KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI

M.Sc INDUSTRIAL MATHEMATICS

THESIS

OPTIMUM PRODUCTION PLANNING PROBLEM

(A CASE STUDY OF ASPECT WATER COMPANY LIMITED IN TECHIMAN MUNICIPALITY)

BY

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DECLARATION

I hereby declare that except for reference to other people's work, which have duly been cited. This submission is my own work towards the Master of Science degree and that, it contains no material neither previously published by another person nor presented elsewhere.

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DEDICATION

I dedicate this research work to my wife Getrude Asiedu, my sister Mary Ameyaa, who made my education a success. Also to my cousin Benjamin and my lovely daughter Precious Nana Akua. I finally dedicate this work to my lovely friend Sandra Obour.

ABSTRACT

All production firms aim at maximizing profit after sales of their products but due to lack of technological and scientific approach in the production setting, many cannot achieve the stated objectives. This study showed the trend of production of sachet water at AWCL which gave the quantity of sachet water produced in each month for the year, 2011. The major objective of this study is to minimize the total cost of production at AWCL using Linear Programming model. The optimal solution to the production planning problem was generated by LP Solver and the demand and supply at each month were determined. The AWCL incurs cost of GH¢1.2355 when producing a bag of sachet water but with the use of linear Programming model, the cost of producing a bag of water was reduced to GH¢0.831519. The analysis also showed that, increasing the wages of regular workers and reducing that of overtime help the company to produce more with minimum cost of production. AWCL should employ more overtime labour when it is necessary to meet the urgent demands from the customers. Instead of employing more manual labour force, the company could have used machinery that can do assembling and packaging of the sachet water. Computer – based planning (scheduling) help the manufacturers to attend to orders from their respective customers easily and to enhance on – time delivery of products. The computerized planning performs better and faster than manual scheduling tools. The analysis showed that the production planning can facilitate the production processes in a way that help the company to streamline the activities that go on during acquisition of raw materials for production and the demands from the customers could be met when the wages of regular labour force are increased.

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CHAPTER ONE

1.0. Introduction

Water is a basic requirement for life and when the resource is to be used for domestic purposes, it should meet some set standards.

In the past two decades, the adverse health among the populace in Ghana and the neighbouring countries were as the result of untreated and improper management of water system. This stems from the fact that water used in our homes and public places are not well treated. As a result of that the contaminated water for human consumption, many people suffered from Cholera and other water borne related diseases (www.WHO.int/water_health/disease/cholera/WWD, 2001)

At the United Nations Millennium summit in 2000 and the Johannesburg Earth summit in 2002, world leaders agreed on the set measurable development targets popularly known as Millennium Development Goals for 2015, which aim at a commitment to reduce to about half the population of people without access to safe drinking water, [www.WHO.int/water_sanitation.../combating_ diseasepart1].

According to the World Health Organization (2004), 1 billion people did not have access to improved water supply in 2002, and 2.4 billion people suffered from diseases caused by contaminated water. About 1.8 million people die from diarrhoea disease and 90% of these deaths are of children under 5 years old (World Health Organization, 2004).

Assessment Report of United Nations Mid-term (United Nations International Children's Emergency Fund and World Health Organization, 2004), 80% of the world's population used an improved drinking water. As the population increases there will be some challenges as regards to the use of improved drinking water.

Besides mortality issues, water-related diseases also prevent people from working and having active lives. The problem of unsafe drinking water in the country is prevalent with associated diseases such as malaria, yellow fever, schistosomiasis (bilharzias), typhoid and diarrhoea.

The renewed global commitments towards the Millennium Development Goals marked for 2015, the importance of locally sourced, low-cost alternative drinking water schemes in contributing to increased sustainable access in rural and peri-urban settings of developing nations cannot be over-emphasized. One of such local interventions in Ghana, where public drinking water supply is endemic is packaged drinking water. This form of packaged water is usually distributed and sold in sachets. Packaged water refers to water that is packaged generally for consumption in a range of vessels including cans, laminated boxes, glass, plastic sachets and pouches, and an iced prepared for consumption .

The demand for sachet water nationwide is much considering the fact that majority of people drink (pure water) sachet water. Most communities in Ghana are presently under-serviced by water utilities due to the inability of the designated Ghana water company limited to meet their needs. Households and public seek other alternative sources which are safe for human consumption. Prominent among these is the sachet water production companies.

The introduction of sachet water in Ghana has helped to solve the problem of contaminated and unhygienic water for public consumption. The production of sachet water is booming and many people are entering into this business that has created a lot of jobs for many people in Ghana. Now, no matter the number of production plants, Ghana cannot cover or meet the demand of sachet water. Considering the selling price of sachet water (Ghp10) which is affordable to every Ghanaian.

The introduction of sachet water system popularly known as “pure water”, its production and distribution channels on the bases of market demands and public consumption are the major concern of every sachet water company in Ghana.

Quality of good safe drinking water is obtained by intense competition among the water companies. Profit is the main goal of every businessman, and demand of sachet water is due to its quality. It therefore encourages the management to develop modern technology for production methodologies in order to remain competitive.

Much has been written about production planning problems in the last few decades. Yet, in spite of the vast body of research, and the fact that many practitioners in operations management are convinced that manual planning (scheduling) is highly capable of improvement, implementations of scheduling techniques in practice are scarce. The lack of innovation in industrial production scheduling in the form of the scientific research which aims at strongly simplified problems which are representative of practice, e.g., small–sized problems, deterministic arrival and processing times pose serious loss to most manufacturing industries in Ghana. Today academia has moved its focus to real–world problems, and many scheduling techniques are now available in standard software. The number of software packages is at hand for production planning (scheduling).

1.1 Background of the study

Safe water is critical to maintaining the good health of people in every country.

It is evident that the introduction of sachet water in Ghana, cholera disease has been reduced drastically in the country with good atmosphere. This is due to emergence of many sachet water companies in Ghana. One of such companies is the Aspect Water Company Limited (AWCL) which is located in Techiman in the Brong Ahafo Region of Ghana. Techiman is one of the municipalities in Brong Ahafo, with the human population of about two hundred and six thousand, eight hundred and fifty six (206,856) and area of 1,053.5 km² = 196.4 inh. /km². (Thomas Brinkoff, Ghana Statistical service/ www.statsghana.gov.gh/docfile/2010). The production of safe drinking water by Aspect Water Company in Techiman Municipality started in the year 2002 and the map of Ghana in Appendix II shows the exact location of town where Aspect Water Company limited is situated. The goal of AWCL is to provide safe drinking water for people in Techiman Municipality in order to reduce or eliminate water related diseases in the area.

AWCL produces sachet water and distributes them to depots in six districts and municipals from Brong Ahafo Region, namely Wenchi, Berekum, Dormaa Ahenkro, Goaso, Bechem and Sunyani. AWCL also supplies sachet water to retailed customers in small containers

Figure 1.1 shows the routes of various depots which AWCL distributes its products

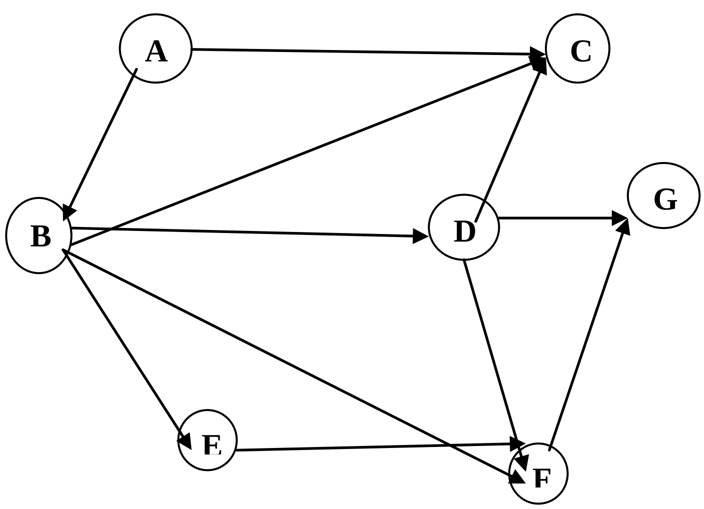

Fig 1.1: AWCL distribution depots.

The Figure 1.1 has seven nodes: A, B, C, D, E, F and G which represents seven towns in six districts in the Brong Ahafo Region.

Node A ——————— Techiman (AWCL) – source

Node B ——————— Sunyani

Node C ——————— Wenchi

Node D ——————— Berekum

Node E ——————— Bechem

Node F ——————— Goaso

Node G ——————— Dormaa Ahenkro

The capacity at each destination is as follows:

(i) Wench – 1500 bags per month

(ii) Sunyani – 2000 bags per month

(iii) Berekum – 1500 bags per month

(iv) Dormaa Ahenkro – 1350 bags per month

(v) Goaso – 1400 bags per month

(vi) Bechem – 1300 bags per month

The total capacity at these six depots is equal to 9050 bags of sachet water supply each month. In order to keep constant stock of sachet water in various depots, AWCL needs to produce maximum amount of products that can be supplied. The demand for sachet water for every month is between 400000 and 500000. AWCL being a profitable company has employed more labourers in order to produce more to furnish the stock in various depots.

The production of sachet water in large quantities and its strong patronage by the public pose serious challenges to the manufacturers. The production in large quantities depends largely on the cost of materials for production, labour cost, inventory cost, managerial cost and control, transportation cost, housing and electricity etc.

1.1.1 Costs of material

Materials are the physical items consumed in producing goods and services. Sachet water produced in either large or small quantities may depend on the raw materials available and machinery. AWCL producing sachet water should be able to produce and deliver safe drinking water in sufficient quantities to satisfy consumer demands. This requires the acquisition of raw materials needed for the production. Material management includes decisions regarding the procurement, control, handling and storage.

The AWCL uses the following raw materials for production: water, membrane filters,

plastic bags and chlorine water

The cost of these raw materials is high and this increases the production cost to the Company

1.1.2 Labour costs

Division of labour is paramount in manufacturing companies. Adding people to an operation may maximize production rate but increase the production cost. There are two important aspects of work – force management: job design which is the specification of job content and of employee skills and training needed to perform that job; work measurement which is the task of estimating operating system output, taking to account the effects of learning (Krajewski/Ritzman, 1992).This means that` operation is constrained by the amount of labour assigned to it. That is, the capacity of both service operations and manufacturing operations is affected by adding or eliminating people. Capacity as a measure of organization's ability to provide customer with demanded services or goods in the amount requested and in a timely manner. Capacity is the maximum rate of production (Vonderembse, 1991).

AWCL is the prominent water company in Techiman municipality, which has employed a lot of workers in various jobs which are packaging, filtering, collecting garbage etc. Because the cost of labour is high, it affects its production rate.

1.1.3 Inventory costs

Inventory is the total amount of goods and/or materials in a store at any given time. The number of sachet water to be produced in a day or a week depends on the stock in the storage areas. Inventory management plays important role in optimizing production. That is, the inventory cost. In the face of uncertain demand, balancing the need for the cost of reductions with finished water inventory management, retail customer demand responsiveness is difficult for Production Manager of Aspect Water Company Limited.

The Production Manager at AWCL needs to know the stock of raw mterials, the finished sachet water at the storage areas and the customer demands in order to control storage loses. The costs that needs to be considered so that AWCL can decide on the amount of stock to have and divide into stock holding costs and stock ordering (and receiving). This will ensure that the AWCL never run out of products.

1.1.4 Transportation costs

To ensure maximum production in AWCL, the amount of water that has to be produced at the source(Si) should be equal to the amount that has to leave the storage areas – demand (dj) to various depots. Because the cost of transporting finished goods is high, it delays supply hence minimize production in AWCL.

1.1.5 Housing and electricity costs

AWCL has a building divided into production and storage rooms with regular power supply. The storage rooms are not adequate to stock the large quantity of sachet water that could be supplied to various depots. Each room has electricity that causes a high electricity consumption in AWCL.

1.2 Statement of the problem

Manufacturing and production of goods is core development of any economy. Before goods are produced, there should be planning on materials to be used, the labour force and the funds for the cost of production.

Planning and scheduling are the most important aspect of decision – making that are used in manufacturing and service industries. Planning involves procurement and production, in transportation and distribution, information processing and communication. Planning can generally be described as allocating a set of resources over time to perform a set of tasks in a production industry.

In production systems, scheduling typically concerns allocating a set of machines to perform a set of work within a certain time period. The result of scheduling is a schedule, which can be defined as: a plan with reference to the sequence of, and time allocated for each item, or operation necessary to complete the item (Vollmann, Berry and Whybark, 1988).

