KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI

Granger-Causality Analysis of Ghana Universities Staff Superannuation Scheme (GUSSS), KNUST

By

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Declaration

I hereby declare that this submission is my own work towards the award of the Master of Science Degree in Industrial Mathematics and that, to the best of my knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the University, except where due acknowledgement had been made in the text.

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Dedication

This work is dedicated to my cherished late father, Mr. Anthony Kwaku Owusu, may his gentle soul rest in the bosom of the lord.

It is also dedicated to my mother, Akosua Serwaa and my lovely daughter Adwoa Owusuwaa Boakye for their love, care and support.

Abstract

The Ghana Universities Staff Superannuation Scheme (GUSSS) like any other pension scheme has income and expenditure patterns. However, whether inflows Granger-cause outflows or outflows Granger-cause inflows is unknown. This study investigates the state of the scheme, stability and long-term behaviors of the scheme, analyze the Granger-causality of inflows and outflows of funds and structure of the scheme. In line with this objectives, secondary data of monthly inflows and outflows of funds for the period 2003 to 2009 were collected. The data was fitted to vector autoregressive (VAR) model of order one(1) and the model parameters were estimated by ordinary least squares(OLS) methods. It was found that the model variables were stationary after first differencing. The system matrix was also found the be stable. The Granger-causality test showed that outflows Granger-cause inflows which means that outflows of funds in the previous month has influence on the inflows of funds in the current month. Again, the impulse response analysis showed that when one standard deviation shock was put to the error term, the model variables initially fluctuated around the zero mean but remained steady and positive in the long-run. We therefore recommend that optimal investment portfolios must be adopted.

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Chapter 1

Introduction

1.1 Background to the study

Everyone needs money to use and save some for retirement or when they can no longer work. Pensioners just as active workers need the following: food, shelter, clothing, and other family and social commitments. Unfortunately, the traditional and customary ways of helping the aged to meet the above needs have broken down due social change resulting from rural-urban migration and international migration. Support from children and relatives are less reliable or totally neglected.

Looking at how sad retirement without adequate income can be, there is the need, all over the world and particularly Ghana, for private and public institutions to put in place an appropriate pension schemes that will ensure brighter and better life for their workers during retirement. It is for this reason that Governments around the world have instituted pension schemes to better the lives of workers after retirement.

In Ghana, several pension schemes have been instituted to cater for the plight of Ghanaian workers after retirement. These include The Social Security and National Insurance Trust(SSNIT), The Ghana Universities Superannuation Staff Scheme (GUSSS),Provident Funds and other Life Insurance Schemes.

The Social Security and National Insurance Trust (SSNIT) was established in 1972 under NRCD 127 to administer the National Social Security Scheme. Prior to 1972, the scheme was administered jointly by the then Department of Pensions and the State Insurance Corporation. Until 1991, the Trust administered a Provident Fund Scheme, which was converted into a Social Insurance Pension

Scheme which was reformed in January 2010 by an Act of Parliament, Act 766. The Act was first enacted on December 12, 2008 to replace the precious-Cap 30 and the SSNIT pension schemes.

The Ghana Universities Staff Superannuation Scheme (GUSSS) was established on 1st January, 1976 by the Public Universities of Ghana namely: University of Ghana(UG), Legon; Kwame Nkrumah University of Science and Technology(KNUST), Kumasi and University of Cape Coast(UCC), Cape Coast which now include University of Education, Winneba; University for Development Studies(UDS), Tamale and University of Mines and Technology(UMT), Tarkwa. The Vice Chancellors of the Universities coordinate the affairs of the scheme.

Ghana Universities Staff Superannuation Scheme, KNUST branch, which is the focus of this research, provides superannuation scheme for its staff after retirement or when they can no longer work.

Membership of the Scheme includes:

- All existing members of GUSSS as at 1st January, 1976.
- University Teachers and Research Fellows
- University Administrative, Library, and Professional Staff of the status comparable with that of University Teachers.
- Any other category of staff permitted by the University Council to be a member of the scheme.

The Scheme is administered by the Director of Finance under the control and supervision of a Management Board consisting of the following:

- A Chairman appointed by the University Council from outside the university.
- Four(4) members appointed by the University Council two(2) of whom shall come from the Council and two(2) from outside the university.
- Two(2) members elected by the Academic Board.

- One(1) non-teaching senior member elected by the senior members.
- One(1) senior staff and one(1) junior staff elected by their members.
- The registrar of the university.

Contributions to the Scheme are recovered directly from members' salaries every month by the Director of Finance and paid into the Bank account of the scheme. Each member contributes a rate determined from time to time by members. No contribution is received from members during the period of leave of absence without pay. The university also contributes in respect of each member a rate fixed by government of the basic salary from the date of joining the scheme until retiring date.

The accounts of the scheme is kept by the Director of Finance of the university under the directive of the management board in such a form to show the state of and condition of the scheme annually.

A member of the scheme qualifies to benefit fully under the scheme if he/she satisfies the compulsory retirement age or voluntary retirement age of which the 55 years is the minimum. In addition to this, a member should have contributed to the scheme for a minimum period of fifteen(15) years. In the event where a member does not qualify for the benefits under the scheme due to the fact that he/she could not attain the minimum pensionable age or serve the minimum contribution period he/she will be paid his contribution (both employers and personal) including interest at a rate equivalent to 91-day Government of Ghana Treasury Bill.

Pension Fund all over the world has one of the most stable source of funds and the KNUST branch of GUSSS is no exception. Cash inflows come from two main sources. The first being the contribution made by members personally and by their employer. The second being the revenue generated from the investments made. In view of this, the sources of funds are stable and extremely predictable.

The scheme like any other pension fund makes some expenditures which is termed as outflows. Cash outflows are in the form of retirement benefits, gratuities, management expenses and bank charges. These expenses are also extremely predictable due to the fact that retirement benefits due members can be estimated far in advance before the actual payment date. However, payments made out of the scheme due to dismissal,vacation of post or premature death of a member in service are unpredictable. Nonetheless, these demands usually represent a small percentage of the fund outflows.

1.2 Problem Statement

Growing old is a universal, irreversible and inescapable. Because of the changing demographics, the age expectancy in Ghana is increasing. It is now about sixty-four (64 years). More and more people are now living longer than before and it is expected that pensioners will continue to live longer.

In view of this, pension fund in which pensioners draw their pension benefits must be robust.

Pension fund receives contributions from members and invest the funds received in order to provide an income for the members in the future.

The fund is to be self-financing and self-sustaining through the contributions of members.

The success of every pension fund thrives on the strength of the regulatory system on which it is established. Pension regulations set out specific guidelines as to how funds are managed and determined investment policies allowed, as well as the supervisory and regulatory guidelines that are put in place to safeguard the fund.

The fund receives income in the form of contribution from members and their employers, spend them in the form of investment and the payment of pension benefits. However, whether inflows Granger-cause outflows or outflows Granger-cause inflows is unknown.

1.3 Objectives of the study

This research aims at investigating:

- the state of the scheme.
- stability and long-term behavior of the scheme.
- Granger-causality analysis of inflows and outflows.
- structural analysis to ascertain the responsiveness of shocks or impulses or innovations to the model.

1.4 Methodology

1.4.1 Sample Data

This is based on Ghana Universities Staff Superannuation Scheme,KNUST. Primary and secondary data on inflows and outflows from 1999 to 2009 were taken for the study.

1.4.2 Statistical Identification of the System

Time series analysis particularly Vector Autoregressive of order one (1) will be used to identify the system dynamics. The data on funds flow will be fitted to VAR (1) and the model parameters will be estimated by Ordinary Least Squares (OLS) method. Granger-causality test will be used to determine the forecast ability of the model Again, structural analysis will be carried out to identify the relationship between inflows and outflows when there is a shock to the error term in the model. All estimations were carried out by a Statistical Software called STATA.

1.5 Justification

The constitution of Ghana clearly specifies that the " state shall provide social assistance to the aged such as will enable them to maintain a decent standard of living" (Republic (1992), article 37, section 6b). It is in this line that GUSSS was instituted to provide a decent retirement benefits to its employees. It is therefore indisputable fact that the study of the relationship between inflows and outflows will afford scheme managers the necessary foresight in their investment options to ensure the sustainability of the scheme.
The findings from the study will also serve as a documentary guide to scheme managers in future amendment of any provisions of the scheme.
Again, the study will form the basis for further research into the activities of GUSSS to ensure continuity of the scheme.

1.6 Organization of the study

This study is organized in five chapters. The first chapter deals with background of the study, problem statement, objectives of the study, methodology, justification and organization of the study. Chapter 2 presents the relevant literature review of the concept of pension, types of pension schemes, risks in pension scheme and investment policies of pension schemes. Chapter 3 presents appropriate methodology for structural and Granger-causality analysis for the funds flow. It basically provides the means upon which the parameters of the model are estimated.
Chapter 4 consists of data modeling and analysis. The final chapter discusses the findings, conclusion and recommendations.

Chapter 2

Literature Review

This chapter contains review of related literature associated with superannuation(pension) and social security; as well as empirical studies that have been done relating to the current study.

Superannuation is a regular payment made into a fund by an employee toward future pension. Thus, a pension is a process by which employers of labour agree to ease the sufferings of its employees' in the long run by putting in place a welfare package that would take care of them when they are labour-inactive, retired or have changed jobs. Olaniyan (2004) takes pension to be a systematic plan by an employer to give benefits to their employees when they decide to leave the job either through retirement or change of job.

There are two types of pension scheme: private and public. A public pension scheme is a social welfare security made to the retired, elderly and those that have changed jobs in the public sector of the economy. According to Heller (1998), the aim of public pension programme is not to raise the savings rate, but rather to provide income security, or at least a minimum income for the elderly.

A private pension scheme is a social security scheme managed and administered by the private sector in order to provide help and relief to elderly and retired employees at the time when they are not economically active. This scheme is defined benefit in nature, as employees save part of their income to receive it with the returns of its investment by the time they have retired or changed jobs. Nearly half of all private sector employees participate in a retirement plan, and the pension costs are approximately 55 per cent of payroll for the sponsoring firms (US Chamber of Commerce, 1994).

Different classifications had been given to pension systems in the pension literature; however, essentially, pension systems ought to be between defined benefit and defined contribution (DC) systems. Kotlikoff (1996) takes pension systems to be between Pay-AS-You-Go (PAYG) and defined contribution fully funded (DCFF) pension programmes.
McGreenvey (1990) also made a distinction between PAYG, DCFF and National Defined Contribution Accounts. However, a wider viewpoint was brought to this classification by Linbeck and Persson (2003). They classified pension systems as being between defined contribution and defined benefit, funded and unfunded, and acturial and non-acturial pension systems.

Due to these different classifications of pension systems, countries move from one regime of classification to another in quest for a suitable and appropriate pension for its employees. All over the world, pension schemes have undergone reforms in recent years to make the systems more sustainable, equitable and growth-enhancing.

Chang and Jaegar (1996) opined that it is generally accepted among the members of the Organization for Economic Cooperation and Development (OECD) that existing pension regimes may be financially unstable, and that as the population ages they require substantial reform to forestall the emergence of large public sector deficits and reductions in national saving. Linbeck and Persson (2003) believe that pension reforms is a result of concern over the long-term financial viability of existing government-operated pension systems.

Hu (2005) empirically examined pension reform, economic growth and financial development. The study used panel data analysis to find a negative relationship in the short run and a positive relationship in the long run, although the results for OECD countries are not very statistically robust. Another empirical test focused on pension fund assets and economic growth. A positive relationship between these two variables is found by the standard economic growth specification; in addition, evidence exists that pensions are a good predictor of economic growth. The panel Granger causality test confirmed this result. The final test deals with the relationship between pension assets and financial development. Hence, the panel correction model and panel Granger causality test suggest that pension fund growth leads to financial development, although some sub-group estimations are not strong.

Khorasanee (1996) used deterministic and stochastic models to assess the risks and benefits of obtaining a pension from a retirement fund by means of

- the purchase of life annuity providing a level monetary income.
- the withdrawal of income from a fund invested in equities.

In each case the projected cash-flows are compared with those from a life annuity providing an income linked to price inflation. He concluded that although each of the two options considered involve significant risks, they may nevertheless be attractive to certain groups of pensioners, in particular those with additional savings held outside the retirement fund.

Higgs and Worthington (2012) estimated economies of scale and scope for 200 large Australian superannuation (pension) funds using a a multiple-output cost function. They separately defined costs in terms of investment expenses - including investment, custodian and asset management fees and operating expenses - comprising management, administration, actuarial, director and trustee fee/charges. The four investment outputs are cash flow-adjusted net asset, the number of investment options, the proportion of total assets in the default strategy and the 5-year rate of return for investment costs, whereas the four operating outputs are cash flow-adjusted net assets, the number of members, net contribution flows and net rollovers of operating costs. They found that economies of scale hold up to at least 300 per cent of current mean fund output in both investment and operating costs. There was very little evidence that economies of scope prevail, generally reflected in the proclivity for many superannuation funds to contract out aspects of both investments and operations.

Dixion (2000) ranked the social security systems in 45 African countries using a comparative evaluation methodology that enables an assessment to be made of the country's statutory social security intention. He concluded that the spread of African social security system design standards are comparable to those of Latin American countries, although the poorest designed African systems are somewhat superior to their Latin American counterparts. They very best designed African social security systems are in North Africa: Tunisia (with its world-class family support program), Algeria and Libya, although Mauritius also stands out.

Palacios and Whitehouse (2006) compared civil pension schemes across countries in terms of benefit provision and cost. They found that in many developing countries, these expenditures have greater fiscal burden than in higher income countries where the tax base is larger. They also compared schemes within the same country covering private sector workers. They finally reviewed key policy issues related to pension schemes covering civil servants as well as other public sector workers, and found that there is particularly little justification for maintaining parallel schemes in the long run.

Iglesias and Palacios (2000) studied how publicly-managed pension funds invested and how their returns compare to relevant benchmarks. The management of these funds have a direct effect on financial sustainability and potential benefit levels. It also has important indirect effects on the overall economy when the funds are large. They found out that publicly managed pension funds (i) are often used to achieve objectives other than providing pensions (ii) are difficult to insulate from political interference and (iii) tend to earn poor rates of return relative to relevant indices. Their findings were consistent across countries of all types, but returns are especially dismal in countries with poor governance. The experience to date suggests that the rationale for prefunding have been seriously undermined by public management of pension reserves. Countries with serious governance problems should probably avoid funding altogether.

