KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI

# LOCATION OF AN ADDITIONAL POLICE STATION IN AFIGYA KWABRE DISTRICT USING THE CONDITIONAL P-MEDIAN APPROACH

BY

EMMANUEL OMENAKO-DANQUAH

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# DECLARATION

I hereby declare that this submission is my own work towards the award of the M. Phil degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the university, except where due acknowledgment had been made in the text.

EMMANUEL OMENAKO-DANQUAH .................... ..................

Student Signature Date

Certified by:

Prof. S. K. AMPONSAH .................... ..................

Supervisor Signature Date

Certified by:

Prof. S. K. AMPONSAH .................... ..................

Head of Department Signature Date

# DEDICATION

This work is dedicated to the Almighty Lord who is my source of strength and knowledge and to my lovely wife Mrs. Susan Omenako-Danquah and sons Kenny B. Omenako-Danquah and Kirk J. Omenako-Danquah who have always stood by me, inspired and supported me.

# ABSTRACT

This study focuses mainly on conditional facility location problems on a network. This thesis discuss the conditional p–median problem on a network. Demand nodes are served by the closest facility whether existing or new. The thesis considers the problem of locating a police station facility (non–obnoxious facility) as a conditional p – median problem, thus some existing facilities are already located in the district. This thesis uses a new formulation algorithm for the conditional p- median problem on a network which was developed by Oded
Berman and Zvi Drezner (2008) to locate an additional police station in Afigya Kwabre district. A 26 node network which had five existing police facilities was used. The result indicated that additional police station should be located at Nanso (node 21) with an optimal objective function value of 62292. The additional facility at Nanso will largely help reduce the pressure on the existing police stations and improved the security of the residents in the district.

# ACKNOWLEDGMENT

My heart is filled with gratitude and praise to the Almighty God for life and bestowing on me wisdom to undertake this research. My sincere gratitude goes to my supervisor Professor S. K. Amponsah for his guidance, counseling and constructive comments, and excellent piece of advice to ensure a successful completion of this work.

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# CONTENTS

# LIST OF ABBREVIATION

DPCU .................................... District Planning Committe Unit

DINAS .........................Dynamic interactive network analysis system

APS .......................................... Advanced Producer Services LB

............................................................Lower Bond

MCLP ................................. Maximum covering location problem PDP

..................................................P-dispersion problem

ARPH .................................Ashanti Regional Police Headquaters

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# CHAPTER 1

# INTRODUCTION

Location simply refers to the act of putting something in place or position where that thing can be identified. A facility is considered as a physical entity that provides services. Facility location problem arise in a wide range of practical applications in different fields of study: economic, management, planning, production and many others. Welch and Duxbury (1997) also classified facilities into three categories: non-obnoxious(desirable), semi-obnoxious and obnoxious(non-desirable).

Almost every public and private sector enterprise that we can think of has been faced with the problem of locating facilities. Government agencies need to determine location of offices and other public services such as schools, hospitals, police stations, fire station ambulance bases, and so on. Industrial firms must determine location for fabrication and assembly plants as well as warehouses. In these cases, the success or failure of facilities depends in part on location chosen for those facilities. Such problems are know as facility location problems.

## 1.1 Bacground of the Study

The Afigya Kwabre District is located in the central part of Ashanti Region of Ghana between Latitudes $6^0$ 50N and Longitudes $1^0$ 40 W and $1^0$ 25W. The District has an area of about 409.4 square kilometers representing 1.68

% of the land area of Ashanti Region. The District is bounded by Kumasi Metropolitan Assembly to the south, Ejura Sekyedumase to the North-West, Atwima Nwabiagya to the South-West, Sekyere South to the North, Offinso Municipal to the West and Kwabre District to the East. According to crime rate report for the year 2012 to 2014 Afigya Kwabre district crime rate has increased from 12.2%

to 16.7 % (Criminal Department(ARPH)). Most of it happened in the towns along the main road through the district to northern part of Ghana and towns on boundaries to Kumasi Metropolitan such as Atimatim, Afrancho and so on. The sitting of a non-obnoxious facility (police station) will help bring down crime rate in Afigya Kwabre district for its inhabitants to feel secure.

## 1.2 Problem Statement

One of the problems facing both the public and the private sector enterprises is how to locate facilities. Institutions site their facilities anywhere, anyhow without first considering how close that facility will be to the people in the community. This work therefore seeks to find the optimal site to locate an additional police in Afigya Kwabre district using conditional P-median model.

## 1.3 Objectives of the study

The objectives of this study:

1. To locate an additional police station using conditional P median model in the Afigya Kwabre District.
2. To locate an additional police station at a suitable site in Afigya Kwabre district and to minimize the distance between the existing facilities and the new facility located.

## 1.4 Methodology

The objective of the study is will locate an additional police station in the Afigya Kwabre district using conditional P- median models. Data on road distance between communities were collected and used. Floyd-Warshall algorithm will be used.

## 1.5 Justification

In recent years, Ghana has been hit with a lot of crime cases such as armed robbery, domestic violence, pick pockets, petty theft and many more. Afigya Kwabre district is no exception, towns like Atimatim, Afrancho and others are fast growing areas in the district where crime is also on increase since they have boundaries with the Kumasi metropolitan Assembly. With an additional police station in the district, it would in turn help improve on the security of the people in the district and the country as a whole.

It is hoped that the result of this study would inform the authorities in the Afigya Kwabre district about the right place to locate a police station in the district. The presence of police personnel in a community or in public areas deter people from going against the law. So this shows that the presence of a police station within a community will go a long way to provide security to the people living there.

If there is a police station in a community, inhabitants and investors will feel secured to establish industries to boost the economic activities. Again it will bring discipline among the youth and the entire community. It will also bring a halt to armed robbery along the major street leading to the northern Ghana and in the Afigya Kwabre district. In the near future a research can be done in location of an additional police station in the other District in Ashanti region.

## 1.6 Scope of the study

The study want to help locate an additional police station in Afigya Kwabre district within the Ashanti region. The study would address the increasing crime rate in the Afigya Kwabre district especially the southern part of the district which shares boundary with the Kumasi Metropolitan Assembly. The study would covered the top twenty-six communities (26) within the district.

## 1.7 Limitation of the study

The research was limited to Afigya Kwabre district in order to ensure effective study as well as reducing the financial burden on the researcher. It is highly believed that other districts in the region as well as in Ghana exhibit common characteristics and share similar plight.

## 1.8 Thesis organization

Chapter one presents the background of the study, problem statement, objective of the study, methodology, justification, scope of the study, limitation of the study and organization of the thesis. The second chapter deals with the literature review. Chapter three presents the research methodology. Data, analysis and results are considered in chapter four . Conclusion and recommendation of the study are in chapter five.

# CHAPTER 2

# LITERATURE REVIEW

## 2.1 Introduction

In location problem we want to locate specific type of facility. Usually we look for the best way to serve a set of communities whose location and demands are known. This implies one needs to decide on:

i. The number and location of the facilities to serve the demand

ii. Size and capacity of each facility

iii. The allocation of the demand points to open facilities

iv. Optimizing some objective location function.

Most location models deals with desirable facilities, such as warehouse, service and transportation centers, emergency services, etc, which interact with the customers and where distance travel is involved. As a consequence, typical criteria for such decision include minimizing some function of the distance between facilities and clients (i. e., average travel time, average response time, cost function of travel or response time, maximum travel time or cost, etc.). However, during the last two decades, those responsible for the overall development of the area, where the new facility is going to be located (i.e. central government, local authorities) as well as those living in the area (population), are showing an increasing interest in preserving the area's quality of life. Hence, new words have been introduced in the location theory, such as: non-obnoxious, obnoxious, semi obnoxious, hazardous, etc. As examples of undesirable facilities we can mention; nuclear and military installations, equipment emitting particular smell or noise, warehouses containing flammable materials, regions containing refuse or waste materials, garbage dumps, sewage plants, correctional centers, etc.

## 2.2 Some Approaches to Facility Location Problem

In Malczewski and Ogryczak (1990) the location of facility is formulated as a multi-objective optimization problem and an interactive approach DINAS, Dynamic interactive network analysis system. Ogryczak and Zawadzki (2002) based on the so called reference point approach is presented. A real application is presented, considering eight sites for potential location and at least four new hospitals to be built, originating in hundred and sixty three alternative location patterns each of them generating many possible allocation schemes. The authors mention that the system can be used to support a group decision - making process making the final decision less subjective. They also observed that during the interactive process the decision – makers have gradually learned about the set of feasible alternatives and in consequence of this leaning process they have change their preference and priorities.

Erkut and Neuman (1992) present a mixed integer linear model for undesirable facility location. The objectives considered are total cost minimization, total opposition minimization and equity minimization.

Ballou (1998) discusses a selected number of facility location methods for strategic planning. He further classifies the more practical methods into a number of categories in the logistics network, which include single–facility location, multi–facility location, dynamic facility location, retail and service location.

Christopher (1972) comprehensively present that whether the problem of depot location is static or dynamic, 'Infinite Set' approaches and 'Feasible Set' approach can be identified. The infinite set approach assumes that a warehouse is flexible to be located anywhere in a certain area. The feasible set approach assumes that only a finite number of known sites are available as warehouse locations. They believe the centre of gravity method is a sort of infinite set model.

Berman and Larson (1985) studied a single-facility location problem in which travel times are stochastic and the facility (e.g. Ambulance) may be relocated at a cost as conditions change. Travel times are scenario-based, and scenario transitions occur according to a discrete-time Markov process. The objective is to choose a facility location for each scenario to minimize expected transportation and relocation costs. The authors show that Hakimi (1964) property applies to this problem and that the problem on a tree is equivalent to the deterministic problem; any scenario can be used to determine the optimal location since Imedian on a tree is independent of the edge of lengths.

They then present a heuristic for the problem on a general network that involves iteratively fixing the location in all but one scenario and solving what amounts to I-median problem. They discuss simple bounds on the optimal objective value of the multi-facility problem.

Berman and LeBlanc (1984), introduce a heuristic for this problem that loops through the scenarios, performs local exchanges within each, and then performs exchanges to link the scenarios in an effort to reduce relocations costs.

Fonseca and Captivo (1996)Fonseca (2006)Fonseca and Captivo (2007) study the location of semi obnoxious facilities as a discrete location problem on a network. Several bi-criteria models are presented considering two conflicting objectives, the minimization of obnoxious effect and the maximization of the accessibility of the community to the closest open facility. Each of these objectives is considered in two different ways, trying to optimize its average value over all the communities or trying to optimize its worst value. The Euclidean distance is used to evaluate the obnoxious effect and the shortest path distance is used to evaluate the accessibility. The obnoxious effect is considered inversely proportional to the weighted Euclidean distance between demand points and open facilities, and demand directly proportional to the population in each community. All the models are solved using Chalmet and Elzinga (1986) non- interactive algorithm for
Bi-criteria Integer Linear Programming modified to an interactive procedure by Ferreira and Paixao (1994). Several equity measures are computed for each nondenominated solution presented to the decision-maker, in order to increase the information available to the decision -maker about the set of possible solutions.

## 2.3 P Centre Location problems

The P-centre model minimizes the maximum distance between any demand point and it nearest facility. This model is introduced under the title p-centre problem which is in fact a minimax problem. In this model the objective is to find locations of p-facilities so that all demands are covered and the nearest facility (coverage distance) is minimized. It can be said that we have relaxed the coverage distance Daskin (1995).

In the p-centre model, each demand point has a weight. These weights may have different interpretations such as time per unit distance, cost per unit distance or loss per unit distance Daskin (1995). So the problem would be seeking a centre to minimize a maximum time, cost or loss. In other words the concern is about the worst case and we want to make it as good as possible Francis and White (1992). Garfinkel and Rao (1977) examined the fundamental properties of the P-centre problem in order to locate a given number of emergency facilities along a road network. He modelled the P-centre problem using integer programming and the problem was successfully solved by using a binary search technique and a combination of exact tests and heuristics.

Chen and Chen (2009) presented a new relaxation algorithm for solving the conditional continuous and discrete p-center problems. In the continuous pcenter problem, the location of the service facilities can be anywhere in the two-dimensional Euclidean space. In the discrete variant there is a finite set of potential service points to choose from. An analogous representation of the discrete p-center problem is the p-center problem on networks. In the p-center problem on networks, both the demand points and the potential service points are located on a weighted undirected graph, and the distance between any two points is the cost of the shortest path between them. They assumed that, there are a finite number of values for the optimal solution of an unconditional p-center problem. They use the assumption to implement the subroutine Get- Next Bound (Lower- Bound) which returns the smallest value, among the possible values for the optimal solution, which is greater than Lower-Bound.

Hassin and Dana (2003) introduced a local search strategy that suits combinatorial optimization problems with a min-max (or max-min) objective. According to this approach, solutions are compared lexicographically rather than by their worst coordinate. They apply this approach to the p-center problem. Based on a computational study, the lexicographic local search proved to be superior to the ordinary local search. This superiority was demonstrated by a worst-case analysis.

