VIBRATION OF HIGH-RISE BUILDINGS

(A CASE STUDY OF UNITY HALL, KNUST)

by

Derick Folson BSc. Computer Science (Hons.)

A thesis submitted to the Department of Mathematics,

Kwame Nkrumah University of Science and Technology

in partial fulfilment of the requirements for the degree

of

MASTER OF SCIENCE

Faculty of Physical Sciences,

College of Science

DECLARATION

I hereby declare that this submission is my own work towards the Master of Science degree and that, to the best of my knowledge it contains no material previously published by another person nor material which has been accepted for award of any other degree of the University, except due acknowledgement has been made in the text

Derick Folson, PG9671206		8th May, 2009
Student's Name& ID	Signature	Date

Certified by:

Mr. E. Agyeman		8/5/09
Supervisor's Name	Signature	Date

Certified by:

Dr. E. Prempeh		11-05-09
Head of Department's Name	Signature	Date

DEDICATION

Dedication to this dissertation is jointly shared by

- My sweet mum, Madam Juliana Agnes Adenkyekye and the entire family, for allowing themselves to be used greatly by the Lord to make me what I am now. God richly bless you.
- My nephews, Kwame Gyimah Addo-Darko, Kwaku Agyaba Afriyie, Kwame Agyaba Afriyie Jnr., and Kofi Agyaba Afriyie, for all the disruptions, which made the research even more challenging and gave me the persistence to carry on.

TABLE OF CONTENTS

Title Page i

Declaration ii

Dedication iii

Table of contents iv

Acknowledgement vi

Abstract vii

CHAPTER ONE 1

1.0 Introduction 1

1.1 Background to the study 1

1.2 Disaster Management 4

1.3 Classification of Disasters 6

1.4 Earthquakes 11

1.5 Earthquakes and Seismicity 13

1.6 Characteristics of Earthquakes 15

1.7 History of Earthquakes 15

1.8 Causes of Earthquakes 19

1.9 Earthquake Measurement 20

1.9.1 Seismic Moment 22

1.9.2 Seismic Energy 25

1.10 Classification of Earthquakes 26

1.11 Building structures 27

1.11.1 Types of structures 29

1.11.1.1 One to Three Storey Buildings 29
1.11.1.2 Multi-Bay and Multi-Storey Buildings 30
1.12 Problem Statement 32
1.13 Objectives 33
1.14 Methodology 33
1.15 Justification 34
1.16 Project limitations 34
1.17 Operational Definitions 35
1.18 Summary 35
CHAPTER TWO 37
2.0 Mathematical models and modeling 37
2.1 Sources of models 38
2.2 Eigenvalues and Eigenvectors 39
2.3 Algorithm Stability of Numerical Eigenvalue Problem 40
2.4 Generalized Eigenvalue Problem 43
2.5 The Eigenvalue of a Pencil 44
2.5.1 The Eigenvalues of a Regular Pencil 44
2.5.2 The Eigenvalues of a Singular Pencil 45
2.6 Floor mass computations 45
2.7 Column/spring constant computations 46
2.8 The Cholesky QR Algorithm 47
2.9 Model Formulation 48
2.9.1 Model Assumptions 50

2.9.2 Displacement under free vibration 50
2.9.3 Absolute and average maximum displacement of a high-rise building 55
2.9.4 Displacement under forced vibration 56
2.10 Solution of Tridiagonal System 62
2.11 Limits of linearity 63
2.12 Summary 64
CHAPTER THREE 65
3.0 Data collection and analysis 65
3.1 Under Free Vibration 66
3.1.1 Case 1: Ignoring Live Load 66
3.1.2 Case 2: Considering Live Load 72
3.2 Algorithm stability test 77
3.3 Under Forced Vibration 77
3.3.1 Case 1: Ignoring Live Load 77
3.3.2 Case 2: Considering Live Load 80
3.4 Simulation of Unity Hall under known forces 82
CHAPTER FOUR 85
4.0 Introduction 85
4.1 Discussion of results 85
4.2 Conclusion 86
4.3 Recommendation 86
REFERENCES 87
APPENDIX (Matlab implementation of the algorithm of the model) 88

ACKNOWLEDGEMENT

The first and most gratitude goes to our Creator and Giver of Life, the Almighty God for giving me strength throughout this harsh period of research.

No text is strictly the creation of its author. I have been dependent from start to finish on Mr. E. Agyeman, a lecturer of Numerical Methods at the postgraduate level at the Mathematics Department of KNUST. My sincere thanks are due to him for enduring endless revisions and wholesale rewrites, helping with content decisions but also giving me free rein when I wanted it.

I would like to acknowledge the support and cooperation I had from my family, for spending so many hours editing and correcting the text and providing me with their variable inputs. I thank them for solidly supporting me throughout this project and keeping their patience even when I lost mine.

There is a saying that some friends are better than brothers, this is the true epitome of a friend, Francis Kwadwo Awuah for his endless support in all my endeavours. God richly bless you for all that you have lost because of me. Such thanks also go to the entire MSc. Mathematics class of 2006/2007 academic year especially, Seidu Ibn Swallah for their immense invaluable assistance to me throughout my study period on campus.

A torrent of appreciation goes to Dr. F. T. Oduro for painfully and yet dedicatedly taking me through this whole project page by page, and providing technical explanations where I lacked.

Of course I accept full responsibility for any errors that might have found their way into the text.

ABSTRACT

Very often, building contractors and civil engineers design and build high-rise buildings oblivious of the maximum possible displacement that each floor can displace due to vibrations. Apparently, building contractors and engineers lay much emphasis on the foundation to the neglect of slabs of each floor of the high-rise building. If the maximum displacement of a high-rise building is known before hand, building experts can run simulations on the building to know the safest amount of load to use on each slab construction that can minimize cost, and yet provide the needed resistance it is designed for.

The structural and sectional drawings of Unity Hall, my case study, were analyzed and the volume of each floor slab computed. The result was multiplied by the density of reinforced concrete to obtain the mass of each slab of the building. The column constants were also computed for use in our problem solving process. The results were then formulated as a symmetric positive generalized eigenvalue problem in terms of mass and stiffness matrices under free vibration and a tridiagonal system under forced vibration. The problem was solved to obtain the maximum possible displacement of each floor of the building. A second solution is obtained by including live load to each floor of the building. The Numerical stability theorem was used to confirm the stability or otherwise of the algorithms employed in this project.

Under free vibration and live load absent, all displacement values fall below 0.5m which shows that the model maintains its linearity under free vibration. However, when live load is present, all displacement values fall below 1.6m, showing that the presence of live

load reduces linearity of the model under free vibration. This goes to suggest that the linearity of the model is compromised when live load is included under free vibration.

Under forced vibration, our model maintains its linearity up to an earthquake of magnitude 1.0 when live load is absent and 1.25 when live load is present. This also confirms that, the presence of students has some effect on the linearity of the model under forced vibration. Above the stipulated magnitudes of 1.0 and 1.25 when live load is absent and present respectively, displacements exceeds 0.5m and 1.6m meaning the linearity assumption becomes no longer tenable and failure of the building becomes eminent.

In spite of the stated magnitude thresholds, it must be noted that if the frequency of the incoming wave coincides with one of the natural frequencies of the building, the amplitude of displacement becomes large, signaling the occurrence of resonance. Where as displacements below 0.5m when live load absent, and 1.6m when live load present, will cause the building to crack, the resonance scenario will cause the building to fail woefully, even to the extent of collapse.

CHAPTER ONE

1.0 INTRODUCTION

Earthquakes are reported quite often in our time. The intensity of earthquakes is gradually increasing with their accompanying destruction. This is more so because the world believes proper land management is achieved when more buildings are compacted into a single or groups of high-rise buildings. Effective though as this may seem, earthquakes has given the world a second thought. This is because if a high-rise building should fall on the epicenter of a quake, the damage is bound to be very enormous as compared to a semi-detached building, for example.

But of particular interest to scientists and engineers is why some high-rise buildings fall under minor earthquake magnitudes while others do not, in spite of the fact that they are of very similar architecture and probably same constructs. This area of research has gain popularity by engineers since the 1989 earthquake in the Marina District of San Francisco, which left some high-rise buildings collapse while others did not. Collapsing buildings are known to cause about eighty percent 80% of earthquake deaths worldwide (Patrick L. Abott, 2006). This calls for immediate attention on our buildings in case of earthquakes especially the high-rise buildings.

1.1 BACKGROUND TO THE STUDY

Earthquakes have been with humanity since time immemorial on this earth. Since 1906 when the off coast of Equador was hit by a devastating earthquake of magnitude 8.8 on the Richter scale with an accompanying heavy loss of lives and property, the world has not relented on its effort to minimize the effect of inevitable future earthquakes. However, all research work in this field has been skewed towards forecasting earthquakes from occurring or knowing before hand when one is bound to occur. In spite of all their effort, the world still experience series of them without their notice Prominent among them was that of the ancient city of Bam in south-eastern Iran, when seventy percent of the city's buildings and about 41,000 people perished representing a third of Bam's population (Microsoft Encarta, 2005).

In our quest to manage our lands judiciously, high-rise buildings, which are known world-wide to be an effective means of curbing the sprouting of estate houses that occupy vast land space that could have been used more profitably. For instance, ten estate houses built for ten separate families can be fused into a one high-rise building of say, ten storeys which will accommodate the same number of families as well as the same land area as one estate house.

Laudable as this idea is, earthquakes are the only threat that makes developers uncertain as to which building style to adapt. This is because in the event of collapse of a high-rise building during earthquake, the degree of disaster will be much higher than a cluster of estate houses. This is so because it is highly impossible for all the cluster of estate houses to fall on the epicenter of an earthquake. High-rise buildings are known to have a higher risk to earthquakes as compared to low buildings because if a single high-rise building falls on the epicenter of an earthquake the loss will be enormous. Efforts in managing and formulating rules that will prevent such buildings from falling in the event of an earthquake have been the headache of many researchers.

Fortunately however, the Mathematical approach has reduced the cost of such research by almost 90% and is even more confident and precise approach than the other means such as the Social Scientists' approach which base most of their findings on available data on disaster pattern of an area. The mathematical approach makes use of Generalized Eigenvalue problems, Simple Harmonic Motion principles, Mathematical Modeling, Differential Equations, and Computational algorithms like the Cholesky factorization.

Almost all eigenvalue problems arising in structural and vibration engineering are of the form $Kx = \lambda Mx$, where M is a symmetric positive definite matrix called the mass matrix, and K is a symmetric matrix called the stiffness matrix. This is called symmetric positive generalized eigenvalue problem

All machines and structures, such as bridges, buildings, and aircraft, possessing mass and stiffness experience vibration to some degree, and their design requires consideration of their oscillatory behavior. Free vibration takes place when a system oscillates due to the forces inherent in the system and without any external forces. Under free vibration such a system vibrates at one or more of its natural frequencies, which are properties of the dynamic system, and depends on the associated mass and stiffness distribution. For forced vibrations, systems oscillate under the excitation of external forces. When such excitation is oscillatory, the system is also forced to vibrate at the excitation frequency.

1.2 DISASTER MANAGEMENT

Almost everyday we witness in the newspaper or on the television, reports of disasters around the world. A disaster is the product of a hazard such as earthquake, flood or windstorm coinciding with a vulnerable situation, which might include communities, cities or villages. There are two main components in this definition: hazard and vulnerability. Without vulnerability or hazard there is no disaster. A disaster occurs when hazards and vulnerability meet. There are several important characteristics that make Disasters different from Accidents. The loss of a sole income earner in a car crash may be a disaster to a family, but only an accident to the community. Variables such as Causes, Frequency, Duration of the Impact, Speed of Onset, Scope of the Impact, Destructive Potential, Human Vulnerability etc determine the difference.

The traditional perception of disaster management has been limited to the idea of "calamity relief", which is seen essentially as a non-plan item of expenditure. However, the impact of major disasters cannot be mitigated by the provision of immediate relief alone, which is the primary focus of calamity relief efforts. Disasters can have devastating effects on the economy for the reason that they cause huge human and economic losses, and can significantly set back development efforts of a region or a State. With the kind of economic losses and developmental setbacks that the country has been suffering year after year, the development process needs to be sensitive towards disaster prevention and mitigation aspects. There is thus the need to look at disasters from a preventive perspective as well.

Disaster management can be defined as the body of policy and administrative decisions and operational activities, which pertain to the various stages of a disaster at all, levels. Broadly disaster management can be divided into pre-disaster and post-disaster contexts. There are three key stages of activity that are taken up within disaster management. They are:

1. Before a disaster strikes **(pre-disaster)**

   Activities taken to reduce human and property losses caused by the hazard and ensure that these losses are also minimized when the disaster strikes. Risk reduction activities are taken under this stage and they are termed as mitigation and preparedness activities.

2. During a disaster **(disaster occurrence)**

   Activities taken to ensure that the needs and provisions of victims are met and suffering is minimized. Activities taken under this stage are called as emergency response activities

3. After a disaster **(post-disaster)**

   Activities taken to achieve early recovery and does not expose the earlier vulnerable conditions. Activities taken under this stage are called as response and recovery activities.

1.3 CLASSIFICATION OF DISASTERS

Disasters come in many form but for the purpose of this study, we categorize them into the following

- **Chemical Emergencies:** Nearly all industries use products containing hazardous materials or chemicals. Improper disposal of nuclear and industrial waste poses most problems to people who leave around them. River sources stand the risk of contamination, which eventually serves as health hazard for communities who use them as their source of drinking water.
- **Dam failure:** Dams all over the world provide great support to humans. These include the production of hydroelectric power, irrigation, potable water, etc Dam failure or levee breeches can occur with little or no warning. They occur when intense storm produces flood in few minutes or even seconds for upstream location. Flash floods occur within six hours of the beginning of a heavy rainfall, and dam failure may occur within hours of the first signs of breaching.
- **Earthquake:** is a series of underground shock waves and movements on the earth's surface caused by natural processes within the earth crust. Earthquakes strike suddenly, violently, and without warning at any time of the day or night. If an earthquake occurs in a populated area, it may cause many deaths and injuries and extensive property damage.
- **Wildfire**: Forest Fire/Wild Fire is one of the destructive natural forces known to mankind. While sometimes caused by lightning, nine out of ten wildfires are human-caused. "Wild Fire" is the term applied to any unwanted and unplanned fire burning

in forest, shrub or grass. Dry conditions at various times of the year and in various parts of the world greatly increase the potential for wildland fires.

- **Flood:** is a temporary inundation of large regions as the result of an increase in reservoir, or of rivers flooding their banks because of heavy rains, high winds, cyclones, storm surge long coast, tsunami, melting snow or dam bursts. Floods are one of the most common hazards in life. However, all floods are not alike. Some floods develop slowly, sometimes over a period of days. But flash floods can develop quickly, sometimes in just a few minutes and without any visible signs of rain. Flash floods often have a dangerous wall of roaring water that carries rocks, mud, and other debris and can sweep away most things in its path. Flooding can also occur when a dam breaks, producing effects similar to flash floods.
- **Hazardous materials:** Chemicals are found everywhere. They purify drinking water, increase crop production, and simplify household chores. But chemicals also can be hazardous to humans or the environment if used or released improperly. Hazards can occur during production, storage, transportation, use, or disposal. Hazardous materials in various forms can cause death, serious injury, long-lasting health effects, and damage to buildings, homes, and other property. Chemical manufacturers are one source of hazardous materials, but there are many others, including service stations, hospitals, and hazardous materials waste sites.
- **Heat:** is a complex phenomenon resulting from a certain combination of temperature, humidity, air movement and duration. Simply stated, a heatwave is an extended period of very high summer temperatures with the potential to adversely affect communities. Heat kills by pushing the human body beyond its limits. In extreme

heat and high humidity, evaporation is slowed and the body must work extra hard to maintain a normal temperature. Older adults, young children, and those who are sick or overweight are more likely to succumb to extreme heat. Conditions that can induce heat-related illnesses include stagnant atmospheric conditions and poor air quality.

- **Hurricane:** A hurricane is a type of tropical cyclone, the generic term for a low-pressure system that generally forms in the tropics. A typical cyclone is accompanied by thunderstorms, and in the Northern Hemisphere, a counterclockwise circulation of winds near the earth's surface. All Atlantic and Gulf of Mexico coastal areas are subject to hurricanes or tropical storms. Parts of the Southwest United States and the Pacific Coast experience heavy rains and floods each year from hurricanes spawned off Mexico. Hurricanes and tropical storms can also spawn tornadoes and microbursts, create storm surges along the coast, and cause extensive damage from heavy rainfall.
- **Landslide:** are slippery masses of rock, earth or debris, which move by force of their own weight down mountain slopes or river banks. In a landslide, masses of rock, earth, or debris move down a slope. Landslides may be small or large, slow or rapid. They are activated by: storms, earthquakes, volcanic eruptions, fires, alternative freezing or thawing, and steepening of slopes be erosion or human modification.
- **Nuclear Power plant emergency:** Nuclear power plants use the heat generated from nuclear fission in a contained environment to convert water to steam, which powers generators to produce electricity. Although nuclear power plants are guarded by very strict and rigid laws and regulations to ensure safety at all times, an accident is inevitable. Such accidents could result in dangerous levels of radiation that could

affect the health and safety of the public living near the nuclear power plant. The potential danger from an accident at a nuclear power plant is exposure to radiation. This exposure could come from the release of radioactive material from the plant into the environment, usually characterized by a plume (cloud-like formation) of radioactive gases and particles. A high exposure to radiation can cause serious illness or death.

- **Terrorism:** Throughout human history, there have been many threats to the security of nations. These threats have brought about large-scale losses of life, the destruction of property, widespread illness and injury, the displacement of large numbers of people, and devastating economic loss. Recent technological advances and ongoing international political unrest are components of the increased risk to national security.
- **Thunderstorm and Lightening:** All thunderstorms are dangerous. Every thunderstorm produces lightning. In the United States, an average of 300 people are injured and 80 people are killed each year by lightning. Although most lightening victims survive, people struck by lightening often report a variety of long-term, debilitating symptoms. Dry thunderstorms that do not produce rain that reaches the ground are most prevalent in the western United States. Falling raindrops evaporate, but lightening can still reach the ground and can start wildfires.
- **Tornado:** Tornadoes are nature's most violent storms. Spawned from powerful thunderstorms, tornadoes can cause fatalities and devastate a neighborhood in seconds. A tornado appears as a rotating, funnel-shaped cloud that extends from a thunderstorm to the ground with whirling winds that can reach 300 miles per hour. Damage paths can be in excess of one mile wide and 50 miles long. Every part of the

world is at some risk from this hazard. Some tornadoes are clearly visible, while rain or nearby low-hanging clouds obscure others. Tornadoes generally occur near the trailing edge of a thunderstorm. It is not uncommon to see clear, sunlit skies behind a tornado.

- **Tsunami:** Tsunamis (pronounced soo-ná-mees), also known as seismic sea waves are a series of enormous waves created by an underwater disturbance such as an earthquake, landslide, volcanic eruption, or meteorite. A tsunami can move hundreds of miles per hour in the open ocean and smash into land with waves as high as 100 feet or more. From the area where the tsunami originates, waves travel outward in all directions. Once the wave approaches the shore, it builds in height. The topography of the coastline and the ocean floor will influence the size of the wave. There may be more than one wave and the succeeding one may be larger than the one before. All tsunamis are potentially dangerous, even though they may not damage every coastline they strike. If a major earthquake or landslide occurs close to shore, the first wave in a series could reach the beach in a few minutes, even before a warning is issued. Areas are at greater risk if they are less than 25 feet above sea level and within a mile of the shoreline. Drowning is the most common cause of death associated with a tsunami.
- **Volcano:** A volcano is a mountain that opens downward to a reservoir of molten rock below the surface of the earth. Unlike most mountains, which are pushed up from below, volcanoes are built up by an accumulation of their own eruptive products. When pressure from gases within the molten rock becomes too great, an eruption occurs. Eruptions can be quiet or explosive. There may be lava flows, flattened landscapes, poisonous gases, and flying rock and ash. Because of their intense heat,

lava flows are great fire hazards. Lava flows destroy everything in their path, but most move slowly enough that people can move out of the way. Fresh volcanic ash, made of pulverized rock, can be abrasive, acidic, gritty, gassy, and odorous. While not immediately dangerous to most adults, the acidic gas and ash can cause lung damage to small infants, to older adults, and to those suffering from severe respiratory illnesses. Ash accumulations mixed with water become heavy and can collapse roofs. Sideways directed volcanic explosions, known as "lateral blasts," can shoot large pieces of rock at very high speeds for several miles. These explosions can kill by impact, burial, or heat. They have been known to knock down entire forests.

- **Winter storm:** Heavy snowfall and extreme cold can immobilize an entire region. Even areas that normally experience mild winters can be hit with a major snowstorm or extreme cold. Winter storms can result in flooding, storm surge, closed highways, blocked roads, downed power lines and hypothermia.

1.4 EARTHQUAKES

Tectonic forces within the earth produce local accumulations of strain that may be released abruptly in the form of seismic energy. Earthquakes result from rapid release of stored elastic strain in the lithosphere, usually in the form of sudden movement of portions of the Earth's crust along faults.

To understand seismicity it is important to consider the physical state of rocks in the solid Earth. Changes in the physical properties of rocks with increasing temperature and pressure (i.e., increasing depth in the Earth) result in decreasing viscosity (a parameter that measures 'resistance to flow') and a transition from 'brittle' to 'ductile' deformation. That is, the outer part of the Earth deforms mainly by fracture, whereas rocks in the deep crust and mantle undergo plastic flow or creep over geologic time scales. The outer (50-100 km) layer of the Earth behaves as more or less rigid ‘plates’ (lithosphere) that can 'drift' on the deeper, less viscous interior (mantle) that is undergoing slow convection.

The boundary between lithosphere and mantle is believed to reflect the change in rheology, which is temperature sensitive. Thus, it is a thermal boundary layer (1300°C isotherm). It may also be a compositional boundary in some places, but in general the lithosphere comprises the crust (oceanic or continental) and the uppermost mantle, which is considered to be 'peridotitic' (olivine-pyroxene rock). Composition of the crust is variable and reflects the geologic processes attending crust formation - essentially mafic (basalt-gabbro) in oceanic regions and sialic (roughly 'andesitic') in continental regions. The crust (especially continental) is lithologically heterogeneous and variable in age (up to 4 billion years old in the oldest regions).

Upwelling mantle convection results in heat transfer toward the surface, which can 'thin' the rigid lithosphere (rift zones), promote melting of the rising mantle rock (mid-ocean ridges or 'hot-spot' volcanoes), and create topographic highs away from which the lithospheric plates tend to drift. Likewise downwelling is associated with subduction of relatively older and cooler oceanic plates.