The production planning in sachet water companies involves raw material acquisition, inventory management control, human labour and available machinery, transportation and distribution of finished products. Every sachet water company is expected to produce large quantity of goods with minimum cost of production.

There are problems emerging everyday in the production and processing industries such as low output in production, inventory loses and high cost of production. The optimal production planning which reduces the cost of production, increases production, manages and control loses of inventory of sachet water production is not much considered in many water manufacturing companies in Ghana. Aspect Water Company Limited is no exception. It produces large quantity of sachet water and supplies them to people in the Brong Ahafo Region.

The costs of production of sachet water and the inventory management control make some Production Managers run at a loss. This is because the raw materials needed to produce the sachet water, equipment, labour and inventory management is cost driven. Inventory management plays an important role in optimization at the production sector. That is, the inventory cost. In the face of uncertain demand, balancing the need for the cost reduction with finished goods inventory management, retail customer demand and responsiveness is difficult for manager in various water companies.

Orders that are released in a production setting have to be transformed into jobs with specified due dates. These jobs often have to be processed in the machines in a work center in a given order or sequence. The distribution channel with labour is not feasible with the objective function. This is due to the fact that the technology has taken over traditional way of production.

Aspect water company Ltd managers are still using the manual way of manufacturing their goods. The cost of labour is very high and the distribution of sachet water to various depots raises the transportation costs and that affects the AWCL's total production. Two Korea trucks (KIA) are assigned to each depot and that brings about the high costs of transportation to the company .The disposal of waste plastic bags has compelled AWCL to employ additional workers to the company. This raises the costs of production of sachet water.

AWCL being the prominent water company in Ghana, which produces more of safe drinking water for human consumption, is always not achieving the expected demand from the customers. This is because the cost of production is high. The company always produces quantity of sachet water less than the demands from its customers.

The problem facing the company is how to establish efficient production schedule that minimizes both total production and inventory (storage) costs while satisfying the customer demands by distributing the products to the various depots.

This study intends to optimize the company's overall production cost and long – term resource allocation based on inventory levels, demand forecast and transportation costs by developing the Linear Programming model on the production planning of sachet water.

1.3 Objectives of the study

The goal of AWCL is to provide safe drinking water for people in Ghana in order to reduce or eliminate water-related diseases. The objectives of the study are:

(i) to study the trend of manufacturing and supply of sachet water at AWCL

(ii) to formulate the problem as a Linear Programming (LP) problem model that will minimize the total production cost.

(iii) to enable AWCL to easily reschedule production in the event of a change in the system (e.g. a reduction in the available production hours at factory due to increased work from products made at that factory).

1.4 Justification of the study

This study justifies the production in general as production planning problem in sachet water companies. It will help to optimize company's production and inventory management levels, customer demands forecast and material acquisition resource. Production planning problem model will help to reduce the scheduling effort in AWCL by arranging the optimal production planning per the constraints. It will also give the information on inventory control and company's capacity available.

1.5 Research Methodology

The production problem involves a single product which is to be manufactured over a number of successive time periods to meet pre – specified demands. Production scheduling problem will be converted to transportation problem by considering the time periods during which production takes place as source, (S) and the time periods in which units will be transported to depots, (d). That is, the number of sachet water produced during the time period i will be taken to be supplies and the number that will be transported to the various depots during the time period j will be taken to be demands. X will be taken as the number of products and C as the cost of production

Therefore x_{ij} will denote the number of units to be produced during period i for transporting during time period j, and C_{ij} will be the unit production cost during time period i plus the cost of storing a unit of product from time period i until time period j. Because units cannot be transported before being produced, C_{ij} is made prohibitively large for $i > j$ to force the corresponding x_{ij} to be zero.

A twelve – months data on AWCL production capacities and expected demands (in units) for the period will be collected for the study. Regular and overtime production cost (in GH¢) including inventory at the beginning of the year will also be gathered for the study.

To achieve the stated objectives, Linear Programming problem model would be used to optimize production at AWCL. Linear programming problem model would be formulated based on the constraints; inventory(x_1), raw material(x_2), regular labour (x_3) overtime labour (x_4),and transportation (x_5) and their corresponding unit costs C_1, C_2, C_3, C_4 and C_5. The objective function will be the cost of production (Z).

The model will be analyzed using the Revised Simplex algorithm and software package such as LP Solver, Excel Solver, MATLAB, FORTRAN, C++, IBM CPLEX, R, GENSTART, SPSS v.16 etc.

1.6 Significance of the study

This study focuses on the raw material costs, labour costs, transportation costs and inventory management problem in the sachet water production industry. It will assist in optimizing company's production by minimizing the cost of production and hence maximizing sachet water production.

The outcome of the study will serve as a source of reference and panacea to all stakeholders in sachet water industries in the bid to improve the quantity in their products as well as minimizing the costs.

1.7 Scope of the study

This study is generally focused on the production planning of sachet water in water companies. The research covers the acquisition of raw materials for production, the inventory management control costs, the labour force at the production setting, the housing and electricity, the managerial costs and the distribution of finished products. Other areas such as waste management system also taken into consideration in the production industries.

1.8 Limitations of the study

In Ghana, there are many areas of production in sachet water companies that needed to be studied. However, this study is focused on: Raw materials cost, inventory cost – stock holding costs and stock ordering (and receiving) costs, transportation cost; and labour cost.

Other areas like managerial cost, housing, electricity and distribution costs will not be considered in the cost of production. The following are limitations encountered during the study:

(i). The data collection on regular and overtime production costs was difficult due to the reluctance to disclose the actual cost incurred by the company.

(ii). The location of the AWCL

1.9 Organization of the study

This research comprises five chapters. The chapters are organized as follows:

Chapter one entails the introduction, background of the study, statement of the problem, proposed model for study, objectives of the study, justification of the study, significance of the study, the scope of the study and the limitations of the study. Chapter two is denoted for review of existing literature on production planning (scheduling). Chapter three presents the research methodology of the study. In chapter four, we shall put forward the data collection and analysis of the data. Chapter five, which is the last chapter of the study, presents the summary conclusions and recommendations of the study.

1.10 Summary

This chapter focused on the introduction of the study, background of the study, the problem statement, objectives of the study and the justification of the study. A brief discussion on research methodology for the study, Significance of the study, Scope of the study, limitations of the study and finally how the work is organized.

The next chapter talks about the relevant literature on production planning.

CHAPTER TWO

LITERATURE REVIEWS

2.0 Chapter overview

This chapter will talk about studies carried out by famous researchers on production planning (scheduling) in manufacturing and industrial practice. In this section, we shall review the relevant researches on production planning using Linear Programming. One of the most popular methods to choose optimum production planning is the method that measures the least cost of production proposed by simplex model. This model minimizes the cost of production such as raw material cost, labour cost, inventory cost and transportation cost.

2.1 Introduction

Before the advent of the sachet water, people have been taken water from various sources that were not hygienic to their body. Sachet water popularly known as pure water served as a good source and means of relieving thirsty and also a good means of reducing poverty among populace. A number of people are involved in the chain of wholesale and retailed that generates income from selling packaged water products. Sachet water popularly called "pure water" has come to stay and it is the highest selling consumable goods in the Ghanaian Market today. There are a lot of problems presently militating against the development of the small scale manufacturing sector in Ghana. Obsolete technologies and machineries, lack of access to modern technology, lack or limited access to management support and technical advisory services, poor access to information on raw materials, infrastructure inadequacy and lack of social support, production development, financial problems, disenabling business environment (poor infrastructure, multiple taxation, etc), and poor economic condition. However, various researchers have put forward means solving problems in the production industries.

This chapter will focus on the use of ICT in the production sector; job – shop scheduling (planning), inventory on material requirement planning (stock – holding and stock – ordering); production planning algorithm, production planning model.

2.2.1 The use of ICT in the production sector

Some of the problems found to be confronting manufacturing industries can be solved by the application of ICT. Information Technology can play an important role in bringing about sustainable economic development. The use of ICT in the production sector of sachet water helps to facilitate production and distribution. Richardson et al, (2006) outlined five main areas of ICT applications in support of firm and rural development. These are: economic development of product, community development, research and education, small and medium enterprises development, and media networks. Sachet water companies make use of ICT in the following ways: online services for information, monitoring and consultation and transaction and processing; commerce. The empirical evidence of the impact of ICT on enterprise performance is at best mixed. In fact in the industrial countries, it is only in 1990s that the empirical evidence has shown ICT to have a substantial effect on firms' profitability levels (Brynjolfsson and Hitt, 1996).

Investment on small scale industries talks less of sachet Water Company – a sub sector of small scale industry in Ghana. An important characteristic of ICT is that they are mostly scale neutral and available to small industries and poor countries as well, although their access is restricted by poor infrastructure and high cost of procuring them. The increase of ICT in enterprises leads to a substitution of IT equipment for other forms of capital and labour and may generate substantial returns for the enterprises that invest in IT and restructure their organization. Extensive research has been conducted in the last 20 years on the business benefits and value generated by ICT investments, and their impact on business performance. From the mid l980s until the mid 1990s, little empirical evidence of a positive and statistically significant relation between ICT investment and business performance. However, though ICT has high return potentials, they may gradually reduce a firm's profitability by integrating markets and exposing sachet water companies to competition.

2.2.2 Job – Shop Scheduling (Planning)

Scheduling is plan of work in any manufacturing company. There are several reviews on production scheduling theory. a review on capacity as a measure of organization's ability to provide customer with demanded services or goods in the amount requested and in a timely manner can be found in Vonderembse (1991); a review on production planning and control system by Burbidge (1990), Production Control (PC) is the function of management which plans, directs and controls the material supply and processing activities in an enterprise'

Herrmann (2006) described the history of production scheduling in manufacturing facilities over the last one hundred (100) years. According to Herrmann (2006), understanding the ways that production scheduling has been done is critical to analyzing existing production scheduling systems and finding ways to improve upon them. The author covered not only the tools used to support decision-making in real-world production scheduling, but also the changes in the production, scheduling systems. He extended the work to the first charts developed by Gantt (1973) to advanced scheduling systems that rely on sophisticated algorithms. Through his findings, he was able to help production schedulers, engineers, and researchers understand the true nature of production scheduling in dynamic manufacturing systems and to encourage them to consider how production scheduling systems can be improved even more. The author did not only review the range of concepts and approaches used to improve production scheduling, but also demonstrate their timeless importance. Lodree and Norman (2006) summarized research related to scheduling personnel where the objective is to optimize system performance while considering human performance limitations and personnel well-being. Topics such as work rest scheduling, job rotation, cross-training, and task learning and forgetting were considered. For these topics, mathematical models and best practices were described. Pfund and Scott (2006) discussed scheduling and dispatching in one of the most complex manufacturing environments-wafer fabrication facilities. These facilities represent the most costly and time-consuming portion of the semiconductor manufacturing process. After a brief introduction to wafer fabrication operations, the results of a survey of semiconductor manufacturers that focused on the current state of the practice and future needs were presented. The authors presented a review of some recent dispatching approaches and an overview on the production planning.

The review on capacity-planning decisions, which are easy to understand because significant capital is usually requested to build capacity, can be found in (Mark and White, 1991). The problem with regard to PC is to determine when and how much to produce in a given manufacturing system in order to satisfy a set of objectives (Liberopoulos and Dallery, 2000). One of the most important activities of PC is what Burbidge (1990) calls ordering. Many manufacturing Companies cannot control the stock – holding and stock – ordering in their shop floor which is one of the functions of shop floor control. Location decisions affect the marketing performance. Serving the customer with manufacturing products is a primary concern in locating facilities. A customer is best served if a company takes advantage of low production cost or low transportation cost (Mark and White, 1991).

Decisions on the operation planning activities in manufacturing industries are in the form of three broad dimensions namely: long – range planning which generally done annually(focusing on a horizon greater than one year), intermediate – range planning(usually covers a period from six to eighteen months, with time increments that are monthly or sometimes quarterly), short – range planning(covers a period from one day or less to six months with time increment usually weekly) found by (Chase, Aquilano and Jacobs, 1998).

Jacobs major contribution on production planning is the tasks that schedulers perform each day. His main concern is on: allocating orders, equipment and personnel to work centers or other specified locations(short – run capacity planning); determining the sequence of order performance (establishing job priorities); initializing performance of the scheduled work (dispatching orders); shop – floor control(production activity control).

Aspect Water company Limited has adopted strategies to determine whether to meet peak demand with overtime shifts, night shifts. Subcontracting is another example of personnel scheduling (Steven Nahmias, 1997). Moreover, improving production scheduling requires that the schedulers manage the bottleneck resource effectively at the storage areas, understanding the problems that occur in the production and distribution centers and decide what to do to handle future uncertainties.