Bruinshooted and Grob (2006) studied how changes in pension incentives affect retirement in the Netherlands. The study used stated rather than a reviewed preference approach, and conducted a field survey questionnaire in the Dutch De Nederlandsche Bank (DNB) Household Survey to derive empirical estimates of pension adjustment and pension wealth effect. They found out that retrenchments of pensions arrangement to the effect of rising the standard retirement age by 1 year induced people to postpone retirement by approximately half a year on average. Retirement postponement varies across people, depending prominently on earnings and non-pension wealth; wealth through earlier retirement whereas they readily accept a lower benefit in case of decrease in pension wealth.

Borsch-Supan et al. (2005) used a multi-economic stimulation model to show the relationship among ageing, pension reform and capital flows. In order to quantify the effects, the study developed a computational general equilibrium model by feeding a multi-country overlapping generation model with detailed long-term demographic projection for seven world regions. The outcome of the simulation indicates that capital flows from fast-ageing regions to the rest of the world will initially be substantial, but that the trend is reversed decumulated savings. They concluded that closed economic models of pension reform missed quantitatively important effects of international capital mobility.

Berkel and Borsch-Supan (2003) investigated the effect of pension reform on retirement decisions in Germany , and focused particularly on the long-term implication of the changes implemented in pension legislation since 1992 and the reform discussed by the Germany Social Security Reform Commission. The results of the simulations indicate that the early retirement pension adjustment factors introduced by the 1992 pension reform will, in the long run, raise the average effective age of retirement for men by somewhat less than 2 years. The across-the-board, 2-year increase in all the relevant age limits proposed would raise the effective average age of retirement of men by approximately 8 months. If the actual adjustment factor is increased from 3.65 to 65 percent year, the effective average retirement age rises by approximately 2 years, the effects are considerably weaker for women.

Jaag et al. (2007) investigated the impact four often-proposed policy measures for sustainable pension: strengthening the tax benefits link, moving from wage to price indexation of benefits, lengthening calculation periods, and introducing more actuarial fairness in pension assessment. The study provided some analytical results and used a computational model to demonstrate the economic and welfare impact of recent pension reform in Australia.

In addition, Stensnesss and Stolen (2007) studied the effects of pension reform on fiscal sustainability, labour supply and equity in Norway. The study used Statistics Norway's dynamic micro-simulation model, MOSART. The result showed that the reform will simulate labour supply and improve public budgets, but will also lead to an increase in inequality in received pension benefit.

In the same vein, Fisher and Keuschnigg (2006) evaluated the effects of pension reform on labour market incentives. They also showed parametric reform in Pay-As-You-Go pension with tax benefit link effects retirement incentives and work incentives of prime-age workers in the presence of a tax benefit link thereby creating a policy trade-off in simulating aggregate labour supply. The article shows how several popular reform scenarios are geared either toward young or old workers, or indeed both groups under appropriate conditions. They also provide a strong characterization of the excess burden of pension insurance and show how it depends on the behavioral supply elasticities on the extensive and intensive margins and the effective tax rates implicit in contribution rates.

Bonin (2009) surveyed the state of the German pension system after a sequence of reforms aimed at achieving long-term sustainability. He argued that in principle, the latest reforms have moved pension provision in Germany from a defined benefit to a defined contribution scheme, and that this move has stabilized pension finances to a great extent. The article further argued that the real economic consequences of the global financial crisis poses threat to the care success factors of the reforms, which are cutting pension levels and raising the mandatory pension age. Finally, the article discussed possible reform measures, including the option to install a fourth pillar providing income retirement through working after pension age.

Salen and Stahlberg (2007) studied the reason why the Swedish pension reform was able to be successively implemented. They argued that governments that do not reform Pay-As-You-Go pension systems will eventually face a pension crisis. In a democracy, reform requires majority support. The problem is that pensions require today's generation to bear the burden for tomorrow's generation. Sweden passed pension legislation that specifies a gradual transition from a public defined benefit plan to a defined contribution plan. They found that a political-economic perspective helps to solve this problem that there are more winners who would vote in favour of the reform than non-winners who would vote against it. The net effect (present value of expected benefits minus present value remaining contributions) of the new and old system contributions of the working generation (age

< 53 years) are reduced by more than expected benefits.

Finseraas (2007) analyzed pension policies in 21 OECD countries in the period 1994-2003, using the OECD's reform intensity score in the area of early retirement with the old-age pension scheme as the dependent variable. The importance of left strength parliament, institutional veto points and corporation is assessed through the use of scatter plots based on ordinary least squares regression. The empirical results show that reform intensity is driven by initial conditions rather than political and institutional variables. Hence, the political elites appear to be able to overcome obstacles to reform and implement necessary changes when there is sufficient pressure for reform.

Olanike and Idowu (2010) used error correction model to evaluate the effect of public pension reform on civil servants in Nigeria. The principal aim of the article was to test the effect and show the long-run relationship that exists between pension scheme reforms and employees' welfare using panel data. They discovered that different reforms had been made to Nigeria's pension scheme over the years, but not all have been able to meet the expectations of the employees in terms of social security and risk aversion in old age. Also, many countries have adopted different pension plans that have resulted in increased social security and wealth of retired and aged employees, but that of Nigeria has been problematic owing to the inadequate disbursement of pension funds and corruption of government officials. They further found that there has been an inverse relationship between pensions and the welfare of employees, and that the same negative relationship exist between years of service of employees and welfare. However, the gratuity paid to public sector employees has a direct relationship with employees' welfare. They subsequently concluded that the pension scheme adopted by Nigeria is not a welfare-enhancing scheme, and that there should be an adjustment to the scheme in terms of its implementation, administration and coordination.

In view of the aforementioned, it is evident that the various studies conducted on pension schemes have affected pension provisions in both developed and developing countries. It is expected that the current study would contribute the above studies and would also improve pension provision on Ghana.

Chapter 3

Methodology

3.1 Introduction

The main aim of this chapter is to explain the basic concepts of difference equation, vector difference equation, eigenvalues and eigenvectors, stability analysis of vector difference equation of order one(1) and ultimately vector autoregressive models.

3.1.1 Difference Equation

Difference equation is an equation involving differences. It also refers to a specific type of recurrence relation. Again, it is a formula for computing an output at a time t based on past and present input samples and past output samples in the time domain.

Difference equation can been seen from at least three points of views: as a sequence of numbers, discrete dynamical system and iterated function. They are all the same but we look at them from different perspective.

1. Difference equation is a sequence of numbers that are generated repeatedly using a rule to relate each number in the sequence to previous numbers in the sequence. For example, the sequence $\{1, 1, 2, 3, 5, 8, 13, 21, ...\}$ is called Fibonacci sequence, generated with the rule $F_{k+2} = F_{k+1} + F_k$ for $k = 0, 1, 2, 3, ...$ and initial values $F_0 = F_1 = 1$.

2. Difference equation is a discrete dynamical system that takes some discrete input signal and produces output signal. For example, the dynamical system $D_k = 2D_{k-1} - 1.5u_k$ take a unit step input

$$u_k = \begin{cases} 0 & \text{for } k = -1, -2, -3, \cdots \\ 1 & \text{for } k = 1, 2, 3, 4, 5, \cdots \end{cases}$$

will produce output of

$$D_k = \frac{3}{2}(1 - 2^{k+1})$$

.

3. Difference equation is an iterated map $I_{k+1} = f(I_k)$ if we see the sequence as iterated function: I_0,$f(I_0)$,$f(f(I_0))$,$f(f(f(I_0)))$, ... The $f(I_0)$ is the first iterate of I_0 under f.The notation $f^m(I_0)$ is the m^{th} *iterate of* I_0 under f. For example, $f^3(I_0) = f(f(f(I_0)))$. The set of all iterates of I_0 is called the orbit of I_0. For instance, the iterated function $I_{k+1} = f(I_k) = I_k^2$ for $I_0 = 1$ will produce orbit $\{1, 1, 1, 1, ...\}$. If $I_0 = 2$, the iterated function generated will be $\{2, 4, 16, 256, 65536, 4294967296...\}$. It is clear that knowing only the rule is not sufficient to know how the sequence behaves. The initial is also very important.

3.1.2 Homogeneous Equation

Equations of the form

$$X_t = \alpha X_{t-1} \tag{3.1}$$

where α is a constant are called homogeneous equations. They involve the terms X_t and X_{t-1}.

3.1.3 Non-Homogeneous Equation

Non-Homogeneous are of the form

$$X_t = \alpha X_{t-1} + c \tag{3.2}$$

where α and c are constants. They are called non-homogeneous due to the extra term c, but α and c are still constants.

3.1.4 Solutions to Difference Equation

A solution of a difference equation is an expression (or formula) that makes the difference equation true for all values of the integer variable t. The nature of a difference equation allows the solution to be calculated recursively. It is easier to see the solution of the difference equation through algebraic equation. For example, if we have a difference equation

$$X_t = aX_{t-1} + b \tag{3.3}$$

with initial value $X_0 = c$, then we can determine the:

$t = 0; X_0 = c$

$t = 1; X_1 = aX_0 + b = ac + b$

$t = 2; X_2 = aX_1 + b$

$X_2 = a(ac + b) + b$

$X_2 = a^2c + b(1 + a)$

$t = 3; X_3 = aX_2 + b$

$X_3 = a(a^2c + b(1 + a) + b)$

$X_3 = a^3c + ab + a^2b + b$

$X_3 = a^3c + b(1 + a + a^2)$

$t = 4; X_4 = aX_3 + b$

$X_4 = a(a^3c + b(1 + a + a^2) + b$

$X_4 = a^4c + ab + a^2b + a^3b + b$

$X_4 = a^4c + b(1 + a + a^2 + a^3)$

$\vdots$

$t = n; X_n = a^nc + b(1 + a + a^2 + ... + a^{n-1})$.

There is an exponential sequence $1+a+a^2+a^3+...+a^{n-1}$ to be summed. This sequence has first term of 1, $n-1$ terms and a common ratio of a.

However, the series

$$\sum_{i=0}^{n-1} a^i = 1 + a + a^2 + ... + a^{n-1}$$

has a closed form

$$\sum_{i=0}^{n-1} a^i = \frac{1-a^n}{1-a},$$

for $a \neq 1$.

Thus, the solution of the difference equation

$$X_t = aX_{t-1} + b, X_0 = c$$

is

$$X_n = a^n c + b(\frac{1-a^n}{1-a}), \quad (3.4)$$

if a $\neq 1$.

Stability Criteria of Equation (3.4):

X_n, will be stable if $|a| < 1$.

X_n, will be unstable if $|a| > 1$.

X_n, will oscillate if $a < 0$.

X_n, will change monotonically if $a > 0$.

3.1.5 Vector Difference Equation

A vector difference equation is a difference equation in which the value of a vector (or matrix) of variables at one point in time is related to its own value at one or more previous points in time using matrices. Sometimes, the time-varying object may itself be a matrix. The order of the equation is the maximum time gap between any two indicated values of the variable vector. For example, $Y_t = AY_{t-1}$ is an example of first-order matrix equation, in which Y is an $(n \times 1)$ vector of variables and A is $(n \times n)$ matrix. The equation $Y_t = AY_{t-1} + BY_{t-2}$ is also an example of a second-order matrix equation in which Y is an $(n \times 1)$ vector of variables, and A and B are $(n \times n)$ matrices.

3.1.6 Homogeneous Vector Difference Equation

The equations $Y_t = AY_{t-1}$ and $Y_t = AY_{t-1} + BY_{t-2}$ are homogeneous vector (or matrix) equations because there are no constant terms added to the end of the equations.

3.1.7 Non-Homogeneous Vector Difference Equation

The equations $Y_t = AY_{t-1} + b$ and $Y_t = AY_{t-1} + BY_{t-2} + b$ with additive constant vector b are examples of non-homogeneous vector equations.

3.1.8 Solution of Vector Difference Equation

Let us assume that we have a homogeneous vector difference equation of the form $Y_t = AY_{t-1}$, then we can iterate and substitute recursively from the initial condition, say $Y_0 = k$, which is the initial value of the vector Y and which must be known in order to find the solution:

t=1; $Y_1 = AY_0 = Ak$

t=2; $Y_2 = AY_1 = AAk = A^2k$

t=3; $Y_3 = AY_3 = AAAk = A^3k$, and so forth.

By induction, we obtain in terms of t:

$$Y_t = A^t k = DQ^t D^{-1} k \qquad (3.5)$$

where D is an $(n \times n)$ matrix whose columns are the eigenvalues of A (assuming the eigenvalues are all distinct) and Q is an $(n \times n)$ diagonal matrix whose diagonal elements are the eigenvalues of A.
A^t shrinks to zero matrix over time if and only if the eigenvalues of A are less than unity in absolute value.

3.2 Eigenvalues and Eigenvectors

Definition 1 *The number λ is an eigenvalue of a square matrix A if and only if $A - \lambda I$ is singular, that is,*

$$det(A - \lambda I) = 0 \tag{3.6}$$

where det *is determinant.*

Definition 2 *A singular matrix is an $n \times n)$ matrix whose determinant is equal to zero.*

3.2.1 Determination of the Eigenvalues of a Matrix

To solve the eigenvalue problem for an $(n \times n)$ matrix A, we fellow these steps:

1. Calculate the determinant of $A - \lambda I$. With λ subtracted along the diagonal, this determinants starts with λ^n or $-\lambda^n$. It is a polynomial in λ of degree n.

2. Find the roots of this polynomial, by solving $det(A - \lambda I) = 0$. The n roots are the eigenvalues of matrix A. They make $A - \lambda I$ singular.

3. For each eigenvalue λ, solve

$$(A - \lambda I)v = 0 \tag{3.7}$$

to find an eigenvector.

Definition 3 *A non-zero vector v which satisfies the equation $(A - \lambda I)v = 0$ is said to be an eigenvector of A corresponding to λ.*

3.2.2 Determination of the Eigenvectors of a Matrix

To calculate the eigenvectors of a matrix, you must first determine the eigenvalues. Substitute one eigenvalue λ into the equation $(A - \lambda I)v = 0$ and solve for v; the resulting non-zero solutions form a set of eigenvectors of A corresponding to the selected eigenvalue. This process is repeated for each of the remaining eigenvalues.

