedule block-times based on fixed percentiles of block-time distributions built from historical data; however, such techniques have not resulted in significantly improved on-time performance (OTP) of the schedule during operations. Thus, from a passenger's perspective, the service-level guarantee of an airline's network continues to be low. We first define two service-level metrics for an airline schedule. The first one is similar to the OTP measure of the U.S. Department of Transportation and they define it as the flight service level. The second metric, called the network service level, is geared toward completion of passenger itineraries. We then develop a stochastic integer programming formulation that optimally perturbs a given schedule to maximum expected profit, while ensuring the two service levels. They also develop a variant of this model that maximizes service levels, while achieving desired network profitability. To solve these models, they developed an efficient algorithm that guarantees optimally. Through extensive computational experiments, using real-world data, they demonstrated that their models and algorithm are efficient and achieve the desired trade-off between service level and profitability.

Serra and Vladimir (1996) used p-median model to locate a facility in Barcelona. A p-median-like model is formulated to address the issue or locating new facilities there is uncertainty. Several possible future scenarios with respect to demand and/or the travel times/distance parameters arc presented. The planner will want a strategy or positioning that will do as "well as possible" over the future scenarios. This paper presents a discrete location model formulation to address this P-median problem under uncertainty. The model is applied to the location of lire stations in Barcelona.

According to Lorry and Richard (1977), in their book "Modeling facility Location Problem as Generalized Assignment Problems", a variety or well-known facility location and location-allocation models are shown to be equivalent to, and therefore solvable as Generalized Assignment Problems (CAP's). The CAP is a 0-1 programming model in which it is desired to minimize the cost or assigning *n* tasks to a subset of *m* agents. Each task must be assigned to one agent but each agent is limited only by the amount of a resource, like time available and the fact that the amount of resource required by a task depends on both the task and the agent performing it. The facility location models considered arc divided into public and private sector models. In the public sector, both p-median and capacity constrained p-median problems are treated. In the p- median problem exactly p of 17 sites must be selected to provide service to all *n*. Each side has an associated weight, e.g., its population and it is desired to minimize the weighted average distance between the *n* sites and their respective service sites. The capacity constrained p-median problem differs only in that there is an upper limit on the sum of the weights or the sites served by each service site. In the private sector they considered both capacitated and incapacitated warehouse location problems in which each customer's demands must be satisfied b) a single warehouse. In addition, they showed how certain types of constraints limiting the site and capacity combinations allowed can be incorporated into these models through their treatment as CAP's. An existing algorithm for the CAP is modified to take advantage of the special structure or these facility location problems and computational results are reported.

Michaelet al. (1997), in an Optimization Model for Location or Subsidized I lousing in Metropolitan Areas. This paper presents an optimization model for evaluation of alternative spatial configurations of rent-subsidized housing in a large metropolitan area as well as associated monetary and nonmonetary impacts. Croups affected by these configurations include residents or subsidized housing, owners or nearby single-family housing, employers and society at large. Since impacts or subsidized housing arc very, localized, the first stage or the model creates potential location patterns for many small geographic areas. The second stage or the model uses local benefits to derive a location scheme for the metropolitan area which balances net social benefit with equity considerations or the geographical impact or subsidized housing. They applied this methodology to a small region in metropolitan Chicago and demonstrate alternative location schemes. Results from this model indicate that there are indeed distinct tradeoffs between the three objectives used in computations.



Pavankurnar et al. (2009), used *Capacitated Facility Location with Distance-Dependent* Coverage under Demand Uncertainty to consider a facility location problem to determine the points of dispensing medicine or supplies in a large-scale emergency. For this problem we consider capacitated facilities, a distance-dependent coverage function and demand uncertainty. We formulate a special case or the maximal covering location problem (MCLP) with a loss function. To account for the distance-sensitive demand and chance-constraints to address the demand uncertainty. This model decides both the locations to open and the supplies assigned to each location. They solved this problem with a locate-allocate heuristic. The illustrated the use or the model by solving a case study or locating facilities to address a large-scale emergency (an anthrax attack) in Los Angeles County.

Lai et al. (2009), portrayed single source facility location problem using genetic algorithm. Single source capacitated facility location problems (SSCFIP) arc basic location-allocation models used in the optimal design or supply chain, computer network and transmission power amongst others. While genetic algorithm (CA) has been successfully applied to many combinatorial optimization problems, it has limited success when applied to solving SSCFLP. This paper proposes a GA solution which adopts an integer-based chromosome encoding approach using roulette wheel selection to allocate the alleles in the chromosome. A case study is conducted which shows that the algorithm is capable or solving the problem within short computational time. Further work in this area could include evaluating the GA developed in this paper with other techniques in terms of solution quality and computational time and using other types of crossovers mutations and selection strategies to determine if there could be further improvement to the GA developed in this paper.

Kevin et al. (2007) also determined optimal police patrol areas with maximal covering and backup covering location models. Their paper presents a new method for determining efficient spatial distributions of police patrol areas. This method employs a traditional maximal covering formulation and an innovative backup covering formulation to provide alternative optimal solutions to police decision makers and to address the lack of objective quantitative methods for police area design in the literature or in practice. This research demonstrates that operations research methods can be used in police decision making presents a new backup coverage model that is appropriate for patrol area design and encourages the integration or geographic information systems and optimal solution procedures. The models and methods are tested with the police geography of Dallas. TX. The optimal solutions are compared with the existing police geography, showing substantial improvement in number of incidents covered as well as total distance traveled.

Edson et al. (2004),a branch-and-price approach to p-median location problems. This paper describes a branch-and-price algorithm for the p-median location problem. The objective is to locate p facilities (medians) such as the sum of the distances from each demand point to its nearest facility is minimized. The traditional column generation process is compared with a stabilized approach that combines the column generation and Lagrangean/surrogate relaxation. The Lagrangean multiplier modifies the reduced cost criterion, providing the selection of" new productive columns at the search tree. Computational experiments are conducted considering especially difficult instances to the traditional column generation and also with some large-scale instances.

There several methods or algorithms or solving location models. In all the solution methods. The major challenge for every location analyst is to identifying the optimal solution. According to Garey and Johnson (1979), attempting a solution with some methods will quite often consume unacceptable computational resources in terms of both computer memory and time and with no guarantee or success. The reason is that even the most basic location models are classified as NP-Hard (non-deterministic polynomial-time or non-polynomial hard) models arc class or problems that arc complex to solve, location analyst must devise other methods to identify optimal solutions. The methods are, grouped under heuristics and Metaheuristics. Under heuristics, we have Greed, Alternate and Vertex Substitution, while Variable Neighborhood Search. Genetic

Algorithms, Scatter Search, Tabu Search, Simulated Annealing and CRASP Mciahcuristic are under Mctahcuristics. There are other methods like Lagrangean, Swap-based Local Search and Discrete Vector Quantization.

### 2.4.1Fixed charge location

Teo(2011) used fixed charge location model to determine the optimal location and types of medical facilities to address to the healthcare needs of the people in barnyan province, Afghanistan. In modeling the healthcare facilities, he created a network model of transportation.

The modes of the network are the villages and small towns in Bamyan and the edges are the available roads in Bamyan. A linear mixed inter location model was used to select the villages in which to place healthcare facilities.

### 2.4.2 The Hub location Model

The hub candidates depend more on their geographical position than on their own elements level. According to PatrickJaillet et al (1996), proposed a new set of formulations for the problem of designing a capacitated airline networks. They proposed heuristics and tested on two data sets. They concluded and made reservation based on the analysis of heuristics solution as opposed to optimal bases. Their main finding was that given a fixed origin destination demand matrix and efficient design suggests the presence of strong connectivity cities, which can be called hub.

Ernest and Krishnaoorthy(1996), Hub location model are node-to-node flows and not simply demands at a particular node. Essentially the objective function is the quadratic on the assignment variable.

In addition, it may not be optimal to assign a node to the nearest hub since the objective functions measured in terms of node-to-node flows and not simply In terms of the cost of assessing the hub systems.



### 2.4.3 The Maximum Location Model

According to Mark (1995), the maximum location problem is generally concerned with the location of undesirable facilities. His goals seek to maximize the demand weighted distance between demand nodes and the facilities to which they are assigned.

According to him, he provided a comprehensive introduction to the art and science of location facilities.

He introduced model-building methods and solution algorithms with objective of demand nodes. The maximum location problem seeks the location of the facilities. Such that the total demand- weighted distance between demand nodes and the facilities to which they are assigned is maximized.

## CHAPTER THREE

## METHODOLOGY

### 3.0 Introduction

This chapter introduces the methods employed to form the basis for analysis in this study. The location analysis method used and solution technique employed is described in detail. The mathematical formulation of the location models and the total distances to be covered and calculations are outlined.



### 3.1 Data

The following data were collected from the Ketu South District Assembly.

- District map
- Population figures of the various town
- Distances (D) between various town
- Sketch of district capital's map showing the relative positions of the ten (10) towns with the distances (in kilometers) between them.

### 3.2 P-median Problem Formulation

The p-median is employed if the objective is to minimizing the weighted distance is the primary goal. This methodology can also be employed, as is the case in this research, to find the minimum weighted distance to locate the hospitals. So the appropriate objective then is to find the minimum of the calculated weighted (wi distances for all the potential sites).

### 3.3 An illustrative example of the p-median methodology

Given the distances an individual should cover to have access to a facility at a selected site as well as the number of people commuting from the various sites to a selected site. For example, if four (4) suburbs A, B, C, D, forming a town are linked with the distances between them and their population as shown in figure 3.1.