Interactions between these moving plates, at their margins, create deformation (tectonism) of three major types:

1. **extension** - plates move apart with the intervening space filled by new igneous material (e.g., mid-ocean ridge basalts)
2. **convergence** - plates collide resulting in subduction (underthrusting) of one plate (usually the denser one), intense deformation and uplift (mountain building), and a particular type of magmatism (volcanic arcs)
3. **strike-slip** - horizontal motion as plates slide by one another (San Andreas); note that because plate margins are not usually straight, friction and obstructions result in significant earthquake activity that may be concentrated in the most constricted zones [www.pub.edu], (accessed 2007 January 4).

1.5 EARTHQUAKES AND SEISMICITY

Seismic Waves

Basically, there are two types of waves generated during an earthquake namely, body waves and surface waves.

Body waves travel through the Earth, emanating from the earthquake focus, or **'hypocenter'** (e.g., a ruptured fault). Body waves are useful in determining the surface location above the earthquake source, or **'epicenter'**, and for determining the amount of energy released, or magnitude. Body waves can be further classified as either:

- compressional or P-waves - these have the highest velocity of all seismic waves (6 *km/s*) and are the first (**P**rimary) waves to be recorded by a seismograph after an earthquake
- shear or S-waves - these oscillate perpendicular to the direction of wave propagation, are slower than P-waves (3.5 *km/s*) and arrive later (**S**econdary)

Surface waves travel along or near the Earth's surface, generally arriving later than body waves. Because they travel along the surface, their effects on society may be significant, including structure collapse, mass movement (landslides), disruption of utilities, and secondary effects such as fires.

One of Charles F. Richter's most valuable contributions to seismology was to recognize that the **seismic waves** radiated by all earthquakes can provide good estimates of their magnitudes. He collected the recordings of seismic waves from a large number of earthquakes, and developed a calibrated system of measuring them for magnitude.

Richter showed that, the larger the intrinsic energy of the earthquake, the larger the **amplitude** of ground motion at a given distance. He calibrated his scale of magnitudes using measured maximum amplitudes of shear waves on seismometers particularly sensitive to shear waves with periods of about one second. The records had to be obtained from a specific kind of instrument, called a **Wood-Anderson seismograph**. Although his work was originally calibrated only for these specific seismometers, and only for earthquakes in southern California, seismologists have developed scale factors to extend Richter's magnitude scale to many other types of measurements on all types of seismometers, all over the world. In fact, magnitude estimates have been made for thousands of Moon-quakes and for two quakes on Mars.

Energy released from an earthquake may be recorded as motions of the Earth's surface by a **seismograph**. Examples of seismograms vary in complexity due to dispersal of seismic energy via reflections at velocity discontinuities and refraction (bending) as waves passes through velocity gradients within the Earth.

1.6 CHARACTERISTICS OF EARTHQUAKES

- They are highly unpredictable in nature.
- It shakes the earth crust for long distances.
- Where as some are a little shiver like a truck passing by, others are very frightening.
- Earthquakes happen along the edge of the tectonic plates.
- They are very destructive and tend to destroy any impediment that comes their way.

1.7 HISTORY OF EARTHQUAKES

Our ancestors believed years ago that giant snakes, turtles, catfish, and even spiders living underneath the ground, caused earthquakes as a result of their movements. Ancient people had many fanciful explanations for earthquakes, usually involving something large and restless living beneath the earth's surface.

Aristotle was one of the first to attempt an explanation of earthquakes based on natural phenomena. He postulated that winds within the earth whipped up the occasional shaking of the earth's surface.

Empirical observations of the effects of earthquakes were rare, however, until 1750, when England was uncharacteristically rocked by a series of five strong earthquakes. These earthquakes were followed on Sunday, November 1, 1755, by a cataclysmic shock and tsunami that killed an estimated 70,000 people, leveling the city of Lisbon, Portugal, while many of its residents were in church. This event marked the beginning of the modern era of seismology, prompting numerous studies into the effects, locations, and timing of earthquakes.

Prior to the Lisbon earthquake, scholars had looked almost exclusively to Aristotle, Pliny, and other ancient classical sources for explanations of earthquakes. Following the Lisbon earthquake, this attitude was jettisoned for one that stressed ideas based on modern observations. Cataloging of the times and locations of earthquakes and studying the physical effects of earthquakes began in earnest, led by such people as John Michell in England and Elie Bertrand in Switzerland.

The hundred or so years following the Lisbon earthquake saw sporadic but increasing studies of earthquake phenomena. These efforts were often spurred on by earthquake catastrophes, such as the 1783 Calabrian earthquakes that killed 35,000 people in the southern toe of Italy. (Charles Davison, 1978)

As communication between various parts of the world became more common, earthquake observations from throughout the world could be combined. Following an earthquake in Chile in 1822, Maria Graham reported systematic changes in the elevation of the Chilean coastline. Observations of coastline changes were confirmed following the 1835 Chilean earthquake by Robert FitzRoy, while Charles Darwin was onshore examining the geology of the Andes.

In the 1850s, 60s, and 70s, three European contemporaries made cornerstone efforts in seismology. Robert Mallet, an engineer born in Dublin who designed many of London's bridges, measured the velocity of seismic waves in the earth using explosions of gunpowder. His idea was to look for variations in seismic velocity that would indicate variations in the properties of the earth. This same method is still used today, for example in oil field exploration. Robert Mallet was also one of the first to estimate the depth of an earthquake underground.

At the same time as Mallet was setting off explosions of gunpowder in England, Alexis Perrey, in France, was making quantitative analyses of catalogs of earthquakes. He was looking for periodic variations of earthquakes with the seasons and with lunar phases. And in Italy, Luigi Palmieri invented an electromagnetic seismograph, one of which was installed near Mount Vesuvius and another at the University of Naples. These seismographs were the first seismic instruments capable of routinely detecting earthquakes imperceptible to human beings.

The foregoing work set the stage for the late 1800s and early 1900s, when many fundamental advances in seismology would be made. In Japan, three English professors, John Milne, James Ewing, and Thomas Gray, working at the Imperial College of Tokyo, invented the first seismic instruments sensitive enough to be used in the scientific study of earthquakes.

In the United States, Grove Karl Gilbert, after studying the fault scarp from the 1872 Owens Valley, California earthquake, concluded that the faults were a primary feature of earthquakes, not a secondary one. Until his time, most people thought that earthquakes were the result of underground explosions and that faults were only a result of the explosion, not a primary feature of earthquakes.

Also in the United States, Harry Fielding Reid took Gilbert's work one step further. After examining the fault trace of the 1906 San Francisco earthquake, Reid deduced that earthquakes were the result of the gradual buildup of stresses within the earth occurring over many years. This stress is due to distant forces and is eventually released violently during an earthquake, allowing the earth to rapidly rebound after years of accumulated strain.

The late 1800s and early 1900s also saw scientific inquiry into earthquakes begun by Japanese researchers. Seikei Sekiya became the first person to be named a professor in seismology; he was also one of the first people to quantitatively analyse seismic recordings from earthquakes. Another famous Japanese researcher from that time is Fusakichi Omori, who, among other work, studied the rate of decay of aftershock activity following large earthquakes. His equations are still in use today.

The twentieth century has seen an increased interest in the scientific study of earthquakes, too involved to discuss here. It should be noted, however, that research into earthquakes has broadened and contributions now come from numerous areas affected by earthquakes, including Japan, the United States, Europe, Russia, Canada, Mexico, China, Central and South America, New Zealand, and Australia, among others [www.projects.crustal.ucsb.edu], (accessed 2007 January 11).

1.8 CAUSES OF EARTHQUAKES

In simple language, earthquakes are caused by faulting, a sudden lateral or vertical movement of rock along a rupture (break) surface. The surface of the Earth is in continuous slow motion. This is plate tectonics--the motion of immense rigid plates at the surface of the Earth in response to flow of rock within the Earth. The plates cover the entire surface of the globe. Since they are all moving they rub against each other in some places (like the San Andreas Fault in California), sink beneath each other in others (like the Peru-Chile Trench along the western border of South America), or spread apart from each other (like the Mid-Atlantic Ridge). At such places the motion isn't smooth as the plates are stuck together at the edges and the rest of each plate is continuing to move, so the rocks along the edges are distorted (what we call "strain"). As the motion continues, the strain builds up to the point where the rock cannot withstand any more bending. With a lurch, the rock breaks and the two sides move. An earthquake is the shaking that radiates out from the breaking rock.

People have known about earthquakes for thousands of years, of course, but they didn't know what caused them. In particular, people believed that the breaks in the Earth's surface-faults-which appear after earthquakes were caused by the earthquakes rather than the cause of them. It was Bunjiro Koto, a geologist in Japan studying a 60-mile long fault whose two sides shifted about 15 feet in the great Japanese earthquake of 1871, who first suggested that earthquakes were caused by faults. Henry Reid, studying the great San Francisco earthquake of 1906, took the idea further. He said that an earthquake is the huge amount of energy released when accumulated strain causes a fault to rupture. He explained that rock twisted further and further out of shape by continuing forces over the centuries eventually yields in a wrenching snap as the two sides of the fault slip to a new position to relieve the strain. This is the idea of "elastic rebound" which is now central to all studies of fault rupture (Gerard Fryer, University of Hawaii, Honolulu).

1.9 EARTHQUAKE MEASUREMENT

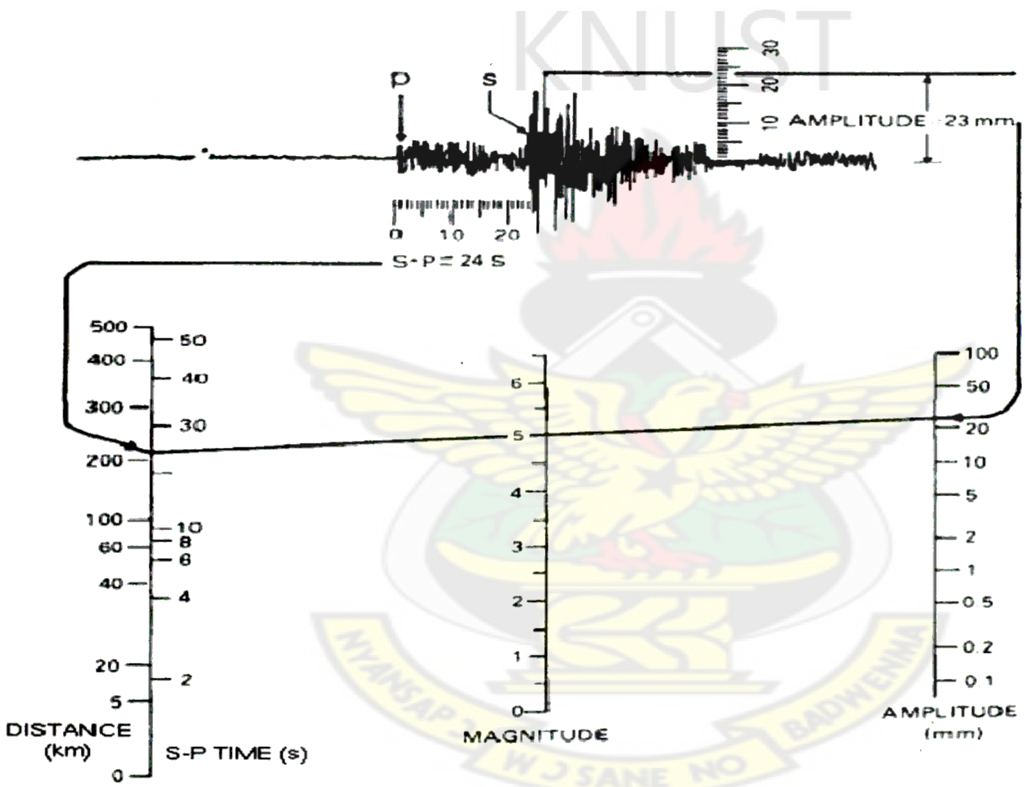
The measurement of an earthquake is very important in seismology. They are measured based on two criteria namely, magnitude and intensity. Magnitude and intensity are scales used to measure the energy and degree of damage that are produced during an earthquake respectively.

Magnitude: a quantitative measurement of the amount of energy released by an earthquake. Each step on the Richter scale represents an increase of 10 times the shaking or rock movement (amplitude) and an increase of 30 times the amount of energy released. The difference between an earthquake that measures 6.5 on the Richter scale and a quake that measures 7.5 is that the 7.5 releases 30 times more energy. It would take 30 magnitude 6.5 quakes to equal the energy output of one 7.5 quake. Even scarier is that, it would take 30 x 20, or 900 magnitude 6.5 quakes to equal just one 8.5 quake.

Intensity: a qualitative assessment of the effects of the earthquake. It is a measure of the degree of observable effects of the movement as well as damage that has occurred using a descriptive scale called Modified Mercalli Scale. The amount of damage is determined by how much the ground shakes, how long the quake lasts, and how well the buildings are constructed.

The surface point above the hypocenter is known as the epicenter. As the distance from an earthquake increases, the arrival time difference between the p-wave and s-waves ('S-P difference') increases proportionately. Thus, epicenter locations can be determined by triangulation using a travel-time curve and data recorded at three or more seismic stations.

The figure below demonstrates hands-on examples of calculating location (as well as Richter magnitude) for real earthquakes using the Richter scale



The scales in the diagram above form what is called a **nomogram** that allows you to do the mathematical computation quickly by eye. The equation for Richter Magnitude is:

$$M_L = \log_{10} A(mm) + (Distance\ correction\ \ factor)$$

Here A is the amplitude, in millimeters, measured directly from the photographic paper record of the **Wood-Anderson** seismometer, a special type of instrument. The *distance factor* comes from a table that can be found in Richter's (1958) book *Elementary Seismology*. The equation behind this nomogram, used by Richter in Southern California, is: $M_L = \log_{10} A(mm) + 3\log_{10}[8\Delta t(s)] - 2.92$

Thus after measuring the wave amplitude you will take its logarithm, and scale it according to the distance of the seismometer from the earthquake, estimated by the S-P time difference Δt in seconds.

Seismologists will then get a separate magnitude estimate from every seismograph station that records the earthquake, and then average them. This accounts for the usual spread of around 0.2 magnitude units that we see reported from different seismological labs right after an earthquake. Each lab is averaging in different stations that they have access to. It may be several days before different organizations will come to a consensus on what was the best magnitude estimate.

1.9.1 SEISMIC MOMENT

Seismologists have more recently developed a standard magnitude scale that is completely independent of the type of instrument. It is called the **moment magnitude**, and it comes from the **seismic moment**.

The idea of the seismic moment is based on elementary physics concept of torque. A torque is a force that changes the angular momentum of a system. It is defined as the force times the distance from the center of rotation. Earthquakes are caused by internal torques, from the interactions of different blocks of the earth on opposite sides of faults.

After some rather complicated mathematics, it can be shown that the moment of an earthquake is simply expressed by:

$$(Moment) = (Rock\ Rigidity) \times (Fault\ Area) \times (Slip\ Dis\tan ce)$$

i.e. $M_o = \mu Ad$

$$(dyne - cm) = \left(\frac{dyne}{cm^2}\right) \times (cm^2) \times (cm)$$

The formula above, for the **moment** of an earthquake, is fundamental to seismologists' understanding of how dangerous faults of a certain size can be.

We will now imagine a chunk of rock on a lab bench, the rigidity, or resistance to shearing, of the rock is a **pressure** in the neighborhood of a few hundred billion dynes per square centimeter. The **pressure** acts over an **area** to produce a force, and it can be seen that the cm-squared units cancel. Now if we guess that the distance the two parts grind together before they fly apart is about a centimeter, then we can calculate the moment, in dyne-cm:

$$M_o = (3 \times 10^{11} \frac{dyne}{cm^2}) \times (10cm) \times (10cm) \times (1cm)$$

$$M_0 = 3 \times 10^{13}\, dyne - cm$$

It is helpful to use scientific notation, since a dyne-cm is really a puny amount of moment.

Now let's consider a second case, the Sept. 12, 1994 Double Spring Flat earthquake, which occurred about 25*km* southeast of Gardnerville. The resultant fault of the quake was 10*km* deep and 15*km* long. The first thing to do, since we are working in centimeters, is to convert the 15-kilometer length and 10*km* depth of that fault to centimeters.

$$\Rightarrow M_o = (3\times10^{11}\frac{dyne}{cm^2})\times(10km)\times\left[\frac{10^5 cm}{km}\right]\times(15km)\times\left[\frac{10^5}{km}\right]\times(30cm)$$

$$M_o = 1.4\times10^{25} dyne-cm$$

Of course this result needs scientific notation even more desperately. We can see that this earthquake, the largest in Nevada in 28 years, had two times ten raised to the twelfth power, or 2 trillion, times as much moment as breaking the rock on the lab table.

There is a standard way to convert a seismic moment to a **magnitude**. The equation is:

$$M_W = \frac{2}{3}\left[\log_{10} M_o(dyne-cm) - 16.0\right]$$

Now let's use this equation (meant for energies expressed in dyne-cm units) to estimate the **magnitude** of the tiny earthquake we can make on a lab table:

$$M_o = 3\times10^{13} dyne-cm \qquad \Rightarrow M_W = \frac{2}{3}\left[\log_{10}(3\times10^{13} dyne-cm) - 16.0\right] \approx -1.7$$

Negative magnitudes are allowed on Richter's scale, although such earthquakes are certainly very small. Next let's take the energy we found for the Double Spring Flat earthquake and estimate its magnitude:

$M_o = 1.4 \times 10^{25}$ $\Rightarrow M_W = \frac{2}{3}\left[\log_{10}(1.4 \times 10^{25}\, dyne - cm) - 16.0\right] \approx 6.1$

The magnitude 6.1 value we get is about equal to the magnitude reported by the UNR Seismological Lab, and by other observers [www.seismo.unr.edu], (accessed 2007 February 3).

1.9.2 SEISMIC ENERGY:

Both the magnitude and the seismic moment are related to the amount of energy that is radiated by an earthquake. Richter, working with **Dr. Beno Gutenberg**, developed a relationship between magnitude and energy. Their relationship is:

$$\log_{10} E_S = 11.8 + 1.5M$$

giving the energy E_S in **ergs** from the magnitude M. Note that E_S is not the total ``intrinsic" energy of the earthquake, transferred from sources such as gravitational energy or to sinks such as heat energy. It is only the amount radiated from the earthquake as seismic waves, which ought to be a small fraction of the total energy transferred during the earthquake process.

More recently, Dr. Hiroo Kanamori came up with a relationship between seismic moment and seismic wave energy. It gives:

$$Energy = (Moment)/20{,}000$$

For this moment is in units of dyne-cm, and energy is in units of ergs. dyne-cm and ergs are unit equivalents, but have different physical meaning.

Let's take a look at the seismic wave energy yielded by our two examples, in comparison to that of a number of earthquakes and other phenomena. For this we'll use a larger unit of energy, the seismic energy yield of quantities of the explosive TNT (Trinitrotoluene):

Table 1.1 Earthquake magnitude and amount of energy released

Richter Magnitude	TNT for Seismic Energy Yield	Example (approximate)
-1.5	6 ounces	Breaking a rock on a lab table
1.0	30 pounds	Large Blast at a Construction Site
1.5	320 pounds	
2.0	1 ton	Large Quarry or Mine Blast
2.5	4.6 tons	
3.0	29 tons	
3.5	73 tons	
4.0	1,000 tons	Small Nuclear Weapon
4.5	5,100 tons	Average Tornado (total energy)
5.0	32,000 tons	
5.5	80,000 tons	Little Skull Mtn., NV Quake, 1992
6.0	1 million tons	Double Spring Flat, NV Quake, 1994
6.5	5 million tons	Northridge, CA Quake, 1994
7.0	32 million tons	Hyogo-Ken Nanbu, Japan Quake, 1995; Largest Thermonuclear Weapon
7.5	160 million tons	Landers, CA Quake, 1992
8.0	1 billion tons	San Francisco, CA Quake, 1906
8.5	5 billion tons	Anchorage, AK Quake, 1964
9.0	32 billion tons	Chilean Quake, 1960
10.0	1 trillion tons	(San-Andreas type fault circling Earth)
12.0	160 trillion tons	(Fault Earth in half through center, OR Earth's daily receipt of solar energy)

1.10 CLASSIFICATION OF EARTHQUAKES

Generally, earthquakes are classified based on their magnitude on the Richter scale. They range from slight to very great as shown in the table below

Table 1.2 Earthquake classifications

Classification	Magnitude on Richter Scale
Slight	Up to 4.9
Moderate	5.0-6.9
Great	7.0-7.9
Very Great	8.0 and above

1.11 BUILDING STRUCTURES

Building Construction refers to procedures involved in the erection of various types of structures. The major trend in present-day construction continues away from handcrafting at the building site and towards on-site assembly of ever larger, more integrated components manufactured away from the site. Another characteristic of contemporary building, related to the latter trend, is the greater amount of dimensional coordination; that is, buildings are designed and components manufactured in multiples of a standard module, which drastically reduces the amount of cutting and fitting required on the building site. A third trend is the production or redevelopment of such large structural complexes as shopping centres, housing estates, entire campuses, and whole towns or sections of cities.

Building construction is the product of private companies and public authorities, with many individuals and organizations involved in the construction of a single structure, from the manufacture of necessary components to final assembly. Building projects by both public and private bodies employ a registered architect or civil engineer under the direction of a project manager, or both, to execute the design and to make sure that it complies with public health, fire, and building regulations as well as the requirements of the owner. The architect or engineer converts these requirements into a set of drawings and written specifications that usually are sent to interested general contractors for bids.

The loads imposed on a building are classified as either "dead" or "live". Dead loads include the weight of the building itself and all major items of fixed equipment. Dead loads always act directly downwards, act constantly, and are additive from the top of the building down. Live loads include wind pressure, seismic forces, vibrations caused by machinery, movable furniture, stored goods and equipment, occupants, and forces caused by temperature changes. Live loads are temporary and can produce pulsing, vibratory, or impact stresses. In general, the design of a building must accommodate all possible dead and live loads to prevent the building from settling or collapsing and to prevent any permanent distortion, excessive motion, discomfort to occupants, or rupture at any point.

The structural design of a building depends greatly on the nature of the soil and underlying geological conditions and human modification of either of these factors.

If a building is to be constructed in an area of a country with a history of earthquake activity, the earth must be investigated to a considerable depth. Faults in the crust of the Earth beneath the soil must obviously be avoided. Certain clay soils have been found to expand 23 cm (9 in) or more if subjected to long cycles of drying or wetting, thus producing powerful forces that can shear foundations and lift lightweight buildings. Some soils with high organic content may, over time, compress under the building load to a fraction of their original volume, causing the structure to settle. Other soils tend to slide under loads.

Soil and geological analyses are necessary, therefore, to determine whether a proposed building can be supported adequately and what would be the most effective and economical method of support.