Example 3.2.1 *Given*

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}.$$

The eigenvalues of A come from $det(A - \lambda I) = 0$:

$$det(A - \lambda I) = \begin{vmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} = \lambda^2 - 4\lambda + 3 = 0.$$

This factors into $(\lambda - 1)(\lambda - 3) = 0$, so the eigenvalues of A are $\lambda_1 = 1$ and $\lambda_2 = 3$.

The eigenvectors come separately by solving $(A - \lambda I)v = 0$ which is $Av = \lambda v$:

$\lambda_1 = 1 : (A - \lambda I)v = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ *gives the eigenvector* $v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$\lambda_2 = 3 : (A - \lambda I)v = \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

gives the eigenvector $v_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

3.3 Stability Analysis of Vector Difference Equation of Order One(1)

The stability of a vector difference equation of a homogeneous system could be summarized in the theorem below.

Let X_t be an $(n \times 1)$ vector at time t and A be a known $(n \times n)$ matrix, then the stability of the homogeneous equation this, $X_t = AX_{t-1}$ depends on the nature of the eigenvalues of A. Thus,

$$X_t = AX_{t-1} \tag{3.8}$$

is

1. **Stable** if all eigenvalues of A satisfy $|\lambda_i| < 1, i = 1, 2, ...$
2. **Neutrally Stable** if some $|\lambda_i| = 1$ and other $|\lambda_i| < 1, i = 1, 2, ...$
3. **Unstable** if at least one eigenvalue has $|\lambda_i| > 1, i = 1, 2, ...$

In the case of **stable**, the powers A^t approach zero and so does the solution $X_t = A^t X_0$.

3.4 Vector Autoreggressive Models

3.4.1 Introduction

One of the most effective, adaptable, and easy to use models, in the three decades, for the analysis of multivariate and bivariate time series is the vector autoregressive (VAR) models. In the past,multivariate simultaneous equation models were used for macroeconometric analysis until Sims (1980) analysis, and the resulting casual impacts of unexpected shocks or innovations to specified variables on the variables in the model are summarized. Impulse response analysis and forecast error variance decompositions are normally used to summarized these casual impacts.

Sims (1980) encouraged the use of vector autoregressive models as an alternatives for describing the dynamic behaviour of economic and financial time series, and for forecasting.

Beside data description and forecasting, VAR models provide a consistent and reliable method for structural inference and policy analysis. Certain assumptions are imposed in structural

3.4.2 Terminology, Notation and General Assumptions

The time series variable Y_t is called integrated of order d, that is, $I(d)$ if stochastic trends can be removed by differencing the variable d times and a stochastic trend stills remains after differencing only $d-1$ times.

Johansen (1995) describes the differencing operator Δ such that $\Delta Y_t = Y_t - Y_{t-1}$, the variable Y_t is $I(d)$ if $\Delta^d Y_t$ is stationary whereas $\Delta^{d-1} Y_t$ still has a stochastic trend.

The symbols $I(0)$ and $I(1)$ denote all variables that are assumed to have no stochastic trend and variables assumed to have stochastic trends respectively.

A K-dimensional vector of time series variables $Y_t = (y_{1t}, ...y_{Kt})'$ is called $I(d)$ or $Y_t \sim I(d)$ for short, if at least one of its elements is $I(d)$. Making use of this terminology,it is possible that some elements of Y_t may be $I(0)$ independently if $Y_t \sim I(1)$.

A set of $I(d)$ variables is called cointegrated if a linear combination exists which is of lower integration order, then the variables have elements of common trend. The $I(d)$ is a term which only refers to the stochastic properties of the variables. To simplify matters, we assume that deterministic elements will normally be at most linear trends of the form $E(Y_t) = \mu_t = \mu_0 + \mu_{1t}$. There is just a constant term in the process if $\mu_1 = 0$. We sometimes assumed that there is no deterministic term if $\mu_t = 0$.

The transpose, inverse, trace, determinant and rank of the matrix A are denoted by A', A^{-1}, $tr(A)$, $det(A)$, and $rk(A)$ respectively. The matrix $(n \times (m+k))$ denotes the matrices $A(n \times m)$ and $B(n \times k)$.

For a matrix $A(n \times m)$ where $n > m$, $A_\perp$ denotes an orthogonal complement, that is, $A'_\perp A = 0$ and $[A : A_\perp]$ is a nonsingular square matrix.

The symbols vec, $\otimes$ and I_n denote the column vectorization operator, the Kronecker product and an $(n \times n)$ identity matrix respectively.

The set of all positive integers, natural numbers and complex numbers are denoted by $\mathbb{Z}$, $\mathbb{N}$, $\mathbb{C}$ respectively.

For a time series variable Y_t, the lag operator L shifts the time index backwards by one period i.e. $\mathbb{L}Y_t = Y_{t-1}$ and $|x|$ denotes the absolute value for the number x.

With regard to distribution and stochastic processes, the following conventions are used. The symbol '$\sim (\mu, \sum)'$ is the abbreviation for 'has a distribution with mean (vector) μ and (co)variance matrix $\sum$' and $N(\mu, \sum)$ denotes a (multivariate) normal distribution with mean (vector) μ and (co)variance matrix $\sum$. Convergence in distribution is denoted by $\rightarrow^d$ and $plim$ stands for the probability limit. Independently and identically distributed is represented by iid. A stochastic process μ_t with $t \in \mathbb{Z}$ or $t \in \mathbb{N}$ is called *white noise* if the $u_t's$ are iid with mean zero, $E(u_t) = 0$ and positive definite covariance matrix $\sum_u = E(u_t u_t')$.

The following abbreviation are also used. DGP, VAR, SVAR, and MA for data generation process, vector autoregression, structural vector autoregression and moving average respectively. Also ML, OLS, GLS, LM, LR, MSE and RMSE stand maximum likelihood, ordinary least squares, generalized least squares, Lagrange multiplier, likelihood ratio, mean squared error and root mean squared error respectively. The natural log is abbreviated as log.

3.4.3 Vector Autoregressive Processes

The Reduced Form Model

Let $Y_t = (Y_{1t}, Y_{2t}, ...Y_{Kt})'$ denote $(K \times 1)$ vector of K related time series variables. The reduced VAR model is of the form

$$Y_t = \mu_t + X_t \tag{3.9}$$

where μ_t is the deterministic part and X_t is entirely the stochastic process with zero mean.

The deterministic term μ_t has at most a linear trend if $\mu_t = \mu_0 + \mu_1 t$. It may also be zero $(\mu_0 = 0)$ or just a constant $(\mu_t = \mu_0)$. It therefore raises doubts when used in the context of forecasting. Because of this, they are not recommendable in applied VAR analysis.

Assuming X_t is a VAR process of order p,then X_t, which is the stochastic part, is of the form

$$X_t = B_1 x_{t-1} + B_2 x_{t-2} + ... + B_p x_{t-p} + u_t \tag{3.10}$$

where B_i are $(K \times K)$ parameter matrices for $i = 1, .., p$ and $u_t = (u_{1t}, u_{2t}, ..., u_{Kt})'$, which is the error process, is a K-dimensional zero mean white noise with (co)variance matrix $E(u_t u_t') = \sum_u$.

In lag operator notation, the VAR(p) can be written as

$$B(L)X_t = u_t \tag{3.11}$$

where $B(L) = I_K - B_1 L - B_2 L^2 - ... - B_p L^p$.

The VAR(p) is stable if

$$det(I_K - B_1 z - B_2 z^2 - ... - B_p z^p) \neq 0 \tag{3.12}$$

for $z \in \mathbb{C}$, $|z| \leq 1$.

Equivalently, if the eigenvalues of the companion matrix have modulus less than one, then the VAR(p) process is stable.

Assuming the VAR(p) process has been initialized in the infinite past, the a stable VAR(p) process is stationary and has time invariant means, variances and covariance structure.

However, if $det(I_K - B_1 z - B_2 z^2 - ... - B_p z^p) = 0$ for $z = 1$, then the VAR(p) is non-stationary and the variable may be cointegrated.

Structural VAR Form

Kilian (2011) proposed as structural model of the form

$$BY_t = v_0^* + v_1^* t + B_1^* y_{t-1} + ... + B_p^* y_{t-p} + v_t \tag{3.13}$$

where $(K \times K)$ matrix B indicates the direct relations, $v_i^* = Bv_i$ for $i = 0, 1$ and $B_j^* = BB_j$ for $j = 1, ..., p$.

$v_t = Bu_t$, which is the error term, is iid white noise with covariance matrix $\sum_v = B \sum_u B'$. The matrix B sometimes has ones on its main diagonal. Naturally, matrix B is chosen such that $\sum_v$ is a diagonal matrix.

A major concern of structural VAR analysis is to identify the relations between the variables or the structural shocks.

3.4.4 Estimation of VAR Models

Stationarity Test Before VAR Estimation

A shock is usually used to describe an unexpected change in a variable or in the value of the error terms at a particular time period.
When we have a stationary system, effect of a shock will die out gradually. But, when we have a non-stationary system, effect of a shock is permanent.
Unit root test is one of the tests used to verify whether or not a time series variable is stationary. There are three(3) common unit root test, namely:

1. Augmented Dickey-Fuller (ADF)
2. Philips-Perron (PP)
3. Dickey-Fuller-GLS (DF-GLS)

The Augmented Dickey-Fuller(ADF) Test

Consider the AR(1) regression model

$$Y_t = \theta Y_{t-1} + \epsilon_t \tag{3.14}$$

The unit root null hypothesis against the stationary alternative corresponds to

$$H_0 : \theta = 1$$

against

$$H_A : \theta < 1$$

The Dickey-Fuller(DF) test is simply the t-test for H_0:

$$\hat{T} = \frac{\hat{\theta} - 1}{SE(\hat{\theta})}$$

where $\hat{\theta}$ is the ordinary least square estimate and the SE($\hat{\theta}$) is the usual standard error estimate.

The DF unit root tests initially assumed that under the unit root null hypothesis, the first difference in the series are serially uncorrelated. Since the first difference of most macroeconomic time series are serially correlated, these tests were of limited value in empirical macroeconomics.
Augmented Dickey-Fuller test was developed to address the problem of serial correlation. The solution is to augment" the test using p lags of the dependent variable. The alternative mode of

$$Y_t = \theta Y_{t-1} + \epsilon_t$$

is now written:

$$\delta Y_t = \theta Y_{t-1} + \sum_{i=1}^{p} \alpha_i \delta Y_{t-i} + \epsilon_t \qquad (3.15)$$

with p > 0.

The Philips-Perron(PP) Test

This test is similar to the ADF test, but it incorporates an automatic correction to the DF procedure to allow for autocorrelated residuals.
The PP test involves fitting

$$Y_t = \theta Y_{t-1} + \epsilon_t$$

and the results are used to calculate the test statistics. PP uses nonparametric transformations to the t-statistics from the original DF regressions such that under the unit root null hypothesis, the transformed statistics (the "z" statistics) have DF distribution. The procedure is is simply:

- regress Y_t on Y_{t-1}
- compute t
- "modify" t to get z
- under the null hypothesis, z's asymptotic distribution is the DF distribution for t

The Dickey-Fuller Generalized Least Squares(DF-GLS) Test

Df-GLS test is essentially a modified ADF test using GLS rationale. All the three(3) tests above give the same result and a rejection of the null hypothesis is always desirable

Classical Estimation of Reduced Form VARs

Let us consider a VAR(p) of the form $Y_t = v_0 + v_1 t + B_1 y_{t-1} + ... + B_p y_{t-p} + u_t$, where $v_0 = (I_K - \sum_{j=1}^{p} B_j)\mu_0 + (\sum_{j=1}^{p} jB_j)\mu_1$ and $v_1 = (I_K - \sum_{j=1}^{p} B_j)\mu_1$.
This can be written in a more compressed form as

$$Y_t = [v_0, v_1, B_1, B_2, ..., B_p]Z_{t-1} + u_t \tag{3.16}$$

where $Z_{t-1} = (1, t, y_{t-1}, ..., y_{t-p})'$.
Assume that the VAR(p) model is covariance stationary, and given a sample of size $T, y_1, ...y_T$ and p presample vector,$y_{-p+1}, ..., y_0$, then the parameters can be estimated efficiently by OLS for each equation separately. The estimator is simply

$$[\hat{v_0}, \hat{v_1}, \hat{B}_1, ...\hat{B}_p] = (\sum_{t=1}^{T} Y_t Z'_{t-1})(\sum_{t=1}^{T} Z_t Z'_{t-1})^{-1} \tag{3.17}$$

The maximum likelihood estimator can also be used to estimate the parameters if Y_t is normally distributed with $u_t \sim N(0, \sum_u)$.
On the other hand, if restrictions are imposed on the parameters, the parameters can be estimated by GLS since the OLS may be ineffective. Let $\theta = vec[v_1, v_2, B_1, ..., B_p]$ and assuming there is a linear restriction of the form

$$\theta = R\lambda \tag{3.18}$$

where R is $((K^2 p + 2K) \times M)$ restriction matrix with rank M which consist of zeros and ones, and λ is the $(M \times 1)$ vector of unrestricted parameters. The GLS estimator for λ is given as

$$\hat{\lambda} = [R'(\sum_{t=1}^{T} Z_t Z'_{t-1} \otimes \sum\nolimits_u^{-1})R]^{-1} R' vec(\sum\nolimits_u^{-1} \sum_{t=1}^{T} y_t Z'_{t-1}) \tag{3.19}$$

The GLS estimator $\hat{\lambda}$ is consistent and asymptotically normally distributed. Practically, the white noise covariance matrix is normally unknown and has to be substituted by an estimator based on an unrestricted estimation of the model.

Bayesian Estimation of Reduced Form VARs

Standard Bayesian methods that are used to estimate linear regression models can also be used to estimate the parameters of the reduced form VAR model. Canova (2007) gave a more thorough description of Bayesian methods VAR analysis. The method will not be discussed here because of its little or no relevance to this present study.

Estimation of Structural VARs

Structural VAR models, properly identified are normally estimated by least squares, maximum likelihood or Bayesian methods. For instance, a properly identified model of the form

$$BY_t = v_0^* + v_1^* t + B_1^* y_{t-1} + ... + B_p^* y_{t-p} + v_t \tag{3.20}$$

can be estimated by the OLS method.