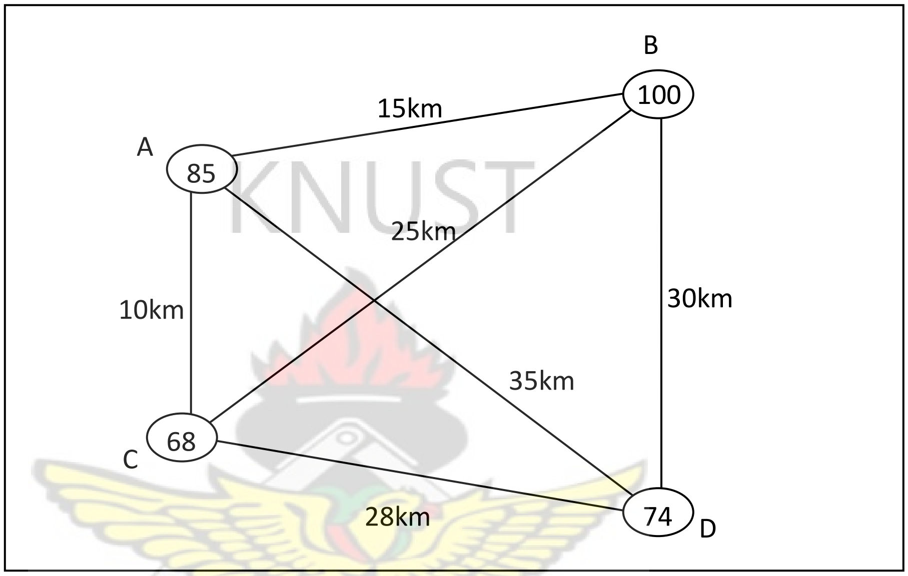



**Figure 3.1 Network of towns**

**Table 3.1: Distance between towns**

| | j | | | | | | |
|---|---|---|---|---|---|---|---|
| | | SITE | 1 | 2 | 3 | 4 | |
| | SITE | TOWN | A | B | C | D | |
| | 1 | A | 0 | 15 | 10 | 35 | |
| i | 2 | B | 15 | 0 | 25 | 30 | |
| | 3 | C | 10 | 25 | 0 | 28 | |
| | 4 | D | 35 | 30 | 28 | 0 | |
| | | | | | | | |

**Table 3.2: Population of the towns**

| Towns | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| | A | B | C | D |
| Population (h) | 85 | 68 | 74 | 100 |

We seek to use p-median to select the most appropriate suburb where this hospital should be sited in order to minimize the overall total distance.

This can be done as follows;



Given the sites distances an individual should cover to have access to a facility at a selected site as well as the number of people commuting from the various sites to a selected site,

$$w_i = \sum_{i,j}^{n} h_i d_{ij} y_{ij}$$

can be used to find the weighted distances and we select the minimum of them as the optimal site. The distance between the various towns is put in a table to form a matrix. The populations (h) of various locations are noted.

$$w_i = \sum_{i,j}^{n} h_i d_{ij} y_{ij}$$

Where $w_1$, $w_2$, $w_3 w_4 \ldots w_{10}$are the total weighted distances of the ten towns.

$d_{ij}$ = the distance between the other towns and the selected town

$h_i$ = the populations of the various towns

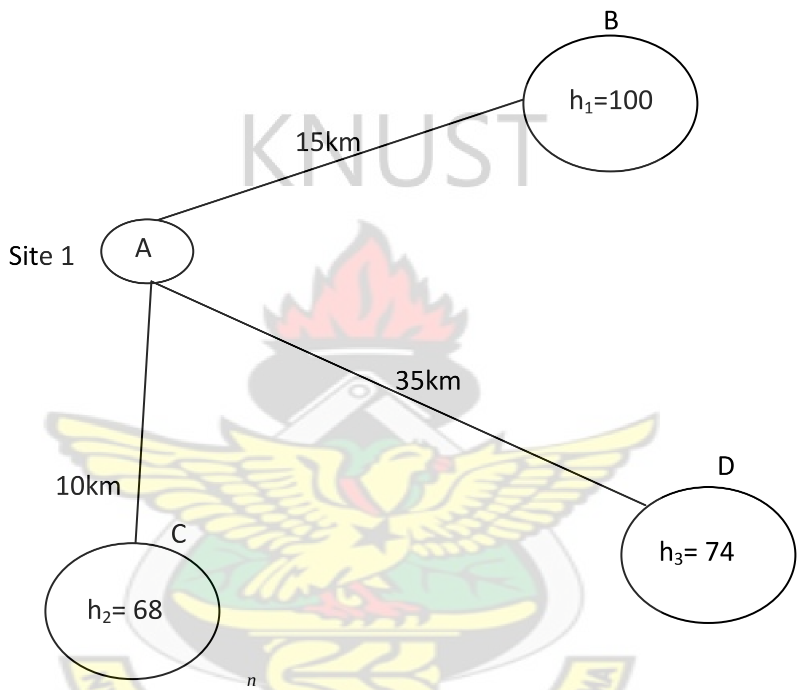
n = 10(nodes)

$w_1$, $w_2$, $w_3$, $w_4$ ... $w_{10}$ are calculated and the minimum of them is selected to locate the facility.

$h_1$, $h_2$, $h_3$, $h_4$... $h_{10}$ are the populations of the towns A, B, C, D,…respectively.

The distance between a town and itself is zero (0), e.g. from D to D is zero (0).

### 3.3.1 Calculating the total weighted distance ($w_1$) of site 1(A)




$$w_1 = \sum_{i,j}^{n} h_j d_{ij} y_{ij} \qquad (1)$$

$$\sum_j x_j = n \qquad (2)$$

$$\sum_j y_{ij} = 1 \qquad (3)$$

$$x_j = \{0,1\} \qquad (4)$$

$$y_{ij} = \{0,1\} \qquad (5)$$

Where

$h_2$, $h_3$ and $h_4$ are 100, 68 and 74 respectively

$d_{12}$, $d_{13}$ and $d_{14}$ are 15, 10 and 35 respectively

$\sum_j y_{ij} = x_j = y_{ij} = n = 1$, because each site would be served by all other sites

$$w_1 = \sum_{i,j}^{n} h_1 d_{ij} y_{ij} \qquad i = 2,3,4 \; and \; j = 2,3,4$$

$$w_1 = d_{12}h_2 + d_{13}h_3 + d_{14}h_4$$

$$w_1 = \sum_{ij}^{n} h_i d_{ij} y_{ij} = 15 \times 100 + 68 \times 10 + 74 \times 35$$

$$w_1 = 4770km$$



Therefore if $w_1$ which is site A is selected, it means that all the people from other suburbs to access the facility are town A and a total distance of 4770km must be covered.

### 3.3.2 Calculating the total weighted distance (w2) of site 2(B)

$$w_1 = \sum_{i,j}^{n} h_j d_{ij} y_{ij}$$

Where

$h_1$, $h_3$ and $h_4$ are 85, 68and 74 respectively

$d_{11}$, $d_{13}$ and $d_{14}$= 15, 25 and 30 respectively

$\sum_j y_{ij} = x_j = y_{ij} = n = 1$, because each site would be served by all other sites

$$w_1 = \sum_{i,j}^{n} h_1 d_{ij} y_{ij} \qquad i = 1,3,4 \; and \; j = 1,3,4$$

$$w_1 = d_{11}h_1 + d_{13}h_3 + d_{14}h_4$$

$$w_1 = \sum_{ij}^{n} h_i d_{ij} y_{ij} = 85 \times 15 + 68 \times 25 + 74 \times 30$$

$$w_2 = 5195km$$

Therefore if $w_2$ which is site B is selected, it means that for all the peoples from other suburbs to access the facility at town B, a total distance of 5195km must be covered.

### 3.3.3 Calculating the total weighted distance (w3) of site 3(C)



$$w_1 = \sum_{i,j}^{n} h_j d_{ij} y_{ij}$$

Where

$h_1$, $h_2$ and $h_4$ are 85, 100 and 74 respectively

$d_{11}$, $d_{12}$ and $d_{14}$ = 10, 25 and 28 respectively

$\sum_j y_{ij} = x_j = y_{ij} = n = 1$, because each site would be served by all other sites

$$w_3 = \sum_{i,j}^{n} h_1 d_{ij} y_{ij} \qquad i = 1,2,4 \; and \; j = 1,2,4$$

$$w_3 = d_{11}h_1 + d_{12}h_2 + d_{14}h_4$$

$$w_3 = \sum_{ij}^{n} h_i d_{ij} y_{ij} = 85 \times 10 + 100 \times 25 + 74 \times 28$$

$$w_3 = 5422km$$

Therefore if $w_3$ which is siteC is selected, it means that for all the peoples from other suburbs to access the facility at town C, a total distance of 5422km must be covered.

### 3.3.4 Calculating the total weighted distance (w4) of site 4(D)

$$w_1 = \sum_{i,j}^{n} h_j d_{ij} y_{ij}$$

Where

$h_1$, $h_2$ and $h_3$ = 100, 85 and 68 respectively

$d_{11}$, $d_{12}$ and $d_{13}$ = 30, 35 and 28 respectively

$\sum_j y_{ij} = x_j = y_{ij} = n = 1$, because each site would be served by all other sites



$$w_4 = \sum_{i,j}^{n} h_1 d_{ij} y_{ij} \qquad i = 1,2,3 \, and \; j = 1,2,3$$

$$w_4 = d_{11}h_1 + d_{12}h_2 + d_{13}h_3$$

$$w_1 = \sum_{ij}^{n} h_i d_{ij} y_{ij} = 30\times100 + 35\times85 + 28\times68$$

$$w_2 = 7879km$$

Therefore if $w_4$ which is site D is selected, it means that for all the peoples from other suburbs to access the facility at town D, a total distance of 7879km must be covered.

Therefore, the minimum of the weighted distances above is 4770km which correspond to site A hence town A is best location for the facility.

The criterion for finding a good location for emergency facilities requires the improvement of the response times. The response time depends on the distance between the emergency facilities and the emergency sites. The aim is to locate these facilities such that the average (total) distance travelled by those who visit or use these facilities is minimized. This measures the effectiveness and efficiency of the emergency facilities. It is clear that people tend to travel to the closet facility regardless of the distance or time travelled. A good way to achieve this is by solving the p-median problem.

The p-median problem consists of determining the location of p emergency facilities to minimize the weighted distance between emergency (demand) points and their closet new emergency facility. A number of authors, such as Berlin et al (1960), Mirchandani (1980), Carson and Batta (1990), Serra and Mirinov (1998), Paluzzi (2004), use the p-median problem solution to locate emergency facilities.