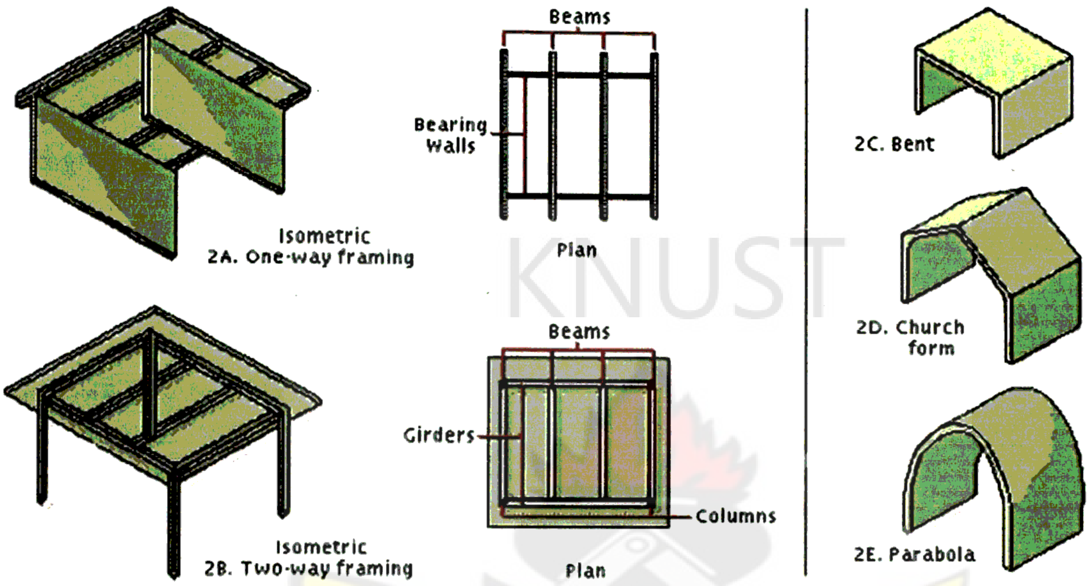
1.11.1 TYPES OF STRUCTURES

1.11.1.1 ONE TO THREE STOREY BUILDINGS

With low buildings the variety of possible shapes is much greater than with taller buildings. In addition to the familiar box shape, which is also used in very tall buildings, low buildings may use cathedral-like forms, vaults, or domes. A simple single-storey structure might consist of a reinforced-concrete slab laid directly on the ground, exterior masonry walls supported by the slab (or by a spread footing cast continuously around the perimeter of the building), and a roof. For low buildings, the use of interior columns between masonry load-bearing walls is still the most common construction method. Spaced columns supported by the slab or by individual spread footings may be used, however; in that case the exterior walls can be supported by or hung between the columns. If the roof span is short, abutting planking made of wood, steel, concrete, or other material can be used to form the roof structure.

As a general rule, the greater the roof span, the more complicated the structure supporting the roof becomes and the narrower the range of suitable materials. Depending on the length of the span, the roof may have one-way framing beams (Figure 2a and 2b) or two-way framing (beams supported on larger girders spanning the longest dimension).

Trusses can be substituted for either method. Trusses, which can be less than 30 cm or more than 9 m deep, are formed by assembling tension and compression members in various triangular patterns. They are usually made of timber or steel, but reinforced concrete may be used.



1.11.1.2 MULTI-BAY AND MULTI-STOREY BUILDINGS

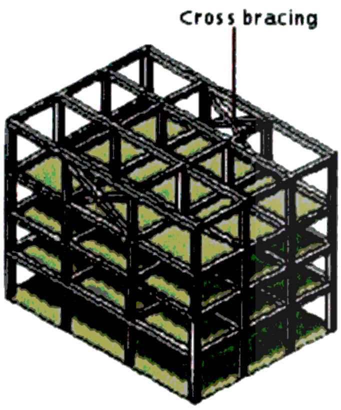
By far the most common form of building structure is the skeleton frame, which consists essentially of the vertical members shown in Figure 2a, 2b, and 2c, combined with a horizontal framing pattern. For tall buildings, the use of load-bearing walls (as in Figure 2a) with horizontal framing members has declined steadily; nonload-bearing curtain walls are used most frequently.

For structures up to 40 storeys high, reinforced concrete, steel, or composite-reinforced concrete and steel can be used in a variety of ways. The basic elements of the steel skeleton frame are vertical columns, horizontal girders spanning the longer distance between columns, and beams spanning shorter distances (Figure 3a). Lateral stability is provided by connecting the beams, columns, and girders; by the support given the structure by the floors and interior walls; and by diagonal bracing or rigid connections between columns, girders, and beams. Reinforced concrete can be used in a similar way, except that concrete shear walls would be used instead of diagonal bracing to provide lateral stability.

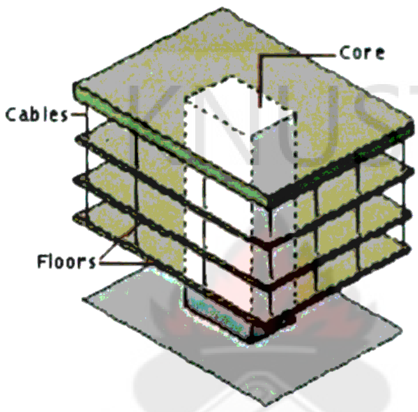
In hanging (Figure 3b), a vertical utility core is built, and strong horizontal roof framing is anchored to the top of the core. Stacking (Figure 3c) is a construction technique in which prefabricated, boxlike units are raised by cranes, placed on top of and alongside each other, and then fastened together.

For buildings over 40 storeys high, steel had been considered the most appropriate material. However, recent advances in the development of high-strength concretes have made concrete competitive with steel. Tall buildings often require more sophisticated structural solutions to resist lateral loads, such as wind and, in some countries, earthquake forces. One of the more popular structural systems is the exterior structural tube, which was used in the construction of the World Trade Center (417*m*/1,368*ft*) in New York. Here, closely spaced columns connected rigidly to the horizontal spandrel beams on the perimeter of the building provided sufficient strength to resist loads and the stiffness to minimize lateral deflections. The structural tube has now been used with concrete and with composite construction consisting of structural steel members encased in reinforced concrete.

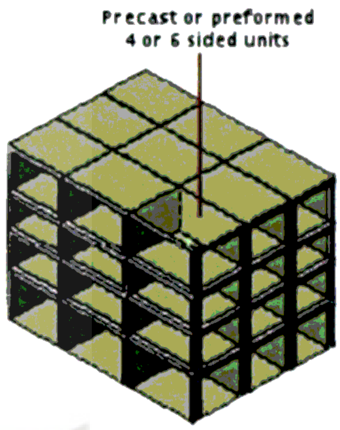
For very tall buildings, the mixing of steel and concrete is commonly used. The high strength-to-weight ratio of steel is excellent for the horizontal spanning members. High-strength concretes can economically provide the compression resistance needed for vertical members. In addition, the mass and internal damping properties of the concrete assist in minimizing vibration effects, which are potential problems in very tall buildings (Microsoft Encarta, 2005).



3a. Skeleton frame



3b. Cable hung



3c. Stacking

1.12 ROBLEM STATEMENT

There has been a lot of research into earthquakes worldwide but all seem to focus on preventing them from occurring, how they occur, causes of it, and predicting when they will occur and even their magnitude on the Richter scale. Not only is such researches capital intensive but also they are abnormally involving and prone to errors. In all these research the effect on certain buildings based on their specific materials construct are not considered. Where as some believe that building firmly rooted to the ground do not suffer much in the event of earthquakes, other researchers believe that it is rather the quality of materials used in the construct that gives the best resistance to a building.

As Ghana has not experienced earthquakes of magnitude higher than 4.9 on the Richter scale, all our attention should not be directed towards preventing earthquakes from occurring but how to manage and prevent losses when they do occur. Not all buildings collapse even at the epicenter and neighbourhood of an earthquake. Why such buildings do not collapse and why very similar ones do is of interest in this research.

The precise possible damage to any high-rise building and the average maximum relative displacement of each floor will the looked into. We will then attempt to make firm decision on the building standards to adapt based on computed values in this project.

1.13 OBJECTIVES

- To model the dynamics of a high-rise building.
- To analyze the possible displacement of various floors of a high-rise building.

1.14 METHODOLOGY

- To formulate the model as a symmetric definite generalized eigenvalue problem in terms of mass and stiffness matrices under free vibration, and a tridiagonal system under forced vibration.
- Use the structural and detailed drawings of a high-rise building as our source of data, as well as the building standards adapted by architects and engineers.
- Run MATLAB-based simulation on the model.

1.15 JUSTIFICATION

Development of a country has swayed from the building of micro and macro stability to the sprouting of edifices for the benefit of the citizens. Such edifices have brought enormous pressure on lands in this country especially in our towns and cities. Unfortunately however, most of these edifices could have been combined into one giant edifice to make more lands available for the nation.

It is therefore important to know the risk levels of high-rise buildings so that we may not be caught off-guard. Mathematical methods will provide the necessary tools to draw sound and safe conclusion on any high-rise building. Such knowledge will help us to decide on which type of high-rise building should be adapted at any particular place based on the known likelihood of an earthquake.

1.16 PROJECT LIMITATIONS

1. The mass of each floor is about 95% accurate. This is because the composite of other materials made use of in the slab construction is not known. These materials include asbestos, felt, and linings for electrical fittings among others which may constitute less than 5% of the slab weight.
2. The relative response to vibrations of each floor was not known and was consequently assumed.

1.17 OPERATIONAL DEFINITIONS

Table 1.3 Operational definitions

Terminology	Definition
Epicenter	It is the point on the (free) surface of the earth vertically above the place of origin (hypocenter) of an earthquake.
Hypocenter or Focus	It is the point with in the earth from which seismic waves originate. Focal depth is the vertical distance between the hypocenter and epicenter.
Magnitude	It is the quantity to measure the size of an earthquake in terms of its energy.
Richter Scale	This is the scale of measure of magnitude of an earthquake.
Intensity	It is the rating of the effects of an earthquake at a particular place, using the Modified Mercalli Scale.
Plate Tectonics	This refers to the motion of immense rigid plates at the surface of the earth in response to flow of rock within the earth.
Seismology	It is a branch of geophysics involving the observation of seismic waves from natural and artificial ground vibrations.
Seismic waves	Vibrations that travel outward from the earthquake fault at speeds of several miles per second. Although fault slippage directly under a structure can cause considerable damage, the vibrations of seismic waves cause most of the destructions during earthquakes.
Faults	The fracture across which displacement has occurred during an earthquake. The slippage may range from less than an inch to 10 yards in a severe earthquake.
Aftershock	An earthquake of similar intensity that follows the main earthquake.
Tectonic force	This is the force that emerges as a result of the interactions between moving plates, at their margins and create deformation.
Seismogram	It is a written or digital recording of energy released from an earthquake as motions of the Earth's surface.

1.18 SUMMARY

Disaster is defined as: "a serious disruption of the functioning of a society, causing widespread human, material, or environmental losses which exceed the ability of the affected society to cope using its own resources." Disaster management can be defined as the body of policy and administrative decisions and operational activities, which pertain to the various stages of a disaster at all, levels.

Disasters are classified into Chemical emergency, Dam failure, Earthquake, Heat, Flood, Hazardous material, Landslides, Fire, Nuclear Power Plant emergency, Terrorism, Thunderstorm, Tornado, Tsunami, Volcano, and Winter storm.

The two types of waves generated during an earthquake are body waves and surface waves. Energy released from an earthquake may be recorded as motions of the Earth's surface by a **seismograph**. The measurement of an earthquake is based on magnitude and intensity. Magnitude and intensity are scales on Richter and Modified Mercalli used to measure the energy and degree of damage produced during an earthquake respectively.

CHAPTERTWO

2.0 MATHEMATICAL MODELS AND MODELING

It is very common to come across word problems in mathematics solved to obtain a single formula or expression. This is only part of mathematical modeling. The broader view of mathematical modeling is that it is about using mathematics to explore topics outside of mathematics. The intimate connection between science and mathematical modeling is eloquently stated by the great contemporary physicist Stephen Hawking as "…take the view…that a theory of physics is just a mathematical model that we use to describe the results of observations. Theory is a good theory if it is an elegant model, if it describes a wide class of observations, and if it predicts the results of new observations"

Mathematical models arise in every field of science. Although the connections between models and physical phenomena in other sciences are not always as strong as in physics, it remains useful to think of theories of science in terms of mathematical models.

A mathematical model is a set of formulas and/or equations based on a quantitative description of real phenomena and created in the hope that the behaviour it predicts will resemble the real behaviour on which it is based. With this definition, a mathematical model could be as simple as a single formula relating two variables or as complicated as a set of equations describing the relationships between a set of unknowns

Mathematical modeling is the art and science of constructing mathematical models and using them to gain insight into physical processes or to make predictions concerning physical processes. The science lies in constructing the mathematical model from the conceptual model, which is an idealized characterization of a real-world situation, and the art lies in determining an appropriate conceptual model.

2.1 SOURCES OF MODELS

It is very common to come across expressions like $y = y_0 - 16t^2$ describing the motion of an object thrown in air or $p = x^3 - 4x^2 - 2x + 12$ also describing a production function, where

$y =$ position of object at any given time t (dependent variable)

$y_0 =$ position of object at time $t = 0$

$t =$ time (independent variable)

$p =$ quantity of the items produced

$x =$ resources employed in the production

The question now becomes, where do such formulas come from? The truth of the matter is that such expressions come from series of experiments conducted at a well equipped physics laboratory. Others also come from physical laws.

Physical Laws

A physical law is a statement that expresses the relationship between quantities closely enough to be taken as exactly true. To qualify as a physical law, a relationship must be observed in a large variety of settings. Scientists do not use measurements only to obtain formulas for specific experiments. A bigger goal is to try to determine basic principles that underlie the measured behaviour. These basic principles are stated in the form of physical laws such as Newton's second law of motion, which is generally given as,

$$F = ma$$

where F is the net force on the object, m is the object's mass, and a is the acceleration that results from the force.

From $Acceleration = \frac{\Delta(velocity)}{\Delta(time)} = \frac{\frac{\Delta(distance)}{\Delta(time)}}{\Delta(time)}$ $\qquad \Rightarrow a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

$$\Rightarrow F = m\frac{d^2x}{dt^2}$$

This expression is a very common modeling formula that we will make use of in our task.

2.2 EIGENVALUES AND EIGENVECTORS

It is a known fact in vector computations that the multiplication of an $n \times n$ matrix by an $n \times 1$ vector yields a new $n \times 1$ vector. Mathematically,

$$A\vec{\eta} = \vec{y}$$

If it is possible to find a constant λ and a vector $\vec{\eta}$ such that

$$A\vec{\eta} = \lambda\vec{\eta}$$

then we shall call λ an eigenvalue and $\vec{\eta}$ the eigenvector of A.

$$\Rightarrow (A - \lambda I)\vec{\eta} = \vec{0}$$

There are two possible solutions to the above equation namely $\vec{\eta} = \vec{0}$ and the second will be an infinitely many nonzero solutions. To find the second case, we will need to determine the values of λ for which we will get

$$\det(A - \lambda I) = 0$$

Once we have the eigenvalues we can then go back and determine the eigenvectors for each eigenvalue. Let's take a look at a couple of quick facts about eigenvalues and eigenvectors.

- **Fact 1**: If A is an $n \times n$ matrix then $\det(A - \lambda I) = 0$ is an n^{th} degree polynomial called the characteristic polynomial.
- **Fact 2**: If $\lambda_1, \lambda_2, ..., \lambda_n$ is the complete list of eigenvalues for A (including all repeated eigenvalues) then
  1. If λ occurs only once in the list then we call λ simple.
  2. If λ occurs $k > 1$ times in the list then we say that λ has multiplicity k.
  3. If $\lambda_1, ..., \lambda_k \ (k \le n)$ are the simple eigenvalues in the list with corresponding eigenvectors $\vec{\eta}^{(1)}, \vec{\eta}^{(2)}, ..., \vec{\eta}^{(k)}$ then the eigenvectors are all linearly independent.
  4. If λ is an eigenvalue of $k > 1$ then λ will have anywhere from 1 to k linearly independent eigenvalues. (Data, 2000).

2.3 ALGORITHM STABILITY OF THE NUMERICAL EIGENVALUE PROBLEM

A matrix A is said to be stable if the eigenvalues of A remain the same or very, very close to it when an entry in A is changed by a very minute amount. In other words, if B represents the matrix which is very close to A as a result of a minute change in an entry of matrix A, then their eigenvalues must be the same in other for us to conclude that the matrix A is stable. Otherwise, it is unstable.

For example, consider matrices $A = \begin{pmatrix} 1 & 1000 \\ 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 1000 \\ -0.001 & 1 \end{pmatrix}$

Matrix B differs from A by only 0.001 in the second row and first column. The eigenvalues of A are 1 with multiplicity of two, but B has no real eigenvalues at all since the characteristic polynomial is $\lambda^2 - 2\lambda + 2$.

What this tells us is that when error enters into the calculation of the entries of a matrix A and we later try to calculate the eigenvalues of A, we must view the results carefully. But if A is symmetric, small changes in A will generally not lead to large changes in the eigenvalues. So in applications involving symmetric matrices, numerical methods are generally quite successful in computing the needed eigenvalues

To make a concrete statement, we define the Frobenius norm of a matrix A as $\| A \|_F = \sqrt{\sum_{1 \le i,j \le n} | a_{ij} |^2}$

STABILITY THEOREM. Let A be an $n \times n$ matrix, and let E be an $n \times n$ ``error matrix." Suppose that $A$ and $E$ are real and symmetric, and set $\bar{A} = A + E$ (that is, $\bar{A}$ is the ``error version" of A). Let $\lambda_1, \ldots, \lambda_n$ be the eigenvalues of A and $\bar{\lambda}_1, \ldots, \bar{\lambda}_n$ be the eigenvalues of $\bar{A}$, then $(\lambda_1 - \bar{\lambda}_1)^2 + (\lambda_2 - \bar{\lambda}_2)^2 + \ldots + (\lambda_n - \bar{\lambda}_n)^2 \le \| E \|_F^2$.

STABILITY COROLLARY. With the same hypotheses of the stability theorem, $| \lambda_k - \bar{\lambda}_k | \le \| E \|_F$ for $k = 1,2,\ldots,n$. This means that the process of finding $\lambda_1, \ldots, \lambda_n$ is stable: Small errors in $A(\| E \|_F$ small) lead to small errors in determination of $\lambda_1,\ldots,\lambda_n(|\lambda_k - \bar{\lambda}_k|$ small). Thus $\sqrt{(\lambda_k - \bar{\lambda}_k)^2} \leq \| E \|_F$

This corollary tells us that if$\| E \|_F$ is less than or equal to a small number then all the computed eigenvalues $\bar{\lambda}_k$ of $A + E$ will be within of the true eigenvalues λ_k of A. That is, small error in real symmetric matrix A leads to small absolute error in a calculation of the eigenvalues. Our initial illustration does not involve a symmetric matrix so let us try one now.

Suppose that we want to calculate the eigenvalues of $A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$ in a computer, which keeps only six significant digits. Assuming the error of 0.000001 in each entry of A, the bound error in calculating the eigenvalues of A is found as follows

Let $E = \begin{pmatrix} \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon \end{pmatrix}$ where $\varepsilon = 0.000001$. Then by the stability corollary

$|\lambda - \bar{\lambda}_k| \leq \sqrt{9(0.000001)^2} = 0.000003$

Of course, this estimate neglects errors introduced by calculations used in a particular method of computing eigenvalues.

The main point has been made: In general, small errors in A need not lead to small errors in computation of $\lambda_1,\ldots,\lambda_n$; however, if A is symmetric, with symmetrically distributed error, then small errors in A will lead to small absolute errors in determination of $\lambda_1,\ldots,\lambda_n$.

Finally, even if the absolute error is small $|\lambda - \bar{\lambda}_k| \leq \varepsilon$ where ε is small, the relative error could be large. For instance, if $\lambda_k = \varepsilon/2$, then $\frac{|\lambda_k - \bar{\lambda}_k|}{|\lambda_k|} = 200\%$ (Rice, 1983)

2.4 GENERALIZED EIGENVALUE PROBLEM

Definition

Given $n \times n$ matrices A and B, find scalars λ and nonzero vectors x such that $Ax = \lambda Bx$. λ is called an eigenvalue, and the vector x is an eigenvector associated with λ for the generalized eigenvalue problem.

It is clear to see that λ is a root of the characteristic equation $\det(A - \lambda B) = 0$. The matrix $A - \lambda B$ is called a matrix pencil and it is conveniently denoted by (A, B).

The rather strange use of the word pencil comes from optics and geometry. An aggregate of (light) rays converging to a point does suggest the sharp end of a pencil and, by a natural extension, the term came to be used for any one-parameter family of curves, spaces, matrices, or other mathematical objects.

Most engineering applications give rise to generalized eigenvalue problems. A majority of eigenvalue problems arising in mechanical vibration are generalized eigenvalue problems. For example eigenvalue problems for vibrations of structures, which is the subject of concern of this project are called symmetric definite generalized eigenvalue problems for the mass and stiffness matrices; $Kx = \lambda Mx$, where M is the mass matrix and K is the stiffness matrix. M and K are usually real symmetric and, furthermore, M is symmetric positive definite.

Definition

If A and B are real symmetric matrices and, furthermore, if B is positive definite, then the generalized eigenvalue problem $Ax = \lambda Bx$ is called the symmetric definite generalized eigenvalue problem.

2.5 THE EIGENVALUE OF A PENCIL

The pencil pair (A, B) is called regular if $\det(A - \lambda B)$ is not identically zero. Otherwise, it is called singular.

2.5.1 The Eigenvalues of a Regular Pencil

Consider a the following conditions for a regular pencil (A, B)

Case 1: If B is nonsingular, then all the eigenvalues of the pair (A, B) are finite and are the same as those of AB^{-1} or $B^{-1}A$

Case 2: If B is singular, then the degree of $p(\lambda) = \det(A - \lambda B)$ is less than n. Let it be r. The r zeros of $p(\lambda)$ are the eigenvalues of the pair (A, B). The convention is to set the $(n-r)$ remaining eigenvalues to be ∞.

2.5.2 The Eigenvalues of a Singular Pencil

If (A,B) is a singular pencil, then because $\det(A-\lambda B)$ vanishes identically, any number λ can be an eigenvalue of (A,B). (Chapra and Canale, 1988, Atkinson, 1993).

2.6 FLOOR MASS COMPUTATIONS

A sample of reinforced concrete used in the floor construction will be taken. This sample will be taken to the lab to find its weight and volume. We will then compute the density of the sample to know the density of reinforced concrete used.

A whole floor of our building is depicted in the diagram below. It is a rectangular box. $B=\{(x,y,z)\mid a\le x\le b, c\le y\le d, r\le z\le s\}$ with function *f*.