3.4.5 Model Specification

Model specification involves the selection of the VAR order and perhaps imposing restrictions on the VAR parameters. The VAR order may be determined using sequential testing procedures or model selection criteria.

Sequential testing proceeds by specifying a maximum reasonable lag order, say P_{max}, and subsequently testing the following sequence of null hypothesis: $H_0 : B_{Pmax} = 0$, $H_0 : B_{Pmax-1} = 0$, and so on. The process stops when the null hypothesis is rejected for the first time, and the appropriate order chosen.

Alternatively model selection criteria can be used. Some of them have the general form

$$C(m) = \log det(\hat{\sum}_m) + c_T\varphi(m) \tag{3.21}$$

where $\hat{\sum} m = T^{-1}\sum_{t=1}^{T} \hat{u}_t\hat{u}_t'$ is the OLS residual covariance matrix estimator for a reduced form VAR model of order m, $\varphi(m)$ is a function of the order m which penalizes large VAR orders, and c_T is a sequence which may depend on the sample size and identifies the specific criterion.

The three most common information criteria are the Akaike's information criterion (Akaike (1973) and Akaike (1974)) which is given by

$$AIC(m) = \log det(\hat{\sum} m) + \frac{2}{T}mK^2 \tag{3.22}$$

where $c_T = \frac{2}{T}$.,

The Hannan-Quinn criterion (Hannan and Quinn (1979),Quinn (1980)) is given by

$$HQ(m) = \log det(\hat{\sum} m) + \frac{2\log\log T}{T}mK^2 \tag{3.23}$$

where $c_T = \frac{2\log\log T}{T}$, and

Schwarz(or Rissanen) information criterion (Schwarz (1978)),Rissanen (1978)) is given by

$$SC(m) = \log det(\hat{\sum} m) + \frac{\log T}{T}mK^2 \tag{3.24}$$

where $c_T = \frac{\log T}{T}$.

In all these criteria $\varphi(m) = mK^2$ is the number of VAR parameters in the model with order m, K is the number of equations and T is the number of observation. According to Lutkepohl (2005), AIC always suggests the largest order, SC chooses the smallest order and HQ is in between but all three criterion may suggest the same log order.

Paulsen (1984) also argues that the order HQ and SC criteria are both consistent with order but AIC tends to overestimate the order asymptotically with a small probability.

The order obtained with sequential testing or model selection criteria is dependent on the choice of P_{max}. A choice of small P_{max} indicates an appropriate model may not be in the set of possibilities and a choice of large P_{max} may result in a large value which may be false.

3.4.6 Model Checking

Formal and informal procedures are used to check whether or not the variables in a VAR model sufficiently represent the DGP. It focuses on the reduced form VAR since the reduced form underlie every structural form, and so any specific reduced form that do not adequately represents DGP, the structural form based on it cannot represent the DGP well. Some of the test are briefly summarized below.

Test For Residual Autocorrelation

The traditional tools for checking residual autocorrelation in VAR models are the Portmanteau and Breusch-Godfrey-LM tests.

Portmanteau Test

The hypothesis of the Portmanteau test is stated as Follows:

H_0: all residual autocovariances are zero i.e. $E(u_t u'_{t-i})$ for $i = 1, 2, ...$

H_1: at least one autocovariances and consequently one autocorrelation is not zero.

The residual autocovariances is,

$$\hat{C}_j = T^{-1} \sum_{t=j+1}^{T} \hat{u}_t \hat{u}'_{t-j} \tag{3.25}$$

where the $\hat{u}_{t's}$ are the mean adjusted residual. The Portmanteau statistics is given by

$$Q_h = T \sum_{j=1}^{h} tr(\hat{C}'_j \hat{C}_0^{-1} \hat{C}_j \hat{C}_0^{-1}) \tag{3.26}$$

or the revised version

$$Q^*_h = T^2 \sum_{j=1}^{h} \frac{1}{T-j} tr(\hat{C}'_j \hat{C}_0^{-1} \hat{C}_j \hat{C}_0^{-1})$$

may also be used.

Breusch-Godfrey-LM Test

The LM test may be seen as a test for zero coefficient matrices in a VAR model for the residuals

$$u_t = A_1 u_{t-1} + ... + A_h u_{t-h} + e_t \tag{3.27}$$

where e_t is a white noise error term. Consequently, a test hypothesis of

$$H_0 : A_1 = ... = A_h = 0$$

versus

$$H_1 : A_i \neq 0$$

for at least one i $\in 1, \cdots, h$ may be used for checking that the u_t is white noise.

(see Lutkepohl (2005),section 4.4.4 for test statistic).
The Portmanteau test is applied mainly to test for autocorrelation of high order whilst the Breusch-Godfrey-LM test is appropriate for testing autocorrelation of low order. Again, for VAR processes with unknown cointegrating rank, the LM test is more appropriate than the Portmanteau test.

Other Popular Tests For Model Adequacy

Nonnormality test are often used for model checking, even though normality is not a necessary condition for the validity of many statistical methods associated with VAR models.
Multivariate normality tests are often used to check residual vector of the VAR model and univariate tests are to investigate normality of the errors of the individual equations.
Appropriate univariate and multivariate tests are available to check conditional heteroskedasticity in the residuals of the VAR models based on data with monthly or higher frequency.
There are also a number of test available to check structural stability if there are changes in the VAR parameters throughout the sample period. Major examples are the so-called Chow tests in which one possible test version considers the null hypothesis of time invariant parameters throughout the sample period versus the likelihood of a change in the parameter values in some period, say T_A (...details in (Lutkepohl, 2005), section 4.6).
After the VAR model has passed through the adequacy tests, it can be used for forecasting and structural analysis subsequently.

3.4.7 Forecasting

Forecasting is one of the main objectives of multivariate/bivariate time series analysis. Forecasting for known VAR processes and later extended to estimated processes are presented.

Forecasting Known VAR Processes

If Y_t is produced by a VAR(p) process of the form

$$Y_t = v_0 + v_1 t + B_1 y_{t-1} + ... + B_p y_{t-p} + u_t,$$

the conditional expectation of Y_{T+h} given Y_t for t $\leq$ T is

$$Y_{T+h|T} = E(y_{T+h}|y_T, y_{T-1}, ...) = v_0 + v_1(T+h) + B_1 y_{T+h-1|T} + ... + B_p y_{T+h-p|T} \tag{3.28}$$

where $Y_{T+j|T} = y_{T+j}$ for $j \leq 0$.

If the white noise u_t is *iid*, $Y_{T+h|T}$ is the optimal, minimum mean squared error (MSE) h-step ahead forecast in period T. The forecasts can be calculated repeatedly for $h = 1, 2, 3, ...$ without difficulty. The h-step forecast error may be expressed as

$$Y_{T+h} - Y_{T+h|T} = u_{T+h} + \phi_1 u_{T+h-1} + ... + \phi_{h-1} u_{T+1} \tag{3.29}$$

where the matrices ϕ_i are determined by repeated substitution

$$\phi_i = \sum_{j=1}^{i} \phi_{i-j} B_j \tag{3.30}$$

for $i = 1, 2, 3, ...$ with $\phi_0 = I_K$ and $B_j = 0$ for $j > p$ (e.g.Lutkepohl (2005) ,chap.2)). Clearly, the reduced form VAR residual is u_t and is the forecast error for 1-step forecast in period $t-1$. The forecasts are unbiased since the errors have mean zero and forecast error covariance or MSE matrix is

$$\sum\nolimits_Y(h) = E[(Y_{T+h} - Y_{T+h|T})(Y_{T+h} - Y_{T+h|T})'] = \sum_{j=0}^{h-1} \phi_j \sum\nolimits_u \phi_j' \tag{3.31}$$

that is $Y_{T+h} - Y_{T+h|T} \sim (0, \sum_Y(h))$.

It is a fact that, the conditional expectation in (3.28) is obtained whenever the conditional expectation of u_{T+h} is zero. Even if the $u_t's$ are just uncorrelated and do not have conditional mean zero, the forecasts obtained recursively from (3.28) are still best *linear* forecasts but may do not be minimum MSE forecasts in a larger class which includes nonlinear forecasts. Again, the deterministic time trend in (3.28) does not add to the inaccuracy of the forecasts in this framework, where no estimation uncertainty is present, while stochastic trends have a substantial effect on the forecast uncertainty.

Forecasting Estimated VAR Processes

If the DGP is unknown and subsequently the VAR model only approximates the true DGP, the forecasts discussed previously will not be available. Let $\hat{Y}_{T+h|T}$ denote a forecast based on a VAR model which is specified and estimated based on the data available, then the forecast error is

$$Y_{T+h} - \hat{Y}_{T+h|T} = (Y_{T+h} - Y_{T+h|T}) + (Y_{T+h|T} - \hat{Y}_{T+h|T}) \tag{3.32}$$

The first term on the right hand side is $\sum_{j+0}^{h-1} \phi_j u_{T+h-1}$ if the true DGP is a VAR process. It includes residuals u_t with $t > T$ only. The second term also involves just $Y_T, Y_{T-1}, ...$ if only variables up to T have been used for model specification and estimation. Consequently, the two terms are independent or at least uncorrelated so that the MSE matrix is

$$\sum\nolimits_{\hat{Y}}(h) = E[(Y_{T+h} - \hat{Y}_{T+h|T})(Y_{T+h} - \hat{Y}_{T+h|T})'] = \sum\nolimits_Y(h) + MSE(Y_{T+h|T} - \hat{Y}_{T+h|T}) \tag{3.33}$$

If the VAR model specified for Y_t properly represents the DGP, the last term on the right-hand side gets closer to zero as the sample size gets larger since the difference $Y_{T+h|T} - \hat{Y}_{T+h|T}$ vanishes asymptotically in probability under standard assumptions. Thus, if the theoretical model fully captures the DGP, specification and estimation uncertainty is not important asymptotically. In finite samples, on the other hand, the precision of the forecasts depend on the precision of the estimators ((Lutkepohl, 2005)

3.4.8 Granger-Causality Analysis

One of the main uses of VAR models is forecasting. The structure of the VAR model provides information about a variables' forecasting ability for other variables. The following intuitive idea of a variable's forecasting ability is due to Granger (1969) and is known as Granger-causality. Granger called a variable Y_{2t} causal for a variable Y_{1t} if the information in past and present values of Y_{2t} is helpful for predicting the variable Y_{1t}, then Y_{2t} is said to Granger-cause Y_{1t}; otherwise it is said to fail to Granger-cause Y_{1t}. This idea is especially easy to implement in a VAR framework. Assuming Y_{1t} and Y_{2t} are produced by a bivariate VAR(p) process,

$$\begin{pmatrix} Y_{1t} \\ Y_{2t} \end{pmatrix} = \sum_{i=1}^{p} \begin{bmatrix} a_{11,i} & a_{12,i} \\ a_{21,i} & a_{22,i} \end{bmatrix} \begin{pmatrix} y_{1,t-i} \\ y_{2,t-i} \end{pmatrix} + \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix} \tag{3.34}$$

Then Y_{2t} is not Granger-causal for Y_{1t} if and only if $a_{12,i} = 0$ for $i = 1, 2, ..., p$. In other words, Y_{2t} is not Granger-causal for Y_{1t} if Y_{2t} does not appear in the (Y_{1t}) equation of the model. The idea of Granger causality does not imply true causality. It only implies forecasting ability (see Lutkepohl (2005), section 2.3.1 for details).

3.4.9 Structural Analysis

The general VAR(p) model has many parameters, and they may be difficult to interpret due to complex interaction and feedback between the variables in the model. The main types of structural analysis summaries are impulse response analysis, forecast error variance decomposition, historical decomposition of time series and analysis of forecast scenarios. The following sections briefly describe them.

3.4.10 Impulse Response Analysis

Impulses, innovations or shocks enter the model of the form

$$Y_t = v_0 + v_1 t + B_1 y_{t-1} + ... + B_p y_{t-p} + ut$$

through the residual vector $u_t = (u_{1t}, ..., u_{Kt})'$. A change in the non-zero component u_t will bring about the same changes in the other variables of the system in the next periods. The marginal effect of a single nonzero element in u_t can be studied conveniently by inverting the VAR representation, and considering the moving average (MA) representation. If the deterministic terms can be ignored since they are not important for impulse response analysis gives

$$Y_t = B(L)^{-1} u_t = \Theta(L) u_t = \sum_{j=0}^{\infty} \Theta_j u_{t-j} \tag{3.35}$$

where $\Theta(L) = \sum_{j=0}^{\infty} \Theta_j L^j = B(L)^{-1}$. The $(K \times K)$ coefficient matrices may be obtained recursively as $\Theta_i = \sum_{j=1}^{i} \Theta_{i-j} B_j$ for $i = 1, 2, ...$ with $\Theta_0 = I_K$ and $B_j = 0$ for $j > p$.

The marginal response of $Y_{n,t+j}$ to a unit impulse u_{mt} is given by the $(n, m)^{th}$ elements of the matrices Θ_j, viewed as a function of j. For this reason, the element of Θ_j represent responses to u_t innovations.

According to Lutkepohl (2005) because the u_t are just the $1 - step$ forecast errors, the impulse responses are sometimes forecast error impulse responses and the corresponding MA is called Wold representation.

The presence of the representation (3.35) makes sure that if the VAR process is stable, then Y_t consists of stationary variables assuming there are no stochastic trends. In that case, $\Theta_j \to 0$ as $j \to \infty$ and the effect of an impulse is transitory. If Y_t has stochastic trend elements, then the Wold MA representation in (3.35) does not exist. For any finite j, Θ_j can be calculated as in the stationary case using the formula in (3.30)

Impulse responses can also be calculated for VAR processes with stochastic trends. For such processes, the marginal effects of a single shock may lead to permanent changes in some or all of the variables.

If an identified structural model of the form

$$BY_t = v_0^* + v_1^* t + B_1^* y_{t-1} + ... + B_p^* y_{t-p} + Av_t$$

is available, then the corresponding residuals are the structural shocks. For stationary process, their corresponding impulse responses can again be obtained by inverting the VAR representation,

$$Y_t = (B - B_1^* L - ... - B_p^* L^p)^{-1} Av_t = \sum_{j=1}^{\infty} \Theta_j B^{-1} Av_{t-j} = \sum_{j=1}^{\infty} \psi_j v_{t-j} \qquad (3.36)$$

where the $\psi_j = \Theta_j B^{-1} A$ contain the structural impulse responses. The latter formulas can also be used for computing structural impulse responses for process with stochastic trend even if the representation (3.36) does not exist.