2.2.3 Inventory on Material Requirement Planning

In production planning, there is a place for problem solving. The planning requires creating a schedule for work that is not in process. However, Steven Nahmias (1997) grouped the production scheduling as Hierarchy of Production Decisions. Materials Requirement Planning (MRP) is one method for meeting specific production goals of finished – goods inventory that should be generated by Master Production Schedule (MPS).

Burbidge defined ordering as the second level of scheduling in production control, which is concerned with regulating the supply of both manufactured parts and bought items, in order to meet the production programmed. This activity is performed by production control systems (PCS), which González and Framinam (2009) defined as being a set of rules defining order release and material flow control in a manufacturing system. PCS are also known as ordering systems according to (Burbidge,*Corresponding author. E-mail: moacir@dep.ufscar.br 1990), a review on production control policies by (Sharma and Aggrawal,2009), material planning methods (Jonsson and Mattsson, 2002), production and material flow control mechanism are the most important parts of production system in any manufacturing industry (Fernandes and Carmo-Silva, 2006), logistics control systems (Ghamari, 2009), material flow control mechanism (Graves et al, 1995), production inventory control policy (Gerathy and Heavey, 2004), and production planning and control systems (MacCarthy and Fernandes, 2000). This is called Systems for Coordination of Orders (SCO), once it is taken into account that the main contribution of such systems is to coordinate the materials and information flow onto the shop floor. SCO schedule or organize material requirements, and/or control the production and purchasing orders release, and/or schedule jobs on machines. From this point, we refer to such systems as SCO. The study also highlights certain insights regarding practical applications of SCO. Comparisons and selection between different SCO have been said by (MacCarthy and Fernandes, 2000) and continue to be (Sharma and Agrawal, 2009; Khojasteh-Ghamari, 2009) an important subject with respect to Production Planning Control research.

Despite the existence of considerable research, the problem as to what is the best choice has not yet been solved. According to Gupta and Snyder (2009), the net results of such research are still inconclusive. The goal of this paper is not to provide such an answer but to contribute in this direction, providing insights regarding SCO application, and relating these insights to characteristics of such systems given by the classification proposed.

Early works on production scheduling focused on developing methods and algorithms for identifying the optimal sequence of the required tasks considering either only one processor(machine) or multiple processors (Baker, 1974). In each of these scheduling methods, the optimal schedule is achieved based on a certain desired goal such as to minimize the total – span to complete all the selected tasks, or to minimize the mean flow of these selected tasks.

The classification framework developed in this paper arose from Burbidge (1968), who divided SCO into three classes:

(i) make-to-order systems;

(ii) stock-controlled systems; and

(iii) programme-controlled systems.

The classification framework proposed is presented further, and is divided into four categories:

(a) Order-controlled systems: There is no stock of final items, once production is carried out according to customers' specifications.

(b) Stock Level-Controlled systems (SLC): The decision about the release of an order is based only on the stock level, which pulls the production.

(c) Flow-Scheduled systems (FS): The release of an order is based on a centralised scheduling drawn up by the PC department. This centralised schedule pushes the production.

(d) Hybrid systems (H):

A Review of Capacity Planning Techniques within Standard Software Packages, Production Planning and Control (WORTMANN, EUWE, TAAL and WIERS, 1996).

2.2.4 Production Planning Model

Another aspect that hampered the implementation of scheduling techniques in practice was that for a number of decades, scheduling techniques needed too much computing power. Most of the scientific research had been directed towards relatively small–scale optimization programs that are highly iterative. Contrarily, nearly all software suppliers considered iterative algorithms to be very risky. Sanderson concludes with the observation that more and better co-ordinateds research on the human factor in scheduling is required. The research reported in the review is widely dispersed over a variety of research journals and the reported works are often carried out in isolation from each other. She also notes that a common research question in much of the literature reviewed: which is better, humans or algorithms is no longer relevant. Humans and algorithms seem to have complementary strengths which could be combined. To be able to do this a sound understanding of the human scheduler is needed (Kleinmuntz, 1990).

A water company marketing sachet water should be able to produce and deliver safe drinking water in sufficient quantities to satisfy consumer demands. This requires the acquisition of physical facilities, the hiring and training of persons or/and production of materials to achieve the desired introduction to wafer fabrication operations, the results of a survey of semiconductor manufacturers that focused on the current state of the practice and future needs were presented. They presented a review of some recent dispatching approaches and an overview of recent

Production planning model has been talked by some renowned writers. An extensive literature on production planning has been developed over almost five decades. In this section, we focus on only a few of these optimization models. Interested readers are referred to the chapter by Missbauer and Uzsoy (2010) in the first volume of this handbook which reviews the basic formulations that are most commonly used in academic research and industrial practice.

A capacitated Material Requirement Planning (MRP)-based model is proposed by Horiguchi et al, (2001).

The goal is to calculate a planned release date for each order during each of its visits to a bottleneck station, and to estimate when the order will be completed. The authors aggregate the times available across machines over discrete time periods (time buckets) that are used to incorporate capacity factors. The model explicitly considers capacity only for specified near-bottleneck stations, and assumes that all other stations have infinite capacity, which is different from the conventional MRP approach. The authors performed two experiments. One examines the effect of the predictability of the capacity model. In their paper, predictability is defined as the deviation of the realized completion time in the simulation model from the predicted completion time in the planning model. The results show that finite capacity planning gives better predictability than dispatching rules such as Critical Ratio (Rose, 2002). The lot with the lowest value has the highest priority. The second experiment tests the effects of using a "safety capacity" in planning, that is, the reduction of the planned capacity of a given station by some amount to keep processing capacity in reserve to deal with unexpected events such as machine breakdowns. Their results show that increasing the safety capacity reduces tardiness and improves predictability, without adversely affecting other performance measures.

There are a wide variety of linear programming-based planning models for production planning. Hackman and Leachman (1989) propose a general production planning framework based on a linear programming model. They take into consideration specific components such as processing and transfer time in order to provide an accurate representation of the production process. However, the time delays in the model do not capture the load-dependent nature of the lead times. Thus, the aspects of production captured in the model are limited. In addition, the LP formulation accommodates non integer values for cycle times as well as planning time buckets of unequal length.

Expanding the model in Hackman and Leachman (1989), Hung and Leachman (1996) incorporate time-dependent parameters representing partial cycle times from job release up to each operation into the LP planning model. Furthermore, the authors provide a framework that iteratively updates the plan through an LP model that develops a plan for a given set of lead times and a simulation model that evaluates the system performance for a given production plan. They estimate the cycle times from the simulation results and show that they can achieve better results by iterating between the LP and the simulation model. It is well known that the relationship between cycle time and machine utilization is nonlinear.

The iterative LP-simulation process provides a good way to approximate such a nonlinear relationship.

Some planning models that try to capture the relationship between cycle time and utilization without resorting to simulation make use of so-called clearing functions (Graves, 1986). Clearing functions express the expected throughput of a machine in a planning period as a function of the expected WIP inventory at the machine over the period. (Our focus on load-dependent cycle times is mainly due to the observation that the load as a function of releases determined as part of the plan affects cycle times that result in scheduling. There are recent studies that try to capture the dependency between the load level and/or utilization and cycle times. In that sense, the underlying approaches can also be viewed as "hybrid" models attempting to link planning and scheduling.)

Missbauer (2002) considered clearing functions for an M/G/1 system. He uses a piece-wise linear approximation for the clearing function to model the effective capacity for bottleneck stations, and considers fixed, load-independent time delays between bottleneck stages to represent the delays at non bottleneck machines. His planning model determines the release plan and uses a short-term order release policy to select specific orders for release into the job shop. The product mix is not considered in their clearing function, i.e., the clearing function only depends on the total planned production quantity, which means the total output of a station can be allocated arbitrarily to different products.

Asmundsson et al, (2006) proposed a clearing function-based planning model, which explicitly considers the product mix. They approximate the clearing function using an empirical approach, together with two sets of constraints enforcing flow conservation for WIP and Finished Goods Inventory (FGI). There is no need for explicit cycle time parameters in their model.

Due to the product mix, different products may have different capacity needs (capacity allocation), and a particular difficulty is estimating the throughput as a function of the product mix currently represented in the WIP. To overcome this, they assume that all products see the same average cycle time, which allows them to use a convex combination of the capacity allocation parameters to approximate the WIP levels of different products, which in turn leads to approximated clearing functions. Exploiting the concavity of clearing functions, they use outer linearization to approximate the functions, which result in an LP model. The objective in their model is to minimize the total production cost, the WIP cost, the FGI holding costs, and the raw material cost. The approximation of the clearing function is also done by simulation with several randomly generated realizations of the demand profile, which are evaluated using the release schedules obtained from the fixed cycle time production planning model of Hackman and Leachman (1989). The authors perform extensive experiments to evaluate the benefit of the clearing function-based model. Different dispatching rules are used to compare planned throughput and actual throughput. Based on these experiments, one of their conclusions is that if planning is done properly, the role of a detailed schedule can be viewed as rescheduling the jobs to adhere to the original production plan that has been distorted by equipment failures and other unpredictable occurrences. Although it is unlikely that the scheduler can restore the original plan in every instance, its ability to do so is highly dependent on the planning algorithm's ability to represent the shop floor dynamics correctly."Such a conclusion shows that we need to consider the coordination between planning and scheduling to achieve better performance, which further motivates our study.

Pahl et al, (2005) gave an extensive survey of planning models, which consider load-dependent cycle times. In addition to the use of clearing functions, there are other approaches. Interested readers are referred to Sect. 3 and the corresponding references in Pahl et al, (2005) for more details.

Model predictive control, or MPC (Qin and Badgwell, 2003), is a method of process control that has been used extensively in processing industries (Kleindorfer et al, 1975).

MPC encompasses a group of algorithms that optimize the predicted future values of the plant output by computing a sequence of future control increments. This optimization model is implemented through a rolling-horizon approach at each sampling time. MPC attempts to model the dependence between the sequence of predicted values of the system output, and the sequence of future control increments.

With knowledge of the system model, disturbance measurements, and historical information of the process, the MPC model calculates a sequence of future control increments that must satisfy appropriate constraints. Vargas-Villami and Rivera (2000) propose a two-layer production control method based on MPC. Extending this work, Vargas-Villami et al, (2003) propose a three-layer version. The first layer, called the adaptive layer, is used to develop a parameter estimation approach. The second layer, the optimizer, solves an MILP model by branch and bound to generate a good-quality production plan. The third layer (direct control) uses dispatching to control the detailed discrete-event reentrant manufacturing line in a simulation model.

The computational results show that the method is less sensitive to initial conditions than “industrial-like” policies examined by Tsakalis et al, (2003). Furthermore, the three-layer approach with the adaptive parameter estimation model achieves reduced variation at high production loads as compared to the two-layer approach. The authors did not perform any cycle time comparisons with existing methods, but point out that an MPC-based model could be a promising tool for planning.

Jaikumar (1974) proposed a methodology, which decomposes the planning and scheduling problem into two subproblems. The first problem is a long range planning problem, which maximizes the profit subject to resource constraints. The Lagrange multipliers obtained in the first problem are used in the objective function of the second short range scheduling model. They propose a heuristic algorithm to reduce the second model to a sequential allocation of production facilities to products. Planning on job scheduling problem is reviewed by Blazewicz et al, (1996).

2.2 Summary

This chapter talked about related literature on the production planning. The review was on the use of ICT in the production sector, Job – shop planning (scheduling), Inventory on Material Requirement Planning (MRP) and Production Planning Model.

The next chapter presents the research methodology of the study.

CHAPTER THREE

METHODOLOGY

3.0 Introduction

The production planning problem at AWCL on raw materials acquisition, inventory management, labour force and the distribution of sachet water to various depots is a prime concern to the operations manager. The cost of production should be less than income after sales of products.

To minimize the cost of production at AWCL, the production setting requires the methods that will streamline the production cost. This can be done by minimizing the total costs of production and maximizing production. The cost of production is minimized by using Mathematical discipline called Linear Programming (LP).

The production problem of Sachet Water at Aspect Water Company Limited can be described as optimum production planning problem. This chapter focuses on sources and data collection, statistical analysis, linear programming and proposed LP model for production of Sachet Water.

3.1 Sources of data collection

The secondary data on monthly production from January to December, 2011 were collected.

The data was collected on expected demand and capacity (both regular and overtime shifts); and the production costs in the year 2011.

3.2 Statistical Analysis:

The unit costs of the inventory (C_1), raw materials (C_2), regular labour (C_3), overtime labour (C_4) and transportation (C_5) were found from the production cost of AWCL for the year, 2011.