We now present the model for the p-median problem. We start with some notation:

I = {l, ..... m} is the set of demand locations, *j* = {l, .... n} is the candidate sites for facilities, $d_y$ is the shortest distance between location *i* and location *j*, $x_y$ = 1 if the customer at location *i* is allocated to the facility at location j and 0 otherwise, $y_j$ = 1 if a facility is established at location} and 0 otherwise, p is the number of facilities to be established, and a, is the population at the demand node *i*. The mathematical formulation is

$$Min \sum_{i=1}^{m} \sum_{j=1}^{n} a_i d_{ij} X_{ij}, \qquad (1)$$

Subject to

$$\sum_{j \in J} x_{ij} =, \forall i \in I \qquad (2)$$

$$\sum_{j \in J} y_j = p \qquad (3)$$

$$x_{ij} \leq y_j \forall j \in J \qquad (4)$$

$$y_j \in \{0,1\},\ x_{ij} \in \{0,1\} \qquad (5)$$

The objective (1) is to minimize the total distance from customers or clients to their nearest facility. Constraint (2) shows that the demand of each customer or client must be met. Constraint (3) shows the number of facilities to be located is p. constraint (4) shows that customers must be supplied from an open facility, and constraint (5) restricts the variables to 0, 1 values.



Several extensions have been proposed by other authors for the p-median model, which improves its efficiency. (Daskin et al, 1988). Extensions to the p-median problem that account for its stochastic nature have been given by Fitzsimmons (1973), Weaver and Church (1985).

**3.4 SOLUTIONS METHODS FOR THE P-MEDIAN PROBLEM**

The p-median problem is a computationally difficult problem to solve (the problem is NP-hard on general networks). Most solution methods are heuristic based because of the large number of variables and constraints that arise for a medium sized network. The heuristics are based on: genetic algorithms simulated annealing, tabu search, node partitioning, node insertion, node substitution and various hybrids (Hosage and Goodchild (1986), Golden and Skiscism (1986), Glover (1990). Some of these heuristics, together with Lagrangian relaxation, which is one of the most successful exact methods, are briefly discussed below.

### 3.4.1Lagrangian Relaxation

Lagrangian relaxation is based on the principle that removing constraints from a problem makes the problem easier to solve. Generally, Lagrangian relaxation removes a constraint and solves the revised problem, which introduces a penalty for violating the removed constraint. The solution procedure for solving the problem is stated below.

The Lagrangian relaxation for the p-median is given as

$$L(\lambda) = \min \sum_i \sum_j d_{ij} x_{ij} + \sum_i \lambda_i \left( 1 - \sum_j x_{ij} \right)$$

subject to constraints (3) - (5)

The expression

$$r_j = \sum_i \min(0, d_{ij} - \lambda_i)$$

is used to minimize the objective function (6) for the fixed values of the Lagrange multipliers.

We then set

$$x_{ij} = \begin{cases} 1 if \ y_j = 1 and \ d_{ij} - \lambda_i \prec 0 \\ 0 otherwise \end{cases}$$

The lower and upper bounds of the objective function are determined by using the variables of modified and unmodified problems respectively. The next step involves the use of subgradient optimization to update the value of the Lagrange multipliers by using the equation below (Daskin 1995):

$$\lambda_i^{m+1} = \max\left\{ 0, \lambda_i^{\ m} - t^m \left( \sum_j x_{ij}^{\ m} - 1 \right) \right\}$$

$$t^m = \frac{A^m(UB = L^m)}{\sum_i \left\{ \sum_j x_{ij}{}^m - 1 \right\}^2}$$

Where $A^m$ is a constant on the $m^{th}$ iteration, $t^m$ is step size at the $m^{th}$ iteration of the Lagrangian procedure, *UB* is the best (smallest) upper bound on the P-median objective function, $L^m$ is the value of the objective function using the solution obtained from the relaxed problem $x_{ij}{}^m$ is the optimal value of the allocation variable at the $m^{th}$ iteration.



An optimal solution is found if the lower bound is equal to the upper bound. Narula et al., (1977) and Galvao (1980) and Beasley (1993) have successfully applied the subgradient optimization to solve a number of problems. However, for the larger problems tested, the computational time is excessively large.

### 3.4.2 Heuristics

In this section, we start our discussion by observing that it is an easy task to assign a set of *m* clients to *p* facilities *J'* with fixed locations. We just determine

$$d_{ij_i}{}^* = \min\{d_{ij}\}, 1 \le i \le m, j \in J' \qquad (1)$$

And assign customer *i* to facility $_{ji}$*. This gives us a tool for generating possible solutions. The procedure is also useful for determining alternative solutions through exchange of facility locations. We can now use the idea above to describe three simple heuristics, which are competitive with other methods.

### 3.4.3 Myopic Algorithm (MA)

The Myopic heuristic is a very greedy type, which works in the following way. First a facility is located in such a way as to minimize the total cost for all customers. Facilities are then added one by one until p is reached. For this heuristic, the location that gives the minimum cost is selected. The main problem with this approach is that once a facility is selected it says in all subsequent solutions. Consequently, the final solution attained may be far from optimal.



### 3.4.4 Neighborhood Search Heuristic (NS)

Maranzana (1964) proposed this heuristic, which is described as follows. We begin with any set of p facility nodes. The demand nodes are then divided into p subsets and, for each subset, a demand node is allocated to the nearest facility node. The node giving the optimal for each subset is found, which results in a new pattern of facility nodes. This process is repeated until the facility nodes pattern remains the same as that in the previous step.

### 3.4.5 Exchange Heuristic (EH)

This is one of the early heuristics developed by Teitz and Bart (1968) for the *p*-median problem. The heuristic starts by choosing an initial set of *p* number of nodes as the solution, and then a node, which is not in the current solution, is selected to substitute for each of the *p* nodes in turn. We find the objective value in each case and compare the changes in the objective function. The substitution leading to the biggest decrease in the objective function is selected and is exchanged for a node in the current solution. This exchange of nodes results in a new solution configuration and this process continues until there is no further improvement in the objective value.

## 3.5 NEW P-MEDIAN HEURISTICS FOR LOCATION EMERGENCY FACILITIES

### 3.5.1 Reduction Heuristics (RH1, RH2, RRH)

In the previous section, the discussion of some of the heuristics (myopic in particular) for the p- median problem uses all the values of the distance matrix without any modification to solve the problem of extreme values (outliers). In this section, we tried to eliminate the problem outliers by using a reduction technique. Outliers can have a strong influence over the final solution. We also eliminate the uncertainty of choosing a good initial solution in the case of the Neighborhood search and Exchange heuristics by using a specific and efficient way of selecting the initial solution for the three new heuristics.

We obtained the initial solution set for the heuristics by first eliminating the outliers and then sum the columns. We then choose the nodes corresponding to the first p nodes of the totals arrange in ascending order. The aim of the heuristics is to eliminate the outliers before using the data. This will enhance a facility to be located at nodes that are not far away from all customers, so the cost of using these facilities is minimized.

We use the initial solution to reduce the distance matrix by setting the nodes that corresponding to the initial set for both rows and columns to zero. This is done with the assumption that customers at those nodes are not charged to uses the facilities. For *RHI*, the columns of the resulting distance matrix are added and the minimum value is chosen for substituting into the initial solution. We finally choose the set with the minimum objective value. In the case of RH2, all the nodes not in the initial solutions are exchanged one-by-one for the nodes in the initial solution. We then choose the facility set with the minimum objective value as the final solution. However, for both heuristics, we choose the initial set as the final solution, if there is no improvement in the objective value after the swapping procedure.



Motivated by the performance of the two new heuristics (RHl and RH2), we extend RH2 and propose a new heuristic, which we call Repeated Reduction Heuristic (RRH). The process of reducing the matrix is similar to RH2 but, in this case, the reduction is done repeatedly until there is no improvement in the final solution.

We describe the three new reduction heuristics for the p-median problem below.

### 3.5.2 Reduction Heuristic one (RH1)

**Step 1**: Set the number of nodes and facilities to be equal to $n$ and $p$ respectively.

**Step 2**: Arrange the n values for each column in ascending order and delete the last α number of values from each column. Next, let the resulting number of nodes be equal to n' (i.e. n' = n - α where α is $p$ for less than twenty nodes, $2p$ for less than thirty nodes, 3p for less than forty nodes etc).

**Step 3**: Sum the first n' values for each column, arrange the values in ascending order, and choose the first $p$ nodes as the initial set.

**Step 4**: Set the columns and rows corresponding to the initial set to zero and sum the columns of the resulting distance matrix.

**Step 5**: Choose the node or nodes corresponding to the minimum value and substitute for the nodes in the initial set.

**Step 6**: Choose the set corresponding to the minimum objective value after substitution procedure reaches the final solution. Otherwise, go to step 3 and choose the initial set as the final solution if that value is lower.



### 3.5.3 Reduction Heuristic Two (RH2)

For RH2, Step I to 4 is the same as *RH1* and the remaining steps are outlined below.

**Step 5**: Substitute all the nodes not in the initial set with the nodes in the initial set.

**Step 6**: Choose the set corresponding to the minimum value as the final solution if that is lower.

We note that the different swapping procedure lead to an improve final solution as compared with RHI (section 5)

### 3.5.4 Repeated Reduction Heuristic (*RRH*)

In this heuristic, we repeatedly use the final solution of *RH2* as the initial set and use step 4 of *RH1*, and 5 and 6 of *RH2*. We continue this until there is no improvement in the final solution. We note that the repeated reduction incorporated in RRH has increased its performance as compared with RH2.

The proposed heuristics are unique in three different ways. First, the methodology is simple and tractable. Second, the elimination of outliers gives a good initial solution. Third, the determination of swapping a node or nodes and the swapping procedure gives a good final solution. We also note that an improvement procedure can be further introduce to reduce the response time.

**Table 3.5.1 Illustrative Example**

| | | | | |
|---|---|---|---|---|
| 0 | 82 | 37 | 51 | 100 |
| 67 | 0 | 78 | 93 | 97 |
| 74 | 18 | 0 | 20 | 49 |
| 20 | 87 | 27 | 0 | 66 |
| 62 | 37 | 51 | 87 | 0 |



We use the data above to illustrate the three new heuristics. To locate two facilities, we eliminate the two greatest values in each column. Hence, we eliminate 67 and 74 in column 1, 82 and 93 in column 4 and 97 and 100 in column 5. Summing the remaining values and arranging them in ascending order gives the following: 2(55), 3 (64),4 (71), 1 (82) and 5 (115). We choose nodes 2 and 3 as the initial solution for RHl, RH2 and RRH. We, therefore, set rows and columns 2 and 3 of the data to zero and we have the following table.