Forecast Error Variance Decompositions

Another tool for investigating the effects of shocks in VAR models are the forecast error variance decompositions. The task of the variance decomposition is to separate the variation in an endogenous variables into the component shocks to the VAR. Thus, the variance decomposition provides information about the relative importance of each random innovation in affecting the variables in the VAR.

In terms of the structural residuals the h-step forecast error of

$$Y_{T+h} - Y_{T+h|T} = u_{T+h} + \Theta_1 u_{T+h-1} + \Theta_{h-1} u_{T+1},$$

can be represented as

$$Y_{T+h} - Y_{T+h|T} = \psi_0 v_{T+h} + \psi_1 v_{T+h-1} + ... + \psi_{h-1} v_{T+1}.$$

Using $\sum_v = I_K$, the forecast error variance of the K^{th} element of Y_{T+h} can be shown to be

$$\sigma_k^2(h) = \sum_{j=0}^{h-1} (\Psi_{k1,j}^2 + ... + \Psi_{kK,j}^2) = \sum_{j=1}^{K} (\Psi_{kj,0}^2 + ... + \Psi_{kj,h-1}^2) \quad (3.37)$$

where $\Psi_{nm,j}$ denotes the $(n,m)^{th}$ element of ψ_j. The quantity $(\Psi_{kj,0}^2 + ... + \Psi_{kj,h-1}^2)$ represents the contribution of the j^{th} shock of the h-step forecast error variance of the variable k. Practically, the relative contributions $(\Psi_{kj,0}^2 + ... + \Psi_{kj,h-1}^2)/\sigma_k^2(h)$ are reported and interpreted for various variables and forecast horizons. A meaningful interpretation of these quantities require that the shocks considered in the decomposition are economically meaningful.

3.4.11 Historical Decomposition of Time Series

Another way of looking at the contributions of the structural shocks to the observed series is opened up by decomposing the series as proposed by (Burbidge and Harrison, 1985). They argue that abandoning deterministic terms and considering the structural MA of equation (3.36), the j^{th} variable can be represented as

$$Y_t = \sum_{i=0}^{\infty}(\Psi_{j1,i}v_{1,t-i} + ... + \Psi_{jK,i}v_{K,t-i}),$$

where $\Psi_{jk,i}$ is the $(j,k)^{th}$ element of the structural MA matrix ψ_i, as previously. Hence,

$$Y_{jt}^{(k)} = \sum_{i=0}^{\infty} \Psi_{jk,i}v_{k,t-i} \tag{3.38}$$

is the effect of the k^{th} structural shock to the j^{th} variable Y_{jt}. Someone may also prefer to explain the effects of the different structural shocks to the j^{th} variable by plotting the $Y_{jt}^{(k)}$ for $k = 1, 2, ..., K$ and $t = 1, 2, ..., T$. In practice, because of the non-availability of these structural shocks, historical decomposition is obviously not possible.

Analysis of Forecast Scenarios

Structural VAR models have also been used for the analysis of different forecast scenarios or conditional forecasts given restrictions for the future values of some of the variables.

In models where all variables are endogenous, fixing the future values of one or more variables may be difficult and one has to assess thoroughly how far the model can be stretched without being nullified. That is, the structural VAR models cannot be expected to reflect the changes induced in the future paths of the variables for arbitrary forecast scenarios (for details, see (Waggoner and Zha, 1999)).

3.4.12 Conclusion

This chapter deals specifically with difference equation, eigenvalues and eigenvectors, stability analysis of vector difference equations of order one(1) and finally vector autoregressive models. Finite order VAR models are popularly used for macroeconomics analysis because they are easy to use. There are several software packages which can be used in performing a VAR analysis and it includes

S-PLUS, Eviews, Stata, etc. The next chapter discusses the results of the study in a precise and concise manner.

Chapter 4

Data Collection, Analysis and Results

4.1 Introduction

In this chapter, we will fit inflows and outflows of the KNUST GUSSS scheme to a VAR model that relates the dependent variables and the independent variables from the data gathered. We will first present summary statistics of the data collected after which the data will be fitted to vector autoregressive model of order one(1). We will then estimate the parameters of the proposed model by ordinary least squares (OLS) method. This is followed by checking the adequacy of the model to ascertain whether or not the VAR model sufficiently represents the data generation process. Granger-causality test will be used to determine the forecast ability of the model. Finally, structural analysis such as impulse response analysis will be carried out to determine the responsiveness of the variables when there is a shock or impulse or innovation to the error terms of the model.

4.2 Data Type and Source

This study mainly depended on secondary data; specifically monthly data was collected from the KNUST GUSSS office in line with certain accounting principles. The data covers 84 months from 2003 to 2009. We could have extended our sample data beyond 2009 but we believe that the introduction of the single spine salary structure (SSSS) could possibly twist the relationship between the variables we want to estimate.

Table 4.1: Categorized Table of Inflows and Outflows

Inflows	Outflows
Contribution from members	Gratuity
Contribution from the University	Investment in Treasury Bills
Interest on Housing Loans	Investment in GUSSS Hostels
Interest on Treasury Bills	Wages and Salaries
Rent from GUSSS Hostels	Audit fees
Interest on Banks Accounts	Bank Charges
	Benefits paid
	Management Expenses
	Running cost of GUSSS

Table 4.2: Summary Statistics of the Variables

Variable	Observation	Mean	Std. Dev.	Minimum	Maximum
Inflows	84	62,954.70	9,648.395	42,017.75	90,698.37
Outflows	84	54,169.76	10,471.49	34,358.72	80,054.21

4.3 Categorization of Variables

We have categorized the variables selected for the study of the GUSSS system into inflow and outflow in the table above.

From table 4.1 above, it is obvious that the GUSSS fund can be viewed as a system of inflows and outflows and therefore the resulting net inflows and net outflows are chosen as our state variables.

4.4 Summary Statistics of the Variables

Table 4.2 below illustrates the summary statistics of the variables. Between 2003 and 2009, the KNUST GUSSS scheme has 84 months of inflows and outflows. Within the same period, the average inflows and outflows recorded were $62,954.70$ and $54,169.76$ Ghana Cedis respectively. The minimum and maximum inflows were $42,017.75$ and $90,698.57$ Ghana Cedis and that of outflows were $34,358.72$ and $80,054.21$ Ghana Cedis respectively within the same period.

4.5 Statistical Identification of the System

In this section, we construct our model and fit the categorized data to the model. We then estimate the model parameters by Ordinary Least Squares (OLS)method. It is important to note however that the parameters estimation is preceded by pre-estimation test and optimal lag length selection.

4.5.1 Construction of Model

The model of the GUSSS system can be expressed in the form

$$X_t = AX_{t-1} + \epsilon_t \tag{4.1}$$

where

X_t = state vector at t^{th} month

A =system matrix

ϵ_t =error term

As we have already indicated above, the GUSSS system has inflow and outflow as state variables and therefore our state vector X_t can be written as

$$X_t = \begin{pmatrix} I_t \\ O_t \end{pmatrix}$$

where

I_t = Inflow at the t^{th} month

O_t = Outflow at the t^{th} month

We begin the identification of the system by fitting the inflow and outflow data to

$$X_t = AX_{t-1} + \epsilon_t \tag{4.2}$$

where

X_t = state vector at t^{th} month

A = system matrix

If

$$A = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix}$$

is an $(n \times n)$ matrix, then equation 4.2 can be written as:

$$\begin{pmatrix} I_t \\ O_t \end{pmatrix} = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix} \begin{pmatrix} I_{t-1} \\ O_{t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{pmatrix} \tag{4.3}$$

The exclusion of a constant term in equations 4.1 and 4.2 mean that the system is a homogeneous model.

Thus, equation 4.3 can be written in two separate equations as:

$$\text{Inflows}_t = \alpha_{11}\text{Inflows}_{t-1} + \alpha_{12}\text{Outflows}_{t-1} + \epsilon_{1t} \tag{4.4}$$

$$\text{Outflows}_t = \alpha_{21}\text{Inflows}_{t-1} + \alpha_{22}\text{Outflows}_{t-1} + \epsilon_{2t} \tag{4.5}$$

The parameters of the equations 4.4 and 4.5 are computed by ordinary least squares (OLS) method built in Stata after which the system matrix A will be subjected to stability analysis in later section.

4.5.2 Testing Stationarity

In fitting the VAR(1) model, the variables in the model ought to be stationary. The stationarity of the variables (i.e. inflow and outflow) is checked using unit root test. Under this test, we shall employ the following methods.

1. Augmented Dickey-Fuller (ADF)

2. Philips-Perron (PP)

Table 4.3: Unit Root Tests of Model Variables at Level

VARIABLE	ADF		PP		DF-GLS	
	Test Statistic	5% Critical Value	Test Statistic	5% Critical Value	Test Statistic	5% Critical Value
Inflow	0.006	1.950	0.008	1.950	7.801	3.065
Outflow	0.048	1.950	0.261	1.950	8.231	3.065

3. Dickey-Fuller-GLS (DF-GLS)

Augmented Dickey-Fuller Unit Root Test Method

Under this test, there are three(3) model equations namely: (i) model with intercept only (ii) model with trend and intercept (iii) model with no trend and no intercept. All these models give the same results but we will limit ourselves to the third model since the model we intend to fit has no intercept and no trend The table below summarizes all the three(3) methods.

HYPOTHESIS:

H_0: The variable has unit root (non stationary)

H_1: The variable does not have unit root (stationary)

GUIDELINES:

When the value of the test statistic is more than 5 % critical value, we reject the null hypothesis and accept the alternate hypothesis but when the absolute value of the test statistic is less than 5 % critical value, we cannot reject the null hypothesis, rather we accept the null hypothesis.

From table(4.2) above, both Augmented Dickey-Fuller and Philips-Perron tests suggest non-stationary of the variables but Dickey-Fuller-GLS suggests stationary variables.

The figures below exhibit the trends of our model variables for the period 2003 to 2009.