The conditional location problem is to locate p new facilities to serve a set of demand points given that q facilities are already located. When q is equal to zero (q = 0), the problem is unconditional. In conditional p – center problems, once the new p locations are determined, a demand can be served either by one of the existing or by one of the new facilities whichever is the closest facility to the demand Berman and Larson (1985). The p-center problem seeks the location of p facilities. Each demand point receives its service from the closest facility. The objective is to minimize the maximal distance for all demand points. The p-center problem consists of choosing p facilities among a set of M possible locations and assigning N clients to them in order to minimize the maximum distance between a client and the facility to which it is allocated.

Elloumi and Terry (2004), presented a new integer linear programming formulation for this min-max problem with a polynomial number of variables and constraints, and show that its LP relaxation provides a lower bound tighter than the classical one. Moreover, they showed that an even better lower bound LB, obtained by keeping the integrability restrictions on a subset of the variables, can be computed in polynomial time by solving at most O(log 2(NM)) linear programs, each having N rows and M columns. They also show that, when

the distances satisfy triangle inequalities, LB is at least one third of the optimal value. Finally, they used the LB in an exact solution method and report extensive computational results on test problems from the literature. For instances where the triangle inequalities are satisfied, their method out performs the running time of other recent exact methods by an order of magnitude. In addition, it is the first one to solve large instances of size up to N = M = 1, 817.

## 2.4 Conditional Location problem

Minieka (2006), stated that, previous treatments of location problems on a graph have been confined to the optimum location of a single facility or the simultaneous optimum location of multiple facilities. The author addresses the problem of optimally locating a facility on a graph when one or more other facilities have already been located in the graph. The author shows that previous solution techniques can be reused if the distances in the graph are judiciously redefined.

Tamir (2001) deal with the location of extensive facilities on trees, both discrete and continuous, under the condition that existing facilities are already located. They require that the selected new server is a subtree, although we also specialize to the case of paths. They study the problem with the two most widely used criteria in Location Analysis: center and median. Their main results under the center criterion are nestedness properties of the solution and subquadratic algorithms for the location of paths and subtrees. For the case of the median criterion they prove that unlike the case where there is no existing facility, the continuous conditional median subtree problem is NP-hard and we develop a corresponding fully polynomial approximation algorithm. They also present subquadratic algorithms for almost all other models.

Wouter(2011), contributed to conditional location by writing; within research on world cities, much attention has been paid to Advanced Producer Services (APS) and their role within both global urban hierarchies and network formation between cities. What is largely ignored is that these APS provide services to firms operating in a range of different sectors. Does sector specific specialization of advanced producer services influence the economic geography of corporate networks between cities? If so, what factors might explain this geographical pattern? This paper investigates these theoretical questions by empirically focusing on those advanced producer services related to the port and maritime sector. The empirical results show that the location of AMPS is correlated with maritime localization economies, expressed in the presence of ship owners and port-related industry as well as APS in general, but not by throughput flows of ports. Based upon the findings, policy recommendations are addressed.

Berman and Drezner (2008) discuss the conditional p-median and p-center problems on a network. Demand nodes are served by the closest facility whether existing or new. Rather than creating a new location for an artificial facility and force the algorithm to locate a new facility there by creating an artificial demand point, the distance matrix was just modified. They suggested solving both conditional problems by defining a modified shortest distance matrix.

Berman and Drezner (2008) described an algorithm to solve conditional location problems (such as the conditional p-median problem or the conditional p-center problem) on networks, where demand points are served by the closest facility whether existing or new. This algorithm requires the one-time solution of a (p+1) unconditional corresponding location problem using an appropriate shortest distance matrix.

# CHAPTER 3

# METHODOLOGY

## 3.1 Facility Location problems

All private sector and government agencies face problem of locating facility. Location of public services such as police station, fire station, hospitals, schools, government officers, airport, lorry station, warehouses and so on. In all these, the success or failure of facilities depends in the locations chosen for those facilities. Such problems are known as facility location problems.

Facility location models are used in a wide variety of applications. Examples include locating hazardous materials sites to minimize exposure to the public, locating lorry station to minimize the distance to board a bus to your destination,locating railroad stations to minimize the variability of delivery schedules, locating automatic teller machines to best serve the banks customers, locating a coastal search and rescue station to minimize the maximum response time to maritime accidents and locating of hospital to best serve the people in the area (Hale and Moberg (2003)). There are different types of facility location problems. Some basic classes of facility location problems are listed below (Berman and Krass (2002))

1. Discrete facility location problem: location problem where the sets of demand points and potential facility location are finite.

2. Continuous facility location problem: location problem in a general space endowed with some metric, example $l_p$ norm. Facility can be located anywhere in the given space.

3. Network facility location problem: location problem which is confined to the links and nodes of an underlying network

4. Stochastic facility location problem: location problem where some parameters, example demand or travel time are uncertain

Models are called dynamic (as opposed to static) if the time element is explicitly represented (Wesolowsky (1973)). This study of the problem was based on, can be classified as discrete. (Current and Schilling (2002)) listed several basic discrete network location models: Covering (including Set covering and Maximal covering), p-center, p-dispersion, p-median, fixed charge, hub and maxisum. Distances or some related measures (example, travel cost or time) are fundamental to such problems. Furthermore, we classify them according to their consideration of distance. The hub and maxisum are based on total or average distance where as p-center, p-dispersion and p-median are based on maximum distance.

## 3.2 Total or Average Distance Model

Most of facility locations in private and public sectors are concerned with the total travel distance between demand nodes and facilities. In the public sector, one might want to locate a network of service providers such as police station and schools in the way that the total distance travel by people must be minimize to reach their closest facility.This approach viewed as an ’efficiency’ objective as opposed to the ’equity’ objective of minimizing the maximum distance, which is mentioned in other models.

I. P-median problem: the p-median problem (Hakimi (1964)) seeks to find the locations of p facilities to minimize the demand weighted total distance between demand nodes and the facilities to which they are assigned.

II. The maxisum location problem: the maxisum location problem seeks to locate p-facilities (undesirable facilities) such that the total demand weighted distance between demand nodes and the facilities to which they are assigned is maximized.

## 3.3 Maximm Distance Models

In most of location problems a maximum distance is a priority. For example in many districts people within a mile of their homes must walk to a police station. Transportation must be provided for those not within this maximum distance. In the facility location literature a priori maximum distances such as these are known as ’covering’ distances. Demand within the covering distance of its closest facility is considered ’covered’. An underlying assumption of this measure of maximum distance is that demand is fully satisfied if the nearest facility is within the coverage distance and is not satisfied if the closet facility is beyond that distance. That is, being closer to a facility than the maximum distance does not improve satisfaction.Some location problems:

1. Set covering location model: The objective of this model is to locate the minimum number of facilities required to "cover" all of the demand nodes (Toregas and Bergman (1971))

2. Maximal covering location problem: The objective of the Maximal covering location problem (MCLP) is to locate a predetermined number of facilities, p, in such a way as to maximize the demand that is covered. Thus, the MCLP assumes that there may not be enough facilities to cover all of the demand nodes. If all nodes cannot be covered, then the model seeks the siting scheme that covers the most demand. (Church and ReVelle (1974))

3. The p-dispersion problem: The p-dispersion (PDP) is only concerned with the distance between new facilities and the objective is to maximize the minimum distance between a pair of facilities. Potential applications of PDP include the siting of military installations attack or locating franchise outlets where separation reduces cannibalization among stores. (Kuby (1987))

4. P-Center Problem: The p-center problem (Hakimi (1964)) addresses the problem of minimizing the maximum distance that demand is from its closest facility given that we are siting a pre-determine number of facilities. There are several possible variations of the basic model. The "vertex" pcenter problem restrict the set of candidate facility sites to the nodes of the network while the ’absolute’ p-center problem permits the facilities to be anywhere along the arcs or the network. Both versions can be either weighted or unweighted. In the unweighted problem, all demand nodes are treat equally. In weighted model, the distance between demand nodes and facilities are multiplied by a weight associated with the demand node. The weight might be represented at a node or more commonly, the level of its demand is an example.

## 3.4 P-Centre problem

The p-center problem which was also introduced first by Hakimi (1964) is to find the facility locations such that the maximum distance between any demand point (customer) and its respective nearest facility is minimized. It has been used to model locations of emergency facilities such as ambulance stations and firehouses, the location of a helicopter to minimize the maximum time to respond to an emergency, and the location of transmitter to maximize the lowest signal level received in a communication network (Carson and Batta (1990)). There are several possible variations of the basic model. If facility locations are restricted to the nodes of the network the problem is referred to as a ’vertex’ p-center problem. Center problems which allow facilities to be located anywhere on the network are known as ’absolute’ p-center problem. Both versions can be either weighted or unweighted. In the weighted problem, the distance between the demand nodes and facilities are multiplied by the weight usually associated with the demand node. In the unweighted problem, all demand nodes are treated equally. Given our previous definitions and the following decision variables

W = the maximum distance between a demand node and the facility to which it is assigned

$$y_{i,j} = \begin{cases} 1 & \text{if the demand node i is assigned to a facility at node j} \\ 0 & \text{if not} \end{cases}$$

The p-center problem can be formulated as follows:

$$Maximize \qquad W \tag{3.1}$$

Subject to:

$$\sum_{j\epsilon J} x_j = p \tag{3.2}$$

$$\sum_{j\epsilon J} y_{ij} = 1 \qquad \forall i\epsilon I \tag{3.3}$$

$$y_{ij} - x_j \geq 0 \qquad \forall i\epsilon I, j\epsilon J \tag{3.4}$$

$$W - \sum_{j\epsilon J} h_i d_{ij} \geq 0 \qquad \forall i\epsilon I \tag{3.5}$$

$$x_j\epsilon\{0,1\} \qquad \forall j\epsilon J \tag{3.6}$$

$$y_{ij}\epsilon\{0,1\} \qquad \forall i\epsilon I, j\epsilon J \tag{3.7}$$

The objective function (3.1) minimizes the maximum demand-weighted distance between each demand node and its closest open facility. Constraint (3.2) stipulates that p facilities are to be located. Constraint set (3.3) requires that each demand node be assigned to exactly one facility. Constraint set (3.4) restricts demand node assignments only to open facilities. Constraint (3.5) defines the lower bound on the maximum demand-weighted distance, which is being minimized. Constraint set (3.6) established the siting decision variable as binary. Constraint set (3.7) requires the demand at demand at a node to be assigned to one facility only. Constraint set (3.7) can be replaced by $y_{ij} \geq 0$ $\forall i\epsilon I; j\epsilon J$because constraint set (3.4) guarantees that $y_{ij} \leq 1$. If some $y_{ij}$ are fractional, we simply assign node i to its closest open facility (Current and Schilling (2002))

## 3.5 The Conditional P-centre

The conditional location problem is to locate p new facilities to serve a set of demand points given that q facilities are already located. When $q = 0$, the problem is unconditional. In the conditional p-center problems, once the new p locations are determined, a demand can be served either by one of the existing or by one of the new facilities whichever is the closest facility to the demand.

## 3.6 Formation of the problem

Consider a network $G = (N,L)$ Where;

$N$ = the set of nodes, $|N| = n$ $L$ = the set of links.

Let d(x, y) be the shortest distance between any $x,yG$, Suppose that there is a set Q ($|Q| = q$) of existing facilities. Let $Y = (Y_1 ...,Y_q)$ and $X = (X_1,X_2,...,X_p)$ be vectors of size q and p respectively, where $Y_i$ is the location of existing facility i and $X_i$ is the location of new facility i. Without any loss of generality we do not need to assume that $Y_i \epsilon N$ .

The conditional p-center location is to minimize;

$$f(x) = \sum_{i=1}^{n} w_i d(X_i), d(Y_i)]$$

Where $(X_i)$ and $(Y_i)$, is the shortest distance from the closet facility in X and Y respectively to the node i. (Berman and Simchi-Levi (1990)).

### 3.6.1 The Alorithm of Berman and Simchi-Levi

The idea is to produce a new location representing all the existing facilities. If a demand point is utilizing the service of an existing facility, it will use the services of the closest existing facility. That is , the distance between a demand point and the new location is the minimum distance among all the existing facilities.

Step 1. Let D be the shortest distance matrix with rows corresponding demands and columns corresponding to potential locations. In order to force the formation of a facility at the new location, a new demand point is considered with a distance of zero from the new potential location and a large distance from all other potential locations.