Table 3.5.2

| | | | | |
|---|---|---|---|---|
| 0 | 0 | 0 | 51 | 100 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 20 | 0 | 0 | 0 | 66 |
| 62 | 0 | 0 | 87 | 0 |

The resulting totals for the non-zero columns give node 1 with the minimum value, so, for RHI, we substitute nodes 2 and 3 with node 1, which results in the possible solution sets of {1,3} and {1 ,2}. We choose {1, 2} since that gives an optimal value of 75.

In the case of RH2 and RRH, we use all the nodes not in the initial solution for substituting for nodes in the initial solution. This gives the possible solution set as follows: {1,2}, {1,3}, {2,4}, {3,4}, {2,5} and {3,5). We choose {l, 2} as the final solution since it gives an optimal value of 75. We continue the same process repeatedly for RRH and now use {1, 2,} as its initial solution, which finally yield {1, 2} as the final solution.

We use the same data to locate three facilities. In this case, we eliminate the three greatest values in each column and sum the values of the remaining columns. This gives the initial solution of 1, 2 and 4. Going through the same process, and setting the rows and columns 1, 2 and 4 to zero, we have the following table.

Table 3.5.3

| | | | | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 49 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 51 | 0 | 0 |

For *RH1*, node 5 has the minimum value, so we substitute node 5 for nodes 1, 2 and 4. Thus, we have the possible sets of {2, 4, 5}; {1, 4, 5} and {l, 2, 5}. We choose {1, 2, 5} as the final solution, which has an optimal value of 38. In the case of RH2 and RRH, we use nodes 3 and 5, which are not in the initial solution for substituting into nodes 1, 2 and 4. This gives the possible solution of {2, 3, 4}, {l, 3, 4}, {l, 2, 3}, {2, 4, 5} {1, 4, 5} and {l, 2, 5}. We finally choose {l, 2, 5} as the final solution, which has an optimal value of 38. For RRH, we again use {l, 2, 5} as the initial solution and continue the process repeatedly. The final solution is {1, 2, 5}. For the Myopic heuristic, we do not eliminate any extreme values, which give the following table.

Table 3.5.4

| | | | | |
|---|---|---|---|---|
| 0 | 82 | 37 | 51 | 100 |
| 67 | 0 | 78 | 93 | 97 |
| 74 | 18 | 0 | 20 | 49 |
| 20 | 87 | 27 | 0 | 66 |
| 62 | 37 | 51 | 87 | 0 |

When we sum all the columns, node 3 has the minimum value of 193. Therefore, one facility is located at node 3. We note that, for the p-median problem, a demand is allocated to the nearest facility. We, therefore, adjust the distance matrix, which gives the following table.

Table 3.3.5

| | | | | |
|---|---|---|---|---|
| 0 | 37 | 37 | 37 | 37 |
| 67 | 0 | 78 | 78 | 37 |
| 0 | 0 | 0 | 0 | 0 |
| 20 | 27 | 27 | 0 | 27 |
| 51 | 37 | 51 | 51 | 0 |

Node 2 has the minimum value of 101 when the columns of the above matrix are added, so, for two facilities, we have nodes 2 and 3 with an objective value of 101.

Similarly, we have adjusted the above matrix after the two facilities were located as shown below.

Table 3.5.6

| | | | | |
|---|---|---|---|---|
| 0 | 37 | 37 | 37 | 37 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 20 | 27 | 27 | 0 | 27 |
| 37 | 37 | 51 | 37 | 0 |

Node 1 has the minimum value when all the columns are added, so, for three facilities, we have nodes 1, 2 and 3 with an objective value of 57. We present, in Table 3.5.7, the results of the example of the three heuristics and Myopic Algorithm. The three heuristics give better results than myopic algorithm.

**Table3.5.7: Results for RH1, RH2, RRH and Myopic**

| P | Solution | | | |
|---|---|---|---|---|
| | RH1, RH2, RRH | | Myopic | |
| | Fac. | Obj. | Fac. | Obj. |
| 2 | {1,2} | 75 | {2, 3} | 101 |
| 3 | {1,2,5} | 38 | {1, 2, 3} | 57 |

**COMPUTATIONAL RESULTS**

The three new heuristics are implemented in C++ and tested on sets of 20 randomly generated data for a [10, 100] matrix with n ranging from 10 to 50 in steps of 10 and p ranging from 2 to 5. The statistic used to measure the quantity of the solution is given as $\frac{H-o}{o} \times 100$ where H is the value given by the implementation of the heuristic and 0 is the optimal value determined by the enumeration method. The value of 0% is considered to be optimal. A small deviation results in a better solution than a large deviation.

Table 3.5.7 gives the performance of the three new heuristics for location 2, 3, 4 and 5 facilities. In Table 3.5.8 below, we have the average values for using ten, twenty, thirty, forty and fifty nodes.

**Table 3.5.8 Average Values for the New Heuristics**

| Number of Nodes (n) | Average Values (%) | | |
|---|---|---|---|
| | *RHI* | *RH2* | *RRH* |
| 10 | 2.22 | 0.79 | 0.32 |
| 20 | 4.87 | 1.96 | 0.72 |
| 30 | 4.38 | 1.65 | 0.66 |
| 40 | 4.60 | 2.27 | 0.87 |
| 50 | 3.04 | 1.00 | 0.49 |



From Table 3.5.8, the average values for RH1 ranges from 2.22% to 4.87%, RH2 ranges from 0.79% to 2.27% and RRH ranges from 0.32% to 0.87%. The values of RRH are almost optimal, which is good rise to acceptable response times.

### 3.6 Comparison of the Repeated Reduction Heuristic (RRH) and some P-Median Heuristics

Motivated by the performance of RRH, we compare the heuristic using data from the literature. We compare this heuristic using the 55 node network data (Swain 1971). The data are given in Colome et al. (2003). The data has been used by authors such as Daskin (1982, 1983), Colome et al. (2003) for testing location problems. The 55-node data set represents 55 communities in the Washington D.C. (USA) area. Demands for each node were generated in pseudo-random manner with most large demands at the centre of the region and most small demand at the outer region.

We compare RRH with the Myopic algorithm (MA), Exchange heuristic (EH) and Neighborhood search (NS) heuristic. We coded the Repeated Reduction Heuristic (RRH) in C++ while the results of the other heuristics were obtained using the SIT ATION software (Daskin, 1995). The solutions of the heuristics were compared with the optimal solutions, which were determined using Lagrangian Relaxation (Daskin, 1995).

**Table 3.5.9: Comparison Performance of RRH and Existing Heuristic using 55-node Data**

| Number of Facilities (P) | MA | NS | EH | RRH |
|---|---|---|---|---|
| | $\frac{H-o}{o}\times 100$ | | | |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 2.3 |
| 4 | 4.0 | 0 | 0 | 0 |
| 5 | 3.5 | 3.5 | 0 | 0 |
| 6 | 5.3 | 5.3 | 2.4 | 2.4 |
| 7 | 6.9 | 3.1 | 0 | 0 |
| 8 | 7.7 | 0.2 | 1.4 | 0 |
| 9 | 7.0 | 0.6 | 0.4 | 0 |

Figure 1: Comparison Performance of Heuristic using 55-node Data

Table 3 and Figure1 show the performance of the new heuristics and the existing ones for the 55- node literature test problem. From Table 2 and Figure 1, the performance measured in terms of the number of optimal solutions gives the rank (from the best to the worst) of RRH, Exchange heuristic, Neighborhood Search heuristic and Myopic heuristic. The new heuristic RRH performs better in the location of all facilities with the exception of the location three and six facilities.

# CHAPTER FOUR

# DATA ANALYSIS AND RESULTS

## 4.1 Data Collection

This chapter is concerned with collection of data and analysis of the data obtained. The data on population of the selected towns and villages in the Ketu south district were obtained from the Ketu south district assembly and the district councils respectively. Also, the distances between town and villages in the Ketu-south district were collected from the district statistical service department.

The figure 4.1 belowillustrates the map of Ketu south district. On the map, the yellow indicates the dried/and areas of the district where people live. The blue colour represents areas of the district which are covered by the lagoon.

In addition, the greycolour on the map shows the areas in the district which are liable to flood. On the other hand, the orange colour illustrate the district capital, major towns, towns and villages in the district, the red lines on the map indicate major roads whereas the dark lines also show the secondary roads in the district.

Conclusively, the thin blue lines also show the rivers and streams in the Ketu south district. All these are shown on the figure 4.1 below.