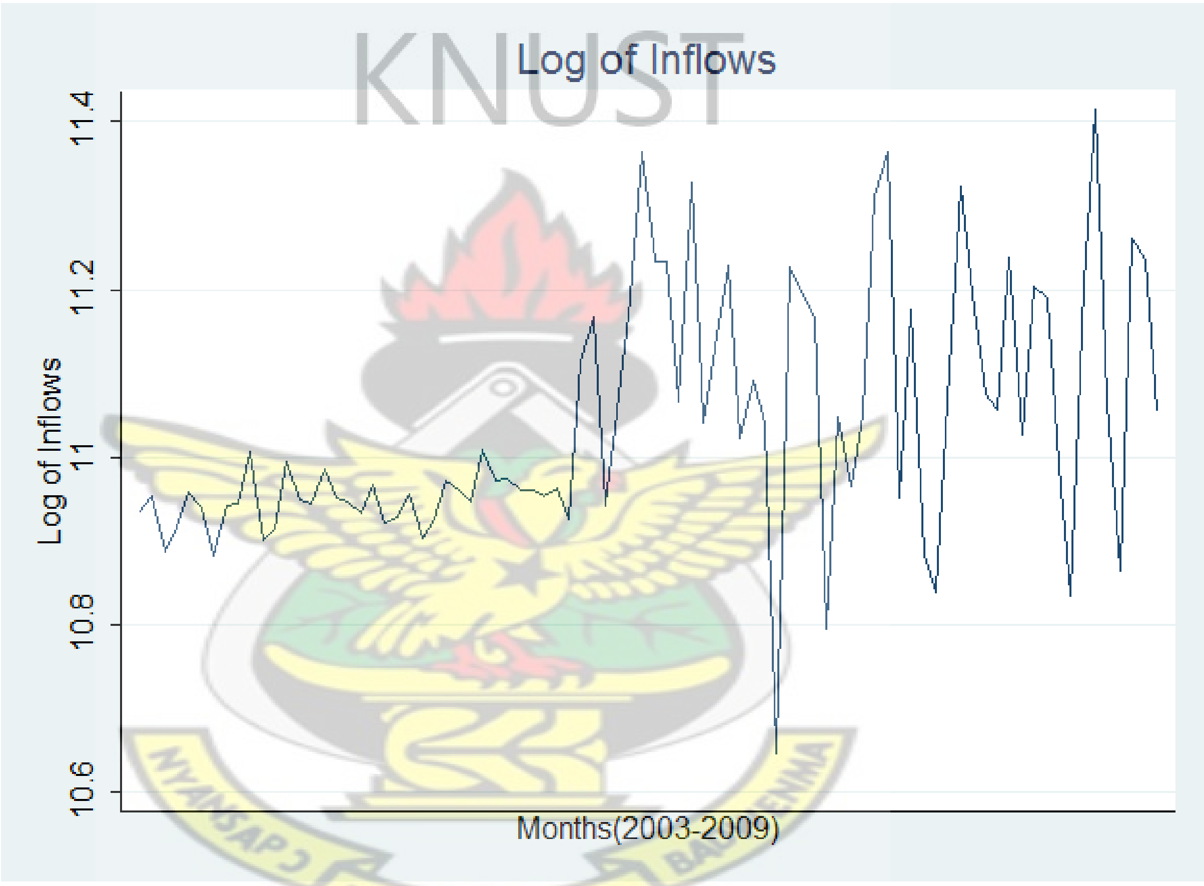

Figure 4.1: Log of Inflows from 2003-2009

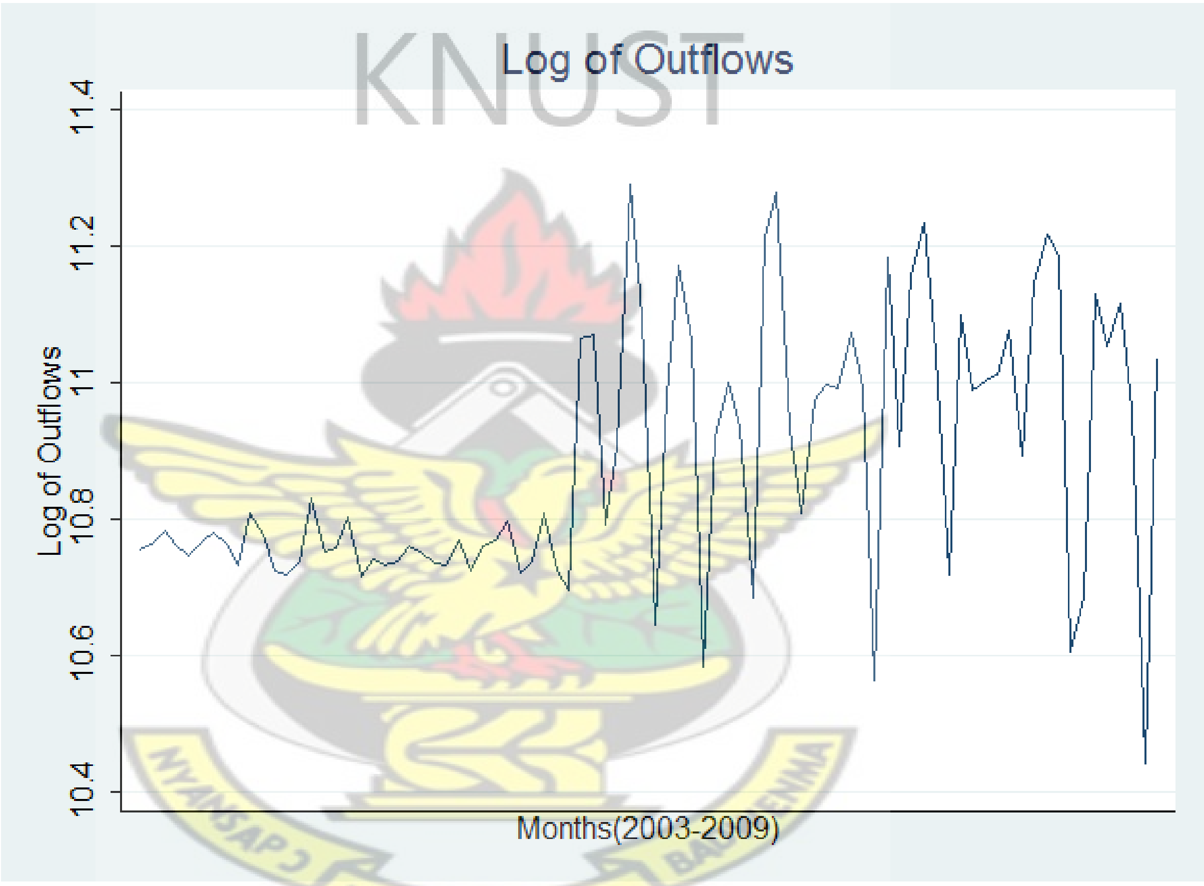

Figure 4.2: Log of Outflows from 2003-2009

Table 4.4: Unit Root Tests of Model Variables after first Differencing

VARIABLE	ADF		PP		DF-GLS	
	Test Statistic	5% Critical Value	Test Statistic	5% Critical Value	Test Statistic	5% Critical Value
Inflow	12.445	1.950	16.452	1.950	12.216	3.067
Outflow	11.891	1.950	16.854	1.950	10.799	3.067

Figures (4.1) and (4.2) above show trends of our variable (inflows and outflows). They do not revolve around a constant mean and therefore variables are non-stationary.

When the variables are non-stationary, they became stationary after taking the first differences.

The table below shows the stationary state of the variables after first differencing.

In table(4.4), our variables look stationary now after first differencing. All the test statistics from all the three(3) unit root tests are more 5% critical value.

Figures (4.3) and (4.4) below show stationary variables after first difference.

In these figures, you would find that variables revolve around a constant mean of zero.

All the tests we have performed have shown that the variables (i.e. inflow and outflow) in our model are stationary. But before we estimate our VAR(1) model we need to select the lag length we intend to use.

4.5.3 Optimum Lag Length Selection

Before we estimate our VAR(1) model, we need to select the number of lags that can be used. Some information criteria are used to select the the optimum lag length. In selecting the optimal lag length, there are some guidelines. We use the value of the Akaike Information Criterion (AIC) because of the fact that the AIC

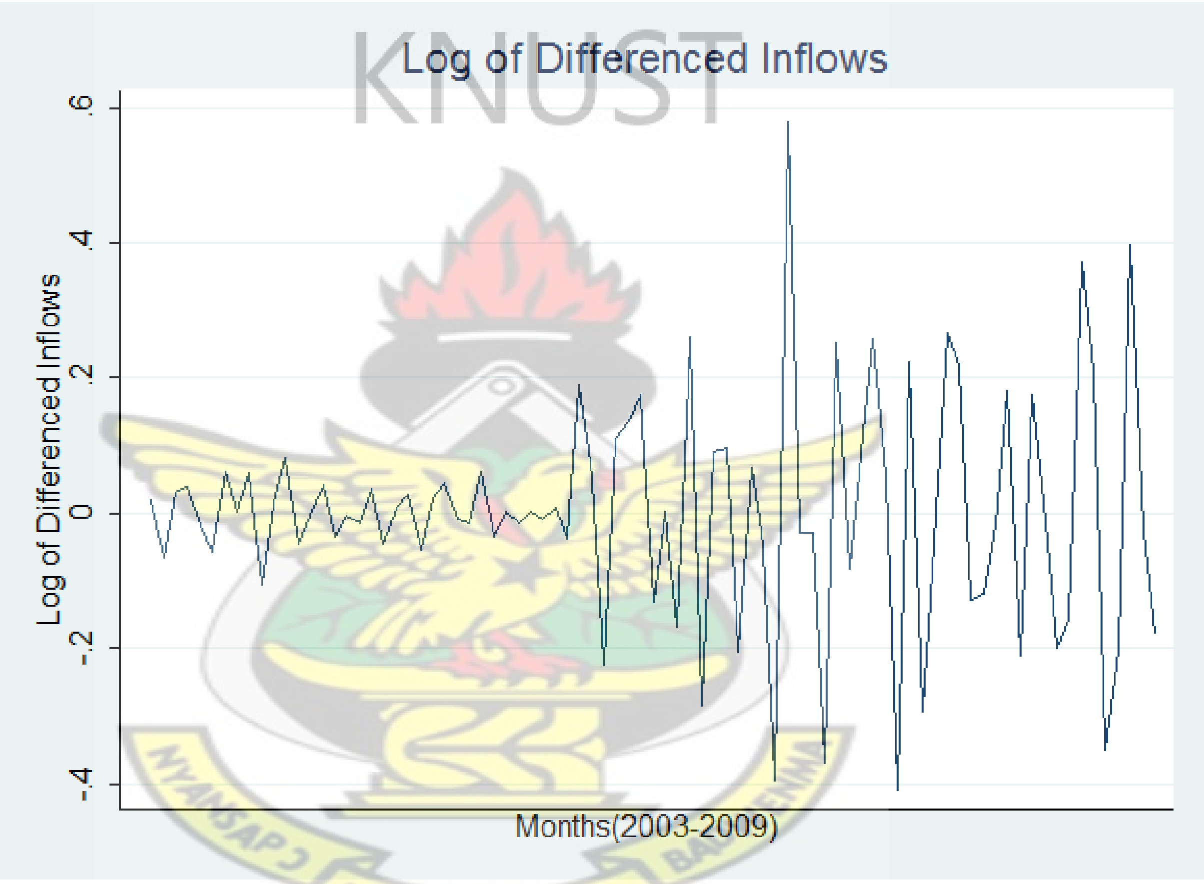

Figure 4.3: Log of Differenced Inflows from 2003-2009

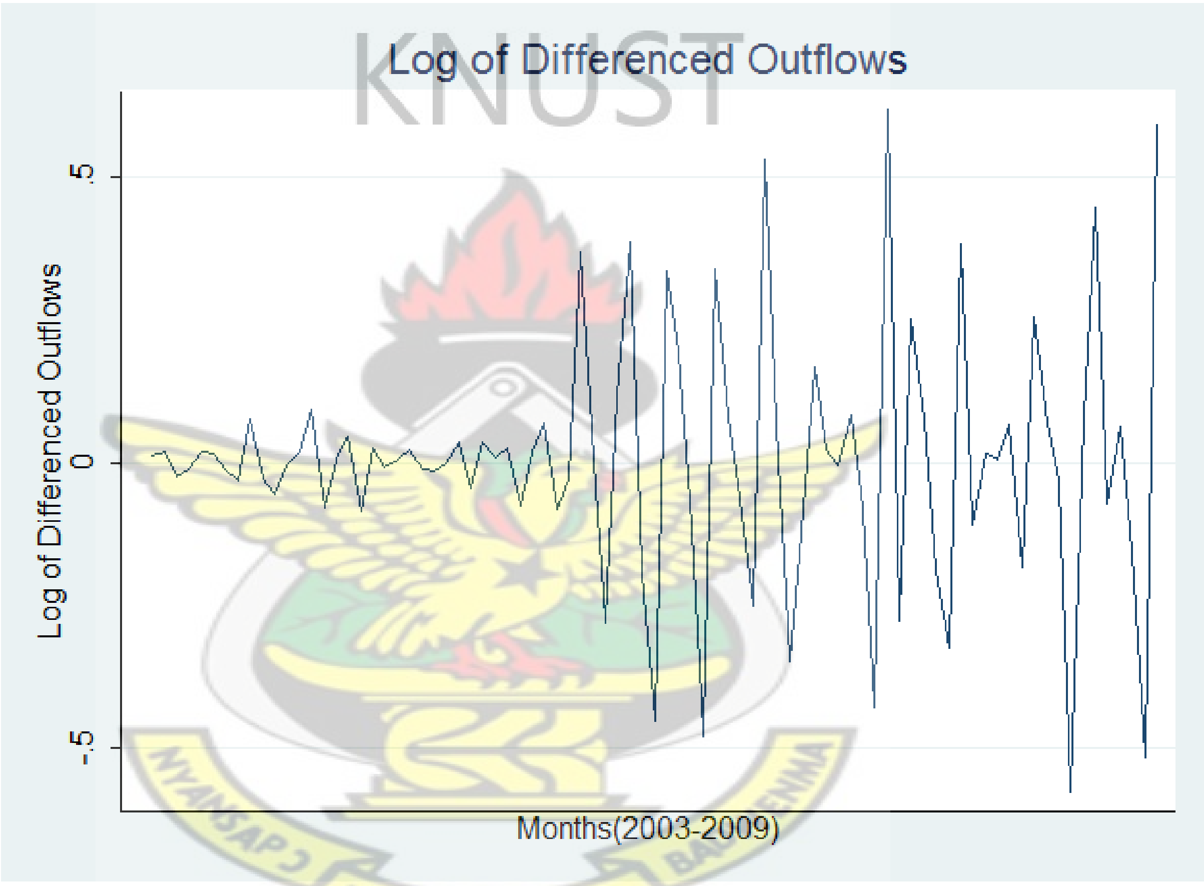

Figure 4.4: Log of Differenced Outflows from 2003-2009

Table 4.5: Lag Length Selection Criteria

Lag	LL	LR	dF	P	FPE	AIC	HQIC	SBIC
0	-1698.96		4		1.0×10^{16}	42.524	42.5479	42.5836
1	-1688.98	19.969	4	0.001	8.7×10^{15}	42.3744	42.4461	42.5531
2	-1676.99	23.97	4	0.000	7.1×10^{15}	42.1742	42.2942	42.4726*
3	-1669.48	15.025	4	0.005	6.5×10^{15}	42.087	42.2541	42.5038
4	-1659.59	19.781*	4	0.001	$5.5 \times 10^{15*}$	41.9397*	42.1546*	42.4757

Endogenous Variables : Inflow Outflow
Exogenous Variable : Constant

method is powerful and efficient with monthly data. The lower the AIC value, the better the model all the time. The table below is the output of the lag length selection.

KEY: LL: Log Likelihood; LR: Likelihood Ratio; dF: Degree of Freedom; P: Probability Value; FPE: Final Prediction Error; AIC: Akaike Information Criterion; HQIC: Hannan-Quinn Information Criterion; SBIC: Schwarz Information Criterion.

In table 4.5 above, it can be seen that at lag 1 the AIC value is 42.3744, at lag 2 the AIC value is 42.1742. It can also be seen that at lag 3 the AIC value is 42.087 and at lag 4 the AIC value is 41.9397.

In line with the guidelines, the Software suggests lag length of 4 for our model. That notwithstanding, outflow of the current month depends on the inflow of the previous month in the model we intend developing. Comparing the AIC values at lags 1 and 4, they are approximately the same and it will not be out of place to settle on lag 1 for our model estimation.

4.5.4 Parameter Estimation

The system (i.e. equation 4.3) is a bivariate vector autoregressive process of order one (VAR(1)). The line representation of equation (4.2) forms exogenous autoregressive models in which the inflow equation has O_{t-1} as its exogenous variable and the outflow equation has I_{t-1} as its exogenous variable. These equations were run separately in Stata by Ordinary Least Squares (OLS) for the parameter

Table 4.6: VAR(1) Model Output

Equation	Parameters	RMSE	R-Square	Chi-Square	Prob > Chi-Square
Inflow	2	0.156673	0.1863	18.77357	0.0001
Outflow	2	0.213474	0.0929	8.39336	0.0150

		Coefficient	Standard Error	z	P-value	[95 % Conf. Interval]
Inflow						
	Inflow L1	-0.2776697	0.1013237	-2.74	0.006	[-0.4762605,-0.790788]
	Outflow L1	-0.2392825	0.0816276	-2.93	0.003	[-0.3992697,-0.0792954]
Outflow						
	Inflow L1	0.0365306	0.1380578	0.26	0.791	[-0.2340577,0.307119]
	Outflow L1	-0.3217942	0.111221	-2.89	0.040	[-0.5397834,-0.103805]

estimates.

VAR(1) Model Estimation Output

The table below shows STATA output of our VAR(1) model of inflow and outflow.

In table 4.6 above, there are two(2) models. The first is Inflow-Outflow model and the second is Outflow-Inflow model. In the first model, the dependent variable is inflow and the independent variables are inflow(L1) and outflow(L1). Inflow(L1) is significant to explain the dependent variable in this model because of the fact that the p-value 0.006 is less than 0.05 level of significance. In the same model, outflow(L1) is also significant because its p-value is 0.003, less than the 0.05 level of significance.