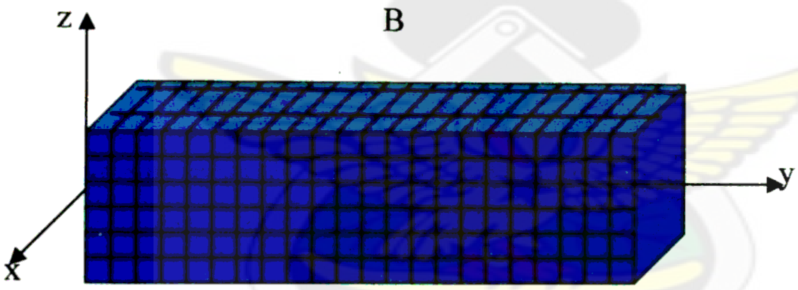


Figure 2.2 Slab volume computation diagram

The first step is to divide B into sub-boxes. We do this by dividing the interval $[a,b]$ into l subintervals $[x_{i-1},x_i]$ of equal width Δx, dividing $[c,d]$ into m subintervals $[y_{j-1},y_j]$ of width Δy, and dividing $[r,s]$ into n subintervals $[z_{k-1},z_k]$ of width Δz. The planes through the endpoints of these subintervals parallel to the coordinate planes divide the box B into lmn sub-boxes.

$B_{ijk}=[x_{i-1},x_i]\times[y_{j-1},y_j]\times[z_{k-1},z_k]$

which are shown in figure 2.6. Each sub-box has volume $\Delta V = \Delta x \Delta y \Delta z$.

Then the triple Riemann sum is given as $\sum_{i=1}^{l}\sum_{j=1}^{m}\sum_{k=1}^{n} f(x_{ijk}^{*}, y_{ijk}^{*}, z_{ijk}^{*})\Delta V$

where the sample point $(x_{ijk}^{*}, y_{ijk}^{*}, z_{ijk}^{*})$ is in B_{ijk}

The triple integral of f over the box B is

$$\iiint_B f(x,y,z)dV = \lim_{l,m,n\to\infty} \sum_{i=1}^{l}\sum_{j=1}^{m}\sum_{k=1}^{n} f(x_{ijk}^{*}, y_{ijk}^{*}, z_{ijk}^{*})\Delta V \quad \text{if this limit exists.}$$

The practical method for evaluating triple integrals is to express them as iterated integrals as follows

Definition

If f is continuous on the rectangular box $B = [a,b]\times[c,d]\times[r,s]$, then

$$\iiint_B f(x,y,z)dV = \int_r^s \int_c^d \int_a^b f(x,y,z)dxdydz$$ (Stewart, 1999).

Once the volume of a whole floor is computed, we can find the mass of that particular floor using the basic formula $mass = density \times volume$

2.7 COLUMNS/SPRING CONSTANTS COMPUTATIONS

The column of the building in our model acts as the spring when it is compared with a spring-mass system. From basic mechanics the angular velocity of an oscillating body is $\omega = 2\pi f$. It can also be shown that the period of oscillation of a spring-mass pendulum with mass m and spring constant k is given by $T = 2\pi\sqrt{\frac{m}{k}}$

Assume the period of oscillation of a simple pendulum of length l and the same particle of mass m is also given as $T = 2\pi\sqrt{\frac{l}{g}}$

$\Rightarrow \frac{m}{k} = \frac{l}{g}$ $\qquad\qquad \therefore k = \frac{mg}{l}$

This means that if the mass of a particular floor is known, the spring/column constant can be computed for each floor of a high-rise building as follows

$k_i = \frac{gm_i}{l}$, for $i = 1,2,3,...$

where $l = length\ of\ floor\ (in\ meters)$

$m = floor\ mass\ (in\ kilograms)$

$g = acceleration\ due\ to\ gravity$

2.8 THE CHOLESKY QR ALGORITHM

Because the eigenvalues of a matrix A are the zeros of the characteristic polynomial $\det(A - \lambda I)$, one would naively think of computing the eigenvalues of A by finding its characteristic polynomial and then computing its zeros by a standard root-finding method. Unfortunately, this is not a practical approach.

A standard practical algorithm for finding the generalized eigenvalues of a symmetric definite pencil is the Cholesky QR algorithm.

Method:

Given a generalized eigenvalue problem $Ax = \lambda Bx$, since B is symmetric positive definite, it admits the Cholesky decomposition $B = LL^T$

$\Rightarrow Ax = \lambda LL^T$

$\Leftrightarrow L^{-1}A(L^T)^{-1}x = \lambda x$

Multiplying through by L^T

$\Rightarrow L^{-1}A(L^T)^{-1}L^T x = \lambda L^T x$

Let $C = L^{-1}A(L^T)^{-1}$ and $y = L^T x$

$\Rightarrow Cy = \lambda y$

$\Leftrightarrow \det(C - \lambda I) = 0$

Definition

Given the symmetric definite pencil (A, B) the corresponding eigenvalues λ_i, and eigenvectors x_i, $i = 1,2,3,\ldots,n$ are computed as follows

- **Step 1:** Find the Cholesky factorization of *B*, such that $B = LL^T$
- **Step 2:** Form $C = L^{-1}A(L^T)^{-1}$ by taking advantage of the symmetry of A.
- **Step3:** Compute the eigenvalues λ_i, $i = 1,2,\ldots,n$ and the orthonormal eigenvectors y_i, $i = 1,2,\ldots,n$ of the symmetric matrix C of the pair (A, B).
- **Step 4:** Compute the eigenvectors x_i corresponding to the eigenvalues λ_i of the pencil (A, B) by solving $L^T x_i = y_i$, $i = 1,2,\ldots,n$

(Householder, 1964, Data, 2000)

2.9 MODEL FORMULATION

Damping Force: No matter the velocity with which an object is released, it will definitely come to rest at a certain time *t*. The object will come to a halt because of damping force acting on it. A damping force is a force that acts to oppose an object's motion. They act in opposite direction to the released item until eventually when the velocity of the object reaches zero, at which point the damping force vanishes.

Restoring Force: When an elastic band is used to tie an object and released while holding the band, the object moves to a certain point and stops momentarily. It will then return to its original point of released without pulling the object. This is possible because the elastic band exerts a restoring force on the object. A restoring force is a force that acts to oppose an object's displacement. The magnitude of the force is an increasing function of displacement rather than speed.

Resonance: It is an effect caused by a vibrating body, setting another body into vibration both at their natural frequencies. In other words, if the frequency of the imposed periodic force is equal to or nearly equal to one of the natural frequencies of a system, resonance results.

Amplitude: The amplitude of a motion is the maximum value of its displacement.

Let us consider a system of n spring masses $m_1, m_2, ..., m_n$, with corresponding spring constants $k_1, k_2, ..., k_n$ under an excitation by a harmonic force $F_1 \sin \omega t$. This system of masses is compared to a block of n high-rise building with the floor masses corresponding to the spring masses. The columns of the building structure correspond to the spring of our model. Hence the model for the spring masses will apply for the high-rise building.

2.9.1 MODEL ASSUMPTIONS

The following assumptions are going to be employed in our model formulation

1. The weight concentration of the building is uniformly concentrated weight at each floor.
2. The weights of the column walls were assumed to be negligible.
3. Displacement is very small. This is to maintain the liner nature of our model.
4. Lateral displacement is assumed rather than vertical displacement.

2.9.2 DISPLACEMENT UNDER FREE VIBRATION

Free vibration takes place when a system oscillates due to the forces inherent in the system and without any external forces. Under free vibration such a system vibrates at one or more of its natural frequencies. Since there is no external force on the system, we set $F(t,y)=0$.

Applying Newton's second law of motion gives

$Total\ \ Force = \text{Re}\,storing\ \ Force + External\ \ Force$

Mathematically,

$$m\frac{d^2 y}{dt^2} = -ky + F(t,y)$$

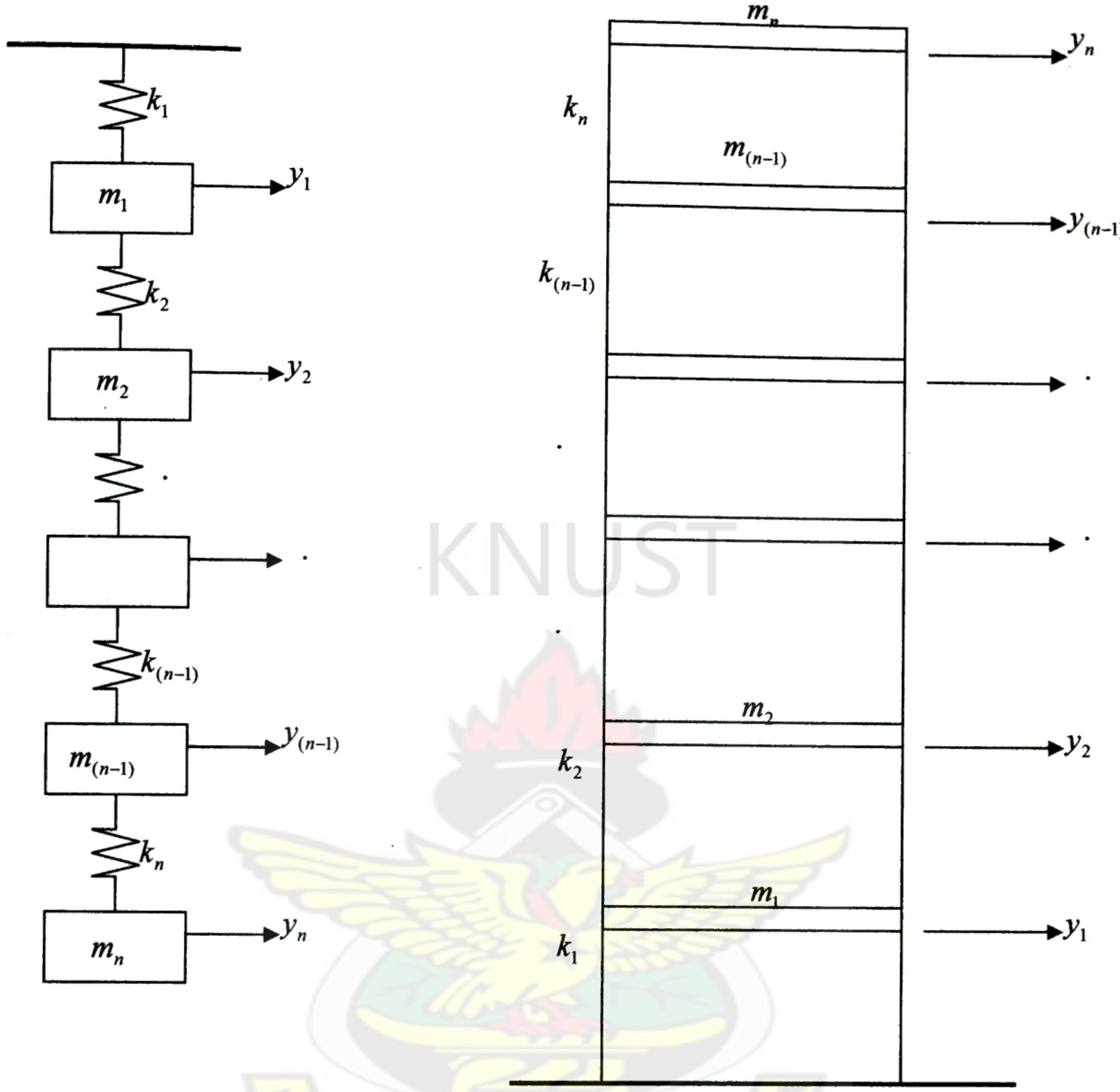


Figure 2.3 Comparison of a building to a helical spring-mass with no external force

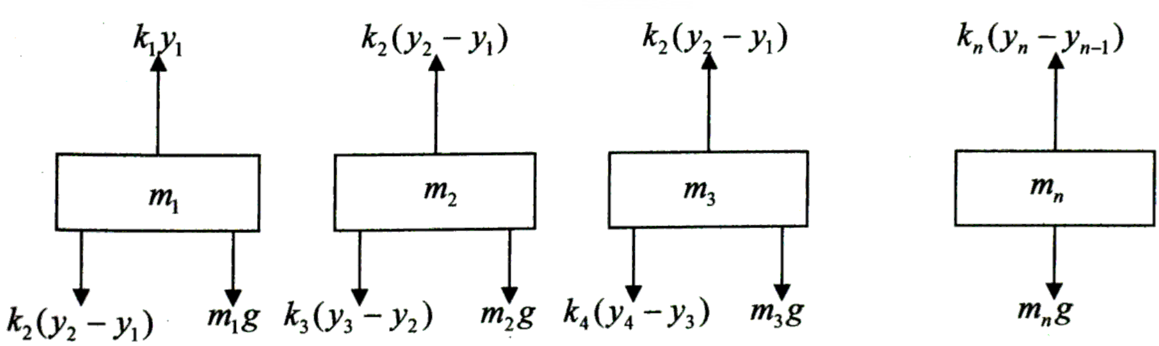


Figure 2.4 Free body diagram for the spring-mass system without external force

From the free body diagram, the equations of motion, by Newton's second law, are:

$$\left.\begin{array}{ll}
m_1\ddot{y}_1 = -k_1 y_1 + k_2(y_2 - y_1) + m_1 g & \text{(1)} \\
m_2\ddot{y}_2 = -k_2(y_2 - y_1) + k_3(y_3 - y_2) + m_2 g & \text{(2)} \\
m_3\ddot{y}_3 = -k_3(y_3 - y_2) + k_4(y_4 - y_3) + m_3 g & \text{(3)} \\
\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot & \\
\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot & \\
m_{n-1}\ddot{y}_{n-1} = -k_{n-1}(y_{n-1} - y_{n-2}) + k_n(y_n - y_{n-1}) + m_{n-1} g & \text{(n-1)} \\
m_n\ddot{y}_n = -k_n(y_n - y_{n-1}) + m_n g & \text{(n)}
\end{array}\right\} \text{(a)}$$

$y_i(t_0) = 0$ for all $i = 1,2,3,\ldots,n$

Suppose we are interested in knowing the displacement of these springs when the system is in an unsteady state-that is when the system attains maximum acceleration. Then by setting the acceleration due to gravity to zero, we obtain: (Franklin, 1968)

$$\left.\begin{array}{ll}
m_1\ddot{y}_1 = -k_1 y_1 + k_2(y_2 - y_1) & (1) \\
m_2\ddot{y}_2 = -k_2(y_2 - y_1) + k_3(y_3 - y_2) & (2) \\
m_3\ddot{y}_3 = -k_3(y_3 - y_2) + k_4(y_4 - y_3) & (3) \\
\cdot \quad \cdot \quad \cdot & \\
\cdot \quad \cdot \quad \cdot & \\
m_{(n-1)}\ddot{y}_{(n-1)} = -k_{(n-1)}(y_{(n-1)} - y_{(n-2)}) + k_n(y_n - y_{n-1}) & (n-1) \\
m_n\ddot{y}_n = -k_n(y_n - y_{(n-1)}) & (n)
\end{array}\right\} \text{(b)}$$

Because we are going to solve the above systems using a matrix approach, it is easier to do so by first rearranging the systems as shown below

$$
\left.
\begin{aligned}
&m_1\ddot{y}_1+(k_1+k_2)y_1-k_2y_2=0 \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (1)\\
&m_2\ddot{y}_2+k_2y_1+(k_2+k_3)y_2-k_3y_3=0 \_\_\_\_\_\_\_\_\_\_\_\_\_\_ (2)\\
&m_3\ddot{y}_3-k_3y_2+(k_3+k_4)y_3-k_4y_4=0 \_\_\_\_\_\_\_\_\_\_\_\_\_\_ (3)\\
&\quad.\qquad\qquad .\qquad\qquad .\\
&\quad.\qquad\qquad .\qquad\qquad .\\
&m_{(n-1)}\ddot{y}_{(n-1)}-k_{(n-1)}y_{(n-2)}+(k_{(n-1)}+k_n)y_{(n-1)}-k_ny_n=0 \_\_\_\_\_\_ (n-1)\\
&m_n\ddot{y}_n-k_ny_{(n-1)}+k_ny_n=0 \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (n)
\end{aligned}
\right\}\quad (c)
$$

We then formulate the immediate systems above in matrix form as

$$
\begin{pmatrix}
m_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & m_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & m_3 & 0 & 0 & 0 & 0 & 0 & 0\\
. & . & . & . & . & . & . & . & .\\
. & . & . & . & . & . & . & . & .\\
. & . & . & . & . & . & . & . & .\\
. & . & . & . & . & . & . & . & .\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{(n-1)} & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_n
\end{pmatrix}
\begin{pmatrix}
\ddot{y}_1\\ \ddot{y}_2\\ \ddot{y}_3\\ .\\ .\\ .\\ .\\ \ddot{y}_{(n-1)}\\ \ddot{y}_n
\end{pmatrix}
+
\begin{pmatrix}
k_1+k_2 & -k_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
k_2 & k_2+k_3 & -k_3 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -k_3 & k_3+k_4 & -k_4 & 0 & 0 & 0 & 0 & 0\\
. & . & . & . & . & . & . & . & .\\
. & . & . & . & . & . & . & . & .\\
. & . & . & . & . & . & . & . & .\\
. & . & . & . & . & . & . & . & .\\
0 & 0 & 0 & 0 & 0 & 0 & -k_{(n-1)} & k_{(n-1)}+k_n & -k_n\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -k_n & k_n
\end{pmatrix}
\begin{pmatrix}
y_1\\ y_2\\ y_3\\ .\\ .\\ .\\ .\\ y_{(n-1)}\\ y_n
\end{pmatrix}
=
\begin{pmatrix}
0\\ 0\\ 0\\ .\\ .\\ .\\ .\\ 0\\ 0
\end{pmatrix}
$$

The above matrix equation is a second order homogenous system of the form $M\ddot{y}+Ky=0$ where $M=diag(m_1,m_2,\ldots,m_n)$ and K, also known as the stiffness matrix

is given as
$$
\begin{pmatrix}
k_1+k_2 & -k_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
k_2 & k_2+k_3 & -k_3 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -k_3 & k_3+k_4 & -k_4 & 0 & 0 & 0 & 0 & 0\\
. & . & . & . & . & . & . & . & .\\
. & . & . & . & . & . & . & . & .\\
. & . & . & . & . & . & . & . & .\\
. & . & . & . & . & . & . & . & .\\
0 & 0 & 0 & 0 & 0 & 0 & -k_{(n-1)} & k_{(n-1)}+k_n & -k_n\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -k_n & k_n
\end{pmatrix}
$$

Since our model is a second order linear differential equation of the form $M\ddot{y} + Ky = 0$ with initial condition $y(t_0) = 0$ its general solution is of the form

$y = A\cos\omega t + B\sin\omega t$

Applying initial condition yields $y = B\sin\omega t$ where B is amplitude of displacement. Hence we generalize displacement and acceleration as follows

$$
\begin{array}{ll}
y_1 = x_1 \sin\omega t & \ddot{y}_1 = -\omega^2 x_1 \sin\omega t \\
y_2 = x_2 \sin\omega t & \ddot{y}_2 = -\omega^2 x_2 \sin\omega t \\
y_3 = x_3 \sin\omega t & \ddot{y}_3 = -\omega^2 x_3 \sin\omega t \\
\cdot \quad \cdot & \cdot \quad \cdot \\
\cdot \quad \cdot & \cdot \quad \cdot \\
y_n = x_n \sin\omega t & \ddot{y}_n = -\omega^2 x_n \sin\omega t
\end{array}
$$

where $x_1, x_2, \ldots, x_n$ are respectively the amplitudes of the masses $m_1, m_2, \ldots, m_n$ and ω denotes the natural frequency. Substituting the above expressions for $y_1, y_2, \ldots, y_n$ and $\ddot{y}_1, \ddot{y}_2, \ldots, \ddot{y}_n$ in equation (c) gives

$$
\left.
\begin{array}{ll}
-m_1 x_1 \omega^2 + (k_1 + k_2)x_1 - k_2 x_2 = 0 & \text{\_\_\_\_\_\_\_\_(1)} \\
-m_2 x_2 \omega^2 + k_2 x_1 + (k_2 + k_3)x_2 - k_3 x_3 = 0 & \text{\_\_\_\_\_\_\_\_(2)} \\
-m_3 x_3 \omega^2 - k_3 x_2 + (k_3 + k_4)x_3 - k_4 x_4 = 0 & \text{\_\_\_\_\_\_\_\_(3)} \\
\cdot \qquad \cdot \qquad \cdot & \\
\cdot \qquad \cdot \qquad \cdot & \\
-m_{(n-1)} x_{(n-1)} \omega^2 - k_{(n-1)} x_{(n-2)} + (k_{(n-1)} + k_n)x_{(n-1)} - k_n x_n = 0 & \text{\_\_\_\_\_\_}(n-1) \\
-m_n x_n \omega^2 - k_n x_{(n-1)} + k_n x_n = 0 & \text{\_\_\_\_\_\_\_\_}(n)
\end{array}
\right\} \text{(d)}
$$

The system of equations above (d) can be simplified in matrix notation as

$$\begin{pmatrix} k_1+k_2 & -k_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ k_2 & k_2+k_3 & -k_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -k_3 & k_3+k_4 & -k_4 & 0 & 0 & 0 & 0 & 0 \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & 0 & 0 & -k_{(n-1)} & k_{(n-1)}+k_n & -k_n \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -k_n & k_n \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ . \\ . \\ . \\ . \\ x_{(n-1)} \\ x_n \end{pmatrix} = \omega^2 \begin{pmatrix} m_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{(n-1)} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_n \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ . \\ . \\ . \\ . \\ x_{(n-1)} \\ x_n \end{pmatrix}$$

The above model is a generalized eigenvalue problem of the form $Kx = \lambda Mx$

where $\lambda_i = \omega_i^{\ 2}$, $i = 1,2,\ldots,n$ are the eigenvalues

2.9.3 ABSOLUTE AND AVERAGE MAXIMUM DISPLACEMENTS OF A HIGH-RISE BUILDING

Let p_{ij} represent the coordinates of participating mode P_{ij}, $E_i = \sum_{j=1}^{n} p_{ji} m_j$ denote mode participation factor of the chosen mode participation p_i due to support excitation, and also let $R_1, R_2, \ldots, R_n$ denote the known maximum relative response of the participating modes. This observation immediately gives the absolute maximum displacement of the participating floors as follows

$$\begin{pmatrix} y_1 \\ y_2 \\ . \\ . \\ y_n \end{pmatrix}_{absolute\,\max} = E_1 R_1 \begin{pmatrix} p_{11} \\ p_{21} \\ . \\ . \\ p_{n1} \end{pmatrix} + E_2 R_2 \begin{pmatrix} p_{12} \\ p_{22} \\ . \\ . \\ p_{n2} \end{pmatrix} + \ldots + E_n R_n \begin{pmatrix} p_{1n} \\ p_{2n} \\ . \\ . \\ p_{nn} \end{pmatrix}$$

The average of the absolute maximum displacement is called the average maximum displacement, which is actually the geometric mean of the absolute maximum displacement. It is computed as follows

$(y_i)_{average\ \max} = \sqrt{(E_1 R_1 p_{i1})^2 + (E_2 R_2 p_{i2})^2 + \ldots + (E_k R_k p_{ik})^2}$ (Golub and Ortega, 1992 Data, 2000)

2.9.4 DISPLACEMENT UNDER FORCED VIBRATION

The impact of an earthquake in areas outside the neighborhood of it is always small compared to the total area very close or directly on the epicenter of the quake. Here, the magnitude of the earthquake becomes the external force acting on the structure. Applying Newton's second law of motion gives

$$Total\ Force = \text{Restoring}\ Force + External\ Force$$

Mathematically,

$$m\frac{d^2 y}{dt^2} = -ky + F(t, y)$$

Since the external force is sinusoidal, it may be represented by $F(t, y) = F_1 \sin \omega t$ where F_1 is the forcing amplitude defined by $F_1 = k_1 y_0$. The parameter y_0 is the amplitude of displacement caused to the moving support by the earthquake and is computed from the formular $M_L = \log_{10} y_0(mm) + 3\log_{10}[8\Delta t(s)] - 2.92$ with the assumption that $\Delta t = 2$.

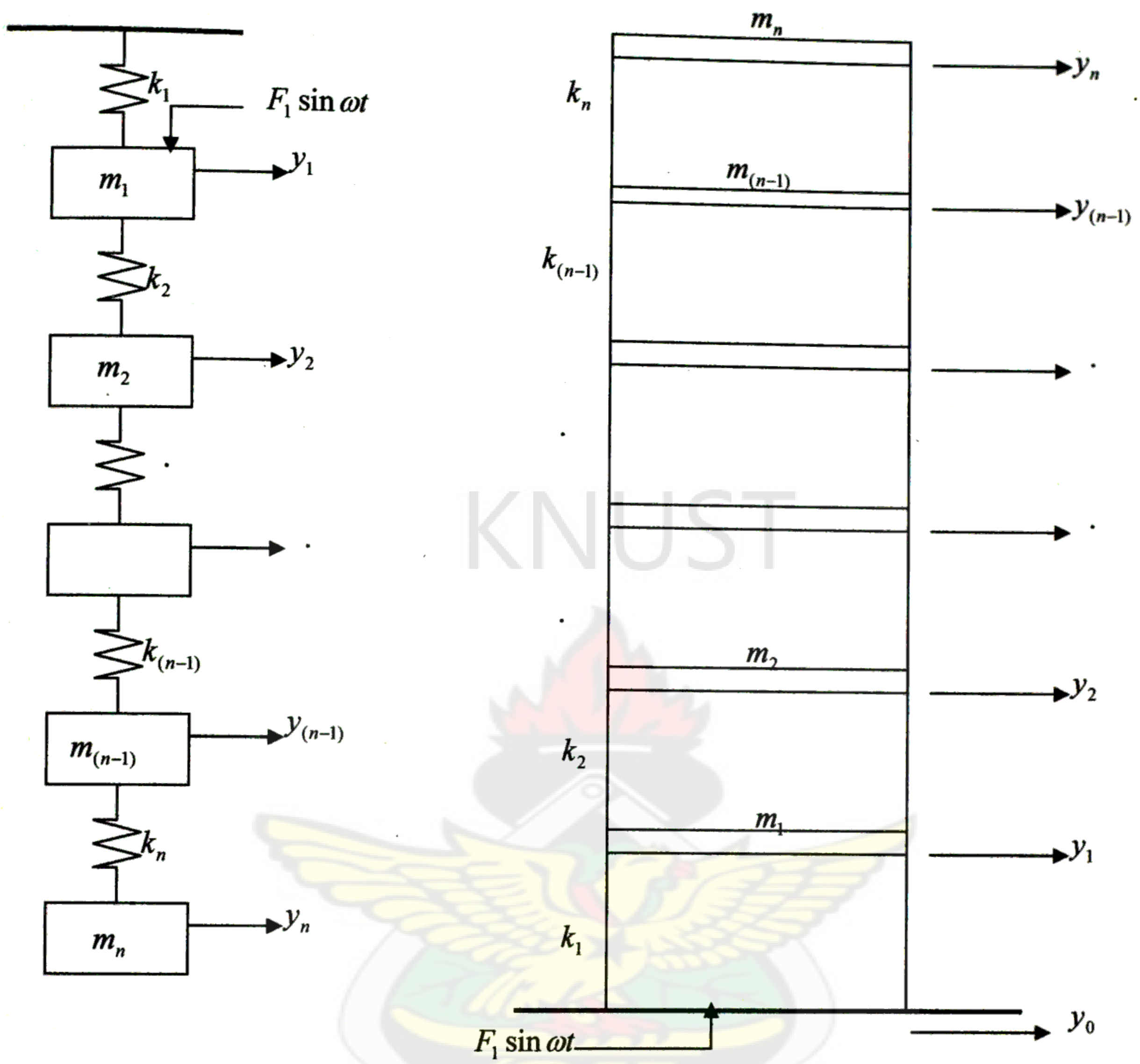


Figure 2.5 Comparison of a building to a helical spring-mass with external force

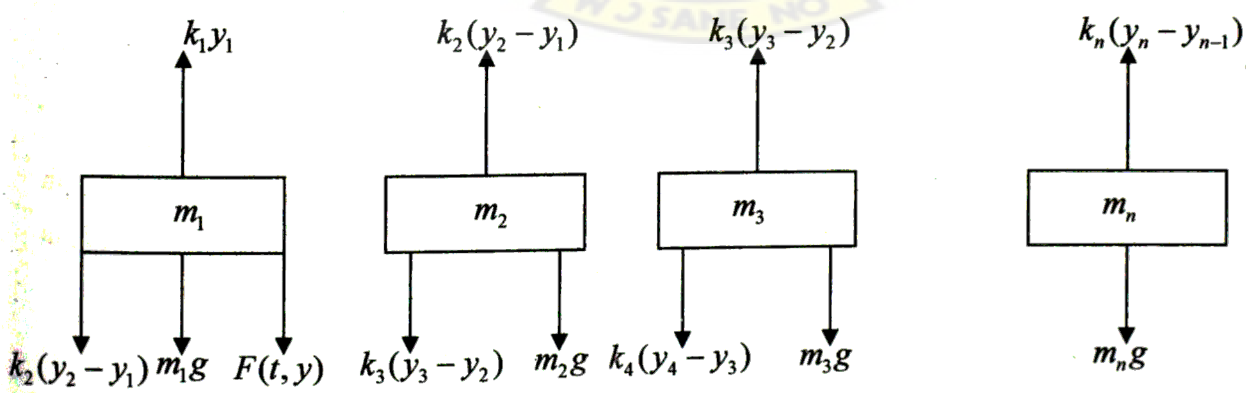


Figure 2.6 Free body diagram for the spring-mass system forced vibration

From the free body diagram, the equations of motion, by Newton's second law, are:

$$\left.\begin{aligned}
&m_1\ddot{y}_1 = -k_1 y_1 + k_2(y_2 - y_1) + m_1 g + F_1 \sin \omega t ________(1)\\
&m_2\ddot{y}_2 = -k_2(y_2 - y_1) + k_3(y_3 - y_2) + m_2 g ________(2)\\
&m_3\ddot{y}_3 = -k_3(y_3 - y_2) + k_4(y_4 - y_3) + m_3 g ________(3)\\
&\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot\\
&\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot\\
&m_{n-1}\ddot{y}_{n-1} = -k_{n-1}(y_{n-1} - y_{n-2}) + k_n(y_n - y_{n-1}) + m_{n-1} g ____(\text{n-1})\\
&m_n\ddot{y}_n = -k_n(y_n - y_{n-1}) + m_n g ________(\text{n})
\end{aligned}\right\} \text{(e)}$$

Suppose we are interested in knowing the displacement of these springs when the system is in an unsteady state-that is when the system attains maximum acceleration. Then by setting the acceleration due to gravity to zero, we obtain:

$$\left.\begin{aligned}
&m_1\ddot{y}_1 = -k_1 y_1 + k_2(y_2 - y_1) + F_1 \sin \omega t ________(1)\\
&m_2\ddot{y}_2 = -k_2(y_2 - y_1) + k_3(y_3 - y_2) ________(2)\\
&m_3\ddot{y}_3 = -k_3(y_3 - y_2) + k_4(y_4 - y_3) ________(3)\\
&\cdot \quad \cdot \quad \cdot\\
&\cdot \quad \cdot \quad \cdot\\
&m_{(n-1)}\ddot{y}_{(n-1)} = -k_{(n-1)}(y_{(n-1)} - y_{(n-2)}) + k_n(y_n - y_{n-1}) ________(n-1)\\
&m_n\ddot{y}_n = -k_n(y_n - y_{(n-1)}) ________(n)
\end{aligned}\right\} \text{(f)}$$

Because we are going to solve the above systems using a matrix approach, it is easier to do so by first rearranging the systems as shown below

$$\left.\begin{array}{ll}
m_1\ddot{y}_1 + (k_1 + k_2)y_1 - k_2 y_2 = F_1 \sin \omega t ____________________ & (1) \\
m_2\ddot{y}_2 + k_2 y_1 + (k_2 + k_3)y_2 - k_3 y_3 = 0 ____________________ & (2) \\
m_3\ddot{y}_3 - k_3 y_2 + (k_3 + k_4)y_3 - k_4 y_4 = 0 ____________________ & (3) \\
\cdot \qquad \cdot \qquad \cdot & \\
\cdot \qquad \cdot \qquad \cdot & \\
m_{(n-1)}\ddot{y}_{(n-1)} - k_{(n-1)}y_{(n-2)} + (k_{(n-1)} + k_n)y_{(n-1)} - k_n y_n = 0 ______ & (n-1) \\
m_n\ddot{y}_n - k_n y_{(n-1)} + k_n y_n = 0 ____________________________ & (n)
\end{array}\right\} \text{(g)}$$

We then formulate the immediate systems above in matrix form as

$$\begin{pmatrix}
m_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & m_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & m_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{(n-1)} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_n
\end{pmatrix}
\begin{pmatrix} \ddot{y}_1 \\ \ddot{y}_2 \\ \ddot{y}_3 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \ddot{y}_{(n-1)} \\ \ddot{y}_n \end{pmatrix}
+
\begin{pmatrix}
k_1+k_2 & -k_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
k_2 & k_2+k_3 & -k_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -k_3 & k_3+k_4 & -k_4 & 0 & 0 & 0 & 0 & 0 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
0 & 0 & 0 & 0 & 0 & 0 & -k_{(n-1)} & k_{(n-1)}+k_n & -k_n \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -k_n & k_n
\end{pmatrix}
\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ y_{(n-1)} \\ y_n \end{pmatrix}
=
\begin{pmatrix} F_1 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 0 \end{pmatrix} \sin \omega t$$

The above matrix equation is a system of second order non-homogenous equations of the form $M\ddot{y}+Ky=c$ where $M=diag(m_1,m_2,\ldots,m_n)$, $c=F_1\sin\omega t$ and

$$K=\begin{pmatrix} k_1+k_2 & -k_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ k_2 & k_2+k_3 & -k_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -k_3 & k_3+k_4 & -k_4 & 0 & 0 & 0 & 0 & 0 \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & 0 & 0 & -k_{(n-1)} & k_{(n-1)}+k_n & -k_n \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -k_n & k_n \end{pmatrix}$$

Our model is a non-homogeneous second order differential equation of the form

$M\ddot{y}+Ky=F(t) \quad y(t_0)=0$

Since F(t) is sinusoidal, the general solution of this differential equation is

$y=A\cos\omega t+B\sin\omega t$

Applying initial condition yields $y=B\sin\omega t$

So in general terms we say that

$$\begin{aligned} y_1 &= x_1\sin\omega t & \ddot{y}_1 &= -\omega^2 x_1\sin\omega t \\ y_2 &= x_2\sin\omega t & \ddot{y}_2 &= -\omega^2 x_2\sin\omega t \\ y_3 &= x_3\sin\omega t & \ddot{y}_3 &= -\omega^2 x_3\sin\omega t \\ &. \quad . & &. \quad . \\ y_n &= x_n\sin\omega t & \ddot{y}_n &= -\omega^2 x_n\sin\omega t \end{aligned}$$

where $x_1,x_2,\ldots,x_n$ are respectively the amplitudes of the masses $m_1,m_2,\ldots,m_n$ and ω denotes the natural frequency. Substituting the above expressions for $y_1,y_2,\ldots,y_n$ and $\ddot{y}_1,\ddot{y}_2,\ldots,\ddot{y}_n$ in equation (f) gives

$$\left.\begin{array}{ll}
-m_1x_1\omega^2+(k_1+k_2)x_1-k_2x_2=F_1 & \text{__________ __________ __(1)} \\
-m_2x_2\omega^2+k_2x_1+(k_2+k_3)x_2-k_3x_3=0 & \text{__________ __________(2)} \\
-m_3x_3\omega^2-k_3x_2+(k_3+k_4)x_3-k_4x_4=0 & \text{__________ __________(3)} \\
\cdot \qquad \cdot \qquad \cdot & \\
\cdot \qquad \cdot \qquad \cdot & \\
-m_{(n-1)}x_{(n-1)}\omega^2-k_{(n-1)}x_{(n-2)}+(k_{(n-1)}+k_n)x_{(n-1)}-k_nx_n=0 & \text{______(}n-1\text{)} \\
-m_nx_n\omega^2-k_nx_{(n-1)}+k_nx_n=0 & \text{__________ __________ ________(}n\text{)}
\end{array}\right\}\text{(h)}$$

The system of equations above (c) can be simplified to obtain

$$\left.\begin{array}{ll}
((k_1+k_2)-m_1\omega^2)x_1-k_2x_2=F_1 & \text{__________ __________ ____(1)} \\
k_2x_1+((k_2+k_3)-m_2\omega^2)x_2-k_3x_3=0 & \text{__________ __________(2)} \\
-k_3x_2+((k_3+k_4)-m_3\omega^2)x_3-k_4x_4=0 & \text{__________ __________(3)} \\
\cdot \qquad \cdot \qquad \cdot & \\
\cdot \qquad \cdot \qquad \cdot & \\
-k_{(n-1)}x_{(n-2)}+((k_{(n-1)}+k_n)-m_{(n-1)}\omega^2)x_{(n-1)}-k_nx_n=0 & \text{______(}n-1\text{)} \\
-k_nx_{(n-1)}+(k_n-m_n\omega^2)x_n=0 & \text{__________ __________ ______(}n\text{)}
\end{array}\right\}\text{(i)}$$

Equation (d) in matrix form is

$$\begin{pmatrix}
(k_1+k_2-m_1\omega^2) & -k_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
k_2 & (k_2+k_3-m_2\omega^2) & -k_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -k_3 & (k_3+k_4-m_3\omega^2) & -k_4 & 0 & 0 & 0 & 0 & 0 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
0 & 0 & 0 & 0 & 0 & 0 & -k_{(n-1)} & (k_{(n-1)}+k_n-m_{(n-1)}\omega^2) & -k_n \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -k_n & (k_n-m_n\omega^2)
\end{pmatrix}
\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ x_{(n-1)} \\ x_n \end{pmatrix}
=
\begin{pmatrix} F_1 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 0 \end{pmatrix}$$

The above matrix can easily be solved to obtain explicit expressions for the amplitudes $x_1, x_2, ..., x_n$ with denominator factor $m_1 \cdot m_2 \cdots m_n(\omega_1^2 - \omega^2) \cdot (\omega_2^2 - \omega^2) \cdots (\omega_n^2 - \omega^2)$ From this, it follows immediately that whenever ω is equal or close to ω_i, for $i = 1,2,..n$ the amplitude becomes arbitrarily large, signaling the occurrence of resonance. In this case when the frequency of the imposed periodic force is equal to one of the natural frequencies of the system, the denominator is zero or close to zero, a situation, which is very alarming for engineers.

2.10 SOLUTION OF TRIDIAGONAL SYSTEM

The resulting system of equations whose matrix equation appears above is known as a tridiagonal system. The solution of linear second order systems of differential equations leads to the solution of systems of algebraic equations in n or $n+2$ unknowns whose coefficients give rise to a tridiagonal system. The solution of such systems is achieved with the following algorithm

Algorithm

Consider the general tridiagonal system shown below

$$\begin{pmatrix} b_1 & c_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_2 & b_2 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_3 & b_3 & c_3 & 0 & 0 & 0 & 0 & 0 \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & 0 & 0 & a_{(n-1)} & b_{(n-1)} & c_{(n-1)} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_n & b_{(n)} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ . \\ . \\ . \\ . \\ x_{(n-1)} \\ x_n \end{pmatrix} = \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ . \\ . \\ . \\ . \\ r_{(n-1)} \\ r_n \end{pmatrix}$$

Step 1: Set $\alpha_1 = \frac{c_1}{b_1}$ and $\gamma_1 = \frac{r_1}{b_1}$

Step 2: For $j = 2(1),\ldots,n$

$$\alpha_j = \frac{c_j}{b_j - a_j\alpha_{(j-1)}} \text{ and } \gamma_j = \frac{r_j - a_j r_{(j-1)}}{b_j - a_j\alpha_{(j-1)}}$$

Step 3: With $c_n = 0$, set $x_n = \gamma_n$

Step 4: For $j = n-1,\ldots,1$

$$x_j = r_j - \alpha_j x_{(j+1)}$$

(Wilkinson, 1965 Coleman and Van Loan, 1988)

The above tridiagonal systems algorithm when employed to solve the system will obtain explicit expression for the amplitudes $x_1, x_2, \ldots, x_n$.

These explicit solutions are of particular interest to an engineer because whenever b_j is equal or close to $a_j\alpha_{(j-1)}$ the amplitude becomes arbitrarily large, for the simple reason that the denominator approaches zero. This results in the occurrence of resonance.

2.11 LIMITS OF LINEARITY

In the derivation of the period of a simple pendulum, the displacement value appeared as $x = l\sin\theta$. It was agreed that whenever θ is small, $\sin\theta \approx \theta$ so that $x = l\theta$

$$\Rightarrow \theta = \frac{x}{l}.$$

From Taylor series expansion of $\sin\theta$,

$$\sin\theta = \sum_{n=0}^{\infty}(-1)^n \frac{\theta^{2n+1}}{(2n+1)!} = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots$$

helps to yield appropriate values for θ to fulfill linearity.

Definition: If x is a real number and x' is the floating-point approximation, then the difference $x' - x$ is called the **absolute error** and the quotient $\frac{x'-x}{x}$, $x \neq 0$ is called the **relative error.**

From $\frac{\partial(l\theta)}{l\theta} = \frac{\Delta x}{x} = \frac{\partial\theta}{\theta} + \frac{\partial l}{l}$, the relative error for θ can easily be computed (Chapra and Canale, 1988)

2.12 SUMMARY

A mathematical model is a set of formulas and/or equations based on a quantitative description of real phenomena and created in the hope that the behaviour it predicts will resemble the real behaviour on which it is based. Mathematical modeling is the art and science of constructing mathematical models and using them to gain insight into physical processes or to make predictions concerning physical processes.

A physical law is a statement that expresses the relationship between quantities closely enough to be taken as exactly true and is the major source of mathematical models.

Most engineering applications give rise to generalized eigenvalue problems. A majority of eigenvalue problems arising in mechanical vibration are generalized eigenvalue problems

CHAPTER THREE

3.0 DATA COLLECTION AND ANALYSIS

The structural drawings of Unity hall was collected and examined. The examinations revealed that the Unity hall building has three different kinds of slabs, namely, the ground floor slab, the intermediate floor slab (i.e. from first floor to eight floor), and the roof top slab.