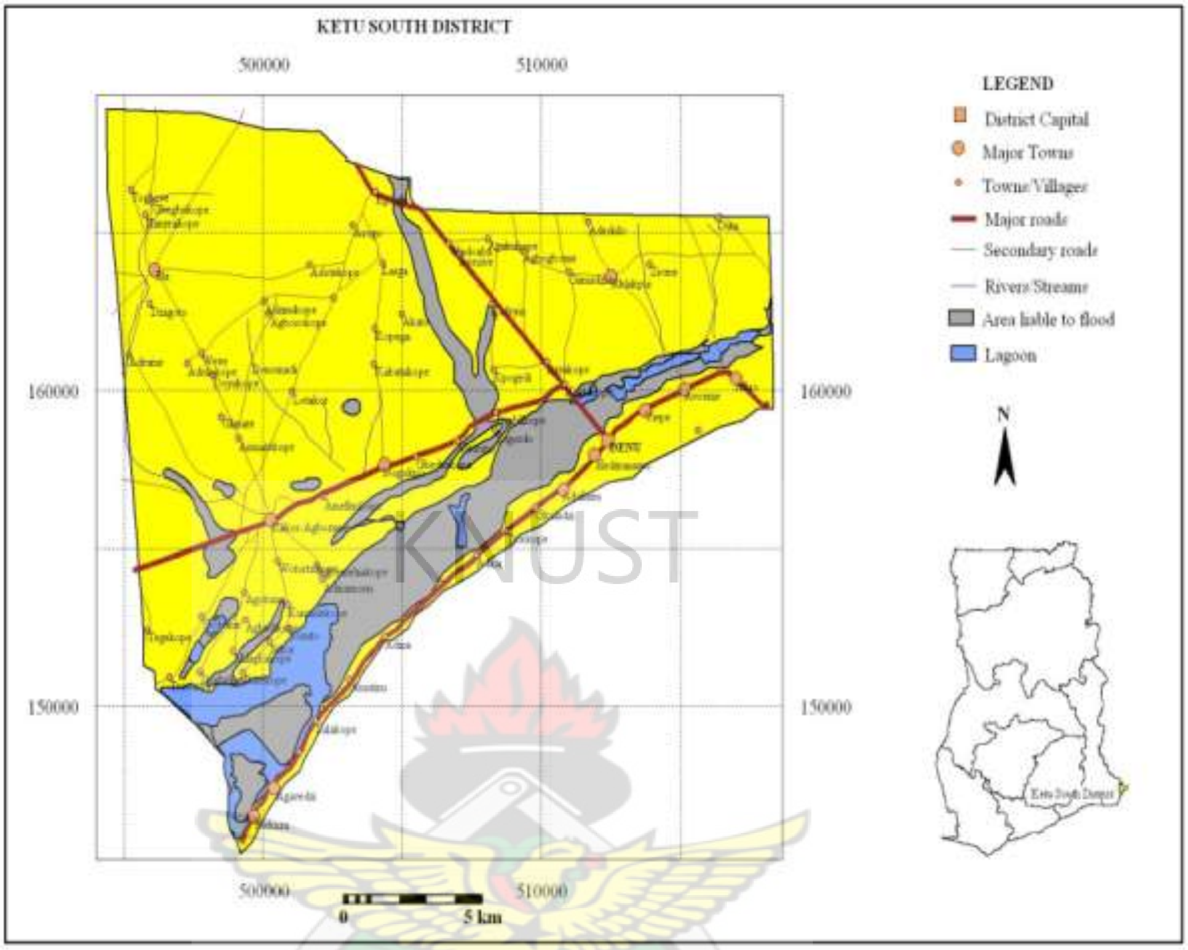



Figure 4.1: Ketu South District

## 4.2 DATA ANALYSIS

The road links connecting the various communities is depicted in Figure 4.2

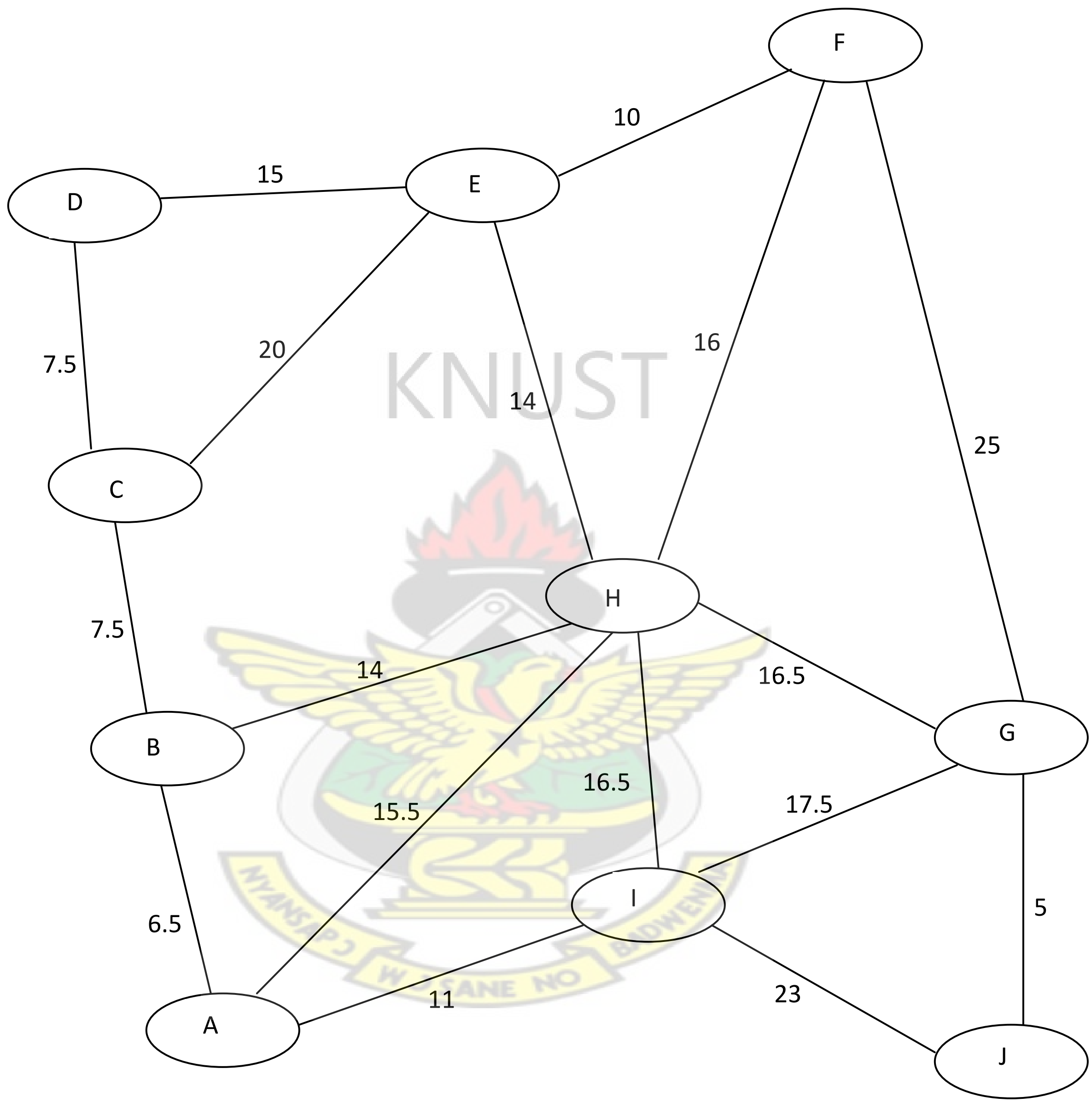



**Figure 4.2: Road links of the ten (10) communities of Ketu South District**

The road distances depicted in Figure 4.2 is shown in Table 4.1

**Table 4.1: Road distances of the communities in kilometers**

| To / From | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| A | 0 | 6.5 | ∞ | ∞ | ∞ | ∞ | ∞ | 15.5 | 11 | ∞ |
| B | 6.5 | 0 | 7.5 | ∞ | ∞ | ∞ | ∞ | 14 | ∞ | ∞ |
| C | ∞ | 7.5 | 0 | 7.5 | 20 | ∞ | ∞ | ∞ | ∞ | ∞ |
| D | ∞ | ∞ | 7.5 | 0 | 15 | ∞ | ∞ | ∞ | ∞ | ∞ |
| E | ∞ | ∞ | 20 | 15 | 0 | 10 | ∞ | 14 | ∞ | ∞ |
| F | ∞ | ∞ | ∞ | ∞ | 10 | 0 | 25 | 16 | ∞ | ∞ |
| G | ∞ | ∞ | ∞ | ∞ | ∞ | 25 | 0 | 16.5 | 17.5 | 5 |
| H | 15.5 | 14 | ∞ | ∞ | 14 | 16 | 16.5 | 0 | 16.5 | ∞ |
| I | 11 | ∞ | ∞ | ∞ | ∞ | ∞ | 17.5 | 16.5 | 0 | 23 |
| J | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 5 | ∞ | 23 | 0 |

The road links together with their respective population or demands ( $h_i$ ) of the communities is also depicted in Figure 4.3

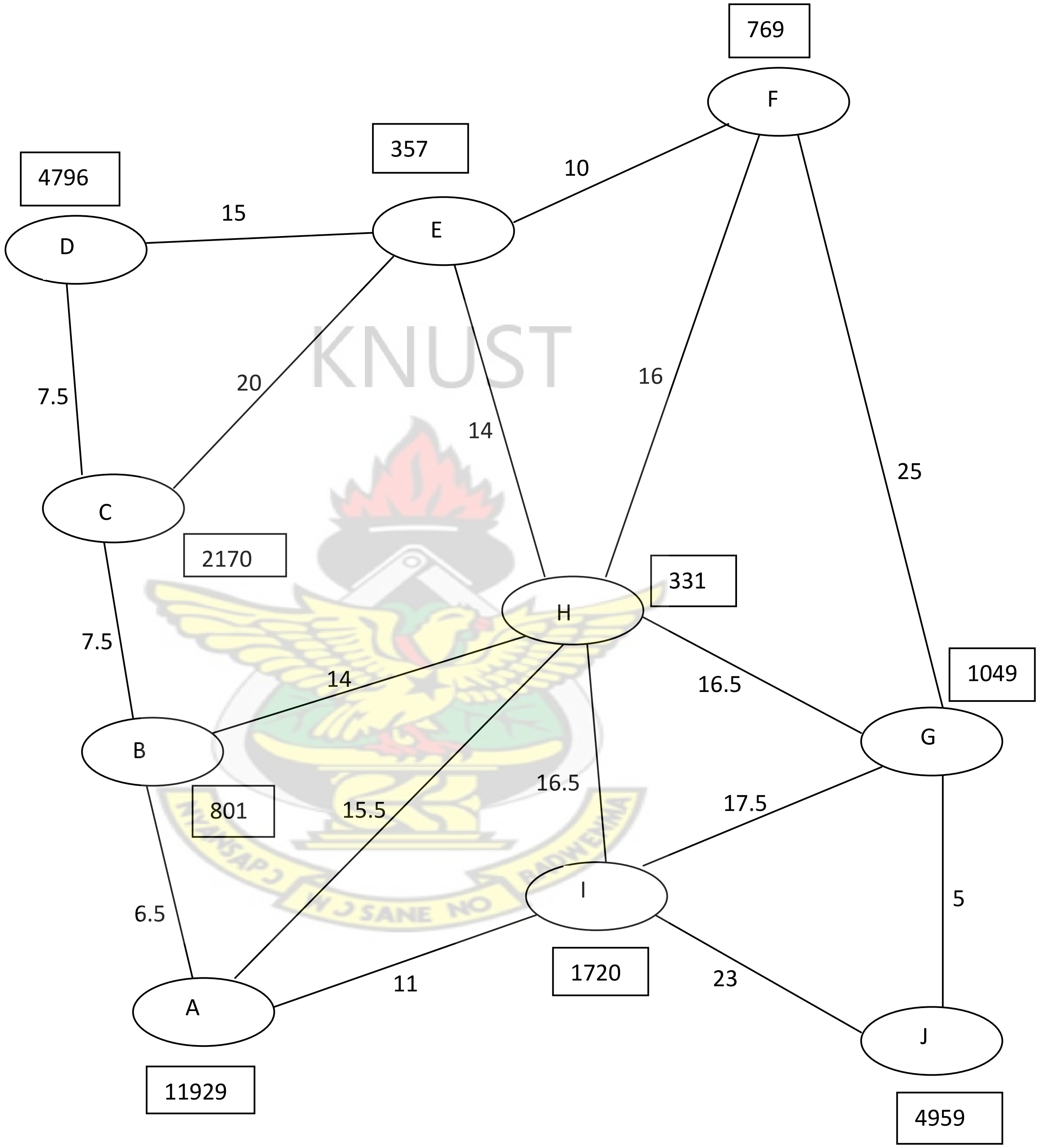



**Figure 4.3: Nodes and their respective populations**

The various demand nodes ($h_i$) and road distances in Figure 4.3 is shown in Table 4.2.