In the second model, the dependent variable is outflow and inflow(L1) and outflow(L1) are the independent variables. Inflow(L1) has p-value 0.791 and it's not significant to explain the dependent variable. Outflow(L1) is significant since its p-value 0.040 is less than 0.05 level of significance.

The z statistic is obtained by the formula:

$$z = \frac{\text{Coefficient}}{\text{Standard Error}}$$

For example,

$$z = \frac{-0.2776697}{0.1013237} = -2.74$$

In reference to equations (4.4) and (4.5),our model can be written as:

$$\text{Inflows}_t = -0.2776697\text{Inflows}_{t-1} - 0.2392825\text{Outflows}_{t-1} + \epsilon_{1t} \quad (4.6)$$

$$\text{Outflows}_t = -0.3217942\text{Outflows}_{t-1} + \epsilon_{2t} \quad (4.7)$$

The system matrix A is thus

$$A = \begin{pmatrix} -0.27766971 & -0.2392825 \\ 0 & -0.3217942 \end{pmatrix}$$

4.6 Model Adequacy

We now check whether or not the VAR(1) model of inflows and outflows sufficiently represent the data generation process. If the model passes the adequacy test, it can be used for forecasting and structural analysis.

4.6.1 Check of Stability Condition Of VAR(1) Estimates

GUIDELINES:

- The VAR(1) process is stationary if the eigenvalues of the system matrix are less one.

- The process is also stable if the eigenvalues of the system matrix have modulus less than one.

Table 4.7: Eigenvalue Stability Condition

Eigenvalue	Modulus
-0.2997319+0.09.85374i	0.313199
-0.2997319-0.09085374i	0.313199

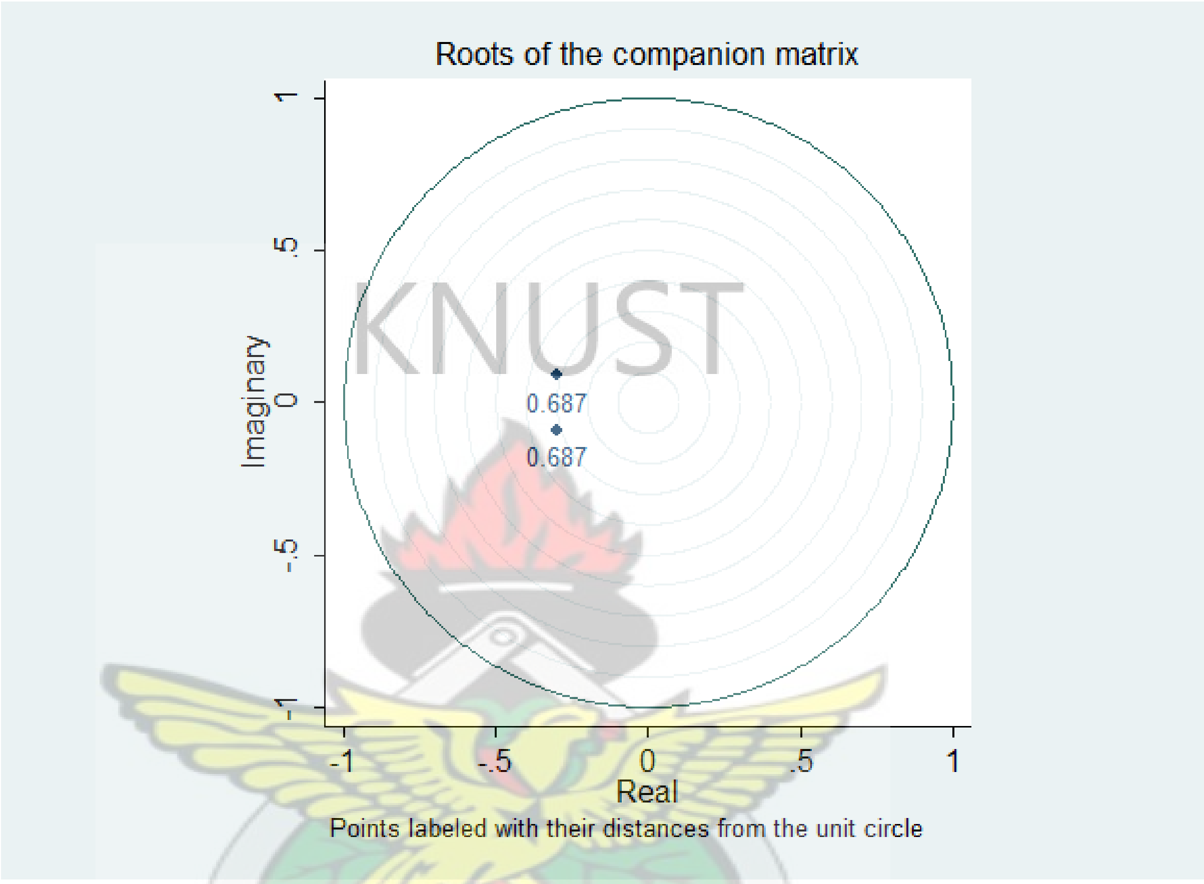

Figure 4.5: Stability Condition of the Estimated VAR Model

All the eigenvalues lie inside the unit circle and therefore the VAR(1) satisfies stability condition.

From table 4.7, the VAR(1) process of inflows and outflows of the KNUST GUSSS scheme is stationary and stable.

A confirmation of the stable VAR is shown in figure (4.5) below.

Table 4.8: Jarque-Bera Test

Equation	Chi-Square	Degree of Freedom	Prob >Chi-Square
Inflow	3.221	2	0.19975
Outflow	8.704	2	0.01288
ALL	11.926	4	0.01791

Table 4.9: Lagranger-Multiplier Test

Lag	Chi-Square	Degree of Freedom	Prob >Chi-Square
1	0.7593	4	0.94382
2	2.3070	4	0.67949
3	6.1416	4	0.18882

Test for Normally Distributed Disturbances

We also check if the residuals of the variables in the VAR(1) model is normally distributed or not.

HYPOTHESIS:

H_0: The residuals are not normally distributed

H_1: The residuals are normally distributed

GUIDELINES:

When the probability value is more than 5%, we cannot reject H_0.

From table (4.8) above, the residuals of inflow are not normally distributed whilst outflow is normally distributed. However, considering all variables in the model, we can say that the residuals of the variables are normally distributed.

Lagranger-Multiplier Test for Residual Autocorrelation

Again, we test whether the residuals of the variables in the VAR model are correlated or uncorrelated.

HYPOTHESIS:

H_0: There is no autocorrelation at lag order

H_1: There is autocorrelation at lag

GUIDELINES:

Table 4.10: Granger-Causality Wald Tests

Equation	Excluded	Chi-Square	Degree of Freedom	Prob > Chi-Square
Inflow	Outflow	8.5931	1	0.003
Inflow	ALL	8.5931	1	0.003
Outflow	Inflow	0.07002	1	0.791
Outflow	ALL	0.07002	1	0.791

When the probability value is more than 5%, we cannot reject H_0.

In table(4.9) above, there are no autocorrelation in the residuals of the variables at lag lengths of 1 to 3. This pattern is always desirable for a good model.

After showing that the VAR(1) process is stationary and stable, and that the residuals are normally distributed with no autocorrelation, we are now in a better position to use our model for forecasting in the next section.

4.6 Forecasting the VAR(1) Process By Granger-Causality Analysis

Our main aim of this study is to determine whether inflows cause outflows or outflows cause inflows. Since our variables are stationary, we can apply Granger-causality test.

A look at our model again.

$$\text{Inflow}_t = -0.2776697\text{Inflow}_{t-1} - 0.2392825\text{Outflow}_{t-1} + \epsilon_{1t} \tag{4.8}$$

$$\text{Outflow}_t = -0.3217942\text{Outflow}_{t-1} + \epsilon_{2t} \tag{4.9}$$

In equations (4.8) and (4.9), Inflow_t and Outflow_t are dependent variables of the current period respectively. Inflow_{t-1} and Outflow_{t-1} are independent variables. We want to investigate whether outflow in the previous month can cause inflow on the current month or inflow in the previous month can cause outflow in the current month or not.

Hypothesis for Model One:

H_0: Outflows in previous month cannot Granger cause inflow in current month

H_1: Outflow in previous month can Granger cause inflows in current month.

Guidelines:

When the probability value is more the 5 %, we cannot reject H_0.

In table(4.10) above, since the p-value 0.003 is less than 0.05, we can reject the null hypothesis and conclude that outflows in previous month can Granger cause inflows in the current month.

Hypothesis for Model Two:

H_0: Inflows in previous month cannot Granger cause outflows in the current month

H_1: Inflows in previous month can Granger cause outflows in the current month.

Guidelines:

When the probability value is more the 5 %, we cannot reject H_0.

Again in table (4.10), the p-value 0.791 is more than 0.05 level of significance,we cannot reject the null hypothesis and conclude that inflows in previous month cannot Granger cause outflows in the current month.

4.7 Structural Analysis By Impulse Response Function

Impulse response function is a shock to the VAR system. It identifies the responsiveness of the dependent variables(endogenous variables) in the VAR when a shock is put to the error terms such ϵ_{1t} and ϵ_{2t} in the equations below.

$$\text{Inflow}_t = -0.2776697\text{Inflow}_{t-1} - 0.2392825\text{Outflow}_{t-1} + \epsilon_{1t} \quad (4.10)$$

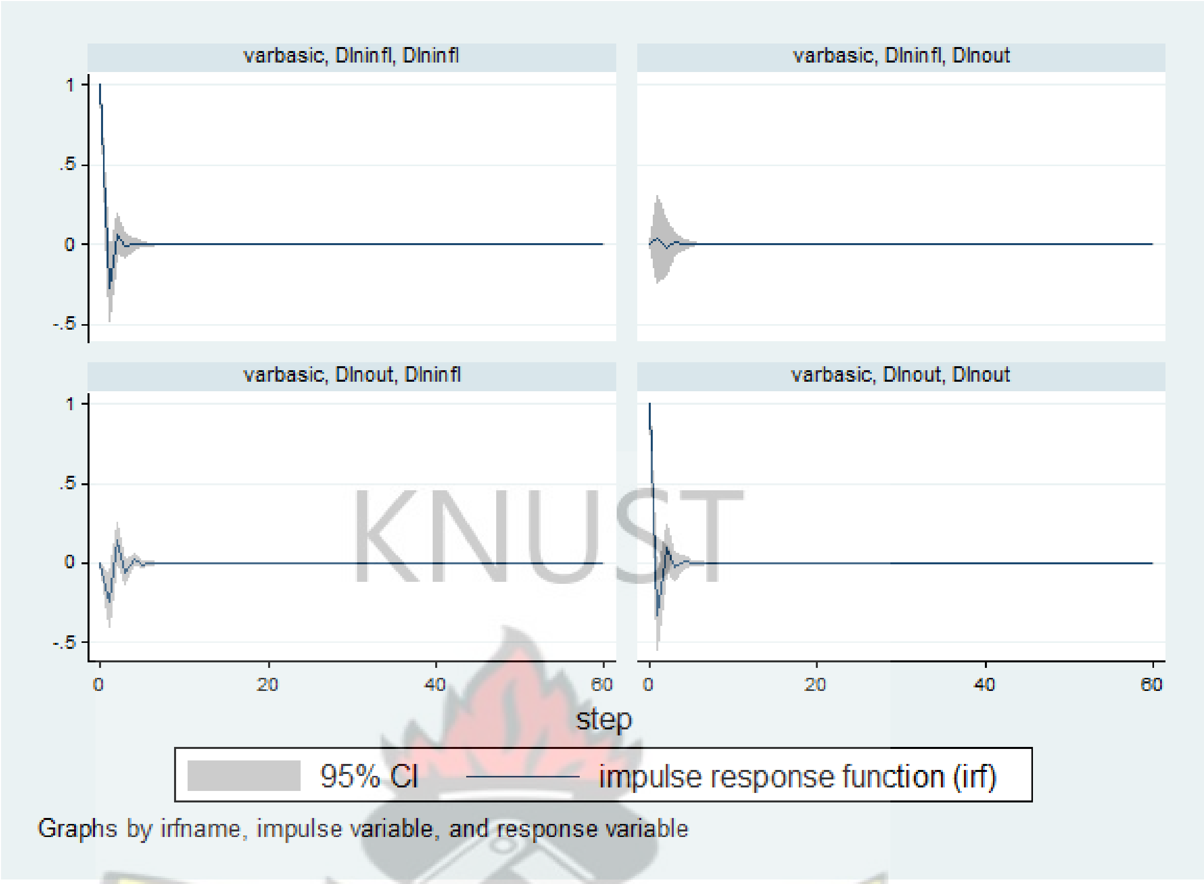

Figure 4.6: Impulse Response Function of Inflows and Outflows

$$\text{Outflow}_t = -0.3217942\text{Outflow}_{t-1} + \epsilon_{2t} \quad (4.11)$$

In the equations 4.10 and 4.11 above, a change or shock in ϵ_{1t} will bring a change in inflows. It will also bring a change in outflows and ultimately a change inflows.

In the same manner, a shock or innovation in ϵ_{2t} will bring about a change in outflows, a change in inflows and subsequently a change in outflows. So a change or shock in ϵ_{1t} or ϵ_{2t} will bring about a change in the whole model.

Our target is to investigate the reaction of the variables(inflows and outflows) when a positive shock of one standard deviation is put to ϵ_{1t} and ϵ_{2t}. The impulse response function is represented in figure 4.6

In figure 4.6, Dlninfl and Dlnout represent log of first difference of inflows and outflows respectively. IRF represents impulse response function and steps represents the number of months into the future.

When one standard deviation shock is put to inflows, the response of outflows is that it rises and falls in the first few months of the first year but becomes positive and steady for the next 60 months.

In the same way, when one standard deviation shock is put to outflows, there will be a fall and rise of inflows in the first few months as response but the fluctuation will die out soon and become positive and steady in the next 60 months into the future.

Chapter 5

Summary of Findings, Conclusion and Recommendation

5.1 Introduction

This chapter summarizes the results of the study and explains any conclusion that have resulted from the statistical analysis of the data. It also covers the recommendations the researcher wishes to put across for further studies which will be essential for stakeholders.

5.2 Summary of Findings

It should be well noted that the findings arrived in this study were based solely on the data obtained from KNUST GUSSS office for the period 2003 to 2009.