3.3.0 Linear Programming (LP)

After the World War II there was a scarcity of industrial material and industrial productivity reached the lowest level. Industrial recession was there and to solve the industrial problem the method linear programming was used to get optimal solution. In industrial world, most important problem for which these techniques are used is how to optimize the profit or how to reduce the costs. As a decision making tool, it has demonstrated its value in various fields such as production, finance, marketing, research development and personnel management. Determination of optimal product mix (a combination of products, which gives maximum profit), transportation schedules, assignment problem and many more. LP involves the planning of activities to obtain an optimal result. Many problems can be formulated as maximizing or minimizing an objective function, given limited resources and competing constraints. If we can specify the objective as linear function of certain variables and constraints on resources as equalities or inequalities on those variables, then we have linear programming. Thomas H. Cormen et al, (2001, 2nd Ed).

LP was first introduced by the Soviet Mathematician, Leonid Kantororich, in the late 1939's as a procedure for solving a production scheduling problem. Hoffmann (1995).

The two most important solution methods of LP are the Graphical methods and Simplex methods. The types of LP problems include Transportation problem, scheduling problem, production problem, assignment problem and transshipment problem. This study will focus on the production planning problem which can be solved using Simplex method.

3.3.1 General form of linear programming:

We wish to optimize a linear function subject to a set of linear inequalities. Given a set of real numbers: $a_1, a_2, a_3, \ldots, a_n$ and a set of variables $x_1, x_2, x_3, \ldots, x_n$.

Linear function f on those variables is defined by

$f(x_1, x_2, \ldots, x_n) = a_1x_1 + a_2x_2 + a_3x_3 + \ldots + a_nx_n =$

$$\sum_{j=1}^{n} \boldsymbol{a_j x_j}$$

If b is a real number and f is a linear function, then the equation $f(x_1, x_2, \ldots, x_n) \leq b$ and $f(x_1, x_2, \ldots, x_n) \geq b$ are linear inequalities. Thomas H. Cormen et al (2001, 2nd Ed).

3.3.2 Terminology

A Linear Programming problem is said be in standard form when it is written:

$$\textbf{maximize} \qquad \sum_{j=1}^{n} \boldsymbol{C_{ij} X_j}$$

Subject to

$$\sum_{j=1}^{n} \boldsymbol{a_{ij} x_{ij}} \leq \boldsymbol{b_i} \qquad \boldsymbol{i = 1, \ldots, m}$$

$$X_i \geq 0, \qquad j = 1, \ldots, n$$

The problem has m variables and n constraints. It may be written using vector terminology as:

$$\text{Maximize} \quad C^T X$$

$$\text{Subject} \quad AX \leq b$$

$$X \geq 0$$

In minimizing the cost function instead of maximizing, it may be rewritten in standard by negating the cost coefficient $C_j(C^T)$.

An LP can be expressed as follows:

Minimize $C_1x_1 + C_2x_2 + \ldots + C_nX_n$

Subject to

$$a_{11}x_1 + a_{12}x_2 \ldots + a_{1n}x_n \leq b_1$$

$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \leq b_2$$

$$\begin{array}{lll} . & & . \\ . & & . \\ . & & . \end{array}$$

$$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \leq b_n$$

$$X_j \geq 0 \text{ for } j = 1, \ldots, n.$$

The objective is the minimization of costs. The vector $c_1,...,c_n$ vector is referred to as the cost vector. The variables $x_1,\ldots,x_n$ have to be determined so that the objective function

$c_1x_1 + \ldots\ldots + c_nx_n$ is minimized.

A general mathematical way of representing a Linear Programming Problem (L.P.P.) is as given below:

Objective function $Z = c_1x_1 + c_2 x_2 + \ldots c_n x_n$

Subjects to

$a_{11}x_1 + a_{12}\,x_2 + a_{13}\,x_3 + \ldots + a_{1j}\,x_j + \ldots + a_{1n}\,x_n\ (\geq, =, \leq)\ b_1$

$a_{21}\,x_1 + a_{22}\,x_2 + a_{23}\,x_3 + \ldots + a_{2j}\,x_j + \ldots + a_{2n}x_n\ (\geq, =, \leq)\ b_2$

. .

. .

. .

$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \ldots + a_{mj}\,x_j \ldots + a_{mn}\,x_n\ (\geq, =, \leq)\ b_m$

and all x_j's are = 0

Where $j = 1, 2, 3, \ldots, n$

Where all c_j' *s*, b_i' s and a_{ij} *'s* are constants and x_j *'s* are decision variables. Any vector x satisfying the constraints of the linear programming problem is called a feasible solution of the problem. Every linear programming problem falls into one of three categories:

i. Infeasible. A linear programming problem is infeasible if a feasible solution to the problem does not exist; that is, there is no vector x for which all the constraints of the problem are satisfied.

ii. Unbounded. A linear programming problem is unbounded if the constraints do not sufficiently restrain the cost function so that for any given feasible solution, another feasible solution can be found that makes a further improvement to the cost function.

iii. Has an optimal solution. Linear programming problems that are not infeasible or unbounded have an optimal solution; that is, the cost function has a unique minimum (or maximum) cost function value. This does not mean that the values of the variables that yield that optimal solution are unique, however.

3.3.3. Duality and Primal problem

Every linear programming problem where we seek to maximize the objective function gives rise to a related problem, called the dual problem, where we seek to minimize the objective function. The two problems interact in an interesting way: every feasible solution to one problem gives rise to a bound on the optimal solution in the other problem. If one problem has an optimal solution, so does the other problem and the two objectives function values are the same. The equations below show a problem in standard form with *n* variables and *m* constraints on the left, and its corresponding dual problem on the right. Consider the primal problem stated in canonical form:

$$\text{Max } \boldsymbol{c^T}\text{x}$$

$$\text{Subject to Ax} \leq \text{b}$$

Where $x = (x_1, x_2, \ldots, x_n)^T$ is any n – vector ,

$c = (c_1, c_2, \ldots, c_n)$ is any n – vector,

$A = (a_{ij})$ is an mxn matrix and

$b = (b_1, b_2, \ldots b_n)^T$ is an m – vector

The associated dual problem:

$$\text{Minimize } b^T\text{w}$$

$$\text{Subject to } A^T w \geq c$$

$$W \geq 0$$

is called the dual of the given problem. The variables in the primal problem are called primal variables and the variables in the dual problem are called dual variables.

REMARKS

We note that the Right Hand Side (RHS) of the primal is the vector of coefficients of the objective function of the dual problem and the vector of coefficients of the objective function of the primal problem is the R.H.S of the dual problem. Also the matrix of coefficients of the constraints in the dual problem is transpose of the matrix of the coefficients of the constraints in the primal problem. Finally all inequalities ≤ in the primal problem are changed to ≥ in the dual problem

3.3.4 The matrix form of LP model

A general LP model in the standard form is the vector Ax = b is written in the matrix form:

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ \cdot \\ b_n \end{pmatrix}$$

The simplex algorithm is an iterative procedure that provides a structured method for moving from one basic feasible solution to another, always maintaining or improving the objective function until an optimal solution is obtained. An Initial Basic Feasible Solution (IBFS) has to be determined first.

The basic algorithm most often used to solve linear programming problems is called the Simplex method.

3.4.1 Graphical solution of LP models

In graphical method, the inequalities (structural constraints) are considered to be equations. This is because; one cannot draw a graph for inequality. Only two variable problems are considered, because we can draw straight lines in two-dimensional plane (*X*- axis and *Y*-axis). More over as we have non – negativity constraint in the problem that is all the decision variables must have positive values always the solution to the problem lies in first quadrant of the graph. Sometimes the value of variables may fall in quadrants other than the first quadrant. In such cases, the line joining the values of the variables must be extended in to the first quadrant.

In optimization, we seek the minimum or maximum of a real – valued function over a subject of R^n . The subset of S of R^n over which the maximum or minimum of a given real – valued function *f* is to be found is usually of the form

$S = \{ x \in R^n : g_i(x) \leq b_i,\ i = 1, 2, 3, \ldots\ldots, p$

$g_i(x) = b_i,\ i = p + 1, p + 2, \ldots, m$

$g_i(x) \geq b_i \quad i = m + 1, m + 2, \ldots, q$

Where $g_i(x)$ is a real valued function bi is a real number for each *i*.

Such a problem can always be reduced to the following:

Minimize $f(x)$

Subject to

$g_i(x) \leq b_i \qquad i = 1, 2, 3, \ldots, p$

$g_i(x) = b_i \qquad i = p + 1, p + 2, \ldots, m$

$g_i(x) \geq 0 \qquad 1 \leq i \leq m$

The set $\boldsymbol{R}_{+}^{\boldsymbol{n}}$ = { x_1, x_2,……………, x_n} ∈ $\boldsymbol{R}_{\boldsymbol{n}}$: $x_i \geq 0$, $1 \leq i \leq n$ is called the non – negative octant. For $n = 2$, $\boldsymbol{R}_{+}^{\boldsymbol{n}}$ is called the first quadrant and for $n = 3$, $\boldsymbol{R}_{+}^{\boldsymbol{n}}$ is called the non – negative octant.

If there exist xi in the first problem which does not satisfy the condition $x_j \geq 0$, define the variables $\boldsymbol{x}_{\boldsymbol{j}}^{+}$ = max(0, x) and $\boldsymbol{x}_{\boldsymbol{j}}^{-}$ = min (0, x_j) , then $\boldsymbol{x}_{\boldsymbol{j}}^{+} \geq 0$, $\boldsymbol{x}_{\boldsymbol{j}}^{-} \geq 0$ and $\boldsymbol{x}_{\boldsymbol{j}}^{+} = \boldsymbol{x}_{\boldsymbol{j}}^{-}$. Replace x_j throughout the first problem by $\boldsymbol{x}_{\boldsymbol{j}}^{+} = \boldsymbol{x}_{\boldsymbol{j}}^{-}$. Do this for every variable xj not satisfying the condition, $x_j \geq 0$.

The result is a new problem containing at most 2^n variables which are non – negative. Hence we can take the general optimization problem in the form.

Minimize $f(x)$

Subject to

$$g_i(x) \leq b_i, \qquad 1 \leq i \leq p$$

$$g_i(x) = b_i, \qquad p + 1 \leq i \leq m$$

$$x_i \geq 0, \qquad 1 \leq i \leq n$$

The function $f(x)$ being minimized is called the objective function. The conditions are called the constraints of the problem. Constraints of the type $g_i(x) = b_i$ are called equality constraints and constraints of the type $g_i(x) \leq b_i$ are called inequality constraints. The constraints $x_i \geq 0$ are called non – negativity constraints.

The set of points in $\boldsymbol{R}^{\boldsymbol{n}}$ satisfying all the constraints of the problem is called the feasible region of the problem. Any point in the feasible region is called feasible point. Let x_o denote a feasible point. If an inequality constraint $g_i(x) \leq b_i$ is satisfied as equality at xo. i.e. $g_i(x) = b_i$, we say that the inequality constraint is binding or active at x_o. If the inequality is satisfied as a strict inequality. i.e.,$g_i(x) < b_i$, we say that the inequality constraint is inactive or not binding at x_o.

3.4.2 THE SIMPLEX METHOD FOR LP MODEL

When the problem is having more than two decision variables, simplex method is the most powerful method to solve the problem. It has a systematic programming, which can be used to solve the problem. However, the problem with two decision variables can be solved using both.

One problem with two variables is solved by using both graphical and simplex method, so as to enable the reader to understand the relationship between the two. Simplex method is an effective and a general procedure for solving linear programming problems.

The Simplex method is the name given to the solution algorithm for solving linear programming problems developed by George Dantzig in 1947. An example is an n – dimensional convex figure that has n + 1 extreme points. Simplex in two dimensions is triangle, and in three dimensions is a tetrahedron. The Simplex method refers to the idea of moving from one extreme point to another on the convex set that is formed by the constant set and non – negativity conditions of the linear programming problem. By solution algorithm we refer to an iterative procedure having fixed computational rules that leads to a solution to the problem in a finite number of steps (i.e. converges to an answer). The simplex method is algebraic in nature and is based upon Gauss – Jordan elimination procedure.

3.4.2.1 SETTING UP THE INITIAL SIMPLEX TABLEAU

In developing a tableau approach for the simplex algorithm, the terms that are used in the initial simplex tableau are defined as follows:

c_j = objective function coefficients for variable j

b_i = right – hand – side coefficients(value) for constraint i

a_{ij} = coefficients of variable j in constraint i

c_B = objective function coefficients of the basic variables

Notation that we will use extensively in the following development of the simplex method is as follows:

c row (the row of objective function coefficients); b row (the column of right – hand – side values of the constraint equations); [A] matrix (matrix with m rows and n columns of the coefficients of the variables in the constraint equations).