**Table 4.2: Demand nodes and road distances of the communities in kilometers**

| $h_i$ | From \ To | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 11929 | A | 0 | 6.5 | ∞ | ∞ | ∞ | ∞ | ∞ | 15.5 | 11 | ∞ |
| 801 | B | 6.5 | 0 | 7.5 | ∞ | ∞ | ∞ | ∞ | 14 | ∞ | ∞ |
| 2170 | C | ∞ | 7.5 | 0 | 7.5 | 20 | ∞ | ∞ | ∞ | ∞ | ∞ |
| 4796 | D | ∞ | ∞ | 7.5 | 0 | 15 | ∞ | ∞ | ∞ | ∞ | ∞ |
| 357 | E | ∞ | ∞ | 20 | 15 | 0 | 10 | ∞ | 14 | ∞ | ∞ |
| 769 | F | ∞ | ∞ | ∞ | ∞ | 10 | 0 | 25 | 16 | ∞ | ∞ |
| 1049 | G | ∞ | ∞ | ∞ | ∞ | ∞ | 25 | 0 | 16.5 | 17.5 | 5 |
| 331 | H | 15.5 | 14 | ∞ | ∞ | 14 | 16 | 16.5 | 0 | 16.5 | ∞ |
| 1720 | I | 11 | ∞ | ∞ | ∞ | ∞ | ∞ | 17.5 | 16.5 | 0 | 23 |
| 4959 | J | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 5 | ∞ | 23 | 0 |

**Table 4.3: selected towns and their respective population**

| Towns | Node | Population |
|---|---|---|
| Klikor - Agbozume | A | 11929 |
| Amutinu | B | 801 |
| Weve | C | 2170 |
| Ehi | D | 4796 |
| Adoteykope | E | 357 |
| Kpoglu | F | 769 |
| Hatsukope | G | 1049 |
| Kabutukope | H | 331 |
| Nogokpo | I | 1720 |
| Denu | J | 4959 |



### 4.3 MODEL FORMULATION

The p-median problem involves placing p facilities so that the total user cost or distance to travel to one of those facilities is minimized. The model can be represented mathematically as follows.

*Let* $w_i$ = *weighted distance for site i*

*i = index for selected site*

*j = index of site for potential facility placement*

*n = number of site(s) to locate facility*

$h_i$ = *demand at node i*

$d_{ij}$ = *distance between node i and node j*

Minimize

$$w_i = \sum_{i,j}^{n} h_i d_{ij} y_{ij} \qquad (1)$$

Subject to

$$\sum_j x_j = p \qquad j = 1, 2\ldots.. \qquad (2)$$

$$\sum_j y_{ij} = 1 \quad \forall_i = \qquad (3)$$



$$j = 1, 2\ldots\ldots..$$

$$x_j = \{0,l\} \qquad (4)$$

$$y_{ij} = \{0,l\} \qquad (5)$$

$x_j$ ={l,0}, where 1 implies a potential facility is located at site j and 0 implies no facility is located at site j

$y_{ij}$={l,0}, where 1 implies site *i* is served by a facility at site *j* and 0 implies site *i* is not served by a facility at site *j*.

The objective function (1) seeks to minimize the total distance covered by other people in the other sites to access the facilities at the selected sites.

$d_{ij}$ = the distance between the other towns and the selected towns

$h_i$ = the populations of various towns

Constraint (2) limits the number of facilities to be located p = 2. Constraint (3) ensures that each node *i*is served by one facility.

Constraints (4) and (5) define the decision variables X and Y. In one centre p-median, both constraints (3) and (4) are equal to one (1).

## 4.4 SHORTEST PATHS BY FLOYD – WARSHALL ALGORITHM

We use Floyd's algorithm to obtain the shortest distance matrix $d(i,j)$ of Figure 4.1. This is shown in Table 4.4

**Table 4.4: Shortest distance matrix $d(i,j)$ of the communities in kilometers**

| To / From | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| A | 0 | 6.5 | 14 | 21.5 | 29.5 | 31.5 | 28.5 | 15.5 | 11 | 33.5 |
| B | 6.5 | 0 | 7.5 | 15 | 27.5 | 30 | 30.5 | 14 | 17.5 | 35.5 |
| C | 14 | 7.5 | 0 | 7.5 | 20 | 30 | 38 | 21.5 | 25 | 43 |
| D | 21.5 | 15 | 7.5 | 0 | 15 | 25 | 45.5 | 29 | 32.5 | 50.5 |
| E | 29.5 | 27.5 | 20 | 15 | 0 | 10 | 30.5 | 14 | 30.5 | 35.5 |
| F | 31.5 | 30 | 30 | 25 | 10 | 0 | 25 | 16 | 32.5 | 30 |
| G | 28.5 | 30.5 | 38 | 45.5 | 30.5 | 25 | 0 | 16.5 | 17.5 | 5 |
| H | 15.5 | 14 | 21.5 | 29 | 14 | 16 | 16.5 | 0 | 16.5 | 21.5 |
| I | 11 | 17.5 | 25 | 32.5 | 30.5 | 32.5 | 17.5 | 16.5 | 0 | 22.5 |
| J | 33.5 | 35.5 | 43 | 50.5 | 35.5 | 30 | 5 | 21.5 | 22.5 | 0 |

### 4.5 Steps in Reduction Heuristics

#### 4.5.1 Reduction Heuristics One (RH1)

**Step 1**: Set the number of nodes and facilities to be equal to *n* and *p* respectively.

**Step 2**: Arrange the n values for each column in ascending order and delete the last a number of values from each column. Next, let the resulting number of nodes be equal to n' (i.e. n' = n - a where a is*p* for less than twenty nodes, 2*p* for less than thirty nodes, 3p for less than forty nodes etc).

**Step 3**: Sum the first n' values for each column, arrange the values in ascending order, and choose the first *p* nodes as the initial set.

**Step 4**: Set the columns and rows corresponding to the initial set to zero and sum the columns of the resulting distance matrix.

**Step 5**: Choose the node or nodes corresponding to the minimum value and substitute for the nodes in the initial set.

**Step 6**: Choose the set corresponding to the objective value after substitution procedure reaches the final solution. Otherwise, go to step 3 and choose the initial set as the final solution if that value is lower.

Using RRH we shall use step 6 as the initial set and use steps 4, 5 and 6. We continue this until there is no improvement in the final solution.

#### 4.5.2 Reduction Heuristic Two (RH2)

For RH2, Step I to 4 is the same as *RH1* and the remaining steps are outlined below.

**Step 5**: Substitute all the nodes not in the initial set with the nodes in the initial set.

**Step 6**: Choose the set corresponding to the minimum value as the final solution if that is lower.

We note that the different swapping procedure lead to an improve final solution as compared with RHI (section 5)

### 4.5.3 Repeated Reduction Heuristic (*RRH*)

In this heuristic, we repeatedly use the final solution of *RH2* as the initial set and use step 4 of *RH1*, and 5 and 6 of *RH2*. We continue this until there is no improvement in the final solution. We note that the repeated reduction incorporated in RRH has increased its performance as compared with RH2.

### 4.5.4 SOLUTION BY REDUCTION HEURISTIC (RH1, RH2, RRH)

The shortest path distance $d(i,j)$ and demand node $h_i$ is shown in Table 4.5

**Table 4.5: Shortest distance matrix $d(i,j)$ of the communities and their respective demands $h_i$**

| $h_i$ | From \ To | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 11929 | A | 0 | 6.5 | 14 | 21.5 | 29.5 | 31.5 | 28.5 | 15.5 | 11 | 33.5 |
| 801 | B | 6.5 | 0 | 7.5 | 15 | 27.5 | 30 | 30.5 | 14 | 17.5 | 35.5 |
| 2170 | C | 14 | 7.5 | 0 | 7.5 | 20 | 30 | 38 | 21.5 | 25 | 43 |
| 4796 | D | 21.5 | 15 | 7.5 | 0 | 15 | 25 | 45.5 | 29 | 32.5 | 50.5 |
| 357 | E | 29.5 | 27.5 | 20 | 15 | 0 | 10 | 30.5 | 14 | 30.5 | 35.5 |
| 769 | F | 31.5 | 30 | 30 | 25 | 10 | 0 | 25 | 16 | 32.5 | 30 |
| 1049 | G | 28.5 | 30.5 | 38 | 45.5 | 30.5 | 25 | 0 | 16.5 | 17.5 | 5 |
| 331 | H | 15.5 | 14 | 21.5 | 29 | 14 | 16 | 16.5 | 0 | 16.5 | 21.5 |
| 1720 | I | 11 | 17.5 | 25 | 32.5 | 30.5 | 32.5 | 17.5 | 16.5 | 0 | 22.5 |
| 4959 | J | 33.5 | 35.5 | 43 | 50.5 | 35.5 | 30 | 5 | 21.5 | 22.5 | 0 |