The results indicate that, all the variables in our system exhibit trends, and therefore non-stationary. But when we took the first difference, the variables became stationary, which is a prerequisite for VAR model estimation.

The system was found to be stable. This is because the eigenvalues of our system matrix

$$A = \begin{pmatrix} -0.27766971 & -0.2392825 \\ 0 & -0.3217942 \end{pmatrix}$$

are less than one and lie inside the unit circle. This is always desirable for Granger causality analysis.

The Granger causality test showed that, during the period under consideration, outflows in the previous Granger cause inflows in the current month. On the other hand, inflows in the previous month does not Granger cause outflows in the current month.

The impulse response analysis also revealed that when one standard deviation shock is put to inflows, outflows rises and falls initially but became positive and steady after fews months. In the same way, when one standard deviation shock is put to outflows, inflows falls and rises initially but became steady and positive thereafter.

5.3 Conclusion

The primary objective of this study was to determine whether inflows in the previous month has influence on outflows of the current month or whether outflows of the previous month has the ability of influencing inflows in the current month of the GUSSS scheme at KNUST.

To be able to determine this, monthly data of inflows and outflows were taken from the GUSSS office at KNUST for the period 2003 to 2009.

The data was fitted to vector autoregressive model of order one(1) and the parameters of the model were estimated by ordinary least squares (OLS) method.

Granger-causality test was performed to determine the direction of causality of the model variables (i.e. inflows and outflows). It was found that outflows in the previous month Granger-cause inflows of the current month during the period under consideration. This means that investments in the form of treasury bills and hostels are prudent measures and must be encouraged since these investments bring in the necessary inflows needed for the scheme's expenditures.

5.4 Recommendation

Based on our findings we therefore recommend that:

- management of the GUSSS scheme at KNUST should continue to invest in treasury bills and hostels as their returns are worthwhile. We therefore recommend that optimal investment portfolios must be adopted.
- future research into KNUST GUSSS scheme should be directed towards finding other investment options that will bring in funds needed to meet the scheme's future expenditures.

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Appendix A

Stata 12 Do File

Getting the Data into Stata 12.

you can use the command insheet using "G:gusssdata.xls" OR

the copy and paste procedure for small data

Transformation of Model Variables Using Logarithms

gen lninfl=log(infl)

gen lnout=log(out)

Plots of Model Variables at Levels

twoway(line lninfl mon),ytitle(Log of Inflows)xtitle(Months(2003-2009))xlabel(none) title(Log of Inflows)

twoway(line lnout mon),ytitle(Log of Outflows)xtitle(Months(2003-2009)) xlabel(none)title(Log of Outflows)

Plots of Model Variables After First Difference

twoway(line Dlninfl mon),ytitle(Log of Differenced Inflows)xtitle(Months(2003-2009))xlabel(none)title(Log of Differenced Inflows)

twoway(line Dlnout mon),ytitle(Log of Differenced Outflows)xtitle(Months(2003-2009))xlabel(none)title(Log of Differenced Outflows)

Stationarity Test of Model Variables at Levels Using Unit Root Method

dfuller lninfl,noconstant regress lags(0)

dfuller lnout,noconstant regress lags(0)

pperron lninfl,noconstant regress

pperron lnout,noconstant regress

```
dfgls lninfl,maxlag(0)
dfgls lnout,maxlag(0)
```

Stationarity Test of Model Variables After First Difference Using Unit Root Method

```
dfuller Dlninfl,noconstant regress lags(0)
dfuller Dlnout,noconstant regress lags(0)
pperron Dlninfl,noconstant regress
pperron Dlnout,noconstant regress
dfgls Dlninfl,maxlag(0)
dfgls Dlnout,maxlag(0)
```

Summary Statistics

```
summarize infl out
```

VAR Model Estimation

```
var Dlninfl Dlnout,noconstant lags(1/1)
varstable,amat(SM) gragh dlabel
varnorm,jbera
varlmar,mlag(3)
vargranger
varbasic Dlninfl Dlnout,lags(1/1)step(60)irf
```

Appendix B

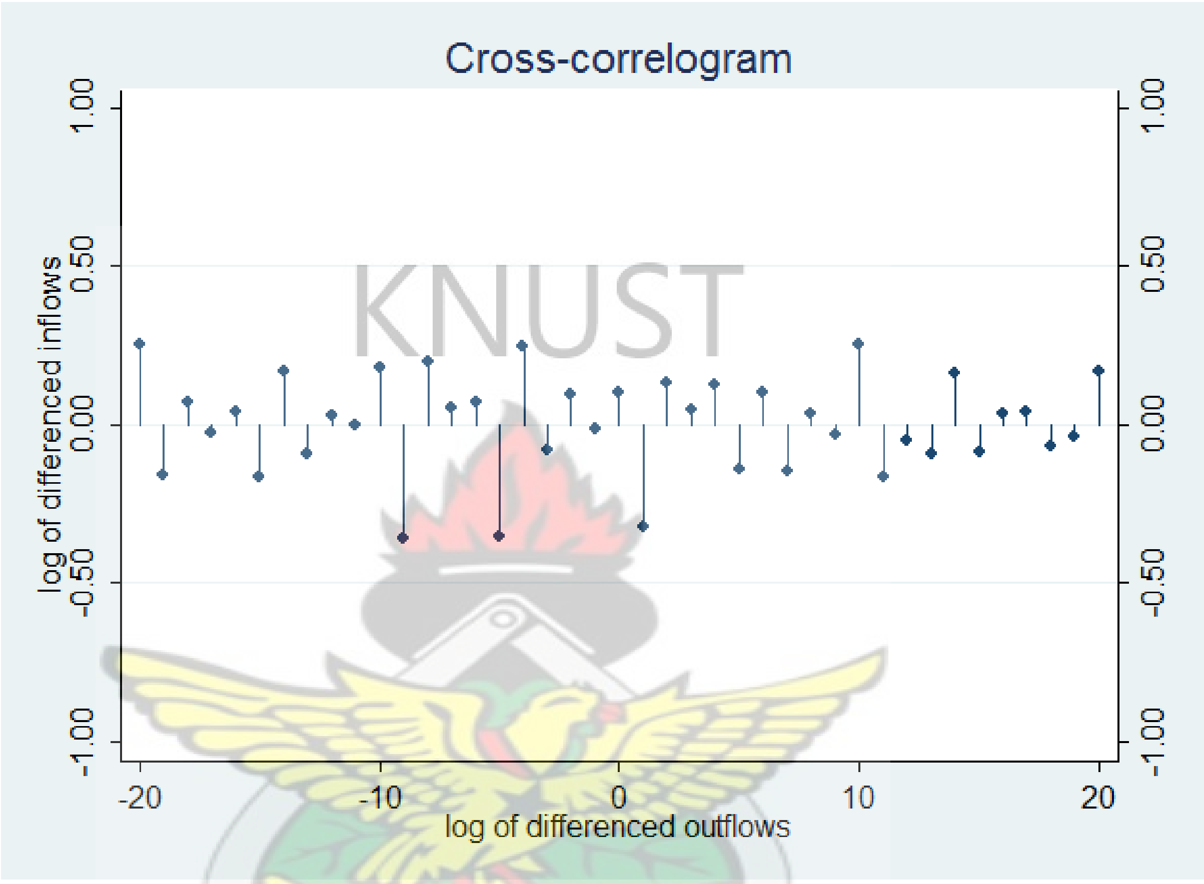

Figure 5.1: Cross-Correlogram for Inflows and Outflows

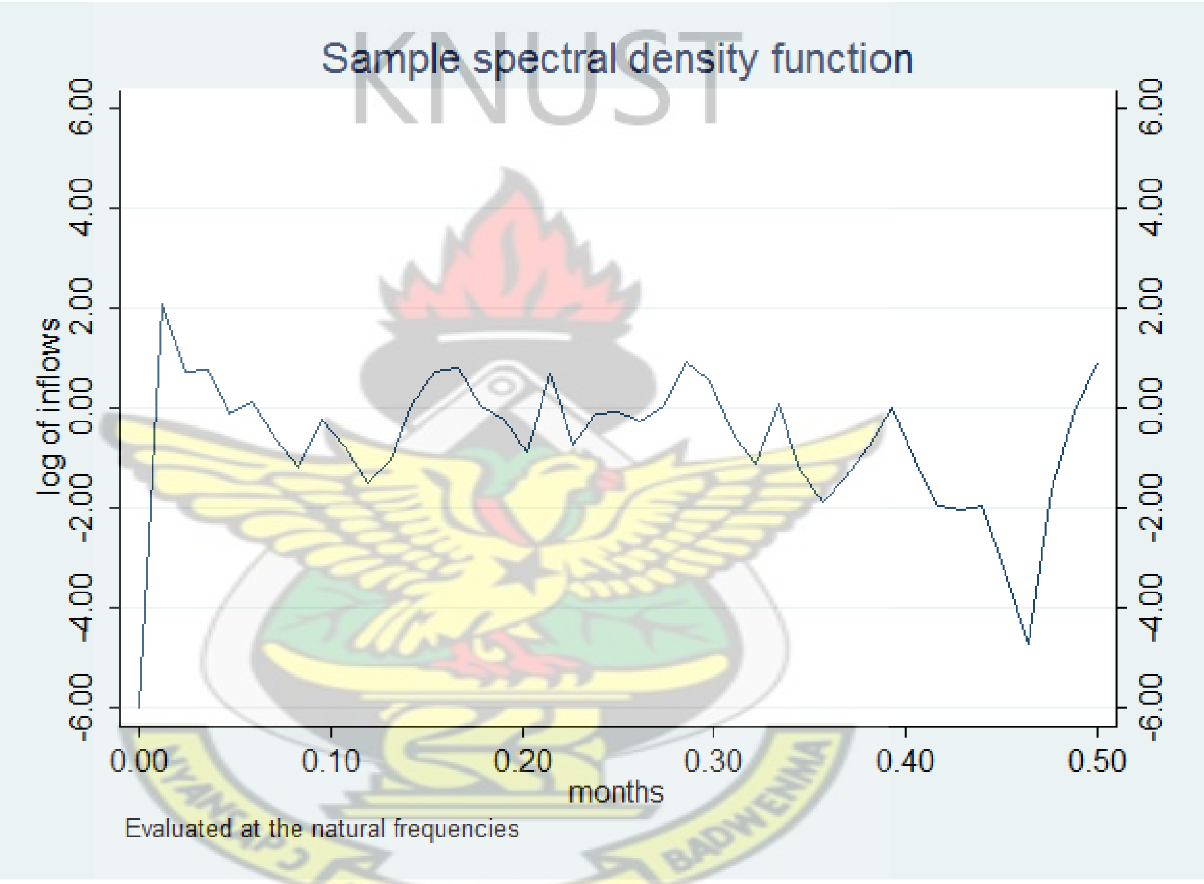

Figure 5.2: Inflows Sample Spectral Density Function

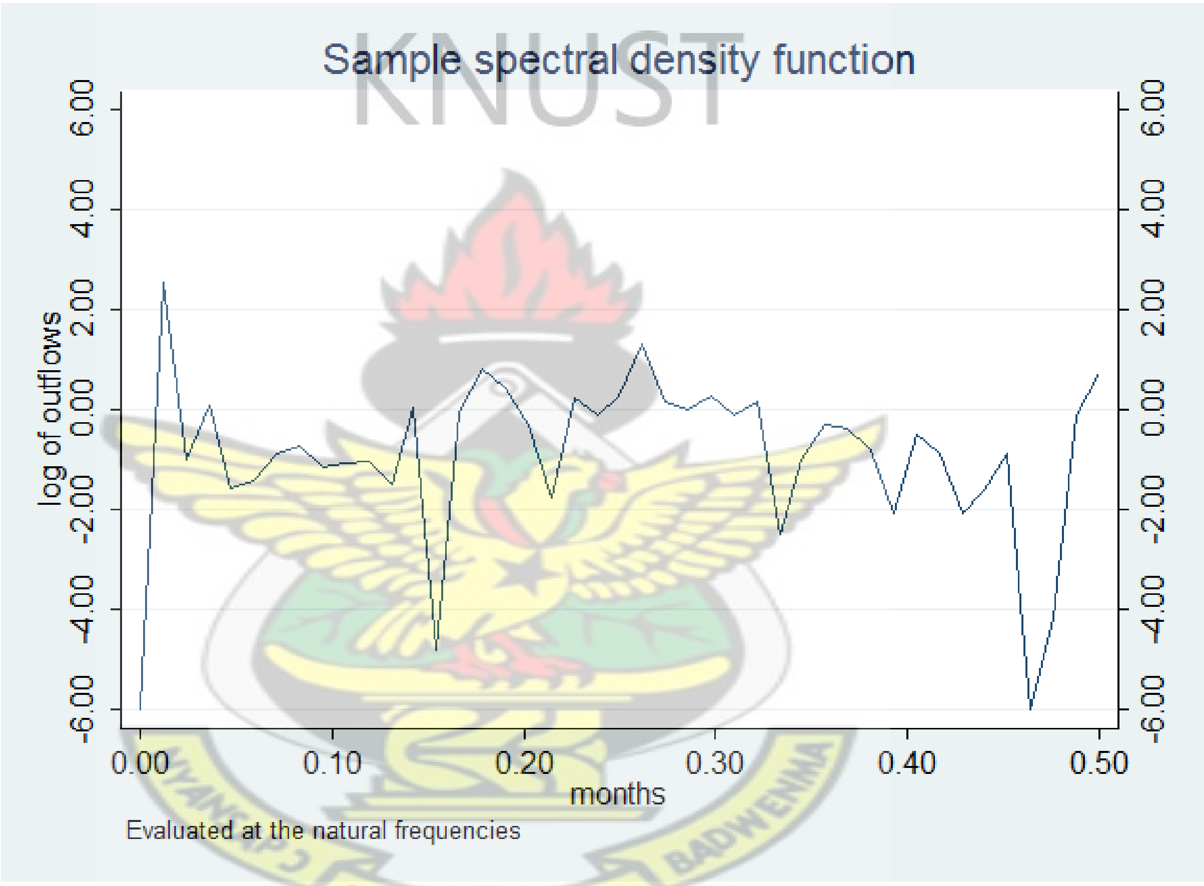

Figure 5.3: Outflows Sample Spectral Density Function