Step 2. The new distance matrix, denoted by $\bar{D}$ , is constructed by adding a new location $a_0$ (a new column) to D which represents the Q existing locations and a new demand point $v_0$ with an arbitrary positive weight . For each

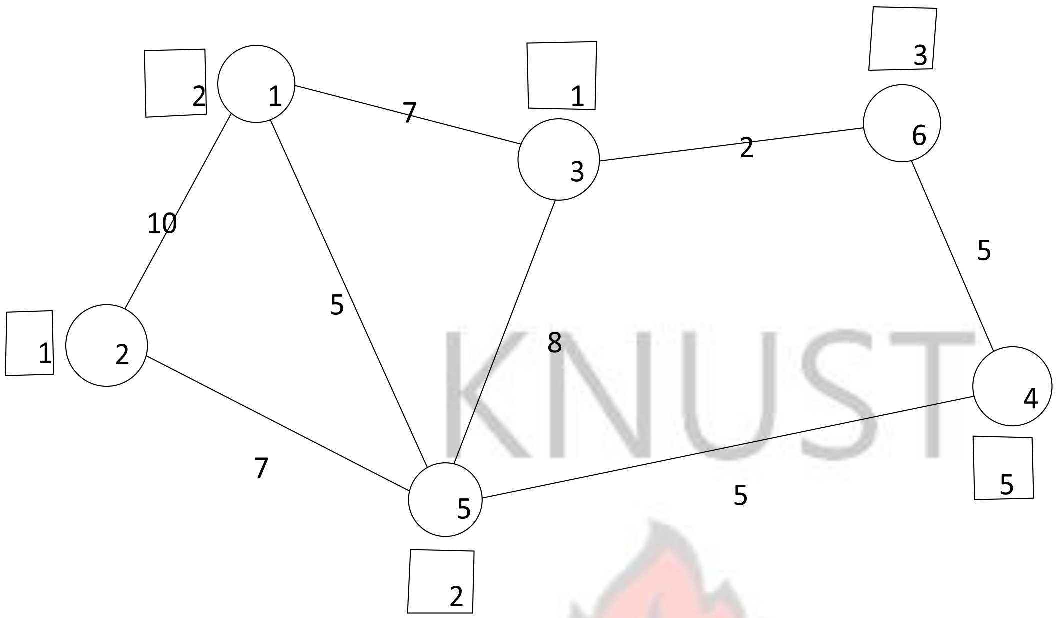
regular demand point (node) i, we have $d(i, a_0) = min_{k \epsilon Q} d_{ik}$ and $d(v_0, a_0)$. For each regular potential location node j, $d(v_0, j)$ = $M$,where M is a large number. Again the nodes in Q and in the potential locations Q are removed

Step 3 Find the optimal new location by using the distance matrix D for the network with objective function

$$min[f(x) = \sum_{i=1}^{n} w_i min\{d(X_i), d(Y_i)\}]$$

To illustrate the approach, we consider the network in figure 3.1 below, where the numbers next to the links are lengths and the numbers next to the nodes are weight. Suppose that the exist facilities are Q=(3, 5, 6) and only one facility is to be located.($p$ = 1)

Figure 3.1: Sample network for p-median




Step 1 : Using Floyd-Warshall algorithm, we obtained the shortest distance matrix D, for the above network, with column 1 and row 1 representing the demand nodes and potential location respectively, and each row represent the interconnected distances

Table 3.1: All pairs shortest path distance matrix, D.

| Demand nodes | Potential location | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | 0 | 10 | 7 | 10 | 5 | 9 |
| 2 | 10 | 0 | 15 | 12 | 7 | 17 |
| 3 | 7 | 15 | 0 | 7 | 8 | 2 |
| 4 | 10 | 12 | 7 | 0 | 5 | 5 |
| 5 | 5 | 7 | 8 | 5 | 0 | 10 |
| 6 | 9 | 17 | 2 | 5 | 10 | 0 |

Step 2 The new distance matrix, denoted by $\bar{D}$, is constructed by adding a new location $a_0$ (a new column) to D which represents the Q existing locations and a new

demand point $v_o$ with an arbitrary positive weight. For each regular demand point(node) i, we have

$d(i, a_o) = min_{k \epsilon Q} d_{ik}$ and $d(v_o,a_o) = 0$.

For each regular potential location node j, $d(v_o,j) = M$, where M is a large number.

$d(i, a_o) = min_{k \epsilon Q} d_{ik}$

$Q = \{3, 5, 6\}$

$i = 1$

$d(1,a_o) = min\{d(1,3)d(1,5)d(1,6)\} =$

$min\{7,5,9\} = 5$ $i = 2$

$d(2,a_o) = min\{d(2,3),d(2,5),d(2,6)\}$

$= min\{15,7,17\} = 7$ $i =$

3

$d(3,a_o) = min\{d(3,3),d(3,5),d(3,6)\}$

$= min\{0,8,2\} = 0$ $i =$

4

$d(4,a_o) = min\{d(4,3),d(4,5),d(4,6)\}$

$= min\{7,5,5\} = 5$ $i =$

5

$d(5,a_o) = min\{d(5,3),d(5,5),d(5,6)\}$

$= min\{8,0,10\} = 0$ $i =$

6

$d(6,a_o) = min\{d(6,3),d(6,5),d(6,6)\}$

$= min\{2,10,0\} = 0$ $d(v_o,a_o) = 0$

$d(v_o,j) = M$ Table 3.2: The

Modified Distance matrix, $\bar{D}$

| Demand nodes | Potential location | | | | | | |
|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | $a_o$ |
| 1 | 0 | 10 | 7 | 10 | 5 | 9 | 5 |
| 2 | 10 | 0 | 15 | 12 | 7 | 17 | 7 |
| 3 | 7 | 15 | 0 | 7 | 8 | 2 | 0 |
| 4 | 10 | 12 | 7 | 0 | 5 | 5 | 5 |
| 5 | 5 | 7 | 8 | 5 | 0 | 10 | 0 |
| 6 | 9 | 17 | 2 | 5 | 10 | 0 | 0 |
| $v_0$ | M | M | M | M | M | M | 0 |

The nodes in Q representing existing facilities nodes are removed. This is shown in the table below

Table 3.3: Modified shortest path distance matrix, $\bar{D}$ with existing facility nodes removed.

| Demand nodes | Potential location | | | |
|---|---|---|---|---|
| | 1 | 2 | 4 | $a_o$ |
| 1 | 0 | 10 | 10 | 5 |
| 2 | 10 | 0 | 12 | 7 |
| 4 | 10 | 12 | 0 | 5 |
| $v_o$ | M | M | M | 0 |

Step 3. Find the optimal new location by using the modified distance matrix $\bar{D}$ and the objective function. Minimize

$$[f(x) = \sum_{i=1}^{n} w_i min\{d(X_i), d(Y_i)\}]$$

Find $min\{d(X_i),d(Y_i)\}$

$X = \{1,2,4,a_o\}$ $Y = \{3,5,6\}$

At $X = 1$

$min\{d(2,1),d(3,1),d(6,1)\}$

$min\{10,7,5,9\}$ = 5

$i$ = 2

$min\{d(2,2),d(3,2),d(5,2),d(6,2)\}$ $min\{0,15,7,17\}$ =

0

$i$ = 3

$min\{d(2,3),d(3,3),d(5,3),d(6,3)\}$ $min\{15,0,8,2\}$ = 0

$i$ = 4

$min\{d(2,4),d(3,4),d(5,4),d(6,4)\}$ $min\{12,7,5,5\}$ = 5

$i$ = 5

$min\{d(1,5),d(3,5),d(5,5),d(6,5)\}$ $min\{5,8,0,10\}$ = 0

$i$ = 6

$min\{d(1,6),d(3,6),d(5,6),d(6,6)\}$ $min\{9,2,10,0\}$ = 0

At $X$ = 2

$i$ = 1

$min\{d(2,1),d(3,1),d(5,1),d(6,1)\}$ $min\{10,7,5,9\}$ = 5

$i$ = 2

$min\{d(2,2),d(3,2),d(5,2),d(6,2)\}$ $min\{0,15,7,17\}$ =

0

$i$ = 3

$min\{d(2,3),d(3,3),d(5,3),d(6,3)\}$ $min\{15,0,8,2\}$ = 0

$i$ = 4

$min\{d(2,4),d(3,4),d(5,4),d(6,4)\}$ $min\{12,7,5,5\}$ = 5

$i$ = 5

$min\{d(2,5),d(3,5),d(5,5),d(6,5)\}$ $min\{7,8,0,10\}$ = 0

$i$ = 6

$min\{d(2,6),d(3,6),d(5,6),d(6,6)\}$ $min\{17,2,10,0\}$ =

0

At $X = 4$

$i = 1$

$min\{d(4,1), d(3,1), d(5,1), d(6,1)\}$ $min\{10,7,5,9\} = 5$

$i = 2$

$min\{d(4,2), d(3,2), d(5,2), d(6,2)\}$ $min\{1,15,7,17\} =$

7

$i = 3$

$min\{d(4,3), d(3,3), d(5,3), d(6,3)\}$ $min\{7,0,8,2\} = 0$

$i = 4$

$min\{d(4,4), d(3,4), d(5,4), d(6,4)\}$ $min\{0,7,5,5\} = 0$

$i = 5$

$min\{d(4,5), d(3,5), d(5,5), d(6,5)\}$ $min\{5,8,0,10\} = 0$

$i = 6$

$min\{d(4,6), d(3,6), d(5,6), d(6,6)\}$ $min\{5,2,10,0\} = 0$

The results are summarized and shown below in Table 3.4 Table 3.4: Optimal new location matrix using the modified shortest distance matrix

| Demand node | Potential location | | |
|---|---|---|---|
| | 1 | 2 | 4 |
| 1 | 0 | 7 | 5 |
| 2 | 5 | 0 | 5 |
| 4 | 5 | 7 | 0 |

Finding the optimal new location using the modified shortest distance, $\bar{D}$ and the

objective function

$$min[f(x) = \sum_{i=1}^{n} w_i min\{d(X_i), d(Y_i)\}]$$

At node 1 i=1

2(0) + 1(7) + 5(5) = 32

At node 2 i=2

2(5) + 1(0) + 5(5) = 35

At node 4 i=4

2(5) +7(1) + 5(0) = 17

From the above objective function values it can be verify that the optimal new location using $\bar{D}$ is node 4 with an objective function value of 17 because it the minimum objective function value. Hence the new location for the facility is node 4.

## 3.7 Berman and Drezner's Alogorithm

A very simple algorithm that solves the conditional p-median problem on a network was discussed by Berman and Drezner (2008). This algorithm requires one-time solution of an unconditional p-median problem using an appropriate shortest distance matrix, rather than creating a new location for an artificial facility, and forcing the algorithm to locate a new facility, thereby creating an artificial demand point. Berman and Drezner's algorithm modify the shortest distance matrix.

Steps

1. Let D be the shortest path distance matrix with rows corresponding to demands and columns corresponding to potential locations.

2. Modified the shortest path distance matrix, from D to $\bar{D}$. That is

$D_{ij} = min\{d_{ij}, min\{d_{ik}\}\} \quad \forall i \epsilon N, j \epsilon N$ (median)

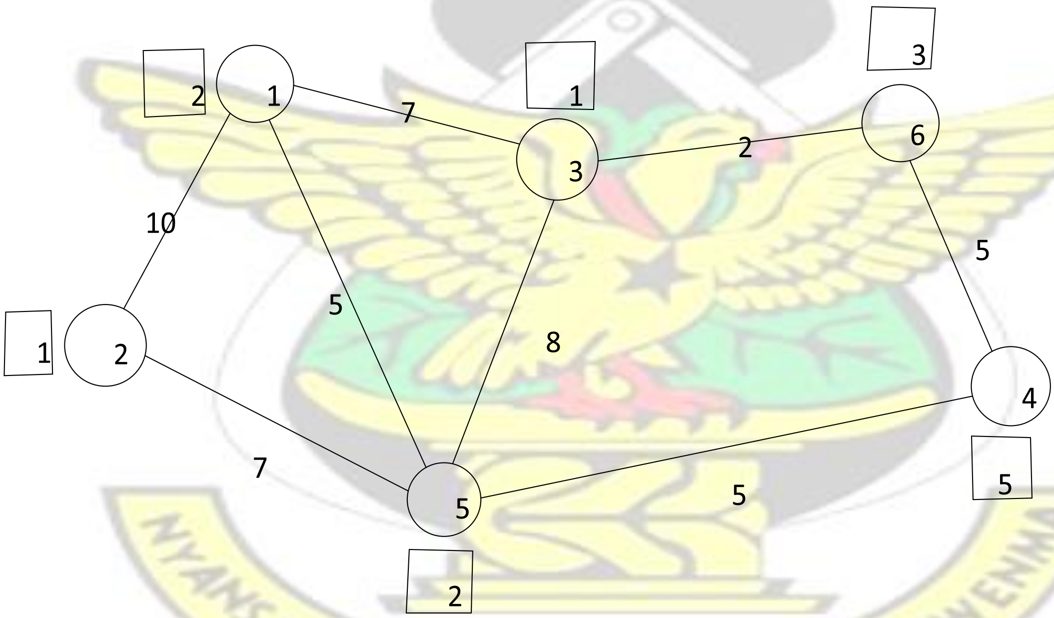
It should be that $\bar{D}$ is not symmetric even when D is symmetric. The unconditional p-median problem using the appropriate $\bar{D}$ solves the conditional p-median problem. This is so since if the shortest distance from node i to the new p facilities is larger than $min_{k \epsilon Q}\{d_{ik}\}$ then the shortest distance to the existing q facility is utilized.

3. Find the optimal new location by using the modified distance matrix $\bar{D}$ for the network with objective function

$$min[f(x) = \sum_{i=1}^{n} w_i\{min\{d(X_i), d(Y_i)\}\}]$$

To illustrate the approach, we consider the network in figure 3.2 below, where the numbers next to the links are lengths and numbers next to the nodes are weight. Suppose that the existing facilities are Q = {3,5,6} and only one facility is to be located. (p=1)

Figure 3.2: Sample network for p-median problem




STEP 1: Using Floyd-Warshall algorithm, we obtained the shortest distance matrix D, for the above network, with column 1 and row 1 representing the demand nodes and potential location respectively, and each other row represents the interconnected distances.