The table below shows the general form of simplex tableau

GENERAL FORM – INITIAL SIMPLEX TABLEAU												
		Decision variables					Slack Variables					
	$\boldsymbol{C_j}$	$\boldsymbol{C_1}$	$\boldsymbol{C_2}$	…	$\boldsymbol{C_n}$		0	0	…	0	Solution	(Objective function coefficients)
$\boldsymbol{C_B}$	Basic Variable	$\boldsymbol{x_1}$	$\boldsymbol{x_2}$	…	$\boldsymbol{x_n}$		1		…	$\boldsymbol{S_m}$		(Headings)
0	$\boldsymbol{S_1}$	$\boldsymbol{a_{11}}$	$\boldsymbol{a_{12}}$	…	$\boldsymbol{a_{1n}}$		1	0	…	0		Constraints coefficients)
…	$\boldsymbol{S_2}$	$\boldsymbol{a_{21}}$	$\boldsymbol{a_{22}}$	…	$\boldsymbol{a_{2n}}$		0	1	…	0		
0	…	…	…	…	…		…	…	…	…		
	$\boldsymbol{S_m}$	$\boldsymbol{a_{m1}}$	$\boldsymbol{a_{m2}}$	…	$\boldsymbol{a_{mn}}$		0	0	…	1		
		$\boldsymbol{Z_1}$	$\boldsymbol{Z_2}$	…	$\boldsymbol{Z_{mn}}$	…	$\boldsymbol{Z_{11}}$	$\boldsymbol{Z_{12}}$	…	$\boldsymbol{Z_{1m}}$	Current value of objective function	
	$\boldsymbol{c_j - Z_j}$	$\boldsymbol{c_1 - Z_1}$	$\boldsymbol{c_2 - Z_2}$	…	$\boldsymbol{c_{mn} - Z_{mn}}$	…	$\boldsymbol{c_{1_1} - Z}$	$\boldsymbol{c_{1_2} - Z_{1_2}}$	…	$\boldsymbol{c_{1_n} - Z_{1_n}}$		Reduced cost (Net contribution)

Table 3.1: General form of Simplex tableau

3.4.2.2 Revised simplex method

Another form of simplex method is the revised simplex which can be represented by matrix form with slack variables.

3.4.2.3 Matrix representation of the simplex table

Suppose that we have a LP model in the standard form:

Max $z = c_1x_1 + c_2x_2 + \text{`}+ c_nx_n$

Subject to

$$a_{11}x_1 + a_{12}x_2 +\ldots + a_{1n}x_n \le b_1$$

$$a_{21}x_1 + a_{22}x_2 +\ldots + a_{2n}x_n \le b_2$$

$$a_{m1}x_1 + a_{m2}x_2 +\ldots+ a_{mn}x_n \le b_m$$

and $x_1, x_2, ..., x_n \ge 0$

Its augmented form is

Max $z = c_1x_1 + c_2x_2 +\ldots + c_nx_n$

Subject to

$$a_{11}x_1 + a_{12}x_2 + \text{`}+ a_{1n}x_n + x_{n+1} = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \text{`}+ a_{2n}x_n + x_{n+2} = b_2$$

$$a_{m1}x_1 + a_{m2}x_2 + \text{`}+ a_{mn}x_n + x_{n+m} = b_m$$ and

slack variables

$x_1, x_2, ..., x_{n,}\ \ x_{n+1}, ..., x_{n+m} \ge 0$

Let

$$c = [\, c_1\ c_2\ \ldots\ c_n\ 0\ \ldots\ 0 \,]$$

$$A = \begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n} & 1 & 0 & 0 \\ a_{21} & a_{22} & \ldots & a_{2n} & 0 & 1 & 0 \\ & & & & & & \\ a_{m1} & a_{m2} & & a_{mn} & 0 & 0 & 1 \end{pmatrix}$$

$$x = \begin{pmatrix} x_1 \\ x_2 \\ \\ X_n \end{pmatrix}, \quad x_s = \begin{pmatrix} x_{n+1} \\ x_{n+2} \\ \\ x_{n+m} \end{pmatrix}, \quad b = \begin{pmatrix} b_1 \\ b_2 \\ \\ b_m \end{pmatrix}$$

Then the system (O) can be written as

Max z = cx

s.t. $Ax \leq b$

$x \geq 0$

The system (A) can be written as

Max z = cx

$$\text{s.t.} \quad [A\ I] \begin{bmatrix} \mathbf{x} \\ \mathbf{xs} \end{bmatrix} = b$$

$$\begin{bmatrix} \mathbf{x} \\ \mathbf{xs} \end{bmatrix} \geq \mathbf{0}$$

The first row of the initial table of the Simplex method can be written in the matrix form

$z - cx = 0$

or

$$[1 - c \;\; 0] \begin{bmatrix} z \\ x \\ xs \end{bmatrix} = 0$$

The initial table of the revised simplex method is

	Z	X_1	…	x_n	X_{n+1}	…	X_{n+m}	RHS
	1	$-c_1$	…	$-c_n$	0	…	0	0
X_{n+1}	0	a_{11}	…	a_{1n}	1	…	0	b_1
	0							
X_{n+m}	0	a_{m1}	…	a_{mn}	0	…	1	b_m

Table 3.2

can be represented by

$$\begin{bmatrix} 1 - c & 0 & 0 \\ 0 \;\; A & I & b \end{bmatrix}$$

which is equivalent to the matrix equation:

$$\begin{pmatrix} \mathbf{1} & -\boldsymbol{c} & \mathbf{0} \\ \mathbf{0} & \mathbf{A} & \mathbf{I} \end{pmatrix} \begin{bmatrix} \mathbf{z} \\ \mathbf{x} \\ \mathbf{xs} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{b} \end{bmatrix}$$

3.4.3 LP PRODUCTION PLANNING

Production problem involves a single product which is to be manufactured over a number of successive time periods to meet pre – specified demands. The transportation problem is to transport the products from the various sources to the various warehouses at minimum cost while satisfying all constraints of productive capacity and demands. Since both production cost and storage cost are known, the objective is to determine a production planning(schedule) which will meet all future demands at minimum total cost(which is inventory cost plus labour costs plus raw material cost plus shipping cost).

Production problem may be converted into transportation problems by considering the time periods during which production takes place at sources S_1, S_2, ……,S_m and the time periods in which units will be shipped to destinations(Depots). The production capacities at source S are taking to be the supplies a_1, a_2 ...a_m in given period i and the demands at the warehouse W_j is d_j.

Let C_{ij} be the production costs per unit during time period i plus the storage cost per unit from time period i until time period j. The problem is to find a production planning, which will meet all demands at minimum total cost, while satisfying all constraints of productive capacity and demands.

3.4.3.1 LP FOR TRANSPORTATION MODEL

Let X_{ij} denote the number of units to be produced during time period i from Si for shipment during time period j to W_j, i = 1,2…….n. Then Xij ≥0 for all i and j.

For each i, the total amount $\sum_{i=1}^{n} X_{ij}$

We consider a set of m supply points from which a unit of the product is produced. But since supply point i can supply at most a_i, units in any given period.

We have $$\sum_{j=1}^{n} X_{ij} \leq a_i$$

$i = 1,2,\ldots,m$ (Supply constraints)

We also consider a set of n demand points to which the product is shipped. Since demand points j must receive at least d_j units of the shipped products.

We have $$\sum_{i=1}^{m} X_{ij} \leq d_j$$

$j = 1,2,\ldots,n$ (Demand constraints)

Since units produced cannot be shipped prior to being produced, C_{ij} is prohibitively large for $i>j$ to force the corresponding X_{ij} to be zero or if shipment is impossible between a given source and destination, a large cost of M is entered.

The total cost of production is given as

$$\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}X_{ij}$$

The general formulation of a production problem is:

Minimize

$$\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}X_{ij}$$

Subject to

$$\sum_{j=1}^{n} X_{ij} \leq a_i$$

$i = 1, 2, \ldots, m$ (Supply constraints)

$$\sum_{i=1}^{m} X_{ij} \leq d_j$$

$j = 1, 2, \ldots, n$ (Demand constraints)

$X_{ij} \geq 0$, $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$

3.4.3.2 The balanced problem

From the supply and demand constraints, if

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} d_j$$

Then total supply equals total demand and the problem is said to be balanced production problem.

The special algorithm works well for the balanced problem. Therefore, the unbalanced problem can always be modeled as an equivalent balanced problem to which the special method can be applied. In a balanced production problem, all the constraints must be binding. If any supply constraints were not binding, then the remaining available products would not be sufficient to meet all demands. Thus, the balanced production problem may be written as:

Minimize

$$\sum_{i=1}^{m}\sum_{j=1}^{n} \boldsymbol{C_{ij}X_{ij}}$$

subject to: $\sum_{j=1}^{n} X_{ij} = a_i \qquad i = 1, 2, \dots, m \quad$ (Supply constraints)

$$\sum_{i=1}^{m} X_{ij} = d_j \qquad j = 1,2, \dots, n \quad \text{(Demand constraints)}$$

Xij ≥ 0, I = 1,2,…, m; j = 1,2,…, n.

3.4.3.3 The Modified Distribution Method (MODI):

The formulation above is solved using a method known as the Modified Distribution Method (MODI). An Initial Basic Feasible Solution (IBFS) is required before the application of the MODI. The IBFS can be obtained by the Northwest corner rule, Vogel's approximation method and the least cost method. MODI aids in obtaining the optimal solution and is established by the following theorem.

Theorem: The theorem states that if we have a basic feasible solution (B.F.S.) consisting of (m+n–1) independent positive allocations and a set of arbitrary numbers *ui* and *vj* (*j* = 1,2 ,..., *n*, *i* = 1,2 ,..., m) such that *Crs* = *ur*+*vs* for all occupied cells (basic variables) then the evaluation corresponding to each empty cell (non basic variables) (*i*, *j*) is given by:

cij = cij − (ui + v j

It is relatively simple to find a basic feasible solution from a balanced production problem. Also, simplex pivots for these problems do not involve multiplication, but reduced to additions and subtraction. For these reasons, it is desirable to formulate a production problem as a balanced production problem. The balanced production problem is specified by the supply, demand and production cost, so the relevant data can be represented in a parameter table in the format of Table 3.1. Since this is a minimization problem, the numbers in the upper right corners in each cell is the unit cost, not revenue. The quantities produced are shown on the right rows whiles demand is shown along the columns.

Table 3.3 The Transportation tableau

Destination

Source	W_1	W_2			W_n	Supply
S_1	C_{11} X_{11}	C_{12} X_{12}	C_{13} X_{13}		C_{1n} X_{1n}	a_1
S_2	C_{21} X_{21}	C_{22} X_{22}	C_{23} X_{23}		C_{2n} X_n	a_2
S_3	C_{31} X_{31}	C_{32} X_{32}	C_{33} X_{33}		C_{3n} X_{3n}	a_3
S_m	C_{m1} X_{m1}	C_{m2} X_{m2}	C_{m3} X_{m3}		C_{mn} X_{mn}	a_m
Demand	d_1	d_2				

It is observed that:

The coefficient of each variable X_{ij} in each constraint is either 1 or 0

The constant on the right hand side of each is an integer

The coefficient matrix A has a certain pattern of 1's and 0's

It can be shown that any linear programming problem with these properties above satisfies the following: thus, if the problem has a feasible solution, then there exist feasible solutions in which all the variables are integers. It is this property on which the modification of the simplex method that provides efficient solution algorithms is based.

The (m + n) conditions;

$$\sum_{j=1}^{n} X_{ij} = a_i \qquad\qquad i = 1,2,\dots,m$$

$$\sum_{i=1}^{m} X_{ij} = d_j \qquad\qquad j = 1,2,\dots,n$$

are dependent since

$$\sum_{i=1}^{m} a_i = \sum_{i=1}^{m}\sum_{j=1}^{n} X_{ij} = \sum_{j=1}^{n} d_j$$

Thus the effective number of constraints on the balanced production problem is (m+n-1). We therefore expect a basic feasible solution of the balanced production problem to have (m+n-1) non-negative entries.