We find demand time distance $[h_i * d(i,j)]$. Thus we multiply row A by $h_A$ and row B by $h_B$, row C by $h_C$ and so on. This is shown in Table 4.6 below

**Table 4.6:** $[\ _i * \boldsymbol{d}(\boldsymbol{i},\boldsymbol{j})]$.

| To / From | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| A | 0 | 77538.5 | 167006 | 256473.5 | 351905.5 | 375763.5 | 339976.5 | 184899.5 | 131219 | 405586 |
| B | 5206.5 | 0 | 6007.5 | 12015 | 22027.5 | 24030 | 24430.5. | 11214 | 14017.5 | 32440.5 |
| C | 30380 | 16275 | 0 | 16275 | 43400 | 65100 | 82460 | 46655 | 54250 | 104160 |
| D | 103114 | 71940 | 35970 | 0 | 71940 | 119900 | 218218 | 139084 | 155870 | 266178 |
| E | 10531.5 | 9817.5 | 7140 | 5355 | 0 | 3570 | 10888.5 | 4998 | 10888.5 | 24276 |
| F | 24223.5 | 23070 | 23070 | 19225 | 7690 | 0 | 19225 | 12304 | 24992.5 | 23070 |
| G | 29896.5 | 31994.5 | 39862 | 47729.5 | 31994.5 | 26225 | 0 | 17308.5 | 18357.5 | 5245 |
| H | 5130.5 | 4634 | 7116.5 | 9599 | 4634 | 5296 | 5461.5 | 0 | 5461.5 | 7116.5 |
| I | 18920 | 30100 | 43000 | 55900 | 52460 | 55900 | 30100 | 28380 | 0 | 38700 |
| J | 168606 | 200839.5 | 238032 | 275224.5 | 337212 | 148770 | 24795 | 106618.5 | 111577.5 | 0 |
| Total | 396008.5 | 466209 | 567204 | 697796.5 | 923263.5 | 824554.5 | 755555 | 551461.5 | 526634 | 906772 |

We wish to locate two facilities, by reduction heuristic in chapter 3, we eliminate the twogreatest values in each column in Table 4.6. Hence we eliminate $103114\ and\ 168606$ in column A, 77538.5 and $200839.5$ in column B, $167006\ and\ 238032$ in column C, $256473.52\ and\ 275224.5$ in column D, $351905.5\ and\ 337212$ in column E, $375763.5\ and\ 148770$ in column F, $339976.5\ and\ 218218$ in column G, 184899.5 and $139084$ in column H, $155870$ and $131219$ in column I, $405586$ and $266178$ in column J. This is shown in Table 4.7.

**Table 4.7: Elimination of outliers**

| To / From | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| A | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| B | 5206.5 | 0 | 6007.5 | 12015 | 22027.5 | 24030 | 24430.5 | 11214 | 14017.5 | 32440.5 |
| C | 30380 | 16275 | 0 | 16275 | 43400 | 65100 | 82460 | 46655 | 54250 | 104160 |
| D | 0 | 71940 | 35970 | 0 | 71940 | 119900 | 0 | 0 | 0 | 0 |
| E | 10531.5 | 9817.5 | 7140 | 5355 | 0 | 3570 | 10888.5 | 4998 | 10888.5 | 24276 |
| F | 24223.5 | 23070 | 23070 | 19225 | 7690 | 0 | 19225 | 12304 | 24992.5 | 23070 |
| G | 29896.5 | 31994.5 | 39862 | 47729.5 | 31994.5 | 26225 | 0 | 17308.5 | 18357.5 | 5245 |
| H | 5130.5 | 4634 | 7116.5 | 9599 | 4634 | 5296 | 5461.5 | 0 | 5461.5 | 7116.5 |
| I | 18920 | 30100 | 43000 | 55900 | 52460 | 55900 | 30100 | 28380 | 0 | 38700 |
| J | 0 | 0 | 0 | 0 | 0 | 0 | 24795 | 106618.5 | 111577.5 | 0 |
| Total | 124,288.5 | 187,831 | 162,166 | 166,098.5 | 234,146 | 300,021 | 197,360.5 | 227,478 | 239,545 | 235,008 |

Summing the remaining values and arranging them in ascending order is shown in table below

| Nodes | A | C | D | B | G | H | E | J | I | F |
|---|---|---|---|---|---|---|---|---|---|---|
| Values | 124288.5 | 162166 | 166098.5 | 187831 | 197360.5 | 227478 | 234146 | 235008 | 239545 | 300021 |

We choose nodes A and C as initial solution for RH1, RH2 and RRH. We therefore set rows and columns of nodes A and C of the data in Table 4.6 to zero (0). This is shown in Table 4.8

**Table 4.8: Setting rows and columns of node A and C to zero**

| To / From | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| A | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| B | 0 | 0 | 0 | 12015 | 22027.5 | 24030 | 24430.5 | 11214 | 14017.5 | 32440.5 |
| C | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| D | 0 | 71940 | 0 | 0 | 71940 | 119900 | 218218 | 139084 | 155870 | 266178 |
| E | 0 | 9817.5 | 0 | 5355 | 0 | 3570 | 10888.5 | 4998 | 10888.5 | 24276 |
| F | 0 | 23070 | 0 | 19225 | 7690 | 0 | 19225 | 12304 | 24992.5 | 23070 |
| G | 0 | 31994.5 | 0 | 47729.5 | 31994.5 | 26225 | 0 | 17308.5 | 18357.5 | 5245 |
| H | 0 | 4634 | 0 | 9599 | 4634 | 5296 | 5461.5 | 0 | 5461.5 | 7116.5 |
| I | 0 | 30100 | 0 | 55900 | 52460 | 55900 | 30100 | 28380 | 0 | 38700 |
| J | 0 | 200839.5 | 0 | 275224.5 | 337212 | 148770 | 24795 | 106618.5 | 111577.5 | 0 |
| Total | 0 | 372,395.5 | 0 | 425,048 | 527,958 | 383,691 | 333,118.5 | 319,906.5 | 341,165 | 397026 |

From Table 4.8, the resulting totals for the non – zero columns give node H with theminimum value of 319,906.5, so for RH1, we substitute node A and C with minimum value of node H, which result in the possible solution set of $\{H, A\}$ $and$ $\{H, C\}$ gives an objective values of $334{,}819.5$ and $378592.5$ respectively. We choose $\{H, A\}$ since that gives an optimal value of$334{,}819.5$.

In the case of RH2 we compare each of the node in the initial solution with all the nodes which are not in the initial solution. This gives the possible solution set as follows: $\{H, A\}, \{H, C\}, \{A, B\}, \{A, D\}, \{A, E\}, \{A, F\}, \{A, G\}, \{A, I\}, \{A, J\}, \{C, B\}, \{C, D\}, \{C, E\}, \{C, F\}, \{C, G\}$, $\{C, I\}, and\ \{C, J\}$. We have the following objective values;

$334{,}819.5, 378592.5, 343159, 268614.5, 337273, 358102, 3324065, 334819.5, 361277, 364661.5,$

corresponding to the solution set above respectively. We chose $\{A, D\}$ since that gives an optimal value of $268614.5$. Comparing the RH1 and RH2 we realized that the different swapping procedure leads to an improved final solution.

The third heuristic RRH is an extension of RH2. In the case of RRH, we choose the final solution of RH2 as the initial solution. In this case $\{A, D\}$is the initial solution. We therefore set rows and columns of nodes A and D of the data in Table 4.8 to zero (0). This is shown in Table 4.9.

**Table 4.9: Solution by RRH**

| To / From | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| A | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 131219 | 405586 |
| B | 0 | 0 | 6007.5 | 0 | 22027.5 | 24030 | 24430.5. | 11214 | 14017.5 | 32440.5 |
| C | 0 | 16275 | 0 | 0 | 43400 | 65100 | 82460 | 46655 | 54250 | 104160 |
| D | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 266178 |
| E | 0 | 9817.5 | 7140 | 0 | 0 | 3570 | 10888.5 | 4998 | 10888.5 | 24276 |
| F | 0 | 23070 | 23070 | 0 | 7690 | 0 | 19225 | 12304 | 24992.5 | 23070 |
| G | 0 | 31994.5 | 39862 | 0 | 31994.5 | 26225 | 0 | 17308.5 | 18357.5 | 5245 |
| H | 0 | 4634 | 7116.5 | 0 | 4634 | 5296 | 5461.5 | 0 | 5461.5 | 7116.5 |
| I | 0 | 30100 | 43000 | 0 | 52460 | 55900 | 30100 | 28380 | 0 | 38700 |
| J | 0 | 200839.5 | 238032 | 0 | 337212 | 148770 | 24795 | 106618.5 | 111577.5 | 0 |
| Total | 0 | 316730.5 | 364228 | 0 | 499418 | 328891 | 197360.5 | 227478 | 239545 | 235008 |

From Table 4.9 the resulting totals for the non – zero columns give node G with the minimum value of 197360.5, we substitute nodes A and D with minimum value of node G and all other nodes which are not in the initial solution set. This result in the possible solution set of {A,D},

$\{G,A\},\{G,D\}$ , $\{A,B\},\{A,C\},\{A,E\},\{A,F\},\{A,H\},\{A,I\},\{A,J\},\{D,B\},\{D,C\},\{D,E\},\{D,F\},\{D,H\},\{D,$

gives an objective values

of $268614.5, 332406.5, 45303$ $343159$,$293939.5, 337273, 358102, 332406.5,$

$$334819.5, 361277, 364661.5, 385961.5, 525604,$$

$752198.5, 643814.5, 453203, 381997.5, 319485.5, and$ $509517.5$ Respectively. We choose {A, D} as our final solution since it gives an optimal solution of $268614.5$.



## 4.6 DISCUSSION

**4.6.1**Summary ofResults andFindings

The following results were obtained from the solution by reduction heuristics $RH_1$, $RH_2$ and repeated reduction heuristics (RRH).