Table 3.5: All pairs shortest path distance matrix, D.

| Demand node | Potential location | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | 0 | 10 | 7 | 10 | 5 | 9 |
| 2 | 10 | 0 | 15 | 12 | 7 | 17 |
| 3 | 7 | 15 | 0 | 7 | 8 | 2 |
| 4 | 10 | 12 | 7 | 0 | 5 | 5 |
| 5 | 5 | 7 | 8 | 5 | 0 | 10 |
| 6 | 9 | 17 | 2 | 5 | 10 | 0 |

STEP 2: Determine a modified shortest distance matrix by:

$$\bar{D}_{ij} = min\{d_{ij}, min_{k \epsilon Q}\{d_{ik}\}\} \qquad \forall i \epsilon N, j \epsilon N$$

For node 1 Q = {3,5,6} $i = 1, j = 1$

$\bar{D}_{11} = min\{d_{11}, min\{d_{13}, d_{15}, d_{16}\}$

$= min\{0, min\{7,5,9\}$

$= min\{0,5\} = 0$ $i = 1, j =$

2

$\bar{D}_{12} = min\{d_{12}, min\{d_{13}, d_{15}, d_{16}\}\}$

$= min\{10, min\{7,5,9\}\}$

$= min\{10,5\} = 5$ $i = 1, j =$

3

$\bar{D}_{13} = min\{d_{13}, min\{d_{13}, d_{15}, d_{16}\}\}$

$= min\{7, min\{7,5,9\}\} =$

$min\{7,5\} = 5$ $i = 1, j = 4$

$\bar{D}_{14} = min\{d_{14}, min\{d_{14}, d_{15}, d_{16}\}\}$

$= min\{10, min\{7,5,9\}\} =$

$min\{10,5\} = 5$ $i = 1, j = 5$

$\bar{D}_{15} = min\{d_{15}, min\{d_{13}, d_{15}, d_{16}\}\}$

$= min\{5,min\{7,5,9\}\} =$

$min\{5,5\} = 5$ $i = 1,j = 6$

$\bar{D}_{16} = min\{d_{16},min\{d_{13},d_{15},d_{16}\}\}$

$= min\{9,min\{7,5,9\}\}$

$= min\{9,5\} = 5$ At

node 2 $i = 2,j = 1$

$\bar{D}_{21} = min\{d_{21},min\{d_{23},d_{25},d_{26}\}\}$

$= min\{10,min\{15,7,17\}\} =$

$min\{10,7\} = 7$ $i = 2,j = 2$

$\bar{D}_{22} = min\{d_{22},min\{d_{23},d_{25},d_{26}\}\}$

$= min\{0,min\{15,7,17\}\} =$

$min\{0,7\} = 0$ $i = 2,j = 3$

$\bar{D}_{23} = min\{d_{23},min\{d_{23},d_{25},d_{26}\}\}$

$= min\{15,min\{15,7,17\}\} =$

$min\{15,7\} = 7$ $i = 2,j = 4$

$\bar{D}_{24} = min\{d_{24},min\{d_{23},d_{25},d_{26}\}\}$

$= min\{12,min\{15,7,17\}\} =$

$min\{12,7\} = 7$ $i = 2,j = 5$

$\bar{D}_{25} = min\{d_{25},min\{d_{23},d_{25},d_{26}\}\}$

$= min\{7,min\{15,7,17\}\} =$

$min\{7,7\} = 7$ $i = 2,j = 6$

$\bar{D}_{26} = min\{d_{26},min\{d_{23},d_{25},d_{26}\}\}$

$= min\{17,min\{15,7,17\}\}$

$= min\{17,7\} = 7$

The results are then summarized and shown in Table 3.6 below with row 1 and column 1 represent potential location and demand node respectively. Other rows represent the interconnecting distances.

Table 3.6: Modified shortest path distance matrix, $\bar{D}$

| Demand nodes | Potential location | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | 0 | 5 | 5 | 5 | 5 | 5 |
| 2 | 7 | 0 | 7 | 7 | 7 | 7 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 5 | 5 | 5 | 0 | 5 | 5 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 |

The existing facility nodes $Q$ = {3,5,6} are removed from the modified shortest path distance matrix, $\bar{D}$ and this is shown in Table 3.6 below.

Table 3.7: Modified shortest path distance matrix, $\bar{D}$ with existing facility nodes removed

| Demand nodes | Potential location | | |
|---|---|---|---|
| | 1 | 2 | 4 |
| 1 | 0 | 5 | 5 |
| 2 | 7 | 0 | 7 |
| 4 | 5 | 5 | 0 |

STEP 3: Find the optimal new location using $\bar{D}$ for the network with the objective function

Minimize

$$f(x) = \sum_{i=1}^{n} w_i min(d(X_i), d(Y_i))$$

Let $X$ = {1,2,4} and $Y$ = {3,5,6} At X=1

$i$ = 1

$min\{d(1,1), d(3,1), d(5,1), d(6,1)\}$
$min\{0,5,5,5\}$ = 0

$i$ = 2

$min\{d(1,2), d(3,2), d(5,2), d(6,2)\}$ $min\{7,7,7,7\}$ = 7

$i = 3$

$min\{d(1,3), d(3,3), d(5,3), d(6,3)\}$ $min\{0,0,0,0\} = 0$

$i = 4$

$min\{d(1,4), d(3,4), d(5,4), d(6,4)\}$ $min\{5,5,5,5\} = 5$

$i = 5$

$min\{d(1,5), d(3,5), d(5,5), d(6,5)\}$ $min\{0,0,0,0\} = 0$

$i = 6$

$min\{d(1,6), d(3,6), d(5,6), d(6,6)\}$ $min\{0,0,0,0\} = 0$

At X=2 $i =$

1

$min\{d(2,1), d(3,1), d(5,1), d(6,1)\}$ $min\{5,5,5,5\} = 5$

$i = 2$

$min\{d(2,2), d(3,2), d(5,2), d(6,2)\}$ $min\{0,7,7,7\} = 0$

$i = 3$

$min\{d(2,3), d(3,3), d(5,3), d(6,3)\}$ $min\{0,0,0,0\} = 0$

$i = 4$

$min\{d(2,4), d(3,4), d(5,4), d(6,4)\}$ $min\{5,5,5,5\} = 5$

$i = 5$

$min\{d(2,5), d(3,5), d(5,5), d(6,5)\}$ $min\{0,0,0,0\} = 0$

$i = 6$

$min\{d(2,6), d(3,6), d(5,6), d(6,6)\}$ $min\{0,0,0,0\} = 0$

At X=4 $i =$

1

$min\{d(4,1), d(3,1), d(5,1), d(6,1)\}$ $min\{5,5,5,5\} = 5$

$i = 2$

$min\{d(4,2), d(3,2), d(5,2), d(6,2)\}$ $min\{7,7,7,7\} = 7$

$i = 3$

$min\{d(4,3), d(3,3), d(5,3), d(6,3)\}$ $min\{0,0,0,0\} = 0$

$i = 4$

$min\{d(4,4), d(3,4), d(5,4), d(6,4)\}$ $min\{0,5,5,5\} = 0$

$i = 5$

$min\{d(4,5), d(3,5), d(5,5), d(6,5)\}$ $min\{0,0,0,0\} = 0$

$i = 6$

$min\{d(4,6), d(3,6), d(5,6), d(6,6)\}$ $min\{0,0,0,0\} = 0$

The results are summarized and shown in table 3.8 with row 1 representing potential location and column 1 representing demand nodes.

Table 3.8: Optimal Location Matrix, using $\bar{D}$

| Demand nodes | Potential location | | |
|---|---|---|---|
| | 1 | 2 | 4 |
| 1 | 0 | 7 | 5 |
| 2 | 5 | 0 | 5 |
| 4 | 5 | 7 | 0 |

finding the optimal new location using the modified shortest distance, $\bar{D}$ and the objective function.

$$min[f(x) = \sum_{i=1}^{n} w_i min\{d(X_i), d(Y_i)\}]$$

At node 1 i=1

$2(0) + 7(1) + 5(5) = 32$

At node 2 i=2

$2(5) + 1(0) + 5(5) = 35$

At node 4 i=4

$2(5) + 7(1) + 5(0) = 17$

From the above objective function values it can be easily be verify that the optimal new location using $\bar{D}$ is node 4 with an objective function value of 17 because it is the minimum objective function value.Hence the new location for the facility is at node 4.

## 3.8 Factor rating method

The factor rating method is popular because a wide variety of factors, from education to recreation to labor skill can be objectively included. When using factor rating method, the following steps must be followed strictly and religiously:

i. Develop a list of relevant factors ii. Assign a weight to each factor to reflect its relative importance in the community.

iii. Develop a scale for each factor (for example 1 to 10 or 1 to 100 points).

iv. Have a related people score each relevant factor using the scale developed in iii above.

v. Multiply the score by the weight assigned to each factor and total the score for each location.

vi. Make a recommended based on the maximum point score, considering the result of qualitative approaches as well.

When a decision is sensitive to minor changes, further analysis of either the weighting or the points assigned may be appropriate. Alternatively, management may conclude that these intangible factors are not the proper criteria on which to base a location decision. Managers therefore place primary weight on the more quantitative aspects of the decision. (Amponsah and Darkwa (2007)).
Table 3.8 illustrate an example of the factor rating analysis of which an assembly must decide among four sites for the construction of a polce station. The assembly selected seven factors listed below as a basis for evaluation and has assigned rating weights on each factor.

Table 3.9: Rating weight of relevant factors and their respective rate on a 1 to 100 basis.

| Factor | Factor name | Rating Weight | Rating of sites | | | |
|---|---|---|---|---|---|---|
| | | | Location A | Location B | Location C | Location D |
| 1 | Proximity to facilities | 3 | 100 | 70 | 80 | 90 |
| 2 | Power-source availability and cost | 4 | 80 | 80 | 100 | 80 |
| 3 | Workforce attitude and cost | 5 | 30 | 60 | 70 | 40 |
| 4 | Population size | 2 | 10 | 80 | 60 | 100 |
| 5 | Community desirability | 2 | 90 | 60 | 80 | 60 |
| 6 | Equipment suppliers in area | 5 | 50 | 50 | 90 | 50 |
| 7 | Economic activities | 1 | 90 | 50 | 60 | 50 |

Table 3.10: Relative scores on factors for a police station

| Factor | Factor name | Rating Weight | Ratio of rate | Rating of sites | | | |
|---|---|---|---|---|---|---|---|
| | | | | Location A | Location B | Location C | Location D |
| 1 | Proximity to facilities | 3 | 0.14 | 14 | 9.8 | 11.2 | 12.6 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 2 | Power-source availability and cost | 4 | 0.18 | 14.4 | 14.4 | 18 | 14.4 |
| 3 | Workforce attitude and cost | 5 | 0.23 | 6.9 | 13.8 | 16.1 | 9.2 |
| 4 | Population size | 2 | 0.09 | 0.9 | 7.2 | 5.4 | 9 |
| 5 | Community desirability | 2 | 0.09 | 8.1 | 5.4 | 7.2 | 4.5 |
| 6 | Equipment suppliers in area | 5 | 0.23 | 11.5 | 11.5 | 20.7 | 11.5 |
| 7 | Economic activities | 1 | 0.05 | 4.5 | 2.5 | 3 | 2.5 |
| TOTAL | | | | 60.3 | 64.1 | 81.6 | 63.7 |

# CHAPTER 4

# DATA COLLECTION, ANALYSIS AND DISCUSSION OF RESULTS

## 4.1 Introduction

A new formulation for the conditional p-median problem Berman and Drezner (2008) would be used to locate a new police station ( p =1) in twenty-six major towns at Afigya Kwabre district. They are Atimatim, Krotobuono, Bronkong, Atimatim, Buoho, Burofoyeduru, Atram, Kodie, Aduman, Adumakasekese, Swedru, Ahodwo, Adwuatia, Hemang, Abrade, Akcom, Nkwantakese, Ahenkro, Kwaman, Nkutam, Nanso, Boanean, Adukro, Atwema, Teterm and Kyekyere.

The district map of Afigya Kwabre district will be used to draw a network for these major towns with the edges being the inter-town distances. The FloydWarshall all pair shortest paths algorithm would be applied to the network to create the shortest path distance matrix and the Berman and Drezner's algorithm would be followed through to solve the problem.

## 4.2 Data collection

The shortest path distances connecting communities is of interest in this study. In view of this a map of Afigya Kwabre district was obtained from District Planning Construction Unit. The major communities in the district were identified and calculated the distances between the major communities by ArcGIS software was used to obtain the interconnected distances between the communities.

A network was formed out of the map. The twenty-six(26) nodes in the network are the towns or communities. The access roads of these major communities are represented by the edges of the network. The numbers attached to the nodes are the respective population of the major communities. This populations depict the weights of each town.