3.4.3.4 The unbalanced problem

Considering the following production problem:

Minimize

$$\sum_{i=1}^{m}\sum_{j=1}^{n} \boldsymbol{C_{ij}X_{ij}}$$

Subject to:

$$\sum_{j=1}^{n} \boldsymbol{X_{ij}} \leq \boldsymbol{a_i} \qquad i = 1,2, \dots, m \ \text{(supply constraints)}$$

The unbalanced problem occurs when

$$\sum_{i=1}^{m} \boldsymbol{a_i} \neq \sum_{j=1}^{n} \boldsymbol{d_j}$$

3.4.3.5 Balancing a production problem if total supply exceeds total demand

If total supply exceeds total demand,

$$\sum_{i=1}^{m} \boldsymbol{X_{ij}} \leq d_j \qquad j = 1,2, \dots, n \ \text{(Demand constraints)}$$

$$X_{ij} \geq \mathbf{0}, \qquad i = 1,2,\dots,m;\ j = 1,2,\dots,n$$

We can balance a production problem by creating a dummy demand point W_F that has a demand equal to the amount of excess supply. Since unit cost of each source or shipments to the dummy demand point are not real shipments, they are assigned a cost of zero. Shipments to the dummy demand point indicate an unused supply capacity. This is illustrated in the table below.

Table 3.4 The unbalanced Transportation tableau

Source	W_1	W_2		W_n	Dummy W_F	Supply
S_1	C_{11}	C_{12}		C_{1n}	0	a_1
S_2	C_{21}	C_{22}		C_{2ns}	0	a_2
S_m	C_{m1}	C_{m2}		C_{mn}	0	a_m
Demand	d_1	d_2				

3.4.3.6 Balancing a production problem if total supply is less than total demand

If total supply is less than total demand, we balance the production problem by creating a factious source Sp whose capacity is strictly the excess of demand over supply and that the unit cost from source to every warehouse is zero. This is illustrated in Table 3.5:

Table 3.5 Transportation tableau

Destination

Source	W_1	W_2	W_n	W_F	Supply
S_1	C_{11}	C_{12}		C_{1n}	a_1
S_2	C_{21}	C_{22}		C_{23}	a_2
S_m	C_{m1}	C_{m2}		C_{mn}	a_m
	0	0		0	
Demand	d_1	d_2		d_n	

3.4.3.7 Finding an initial basic feasible solution

Three methods can be used to find the initial basic feasible solution for a balanced transportation can be used to find the initial basic feasible solution for a balanced transportation problem.

These are;

(i) Northwest Corner Methods

(ii) Minimum or least cost methods

(iii)Vogel's Approximation methods

The solution obtained under each of the three methods is not optimal

3.4.3.8 The North West Corner Rule for finding an initial basic feasible solution

In this method, we choose the entry in the upper left hand corner (Northwest corner) of the transportation tableaus, i.e the shipment from source 1 to warehouse (depot) 1. Use this to supply as much of the demand at W_1 as possible. Record the shipment with a circled number in the cell. If the supply at S_1 is not used up by the allocation use the remaining supply to fill the remaining demands at W_2, W_3… In that order until supply at S_1 is used up, record all shipments in circles in appropriate cells. When one supply is used up, go to the next supply and start filling the demands beginning with the first warehouse in that row, where there is still a demand unfilled, recording in circled numbers all allocations.

In certain cases, a degenerate situation arises and the solution is not a BFS because it has fewer than (m+n-1) cells in the solution. This occurs because at some point during the allocation when a supply is used up, there is no cell with unfulfilled demand in the column. In the non-degenerate case, until the end, whenever a supply is used up, there is always an unfulfilled demand in the column.

The northwest corner method still yields a BFS even in the case of degeneracy, if it is modified as follows: having obtained a solution which is not basic, choose some empty cells and add the solution with circled zeros in them to produce a BFS, i.e,

The total number of cells with allocations should be (m+n-1)

There should be no circuits among the cells of the solution.

3.4.3.9 Least Cost Method

The Northwest Corner method does not utilize production cost per unit, so it can yield an initial basic feasible solution that has a very high production cost. Then determining an optimal solution may require several pivots or iterations. The minimum cost method used the production costs in an effort to produce a basic feasible solution that has a lower total cost. Fewer pivots or iterations will then be required to find the problem's optimal solution. Under the Minimum Cost Method, we find the variable with the smallest or least unit cost (call it X_{ij}). Then assign X_{ij} the largest possible value, min (a_i d_j). Cross out row i and column j and reduce the supply or demand of the non-crossed out row or column by the value of X_{ij} next minimum unit cost and repeat the procedure, without violation any of the supply and demand constraints.

We continue until there is only one cell that can be chosen. In this case, cross out both cell's row and column. With the exception of the last variables with small costs to be basic variables, it does not always yield a basic feasible solution with a relatively low total production cost. In relatively high shipping cost. Thus the Minimum Cost Method would yield a costly basic feasible solution. Vogel's Approximation Method for finding a basic feasible solution usually avoids extremely high shipping cost.

3.4.4.0 Vogel's approximation method

Begins by computing for each row and column, a "penalty" equal to the difference between the smallest costs in the row and column and the second smallest. Row penalties are shown along the right of each row and column penalties are shown below each column. Next we find the row or column with the largest penalty. The method is a variant of the least cost method and based on the idea that if for reason, the allocation cannot be made to the least unit cost cell via row or column then, it is made to the next least cost cell in that row or column and the appropriate penalty paid for not being able to make the best allocation. We choose the cell with the greatest row and column penalties. Allocate as much to this cell as the row supply or column demand will allow. This means either a supply is exhausted or a demand is satisfied. In either case, delete the row of the exhausted supply or the column of the satisfied demand. We re-compute new penalties (using only cells that do not lie in a crossed-out row or column), and repeat the procedure until only one uncrossed cell remains. We set this variable equal to the supply or demand associated with the variable, and cross out the variable's row and column, A BFS has now been obtained.

The Vogel's Approximation Method provides a BFS which is close to optimal or is optimal and this performs better than the Northwest Corner or the Minimum Cost Method. Vogel's Approximation Method may lead to an allocation with fewer than (m+n-1) non-empty cells even in the non-degenerate case. To obtain the right number of cells in the solution, we add enough zero entries to empty cells, avoiding the generation of circuits among the cells in the solution.

Of the three methods discussed, the Northwest Corner Method requires the least effort and Vogel's method requires the most effort. Extensive research (Glover et al, 1974) has shown that when Vogel's method is used to find an initial basic feasible solution, it usually takes substantially fewer pivots or iterations than the other two methods. For this reason, the Northwest Corner and the Minimum or Least Cost methods are rarely used to find a basic feasible solution to a large production problem.

3.4.4.1 Improving solution to optimality

The solutions obtained under the three methods are feasible, but not optimal. To obtain optimality, we improve on these solutions using two methods. These are:

The steppingstone method

The Modified Distribution method (MODI)

3.4.4.2 The steppingstone method

Suppose that we have a basic feasible solution, consisting of non-negative allocations in (m+n-1) cells, we cells the cells which are not in the basic feasible solution unoccupied cells. Then for each unoccupied cell, a unique circuit begins and ends the cell, consisting of that unoccupied cell and other cells all of which are occupied such that each row or column in the tableau either contains two or none of the cells of the circuit.

3.4.4.3 Test for optimality

To test the current basic feasible solution for optimality, we take each of the unoccupied cells in turns and place one unit allocation in it. This is indicated by just the sign “-“ and “+”. Following the unique circuit containing this cell as described above place alternately the signs “-“ and “+” until all the cells of the circuit are covered. Knowing the unit cost of each cell, we compute the total change in cost produced by allocation of one unit in the empty cell and the corresponding placements in the other cells of the circuit.

This change in cost is called improvement index of the unoccupied cell. If the improvement index of each unoccupied cell in the given basic feasible solution is nonnegative then the current basic feasible solution is optimal since every reallocation increases the cost. If there is at least one unoccupied cell with a negative improvement index then a reallocation to produce a new basic feasible solution with a lower cost is possible and so the current basic feasible solution is not optimal. Thus the current basic feasible solution with a lower cost is possible and so the current basic feasible solution is not optimal. Thus the current basic feasible solution is optimal if and only each unoccupied cell has a non-negative improvement index.

3.4.4.4 Improvement to optimality

If there exists at least one unoccupied cell in a given basic feasible solution which has a negative improvement index, then, the basic feasible solution is not optimal.

To improve on this solution, we find the unoccupied cell with the most negative improvement index of say N using the circuit that was used in the calculation of its improvement index, find the smallest allocation in the cells of the circuit with sign "-". Call this smallest allocation k. subtract k from the allocations in circuit with the sign"-" and add it to all the allocations in the cells in the circuit with the sign "+". This has the effect of satisfying the constraints on demand and supply in the transportation tableau. Since the cell which carried the allocation k now has a zero allocation, it is deleted from the solution and is replaced by the cell in the circuit which was originally unoccupied and now has an allocation k. The result of each reallocation is new basic feasible solution. The cost of this new basic feasible solution in

N is less than the cost of the previous basic feasible solution. This new basic feasible solution is tested for optimality and the whole procedure repeated until an optimal solution is attained

3.4.4.5 The modified distribution method (MODI)

Consider the balanced production problem below:

Table 3.6 Balanced Transportation Tableau

Warehouse

Source	W_1	W_2	W_n	W_F	Supply
S_1	C_{11} X_{11}	C_{12} X_{12}		C_{1n} X_{1n}	a_1
S_2	C_{21} X_{21}	C_{22}		C_{2n} X_{2n}	a_2
S_3	C_{31} X_{31}	C_{32} X_{32}		C_{3n} X_{3n}	a_3
Sm	C_{m1} X_{m1}	C_{m2} X_{m2}		C_{mn} X_{mn}	a_m
Demand	d_1	d_2		d_n	

Assuming an initial basic feasible solution is obtained. Then (m+n-1) cells are occupied.

3.4.4.6 Test for optimality

For each occupied cell (i, j) of the transportation tableau, compute a row index U_i, and column index V_j such that $C_{ij} = U_i - V_j$. Since there are (m+n-1) occupied cells, it follows that there are (m+n-1) of these equations. Since there are (m+n) row and column indices altogether, it follows that by prescribing any arbitrary value for one of them, say U = 0, then the equations are solved for the remaining (m+n-1) unknowns U_i, V_j. With all the U_i, V_j known, we compute for each unoccupied cell such that the evaluation factor $e_{st} = C_{st} - u_s - v_t$

It can be shown that the evaluation factors are the relative cost factors corresponding to the non-basic variables when the simplex method is applied to the transportation problem. Hence the current basic feasible solution is optimal if and only if $e_{st} \geq 0$ for all unoccupied cells (s,t), since the production problem is a minimization problem. If there are unoccupied cells with negative evaluation factors, then current basic feasible solution is not optimal and needs to be improved.

3.4.4.7 Improvement to optimality

To improve the current non-optimal basic feasible solution we find the unoccupied cell with the most negative evaluation factor, construct its circuit and adjust the values of the allocation in the cells of the circuit in exactly the same way as was done in the steppingstone method. This yields a new basic feasible solution. With a new basic feasible solution available, the whole process is repeated until optimality is attained.

REMARKS

The fact the circuit is not constructed for every unoccupied cell makes the modified distribution Method (MODI) more efficient than the steppingstone method. In fact the Modified Distribution Method (MODI) is currently the most efficient method of solving the production problem.

3.5. Basic Assumptions of LP

The following are some important assumptions made in formulating a linear programming model:

(i). It is assumed that the decision maker here is completely certain (*i.e.*, deterministic conditions) regarding all aspects of the situation, *i.e.*, availability of resources, profit contribution of the products, technology, courses of action and their consequences etc.

(ii). It is assumed that the relationship between variables in the problem and the resources available i.e., constraints of the problem exhibits linearity. Here the term linearity implies proportionality and additivity. This assumption is very useful as it simplifies modeling of the problem.

(iii). We assume here fixed technology. Fixed technology refers to the fact that the production requirements are fixed during the planning period and will not change in the period.

(iv). It is assumed that the profit contribution of a product remains constant, irrespective of level of production and sales.

(v). It is assumed that the decision variables are continuous. It means that the companies manufacture products in fractional units. For example, company manufacture 2.5 vehicles,

(vi). Non – integer values of decision variables are accepted. This is referred to as the assumption of divisibility.

(vii). It is assumed that only one decision is required for the planning period. This condition shows that the linear programming model is a static model, which implies that the linear programming problem is a single stage decision problem. (Note: Dynamic Programming problem is a multistage decision problem).

(viii). All variables are restricted to nonnegative values (i.e., their numerical value will be ≥ 0).

3.6 Summary

In this chapter, we considered the modeling of the production problem into graphical method, simplex methods and transportation model for easy computation.

The next chapter will focus on analyzing the expected demands, inventory and regular production and overtime capacities for the year. Here, LP Solver will then be used to find the optimal production planning.

CHAPTER FOUR

DATA COLLECTION AND ANALYSIS

4.0 Introduction

This chapter presents data collection and analysis of the study.

Aspect Water Company Limited produces and sells innovative, high quality and consumable sachet water products. AWCL supplies sachet water to its customers in the Brong Ahafo Region and the nation at large. The company produces ‘pure water’ based on orders from its registered customers and other retailers.

The quantity of sachet water produced per day depends on the number of workers at the production room and raw materials available. These jobs often have to be processed on the machines in a production room.

Unexpected events on the shop floor, such as machine breakdowns, reduction of human workforce due to sickness or absenteeism has to be taken into consideration, since they may reduce the quantity to be produced in a day. The variables are inventory (x_1), raw materials (x_2), the regular time labour (x_3) , overtime labour (x_4) and transportation (x_5).

Detailed planning (scheduling) of tasks to be performed helps maintain efficiency and control of operations. This chapter will focus on computational procedure and data analysis (implementation of proposed models, solution to the models using LP solver package), findings and discussions.

4.1 Computational Procedure and Data Analysis

Tables 4.1, 4.2 and 4.3 show the company's production capacity (regular and overtime) and expected demands (in bags) for sachet water from January – December, 2011.The variable quantities for production and the production cost for each variable.

Table 4.1: Expected demand and capacity of sachet water for the year, 2011

Month	Sachet Water Demand (bags)	Regular Time shift Capacity (bags)	Overtime Shift Capacity (bags)
January	15000	6933	3467
February	15000	6667	3333
March	15000	6934	3466
April	14334	6915	3455
May	14034	6934	3436
June	14167	6778	3392
July	15000	7000	3500
August	15000	6895	3445
September	14834	6934	3466
October	14500	6933	3467
November	14667	6912	3458
December	14367	6933	3467

Source: Aspect Water Company Ltd

Table 4.2: The production quantity of sachet water for the year, 2011

Month	Jan	Feb	March	April	May	June	July	Aug	Sept	Oct	Nov	Dec
Inventory	10000	10100	10000	10000	10000	10000	11200	10000	12000	11000	11000	11200
Raw material	1000	1100	1000	1120	1000	1150	1120	1000	1200	1000	1120	1300
Regular labour	2080	1950	1976	2080	1950	2028	2080	2080	2080	1950	1820	2080
Overtime labour	1040	1040	1040	1040	988	910	1040	1040	1040	1040	1040	1040
Transportation	10400	10000	10400	10370	10400	10170	10500	10340	10370	10400	10370	10400

Source: Aspect Water Company Ltd

Table 4.3. The production cost in (Gh¢) for the year, 2011

Month	Jan	Feb	March	April	May	June	July	Aug	Sept	Oct	Nov	Dec
Inventory	550	600	550	560	550	600	700	580	650	600	610	650
Raw material	8000	8100	8500	8960	8500	9200	9960	8800	10560	9000	10080	11700
Regular labour	31200	31200	31200	31200	31200	31200	31200	31200	31200	31200	31200	31200
Overtime labour	15600	15600	15600	15600	15600	15600	15600	15600	15600	15600	15600	15600
Transportation	71760	71760	71760	71760	71760	71760	71760	71760	71760	71760	71760	71760

Source: Aspect Water Company Ltd

The production takes place at both regular and overtime shifts for each of the twelve months. Since the demand for sachet water each month is greater than the supply, each of these months is a source. The inventory at the storage served as work in progress (WIP). The company works with maximum number of one hundred and twenty workers a day.

It was clear that the company incurred the total production costs from the following units: Regular unit cost of GH¢0.30, overtime unit cost of GH¢0.15, raw material unit cost of, GH¢0.0897, inventory unit cost of GH¢0.0058 and transportation unit cost of GH¢0.69 giving the total production cost of GH¢1.2355 per bag for producing 124,150 bags of sachet water for the one year period. The company sells a bag of sachet water to its customers for GH¢1.50.

The production problem is modelled as the Linear Programming model in order to minimize the total cost of production whilst satisfying the demand.

4.1.1 Implementation of Model

We use the imperial data in Table 4.2 to implement the Proposed LP model formulated in chapter three.

The proposed model involves the planning(scheduling) formulation taking into account the unit cost of production, C_{ij} , the supply at a_i at source S_i and the demand d_j at destinations(depots) for $i \in (1,2,\ldots,12)$. The problem is:

$$\text{Minimize } Z = \sum_{j=1}^{n} C_{ij} X_{ij}$$

Subject to:

$$\sum_{j=1}^{n} a_{ij} x_{ij} \leq b_i$$

$$Xj \geq 0, \quad j = 1,2,\ldots,n$$

The problem is formulated as:

Minimize $Z = 0.0058x_1 + 0.0897x2 + 0.30x_3 + 0.15x_4 + 0.69x_5$

Subject to:

$$10000x_1 + 1000x_2 + 2080x_3 + 1040x_4 + 10400x_5 \leq 15000$$

$$10100x_1 + 1100x_2 + 1950x_3 + 1040x_4 + 10000x_5 \leq 15000$$

$$10000x_1 + 1000x_2 + 1976x_3 + 1040x_4 + 10400x_5 \leq 15000$$

$$10000x_1 + 1120x_2 + 2080x_3 + 1040x_4 + 10370x_5 \geq 14334$$

$$10000x_1 + 1000x_2 + 2080x_3 + 988x_4 + 10400x_5 = 14034$$

$$10000x_1 + 1150x_2 + 1950x_3 + 910x_4 + 10170x_5 \leq 14167$$

$$11200x_1 + 1120x_2 + 2080x_3 + 1040x_4 + 10500x_5 \leq 15000$$

$$10000x_1 + 1000x_2 + 2080x_3 + 1040x_4 + 10340x_5 \leq 15000$$

$$12000x_1 + 1200x_2 + 2080x_3 + 1040x_4 + 10400x_5 \geq 14834$$

$$11000x_1 + 1000x_2 + 1950x_3 + 1040x_4 + 10400x_5 \geq 14500$$

$$11000x_1 + 1120x_2 + 1820x_3 + 1040x_4 + 10370x_5 = 14667$$

$$11200x_1 + 1300x_2 + 2080x_3 + 1040x_4 + 10400x_5 \geq 14367$$

4.1.2 Solution of Production Planning Model

LP solver was used to find the solution of the planning model. LP solver is a windows package which can be used to obtain the optimal solution to production planning problem. It is an optimization package intended for solving linear, integer and other programming problems. LP solver is based on the efficient implementation of the modified Simplex method that solves large scale problems.

Before using the LP solver, an initial table is created. This is found in Appendix III. Each cell in the table contains the objective cost, the quantities of inventory, raw materials, regular labour, overtime labour, transportation (supply) and the demand. The slack variables occupy the remaining cells in the table. For a solution to the production problem to exist, the total demand should be equal to the total supply. The production for each month was used as the supply to satisfy the demand from the customers. The production at AWCL varied in twelve months period. There are eight (8) iterations generated by the software for which the feasible solution was found after six (6) iterations. The initial and the final tableaus are shown in Appendix III and IV.

The total production output according to Table 4.5 is 173788 and the total demand is 175903. Since the total supply is less than total demand, dummy supply of 49638(i.e. 173788 - 124150) is created to balance the production problem. The optimal solutions to the problem are shown in Table 4.5.

Table 4.5. Optimal Solutions generated by LP solver

Decision variable	Solution Variable	Unit Cost	Total contribution	Reduced Cost
Inventory (x_1)	0.508605	0.0058	0.002949909	0
Raw material (x_2)	2.23042	0.0897	0.2000686774	0
Regular labour (x_3)	0.675403	0.30	0.2026209	0
Overtime labour (x_4)	0.869185	0.15	0.13037775	0
Transportation (x_5)	0.428263	0.69	0.29550147	0

Min (Z) = 0.831519

4.1.3 Interpretation of Results

From the Table 4.5, the decision variables are the inventory (x_1), raw material (x_2), regular labour (x_3), overtime labour (x_4) and transportation (x_5). $x_1 = 0.508605$, $x_2 = 2.23042$, $x_3 = 0.675403$, $x_4 = 0.869185$ and $x_5 = 0.428263$.

The optimal solution of the production problem is given by:

$0.0058(0.508605) + 0.0897(2.23042) + 0.30(0.675403) + 0.15(0.869185) + 0.69(0.428263) = 0.831519$. Thus the cost of producing a bag of sachet water is GH0¢0.831519 developed by the model and cost incurred by the company is GH1¢1.2355 per bag.

The monthly production output and demands generated by LP solver are also shown in the Table 4.6.

Table 4.6: The optimal constraints (values) generated by LP Solver

Month	Quantity Supplied	Quantity Demanded	Surplus	Shortage
January	14079	15000	-	921
February	14094	15000	-	904
March	14009	15000	-	991
April	14334	14334	-	-
May	14034	14034	-	-
June	14115	14167	-	52
July	15000	15000	-	-
August	14054	15000	-	946
September	15543	14834	709	-
October	14500	14500	-	-
November	14667	14667	-	-
December	15359	14367	992	-

From Table 4.6, the company had shortages in January, February, March, June, and August. However, there was some surplus in September and December.

4.2. Changing the coefficients of Regular labour (x_3) and Overtime labour (x_4) in the objective function.

When the coefficients of the variables x_3 and x_4 of the objective function were changed the optimal solutions are shown in the table below.

Table 4.7: The optimal solution generated by the LP Solver

Decision Variable	Solution Variable	Unit Cost	Total contribution	Reduced Cost
Inventory (x_1)	0.508605	0.0058	0.002949909	0
Raw material (x_2)	2.23042	0.0897	0.200068674	0
Regular labour (x_3)	0.675403	0.35	0.23639105	0
Overtime labour (x_4)	0.869185	0.10	0.0869185	0
Transportation (x_5)	0.428263	0.69	0.29550147	0

Min (Z) = 0.821830

From the Table 4.7, the optimal solution given by the model when the variables x_3 and x_4 are perturbed. That is, 0.0058(0.508605) + 0897(2.23042) + 0.35(0.675403) + 0.10(0.869185) + 0.69(0.428263) = 0.821830.

The monthly production output and demands generated by LP solver were not changed when the coefficients of x_3 and x_4 were changed but there was significant changed in the cost of production as shown in the Table 4.7.

4.2.1: Sensitivity Analysis of the Proposed Model

Sensitivity (or post – optimal) analysis of the proposed model allows us to observe the effect of changes in the parameters of the LP problem on the optimal solution.

The following table shows the post – optimal solution when the coefficients of the objective function changed.

Table 4.8: Post – Optimal Solution generated by LP Solver

Decision Variable	Current Cost	Min Cost	Max Cost	Str Vector
Inventory (x_1)	0.0058	0.0058	0.0058	0
Raw material (x_2)	0.0897	0.0897	0.0897	0
Regular labour (x_3)	0.35	0.269792	0.425679	0.35
Overtime labour (x_4)	0.10	0.0770835	0.121622	0.10
Transportation (x_5)	0.69	0.69	0.69	0

Scale Factor Range: Min = - 0.229165 Max = 0.216224

4.2.2 Interpretation of Results

From the Table 4.8, changing the unit costs of regular labour and overtime labour gave the costs of 0.269792 and 0.0770835 respectively. This means the company could have minimized the cost further if the costs of regular and overtime labour were changed.

4.3. Findings

All constraints and optimality conditions were satisfied and a solution was found after eight (8) iterations. From Tables 4.6 and 4.8, the total production in September and December were greater than the demands. The company produced more than what was demanded from the customers. The higher quantities produced by AWCL in those months were as a result of large quantity of raw materials available for production. The months January, February, March, June, and August products were less than their demands but greater than the supplies. For the months April, May, July, October and November the demands were satisfied by producing the same quantities. Comparing the months April and June, the quantity of raw materials for production were in June was larger than that in April but the production output in April was bigger than that in June. This is because the number of workers employed in April was larger than that in June.

Thus the quantity of water to be produced by AWCL depends on the available raw materials and the number of labourers.

The optimal solution computed gave the total cost of production by GH¢0.831519.Thus: 0.0058(0.508605) +0.0897(2.23042) + 0.30(0.675405) + 0.15(0.869185) + 0.69(0.428263) = 0.831519.

When the coefficients of the unit costs of the regular labour and Overtime labour (i.e. regular labour unit cost of GH¢0.30 changed to GH¢0.35, overtime labour unit cost of GH¢0.15 changed to GH0.10) were changed, their total contributions were also changed but that of inventory, raw materials and transportation costs remained unchanged as shown in table 4.7.

The optimal solution generated by LP Solver during perturbation analysis gave the minimum cost of GH¢ 0.821830.

The total contribution for inventory is 0.002949909 which gives the same value when perturbed. Raw materials total contribution from the analysis is 0.2000686774 which has the same value during perturbation analysis. Regular time labour total contribution shows tremendous change in value when perturbed (0.2026209 is the contribution and 0.23639105 is the perturbation result). Overtime labour contribution from the analysis 0.13037775 and when perturbed gave the value 0.0869185.