This summarized in table 4.10 below

**Table 4.10 solution by $RH_1$, $RH_2$ and RRH**

| | Facility | Objective function |
|---|---|---|
| $RH_1$ | {A, H} | 334819.5 |
| $RH_2$ | {G, A} | 268614.5 |
| RRH | {A, D} | 268614.5 |

In locating two hospitals facilities in Ketu south district results from the $RH_1$, $RH_2$and RRH, were shown in the table4.10. Node {A, H} gave a minimum objective function of 334819.5 from $RH_1$. This implies that when using $RH_1$ the two facilities should be sited at klikor-Agbozume with a population of 11929 and kabutukope with population of 331 respectively. Also from Table 4.10 both $RH_2$ and RRH gave and improve optimal solution of 268614.5 occuring at node {A, D}. This means that the two facilities should be sited at klikor-Agbozume and Ehi with a total population of 11929 and 4796.

Again, we used repeated reduction heuristics RRH algorithm to determine the optimal location of the two hospitals to be sited at Ketu south district when considering 10 towns and villages in the district. According to the model (RRH), the two hospitals should be sited at Klikor-Agbozume and Ehi with an overall total 11729 and 4796 respectively. The total population of the ten (10) selected communities is 28881.

# CHAPTER FIVE

# CONCLUSION AND RECOMMENDATION

## 5.1 CONCLUSION

The main objectives of locating this two facilities is to optimally locate a central site in the Ketu south district which will be closer to all the surrounding villages and towns in the district which will cut down cost and distance of travelling to access the facilities (hospitals).



In the analysis, we use RRH to solve the problem which gives the improved optimal solution of 268614.5 occurring at node (A, D). This implies that the two facilities should be sited at Klikor-Agbozume and Ehi.

Again, the two hospitals should be sited at Kilkor-Agbozume and Ehi with an overall total population of 11929 and 4796 respectively. The population of the ten (10) communities selected is 28881.

## 5.2 RECOMMENDATION

This thesis seeks to find a common places or locations (sites) to locate two facilities (hospitals)inKetu south district which will minimize the cost and the distances which the people need to travel to optimally access the facilities.

1. Based on the findings, I recommend that two facilities (hospitals) should situate at Kilkor-Agbozume and Ehi.

2. Other researchers should use the same methods to locate different facilities to ascertain the efficiency of the RRH algorithm.
3. The government should site facilities using facility location models rather than no political grounds.

Based on the finding I recommend that in future the two facilities (hospitals) should located at Kilkor-Agbozume and Ehi which are seen to be the best site for the location of the two facilities.

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## APPENDICES

### SOLUTION BY MYOPIC ALGORITHM

The shortest path distance and demand node is shown in Table 4.5

Table 4.5: Shortest distance matrix of the communities and their respective demands

| | From<br>To | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 11929 | A | 0 | 6.5 | | | | | | 15.5 | 11 | |
| 801 | B | 6.5 | 0 | 7.5 | | | | | 14 | | |
| 2170 | C | | 7.5 | 0 | 7.5 | 20 | | | | | |
| 4796 | D | | | 7.5 | 0 | 15 | | | | | |
| 357 | E | | | 20 | 15 | 0 | 10 | | 14 | | |
| 769 | F | | | | | 10 | 0 | 25 | 16 | | |
| 1049 | G | | | | | | 25 | 0 | 16.5 | 17.5 | 5 |
| 331 | H | 15.5 | 14 | | | 14 | 16 | 16.5 | 0 | 16.5 | |
| 1720 | I | 11 | | | | | | 17.5 | 16.5 | 0 | 22.5 |
| 4959 | J | | | | | | | 5 | | 22.5 | 0 |

We use myopic algorithm to find first four medians. We find demand time distance. Thus we multiply row A by and row B by, row C by and so on. By summing the entries in each column we obtain the values of. The smallest value gives the solution to the 1 – median problem. This is shown in Table 4.6.

**Comparing the total results in Table 4.6 the minimum value is 466209 and it occurs** at node B. Hence to locate one hospital in the district, location B is the optimal site. We compute for each node location pair to get the location of the second facility. Table 4.7 shows the column total corresponding to.

**Comparing the total results in Table 4.7 the minimum value is and it occurs** at node A. Hence the second hospital in the district would be located at site A. To locate the third facility we compute for each node location pair. Table 4.8 shows the column total corresponding to.

**Comparing the total results in Table 4.8 the minimum value is and it occurs** at node J. Hence the third hospital in the district would be located at site J. To locate the Fourth facility we compute for each node location pair. Table 4.9 shows the column total corresponding to.

Table 4.6: The first myopic median $[h_i * d(i,j)]$.

| To / From | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| A | 0 | 77538.5 | 167006 | ∞ | 351905.5 | 375763.5 | 339976.5 | 184899.5 | 131219 | 405586 |
| B | 5206.5 | 0 | 6007.5 | 12015 | 22027.5 | 24030 | 24430.5 | 11214 | 14017.5 | 32440.5 |
| C | 30380 | 16275 | 0 | 16275 | 43400 | 65100 | 82460 | 46655 | 54250 | 104160 |
| D | ∞ | 71940 | 35970 | 0 | 71940 | 119900 | 218218 | 139084 | 155870 | 266178 |
| E | 10531.5 | 9817.5 | 7140 | 5355 | 0 | 3570 | 10888.5 | 4998 | 10888.5 | 24276 |
| F | 24223.5 | 23070 | 23070 | 19225 | 7690 | 0 | 19225 | 12304 | 24992.5 | 23070 |
| G | 29896.5 | 31994.5 | 39862 | 47729.5 | 31994.5 | 26225 | 0 | 17308.5 | 18357.5 | 5245 |
| H | 5130.5 | 4634 | 7116.5 | 9599 | 4634 | 5296 | 5461.5 | 0 | 5461.5 | 7116.5 |
| I | 18920 | 30100 | 43000 | 55900 | 52460 | 55900 | 30100 | 28380 | 0 | 38700 |
| J | 168606 | 200839.5 | 238032 | 275224.5 | 337212 | 148770 | 24795 | 106618.5 | 111577.5 | 0 |
| Total | ∞ | 466209 | 567204 | ∞ | 923263.5 | 824554.5 | 755555 | 551461.5 | 526634 | 906772 |

Table 4.7: The Second Myopic median $[h_i * \min\{d(i,j); d(i,B)\}]$

| To / From | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| A | 0 | 77538.5 | 77538.5 | 77538.5 | 77538.5 | 77538.5 | 77538.5 | 77538.5 | 77538.5 | 77538.5 |
| B | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | 16275 | 16275 | 0 | 16275 | 16275 | 16275 | 16275 | 16275 | 16275 | 16275 |
| D | 71940 | 71940 | 35970 | 0 | 71940 | 71940 | 71940 | 71940 | 71940 | 71940 |
| E | 9817.5 | 9817.5 | 7140 | 5355 | 0 | 3570 | 9817.5 | 4998 | 9817.5 | 9817.5 |
| F | 23070 | 23070 | 23070 | 19225 | 7690 | 0 | 19225 | 12304 | 23070 | 23070 |
| G | 29896.5 | 31994.5 | 31994.5 | 31994.5 | 31994.5 | 26225 | 0 | 17308.5 | 18357.5 | 5245 |
| H | 4634 | 4634 | 4634 | 4634 | 4634 | 4634 | 4634 | 0 | 4634 | 4634 |
| I | 18920 | 30100 | 30100 | 30100 | 30100 | 30100 | 30100 | 28380 | 0 | 30100 |
| J | 168606 | 200839.5 | 200839.5 | 275224.5 | 200839.5 | 148770 | 24795 | 106618.5 | 111577.5 | 0 |
| Total | $\boxed{343159}$ | 466209 | 567204 | 385961.5 | 1138858 | 1076899 | 952171.5 | 1033209 | 1031056.5 | 936466.5 |

Table 4.8: The third myopic median $[h_i * \min\{d(i,j); d(i,B); d(i,A)\}]$

| To / From | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| A | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| B | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | 16275 | 16275 | 0 | 16275 | 16275 | 16275 | 16275 | 16275 | 16275 | 16275 |
| D | 71940 | 71940 | 35970 | 0 | 71940 | 71940 | 71940 | 71940 | 71940 | 71940 |
| E | 9817.5 | 9817.5 | 7140 | 5355 | 0 | 3570 | 9817.5 | 4998 | 9817.5 | 9817.5 |
| F | 23070 | 23070 | 23070 | 19225 | 7690 | 0 | 19225 | 12304 | 23070 | 23070 |
| G | 29896.5 | 29896.5 | 29896.5 | 29896.5 | 29896.5 | 26225 | 0 | 17308.5 | 18357.5 | 5245 |
| H | 4634 | 4634 | 4634 | 4634 | 4634 | 4634 | 4634 | 0 | 4634 | 4634 |
| I | 18920 | 18920 | 18920 | 18920 | 18920 | 18920 | 18920 | 18920 | 0 | 18920 |
| J | 168606 | 168606 | 168606 | 168606 | 168606 | 148770 | 24795 | 106618.5 | 111577.5 | 0 |
| Total | 343159 | 343159 | 288236.5 | 262911.5 | 317961.5 | 290334 | 165606.5 | 248364 | 255671.5 | $\boxed{149901.5}$ |

Table 4.9: The fourth myopicmedian$h_i * \min\{d(i,B); d(i,A); d(i,J); d(i,j)\}$.

| To / From | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| A | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| B | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | 16275 | 16275 | 0 | 16275 | 16275 | 16275 | 16275 | 16275 | 16275 | 16275 |
| D | 71940 | 71940 | 35970 | 0 | 71940 | 71940 | 71940 | 71940 | 71940 | 71940 |
| E | 9817.5 | 9817.5 | 7140 | 5355 | 0 | 3570 | 9817.5 | 4998 | 9817.5 | 9817.5 |
| F | 23070 | 23070 | 23070 | 19225 | 7690 | 0 | 19225 | 12304 | 23070 | 23070 |
| G | 5245 | 5245 | 5245 | 5245 | 5245 | 5245 | 0 | 5245 | 5245 | 5245 |
| H | 4634 | 4634 | 4634 | 4634 | 4634 | 4634 | 4634 | 0 | 4634 | 4634 |
| I | 18920 | 18920 | 18920 | 18920 | 18920 | 18920 | 18920 | 18920 | 0 | 18920 |
| J | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Total | 149901.5 | 149901.5 | 94979 | $\boxed{69654}$ | 124704 | 120584 | 140811.5 | 129682 | 130981.5 | 149901.5 |