Table 4.1: Major communities in Afigya Kwabre District and their respective nodes

| Town | Node | Town | Node | Town | Node |
|---|---|---|---|---|---|
| Afrancho | 1 | Adumakasekese | 10 | Kwaman | 19 |
| Krotobuono | 2 | Swedru | 11 | Nkutam | 20 |
| Bronkong | 3 | Ahodwo | 12 | Nanso | 21 |
| Atimatim | 4 | Adwuatia | 3 | Boamang | 22 |
| Buoho | 5 | Hemang | 14 | Adukro | 23 |
| Burofoyeru | 6 | Abrade | 15 | Atwema | 24 |
| Atram | 7 | Akcom | 16 | Teterm | 25 |

| | | | | | |
|---|---|---|---|---|---|
| Kodie | 8 | Nkwantakese | 17 | Kyekyere | 26 |
| Aduman | 9 | Ahenkro | 18 | | |

Table 4.2: Major communities in Afigya Kwabre District and their respective population

| Town | Population | Town | Population | Town | Population |
|---|---|---|---|---|---|
| Afrancho | 7548 | Adumakasekese | 5565 | Kwaman | 1912 |
| Krotobuono | 1823 | Swedru | 1121 | Nkutam | 1098 |
| Bronkong | 2111 | Ahodwo | 986 | Nanso | 851 |
| Atimatim | 23948 | Adwuatia | 998 | Boamang | 873 |
| Buoho | 5419 | Hemang | 1089 | Adukro | 1225 |
| Burofoyeru | 1075 | Abrade | 977 | Atwema | 760 |
| Atram | 799 | Akcom | 1125 | Teterm | 802 |
| Kodie | 3905 | Nkwantakese | 1509 | Kyekyere | 939 |
| Aduman | 1204 | Ahenkro | 5834 | | |

Source: DPCU Construct 2013

The nodes of the network were developed in a matrix form. Communities which have direct road link are indicated with their respective distance, whereas communities with no direct road link are indicated with a dash. The matrix formed a rectangle matrix of order 26 by 26.

Figure 4.1: Network of 26 towns in Afigya Kwabre district

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 2 | 3 | - | 4 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 2 | 2 | 0 | 2 | 2 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 3 | 3 | 2 | 0 | - | - | 3 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 4 | - | - | - | 0 | - | 5 | - | - | - | 17 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 5 | 4 | - | - | - | 0 | - | - | 7 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 6 | - | - | 3 | 5 | - | 0 | 4 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 7 | - | - | - | - | - | 4 | 0 | 8 | 9 | 15 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 8 | - | - | - | - | 7 | - | 8 | 0 | 23 | - | - | - | - | - | - | 13 | 29 | - | - | - | - | - | - | - | - | - |
| 9 | - | - | - | - | - | - | 9 | 23 | 0 | - | 10 | - | - | - | - | 17 | - | - | - | - | - | - | - | - | - | - |
| 10 | - | - | - | 17 | - | - | 15 | - | - | 0 | 5 | - | - | 11 | - | - | - | - | - | - | - | - | - | - | - | - |
| 11 | - | - | - | - | - | - | - | - | 10 | 5 | 0 | 7 | - | - | - | - | - | - | 28 | - | - | - | - | - | - | - |
| 12 | - | - | - | - | - | - | - | - | - | - | 7 | 0 | 9 | - | - | - | - | - | 19 | - | - | - | - | - | - | - |
| 13 | - | - | - | - | - | - | - | - | - | - | - | 9 | 0 | 8 | - | - | - | - | - | - | - | - | - | - | - | - |
| 14 | - | - | - | - | - | - | - | - | - | 11 | - | - | 8 | 0 | 1 | - | - | - | - | - | - | - | - | - | - | - |
| 15 | - | - | - | - | - | - | - | - | - | - | - | - | - | 1 | 0 | - | - | - | - | - | - | - | - | - | - | - |
| 16 | - | - | - | - | - | - | - | 13 | 17 | - | - | - | - | - | - | 0 | 8 | 12 | - | - | - | - | - | - | - | - |
| 17 | - | - | - | - | - | - | - | 29 | - | - | - | - | - | - | - | 8 | 0 | - | - | - | - | - | - | - | - | - |
| 18 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | 12 | - | 0 | 8 | - | - | - | - | - | - | - |
| 19 | - | - | - | - | - | - | - | - | - | - | 28 | 19 | - | - | - | - | - | 8 | 0 | 11 | 31 | - | - | - | - | - |
| 20 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | 11 | 0 | 9 | - | - | - | - | - |
| 21 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | 31 | 9 | 0 | 1 | - | - | - | - |
| 22 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | 1 | 0 | 10 | - | 27 | - |
| 23 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | 10 | 0 | 2 | - | - |
| 24 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | 2 | 0 | 19 | - |
| 25 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | 19 | 0 | 33 |
| 26 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | 33 | 0 |

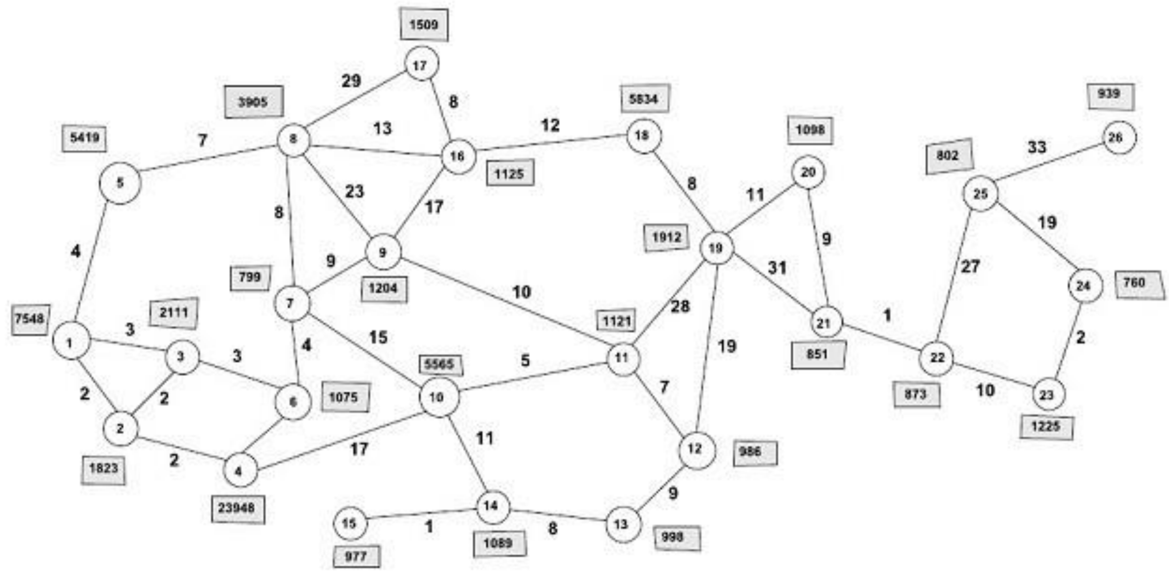



Figure 4.2: Matrix of Network Indicating Towns and their Pair of Distance

# 4.3 Data Analysis

The Floyd–Warshall All Pair Shortest Path algorithm was applied to the matrix in Figure 4.2 to obtain the all pairs shortest path distance matrix D, shown in Table 4.3.

Row 1 and Column 1 represent the potential location and demand nodes respectively. The other rows also represent the inter–community distances. The MATLAB code for the Floyd–Warshall algorithm used to obtain the all pair shortest path distance is shown Appendix B.

Table 4.3: Summary of Shortest Distance Matrix between Pair of Nodes, D

| | 1 | 2 | 3 | . | . | . | 13 | 14 | 15 | . | . | . | 23 | 24 | 25 | 26 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 2 | 3 | . | . | . | 40 | 32 | 33 | . | . | . | 75 | 77 | 92 | 125 |
| 2 | 5 | 0 | 2 | . | . | . | 38 | 30 | 31 | . | . | . | 80 | 82 | 97 | 130 |
| 3 | 3 | 2 | 0 | . | . | . | 40 | 32 | 33 | . | . | . | 78 | 80 | 95 | 128 |
| . | . | . | . | . | . | . | . | . | . | . | . | . | . | . | . | . |
| . | . | . | . | . | . | . | . | . | . | . | . | . | . | . | . | . |
| . | . | . | . | . | . | . | . | . | . | . | . | . | . | . | . | . |
| 13 | 43 | 38 | 40 | . | . | . | 0 | 8 | 9 | . | . | . | 59 | 61 | 76 | 109 |
| 14 | 35 | 30 | 32 | . | . | . | 8 | 0 | 1 | . | . | . | 67 | 69 | 84 | 117 |
| 15 | 36 | 31 | 33 | . | . | . | 9 | 1 | 0 | . | . | . | 68 | 70 | 85 | 118 |
| . | . | . | . | . | . | . | . | . | . | . | . | . | . | . | . | . |
| . | . | . | . | . | . | . | . | . | . | . | . | . | . | . | . | . |
| . | . | . | . | . | . | . | . | . | . | . | . | . | . | . | . | . |
| 23 | 75 | 77 | 78 | . | . | . | 59 | 67 | 68 | . | . | . | 0 | 2 | 21 | 54 |
| 24 | 77 | 79 | 80 | . | . | . | 61 | 69 | 70 | . | . | . | 2 | 0 | 19 | 52 |
| 25 | 92 | 94 | 95 | . | . | . | 76 | 84 | 85 | . | . | . | 21 | 19 | 0 | 33 |
| 26 | 125 | 127 | 128 | . | . | . | 109 | 117 | 118 | . | . | . | 54 | 52 | 33 | 0 |

## 4.4 Model Formlation

Berman and Drezner's algorithm (2008) is used to solve the problem. This algorithm requires a one-time solution of an unconditional p- median problem using an appropriate shortest distance matrix. I begin by formulating the conditional p-median problem as

$min[f(x) = \sum_{i=1}^{n} w_i min\{d(X_i), d(Y_i)\}]$

Let $G = (N,L)$ be a network with N being the set of nodes, $|N| = n$ and L being the set of links. Consider a non-negative number $w_i$ that represent the demand weight at node $iN$. Let $d_{xy}$ be the shortest distance between any two nodes $x,yG$.

Suppose that there is a set $Q(|Q| = q)$ of existing facilities. Let $Y = (Y_1 ..., Y_q)$ and $X = (X_1, X_2, ..., X_p)$ be vectors of size q and p respectively, where $Y_i$ is the location of existing facility i and $X_i$ is the location of new facility i. Where $d(Y_i)$ and $d(X_i)$ is the shortest distance from the closest facility in Y and X respectively to node i. Without any loss in generality i do not need to assume that $Y_i \epsilon N$.

With existing police station at Teterm, Ahenkro and Kodie, police post at Boamang and Kyekyewere. These communities form the set of existing facilities, thus node 25, node 18, node 8, node 22 and node 26 respectively. This gives Y = { 8,18,22,25,26}. The remaining nodes also form the set of potential location of new facilities.

Thus X= {1,2,3,4,5,6,7,9,10,11,12,13,14,15,16,17,19,20,21,23,24}. Where i={1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,2,25,26}

## 4.5 Algorithm used to solve the problem

Steps

1. Let D be the shortest path distance matrix with rows corresponding to demands and columns corresponding to potential locations.

2. Modified the shortest path distance matrix from D to $\bar{D}$. That is $\bar{D}_{ij} = min\{d_{ij}, min\{d_{ik}\}\} \quad \forall i \epsilon N, j \epsilon N,$ where k belongs to the set of existing facilities. It should be noted that $\bar{D}$ is not symmetric even when D is symmetric.

3. Remove the nodes in Q and the Potential location in Q

4. Find the optimal new location by the modified distance matrix $\bar{D}$. For the network with objective function

$$min[f(x) = \sum_{i=1}^{n} \quad w_i min\{d(X_i), d(Y_i)\}]$$

## 4.6 Computation and Results

Step 1: The Floyd-Warshall all pair shortest path algorithm was applied to the matrix in table in figure 4.2 to obtained the shortest distance matrix between each pair of node as displayed in Table 4.3 The matrix shows the length of the shortest path between respective nodes.

Step 2: A modified shortest distance matrix $\bar{D}$ is determine by using the formulation $\bar{D}_{ij}=min\{d_{ij}, min\{d_{ik}\}\} \quad \forall i\epsilon N, j\epsilon N, k\epsilon Q$,

where $Q$ = {8,18,22,25,26} and

i,j={1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26}.