That means when the wages of overtime labour were reduced and the regular labour increased, the cost of producing a bag of sachet water would reduced.

The optimal solution generated by the LP Solver gives the total minimum cost for production thereby increasing the total production output for the company.

4.4. Summary

This chapter focused on the computational procedure and analysis of data, implementation of proposed model, solution to the models using LP solver Package, Sensitivity analysis and the findings.

The next chapter talks about discussions, conclusions and recommendations.

CHAPTER FIVE

SUMMARY CONCLUSIONS AND RECOMMENDATIONS

5.0. Introduction

Aspect Water Company Ltd is the largest producer of sachet water in the Brong Ahafo Region which serves as the source of safe drinking water for human consumption. The company's percentage profit after sales of one bag of sachet water is 21.40% which is smaller than the expected profit from the production. Though the company makes a slight gain after sales, it can still reduce the cost of production.

All production firms aim at maximizing profit after sales of their products but due to lack of technological and scientific approach in the production setting, many cannot achieve them.

5.1 Summary of main findings

This study showed the trend of production of sachet water at AWCL in Table 4.6 which gave the quantity of sachet water produced in each month for the year, 2011.the cost of production has been reduced to the minimum. The major objective of this study is to minimize the total cost of production at AWCL using Linear Programming model.

The secondary data was collected on monthly production capacities and demands of sachet water from customers in bags. The data was formulated and analysed as Linear Programming model. The optimal solution to the production planning problem was generated by LP Solver. The demand and supply at each month were determined using the LP solver.

The AWCL incurs cost of GH¢1.2355 when producing a bag of sachet water but with the use of linear Programming model, the cost of producing a bag of water was reduced to GH¢0.831519. The analysis also showed that, increasing the wages of regular workers and reduce that of overtime helps the company to produce more with minimum cost of production.

The use of the model showed how the monthly production should be done in order to reduce the total cost of production. It also showed the number of bags of sachet water the company could have produced and supplied to the customers to satisfy the monthly demands.

The following were the production output for AWCL for the year, 2011 generated by LP Solver.

(i). 14079 bags of sachet water were used to satisfy the demand for January.

(ii). 14094 bags of sachet water were supplied to the customers in February.

(iii). In March, 14009 bags were produced instead of 15000 bags demanded.

(iv). The AWCL was able to meet the demand of 14334 bags of sachet water in April.

(v). 14034 bags were supplied to the customers in May. Here, the company was able to satisfy the demand from the customers.

(vi). 14115 bags were produced to clear the demand in June.

(vii). 15000 bags of sachet water were produced by AWCL to satisfy the demand of 15000 bags in July.

(viii). In August, the demand from the customers was 15000 but the company supplied 14054 bags.

(ix). The demand for water in September was the highest production by the company. The company supplied 15543 bags of sachet water to satisfy the demand of 14834 bags.

(x). The demand in October was also met by the company. The supply was 14500 bags.

(xi). There were14667 bags of sachet water were produced to balance the demand in November.

(xii). The month December was the second highest production by the company. The supply was greater than the demand from the customers. The supply was 1535 bags and the demand was 14367 bags of sachet water.

5.2 Discussions and Conclusions

The total production cost for the company was GH¢214715.074 and the minimum total cost of production cost from the findings gave GH¢144508.024, the percentage reduction of 32.70%. From the findings, the Aspect Water Company Limited could have reduced the total production by GH¢ 0.403981(32.70%) gone by the model.

The cost of overtime labour was higher as expected. The company could maximize total profit after sales of its products if it reduces the number of workers for overtime labour and increases the wages of workers engaged in regular production, therefore ensures optimum utilization of human and plant capacities that would bring about some savings for the company thereby reducing the cost of labour to the minimum. AWCL should employ more overtime labour when it is necessary to meet the urgent demands from the customers.

Instead of employing more manual labour force, the company could have used machinery that can do assembling and packaging of the sachet water.

Orders from customers could have been increased in April, May, September, October and December if proper communication has been done. The use of ICT for processing information from the customers helps to ascertain the required orders from the regular customers and also to study the market trend before production. This prompts the company to increase or reduce its production (That is, when and what to produce) satisfy the demands.

Computer – based planning (scheduling) help the manufacturers to attend to orders from their respective customers easily and to enhance on – time delivery of products.

The computerized planning performs better and faster than manual scheduling tools.

From the findings, it is clear that the efficient planning (scheduling) could have reduced production cost whilst satisfying the demands from the customers.

The analysis showed that the production planning can facilitate the production processes in way that help the company to streamline the activities that go on during acquisition of raw materials for production.

The use of technology in the manufacturing and production industries helps the companies or firms to produce more with minimum cost.

5.3 Recommendations

Linear Programming models solve all the production planning problems by increasing the production capacities, minimizing the cost and hence maximizing profits in the production industries and firms.

It is recommended that production companies, including AWCL should incorporate the Linear Programming model in their production.

In line with the findings of this research, Sachet Water Companies should adopt ICT devices to enhance their performance. The expertise should be mandated to train the existing worker (especially those at the lower level) on the job – shop for maximum productivity and also to face the challenges of new technology.

Also, AWCL should invest more on ICT capital and ICT labour in order to enhance their profitability.

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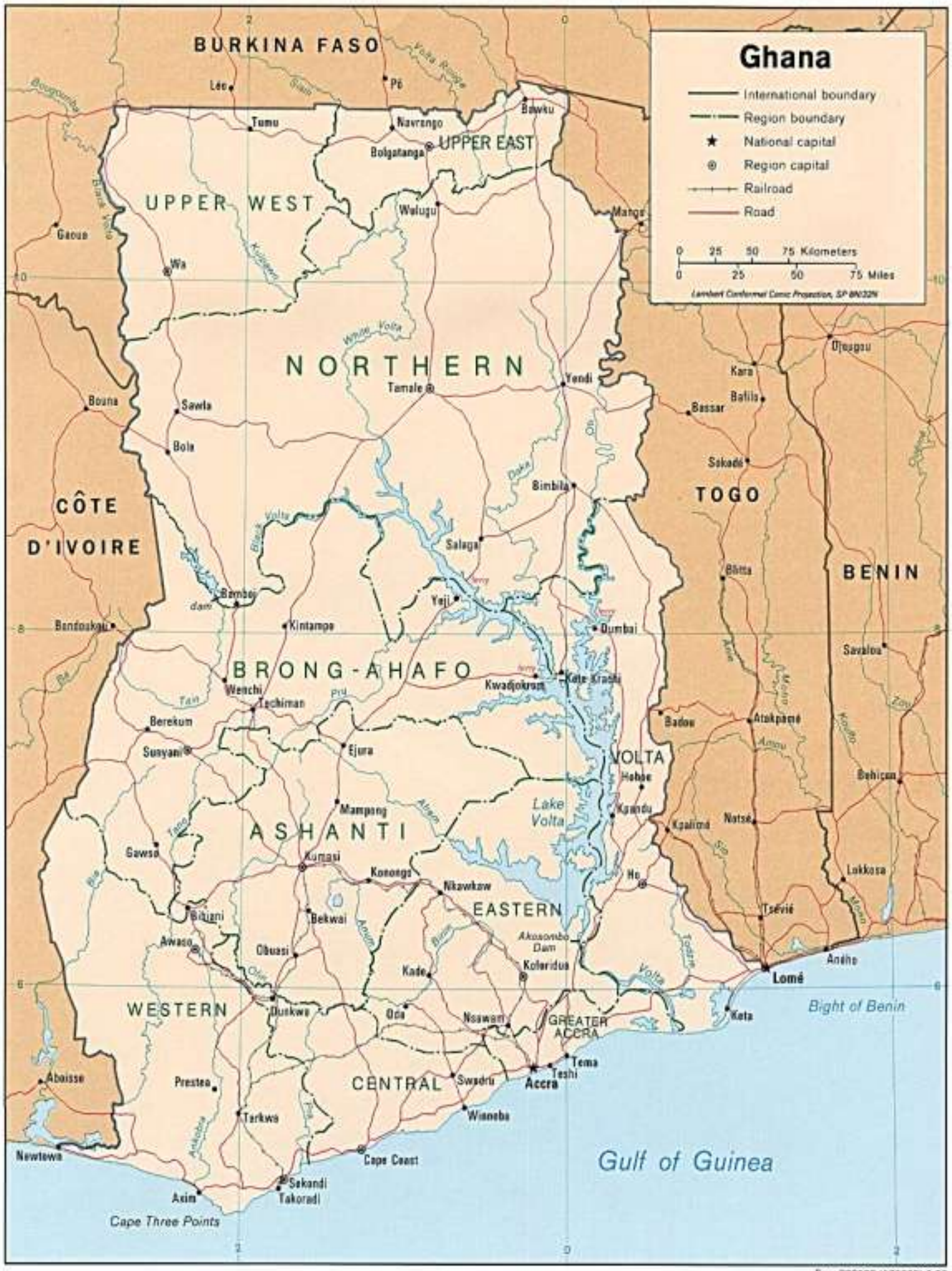
APPENDIX I

Expected demand and capacity of sachet water for the year, 2011

Month	Sachet Water Demand(bags)	Regular Time Shift Capacity(bags)	Overtime Shift Capacity(bags)
January	15000	6933	3467
February	15000	6667	3333
March	15000	6934	3466
April	14334	6915	3455
May	14034	6934	3436
June	14167	6778	3392
July	15000	7000	3500
August	15000	6895	3445
September	14834	6934	3466
October	14500	6933	3467
November	14667	6912	3458
December	14367	6933	3467

APPENDIX II

Map of Ghana showing where Techiman is located



APPENDIX III

INITIAL TABLEAU TO SOLVED BY LP SOLVER

Basis	X1	X2	X3	X4	X5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15	S16	S17	S18	S19	S20	S21	RHS
S6	10000	1000	2080	1040	10400	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	15000
S7	10100	1100	1950	1040	10000	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	15000
S8	10000	1000	1976	1040	10400	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	15000
S16	10000	1120	2080	1040	10370	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	14334
S17	10000	1000	2080	988	10400	0	0	0	-1	0	0	0	0	0	0	0	1	0	0	0	0	14034
S10	10000	1150	1950	910	10170	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	14167
S11	11200	1120	2080	1040	10500	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	15000
S12	10000	1000	2080	1040	10340	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	15000
S18	12000	1200	2080	1040	10400	0	0	0	0	0	0	0	-1	0	0	0	0	1	0	0	0	14834
S19	11000	1000	1950	1040	10400	0	0	0	0	0	0	0	0	0	-1	0	0	0	1	0	0	14500
S20	11000	1120	1820	1040	10370	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	1	0	14667
S21	11200	1300	2080	1040	10400	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	1	14367
OBJ	-65200	-6740	-12090	-6188	-162340	0	0	0	1	0	0	0	1	1	1	0	0	0	0	0	0	86736

APPENDIX IV

OPTIMAL SOLUTION GENERATED BY LP SOLVER

Basis	X1	X2	X3	X4	X5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15	RHS
S6	0	0	0	0	0	1	0	0	569/668	0	639/1336	0	0	783/835	0	920.802
S7	0	0	0	0	0	0	1	0	1.20584	0	0.0357036	0	0	0.537605	0	906.008
S8	0	0	0	0	0	0	0	1	0.33324	0	0.860928	0	0	0.887904	0	991.044
X2	0	1	0	0	0	0	0	0	-5/5344	0	0.00370135	0	0	0.0081018	0	2.23042
S15	0	0	0	0	0	0	0	0	-0.288174	0	1.78001	0	0	1.34335	1	991.649
S10	0	0	0	0	0	0	0	0	-1.0101	1	-1.05603	0	0	-2.4197	0	52.5365
X3	0	0	1	0	0	0	0	0	-0.00114003	0	0.0036918	0	0	-2/4175	0	0.675403
S12	0	0	0	0	0	0	0	0	6/7/668	0	0.41003	1	0	1.00671	0	946.498
X1	1	0	0	0	0	0	0	0	47/66800	0	0.0095687	0	0	-0.000124551	0	0.508605
X4	0	0	0	1	0	0	0	0	-0.0163807	0	-0.00919795	0	0	-0.0180332	0	0.869185
S13	0	0	0	0	0	0	0	0	123/334	0	1453/668	0	1	362/835	0	708.491
X5	0	0	0	0	1	0	0	0	1/835	0	-0.00113772	0	0	24/20875	0	0.428263
OBJ	0	0	0	0	0	0	0	0	0.00205261	0	0.000723409	0	0	0.00132939	0	0.831519