These slabs are represented schematically as below

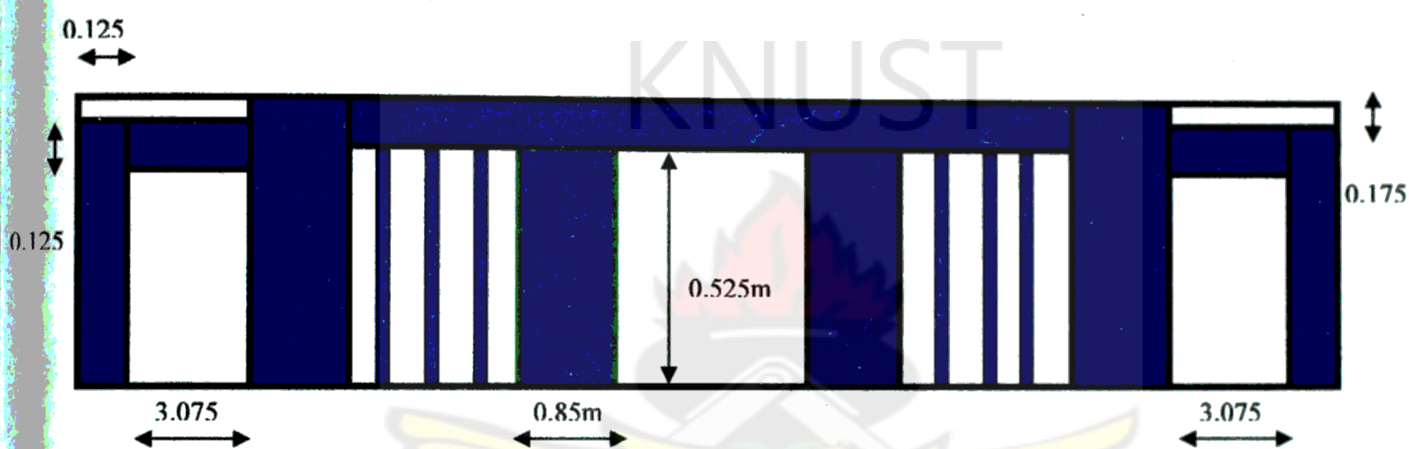


Fig. 3.1 Ground floor dimensions

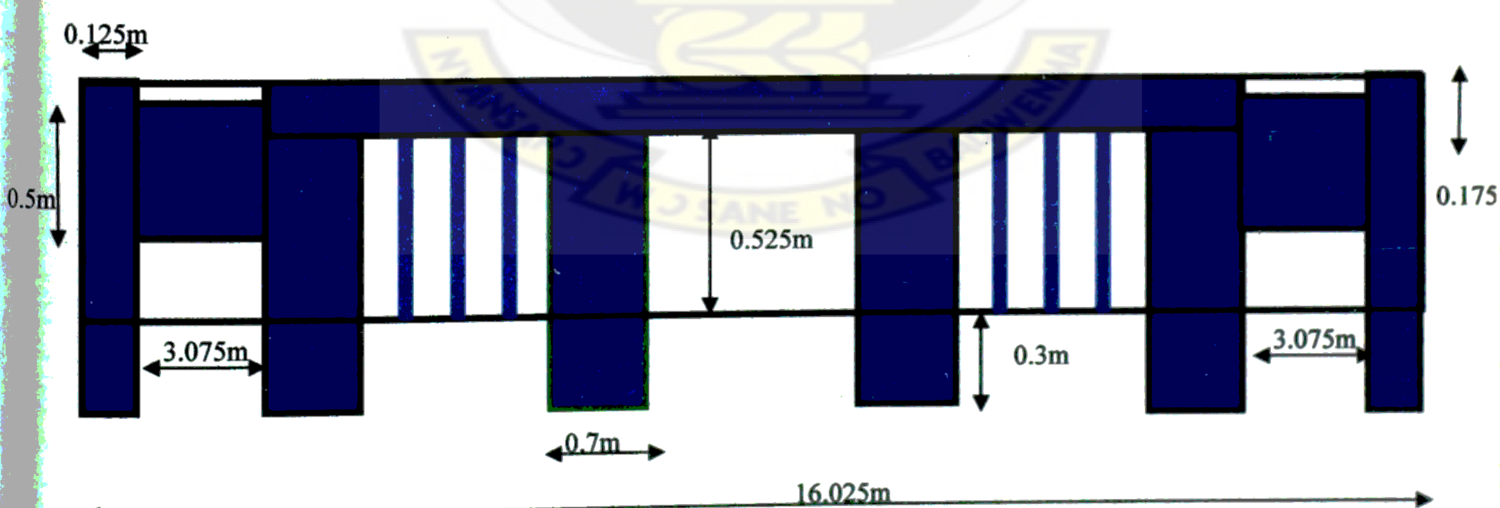


Fig. 3.2 Subsequent floor dimensions

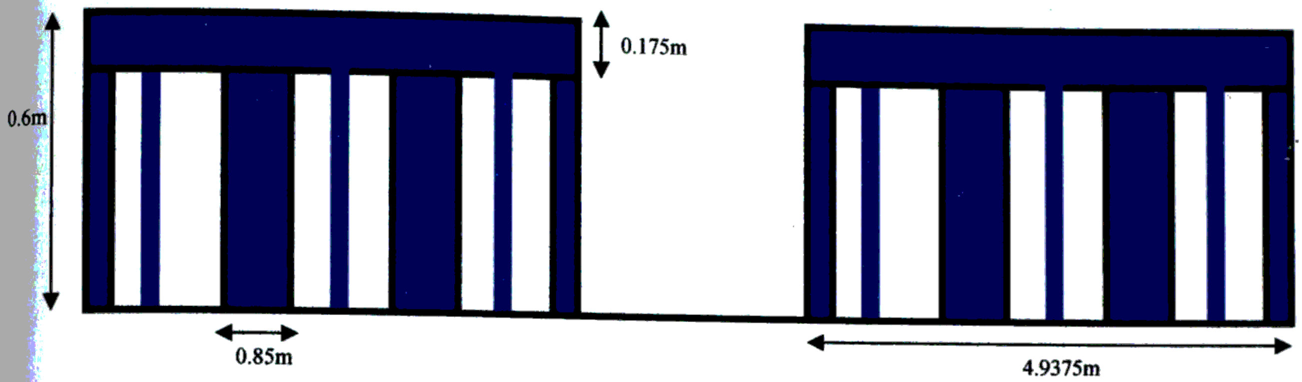


Fig. 3.3 Roof top dimensions

Mathematical methods were applied to these data dimensi ons and obtained the table below

Table 3.1 Extraction of data into tables

Unity Hall						
Floor	**Length (m)**	**Breadth (m)**	**Volume (m^3)**	**Mass (kg)**		**Spring constant (N/m)**
				Dead load	**Live load**	
Ground	51.6	16.025	259.806	62353.44	14280	2.2×10^5
1-8	51.6	16.025	397.5135	95403.24	14280	3.4×10^5
Roof top	51.6	9.875	191.14575	45874.98	-	1.6×10^5

3.1 UNDER FREE VIBRATION

3.1.1 Case 1: Ignoring Live Load

Mass matrix

$$M = 10^4 \begin{pmatrix} 6.2353 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 9.5403 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 9.5403 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 9.5403 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 9.5403 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 9.5403 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 9.5403 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9.5403 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9.5403 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4.5875 \end{pmatrix};$$

Stiffness matrix

$$K = 10^5 \begin{pmatrix} 5.6 & -3.4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3.4 & 6.8 & -3.4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -3.4 & 6.8 & -3.4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3.4 & 6.8 & -3.4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3.4 & 6.8 & -3.4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3.4 & 6.8 & -3.4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -3.4 & 6.8 & -3.4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -3.4 & 6.8 & -3.4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & & -3.4 & 5.0 & -1.6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.6 & 1.6 \end{pmatrix}$$

From $M = LL^T$

$$\Rightarrow L = \begin{pmatrix} 249.7058 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 308.8738 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 308.8738 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 308.8738 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 308.8738 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 308.8738 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 308.8738 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 308.8738 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 308.8738 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 214.1845 \end{pmatrix}$$

$$C = \begin{pmatrix} 8.8774 & -4.3402 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4.3402 & 7.0175 & -3.5088 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -3.5088 & 7.0175 & -3.5088 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3.5088 & 7.0175 & -3.5088 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3.5088 & 7.0175 & -3.5088 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3.5088 & 7.0175 & -3.5088 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -3.5088 & 7.0175 & -3.5088 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -3.5088 & 7.0175 & -3.5088 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3.5088 & 5.1960 & -2.4331 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2.4331 & 3.5088 \end{pmatrix}$$

The eigenvalues of C, which are the eigenvalues of the pencil, (K, M) are 13.5271, 12.0421, 8.3144, 8.3144, 9.6378, 6.6940, 4.4513, 0.1165, 1.0127, and 2.5946.

The natural frequencies corresponding to these eigenvalues are

$$f = \begin{pmatrix} 0.5851 & 0.5521 & 0.4587 & 0.4587 & 0.4939 & 0.4116 & 0.3357 & 0.0543 & 0.1601 & 0.2563 \end{pmatrix}^T$$

The approximate eigenvectors of (K, M) associated with these eigenvalues are

$$\begin{pmatrix} -0.0810 \\ 0.0868 \\ -0.2613 \\ 0.3980 \\ -0.4770 \\ 0.4870 \\ -0.4265 \\ 0.3042 \\ -0.1379 \\ 0.0335 \end{pmatrix} \quad \begin{pmatrix} 0.1851 \\ -0.1350 \\ 0.4223 \\ -0.4698 \\ 0.2504 \\ 0.1112 \\ -0.4097 \\ 0.4754 \\ -0.2711 \\ 0.0773 \end{pmatrix} \quad \begin{pmatrix} -0.7071 \\ -0.0917 \\ -0.3260 \\ 0.1046 \\ 0.1060 \\ -0.0632 \\ -0.0265 \\ 0.0287 \\ 0.0000 \\ -0.0061 \end{pmatrix} \quad \begin{pmatrix} -0.7071 \\ -0.0917 \\ -0.3260 \\ 0.1046 \\ 0.1060 \\ -0.0632 \\ -0.0265 \\ 0.0287 \\ 0.0000 \\ -0.0061 \end{pmatrix} \quad \begin{pmatrix} -0.3302 \\ 0.0579 \\ -0.4517 \\ 0.2795 \\ 0.2430 \\ -0.4609 \\ 0.1012 \\ 0.3854 \\ -0.3890 \\ 0.1544 \end{pmatrix}$$

$$\begin{pmatrix} -0.3229 \\ -0.1624 \\ -0.4144 \\ 0.1242 \\ 0.4258 \\ -0.0850 \\ -0.4337 \\ 0.0450 \\ 0.4378 \\ -0.3344 \end{pmatrix} \quad \begin{pmatrix} 0.2068 \\ 0.2109 \\ 0.4101 \\ 0.0890 \\ -0.3450 \\ -0.3413 \\ 0.0953 \\ 0.4111 \\ 0.2053 \\ -0.5300 \end{pmatrix} \quad \begin{pmatrix} -0.0269 \\ -0.0544 \\ -0.1402 \\ -0.2214 \\ -0.2953 \\ -0.3594 \\ -0.4115 \\ -0.4500 \\ -0.4735 \\ -0.3396 \end{pmatrix} \quad \begin{pmatrix} 0.0833 \\ 0.1509 \\ 0.3612 \\ 0.4672 \\ 0.4385 \\ 0.2831 \\ 0.0461 \\ -0.2043 \\ -0.3957 \\ -0.3857 \end{pmatrix} \quad \begin{pmatrix} 0.1433 \\ 0.2074 \\ 0.4387 \\ 0.3456 \\ -0.0031 \\ -0.3495 \\ -0.4374 \\ -0.2019 \\ 0.1829 \\ 0.4867 \end{pmatrix}$$

The eigenvectors x_i corresponding to the eigenvalues λ_i of the pencil (A, B) is obtained by solving $L^T x_i = y_i$, $i = 1,2,\ldots,n$

$$x_1 = \begin{pmatrix} -0.0003 \\ 0.0003 \\ -0.0008 \\ 0.0013 \\ -0.0015 \\ 0.0016 \\ -0.0014 \\ 0.0010 \\ -0.0004 \\ 0.0002 \end{pmatrix} \quad x_2 = \begin{pmatrix} 0.0007 \\ -0.0004 \\ 0.0014 \\ -0.0015 \\ 0.0008 \\ 0.0004 \\ -0.0013 \\ 0.0015 \\ -0.0009 \\ 0.0004 \end{pmatrix} \quad x_3 = \begin{pmatrix} -0.0028 \\ -0.0003 \\ -0.0011 \\ 0.0003 \\ 0.0003 \\ -0.0002 \\ -0.0001 \\ 0.0001 \\ 0.0000 \\ 0.0000 \end{pmatrix} \quad x_4 = \begin{pmatrix} -0.0028 \\ -0.0003 \\ -0.0011 \\ 0.0003 \\ 0.0003 \\ -0.0002 \\ -0.0001 \\ 0.0001 \\ 0.0000 \\ 0.0000 \end{pmatrix} \quad x_5 = \begin{pmatrix} -0.0013 \\ 0.0002 \\ -0.0015 \\ 0.0009 \\ 0.0008 \\ -0.0015 \\ 0.0003 \\ 0.0012 \\ -0.0013 \\ 0.0007 \end{pmatrix}$$

$$x_6 = \begin{pmatrix} -0.0013 \\ -0.0005 \\ -0.0013 \\ 0.0004 \\ 0.0014 \\ -0.0003 \\ -0.0014 \\ 0.0001 \\ 0.0014 \\ -0.0016 \end{pmatrix} \quad x_7 = \begin{pmatrix} 0.0008 \\ 0.0007 \\ 0.0013 \\ 0.0003 \\ -0.0011 \\ -0.0011 \\ 0.0003 \\ 0.0013 \\ 0.0007 \\ -0.0025 \end{pmatrix} \quad x_8 = \begin{pmatrix} -0.0001 \\ -0.0002 \\ -0.0005 \\ -0.0007 \\ -0.0010 \\ -0.0012 \\ -0.0013 \\ -0.0015 \\ -0.0015 \\ -0.0016 \end{pmatrix} \quad x_9 = \begin{pmatrix} 0.0003 \\ 0.0005 \\ 0.0012 \\ 0.0015 \\ 0.0014 \\ 0.0009 \\ 0.0001 \\ -0.0007 \\ -0.0013 \\ -0.0018 \end{pmatrix} \quad x_{10} = \begin{pmatrix} 0.0006 \\ 0.0007 \\ 0.0014 \\ 0.0011 \\ 0.0000 \\ -0.0011 \\ -0.0014 \\ -0.0007 \\ 0.0006 \\ 0.0023 \end{pmatrix}$$

The mode participating factor of the chosen mode p_i due to support excitation is given

as $E_i = \sum_{j=1}^{n} p_{ji} m_i$

$$E_1 = m_1 p_{11} + m_2 p_{21} + m_3 p_{31} + \ldots + m_{10} p_{101} = -21.3108$$

$$E_2 = m_1 p_{12} + m_2 p_{22} + m_3 p_{32} + \ldots + m_{10} p_{102} = 33.3823$$

$$E_3 = m_1 p_{13} + m_2 p_{23} + m_3 p_{33} + \ldots + m_{10} p_{103} = -227.3430$$

$$E_4 = m_1 p_{14} + m_2 p_{24} + m_3 p_{34} + \ldots + m_{10} p_{104} = -488.0683$$

$$E_5 = m_1 p_{15} + m_2 p_{25} + m_3 p_{35} + \ldots + m_{10} p_{105} = -609.9620$$

$$E_6 = m_1 p_{16} + m_2 p_{26} + m_3 p_{36} + \ldots + m_{10} p_{106} = -781.5557$$

$E_7 = m_1 p_{17} + m_2 p_{27} + m_3 p_{37} + \ldots + m_{10} p_{107} = -616.2667$

$E_8 = m_1 p_{18} + m_2 p_{28} + m_3 p_{38} + \ldots + m_{10} p_{108} = -1438.6857$

$E_9 = m_1 p_{19} + m_2 p_{29} + m_3 p_{39} + \ldots + m_{10} p_{109} = -1146.2876$

$E_{10} = m_1 p_{19} + m_2 p_{29} + m_3 p_{39} + \ldots + m_{10} p_{1010} = -949.8482$

Assuming the maximum relative response of each floor as follows $R_1 = 0.4m$, $R_2 = R_3 = R_4 = R_5 = R_6 = R_7 = R_8 = R_9 = 0.2m$ and $R_{10} = 0.1m$

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \end{pmatrix}_{abs.\max} = \begin{pmatrix} 0.2592 \\ -0.0877 \\ -0.0095 \\ -0.3130 \\ -0.1297 \\ 0.3629 \\ 0.3845 \\ 0.1744 \\ 0.2342 \\ 0.4521 \end{pmatrix} \qquad \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \end{pmatrix}_{average.\max} = \begin{pmatrix} 0.2174 \\ 0.1081 \\ 0.2633 \\ 0.2382 \\ 0.2538 \\ 0.2532 \\ 0.2607 \\ 0.2574 \\ 0.3053 \\ 0.4258 \end{pmatrix}$$

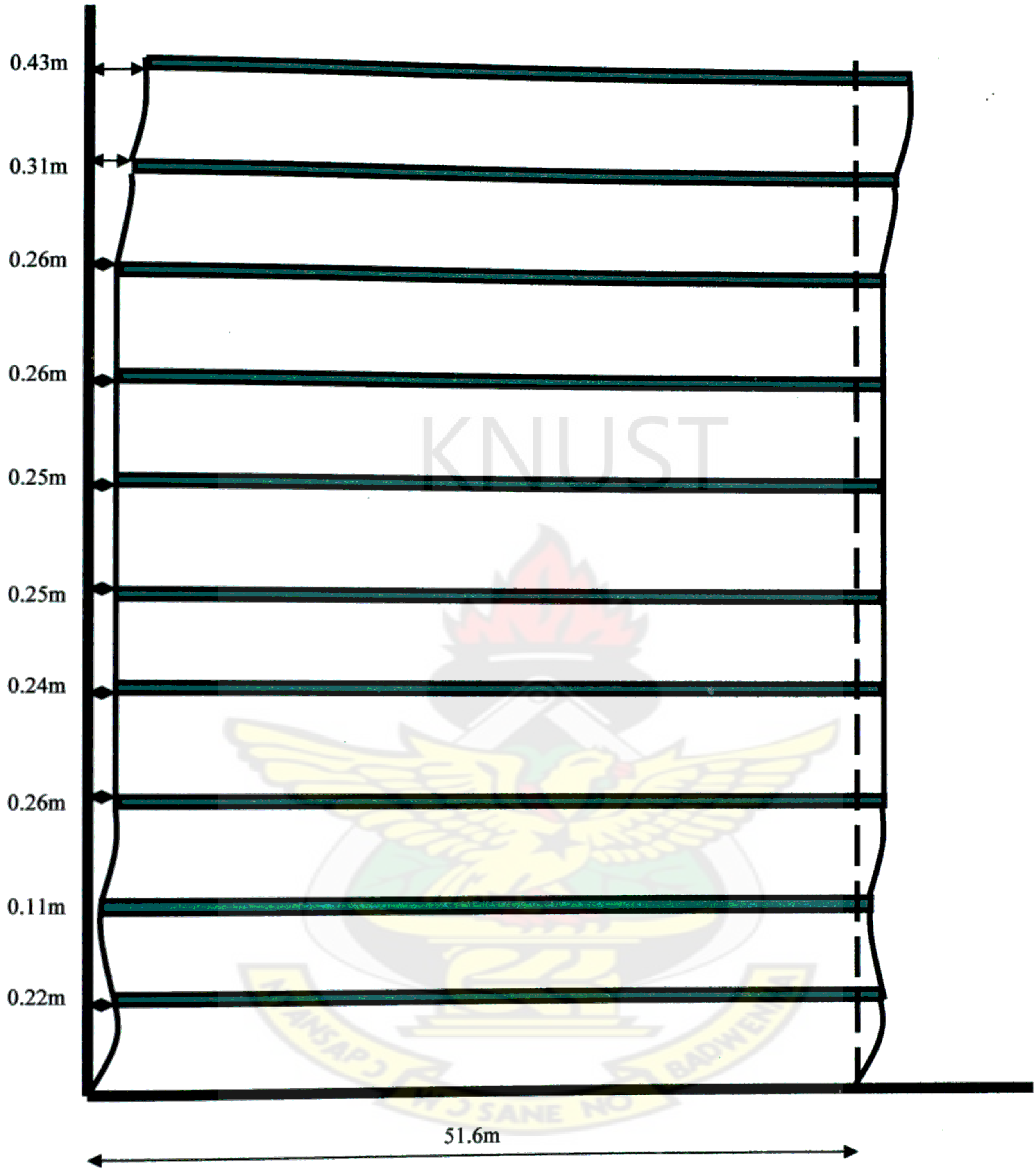


Figure 3.3 Schematic diagram of the maximum displacement of the building when live load is ignored

3.1.2 Case 2: Considering Live Load

Number or rooms on each floor = 28

Average number of students per room = 6

Average weight per student = $60kg$

Number of double beds per room = 2

Average weight per double bed = $75kg$

Live load per floor $= 28(6 \times 60 + 2 \times 75) = 14280kg$

Mass matrix

$$M = \begin{pmatrix} 76633 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 109683 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 109683 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 109683 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 109683 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 109683 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 109683 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 109683 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 109683 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 45875 \end{pmatrix}$$

From $M = LL^T$

$$\Rightarrow L = \begin{pmatrix} 276.8267 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 331.1842 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 331.1842 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 331.1842 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 331.1842 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 331.1842 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 331.1842 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 331.1842 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 331.1842 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 214.1845 \end{pmatrix}$$

$$
C=\begin{pmatrix}
7.2231 & -3.6512 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
3.6512 & 6.1039 & -3.0520 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -3.0520 & 6.1039 & -3.0520 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & -3.0520 & 6.1039 & -3.0520 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -3.0520 & 6.1039 & -3.0520 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & -3.0520 & 6.1039 & -3.0520 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -3.0520 & 6.1039 & -3.0520 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & -3.0520 & 6.1039 & -3.0520 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -3.0520 & 4.5195 & -2.2692\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2.2692 & 3.5088
\end{pmatrix}
$$

The eigenvalues of C, which are the eigenvalues of the pencil, (K,M) are 11.7730, 10.5089, 6.8944, 6.8944, 8.4975, 6.0152, 4.0603, 0.1029, 0.8986 and 2.3336

The natural frequencies corresponding to these eigenvalues are

$$
f=\begin{pmatrix}0.5459 & 0.5157 & 0.4177 & 0.4177 & 0.4638 & 0.3902 & 0.3206 & 0.0510 & 0.1508 & 0.2430\end{pmatrix}^T
$$

The approximate eigenvectors of (K,M) associated with these eigenvalues are

$$
\begin{pmatrix}-0.0757\\0.0943\\-0.2657\\0.3993\\-0.4760\\0.4848\\-0.4246\\0.3039\\-0.1399\\0.0384\end{pmatrix}
\quad
\begin{pmatrix}0.1719\\-0.1547\\0.4288\\-0.4643\\0.2413\\0.1160\\-0.4087\\0.4739\\-0.2753\\0.0892\end{pmatrix}
\quad
\begin{pmatrix}-0.7071\\-0.0637\\-0.3379\\0.0768\\0.1267\\-0.0496\\-0.0435\\0.0238\\0.0126\\-0.0136\end{pmatrix}
\quad
\begin{pmatrix}-0.7071\\-0.0637\\-0.3379\\0.0768\\0.1267\\-0.0496\\-0.0435\\0.0238\\0.0126\\-0.0136\end{pmatrix}
\quad
\begin{pmatrix}-0.3117\\0.1088\\-0.4583\\0.2506\\0.2617\\-0.4559\\0.0958\\0.3807\\-0.3944\\0.1794\end{pmatrix}
$$

$$\begin{pmatrix} -0.3433 \\ -0.1136 \\ -0.4140 \\ 0.1015 \\ 0.4169 \\ -0.0894 \\ -0.4195 \\ 0.0772 \\ 0.4218 \\ -0.3819 \end{pmatrix} \quad \begin{pmatrix} 0.2319 \\ 0.2009 \\ 0.4119 \\ 0.0750 \\ -0.3617 \\ -0.3172 \\ 0.1494 \\ 0.4172 \\ 0.1300 \\ -0.5348 \end{pmatrix} \quad \begin{pmatrix} 0.0282 \\ 0.0550 \\ 0.1418 \\ 0.2239 \\ 0.2984 \\ 0.3628 \\ 0.4151 \\ 0.4533 \\ 0.4763 \\ 0.3173 \end{pmatrix} \quad \begin{pmatrix} -0.0882 \\ -0.1528 \\ -0.3662 \\ -0.4718 \\ -0.4385 \\ -0.2760 \\ -0.0323 \\ 0.2209 \\ 0.4091 \\ 0.3557 \end{pmatrix} \quad \begin{pmatrix} 0.1573 \\ 0.2107 \\ 0.4485 \\ 0.3434 \\ -0.0243 \\ -0.3734 \\ -0.4370 \\ -0.1665 \\ 0.2314 \\ 0.4467 \end{pmatrix}$$

The eigenvectors x_i corresponding to the eigenvalues λ_i of the pencil (A, B) is obtained by solving $L^T x_i = y_i$, $i = 1,2,\ldots,n$

$$x_1 = \begin{pmatrix} -0.0003 \\ 0.0003 \\ -0.0009 \\ 0.0013 \\ -0.0015 \\ 0.0016 \\ -0.0014 \\ 0.0010 \\ -0.0005 \\ 0.0002 \end{pmatrix} \quad x_2 = \begin{pmatrix} 0.0007 \\ -0.0005 \\ 0.0014 \\ -0.0015 \\ 0.0008 \\ 0.0004 \\ -0.0013 \\ 0.0015 \\ -0.0009 \\ 0.0004 \end{pmatrix} \quad x_3 = \begin{pmatrix} -0.0028 \\ -0.0002 \\ -0.0011 \\ 0.0002 \\ 0.0004 \\ -0.0002 \\ -0.0001 \\ 0.0001 \\ 0.0000 \\ -0.0001 \end{pmatrix} \quad x_4 = \begin{pmatrix} -0.0028 \\ -0.0002 \\ -0.0011 \\ 0.0002 \\ 0.0004 \\ -0.0002 \\ -0.0001 \\ 0.0001 \\ 0.0000 \\ -0.0001 \end{pmatrix} \quad x_5 = \begin{pmatrix} -0.0012 \\ 0.0004 \\ -0.0015 \\ 0.0008 \\ 0.0008 \\ -0.0015 \\ 0.0003 \\ 0.0012 \\ -0.0013 \\ 0.0008 \end{pmatrix}$$

$$x_6 = \begin{pmatrix} -0.0014 \\ -0.0004 \\ -0.0013 \\ 0.0003 \\ 0.0013 \\ -0.0003 \\ -0.0014 \\ 0.0003 \\ 0.0014 \\ -0.0018 \end{pmatrix} \quad x_7 = \begin{pmatrix} 0.0009 \\ 0.0007 \\ 0.0013 \\ 0.0002 \\ -0.0012 \\ -0.0010 \\ 0.0005 \\ 0.0014 \\ 0.0004 \\ -0.0025 \end{pmatrix} \quad x_8 = \begin{pmatrix} 0.0001 \\ 0.0002 \\ 0.0005 \\ 0.0007 \\ 0.0010 \\ 0.0012 \\ 0.0013 \\ 0.0015 \\ 0.0015 \\ 0.0015 \end{pmatrix} \quad x_9 = \begin{pmatrix} -0.0004 \\ -0.0005 \\ -0.0012 \\ -0.0015 \\ -0.0014 \\ -0.0009 \\ -0.0001 \\ 0.0007 \\ 0.0013 \\ 0.0017 \end{pmatrix} \quad x_{10} = \begin{pmatrix} 0.0006 \\ 0.0007 \\ 0.0015 \\ 0.0011 \\ -0.0001 \\ -0.0012 \\ -0.0014 \\ -0.0005 \\ 0.0007 \\ 0.0021 \end{pmatrix}$$

The mode participating factor of the chosen mode p_i due to support excitation is given as $E_i = \sum_{j=1}^{n} p_{ji} m_i$

$$E_1 = m_1 p_{11} + m_2 p_{21} + m_3 p_{31} + ... + m_{10} p_{101} = -18.0437$$

$$E_2 = m_1 p_{12} + m_2 p_{22} + m_3 p_{32} + ... + m_{10} p_{102} = 30.7270$$

$$E_3 = m_1 p_{13} + m_2 p_{23} + m_3 p_{33} + ... + m_{10} p_{103} = -227.3960$$

$$E_4 = m_1 p_{14} + m_2 p_{24} + m_3 p_{34} + ... + m_{10} p_{104} = -485.5189$$

$$E_5 = m_1 p_{15} + m_2 p_{25} + m_3 p_{35} + ... + m_{10} p_{105} = -590.0691$$

$$E_6 = m_1 p_{16} + m_2 p_{26} + m_3 p_{36} + ... + m_{10} p_{106} = -763.4594$$

$$E_7 = m_1 p_{17} + m_2 p_{27} + m_3 p_{37} + ... + m_{10} p_{107} = -602.2372$$

$$E_8 = m_1 p_{18} + m_2 p_{28} + m_3 p_{38} + ... + m_{10} p_{108} = 222.2409$$

$$E_9 = m_1 p_{19} + m_2 p_{29} + m_3 p_{39} + ... + m_{10} p_{109} = -65.7332$$

$$E_{10} = m_1 p_{110} + m_2 p_{210} + m_3 p_{310} + ... + m_{10} p_{1010} = 141.1426$$

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \end{pmatrix}_{abs.\max.} = \begin{pmatrix} 6.6981 \\ -0.3262 \\ 4.0877 \\ -2.2964 \\ -2.5452 \\ 2.9069 \\ 0.1576 \\ -4.8628 \\ -2.2751 \\ 4.1940 \end{pmatrix} \qquad \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \end{pmatrix}_{average.\max.} = \begin{pmatrix} 1.3882 \\ 0.3957 \\ 1.1870 \\ 0.5288 \\ 1.0149 \\ 0.8747 \\ 0.8697 \\ 0.9260 \\ 1.0418 \\ 1.5754 \end{pmatrix}$$

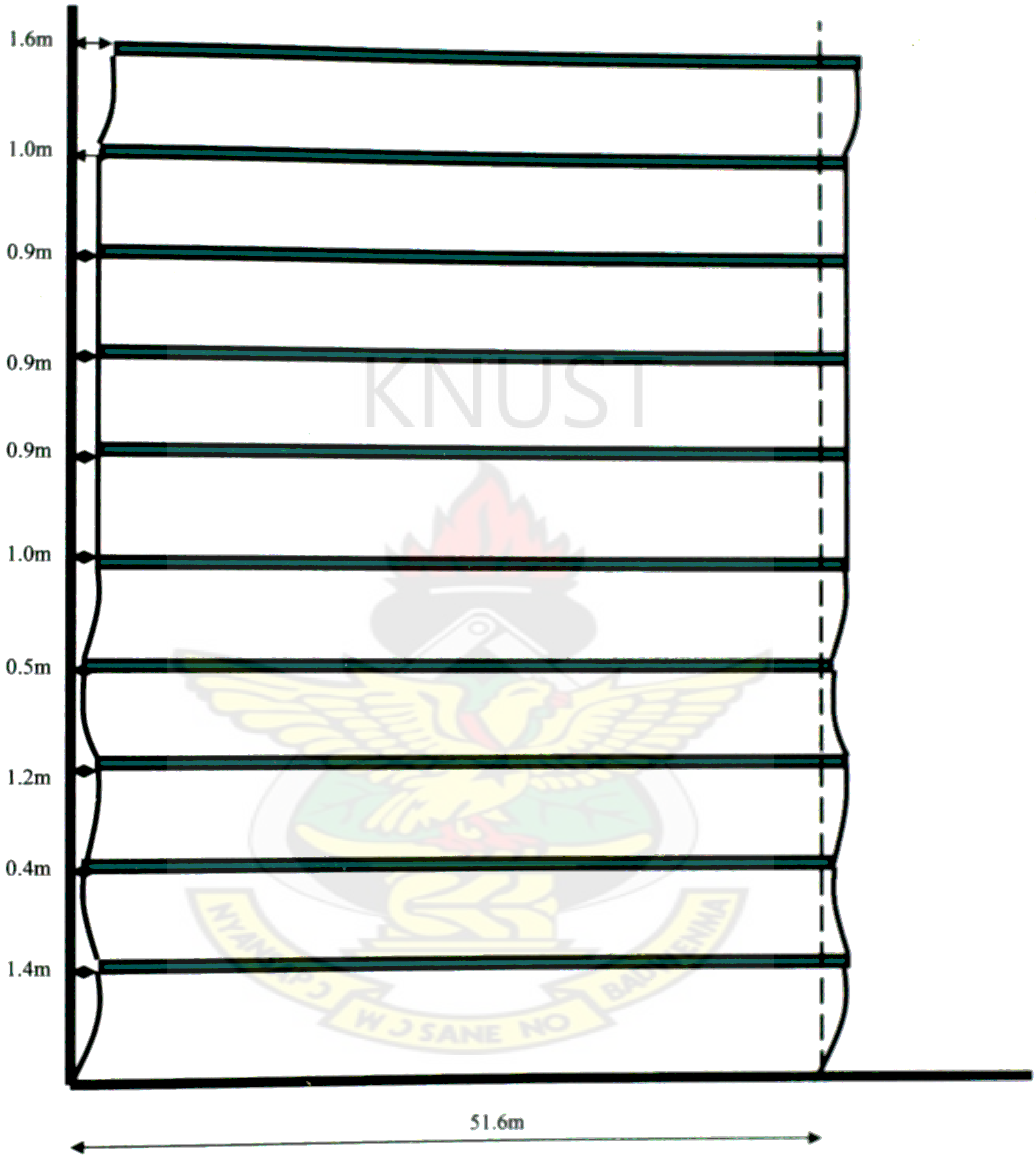


Figure 3.4 Schematic diagram of the maximum displacement of the building when live load is considered

3.2 ALGORITHM STABILITY TEST

The error matrix is calculated by taking the difference between the mass matrices (when we ignored live load and that of when the live load was considered).

$$E = \begin{pmatrix} -1.6542 & 0.6889 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.6889 & -0.9136 & 0.4568 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.4568 & -0.9136 & 0.4568 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.4568 & -0.9136 & 0.4568 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.4568 & -0.9136 & 0.4568 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.4568 & -0.9136 & 0.4568 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.4568 & -0.9136 & 0.4568 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.4568 & -0.9136 & 0.4568 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.4568 & -0.6765 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1639 & 0 \end{pmatrix}$$

$$(\lambda_1-\bar{\lambda}_1)^2+(\lambda_2-\bar{\lambda}_2)^2+\dots+(\lambda_n-\bar{\lambda}_n)^2=11.4557 \qquad \|E\|_F^2=\sum_{j=1}^{n}\sum_{i=1}^{n}|\varepsilon_{ij}|^2=12.9618$$

Hence the stability theorem, which states that for stability to be assured, the condition

$(\lambda_1-\bar{\lambda}_1)^2+(\lambda_2-\bar{\lambda}_2)^2+\dots+(\lambda_n-\bar{\lambda}_n)^2 \le \|E\|_F^2$ must be satisfied.

3.3 UNDER FORCED VIBRATION

3.3.1 Case 1: Ignoring live load

The resultant generalized eigenvalues problem is

$$10^5\begin{pmatrix} 4.7559 & -3.3475 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3.3475 & 5.5024 & -3.3475 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -3.3475 & 5.5024 & -3.3475 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3.3475 & 5.5024 & -3.3475 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3.3475 & 5.5024 & -3.3475 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3.3475 & 5.5024 & -3.3475 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -3.3475 & 5.5024 & -3.3475 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -3.3475 & 5.5024 & -3.3475 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3.3475 & 3.7646 & -1.6096 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.6096 & 1.0362 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \end{pmatrix}=10^6\begin{pmatrix} 4.4428 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Applying the algorithm for tridiagonal system described earlier

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \end{pmatrix} = \begin{pmatrix} 7.8628 \\ -2.1009 \\ 4.4095 \\ 9.3490 \\ 10.9579 \\ 8.6631 \\ 3.2821 \\ -3.2682 \\ -8.6542 \\ -13.4434 \end{pmatrix} \qquad \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \end{pmatrix}_{avg.\max} = \begin{pmatrix} 7.8628 \\ 2.1009 \\ 4.4095 \\ 9.3490 \\ 10.9579 \\ 8.6631 \\ 3.2821 \\ 3.2682 \\ 8.6542 \\ 13.4434 \end{pmatrix}$$

Displacement(x)	Four-digit decimal floating point number x'	Relative error $\frac{\lvert x'-x \rvert}{\lvert x \rvert}$	$\frac{\partial l}{l}$	$\frac{\partial \theta}{\theta}$
7.8626	0.7863×10^1	5.0874×10^{-5}	1.75×10^{-2}	1.7493×10^{-2}
2.1009	0.2101×10^1	4.1599×10^{-5}	1.75×10^{-2}	1.7502×10^{-2}
4.4095	0.4410×10^1	1.1339×10^{-4}	1.75×10^{-2}	1.7431×10^{-2}
9.3490	0.9349×10^1	5×10^{-5}	1.75×10^{-2}	1.7494×10^{-2}
10.9579	0.1096×10^2	1.9164×10^{-4}	1.75×10^{-2}	1.7352×10^{-2}
8.6631	0.8663×10^1	1.1543×10^{-5}	1.75×10^{-2}	1.7532×10^{-2}
3.2821	0.3282×10^1	3.0468×0^{-5}	1.75×10^{-2}	1.7514×10^{-2}
3.2682	0.3268×10^1	6.1196×10^{-5}	1.75×10^{-2}	1.7483×10^{-2}
8.6542	0.8654×10^1	2.3110×10^{-5}	1.75×10^{-2}	1.7521×10^{-2}
13.4434	0.1344×10^2	2.5291×10^{-4}	1.75×10^{-2}	1.7291×10^{-2}

Observing critically the relative error for θ shows that it is very close to the relative error for l as a result of the minute values for the relative error for displacement x.

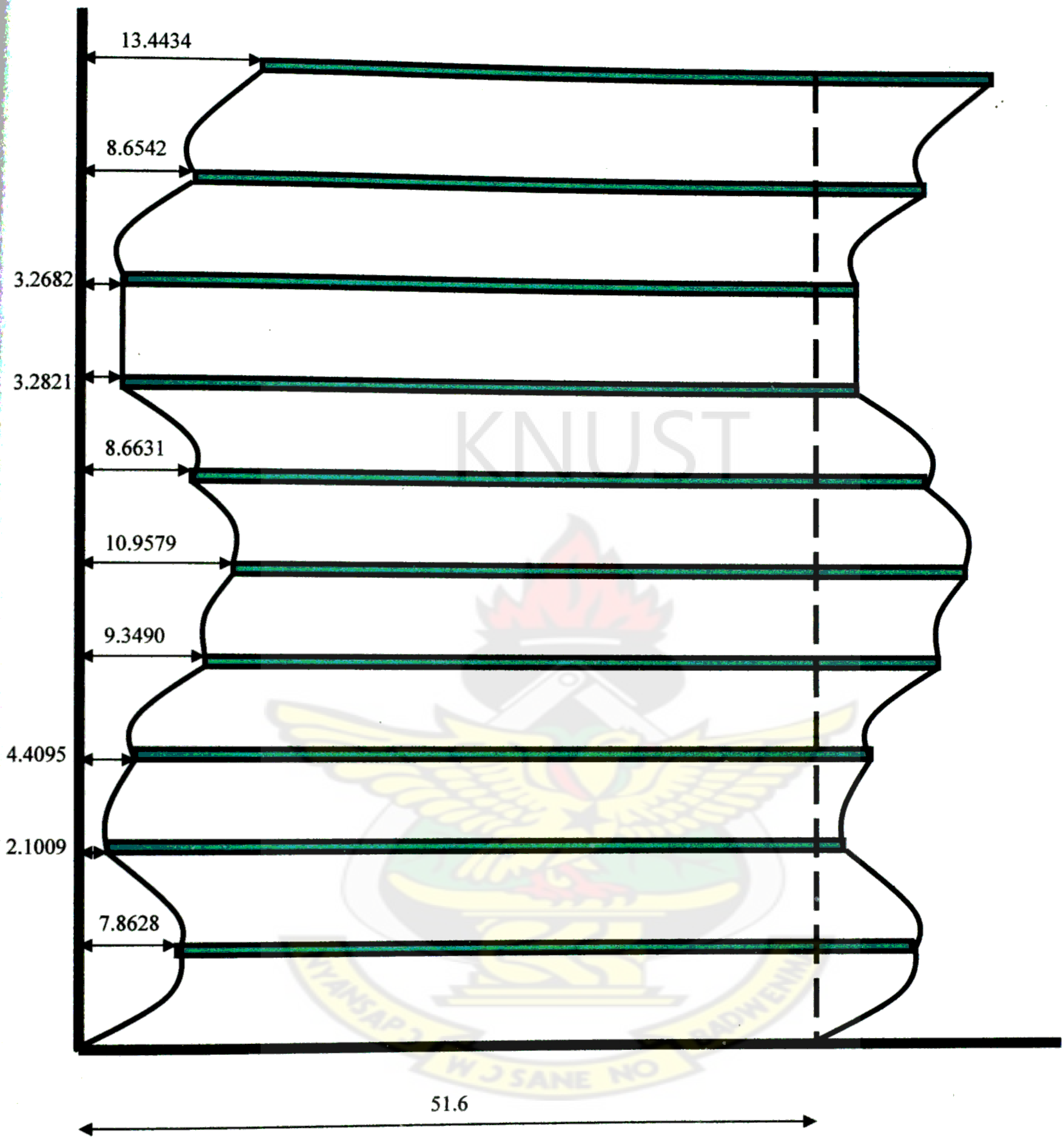


Figure 3.5 Schematic diagram of the maximum displacement of the building with no live load, when subjected to an earthquake of magnitude 2.0 on the Richter scale

3.3.2 Case 2: Considering live load

The resultant generalized eigenvalues problem is

$$10^{5}\begin{pmatrix} -4.2661 & -3.3475 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3.3475 & -6.0241 & -3.3475 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -3.3475 & -2.0595 & -3.3475 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3.3475 & -2.0595 & -3.3475 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3.3475 & -3.8180 & -3.3475 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3.3475 & -1.0952 & -3.3475 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -3.3475 & 1.0489 & -3.3475 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -3.3475 & 5.3896 & -3.3475 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3.3475 & 2.7790 & -1.6096 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.6096 & -0.0343 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \end{pmatrix} = 10^{6}\begin{pmatrix} 4.4428 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Applying the algorithm for tridiagonal system described earlier

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \end{pmatrix} = \begin{pmatrix} -7.1919 \\ -4.1065 \\ 0.1981 \\ 3.9846 \\ -2.6496 \\ -0.9626 \\ 2.9646 \\ 1.8915 \\ 0.0809 \\ 3.7940 \end{pmatrix} \qquad \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \end{pmatrix}_{avg.\max} = \begin{pmatrix} 7.1919 \\ 4.1065 \\ 0.1981 \\ 3.9846 \\ 2.6496 \\ 0.9626 \\ 2.9646 \\ 1.8915 \\ 0.0809 \\ 3.7940 \end{pmatrix}$$

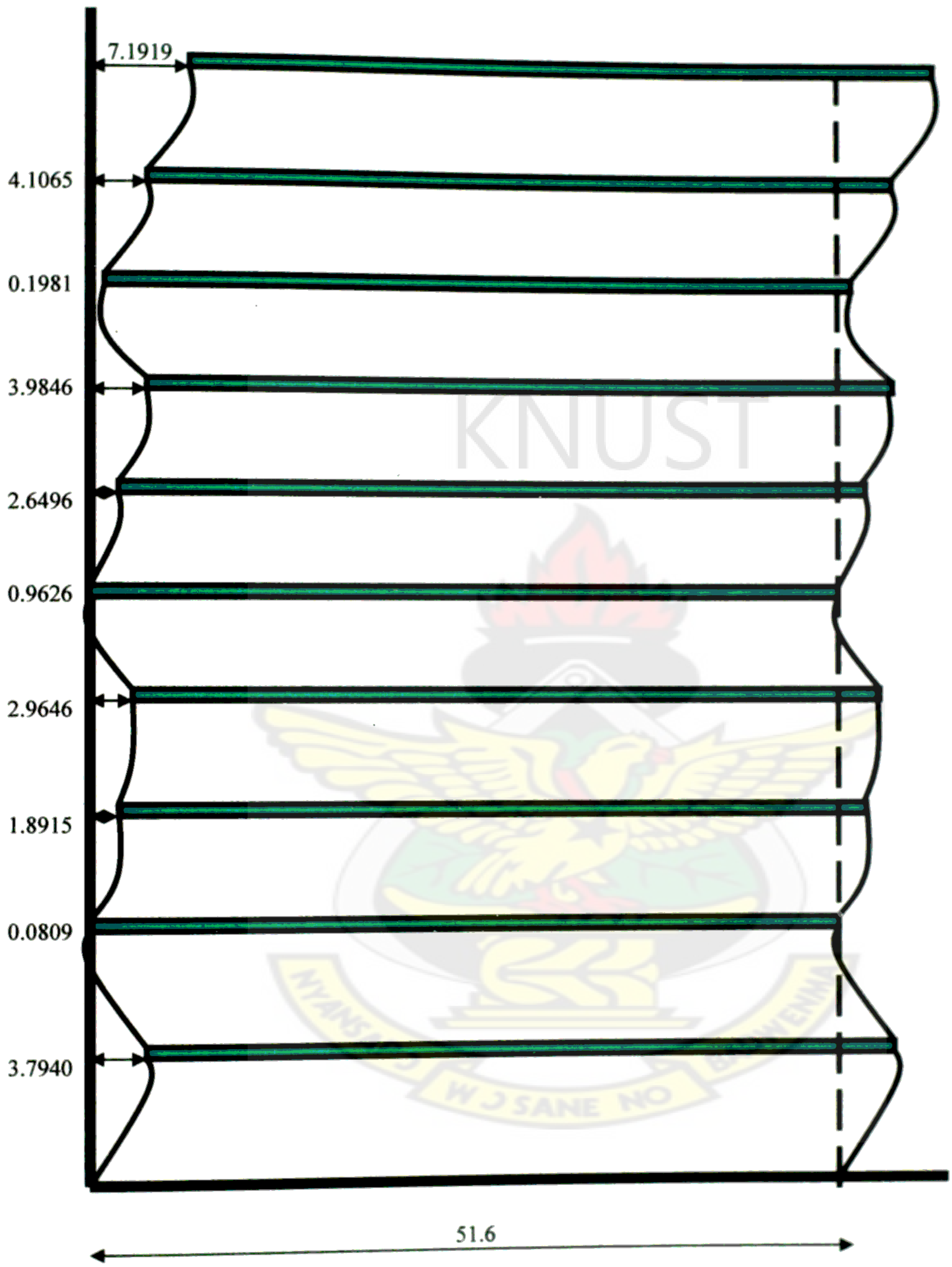


Figure 3.6 Schematic diagram of the maximum displacement of the building with live load included, when subjected to an earthquake of magnitude 2.0 on the Richter scale

3.4 SIMULATION OF UNITY HALL UNDER KNOWN FORCES

The maximum possible displacement when no live load is present in meters is

$$(0.2174 \quad 0.1081 \quad 0.2633 \quad 0.2382 \quad 0.2538 \quad 0.2532 \quad 0.2607 \quad 0.2574 \quad 0.3053 \quad 0.4258)^T$$

Table 3.2(a) Simulation results when live load is ignored

QUAKE MAGNI TUDE	LIVE LOAD IGNORED MAXIMUM DISPLACEMENT UNDER A KNOWN FORCE
0.25	$(0.1398 \quad 0.0374 \quad 0.0784 \quad 0.1663 \quad 0.1949 \quad 0.1541 \quad 0.0584 \quad 0.0581 \quad 0.1539 \quad 0.2391)^T$
0.50	$(0.2486 \quad 0.0664 \quad 0.1394 \quad 0.2956 \quad 0.3465 \quad 0.2740 \quad 0.1038 \quad 0.1034 \quad 0.2737 \quad 0.4251)^T$
0.75	$(0.4422 \quad 0.1181 \quad 0.2480 \quad 0.5257 \quad 0.6162 \quad 0.4872 \quad 0.1846 \quad 0.1838 \quad 0.4867 \quad 0.7560)^T$
1.00	$(0.7863 \quad 0.2101 \quad 0.4409 \quad 0.9349 \quad 1.0958 \quad 0.8663 \quad 0.3282 \quad 0.3268 \quad 0.8654 \quad 1.3443)^T$
1.25	$(1.3982 \quad 0.3736 \quad 0.7841 \quad 1.6625 \quad 1.9486 \quad 1.5405 \quad 0.5836 \quad 0.5812 \quad 1.5390 \quad 2.3906)^T$
1.50	$(2.4864 \quad 0.6644 \quad 1.3944 \quad 2.9564 \quad 3.4652 \quad 2.7395 \quad 1.0379 \quad 1.0335 \quad 2.7367 \quad 4.2512)^T$
1.75	$(4.4216 \quad 1.1814 \quad 2.4796 \quad 5.2573 \quad 6.1621 \quad 4.8716 \quad 1.8456 \quad 1.8379 \quad 4.8666 \quad 7.5598)^T$
2.0	$(7.8628 \quad 2.1009 \quad 4.4095 \quad 9.3490 \quad 10.9579 \quad 8.6631 \quad 3.2821 \quad 3.2682 \quad 8.6542 \quad 13.4434)^T$

The maximum possible displacement when live load is present, given in meters is

$$(1.3882 \quad 0.3957 \quad 1.1870 \quad 0.5288 \quad 1.0149 \quad 0.8747 \quad 0.8697 \quad 0.9260 \quad 1.0418 \quad 1.5754)^T$$

Table 3.2(b) Simulation results when live load is considered

QUAKE MAGNI TUDE	LIVE LOAD CONSIDERED MAXIMUM DISPLACEMENT UNDER A KNOWN FORCE
0.25	$(0.1279 \quad 0.0730 \quad 0.0035 \quad 0.0709 \quad 0.0471 \quad 0.0171 \quad 0.0527 \quad 0.0336 \quad 0.0014 \quad 0.0675)^T$
0.50	$(0.2274 \quad 0.1299 \quad 0.0063 \quad 0.1260 \quad 0.0838 \quad 0.0304 \quad 0.0937 \quad 0.0598 \quad 0.0026 \quad 0.1200)^T$
0.75	$(0.4044 \quad 0.2309 \quad 0.0111 \quad 0.2241 \quad 0.1490 \quad 0.0541 \quad 0.1667 \quad 0.1064 \quad 0.0045 \quad 0.2134)^T$
1.00	$(0.7192 \quad 0.4106 \quad 0.0198 \quad 0.3985 \quad 0.2650 \quad 0.0963 \quad 0.2965 \quad 0.1892 \quad 0.0081 \quad 0.3794)^T$
1.25	$(1.2789 \quad 0.7302 \quad 0.0352 \quad 0.7086 \quad 0.4712 \quad 0.1712 \quad 0.5272 \quad 0.3364 \quad 0.0144 \quad 0.6747)^T$
1.50	$(2.2743 \quad 1.2986 \quad 0.0626 \quad 1.2600 \quad 0.8379 \quad 0.3044 \quad 0.9375 \quad 0.5982 \quad 0.0256 \quad 1.1998)^T$
1.75	$(4.0443 \quad 2.3092 \quad 0.1114 \quad 2.2407 \quad 1.4900 \quad 0.5413 \quad 1.6671 \quad 1.0637 \quad 0.0455 \quad 2.1335)^T$
2.00	$(7.1919 \quad 4.1065 \quad 0.1981 \quad 3.9846 \quad 2.6496 \quad 0.9626 \quad 2.9646 \quad 1.8915 \quad 0.0809 \quad 3.7940)^T$

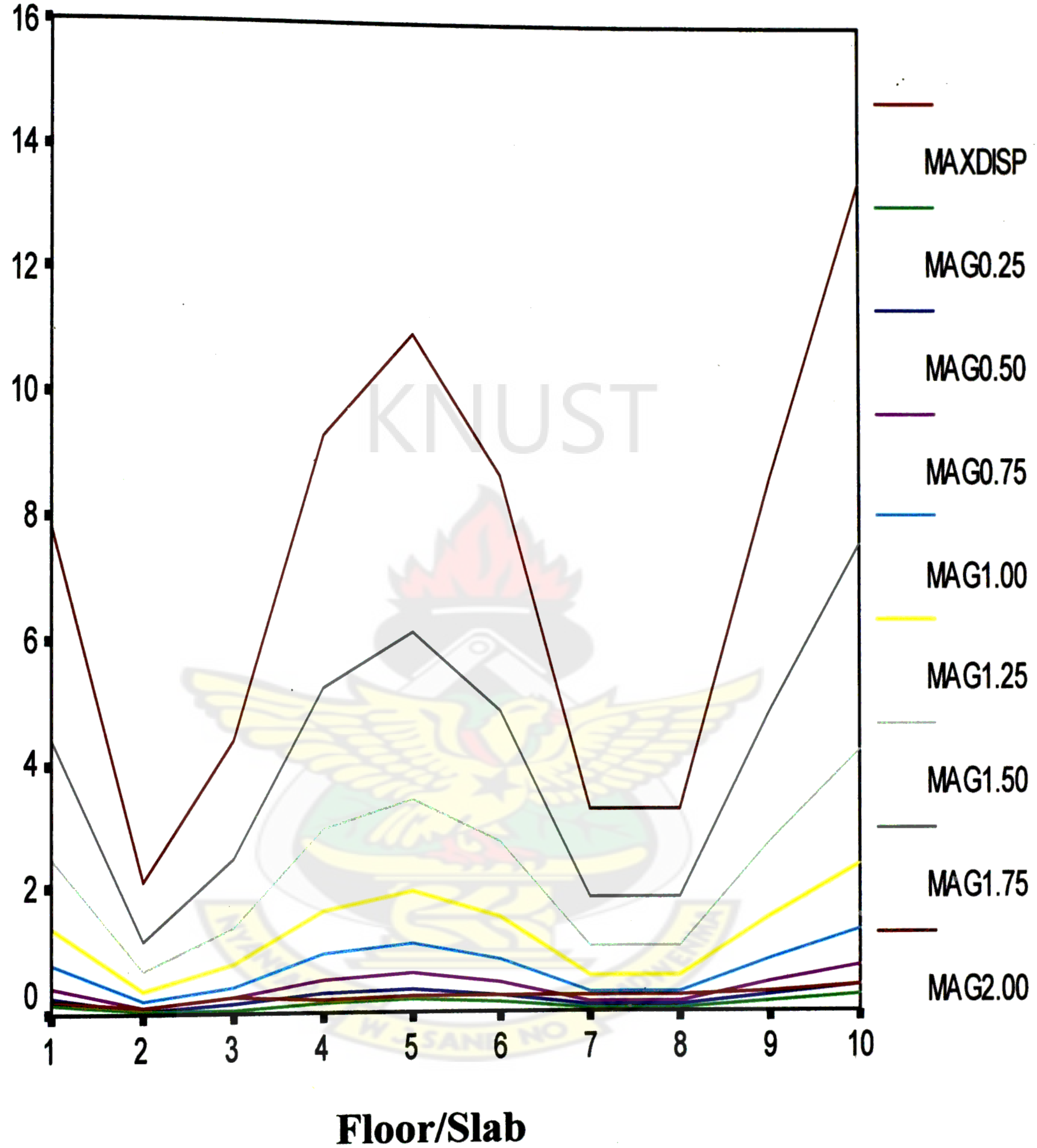


Figure 3.7(a) Graph showing simulation results when live load is ignored

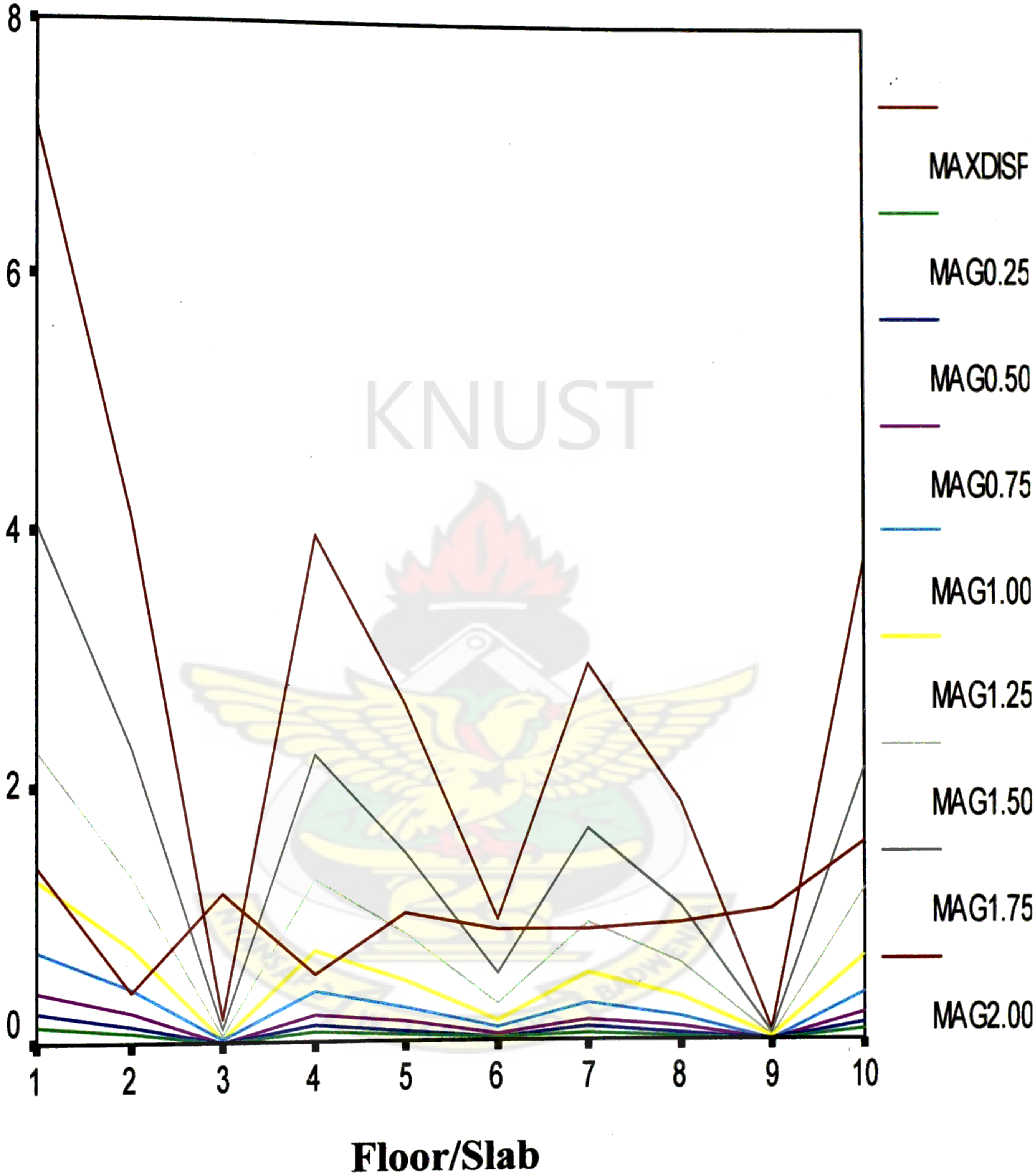


Figure 3.7(b) Graph showing simulation results when live load is considered

CHAPTER FOUR

4.0 INTRODUTION

This chapter seeks to discuss in detail the results of the computations of chapter three. It will also list the conclusions drawn based on the computations and the discussion of results. The recommendation of the project is provided for the benefit of society.

4.1 DISCUSSION OF RESULTS

Under free vibration and live load absent, all displacement values fall below 0.912*m* which shows that the model maintains its linearity under free vibration. However, when live load is present, some displacement values exceeded 0.912*m*, showing that the presence of live load reduces linearity of the model under free vibration. This goes to suggest that the linearity of the model is compromised when live load is included.

Under forced vibration, our model maintains its linearity up to an earthquake of magnitude 1.0 when live load is present and 1.25 when live load is absent This also confirm the allusion that, the presence of students have little effect on the linearity of the model under forced vibration. Above the stipulated magnitudes of 1.0 and 1.25 when live load is absent and present respectively, displacements exceeds 0.912*m* meaning the linearity assumption becomes no longer tenable and failure of the building becomes eminent.

In spite of the stated magnitude thresholds, it must be noted that if the frequency of the incoming wave coincides with one of the natural frequencies of the building, the amplitude of displacement becomes large, signaling the occurrence of resonance. Where as displacements above $0.912m$ will cause the building to crack, the resonance scenario will cause the building to fail woefully, even to the extent of collapse.

4.2 CONCLUSION

- Unity Hall has been modeled as a linear mechanical system and studied under conditions of
  1. Free vibration.
  2. Forced vibration.
- The linearity assumption is represented by bounds of θ in the range $-0.32 \leq \theta \leq 0.32$ radians.
- Unity Hall can maintain its linearity up to an earthquake of magnitude *1.0* when live load is absent and *1.25* when live load is present.
- The presence of live load in the building is likely to improve its tolerance under earthquake conditions.
- If the frequency of the imposed periodic force coincides with on of the modal frequencies of the building, the amplitudes become arbitrarily large, signaling the occurrence of resonance.

4.3 RECOMMENDATION

Simulation should be run for the various equations generated in this thesis to generate appropriate construction details to adapt in building high-rise buildings to be able to maintain its linearity and also minimize cost.

REFERENCES

1. Glenn Ledder, *Differential Equations: A Modeling Approach*, McGraw Hill, 2005
2. James Stewart, *Calculus*, International Thomson Publishing Inc., 1999
3. Biswa Nath Data, *Numerical Linear Algebra and Applications*, International Thomson Publishing Inc., 2000
4. Paul Dawkins, *Lecture Notes on Differential Equations*, http://tutorial.math.lamar.edu, 2005
5. Bureau of Indian Standards, *Status Report on Standardization Efforts in the Area of Mitigation of Natural Hazards*, 2006
6. Charles Davidson, *The Founder of Seismology*, Amo Press, New York 1978.
7. John H. Matthews, Kurtis D. Fink, *Numerical Methods Using MATLAB*, Pearson Education, Inc. 2004
8. Wilkinson, James H. *The Algebraic Eigenvalue Problem*, Oxford: Clarendon Press, 1965
9. Schneider, Hans and George Philip Baker, *Matrices and Linear Algebra*, New York: Dover Publications, 1989
10. Householder A.S. *The Theory of Matrices in Numerical Analysis*. New York: Dover Publications, 1964
11. Franklin, Joel N. *Matrix Theory*. Englewood Cliffs, New Jersey: Prentice Hall, 1968
12. Atkinson, Kendall. *Elementary Numerical Analysis*; New York. John Wiley, 1993
13. Cheney, Ward and David Kincaid, *Numerical Mathematics and Computing*, Third Edition, Pacific Grove, California: Brooks/Cole, 1994

14. Golub, Gene H., and James M. Ortega, Scientific Computing and Differential Equation: An Introduction to Numerical Methods. San Diego: Academic Press, 1992

15. Rice, John R. Numerical Methods, Software and Analysis. New York: McGraw-Hill, 1983

16. Ortega, James M., and William G. Poole Jr. Numerical Methods for Differential Equations. Mass: Pitman Publishing, 1981

17. T. Coleman and Charles Van Loan. Handbook for Matrix Computations. Philadelphia: SIAM, 1988

18. Stewart G.W. An Introduction to Matrix Computations. New York: Academic Press, 1973

19. Patrick L. Abbott, Natural Disasters, Fifth edition, McGraw-Hill, 2006

20. Philip E. Gill, Walter Murray, and Margaret H. Wright. Numerical Linear Algebra and Optimization. Reading, Mass: Addison Wesley, 1991

21. Chapra, Steven C, and Raymond P. Canale. Numerical Methods for Engineers, Second edition. New York: McGraw Hill, 1988

22. O'Neil, Peter V. Advanced Engineering Mathematics, Third edition. Belmont, California: Wadsworth, 1991

23. Braun, M. Differential Equations and Their Applications. New York: Springer-Verlag, 1978

24. Luenberger, David G. Introduction to Linear and Nonlinear Programming. New York: Addison-Wesley, 1973

APPENDIX

MATLAB IMPLEMENTATION OF THE ALGORITHM FOR THE MODEL

```
clc
display('-----------------------------------------------------------')
display('THIS IS THE CODING FOR A THESIS IN PARTIAL FULFILMENT OF A
MASTER')
display('OF SCIENCE DEGREE IN MATHEMATICS, PRESENTED TO THE
GRADUATE SCHOOL')
display('OF KNUST BY DERICK FOLSON (BSC. COMPUTER SCIENCE)')
display('-----------------------------------------------------------')
g=10;
pi=22/7;
h=input('What is the height of each floor in meters?');
if h<=0
  display('Remember you are entering height of a floor of a building! Run the
programme again')
break
end
n=input ('What is the size of matrix or total number of floors?');
if n<=1
  disp('Please, we are dealing with storey buildings! Run the programme again')
  break
end
```

```
M=zeros(n);%holds the mass matrix when live load is absent
M1=zeros(n);%holds the mass matrix when live load is considered
M2=zeros(n);%holds the error matrix
K=zeros(n);%holds the stiffness matrix
R=zeros(n,1);%holds the relative response to displacement
F=zeros(n,1);%holds the quake magnitude vector
r=zeros(n,1);%holds the displacements under a quake
D1=zeros(n,1);%holds the eigenvalues when liveload is absent
D2=zeros(n,1);%holds the eigenvalues when live load is present
F1=zeros(n,1);%holds the natural frequencies when live load is absent
F2=zeros(n,1);%holds the natural frequencies when live load is present
Y=zeros(n,1);%holds the right hand side of the stability theorem resu,lts
for i=1:n
   M(i,i)=input('enter mass');
   if M(i,i)<=0
      disp('a floor mass cannot have this value')
      break
   end
end
display('---------------------MASS MATRIX-------------------------------')
M
sk=zeros(n,1);
for i=1:n
```

```
    sk(i)=M(i,i)*g/h;
end
display('----------------------COLUMN CONSTANTS-------------------------')
sk
sk(n+1)=0;
for i=1:n
    K(i,i)=sk(i)+sk(i+1);
end
K;
for i=3:n
    K(2,1)=sk(2);
    K(2,3)=-sk(3);
    K(1,2)=-sk(2);
    K(i,n+1)=0;
    K(i,i-1)=-sk(i);
    K(i,i+1)=-sk(i+1);
end
K(:,n+1)=[];
display('---------------------------STIFFNESS MATRIX---------------------')
K
display('------------------------CHOLESKY FACTORIZATION------------------')
L=chol(M)
display('------------------------SYMMETRIC MATRIX-----------------------')
```

```
C=inv(L)*K*inv(L')
[V,D]=eig(C)
for i=1:n
    D1(i)=real(D(i,i));
    F1(i)=sqrt(D1(i))/(2*pi);
end
D1;
display('---------------NATURAL FREQUENCIES----------------------------')
F1
for i=1:n
    R(i)=input('enter relative response');
end
R
    x=zeros(n,1);
    sum=0;
    sum1=0;
    sum2=0;
for i=1:n
    x=inv(L')*real(V(:,i));
    for i=1:n
        E=M(i,i)*x(i);
        sum=sum+E;
    end
```

```
    display('-------------EIGENVECTOR-----------------')
    x
    display('----------------E-----------------------')
    sum
    sum1=sum1+sum*R(i)*x;
    sum2=sum2+(sum*R(i)*x).^2;
end
display('---------------------ABSOLUTE MAXIMUM-----------------------')
absmax=sum1
display('------------------------------------------------------------')
display('---------------------AVERAGE MAXIMUM DISPLACEMENT-------------')
display('------------------------------------------------------------')
avgmax=sqrt(sum2)
max(avgmax);
for i=1:n
    M2(i,i)=input('enter mass of live load');
end
M1=M2+M;
display('----------------DEAD AND LIVE LOAD MASS MATRIX----------------')
M1
display('----------------------CHOLESKY FACTORIZATION----------------')
L1=chol(M1)
display('----------------------SYMMETRIC MATRIX---------------------')
```

```
C1=inv(L1)*K*inv(L1')

[V,D]=eig(C1)

for i=1:n

   D2(i)=real(D(i,i));

   F2(i)=sqrt(D2(i))/(2*22/7);

end

D2;

display('------------------NATURAL FREQUENCIES OF FLOORS-----------------')

F2

 sum3=0;

   sum4=0;

   sum5=0;

for i=1:n

   x=inv(L')*real(V(:,i));

   display('----------------EIGENVECTORS OF THE PENCIL (K,M )-------------')

   x

   for i=1:n

      E=M(i,i)*x(i);

      sum3=sum3+E;

      sum4=sum4+sum3*R(i)*x;

      sum5=sum5+(sum3*R(i)*x).^2;

   end

   display('-----------E1------------')
```

```
    sum3
end
display('---------------------ABSOLUTE MAXIMUM-----------------------')
absmax=sum4
display('-----------------------------------------------------------')
display('---------------------AVERAGE MAXIMUM DISPLACEMENT-------------')
display('-----------------------------------------------------------')
avgmax=sqrt(sum5)
max(avgmax);
display('---------------------ERROR MATRIX-----------------------')
ER=C1-C
display('-----------------ALGORITHM STABILITY TEST-----------------')
Y=(D1-D2).^2
sum6=0;
for i=1:n
   sum6=sum6+Y(i);
end
sum6
s=(norm(ER,'fro'))^2
if sum6<=s
   display('WE HAVE A STABLE SYSTEM')
else
   display('WE HAVE AN UNSTABLE SYSTEM')
```

```
end

c=input('what magnitude do you want to subject the building to?');

t=2

F(1)=10^(c+2.92-3*log10(8*t));


display('----------------------QUAKE MAGNITUDE VECTOR-------------------')

F

for i=1:n

   K(i,i)=K(i,i)-M(i,i)*real(D(i,i));

end

display('---------------------RESULTANT TRIDIAGONAL SYSTEM--------------')

K

display('----------------------DISPLACEMENT UNDER EARTHQUAKE------------')

r=real(inv(K)*F)


display('------------------ABSOLUTE MAXIMUM DISPLACEMENT----------------')

real(r)

display('------------------------------------------------------------')

display('------------------AVERAGE MAXIMUM DISPLACEMENT----------------')

display('------------------------------------------------------------')

abs(r)

for i=1:n

   K(i,i)=K(i,i)-M1(i,i)*real(D(i,i));
```

```
end

display('---RESULTANT TRIDIAGONAL SYSTEM WHEN LIVE LOAD IS
CONSIDERED----')

K

display('---DISPLACEMENT UNDER EARTHQUAKE WHEN LIVE LOAD IS
CONSIDERED---')

r=real(inv(K)*F)


display('------------------ABSOLUTE MAXIMUM DISPLACEMENT----------------')

real(r)

display('--------------------------------------------------------------')

display('------------------AVERAGE MAXIMUM DISPLACEMENT-----------------')

display('--------------------------------------------------------------')

abs(r)
```