The MATLAB code used to obtain the modified shortest distance matrix is shown in Appendix B

For node 1 $i$ =

1,$j$ = 1

$\bar{D}_{11} = min\{d_{11}, min\{d_{1,8}, d_{1,18}, d_{1,22} d_{1,25}, d_{1,26}\}$

$=min\{0, min\{11,36,65,92,125\}$

$=min\{0,11\} = 0$ $i = 1, j = 2$

$\bar{D}_{1,2} = min\{d_{1,2}, min\{d_{1,8}, d_{1,18}, d_{1,22} d_{1,25}, d_{1,26}\}$

$=min\{2, min\{11,36,65,92,125\}$

$=min\{2,11\} = 2$ $i = 1, j = 3$

$\bar{D}_{1,3} = min\{d_{1,3}, min\{d_{1,8}, d_{1,18}, d_{1,22} d_{1,25}, d_{1,26}\}$

$=min\{3, min\{11,36,65,92,125\}$

$=min\{3,11\} = 3$ $i = 1, j = 4$

$D^{-}_{1,4} = min\{d_{1,4}, min\{d_{1,8}, d_{1,18}, d_{1,22} d_{1,25}, d_{1,26}\}$

$= min\{4, min\{11,36,65,92,125\}$
$= min\{4,11\} = 4$ $i =$

$1, j = 5$

$D^{-}_{1,5} = min\{d_{1,5}, min\{d_{1,8}, d_{1,18}, d_{1,22} d_{1,25}, d_{1,26}\}$

$= min\{4, min\{11,36,65,92,125\}$

$= min\{4,11\} = 4$ $i = 1, j = 6$

$D^{-}_{1,6} = min\{d_{1,6}, min\{d_{1,8}, d_{1,18}, d_{1,22} d_{1,25}, d_{1,26}\}$

$= min\{6, min\{11,36,65,92,125\}$

$= min\{6,11\} = 6$ $i = 1, j = 7$

$D^{-}_{1,7} = min\{d_{1,7}, min\{d_{1,8}, d_{1,18}, d_{1,22} d_{1,25}, d_{1,26}\}$

$= min\{10, min\{11,36,65,92,125\}$

$= min\{10,11\} = 10$ $i = 1, j = 8$

$D^{-}_{1,8} = min\{d_{1,8}, min\{d_{1,8}, d_{1,18}, d_{1,22} d_{1,25}, d_{1,26}\}$

$= min\{11, min\{11,36,65,92,125\}$

$= min\{11,11\} = 11$ $i = 1, j = 9$

$D^{-}_{1,9} = min\{d_{1,9}, min\{d_{1,8}, d_{1,18}, d_{1,22} d_{1,25}, d_{1,26}\}$

$= min\{19, min\{11,36,65,92,125\}$

$= min\{19,11\} = 11$ $i = 1, j = 10$

$D^{-}_{1,10} = min\{d_{1,10}, min\{d_{1,8}, d_{1,18}, d_{1,22} d_{1,25}, d_{1,26}\}$

$= min\{21, min\{11,36,65,92,125\}$

$= min\{21,11\} = 11$ $i = 1, j = 11$

$D^{-}_{1,11} = min\{d_{1,11}, min\{d_{1,8}, d_{1,18}, d_{1,22} d_{1,25}, d_{1,26}\}$

$= min\{26, min\{11,36,65,92,125\}$

$= min\{26,11\} = 11$
$i = 1, j = 12$

$D^{-}_{1,12} = min\{d_{1,12}, min\{d_{1,8}, d_{1,18}, d_{1,22} d_{1,25}, d_{1,26}\}$

$=min\{33,min\{11,36,65,92,125\}$

$=min\{33,11\} = 11$ $i = 1, j = 13$

$\bar{D}_{1,13} = min\{d_{1,13},min\{d_{1,8},d_{1,18},d_{1,22}d_{1,25},d_{1,26}\}$

$=min\{40,min\{11,36,65,92,125\}$

$=min\{40,11\} = 11$ $i = 1, j = 14$

$\bar{D}_{1,14} = min\{d_{1,14},min\{d_{1,8},d_{1,18},d_{1,22}d_{1,25},d_{1,26}\}$

$=min\{32,min\{11,36,65,92,125\}$

$=min\{32,11\} = 11$ $i = 1, j = 15$

$\bar{D}_{1,15} = min\{d_{1,15},min\{d_{1,8},d_{1,18},d_{1,22}d_{1,25},d_{1,26}\}$

$=min\{33,min\{11,36,65,92,125\}$

$=min\{33,11\} = 11$ $i = 1, j = 16$

$\bar{D}_{1,16} = min\{d_{1,16},min\{d_{1,8},d_{1,18},d_{1,22}d_{1,25},d_{1,26}\}$

$=min\{24,min\{11,36,65,92,125\}$

$=min\{24,11\} = 11$ $i = 1, j = 17$

$\bar{D}_{1,17} = min\{d_{1,17},min\{d_{1,8},d_{1,18},d_{1,22}d_{1,25},d_{1,26}\}$

$=min\{32,min\{11,36,65,92,125\}$

$=min\{32,11\} = 11$ $i = 1, j = 18$

$\bar{D}_{1,18} = min\{d_{1,18},min\{d_{1,8},d_{1,18},d_{1,22}d_{1,25},d_{1,26}\}$

$=min\{36,min\{11,36,65,92,125\}$

$=min\{36,11\} = 11$ $i = 1, j = 19$

$\bar{D}_{1,19} = min\{d_{1,19},min\{d_{1,8},d_{1,18},d_{1,22}d_{1,25},d_{1,26}\}$

$=min\{44,min\{11,36,65,92,125\}$

$=min\{44,11\} = 11$ $i = 1, j = 20$

$\bar{D}_{1,20} = min\{d_{1,20},min\{d_{1,8},d_{1,18},d_{1,22}d_{1,25},d_{1,26}\}$

$=min\{55,min\{11,36,65,92,125\}$

$=min\{55,11\} = 11$ $i = 1, j = 21$

$\bar{D}_{1,21} = min\{d_{1,21}, min\{d_{1,8}, d_{1,18}, d_{1,22} d_{1,25}, d_{1,26}\}$

$= min\{64, min\{11,36,65,92,125\}$

$= min\{64,11\} = 11 \quad i = 1, j = 22$

$\bar{D}_{1,22} = min\{d_{1,22}, min\{d_{1,8}, d_{1,18}, d_{1,22} d_{1,25}, d_{1,26}\}$

$= min\{65, min\{11,36,65,92,125\}$

$= min\{65,11\} = 11 \quad i = 1, j = 23$

$\bar{D}_{1,23} = min\{d_{1,23}, min\{d_{1,8}, d_{1,18}, d_{1,22} d_{1,25}, d_{1,26}\}$

$= min\{75, min\{11,36,65,92,125\}$

$= min\{75,11\} = 11 \quad i = 1, j = 24$

$\bar{D}_{1,24} = min\{d_{1,24}, min\{d_{1,8}, d_{1,18}, d_{1,22} d_{1,25}, d_{1,26}\}$

$= min\{77, min\{11,36,65,92,125\}$

$= min\{77,11\} = 11 \quad i = 1, j = 25$

$\bar{D}_{1,25} = min\{d_{1,25}, min\{d_{1,8}, d_{1,18}, d_{1,22} d_{1,25}, d_{1,26}\}$

$= min\{92, min\{11,36,65,92,125\}$

$= min\{92,11\} = 11 \quad i = 1, j = 26$

$\bar{D}_{1,26} = min\{d_{1,26}, min\{d_{1,8}, d_{1,18}, d_{1,22} d_{1,25}, d_{1,26}\}$

$= min\{125, min\{11,36,65,92,125\}$

$= min\{125,11\} = 11$

The results are summarized and shown in Table 4.4 below with row 1 and column 1 represent potential location and demand node respectively. Other rows and columns represent the inter-communities distances

Table 4.4: Summary of Modified shortest path distance matrix, $\bar{D}$

| Demand nodes | Potential location | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | . | . | . | 13 | 14 | 15 | . | . | . | 23 | 24 | 25 | 26 |

| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 2 | 3 | . | . | . | 11 | 11 | 11 | . | . | . | 11 | 11 | 11 | 11 |
| 2 | 5 | 0 | 2 | . | . | . | 16 | 16 | 16 | . | . | . | 16 | 16 | 16 | 16 |
| 3 | 3 | 2 | 0 | . | . | . | 14 | 14 | 14 | . | . | . | 14 | 14 | 14 | 14 |
| . | . | . | . | . | . | . | . | . | . | . | . | . | . | . | . | . |
| . | . | . | . | . | . | . | . | . | . | . | . | . | . | . | . | . |
| . | . | . | . | . | . | . | . | . | . | . | . | . | . | . | . | . |
| 13 | 36 | 36 | 36 | . | . | . | 0 | 8 | 9 | . | . | . | 36 | 36 | 36 | 36 |
| 14 | 34 | 30 | 32 | . | . | . | 8 | 0 | 1 | . | . | . | 34 | 34 | 34 | 34 |
| 15 | 35 | 31 | 33 | . | . | . | 9 | 1 | 0 | . | . | . | 35 | 35 | 35 | 35 |
| . | . | . | . | . | . | . | . | . | . | . | . | . | . | . | . | . |
| . | . | . | . | . | . | . | . | . | . | . | . | . | . | . | . | . |
| . | . | . | . | . | . | . | . | . | . | . | . | . | . | . | . | . |
| 23 | 10 | 10 | 10 | . | . | . | 10 | 10 | 10 | . | . | . | 0 | 2 | 10 | 10 |
| 24 | 12 | 12 | 12 | . | . | . | 12 | 12 | 12 | . | . | . | 2 | 0 | 12 | 12 |
| 25 | 0 | 0 | 0 | . | . | . | 0 | 0 | 0 | . | . | . | 0 | 0 | 0 | 0 |
| 26 | 0 | 0 | 0 | . | . | . | 0 | 0 | 0 | . | . | . | 0 | 0 | 0 | 0 |

From the Table 4.4, it can be seen that the existing facility nodes Q = {8, 18, 22, 25, 26} has a minimum road distance of zero between them. Hence the set of demand nodes and potential location of existing facility are removed from the modified shortest path distance matrix $\bar{D}$ and this is shown in table 4.5 below Table 4.5: Summary of Modified shortest path distance matrix, $\bar{D}$

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 19 | 20 | 21 | 23 | 24 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 2 | 3 | 4 | 4 | 6 | 10 | 11 | 11 | 11 | 11 | 11 | 11 | 11 | 11 | 11 | 11 | 11 | 11 | 11 | 11 |
| 2 | 5 | 0 | 2 | 2 | 9 | 5 | 9 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 |
| 3 | 3 | 2 | 0 | 4 | 7 | 3 | 7 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 |
| 4 | 7 | 2 | 4 | 0 | 7 | 5 | 7 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 |
| 5 | 4 | 6 | 7 | 7 | 0 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 |
| 6 | 6 | 5 | 3 | 5 | 10 | 0 | 4 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 |
| 7 | 8 | 8 | 7 | 8 | 8 | 4 | 0 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 |
| 9 | 17 | 17 | 16 | 17 | 17 | 13 | 9 | 0 | 15 | 10 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 |

| | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 23 | 19 | 21 | 17 | 23 | 19 | 15 | 15 | 0 | 5 | 12 | 19 | 11 | 12 | 23 | 23 | 23 | 23 | 23 | 23 | 23 |
| 11 | 27 | 24 | 26 | 22 | 27 | 23 | 19 | 10 | 5 | 0 | 7 | 16 | 16 | 17 | 27 | 27 | 27 | 27 | 27 | 27 | 27 |
| 12 | 27 | 27 | 27 | 27 | 27 | 27 | 26 | 17 | 12 | 7 | 0 | 9 | 17 | 18 | 27 | 27 | 27 | 27 | 27 | 27 | 27 |
| 13 | 36 | 36 | 36 | 36 | 36 | 36 | 34 | 26 | 19 | 16 | 9 | 0 | 8 | 36 | 36 | 36 | 36 | 36 | 36 | 36 | 36 |
| 14 | 34 | 30 | 32 | 28 | 34 | 30 | 26 | 26 | 11 | 16 | 17 | 8 | 0 | 1 | 34 | 34 | 34 | 34 | 34 | 34 | 34 |
| 15 | 35 | 31 | 33 | 29 | 35 | 31 | 27 | 27 | 12 | 17 | 18 | 9 | 1 | 0 | 35 | 35 | 35 | 35 | 35 | 35 | 35 |
| 16 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 0 | 8 | 12 | 12 | 12 | 12 | 12 |
| 17 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 0 | 20 | 20 | 20 | 20 | 20 |
| 19 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 0 | 8 | 8 | 8 | 8 |
| 20 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 0 | 9 | 10 | 10 |
| 21 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |
| 23 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 0 | 2 |
| 24 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 2 | 0 |

Step 4: Find the optimal new location for the police station using the modified distance matrix, $\bar{D}$ with existing facility nodes Y={8,18,22,25,26} removed from the network with the objective function:

$$min[f(x) = \sum_{i=1}^{n} w_i min\{d(X_i), d(Y_i)\}]$$

Let i= {1,2,3,4,5,6,7,9,10,11,12,13,14,15,16,17,19,20,21,23,24}. and X={1,2,3,4,5,6,7,9,10,11,12,13,14,15,16,17,19,20,21,23,24}

The optimal new location of the police station is now found by using the modified shortest distance matrix and the objective function:

Minimise $f(x) = \sum_{i=1}^{n} w_i min\{d(X_i), d(Y_i)\}$

At X=1 (Potential location 1)

7548(0)+1823(2)+2111(3)+23948(4)+5419(4)+1075(6)+799(10)+1204(11)+5565(11)

+1121(11)+986(11)+998(11)+1089(11)+977(11)+1125(11)+1509(11)+

1912(11)+1098(11)+851(11)+1225(11)+760(11)=366507

At X=2 (Potential location 2)

7548(5)+1823(0)+2111(2)+23948(2)+5419(9)+1075(5)+799(9)+1204(16)+5565(16)

+1121(16)+986(16)+998(16)+1089(16)+977(16)+1125(16)+1509(16)+

1912(16)+1098(16)+851(16)+1225(16)+760(16)=477917

At X=3 (Potential location 3)

7548(3)+1823(2)+2111(0)+23948(4)+5419(7)+1075(3)+799(7)+1204(14)+5565(14)

+1121(14)+986(14)+998(14)+1089(14)+977(14)+1125(14)+1509(14)+

1912(14)+1098(14)+851(14)+1225(14)+760(14)=583880

At X=4 (Potential location 4)

7548(7)+1823(2)+2111(4)+23948(0)+5419(7)+1075(5)+799(7)+1204(17)+5565(17)

+1121(17)+986(17)+998(17)+1089(17)+977(17)+1125(17)+1509(17)+

1912(17)+1098(17)+851(17)+1225(17)+760(17)=460967

At X=5 (Potential location 5)

7548(4)+1823(6)+2111(7)+23948(7)+5419(0)+1075(7)+799(7)+1204(7)+5565(7)

+1121(7)+986(7)+998(7)+1089(7)+977(7)+1125(7)+1509(7)+

1912(7)+1098(7)+851(7)+1225(7)+760(7)= 379601

At X=6 (Potential location 6)

7548(6)+1823(5)+2111(3)+23948(5)+5419(10)+1075(0)+799(4)+1204(12)+5565(12)

+1121(12)+986(12)+998(12)+1089(12)+977(12)+1125(12)+1509(12)+

1912(12)+1098(12)+851(12)+1225(12)+760(12)= 495970

At X=7 (Potential location 7)

7548(8)+1823(8)+2111(7)+23948(8)+5419(8)+1075(4)+799(0)+1204(8)+5565(8)

+1121(8)+986(8)+998(8)+1089(8)+977(8)+1125(8)+1509(8)+

1912(8)+1098(8)+851(8)+1225(8)+760(8)=492341

At X=9 (Potential location 9)

7548(17)+1823(17)+2111(16)+23948(17)+5419(17)+1075(13)+799(9)+1204(0)+5565(15)

+1121(10)+986(17)+998(17)+1089(17)+977(17)+1125(17)+1509(17)+

1912(17)+1098(17)+851(17)+1225(17)+760(17)=1021183

At X=10 (Potential location 10)

7548(23)+1823(19)+2111(21)+23948(17)+5419(23)+1075(19)+799(15)+1204(15)+5565(0)
+1121(5)+986(12)+998(19)+1089(11)+977(12)+1125(23)+1509(23)+

1912(23)+1098(23)+851(23)+1225(23)+760(23)=1089937

At X=11 (Potential location 11)

7548(27)+1823(24)+2111(26)+23948(22)+5419(27)+1075(23)+799(19)+1204(10)+5565(5)

+1121(0)+986(7)+998(16)+1089(16)+977(17)+1125(27)+1509(27)+

1912(27)+1098(27)+851(27)+1225(27)+760(27)=1341237

At X=12 (Potential location 12)

7548(27)+1823(27)+2111(27)+23948(27)+5419(27)+1075(27)+799(26)+1204(17)+5565(12)

+1121(7)+986(0)+998(9)+1089(17)+977(18)+1125(27)+1509(27)+

1912(27)+1098(27)+851(27)+1225(27)+760(27)=1557957

At X=13 (Potential location 13)

7548(36)+1823(36)+2111(36)+23948(36)+5419(36)+1075(36)+799(34)+1204(26)+5565(19)

+1121(16)+986(9)+998(0)+1089(8)+977(36)+1125(36)+1509(36)+

1912(36)+1098(36)+851(36)+1225(36)+760(36)=2049443

At X=14 (Potential location 14)

7548(34)+1823(30)+2111(32)+23948(28)+5419(34)+1075(30)+799(26)+1204(26)+5565(11)

+1121(16)+986(17)+998(8)+1089(0)+977(1)+1125(34)+1509(34)+

1912(34)+1098(34)+851(34)+1225(34)+760(34)=1046571

At X=15 (Potential location 15)

7548(35)+1823(31)+2111(33)+23948(29)+5419(35)+1075(31)+799(27)+1204(27)+5565(12)

+1121(17)+986(18)+998(9)+1089(1)+977(0)+1125(35)+1509(35)+

1912(35)+1098(35)+851(35)+1225(35)+760(35)=1772375

At X=16 (Potential location 16)

7548(12)+1823(12)+2111(12)+23948(12)+5419(12)+1075(12)+799(12)+1204(12)+5565(12)

+1121(12)+986(12)+998(12)+1089(12)+977(12)+1125(0)+1509(8)+

1912(12)+1098(12)+851(12)+1225(12)+760(12)= 734208

At X=17 (Potential location 17)

7548(20)+1823(20)+2111(20)+23948(20)+5419(20)+1075(20)+799(20)+1204(20)+5565(20)

+1121(20)+986(20)+998(20)+1089(20)+977(20)+1125(20)+1509(0)+
1912(20)+1098(20)+851(20)+1225(20)+760(20)=123268

At X=19 (Potential location 19)

7548(8)+1823(8)+2111(8)+23948(8)+5419(8)+1075(8)+799(8)+1204(8)+5565(8)

+1121(8)+986(8)+998(8)+1089(8)+977(8)+1125(8)+1509(8)+

1912(0)+1098(8)+851(8)+1225(8)+760(8)=366060

At X=20 (Potential location 20)

7548(10)+1823(10)+2111(10)+23948(10)+5419(10)+1075(10)+799(10)+1204(10)+5565(10)

+1121(10)+986(10)+998(10)+1089(10)+977(10)+1125(10)+1509(10)+

1912(10)+1098(0)+851(9)+1225(10)+760(10)=388099

At X=21 (Potential location 21)

7548(1)+1823(1)+2111(1)+23948(1)+5419(1)+1075(1)+799(1)+1204(1)+5565(1)

+1121(1)+986(1)+998(1)+1089(1)+977(1)+1125(1)+1509(1)+

1912(1)+1098(1)+851(0)+1225(1)+760(1)=62292

At X=23 (Potential location 23)

7548(10)+1823(10)+2111(10)+23948(10)+5419(10)+1075(10)+799(10)+1204(10)+5565(10)

+1121(10)+986(10)+998(10)+1089(10)+977(10)+1125(10)+1509(10)+

1912(10)+1098(10)+851(10)+1225(0)+760(2)=591990

At X=24 (Potential location 24)

7548(12)+1823(12)+2111(12)+23948(12)+5419(1)+1075(12)+799(12)+1204(12)+5565(12)

+1121(12)+986(12)+998(12)+1089(12)+977(12)+1125(12)+1509(12)+

1912(12)+1098(12)+851(12)+1225(12)+760(0)=688987

## 4.7 Discssion of Results

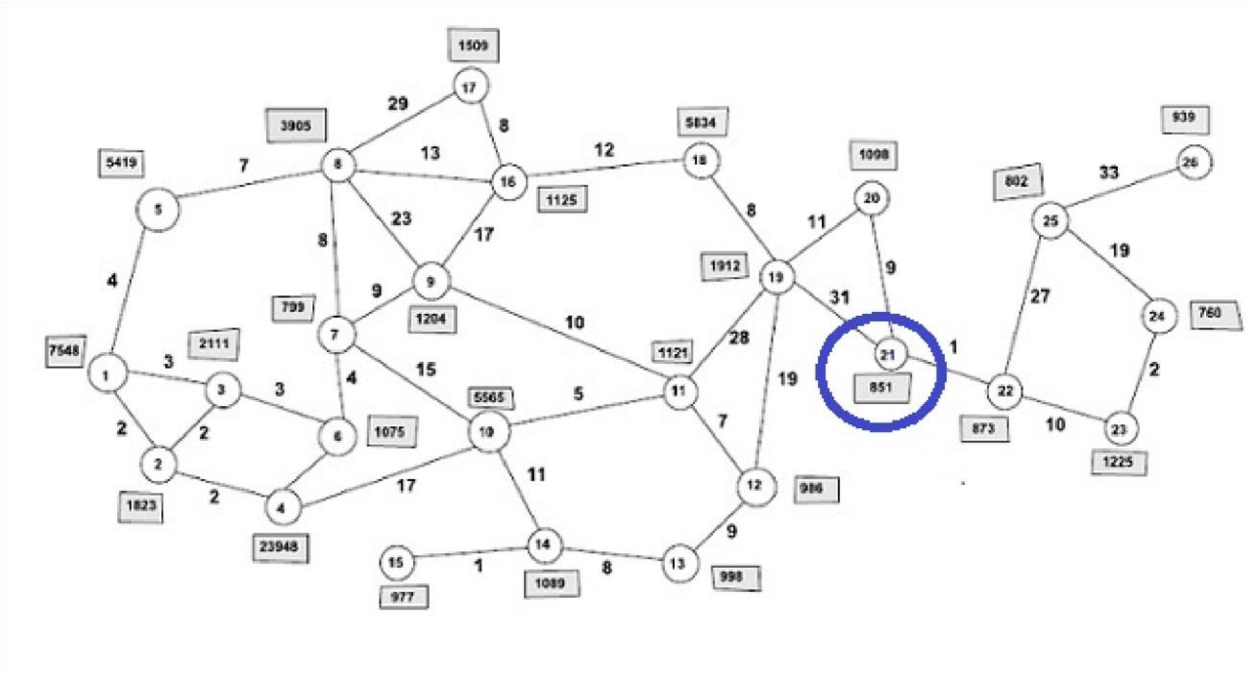
Considering the twenty-six nodes network and solving the conditional p–median problem with Q = {8, 18, 22, 25, 26} and P = 1. The optimal new location using the modified Shortest distance, thus by using the Berman and Drezner's algorithm, the new optimal location of police station can be located at node 21 ,(Nanso) with the minimum objective function value of 62292 .

# CHAPTER 5

# CONCLUSION AND RECOMMENDATION

## 5.1 Conclusion

The objective of this study was to model the location of an additional police station




using the conditional p – median model and find an optimal location for the police station in Afiyga Kwabre district. The data obtained from the district assembly was modeled into a conditional p–median problem and the Berman and Drezner's algorithm (2008) was used to solve the problem. The results as discussed in chapter 4, section 4.6 showed that the additional police station should be located at Nanso (node 21). The demand – weighted total (or average) distance using the Berman and Drezner's algorithm is 62292 because it is the minimum objective function value.

The additional police station will optimally serve the twenty-six major towns in the district as well as the various communities in the district. The new police station will help to reduce crime rate in the district and reduced the pressure on the existing police stations and police posts in the district. This will also help improve the quality of service provided to the inhabitants and their security in

the district.

Figure 5.1: Network of 26 towns in Afigya Kwabre district indicating the location of the additional police station

## 5.2 Recommendation

In view of the results obtained in the study the following recommendation are made :

1. The Afigya Kwabre district assembly is recommended to build an additional police station based on this study at Nanso to help reduce the pressures on the existing police post and police station facilities.

2. Private organizations and Non Governmental organization who will like to invest in the establishment of a police station in the district could use this study to optimally locate the police station at Nanso.

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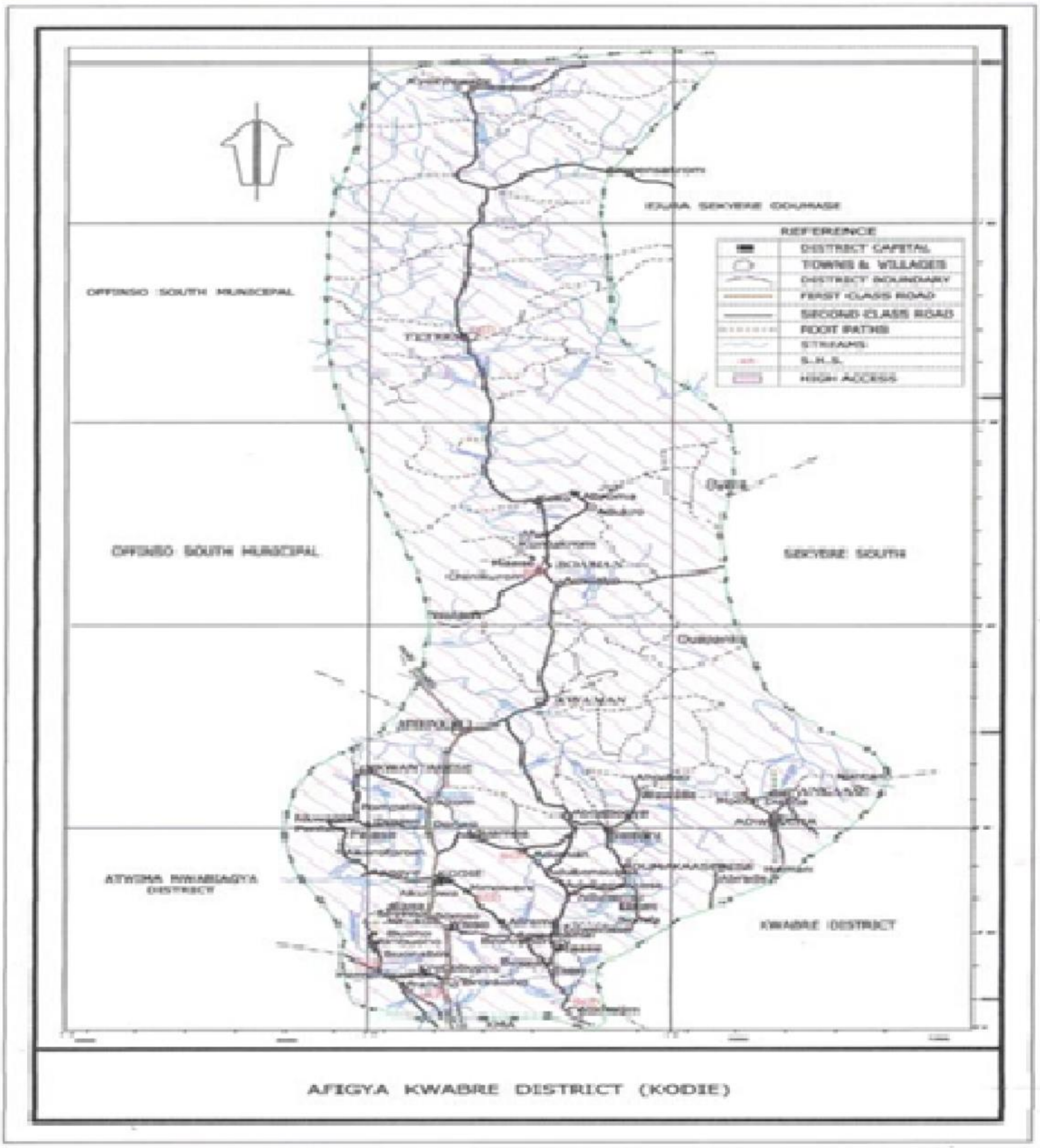
# APPENDIX A

Figure 5.2: Map of Afigya Kwabre district





# APPENDIX B

## 5.3 MATLAB CODE FOR FLOYD-WARSHALL ALGORITHM

```
function floyd_mat = floyd_warshall(A,thestart,theend)
%close all; clc
%keeping a copy of the original ending node new_theend=theend;
%Obtaining the dimension of the matrix A [r c] = size(A);
%creating an empty array to store the predecessor matrix pred_mat = []; if
nargin < 3 %checking the number of input arguments disp(' ') elseif
or(thestart,theend) > r disp('The node you entered does not exist') elseif
or(thestart,theend) < 0 disp('Node can only be positive')
else
for i = 1:r for j = 1:r
if A(i,j) = 0
pred_mat(i,j) = i;
else
pred_mat(i,j) = 0;
end end end
%Floyd_Warshall starts its work here
for k = 1:r for i = 1:r
for j = 1:r if (A(i,k) + A(k,j)) < A(i,j)
A(i,j) = A(i,k) + A(k,j); % Update the
predecessor matrix pred_mat(i,j) =
pred_mat(k,j); end end end end
floyd_mat = A;
%Array for storing the path thepath = [];
while (thestart = theend) thep =
```

pred_mat(thestart,theend); thepath =

[thepath thep]; theend =

pred_mat(thestart,theend); end thepath =

fliplr(thepath);% end

% Let us add the last figure in the route thepath(end+1) =

new_theend;

# APPENDIX C

Figure 5.3: All shortest path distance matrix, D

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 2 | 3 | 4 | 4 | 6 | 10 | 11 | 19 | 21 | 26 | 33 | 40 | 32 | 33 | 24 | 32 | 36 | 44 | 55 | 64 | 65 | 75 | 77 | 92 | 125 |
| 2 | 5 | 0 | 2 | 2 | 9 | 5 | 9 | 16 | 18 | 19 | 24 | 31 | 38 | 30 | 31 | 29 | 37 | 41 | 49 | 60 | 69 | 70 | 80 | 82 | 97 | 130 |
| 3 | 3 | 2 | 0 | 4 | 7 | 3 | 7 | 14 | 16 | 21 | 26 | 33 | 40 | 32 | 33 | 27 | 35 | 39 | 47 | 58 | 67 | 68 | 78 | 80 | 95 | 128 |
| 4 | 7 | 2 | 4 | 0 | 11 | 5 | 9 | 17 | 18 | 17 | 22 | 29 | 36 | 28 | 29 | 30 | 38 | 42 | 48 | 59 | 68 | 69 | 79 | 81 | 96 | 129 |
| 5 | 4 | 6 | 7 | 8 | 0 | 10 | 14 | 7 | 23 | 25 | 30 | 37 | 44 | 36 | 37 | 20 | 28 | 32 | 40 | 51 | 60 | 61 | 71 | 73 | 88 | 121 |
| 6 | 6 | 5 | 3 | 5 | 10 | 0 | 4 | 12 | 13 | 19 | 23 | 30 | 38 | 30 | 31 | 25 | 33 | 37 | 45 | 56 | 65 | 66 | 76 | 78 | 93 | 126 |
| 7 | 10 | 9 | 7 | 9 | 14 | 4 | 0 | 8 | 9 | 15 | 19 | 26 | 34 | 26 | 27 | 21 | 29 | 33 | 41 | 52 | 61 | 62 | 72 | 74 | 89 | 122 |
| 8 | 11 | 13 | 14 | 15 | 7 | 12 | 8 | 0 | 17 | 23 | 27 | 34 | 42 | 34 | 35 | 13 | 21 | 25 | 33 | 44 | 53 | 54 | 64 | 66 | 8 | 114 |
| 9 | 19 | 18 | 16 | 18 | 23 | 13 | 9 | 17 | 0 | 15 | 10 | 17 | 26 | 26 | 27 | 17 | 25 | 29 | 36 | 47 | 56 | 57 | 67 | 69 | 8 | 117 |
| 10 | 24 | 19 | 21 | 17 | 28 | 19 | 15 | 23 | 15 | 0 | 5 | 12 | 19 | 11 | 12 | 32 | 40 | 39 | 31 | 42 | 51 | 52 | 62 | 64 | 79 | 112 |
| 11 | 29 | 24 | 26 | 22 | 23 | 23 | 19 | 27 | 10 | 5 | 0 | 7 | 16 | 16 | 17 | 27 | 35 | 34 | 26 | 37 | 46 | 47 | 57 | 59 | 74 | 107 |
| 12 | 36 | 31 | 33 | 29 | 40 | 30 | 26 | 34 | 17 | 12 | 7 | 0 | 9 | 17 | 18 | 34 | 42 | 27 | 19 | 30 | 39 | 40 | 50 | 52 | 67 | 100 |
| 13 | 43 | 38 | 40 | 36 | 47 | 38 | 34 | 42 | 26 | 19 | 16 | 9 | 0 | 8 | 9 | 43 | 51 | 36 | 28 | 39 | 48 | 49 | 59 | 61 | 76 | 109 |
| 14 | 35 | 30 | 32 | 28 | 39 | 30 | 26 | 34 | 26 | 11 | 16 | 17 | 8 | 0 | 1 | 43 | 51 | 44 | 36 | 47 | 56 | 57 | 67 | 69 | 84 | 117 |
| 15 | 36 | 31 | 33 | 29 | 40 | 31 | 27 | 35 | 27 | 12 | 17 | 18 | 9 | 1 | 0 | 44 | 52 | 45 | 37 | 48 | 57 | 58 | 68 | 70 | 85 | 118 |
| 16 | 24 | 26 | 27 | 28 | 20 | 25 | 21 | 13 | 17 | 32 | 27 | 34 | 43 | 43 | 44 | 0 | 8 | 12 | 20 | 31 | 40 | 41 | 51 | 53 | 68 | 101 |
| 17 | 32 | 34 | 35 | 36 | 28 | 33 | 29 | 21 | 25 | 40 | 35 | 42 | 51 | 51 | 52 | 8 | 0 | 20 | 28 | 39 | 48 | 49 | 59 | 61 | 76 | 109 |
| 18 | 36 | 38 | 39 | 40 | 32 | 37 | 33 | 25 | 29 | 39 | 34 | 27 | 36 | 44 | 45 | 12 | 20 | 0 | 8 | 19 | 28 | 29 | 39 | 41 | 59 | 89 |
| 19 | 44 | 46 | 47 | 48 | 40 | 45 | 41 | 33 | 36 | 31 | 26 | 19 | 28 | 36 | 37 | 20 | 28 | 8 | 0 | 11 | 20 | 21 | 31 | 33 | 48 | 61 |
| 20 | 55 | 57 | 58 | 59 | 51 | 56 | 52 | 44 | 47 | 42 | 37 | 30 | 39 | 47 | 48 | 31 | 39 | 19 | 11 | 0 | 9 | 10 | 20 | 22 | 37 | 70 |
| 21 | 64 | 66 | 67 | 68 | 60 | 65 | 61 | 53 | 56 | 51 | 46 | 39 | 48 | 56 | 57 | 40 | 48 | 28 | 20 | 9 | 0 | 1 | 11 | 13 | 28 | 61 |
| 22 | 65 | 67 | 68 | 69 | 61 | 66 | 62 | 54 | 57 | 52 | 47 | 40 | 49 | 57 | 58 | 41 | 49 | 29 | 21 | 10 | 1 | 0 | 10 | 12 | 27 | 60 |
| 23 | 75 | 77 | 78 | 79 | 71 | 76 | 72 | 64 | 67 | 62 | 57 | 50 | 59 | 67 | 68 | 51 | 59 | 39 | 31 | 20 | 11 | 10 | 0 | 2 | 21 | 54 |
| 24 | 77 | 79 | 80 | 81 | 73 | 78 | 74 | 66 | 69 | 64 | 59 | 52 | 61 | 69 | 70 | 53 | 61 | 41 | 33 | 22 | 13 | 12 | 2 | 0 | 19 | 52 |
| 25 | 92 | 94 | 95 | 96 | 88 | 93 | 89 | 81 | 84 | 79 | 74 | 67 | 76 | 84 | 85 | 68 | 78 | 56 | 48 | 37 | 28 | 27 | 21 | 19 | 0 | 33 |
| 26 | 125 | 127 | 128 | 129 | 121 | 126 | 122 | 114 | 117 | 112 | 107 | 100 | 109 | 117 | 118 | 101 | 109 | 89 | 81 | 70 | 61 | 60 | 54 | 52 | 33 | 0 |

# APPENDIX D

Figure 5.4: Modified matrix $\bar{D}$

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 2 | 3 | 4 | 4 | 6 | 10 | 11 | 11 | 11 | 11 | 11 | 11 | 11 | 11 | 11 | 11 | 11 | 11 | 11 | 11 | 11 | 11 | 11 | 11 | 11 |
| 2 | 5 | 0 | 2 | 2 | 9 | 5 | 9 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 |
| 3 | 3 | 2 | 0 | 4 | 7 | 3 | 7 | 1 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 1 | 14 | 14 | 14 | 14 | 14 | 14 | 14 |
| 4 | 7 | 2 | 4 | 0 | 7 | 5 | 7 | 7 | 1 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 |
| 5 | 4 | 6 | 7 | 7 | 0 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 |
| 6 | 6 | 5 | 3 | 5 | 10 | 0 | 4 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 |
| 7 | 8 | 8 | 7 | 8 | 8 | 4 | 0 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 9 | 17 | 17 | 16 | 17 | 17 | 13 | 9 | 17 | 0 | 15 | 10 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 1 | 17 | 17 | 17 | 17 | 17 | 17 |
| 10 | 23 | 19 | 21 | 17 | 23 | 19 | 15 | 23 | 15 | 0 | 5 | 12 | 19 | 11 | 12 | 23 | 23 | 23 | 23 | 23 | 23 | 23 | 230 | 23 | 23 | 23 |
| 11 | 27 | 24 | 26 | 22 | 27 | 23 | 19 | 27 | 10 | 5 | 0 | 7 | 16 | 16 | 17 | 27 | 27 | 27 | 27 | 27 | 27 | 27 | 27 | 27 | 27 | 27 |
| 12 | 27 | 27 | 27 | 27 | 27 | 27 | 26 | 27 | 17 | 12 | 7 | 0 | 9 | 17 | 18 | 27 | 27 | 27 | 27 | 27 | 27 | 27 | 27 | 27 | 27 | 27 |
| 13 | 36 | 36 | 36 | 36 | 36 | 36 | 34 | 36 | 26 | 19 | 16 | 9 | 0 | 8 | 9 | 36 | 36 | 36 | 28 | 36 | 36 | 36 | 36 | 36 | 36 | 36 |
| 14 | 34 | 30 | 32 | 28 | 34 | 30 | 26 | 34 | 26 | 11 | 16 | 17 | 8 | 0 | 1 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 | 34 |
| 15 | 35 | 31 | 33 | 29 | 35 | 31 | 27 | 35 | 27 | 12 | 17 | 18 | 9 | 1 | 0 | 35 | 35 | 35 | 35 | 35 | 35 | 35 | 35 | 35 | 35 | 35 |
| 16 | 12 | 121 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 0 | 8 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 |
| 17 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 8 | 0 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 |
| 18 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 19 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 0 | 8 | 8 | 8 | 8 | 8 | 8 | 8 |
| 20 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 0 | 9 | 10 | 10 | 10 | 10 | 10 |
| 21 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 22 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 23 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 0 | 2 | 10 | 10 |
| 24 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 2 | 0 | 12 | 12 |
| 25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |