KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY

INSTITUTE OF DISTANCE LEARNING

TOPIC

SIMULATED ANNEALING IN TELECOMMUNICATION NETWORK PLANNING

A Thesis Submitted to the School of Graduate Studies and Research In Partial Fulfillment of the Requirements For the Degree of Master of Science in Industrial Mathematics Institute of Distance Learning Kwame Nkrumah University of Science and Technology

BY

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DECLARATION

I hereby declare that this submission is my own work towards the Master of Science degree and that, to the best of my knowledge it contains no material previously published by another person nor material which has been accepted for award of any other degree of the university, except where due acknowledgement has been made in the text.

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ABSTRACT

This thesis proposes a Simulated Annealing (SA) based method for planning of fixed wireless access (FWA) in telecommunication network planning. It may be introduced as a combinatorial optimization method that determines the location and capacity of base stations and end-users in a design space. The aim of this thesis is to develop a model that can locate base stations in design space and connect end-users to the base stations and solve the model within one hour. This thesis describes the mathematical model for the base station location problem for fixed wireless access (FWA) while minimizing the number of not connected end-users. The Simulated Annealing is implemented in C-programming language and visualization of the results is made using functions developed in the program Matlab. All data used in these tests are from a 20 times 20-kilometer square design area. The data simulating end-users are randomly chosen among companies and institutions within the design space. The model is successfully applied to planning of a real fixed wireless access network.

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DEDICATION

To my mother, Ulanda Puoza.

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LIST OF MAJOR ACRONYMS

FWA	Fixed Wireless Access
PMP	Point-to-Multipoint
IP	Internet Protocol
ATM	Asynchronous Transfer Mode
PSTN	Public Switching Telephone Network
BAS	Broadband Access System
bps	bits pr. Second
GSM	'Global System of Mobile communications' or 'Groupe Spéciale Mobile'
UMTS	Universal Mobile Telecommunication System
LOS	Line of Sight
C/I	Carrier to interference signal ratio
ISP	Internet Service Provider
BS	Base Station
CDMA	Code Division Multiple Access
MIP	Mixed Integer Programming
ISDN	Integrated Services Digital Network
LP	Linear Programming
NP	Nondeterministic Polynomial-time

CHAPTER ONE

INTRODUCTION

1.1 Radio Network Planning and Resource Optimization

With new wireless communication technologies and the increasing size of radio networks, the tasks of network planning and resource optimization are becoming more and more challenging. This is firstly because the radio resource is scarce these days due to the increasing number of subscribers and the many different types of networks operating within the limited frequency spectrum. Secondly, deploying and operating a large network is expensive and requires careful network dimensioning to ensure high resource utilization. As a consequence, manual network design and tuning for improving radio resource allocation are most likely to fail in current and future networks. This necessitates developing automated tools and optimization algorithms that are able to tackle the difficult task. Furthermore, radio network planning and resource optimization can clearly benefit from the well-established optimization theory due to similarities in the objective-oriented way of approaching a problem, selecting the best solution from a number of possible solutions and dealing with many restrictions. In fact, many of the network planning and resource optimization problems can be viewed as specific applications of classical optimization problems.

In this thesis, optimization is considered as the main approach to designing and improving performance of wireless networks such as Fixed Wireless Access Network (FWAN), Universal Mobile Telecommunications System (UMTS), Wireless Local Area Networks (WLANs) and ad hoc networks. The goal is to identify relevant problems for Fixed Wireless Access Network (FWAN) technologies, formalize the problems, and find reasonable solution approaches. First, however, we shall discuss what radio network planning and optimization are about, the type of problems they typically address and the typical optimization techniques that can be utilized to solve these problems.

1.1.1 Planning, Optimization or Both?

The tremendous popularity of wireless networks has attracted the attention of many researchers into planning and optimization of wireless networks and radio resources. Network planning refers to the process of designing a network structure and determining network elements subject to various design requirements. Network planning is associated with network dimensioning and detailed planning, i.e., two network life phases both of which are very important since an implemented plan imposes further hard constraints on network performance. In cellular networks, for example, these constraints are very often associated with hard capacity. Network performance (capacity) limitation due to resource exhaustion in a particular situation, e.g., high interference and/or heavy load, is often referred to as soft capacity. The goal of resource planning is to provide a network with the sufficient amount of resources and ensures its effective utilization, whilst achieving a certain minimum amount of hard capacity is usually a task for network planning.

Traditionally, planning is usually viewed as a static task. However, with heterogeneous radio environments, the concept of dynamic network reconfigurability featuring the concept of software defined radio has recently gained popularity. Dynamic network planning, by which the network infrastructure and the network mechanisms are to be defined dynamically, has become an attractive research area.

Network optimization amounts to finding a network configuration to achieve the best possible performance. The goal of resource optimization is to achieve the best possible resource utilization. The boundary between the two areas, network optimization and resource optimization, is even tighter than that between network planning and resource planning. Moreover, in practice, resource optimization is very often a part of network optimization. The main difference between the two concepts is that optimization tasks related to network infrastructure are typically associated with network optimization rather than resource optimization.

The applicability of optimization techniques for ensuring good network performance is quite intuitive, both for planning a network and/or radio resources, and for optimizing them during operation and maintenance, provided that a reasonable trade-off between the model complexity and reality can be found. Moreover, due to network complexity, its size, and the necessity of dealing with many factors and control parameters, the planning and optimization tasks are often beyond the reach of a manual approach. As a result, the latest trend is automated wireless network planning and optimization, initiated by operators of cellular networks but well spread in industrial and research societies. The trend implies using computer systems to generate network design decisions with the minimum amount of human assistance. Such planning systems can clearly benefit from incorporating different optimization modules implementing optimization algorithms. For automated optimization, optimization algorithms are not just a part of the system but are the core of the system. In addition to making the network design process time-efficient, planning and optimization tools can significantly reduce network deployment, operation and maintenance costs [1].

1.1.2 Some Classical Optimization Problems in Radio Network Design and Recent Trends

Due to historical reasons and technological and technical aspects of radio communications, the architecture of wireless networks traditionally has had some infrastructure, although infrastructureless networks are becoming more and more popular nowadays. In early networks, the infrastructure was formed by only one radio base station serving the entire intended area. Because of the simple architecture and low flexibility in terms of radio network configuration, radio network design was more focused on system capability and related technical issues, whereas the network planning itself did not receive much attention.

Network planning became a more challenging task with extended network architectures and more radio base stations. Initially, the task involved only two decisions that had to be made in the planning phase: how many radio base stations were needed and where they should be located. This task has been known as the radio base station location problem. In the optimization context, this engineering task, which often also involves cost optimization, can be viewed as a facility location problem, one of the classical problems in the field of mathematical programming, but with some more interdependencies and requirements imposed by radio access technology. The objective is to find a subset of a given set of candidate locations such that the total cost is minimized and the entire area is covered. Note that if feasible solutions are defined in continuous space and the link quality is not uniquely defined by distance between facilities and users, even the single-facility location problem may be computationally difficult and belong to a class of nondeterministic polynomial-time hard or *NP-hard* problems [2].

Frequency assignment is another classical problem (or, actually, a family of problems) in radio network planning and optimization. The problem was brought into focus with the introduction of the second generation (2G) cellular networks; in particular, Global System for Mobile Communications (GSM) networks, which use a combination of Frequency Division Multiple Access (FDMA), Time Division Multiple Access (TDMA), and random access. There have been many variations of the frequency assignment problems. The basic problem amounts to finding a frequency assignment, which is feasible to assignment constraints and interference constraints. Other examples are the minimum interference problem and the minimum span frequency assignment problem. In the first problem, co-channel and adjacent channel interference are minimized. The objective of the second problem is to minimize the difference between the highest and the lowest frequency used in solution. Frequency assignment is typically viewed as a graph coloring problem [1].

The radio base station location problem and the frequency assignment problem have been the most studied problems in the context of cellular radio network planning and optimization. Among the others, less studied, problems is topological network design where the goal is to design network topology of minimum cost that is able to connect the candidate base stations to a fixed (wired) telephone network [3]. The problem involves decisions not only on locations of radio base stations but also on topology of the wired network, i.e., routers, switches, hubs, gateways, etc.

Numerous problems have been considered for optimizing dynamic behavior of radio networks, with and without infrastructure. Among them, the power control problem, in which the power assignment is to be decided such that the network capacity is maximized, has been attracting a lot of attention in research. The problem is particularly interesting for interference-limited networks with a small frequency reuse factor, e.g., networks based on Wideband Code Division Multiple Access (WCDMA) [1].

Computational complexity of models is an important factor that has often a direct effect on its application. A detailed but intractable mathematical model can be sometimes as useless from the practical point of view as an oversimplified but easily solvable model. This fact has to be considered when developing models. Unfortunately, in most cases models developed for radio network planning and optimization tend to be difficult to solve. In fact, all the optimization problems (at least in their general forms) that have been discussed so far belong to the class of *NP*-hard problems. This means that the existence of polynomial-time algorithms (polynomial with respect to problem size) that can solve these problems is very unlikely, although this has not yet been proven. Moreover, the problem instances tend to be very large in realistic scenarios. This results in that manual parameter adjustment in such networks becomes a tedious task making even more challenging network planning and optimization of radio resources. In such situations exact algorithms typically do not help and even obtaining good approximations can be very difficult. Furthermore, very simple strategies based on one-at-a-time parameter manipulation are not very effective either. Thus, the importance of designing efficient optimization algorithms is not only in solving the problems but also in contributing to the problem application area and thus strengthening the link between theory and practice.

To this point, we have presented a number of typical problems that have been studied in the context of radio network planning and optimization. Although the basic building blocks of models have remained the same (i.e., the classical optimization models are typically adopted by many applications), the optimization problems and modeling approaches have been changing over time in line with technological and scientific trends. Thus, we can distinguish between several major steps in the history of network modeling and optimization. In the first step, the modeling approach mainly followed the trend of oversimplification of reality. Although this was a very important step for establishing relation between the optimization theory and network design, the necessity of more realistic models and practically applicable results gave rise to a new trend — joint optimization of several network aspects. This approach has clearly become superior to the previously used sequential optimization approach that exploits model simplicity.

The next trend has been cross-layer design and optimization [4], which actually extends the concept of joint optimization by considering the information flows across the network layers to enable solutions that are globally optimal to the entire system and thus facilitating the optimal layer design. This has been an important step towards decreasing the gap between the optimization state-of-the-art and modeling realism. The most recent trend in network optimization is considering layering as optimization decomposition [5] by which the overall communication network is modeled by a master problem where each layer corresponds to a decomposed sub-problem, and the interfaces among layers are quantified as functions of the optimization variables coordinating the sub-problems. Although the problem decomposition techniques have been widely used for planning and designing networks for quite some time, the new concept is important by itself since it focuses on network architecture but at the same time facilitates distributed control and cross-layer resource allocation. The main difference between the cross-layer design and optimization and the concept of layering as optimization decomposition is that the latter provides also insights into network architecture and layering.

1.2 Mathematical Programming as an Optimization Tool

The term "mathematical programming" refers to a planning process that allocates resources in the best possible, or optimal, way minimizing the costs and maximizing the profits. The mathematical programming approach is to construct a mathematical model, program, to represent the problem. In a mathematical model, variables are used to represent decisions, and the quality of decisions is measured by the objective function. Any restrictions on the values of decision variables are expressed by equations and inequalities.

1.2.1 Linear Programming

One of the most important areas of mathematical programming is linear programming (LP). The main precursor to LP is considered to be the work published by Leonid Kantorovich in 1939 [6] which laid out the main ideas and algorithms of linear programming. The latter was viewed as a tool for economic planning. The key assumption of LP is that all functions in the model, i.e., objective function and constraint functions, are linear and all variables are continuous. If all or some of the variables are constrained to be integers, the problem is a subject of study for linear integer programming and mixed-integer programming (MIP), respectively. Mathematical programming problems in which some of the constraints or the objective functions are nonlinear are studied by nonlinear programming.

In a compact way, a typical minimization LP formulation can be represented as follows,

$$\min\left\{c^T x : Ax \geq b, x \in R_+\right\},$$

where c is a row vector of costs, x is a column vector of nonnegative real variables, A is a matrix, and b is a column vector. (Converting a maximization linear program to the formulation above is straightforward.) The set of feasible solutions to the system of linear inequalities defines a convex polyhedron. One of the commonly used methods for solving linear programs is the simplex method, originally proposed by George Dantzig. The simplex method utilizes the concept of a simplex and the idea that in the case of closed convex polyhedron the optimum occurs either at a vertex of the polyhedron (and is then unique) or on its edge or face (and is then non-unique). The method finds the optimal solution by moving along the edges of the polyhedron from one vertex to another, adjacent, vertex such that the objective function value does not worsen. In each iteration, a simplex is specified by the set of dependent (basic) variables. A pivot rule is used to decide on the next move if there are several alternatives [1].

1.2.2 Integer and Mixed-Integer Programming

Integer programming and mixed-integer programming extend LP to deal with integrality constraints. The focus of integer programming is on integer problems where decision variables may only have integer values. If only some of the variables are required to have integer values, the model is referred to as a mixed-integer programming model. Below is a typical minimization MIP formulation,

$$\min\left\{c^T x + h^T y : Ax + Gy \geq b, x \in R_+, y \in Z_+\right\},$$

where x, c, b, and A as previously defined, y is a column vector of nonnegative integer variables, h is a row vector of costs of y-variables, and G is a matrix. Note that a pure integer program is the special case of the formulation above with no x-variables. An integer program where all variables are binary is called *0-1* or binary integer program.

An area closely related to mathematical programming is combinatorial optimization which studies problems involving finding the best (with respect to a given objective) solution out of a discrete set of feasible solutions. A combinatorial optimization problem can often be formulated as an integer or binary integer program.

Below are some examples of typical integer and mixed-integer programs that arise in radio network planning?

Set Covering. In the classical set covering problem, we are given a ground set of M elements and a collection N of subsets of M, the goal is to choose the minimum number of subsets that together cover the entire ground set. The problem has a typical application in radio network coverage planning where the ground set is represented by points that are to be covered, and the collection of subsets represents a set of candidate sites. However, since installation costs as well as operation and maintenance costs typically vary by site, in wireless network planning it is more common to consider the minimum-cost set covering problem where a non-negative cost is associated with each subset.

The corresponding binary integer programming formulation is given below.

$$\min \sum_{j \in N} c_j x_j$$
$$wrt$$
$$\sum_{j \in N} a_{ij} x_j \geq 1 \cdots\cdots\cdots\cdots\cdots\cdots\cdots i \in M$$
$$x_j \in \{0,1\} \cdots\cdots\cdots\cdots\cdots\cdots\cdots j \in N$$

where x_j is a binary variable that equals one if and only if subset j is selected, and a_{ij} is the element of incidence matrix A such that $a_{ij} = 1$ if and only if element i is covered by subset j.

Facility Location: Let M be a set of possible facility locations and N be a set of clients. Suppose there is a fixed cost f_i of opening a facility at location i, and there is a transportation cost of c_{ij} associated with every facility $i \in M$ and every client $j \in N$. The problem is to decide which facilities to open, and which facility serves each client so as to minimize the sum of the fixed and transportation costs, and every client is assigned to exactly one facility. The problem is known as the uncapacitated facility location problem. Note that the problem is similar to the set covering problem except for the addition of the transportation costs. Below is an integer programming formulation of the problem.

$$\min \sum_{i \in M} \sum_{j \in N} c_{ij} x_{ij} + \sum_{i \in M} f_i y_i$$
$$wrt$$
$$x_{ij} \leq y_i \cdots\cdots\cdots\cdots\cdots\cdots\cdots i \in M, j \in N$$
$$\sum_{i \in M} x_{ij} = 1 \cdots\cdots\cdots\cdots\cdots\cdots\cdots j \in N$$
$$x_{ij} \in \{0,1\} \cdots\cdots\cdots\cdots\cdots\cdots\cdots i \in M, j \in N$$
$$y_i \in \{0,1\} \cdots\cdots\cdots\cdots\cdots\cdots\cdots i \in M$$

In the formulation, variable y_i is one if and only if facility i is used, and x_{ij} is one if and only if client j is assigned to facility i. Note that in a general formulation of the facility location problem, x-variables are continuous, i.e., $x_{ij} \geq 0$, $i \in M$, $j \in N$; however, due to the single assignment property [7] of the uncapacitated facility location problem, a client is always entirely served by the closest facility.

A slight variation of the model presented above is the capacitated facility location problem in which each facility i has maximum capacity Q_i and each client $j \in N$ has demand d_j. Thus, in the capacitated facility location problem constraints

$$\sum_{j \in N} d_j x_{ij} \leq Q_i y_i \cdots\cdots\cdots\cdots\cdots\cdots\cdots i \in M$$

are typically used. These constraints make redundant the first set of constraints in the formulation of the uncapacitated facility location problem. The redundant constraints are, however, usually kept to make the formulation more efficient (to strengthen its LP relaxation, in particular). Moreover, x-variables are continuous in the classical capacitated facility location problem.

Both types of the facility location problems are used in wireless network planning. One application is the problem of minimizing installation and maintenance costs of base stations in a network providing fixed rate services such that each client gets served. Maximum k-facility location problem [8] is a variation of the uncapacitated facility location problem, where the sum of link performance metrics is maximized and the number of facilities is at most k. This type of model has been used for maximizing net user throughput.

1.2.3 Solution Methods and Techniques for Integer and Mixed-Integer Programs

In general, integer programs are much harder to solve than LPs. The integrality constraints in MIPs are therefore also often considered as the complicating part. The simplest approach to tackle a pure integer programming problem is to explicitly enumerate all possibilities (if they are finite). However, only the smallest instances could be solved by such an approach due to the effect known as combinatorial explosion that describes the rapidly accelerating increase in the number of combinations with the increased number of control parameters. In this section, we present the most common approaches used to tackle integer programs and MIPs.

Relaxation and Bounding

Relaxation is an important mathematical programming approach which is used to replace a "difficult" optimization problem by a simpler one by either removing some of the constraints or substituting them with other more easily handled constraints. For a minimization problem, the solution obtained from the relaxation of the original problem gives a lower bound on the optimal solution to the original problem. For maximization, relaxation gives an upper bound. The bound obtained from the relaxation is called a dual bound. In minimization problems, any feasible solution is an upper bound on the integer optimum (a lower bound in maximization problems). This bound is also called a primal bound. The bounds can be weak or strong depending on how far they are from the optimal solution to the original problem. The relative gap, called duality gap, between the primal and the dual bounds is usually used to estimate the quality of the obtained feasible solution. The gap depends on the efficiency of relaxation, solution algorithms, problem size and its complexity [1].

One of the common relaxation techniques is *LP*-relaxation, which involves solving the original problem without integrality constraints, i.e., by treating all variables as continuous variables. A straightforward approach for generating an integer solution from an LP-solution is rounding of non-integer values in the resulting LP solution. The drawbacks of this approach are that the solution obtained by rounding is not necessarily feasible and/or it may be far from optimal.

Another relaxation technique is Lagrangian relaxation [9]. Lagrangian relaxation uses the idea of relaxing some constraints by bringing them into the objective function with associated Lagrange multipliers. In a minimization problem, Lagrangian dual refers to the problem of maximizing the dual bound with respect to the Lagrange multipliers. Properties of the Lagrangian dual can be found in [10], and its application to integer programming was explored in [11]. Sub-gradient method is very often used to solve the Lagrangian dual problems. By this method, the sub-gradient direction is obtained by minimizing all the sub-problems and then the multipliers are updated along the sub-gradient direction. Motivated by the fact that the sub-gradient method may be very computationally expensive for large problems, some variations of the sub-gradient method have been proposed. The examples are the interleaved sub-gradient method, which minimizes only one sub-problem per iteration to obtain a direction and then updates the multipliers, and the surrogate sub-gradient method, which utilize the idea that only near optimization of one sub-problem is necessary to obtain a proper direction. Other approaches, such as the analytic center cutting-plane method, augmented Lagrangian algorithms, and bundle methods have also been proposed [1].

Problem Decomposition

Problem decomposition is an approach exploiting the model structure to decompose the problem into smaller and easier-to-solve sub-problems. The classical decomposition methods are Lagrangian decomposition, Dantzig-Wolfe decomposition, and Benders decomposition. By Lagrangian decomposition, also known as variable splitting, a set of copies of the original variables is introduced for a subset of constraints, and then Lagrangian duality is applied by relaxing the constraints that set equivalence between the original variables and the copies. In Benders decomposition, problem variables are partitioned into two sets, master problem variables that are typically complicating variables (e.g., integer variables in a MIP) and sub-problem variables. The Benders algorithm iteratively solves a master problem, which assigns tentative values for the master problem variables, and a sub-problem obtained by fixing the master problem variables to the tentative values. Solutions to the sub-problems are used for generating inequalities that cut off non-optimal assignments, called Benders cuts, that, being added to the master problems which is to be then resolved, narrow down the search space of the master problem variables. The key idea of Dantzig-Wolfe decomposition is to reformulate the problem by substituting the original variables with a convex combination of the extreme points of a substructure of the formulation. The resulting problem formulations consists of sub-programs (slave programs) corresponding to its independent parts and a master program that ties together the subprograms. When solving the master problem, column generation can be used to deal with a large number of variables. The main principle of column generation algorithms is to never list explicitly all of the columns (extreme points) of the problem formulation, but rather to generate them only as "needed". Note that for LP, applying Dantzig-Wolfe decomposition is the same as applying Benders decomposition to the dual problem. Originally, Dantzig-Wolfe decomposition was intended for solving LPs, but later it has also been adapted for integer and mixed-integer programs [1].

Branch-and-Bound and Its Extensions

An important technique in integer and mixed-integer programming, branch-and-bound, is based on the divide-and-conquer principle that was originally presented in [12] but currently has a lot of extensions. The branch-and-bound technique is a procedure by which in each iteration a sub-problem (the original problem in the first iteration) is subdivided into smaller sub-problems by partitioning the set of feasible solutions into smaller subsets. This can be done by restricting the range of the integer variables (for binary variables, there are only two possible restrictions: setting the variable to either 0 or 1). In general, with respect to a variable with lower bound l and upper bound u, the problem will be divided into two sub-problems with ranges l to q and $q + 1$ to u, respectively. LP relaxation can be used to obtain bounds on the optimal integer solution. If the optimal solution to a relaxed problem is (coincidentally) integral, it is an optimal solution to the sub-problem, and the value can be used to terminate searches of sub-problems whose lower bounds are higher. Conquering is done by bounding how good the best solution in the subset can be, and discarding the subset if it's bound indicates that it cannot possibly contain an optimal solution for the original problem.

A technique, by which cutting planes are embedded into a branch-and-bound framework, is known as branch-and-cut. For branch and cut, the lower bound can be, for example, provided by the LP relaxation of the integer program. The optimal solution to this linear program is at an extreme point (vertex) of the feasible region. If the optimal solution to the LP is not integral, this algorithm searches for a constraint, which is violated by this solution, but is not violated by any optimal integer solutions, i.e., a cutting plane. When this constraint is added to the LP, the old optimal solution is no longer valid, and so the new optimal solution will be different, potentially providing a better lower bound. Cutting planes are added iteratively until either an integral solution is found or it becomes impossible or too expensive to find another cutting plane. In the latter case, a traditional branch operation is performed and the search for cutting planes continues on the sub-problems.

Branch-and-price combines branch-and-bound with column generation. This method is used to solve integer programs where there are too many variables to be handled efficiently all together. Thus, only a subset of variables is maintained and columns are generated as needed while solving the linear program. Columns with profitable reduced costs are added to the LP relaxation; if no such column exists, the solution is optimal. If the LP solution does not satisfy the integrality constraint, branching is applied [1].

Dynamic Programming

Another technique often used in integer programming is dynamic programming which provides a systematic procedure for determining the optimal sequence of interrelated decisions. The dynamic programming method can be applied to problems that have optimal substructure, i.e., optimal solutions of sub-problems can be used to find the optimal solution of the overall problem. For example, the shortest path from one vertex in a graph to another one can be found by first computing the shortest path to the goal from all adjacent vertices, and then using the result to successfully pick the best overall path.

The main idea of the algorithm is that the problem can be broken into smaller sub-problems that can be recursively solved to optimality. The found optimal solutions can then be used to construct an optimal solution for the original problem. The sub-problems themselves are solved by dividing them into sub-subproblems, and so on, until a simple and easy-to-solve case is reached.

Since the optimal solution is calculated recursively from the optimal solutions to slightly different problems, an appropriate recursive relationship for each individual problem needs to be formulated. On the other hand, if such a recursion exists, we obtain great computational savings over using exhaustive enumeration to find the best combination of decisions, especially for large problems.

Heuristics

Because most of practical problems and many interesting theoretical problems are *NP*-hard, heuristics and approximation algorithms play an important role in applied integer programming. Such algorithms are used to find suboptimal solutions when the time or cost required to find an optimal solution to the problem would be very large [1]. A heuristic is typically a simple intuitively designed procedure that exploits the problem structure and does not guarantee an optimal solution. A meta-heuristic ("meta" means "beyond") is a general high-level procedure that coordinates simple heuristics and rules to find good approximate (or even optimal) solutions to computationally difficult combinatorial optimization problems. A meta-heuristic does not automatically terminate once a locally optimal solution is found.

Greedy heuristics are simple iterative heuristics specifically designed for a particular problem structure. A greedy heuristic starts with either a partial or infeasible solution and then constructs a feasible solution step by step based on some measure of local effectiveness of the solutions. In each iteration, one or more variables are assigned new values by making greedy choices. The procedure stops when a feasible solution is generated. As an extension of greedy heuristics, a large number of local search approaches have been developed to improve given feasible solutions.

Lagrangian heuristics exploit the solution process of the Lagrangian dual in order to obtain feasible solutions to the original problem. Most Lagrangian heuristics proposed so far attempt to make the optimal solution to the Lagrangian relaxation feasible, e.g., by means of a simple heuristic.

Local search is a family of methods that iteratively search through the set of solutions. Starting from an initial feasible solution, a local search procedure moves from one solution optimal within a neighboring set of solutions; this is in contrast to a *global optimum*, which is the optimal solution in the whole solution space solution to a neighboring solution with a better objective function until a local optimum is found or some stopping criteria are met. The next two algorithms, simulated annealing and tabu search, enhance local search mechanisms with techniques for escaping local optima.

Simulated annealing is a probabilistic meta-heuristic derived from statistical mechanics [13]. This iterative algorithm simulates the physical process of annealing, in which a substance is cooled gradually to reach a minimum-energy state. The algorithm generates a sequence of solutions and the best among them becomes the output. The method operates using the neighborhood principle, i.e., a new solution is generated by modifying a part of the current one and evaluated by the objective function (corresponding to a lower energy level in physical annealing). The new solution is accepted if it has a better objective function value. The algorithm also allows occasional non-improving moves with some probability that decreases over time, and depends on an algorithm parameter and the amount of worsening. A non-improving move means to go from one solution to another with a worse objective function value. This type of move helps to avoid getting stuck in local optimum. It has been proved that with a sufficiently large number of iterations and a sufficiently small final temperature, the simulated algorithm converges to a global optimum with a probability close to one. However, with these requirements, the convergence rate of the algorithm is very low. Therefore, in practice it is more common to accelerate the algorithm performance to obtain fast solution approximations [1].

Tabu search is a meta-heuristic technique that operates using the following neighborhood principle. To produce a neighborhood of candidate solutions in each iteration, a solution is perturbed a number of times by rules describing a move. The best solution in the neighborhood replaces the current solution. To prevent cycling and to provide a mechanism for escaping locally optimal solutions, some moves at one iteration may be classified as tabu if the solutions or their parts, or attributes, are in the tabu list (the short-term memory of the algorithm), or the total number of iterations with certain attributes exceeds a given maximum (long-term memory). There are also aspiration criteria which override the tabu moves if particular circumstances apply.

Genetic algorithm are probabilistic meta-heuristics that mimic some of the processes of evolution and natural selection by maintaining a population of candidate solutions, called individuals, which are represented by strings of binary genes. A genetic algorithm starts with an initial population of possible solutions and then repeatedly applies operations such as crossover, mutation, and selection to the set of candidate solutions. A crossover operator generates one or more solutions by combining two or more candidate solutions, and a mutation operator generates a solution by slightly perturbing a candidate solution. Thus, the population of solutions evolves via processes which emulate biological processes. Introduced by Holland [14], the basic concept is that the strong species tend to adapt and survive while the weak ones tend to die out.

1.3 Background to the Thesis

A typical connection from the end-users to a plain old telephone system or an Internet service provider (ISP) is via fixed lines [15]. Another option is to use Fixed Wireless Access (FWA) which provides a fast establishment and/or expansion of the connection between operator and end-user. This thesis considers the planning of networks starting at a level where location and demand of each end-user and location of potential base station sites within the service area are known. Today, the radio network planning is done manually. Depending on the size of the desired network the process takes between 3 to 5 days for one person. This gives rise to promote use of computers utilizing operational research to speed-up this process.

The FWA system applied in this master thesis is a proposed Vodafone MINI-LINK Broadband Access System (MINI-LINK BAS). The MINI-LINK BAS system provides connection between IP/PSTN/ATM backbone network and the end-user service terminals. The backbone network is connected to the base stations and one base station can be connected to a number of end-users (Point to Multipoint access (PMP)). One base station can host up to 6 sectors and each sector has a capacity of 37 Mbps Gross bit rate full duplex. One sector covers the end-users in an area within an angle of 90^0 with a maximum transmission range at approximately 5 km. This means that a base station with 4 sectors have a total potential coverage area that can be approximated by a circle with centre at the base station and a maximum radius of five kilometres (5 km).

1.4 Statement of the Problem

One of the major tasks in Telecommunication Network Planning for fixed Wireless Access is identifying the best location of Base Stations and connecting end-users to the base stations at a minimum cost. Hence the object is to minimize the number of base stations while connecting the maximum number of end-users.

The primary cost of the network is the cost of establishing base stations. Hence, the object is to minimize the number of base stations while maintaining 'sufficient coverage'. 'Sufficient coverage' is a matter of definition similar to the success parameter. The operator decides whether the network requested has to cover all potential end-users, or that the success parameter of e.g. 80% capacity utilization for base stations is acceptable. There exists other ways of defining the best network for the actual operator but these two are the most common. In the typical real-life planning process the first question in the inquiry is the cost of covering all end-users with sufficient capacity. Second question is how many end-users are covered with a lower number of base stations, and is this number of end-users above operator's minimum service limit. This means that it is desirable to design the planning model in a way where both planning with unlimited and fixed numbers of base stations are possible.

Current computer based tools for assisting in the planning process are only capable of computing the coverage once the base stations have been placed, and not placing the base stations themselves.

This gives the primary task of this project; construct a model that can place base stations and connect end-users to base stations. Firstly it is necessary to define limitations and assumptions in the model in order to make the problem manageable.

When dealing with radio signals it is important to take radio propagation loss between transmitter and receiver into account. Unfortunately it is very complicated to compute this value due to its dependency of the topography of the surface in the coverage area e.g. vegetation, buildings etc. Hence, in this model it is decided to reduce the planning area from a 3-dimensional into a 2-dimensional plan. Connections between base stations and end-users are pre-computed in the 3-dimensional plan in order to identify where line of sight (LOS) between transmitting and receiving antennas exists. Finally, in the 2-dimensional model the propagation loss is assumed linear dependent of transmission distance and transmission power, measured on a logarithmic scale. End-users are located in the planning area and identified by the coordinates and their demand measured in bps. Potential sites for base stations are also given in the plan, identified by their coordinates.

Connecting end-users to base stations is done by connecting the end-users to the nearest base station. This is done while monitoring whether the sum of end-user demand is less than or equal to, the maximal capacity of the base station and that the distance between the base station and the end-user is less than 5 km. To do this it is necessary to simplify the model of the base station in a way where the coverage area of each base station is assumed to be one circular area instead of four sectors of 90 degrees. The maximal capacity of the base station is simply computed as the product of the number of sectors and the maximal capacity of each sector. The distance to the most distant end-user from each base station defines the radius of the coverage circle of the base station. It is desirable that the overlap between any pair of coverage circles is minimal due to interference [15].

1.5 Research Goal and Objectives

The primary thrust of this research is aimed at creating mathematical models for task of planning process, identifying the necessary input data and defining planning parameters that identify the quality of the network. The planning of the network can be divided in to two separate steps as follows:

Create a network with total coverage and sufficient capacity for all end-users in the area while maximizing the minimal load on each base station and minimizing the overlap between coverage circles.

The output of this step is a plan that gives the number of base stations, the location of each base station and all base station-end-user connections. This information gives the operator an opportunity to decide if the number of base stations is acceptable or that another optimization with a fixed number of base stations has to be made. An optimization with a fixed number of base stations is as follows:

Find maximal coverage with a fixed number of base stations while maximizing the minimal load on each base station, and minimizing the overlap between circles.

When step 2 has been performed it should be checked whether the number of end-users connected is above the minimum service limit. The planning is performed, aiming at covering the expected demand after a period of e.g. 10 years. Identifying milestones for the rollout plan is done by ranking the base stations by the load on each base station. Base stations are then established successively starting with the one having the most load. This means that milestones for year 4 and 6 are given by the pace of the rollout progress and not by a separate optimization aiming at year 4 and 6.

The overall objective of this research is to Develop and implement a mathematical model that can locate base stations, connect end-users to base stations and find a way to solve the model spending less time than solving the location problem manually.

1.6 Research Methodology

In this research, realistic mathematical models for base station location for Fixed Wireless Access (FWA) are formulated and solved using the Meta-heuristics, Simulated Annealing Algorithm. It goes beyond developing Meta-heuristic to solve simple strategies to optimize the base station location problem. Instead, the approach is general for the planning of any wireless communication networks such as GSM, UMTS and others.

The idea is to create a core C-program that, with the correct input data files is able to assist in the planning process. The visualization of the result is made using function developed in the program MatLab. *Figure 1.1* depicts the process under consideration.

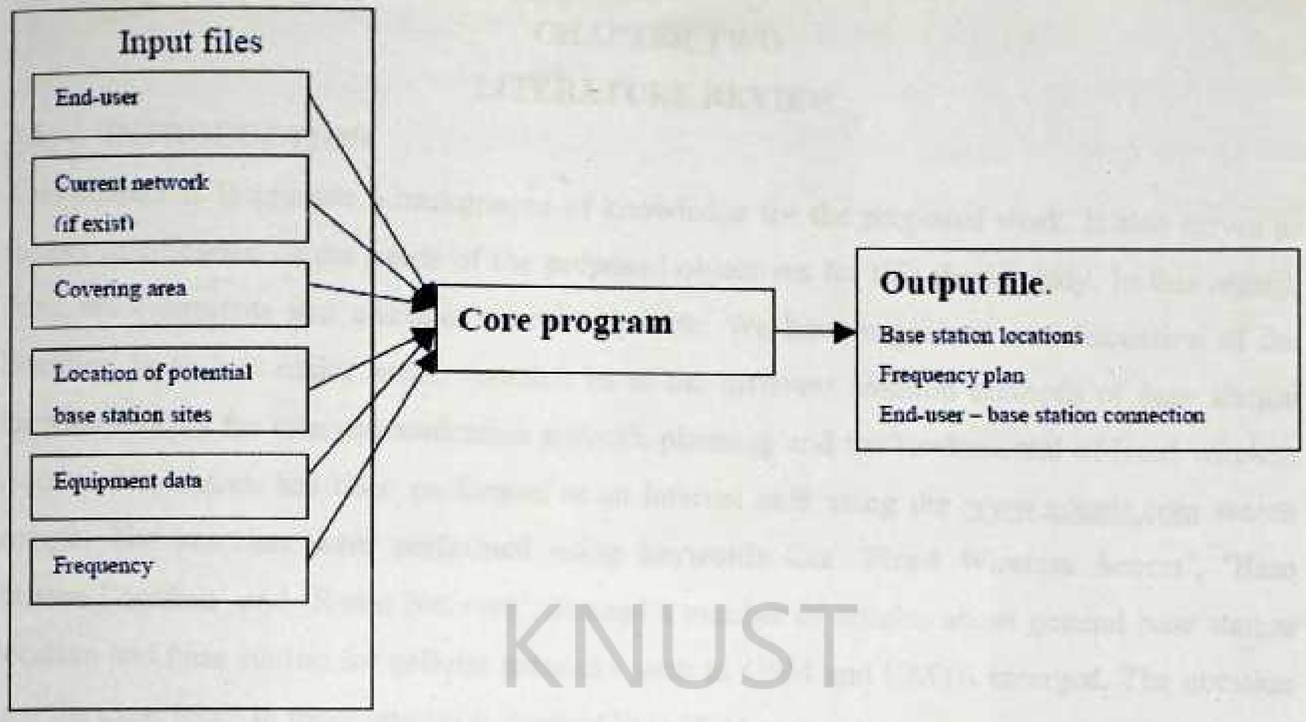

Figure1: Planning Process for Base Station Location

1.8 Organization of the Thesis

This research is structured as follows: Chapter one is an introduction to the research and the background. It has also served as an introduction to the specific problems that have addressed and the solution procedures that will be used to solve them. We provide a summary of existing literature in Chapter two. Chapter three introduces the mathematical models for base stations location problem in network planning and the required input data files to the model are analysed. The problems for the models are solved and a series of test run conducted and the results discussed in chapter four. Finally, in chapter five, we will compare and contrast the results to the specific problem and possible future research.

CHAPTER TWO

LITERATURE REVIEW

2.1 INTRODUCTION

This chapter is to provide a background of knowledge for the proposed work. It also serves as further justification of the needs of the proposed objectives for this thesis study. In this regard, there are comments and critiques on related work. We have organized our discussion of the literature in to two major areas: research in to the different solution methods of base station location models for telecommunication network planning and the fundamental of fixed wireless Access. The search has been performed at an internet café using the www.google.com search engine. The searches were performed using keywords like 'Fixed Wireless Access', 'Base Station Location' and 'Radio Network'. Instead a number of articles about general base station location and base station for cellular networks such as GSM and UMTS emerged. The question is if the experience in these articles is applicable in FWA networks.

All results and illustrations in this section are from the respective articles.

The usage of telecommunication networks has considerably changed in the last decade in Ghana. The traditional concept of telecommunication networks exclusively dedicated to the telephony is now completely outdated. The new telecommunication networks will carry without distinction voice, video and data traffic. Most of the telecommunication is carried over the internet. This revolution started a few years ago, is characterized by a spectacular growth in traffic (the traffic doubles every ten months). Another critical factor is the evolution of networks in the world, is the liberation of telecommunication markets, forcing all historical operators to adapt very quickly to the new competitive environment. The context for the development of the next generation networks has change considerably giving rise to many new problems, among which the optimal choice of locations for base stations remains a crucial one, especially when combined with other features of the new telecommunication networks.

2.2 BASE STATION LOCATION FOR FIXED WIRELSS ACCESS

The mobile (cellular) phone system works as a network containing base stations. Within each cell, a base station (with an antenna) can link with a number of handsets (mobile phones). The mobile phones and the base stations communicate with each other, sharing a number of operation frequencies. Other transmission links connect this base station with switches connecting to base stations in other cells, or with switches connected to conventional phones. The cell exists in order to permit re-use of frequencies – the same frequency can be used in different cells (given a sufficient distance). The cell is the unit of a cellular communication system. A certain number of cells are chosen to install switches, which route call to another base station or to a public switched telephone network.

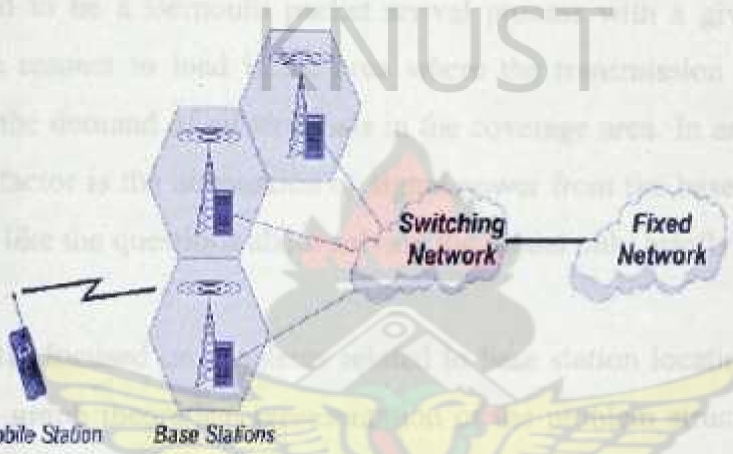

Figure 2.1: A simple architecture of wireless cellular network

The design of mobile networks often involves problems of devices location (BTS, multiplexers, switches etc.) There are works specifically related to the design of the BTS-switch structures, like the BTS location problem in which the objective is to obtain the optimal location of BTSs in a grid, such that the radio coverage of the considered grid is maximal. Another important problem, directly related to the BTS-switch structure in mobile networks, is the assignment of cells to switches problem. This problem considers that the BTSs and switches of the network are already positioned, and its objective is to assign each BTS to a switch, in such a way that a capacity constraint has to be fulfilled. The objective function is then formed by two terms: first the sum of the distances from the BTS to the switches must be minimal, and there is another term related to handovers between cells assigned to different switches, which must be minimized. In addition, there are other location problems related to the design of communications network, either mobile networks or computer networks.

Yang and Ephremides (2000) worked with the problems related to the selection of location and transmission power of base stations while maximizing the minimum throughput among the mobiles. The major issue of the article is to model the movements of the mobile terminals and argue that this simulation is correct. The design space is discretized into a grid of legal points. The movement of the mobile terminals is simulated by a random walk between points. Over time the movements of the mobile terminals converge towards a steady state. The traffic to each terminal is assumed to be a Bernoulli packet arrival process with a given rate. The maximal coverage area with respect to load is the area where the transmission capacities of the base stations can match the demand of all terminals in the coverage area. In areas where there is low traffic the limiting factor is the attenuation of signal power from the base station. The modeling of the signal loss is like the questions about solving the model only briefly touched.

Calegari et al, (2001) focused on problems related to base station location in UMTS networks. The authors used a graph theoretical representation of the problem structure. The design space was tiled using a grid. A node represents each tile in the grid and an edge connects the node with each of the base stations that can cover the node. See *Figure 2.2*. This gives a bipartite graph with tile nodes on one side and base station nodes on the other. The solution method suggested in the articles is the graph theoretical problem 'Minimum dominating set'. This means find the minimum set of base stations that covers all end-users. This problem has been proved to be NP-hard. The primary aim of the article is to describe a solution method based on a genetic algorithm. The results from a test run on a 70 km x 70 km planning area did not produce satisfactory solutions but the algorithms are still under development. One area of special interest is to reduce the size of bipartite graph.

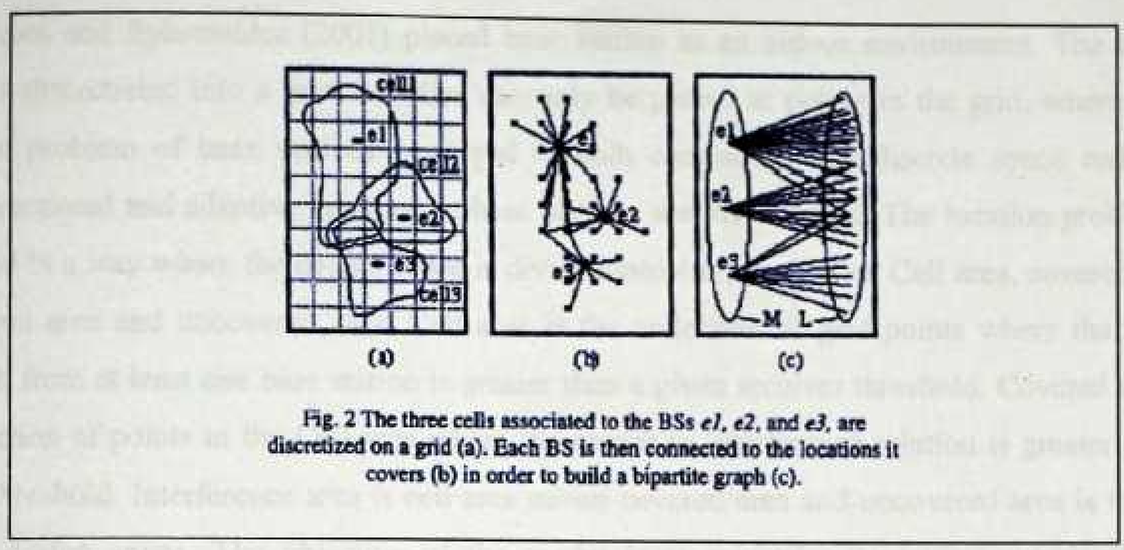

Figure 2.2: A Bipartite Graph with Tile Nodes

The graph theoretical approach in article [20] is different. Here the propagation model added to each potential base station site is based on one type of antenna. A graph is created where a node represents each potential base station site. If two sites have a coverage area in common that is over a given threshold, an edge is added between the respective nodes see *Figure 2.3*. In this graph the solution is given by the graph theoretical problem 'Maximum independent set'. This means find a maximum set of base stations that are not connected. The problem is solved using 7 different stepwise heuristic methods and the results are analyzed to find the method that gives the best coverage. The article pays most attention to select the best overlap threshold value. According to the article the success parameter when evaluating a given overlap threshold value is either the number of selected base stations or the relative coverage.

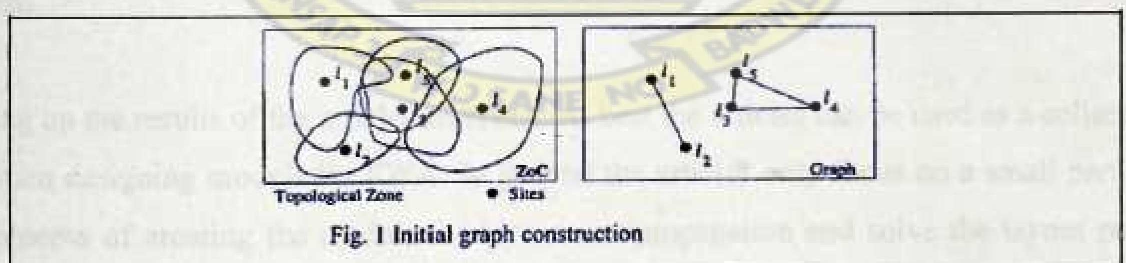

Figure 2.3: Initial Graph Construction

Stamatelos and Ephremides (2001) placed base station in an indoor environment. The design space is discretizied into a grid. Mobiles can only be placed at points in the grid, whereas the location problem of base stations is solved in both continuous and discrete space and both omnidirectional and adaptive antennas at base stations are investigated. The location problem is modeled in a way where the design space is divided into 4 types of area. Cell area, covered area, interfered area and uncovered area. Cell area is the collection of grid points where the signal strength from at least one base station is greater than a given receiver threshold. Covered area is a collection of points in the Cell area where the carrier to interference relation is greater than a given threshold. Interference area is cell area minus covered area and uncovered area is the rest of the design space. The objective of the model is to minimize a convex combination of Uncovered and Interference area by finding the location of each base station and the transmission power. The methods proposed for solving the model in continuous space are steepest descent and Downhill simplex. The Steepest descent method makes use of the gradient of the objective function to find the next step of the algorithm. The downhill simplex is similar to the Nelder-Mead simplex method mentioned in [19]. In the discrete space where most work in this article has been done, the model is solved using the Hopfield and Tank neural network. In test runs placing 3 base stations with omnidirectional antennas the steepest descent method gave worst results. Best results were achieved with the Hopfield and Tank neural network. Of the 200 simulations with neural network only 3% did not converge. All results were within 4% from optimal solution. In the test run with adaptive antennas at the base stations the Nelder- Mead simplex method produced a solution 50 % from optimal solution. The steepest descent algorithm failed to converge but the neural network produced solutions between 17 and 28 % from optimum.

Summing up the results of the articles searched. At best the articles can be used as a collection of ideas when designing models for FWA. In general the articles only focus on a small part of the entire process of creating the models for layout and propagation and solve the layout problem whereas the rest of the process is only sparsely described.

The steady state of the moving mobiles in [16] illustrates the theoretical similarities between mobile systems and FWA systems. In [17] the similarity between FWA and cellular networks where tiles are replaced by end-users is obvious using this representation. As in the FWA case the objective in the UMTS case is to find a set of base stations that cover a maximum number of tiles/end-users using a minimum number of base stations.

2.3 LOCATION OF CONCENTRATORS/BASE STATIONS USING SIMULATED ANNEALING

In section 2.3.1 we present the simulated annealing algorithm and a set of rules that can be applied to the problems described in locating base station in network planning. Sections 2.3.2, 2.3.3 and 2.3.4 respectively deal with the uncapacitated facility location problem (UFLP), capacitated facility location problem (CCLP) and terminal assignment problem (TA) in telecommunication network planning.

2.3.1 Simulated Annealing

Simulated Annealing was first introduced in 1983 as an intriguing technique for optimizing functions of many variables [13]. Simulated annealing is a heuristic strategy that provides a means for optimization of *NP* complete problems: those for which an exponentially increasing number of steps are required to generate the/an exact answer. Although such a heuristic (logical) approach can't guarantee to produce the exact optimum, an acceptable optimum can be found in a reasonable time, while keeping the computational expense dependent on low powers of N, the dimension of the problem. Simulated annealing is based on an analogy to the cooling of heated metals. In any heated metal sample the probability of some cluster of atoms at a position, r_i, exhibiting a specific energy state, $E(r_i)$, at some temperature T, is defined by the Boltzmann probability factor:

$$P\left[E\left(r_i\right)\right] = e^{-\left[\frac{E(r_i)}{k_B T}\right]}$$

where k_B is Boltzmann's constant. As a metal is slowly cooled, atoms will fluctuate between relatively higher and lower energy levels and allowed to equilibrate at each temperature *T*. The material will approach a *ground state*, a highly ordered form in which there is very little probability for the existence of a high energy state throughout the material.

If the Energy function of this physical system is replaced by an objective function, $f(X)$, that is dependent on a vector of design variables, X, then the slow progression towards an ordered ground state is representative of a progression to a global optimum. To achieve this, a control parameter *T*, analogous to a temperature, and a constant *C*, analogous to Boltzmann's constant, must be specified for the optimization problem. In standard iterative improvement methods, a series of trial points is generated until an improvement in the objective function is noted in which case the trial point is accepted. However, this process only allows for downhill movements to be made over the domain.

In order to generate the annealing behaviour, a secondary criterion is added to the process. If a trial point generates a large value of the objective function then the probability of accepting this trial point is determined using the Boltzmann probability distribution:

$$P[acceptX_t] = e^{-\left[\frac{f(X_t)-f(X_o)}{CT}\right]}$$

, where X_0 is the initial starting point. This probability is compared against a randomly generated number over the range [0.…1]. If $P[acceptX_t] \geq random[0.....1]$ then the trial point is accepted. This dependence on random numbers makes simulated annealing a stochastic method. Various implementations will use various methods of random number generation; an example is the Lehmer generator [22]. Repeating this iterative improvement many times at each value of the control parameter *T*, the methodical thermal rearrangement of atoms within a metal at a temperature *T* is simulated [21]. This iterative improvement forms the *inner loop* of the method. The variation of the control parameter *T* is contained in an *outer loop*. The initial value of the control parameter is suitably high and a methodology for decrementing *T* is applied. *Figure 2.4* provides a flowchart representation of the annealing algorithm.

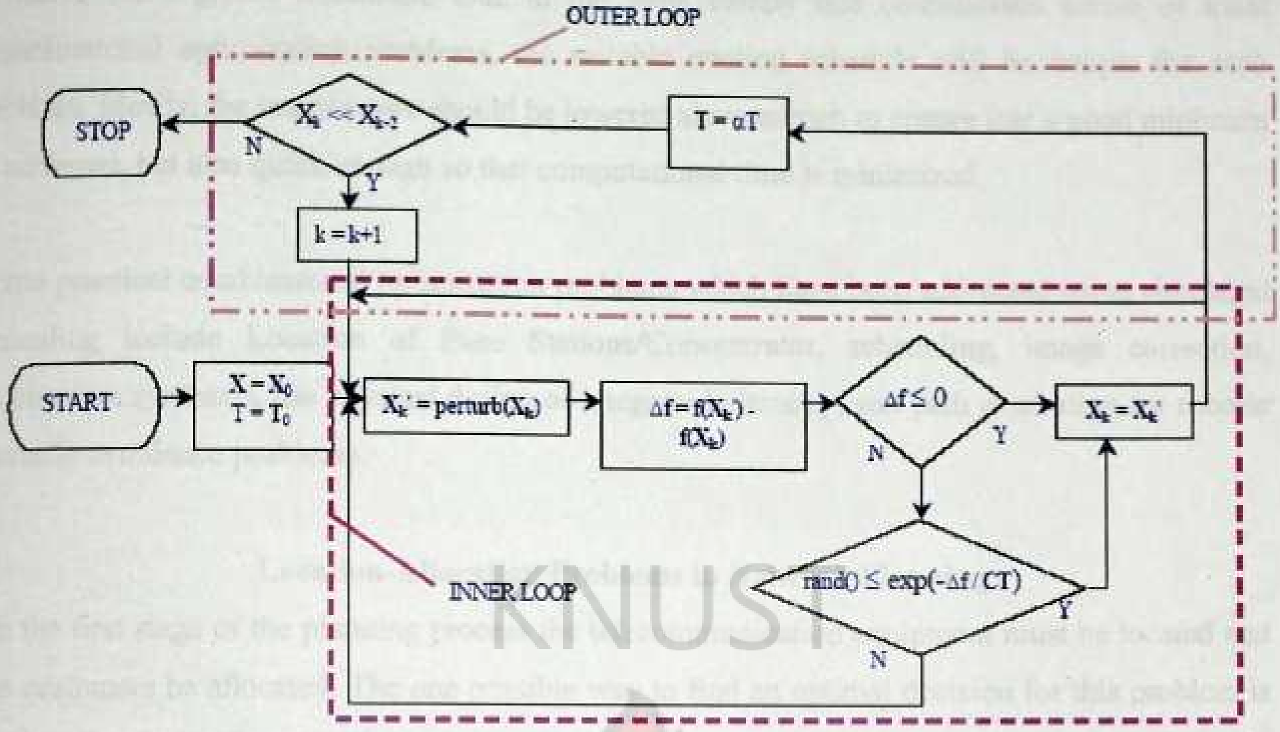

Figure 2.4: A Flowchart Representation of the Annealing Process.

Application of the annealing strategy to any engineering optimization problem requires definition of four major components.

Problem Configuration: a definition of the suitable domain over which the optimum can be sought. This knowledge is often expressed in the form of constraint equations.

Neighborhood Configuration: a method of iteratively perturbing the design vector to create new trial points.

Objective function: a scalar equation that weighs all of the design variables to provide a measure of the goodness for each trial point.

Cooling / Annealing Schedule: a methodology for specifying the maximum number of inner loop iterations and the manner in which the control parameter will be decremented in each iteration of the outer loop.

Often the most difficult step in the annealing process is the development of an appropriate cooling schedule. To ensure the success of the optimization, the temperature (control parameter) must be controlled so that it is large enough to move off a local minimum, but small enough not to move off a global minimum. Due to the wide variety and complicated nature of most combinational optimization problems, the suitable cooling schedule will be unique for each problem. Ideally, the temperature should be lowered slow enough to ensure that a good minimum is achieved, but also quick enough so that computational time is minimized.

Some practical combinatorial optimization problems which have been addressed using simulated annealing include Location of Base Stations/Concentrator, scheduling, image correction, mechanism synthesis, the physical design of integrated circuitry, and path generation for robotic obstacle avoidance problems.

Location-Allocation Problems in Network Planning

On the first stage of the planning process the telecommunication equipment must be located and the customers be allocated. The one possible way to find an optimal decision for this problem is to use cheaper communication equipment. In most cases this equipment consists of simple multiplexers and base stations. There are many known base station location problems. Some of them are the uncapacitated facility location problem and the capacitated facility location problem.

2.3.2 The uncapacitated facility location problem

The uncapacitated location models for telecommunication networks are concerned with the location of base station/concentrators and the assignment of terminal/end-users to these base stations/concentrators assuming that there is no capacity constraint for the base stations/concentrators. It is also often called a simple plant location problem (SPLP).

The uncapacitated facility location problem (UFLP) is a problem for the design of uncapacitated telecommunication networks [23]. These models are generalizations of the UFLP o they include the UFLP as a subproblem. The UFLP can be stated as follows: given the set of terminals *N* and the set of possible locations for the base station/concentrators *M*, determine the number and the location of base station/concentrators and assign the terminal/end-users to these base station/concentrators. The goal is to minimize the sum of the cost of installing base station/concentrators and the cost of serving terminals via the base station/concentrators. The formulation of the UFLP is as follows [23]:

$$\min \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} + \sum_{j \in J} f_j y_j$$

$$wrt$$

$$\sum_{j \in J} x_{ij} = 1 \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (a)$$

$$x_{ij} - y_j \leq 0 \cdots \cdots i \in I, j \in J \cdots \cdots \cdots \cdots \cdots \cdots (b)$$

$$x_{ij} \in \{0,1\} \cdots \cdots i \in I, j \in J \cdots \cdots \cdots \cdots \cdots \cdots (c)$$

$$y_j \in \{0,1\} \cdots \cdots j \in J \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (d)$$

I is a set of terminal/end-user locations. *J* is a set of sites where base stations can be located. c_{ij} is the cost of assigning terminal/end-user *i* to base station *j*. f_j is the setting-up cost of a base station at site *j*. The binary variable y_j is either 1 or 0 according to whether a base station/concentrator location is established or not. The binary variable x_{ij} is 1 if terminal/end-user *i* is connected to base station *j* and 0 otherwise. The constraints (a) guarantee that every terminal/end-user is connected to one and only one base station, whereas (b) say that a terminal/end-user can only be assigned to site *j* if this site contains a base station.

2.3.3 The capacitated facility location problem

Capacitated models are developed to decide about the locations of base stations/concentrators and the assignment of terminals without violating the capacity constraints of the base stations/concentrators [23]. This capacity can be defined in terms of the demand of the terminal assigned to a base station/concentrator or it may be in terms of the demands of the terminals. These problems are the same as the capacitated facility location problem with singles sourcing, i.e. each demand point can be assigned to a single facility and it is also called the capacitated base station/concentrator location problem (CCLP).

The CCLP is defined as follows [23]: given a set of terminal N with known demands d_i for each terminal $i \in N$, and a set of possible locations for base stations/concentrators M with capacities Q_j for each $j \in M$, the aim is to install base stations/concentrators and assign each terminal to exactly one base stations/concentrators so that the total cost of installing base stations/concentrators and the cost of assigning terminals to base stations/concentrators is minimized and the capacities of the base stations/concentrators are sufficient to supply the demand of terminals assigned to them. The other parameters and variables are defined in the UFLP. The CCLP can be formulated as follows:

$$\boxed{\begin{array}{l} \min \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} + \sum_{j \in J} f_j y_j \\ \quad w\,r\,t \\ \sum_{j \in J} x_{ij} = 1 \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (e) \\ \sum_{i \in I} x_{ij} - k_j y_j \leq 0 \cdots \cdots \quad j \in J \cdots \cdots \cdots \cdots \cdots \cdots (f) \\ x_{ij} \in \{0,1\} \cdots \cdots \cdots \cdots \cdots \quad i \in I, j \in J \cdots \cdots \cdots \cdots (g) \\ y_j \in \{0,1\} \cdots \cdots \cdots \cdots \cdots \quad j \in J \cdots \cdots \cdots \cdots \cdots \cdots (h) \end{array}}$$

CCLP is the same as UFLP except for constraints (f). They say that a terminal can only be connected to an established concentrator location and that no more than K_j terminals can be connected to a concentrator at site j.

2.3.4 Terminal Assignment Problem

The objective of the terminal assignment (TA) problem involves determining minimum cost links to form a network by connecting a given collection of terminals/end-users to a given collection of base stations/concentrators [24]. The terminals/end-users sites and base stations/concentrators sites have ~~fixed~~ location and are known. The capacity requirement of each terminals/end-users is known and may vary form one terminals/end-users to another. The capacities of all base stations/concentrators are also known. The problem is now to identify for each terminal/end-user the base station/concentrator to which it should be assigned, under two constraints, in order to minimize the total cost. The two constraints imposed on the TA problem are:

- Each terminal/end-user must be connected to one and only one of the base stations/concentrators.
- The aggregate capacity requirement of the terminals/end-users connected to any one base station/concentrator must not exceed the capacity of that base station/concentrator.

The intractability of this problem(s) is motivation for the pursuit of heuristics that produce approximate, rather than exact, solutions.

The following is a formal definition of the terminal assignment problem where we make use of Stinson's terminology for combinational optimization problem [24].

Problem instance

Terminals:	$l_1, l_2 \ldots\ldots\ldots l_T$
Weights:	$w_1, w_2 \ldots\ldots w_T$
Base station/Concentrator:	$r_1, r_2, \ldots\ldots r_c$
Capacities:	$p_1, p_2 \ldots. p_c$

w_i is the weight, or capacity requirement of terminal l_i.The weights and capacity are positive integers and $w_i \prec \min\{p_1, p_2 \ldots. p_c\}$ for $i = 1, 2 \ldots\ldots\ldots T$ .The T terminals and C concentrator are placed on the Euclidean grid that is l_i has coordinates (l_{i1}, l_{i2}) and r_j is located at (r_{j1}, r_{j2}).

Feasible Solution

Assign each terminal to one of the concentrators such that no concentrator exceeds its capacity. In other words, a feasible solution to the terminal assignment problem is:

A vector $\vec{x} = x_1, x_2 ....x_T$ where $x_i = j$ means that the ith terminal is assigned to concentrator j such that $1 \le x_i \le C$ and x_i is an integer, for $i = 1,2.........T$ (i.e., all terminal have to be assigned) and

$$\sum_{i \in R_j} w_i \le p_j \quad \text{for } j = 1,2.........c$$

(i.e., capacity of concentrator is not exceeded) where $R_j = \{i \mid x_i = j\}$, i.e. R_j represents the terminals that are assigned to coordinator j.

<u>Objective Function:</u>

A function

$$Z(\vec{x}) = \sum_{i=1}^{T} \cos t_{ij}$$

where $\vec{x} = x_1, x_2 ....x_T$ is a solution and $x_i = j$ and

$$\cos t_{ij} = round\left\{\sqrt{\left(l_{i1} - r_{j1}\right)^2 + \left(l_{i2} - r_{j2}\right)^2}\right\}$$

for $1 \le i \le T$ i.e., the result of rounding the distance between terminal i and concentrator j .in other words, $Z(\vec{x})$ denotes the overall cost of assigning individual terminal to concentrators according to the solution represented by $\vec{x}$.

<u>Optimal solution</u>

A feasible vector $\vec{x}$ that yields the smallest possible $Z(\vec{x})$

Example 1

Tables indicate a collection of T=10 terminals sites and C=3 concentrator sites. The weight requirement and the coordinates based on a 100x100 Euclidean grid for each terminal site are specified in *table 2.1*. The coordinates for the concentrator sites and their capacities are listed in *table 2.2.* The cost of assigning a terminal to a concentrator is the Euclidean distance between them rounded to the nearest integer [24].

Table 2.1: Terminal Capacity Requirement (weight) and Terminal Coordinates

Terminal (l_i)	**Weight (w_i)**	**Coordinates**
1	5	(54,28)
2	4	(28,75)
3	4	(84,44)
4	2	(67,17)
5	3	(90,41)
6	1	(68,67)
7	3	(24,79)
8	4	(38,59)
9	5	(27,86)
10	4	(07,76)

Table 2.2: Concentrator Capacities and Coordinates

Concentrator	**Capacity (p_i)**	**Coordinates**
1	12	(19,76)
2	14	(50,30)
3	13	(23,79)

Figure 2.5 illustrates an assigment of the first 9 terminals which connot be extended to the 10th terminal without intriducing infeasibilty, that is none of the 3 concentrators ia able to service the capacity requirement of terminal l_{10}

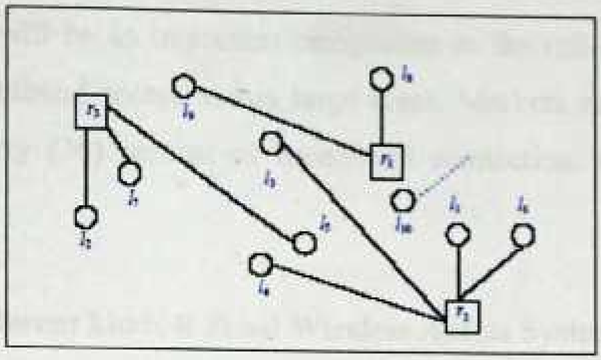

Figure 2.5: Illustration of Assignment Problem

The tatal cost computed as the sum od the cost for the 9 selected links is 223.

The problem has feasible solution though by inter-changing terminals l_4 and l_8 for instance, and assigning them to concentrator r_1 and r_2 respectively, r_1 will have enough room to accommodate l_{10}

2.4 TECHNOLOGY OF FIXED WIRELESS ACCESS

Fixed wireless Access is a variant of wireless broadband, where a radio link is used instead of a cable or fibre for the transmission of voice and data. There are numerous acronyms for the use of wireless technology in local loop applications, as well as a variety of applications which require differences in the choice of wireless technology to optimize the economics, performance and spectrum utilization for each one. Fixed wireless Access can, for example, be used for rapid internet access and video conferences. The communication goes from a transmitter to a fixed terminals fitted, for example, on a building roof, in contrast to mobile telephony where the communication goes from a transmitter to mobile terminals [25].

Fixed Wireless Access is suitable for broadband access in areas where a roll-out with fibre or other infrastructure is expensive, for example in sparsely populated areas. This is due to the fact that all users, following installation of a central base station, can get broadband access within the base station's area of coverage. Broadband access via fibre requires a cable to be laid to each user, which also entails higher installation cost than Fixed Wireless Access.

It is predicted that FWA will be an important component in the roll-out of broadband, as the technology facilitates broadband access across large areas. Markets estimates have shown that between four (4) and thirty (30) percent of broadband connection may comprise radio-base solution [25].

Different kinds if Fixed Wireless Access Systems

2.4.1 Point to Multi-Point

A Fixed Wireless Access system comprises, in its simplest form, a transmitter and a number of receivers. The transmitter often comprises of a central base station and the receiver is small equipment located with the user [25]. Those users located within the area covered by the base station can imply be offered broadband access after receiver equipment is installed with them. This kind of system is known as point to multi-point.

2.4.2 Multi-Point to Multi-Point

There are also FWA systems that have not been based on the use of large base station. In these systems, each user equipment constitutes a mini base station, which communicates with other user equipment. In this way a network of user equipment is created. New customers can be connected in proximity to the existing network. When new customers are connected, the area covered by the network increases simultaneously. This kind of system is called a mesh or multi-point to multi-point [25].

CHAPTER THREE

PROBLEM REPRESENTATION AND DATA ANALYSIS

3.0 Introduction

The purpose of this thesis is to construct a mathematical model to represent the problem i.e. the location of base station in telecommunication network planning for Fixed Wireless Access. In a mathematical model, variables are used to represent decisions, and the quality of decisions is measured by the objective function. Any restrictions on the values of decision variables are expressed by equations and inequalities.

Conceptual model

3.1 Design space

Design space for a FWA network is the geographical area that is sought covered. In the model this area is represented as a 2-D coordinate system.

3.2 End-users

End-users are the final consumers in the network. Their demands are voice, video and data services. The traffic load for these services is dynamic and it is represented by the average traffic load and the peak traffic, both in bits/sec. The load is the expected load of the end-user at a given year. The location of the end-user is either a company or a private domicile. In the model, each end-user address is converted into a coordinate set in the 2-D model design space. End-users are identified by the index *i* in the model. The demand of each end-user is given by his/her average load and is the one used in this planning.

3.3 Base station site

Potential base station sites are locations where it is possible to place a base station. These sites are indexed by *j* and *k* and represented by a geographical coordinate set. In the model these values are converted to a set of coordinates in the 2-D model design space. The capacity of each base station is measured in bits/sec. Like in the case of end-users the capacity is converted into a single number.

3.4 Radio propagation

The attenuation of radio signals is the limiting factor when considering the possible distance between base station and end-user. The attenuation is dependent of factors like obstructing buildings, rain attenuation, etc. For FWA the requirement for connection between base station and end-user is line of sight, LOS. Simplified, 'LOS' means it is possible for the base station antenna and the end-user antenna to 'see' each other. In the model the radio attenuation measured in dB is assumed linear dependent of the Euclidean distance between base station and end-user. A potential connection between a base station and an end-user requires that the Euclidean distance is below maximum transmission distance due to signal attenuation and where LOS exists. The signal strength at the end-user is assumed to be the transmitted power minus an attenuation constant multiplied by the Euclidean distance between end-user and base station.

3.5 Interference

When the network consists of more than one base station using the same frequency or one of the two adjacent frequencies, there is interference. Interference is measured as the relationship between the Carrier signal from the assigned base station and the sum of Interfering signals from other base stations, C/I. The interference is only interesting at the points where end-users are located. In this model the C/I is computed as the ratio of the signal strength at each end-user from its assigned base station and by the sum of signal strengths at the end-user, from all the rest of the base stations. The signal strength at an end-user from a given base station is computed according to the model mentioned in the preceding section 'Radio propagation' likewise is the interfering signal strength computed.

3.6 Optimization parameters

In step one in the network design process, the aim for the radio planner is to create a base station layout that ensures that all end-users are connected to a base station. This can be done numerous ways. Operators can have different quality targets toward the layout of the network such as "a maximal number of end-users must be covered" or "the load on each base station must be maximal". Most of these targets can be expressed using the three parameters: active base stations, end-users and load on base stations.

The most expensive components in an FWA-network are the base stations including the cost of the sites. This makes it attractive to minimize the number of base stations in order to reduce the costs of the network.

The load on each base station expresses the future options to develop the network, besides the potential revenue on the individual base station. The decision of placing a base station at a given location, or not, is made on the basis of the expected load at the location. Due to the initial cost of establishing base stations it is not attractive to establish a base station if the expected load at the location is less than a given threshold. In order to prepare the network for future expansion in the number of end-users using the established network is it desirable to have a close to even load on all base stations.

Finally the total number of end-users connected to base stations is an important parameter. The number of connected end-users is obviously closely related to the load on the base stations.

This gives three options for objective functions where the selected parameter is optimized. Instead of utilizing only one of the objective functions it is possible to use all three in a three-criterion solution method. Another option is to add a 'cost' on each parameter and then optimize the total 'cost' of the network. A special case of this function is where the sum of 'cost' constants is *1*. This special case is called a convex combination.

Regardless of the objective function chosen the function is optimized with respect to a lot of constraints. These constraints are imposed by the system limitations and expressed mathematically.

3.6.1 Objective functions

- Maximizes the minimal load variable.
- Minimizes the number of base stations (in step one only)
- Maximizes the number of end-users connected (in step two only)
- Optimize a convex combination of two or three of the parameters, number of active base stations, number of connected end-users and maximal minimal load on a base station.
- Minimize the total 'cost' of the network.

3.6.2 Constraints

- The sum of capacity demands connected to one base station must be larger than or equal to the minimum load variable or a minimum load constant.
- The sum of capacity demands connected to one base station must be less than or equal the maximum base station capacity.
- Each end-user must be connected to only one base station.
- End-users can only be connected to active base stations.
- End-users can only be connected to base stations if it is possible to get line of sight.
- The C/I at each end-user must be larger than or equal to a given threshold value.
- The signal strength at each end-user from the assigned base station must be larger than or equal to a given threshold value.

3.6.3 Model variables

When modeling these objective functions and constraints a number of factors are identifying base stations, capacity, legal connections, etc. The definitions of these factors are as follows:

Decision factors:

The optimization algorithm can adjust these variables within the given range, in order to achieve the best solution.

Conditional factors:

These values vary when decision variable values are altered, identifying the conditions of the system.

Structural factors:

These values are assumed constant within the time interval considered. Hence, the model cannot change these values.

Environmental factors:

Factors that are controlled outside the system considered but can affect the conditions of the system. These factors do not appear in this thesis.

The variables in the model are as follows:

Table 3.1: Variables used in the Model

Factor	Symbol	Role	Range
End-user id	i	--	$\{1,2,..,(I-1),I\}$
End-user coordinate	x_i,y_i	Structural factor	$[0;X_{max}, 0;Y_{max}]$
End-user demand	d_i	Structural factor	$[0;CAP_{lim}]$
Base station site id	j,k	--	$\{1,2,..,(J-1),J\}$
Base station coordinate	x_j,y_j	Structural factor	$[0;X_{max}, 0;Y_{max}]$
Transmission power at base station	P_j	Decision factor	$[0;P_{max}]$
Base station at site	b_j	Decision factor	$\{0,1\}$
Base station capacity	CAP_{lim}	Structural factor	R_+
Legal connections	k_{ij}	Structural factor	$\{0,1\}$
End-user – base station connec.	c_{ij}	Decision factor	$\{0,1\}$
Minimum number of end-user base station connection	C_{min}	Decision factor	$\{0,1..(I-1),I\}$
Signal at end-user from base station	s_{ij}	Conditional factor	$[0;p_j]$
Minimum C/I	C/I_{lim}	Structural factor	R
Minimum signal strength	SIG_{lim}	Structural factor	R
Load on base station	L	Conditional factor	$[0;CAP]$
Min. load on each base station	L_{lim}	Decision factor	$[0;CAP]$
Distance between end-user and base station	$Dist_{ij}$	Structural factor	R_+
Signal attenuation constant	$Att.$	Structural factor	R
Distance between two Base stations j,k	$O_{j,k}$	Structural factor	R_+
Minimum distance between two base stations	O_{min}	Structural factor	R_+
Objective value	Z	Conditional factor	R
Various costs	Q	--	R
'Big model constant'	M	--	$\rightarrow\infty$
Distance between base station j and most distant end-user connected to j	r_j	Conditional factor	$[0;Max\_dist]$
Density weight factor	w_i	Structural factor	R_+

Mathematical model

3.7 Optimization models

When designing mathematical models, the aim is to create the equations that identify the constraints of the system and the objective function that expresses the value of the system. When these equations have been formulated the model can give the answer whether a given parameter (decision factor) setting is legal or not and identify the value of the system as a single number. The model can now be used to find the parameter setting that gives the best system value by optimizing the objective function.

As mentioned, three parameters identify the value of the network design. These parameters are 'minimal load on base stations in the solution' named L, 'number of connected end-users' found as the sum of $c_{i,j}$-variables and 'number of active base stations' found as the sum of b_j-variables.

Despite it is only possible to optimize one parameter at a time none of the parameters can be isolated and optimized without considering the two other parameters. When one parameter acts as the objective function at least one of the other parameters has to be inserted into the model with an upper and lower bound. When setting the parameter bounds in the model, care must be taken not to make the model infeasible. This is regardless of the number of parameters inserted in the model is one or two. E.g. consider a situation where the minimal load on base stations is sought maximized. The number of base stations is bounded to a value where the product of base stations capacity and number of base stations is less than the total sum of end-user demands. In this situation the model is infeasible if all end-users must be covered according to the constraint on the number of end-users. However, removing the constraint on the number of base stations will not guarantee that the model is feasible. Depending on the distribution of the end-users the model could still be infeasible due to the constraints regarding interference. When using a multi-criterion solution method the valid combinations and ranges of these bounds are identified as a part of the process.

3.7.1 Maximizing minimal load

In step one of the optimization process the aim is to cover all end-users and to have a maximal minimal load L on each base station. This outlines a model with an object function that maximizes the minimal load and a constraint that set the sum of end-users equal to the total number of end-users. The sum of the number of base stations can be left unconstrained.

One obvious element in getting a maximal minimal load L on the base stations is to distribute the total load over a minimal number of base stations b_j. The minimal number of base stations that can cover all end-users is found by dividing the total sum of end-user capacity demands with the base station capacity. This number is then rounded up to the nearest integer. One upper limit of L is found as the total sum of end-user capacity demands divided by the minimum number of base stations needed. Intuitively, this objective function should provide a solution with a minimum number of base stations but it is not necessarily the case. The reason is that end-users are not evenly distributed in design space and base stations can only be located at discrete locations.

3.7.2 Maximizing number of connected end-users

In step two of the design process the idea is to choose a fixed number B of base stations and then find the maximal number of end-users that can be connected to B base stations among the available. The obvious objective function for this optimization is a model that maximizes the number of connected end-users. This model needs a bound on the number of active base stations.

A constraint that sets a minimum for the load on each base station can be added to the model. However, here it is important to notice that if the minimal load is set too high the model is infeasible. Omitting the load constraint, the model is always feasible. Though, the load on some of the base stations could end with zero.

3.7.3 Minimize the number of base stations

A model that minimizes the total number of base stations while keeping the number of connected end-users above the lower bound will at the same time implicitly maximize the average load on the base stations. A minimal number of active base stations require as many end-users as possible at each active base station. Whereas getting a result with maximal minimal load is unlikely the case. Again this is due to end-users not being evenly distributed in design space and it is only possible to locate base stations at discrete locations.

This objective function is not directly necessary in none of the two planning steps but is usable if the operator requires coverage of e.g. 90% of the end-users and wants to know how many base stations are needed. Summing up on these three relations, they indicate that a good final solution is a tradeoff between these three parameters. Methods considered for finding a good or best solution in this paper are the cost function and the multi-criterion solution method.

3.7.4 Static multi-criterion solution method

A multi-criterion method is a series of optimization. In each optimization one parameter is selected to be in the objective function. Some or all of the remaining parameters are inserted in the model, each one limited by two equations that sets the upper and lower bound for the parameter. This means finding the best solution for parameter *A* while maintaining given values for parameter *B, C, etc. Figure 3.1* illustrates the result of an imaginary 2-parameter example. The objective function optimizes parameter *B* and a constraint bounds parameter *A* to be greater than or equal to a given value in each optimization process. The figure shows the optimized value of parameter *B* at each fixed value of parameter *A*.

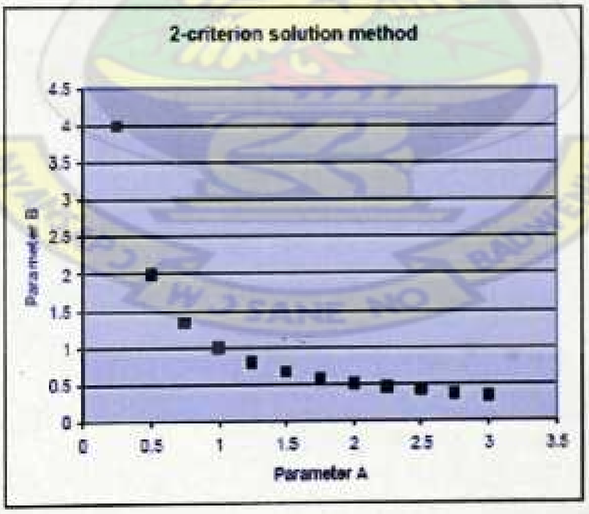

Figure 3.1: Two (2)-Criterion Solution Method

As can be seen from *Figure 3.1* no solution exists for values of A greater than 3. Equally, for values greater than 4 for parameter B the problem is infeasible. In the actual network *Figure 3.1* could illustrate the result of a series of optimizations where one of the parameters is uncontrolled. Consider a situation where the aim is to maximize the minimal load while maintaining a fixed number of end-users connected. In this case let parameter B illustrate maximal minimal load and parameter A be the number of end-users connected. At each optimization the number of end-users is fixed at a value within the desired range and after each optimization the result is plotted as a coordinate set of (end-users connected, maximal minimal load). Due to the fact that end-users are not uniformly distributed in design space, it is very likely that the maximal minimal load is low if the number of end-users connected is high and vice versa. Note that in this situation it is not necessary to control the number of base stations though it is possible.

For the actual problems with 3 parameters the process is technically the same but naturally becomes more time consuming due to the increased number of combinations of parameter settings.

The total model for finding the maximal minimal load in step one in the optimization process is shown in *Model 1*. Notice that L in this model is a variable.

$$Z = Max(L)$$

$$wrt$$

$$\sum_i c_{ij}.d_i + (1 - b_j).M \cdots\cdots\cdots \geq L \cdots\cdots\cdots\cdots \forall j \cdots i \in I, j \in J \cdots (5)$$

$$\sum_i c_{ij}.d_i \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \leq CAP_{\lim}.b_j \cdots \forall j \cdots i \in I, j \in J \cdots (6)$$

$$\sum_j c_{ij} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots = 1 \cdots\cdots\cdots\cdots \forall j \cdots i \in I, j \in J \cdots (7)$$

$$c_{ij} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \leq k_{ij}.b_j \cdots\cdots\cdots \forall j \cdots i \in I, j \in J \cdots (8)$$

$$\left[\sum_j s_{ij}.\left(1 - c_{ij}\right).k_{ij}\right].C/I_{\lim} \leq \sum_j \left(s_{ij}.c_{ij}\right) \cdots \forall j \cdots i \in I, j \in J \cdots (9)$$

$$\sum_j s_{ij}.c_{ij} \cdots\cdots\cdots\cdots\cdots\cdots\cdots \geq SIG_{\lim} \cdots\cdots\cdots \forall j \cdots i \in I, j \in J \cdots (10)$$

$$b_j.k_{ij}.\left(p_j - dist_{ij}.Att\right) \cdots = s_{ij} \cdots\cdots\cdots\cdots\cdots\cdots \forall j \cdots i \in I, j \in J \cdots (11)$$

$$b, c \in \{0,1\}; p \in [0, P_{\max}]; s \in R; L \in [0, CAP_{\lim}]$$

Model 1

Due to multiplication of two or more variables in *Equation 9, 10* and *11* this model is non-linear and hereby not necessarily optimally solvable.

Step two in the optimization process is to find the maximal number of end-users connected to a given number of base stations. A model similar to *Model 1* with a few adjustments is used for this purpose. The objective function in this case is maximizing the number of end-users connected. *Equation 7* is changed from equality to an inequality. Finally, a constraint is added that ensures that the sum of b_j-variables is equal to the desired number of base stations. Additionally, a constraint that controls the minimum load L on base stations is added. Unfortunately it is not possible prior to the optimization to give a value combination for number of base stations and minimum load L that necessarily makes the model feasible.

Rethinking the modeling of step two could as well be a model similar to *Model 1* where *Equation 7* is changed from equality to an inequality and another constraint is added. This constraint set a lower bound to the sum of connected end-users C_{lim} but leaves the number of base stations unconstrained. As mentioned earlier, this model will implicitly reduce the number of base stations, though not necessarily leading to a global minimum. The operator has to set a range for the desired coverage instead of setting a fixed number of base stations. Using the static multi-criterion solution method, one optimization is made for each value of the C_{lim}-value within the given range set by the operator. For each setting of the C_{lim} the optimization will provide an optimized value for L and the number of base stations. When optimizations with a sufficient number of C_{lim}-values within the given range have been performed, the operator's tradeoff can be performed on the basis of the network options.

3.7.5 Minimal cost function method

The basic idea of the cost function method is to use costs to guide the model towards a solution with the desired qualities. When using the minimal cost function the most difficult task is to put a cost on each parameter. E.g. the cost of a base station can be set relatively easily by using the cost of the base station in money. It gets more complicated when it comes to the cost of the parameter end-user and load.

One option to put a cost on the parameter end-user is to use the loss of income from each not connected end-user. Then the optimization model will weight the cost of adding an extra base station and get the extra income when extra end-users get connected.

Adding costs to load can be done in two ways. One option is to add a cost on load deviation from a given desired load on each base station. The problem here is that the difference between the desired load and the actual load can be either positive or negative. Using 'absolute value' or 'difference squared' functions to avoid this problem makes the objective function non-linear. The effect of this cost is that it finds the minimal sum of deviations from the given load level. Another option is to add a cost to the non-utilized capacity at each base station. This method is relatively easy to implement by multiply the cost value and the difference between capacity and load on each active base station. The cost value in connection with non-utilized demand can be set as the lack of income of each non-utilized kbps.

The major disadvantage with such a formulation of the objective function is that it can be difficult to find the correct weight of the cost values in order to get the desired effect on the network design. Thus, summing disadvantage costs gives a total disadvantage and no information about disadvantage deviation. This hides undesired variations in e.g. load on different base stations. Hence, this objective function maximizes the average load with no respect to deviation, whereas maximizing the minimal load objective function both maximizes the minimal load and minimizes the deviation between load on base station.

The mathematical formulation of the minimum cost model is as follows:

$$Min\left[\sum_j\left(CAP_{\lim}.b_j-\sum_i c_{ij}.d_i\right).Q_L+\sum_j b_j.Q_{BS}+\sum_{i,j}c_{ij}.Q_{EU}\right]$$

$$wrt$$

$$\sum_i c_{ij}.d_i\cdots\cdots\cdots\cdots\cdots\cdots\cdots\le CAP_{\lim}.b_j\cdots\forall j\cdots i\in I, j\in J\cdots(6)$$

$$\sum_j c_{ij}\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots=1\cdots\cdots\cdots\cdots\forall i\cdots i\in I, j\in J\cdots(7)$$

$$c_{ij}\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\le k_{ij}.b_j\cdots\cdots\cdots\forall i,j\cdots i\in I, j\in J\cdots(8)$$

$$\left[\sum_j s_{ij}.(1-c_{ij}).k_{ij}\right].C/I_{\lim}\le\sum_j(s_{ij}.c_{ij})\cdots\forall i\cdots i\in I, j\in J\cdots(9)$$

$$\sum_j s_{ij}.c_{ij}\cdots\cdots\cdots\cdots\cdots\cdots\cdots\ge SIG_{\lim}\cdots\cdots\cdots\forall i\cdots i\in I, j\in J\cdots(10)$$

$$b_j.k_{ij}.(p_j-dist_{ij}.Att)\cdots=s_{ij}\cdots\cdots\cdots\cdots\cdots\forall i,j\cdots i\in I, j\in J\cdots(11)$$

$$b,c\in\{0,1\};L\in[0,CAP]$$

Model 2

As can be seen, most of this model is equal to the maximizing minimal load model apart from a minimum load limit, *Equation 5*, and the object function, *Equation 4*. Similarly, the model is non-linear due to *Equation 9, 10* and *11*. In this formulation the model will find a solution that covers all end-users at the minimal cost with respect to the given costs Q.

3.7.6 Objective functions

The exact mathematical formulations of the object functions mentioned in the previous section are as follows:

$$Z = Max(L) \tag{1}$$

This function maximizes the minimum load variable L. Hence, at least one constraint has to be added to the model that bounds either minimum number of end-users connected or minimum number of active base stations.

$$Z = Min\sum_{j} b_j \cdots j \in J \tag{2}$$

This function minimizes the number of active base stations. When using this function with a constraint for minimum number of end-users connected, the average load on base stations is maximized. The difference from using *Equation 1* is that in *Equation 1* the minimum load is maximized whereas with this equation and the minimum number of end-users-constraint, the minimum load is uncontrolled. Modifying this objective function to maximize the sum of base station variables and use it in connection with a minimum load constraint it finds the maximal set of profitable base stations.

$$Z = Max\sum_{i,j} c_{ij} \cdots\cdots \forall i, j \cdots i \in I, j \in J \tag{3}$$

This function maximizes the number of end-users connected. Like *Equation 1* and *Equation 2* this objective function needs at least one constraint to control either the load on each base station or the number of base stations.

$$Z = Min\left\{\sum_{j}\left[\left(CAP_{\lim} - \sum_{i} c_{ij}.d_i\right).Q_L.b_j\right] + \sum_{j} b_j.Q_{BS} + \sum_{i,j} c_{ij}.Q_{ij}\right\} \cdots\cdots i \in I, j \in J \tag{4A}$$

$$Z = Min\left[\sum_{j}\left(CAP_{\lim}.b_j - \sum_{i} c_{ij}.d_i\right).Q_L + \sum_{j} b_j.Q_{BS} + \sum_{ij} C_{ij}.Q_{ij}\right] \cdots\cdots i \in I, j \in J \tag{4B}$$

These functions minimize the total cost of the network system. In *Equation 4A*, the first component, the demand connected to a base station j is subtracted from the capacity CAP_{lim}. This non-utilized capacity is multiplied a cost constant Q_L and the binary variable b_j that indicates whether a base station capacity is available or not. Finally summed over base stations j. Unfortunately, this construction results in a multiplication of the variables c_{ij} and b_j which makes the object function non-linear. It is crucial to have the binary bj variable present otherwise will non-active base stations contribute in a negative way with the total capacity.

Another way of adding a cost to non-utilized capacity is displayed in *Equation 4B*. Here the total demand connected to the base station j is subtracted from the capacity constant multiplied by the base station variable b_j. This formulation is possible due to a constraint that makes sure that demand only can be connected to active base stations. This gives, if the b_j is '0' then the total demand is also '0'.

The rest of *Equation 4A* and *Equation 4B* are identical.

In the second component establishing a base station, $b_j = 1$, is multiplied by a cost and summed over all base stations.

In the last component, the binary variable $c_{i,j}$, indicating if end-user i is connected to base station j, is multiplied with the negative cost of not connecting end-user i, summed over all combinations of i and base station j.

The advantage with this formulation is the flexibility of the object function. Any variable in the model can be priced and be added as a component in the object function. The layout of the function in *Equation 4A and 4B* is only one suggestion on how to price less desired elements in the network.

3.7.7 Constraints

The system constraints imposed by the system like maximal transmission range, total bit rate capacity, etc. must be respected. These constraints formulated mathematically are the following.

$$\sum_i c_{ij}.d_i + (1-b_j).M \geq L \cdots \forall j \cdots i \in I, j \in J \quad (5)$$

The connections between end-user i and base station site j multiplied with the demand at end-user i summed over end-users i, must be greater than or equal to the minimum load variable L. If base station j is inactive, $bj = 0$, the second component add a large constant, M, to the zero-demand and hereby makes the equation valid. The model generates one equation for each base station site j. This constraint ensures that the sum of loads from all end-users connected to base station j is above or equal to the minimum limit L if a base station exist at site j.

$$\sum_i c_{ij}.d_i \leq CAP_{\lim}.b_j \cdots \forall j \cdots i \in I, j \in J \quad (6)$$

The connections between end-user i and base station site j, multiplied with the demand at end-user i summed over end-users i, must be less than or equal to CAP_{lim} multiplied by the base station site j. This equation generates one inequality for each base station site j. This ensures that the sum of loads from all end-users connected to base station j is below or equal to the capacity at base station j.

$$\sum_j c_{ij} = 1 \cdots \forall j \cdots i \in I, j \in J \quad (7)$$

The connections between end-user i and base station site j summed over base station sites j must be equal to one. The model generates one equation for each end-user. When c_{ij} is binary, this ensures that each end-user is assigned to exactly one base station site only and due to the equal sign each end-user does get assigned to a base station. In a model that strives toward a solution without all end-users, the equation must be changed from an equality to an inequality.

$$c_{ij} \leq k_{ij}.b_j \cdots \forall j \cdots i \in I, j \in J \quad (8)$$

The connection between end-user i and base station site j must be less than or equal to the product of the legal connection identifier k_{ij} between end-user i and base station site j and the active base station identifier b_j. The model generates one equation for each combination of i and *j*. The equation ensures that end-users only get connected to active base stations using legal connections.

$$C/I_{\lim} \le \frac{s_{ij}.c_{ij}}{\left[\sum_j s_{ij}.(1-c_{ij}).k_{ij}\right]} \cdots \forall j \cdots i \in I, j \in J \tag{9A}$$

$$\left[\sum_j s_{ij}.(1-c_{ij}).k_{ij}\right].C/I_{\lim} \le s_{ij}.c_{ij} \cdots \forall j \cdots i \in I, j \in J \tag{9B}$$

In *Equation 9A*, the signal power at end-user *i* from base station site *j* assigned to the end-user divided by the sum of signal powers at end-user *i* from base station sites *j* *not* assigned to the end-user *i* must be greater than or equal to C/I_{lim}. However, equations containing divisions with variables are always non-linear. The way to avoid this is to multiply both sides of the inequality sign with the denominator. This has been done in *Equation 9B*. The model generates one equation for each end-user. This ensures that carrier to interference ratio is above or equal to the threshold value. Due to the multiplication of the two variables c_{ij} and s_{ij}, use of this equation will make the model non-linear.

$$\sum_j s_{ij}.c_{ij} \ge SIG_{\lim} \cdots \forall j \cdots i \in I, j \in J \tag{10}$$

The signal strength at end-user *i* from its assigned base station *j* must be greater than or equal to the minimum signal strength limit. The model generates one inequality for each end-user *i*. This ensures that the signal strength at each end-user from the assigned base station is sufficient. Due to multiplication of the two variables c_{ij} and s_{ij} use of this equation will make the model non-linear.

$$b_j.k_{ij}.\left(p_j - dist_{ij}.Att\right) = s_{ij} \cdots \forall j \cdots i \in I, j \in J \tag{11}$$

The output power at base station *j* minus the attenuation product must be greater than or equal to the signal strength from base station *j* at the end-user *i*. The attenuation product is assumed as a constant multiplied by the distance between base station *j* and end-user *i*. The signal strength is only computed at each end-user from active base stations, $b_j = 1$ and at legal connections, $k_{ij} = 1$. This equation sets the output power value for each base station measured in dBm. Due to the multiplication of the two variables b_j and p_j use of this equation will make the model non-linear.

$$\sum_{i,j} c_{ij} \geq c_{\min} \cdots \forall i, j \cdots i \in I, j \in J \tag{12}$$

The sum of end-user-base station connections summed over end-users *i* and base stations *j* must be greater than or equal to the value C_{min}. This equation generates only one constraint containing all potential end-user-base station connections. This equation does only have a function if the equality in *Equation 7* is changed to an inequality. This equation ensures that at least *Cmin* end-users get connected to a base station.

DATA ANALYSIS

An analysis of the design space with end-user and base stations sites prior to problem solving can reveal important information that can be very useful when optimizing.

3.8 Input data

The input data for the model is provided by the operator and are typically a spreadsheet with columns containing end-user id, an end-user value proportional to the end-user demand and an (x, y)-coordinate set of the end-user location. Equally, the potential base station sites have an id code and an (x, y)-coordinate set. The provider of the radio equipment supplies the input data regarding the equipment: The minimum and maximum power output, the need for signal strength at the end-user, the signal attenuation factor and the minimum C/I ration accepted.

3.9 Refining the data

From the given set of data a number of pre-run computations are necessary in order to make the data useable in the model.

3.9.1 Distance table

The distances between end-users and base stations are simply computed as Euclidean distances.

$$dist_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}, \forall i, j \tag{13}$$

3.9.2 Legal connection table

A connection in the network is legal if the signal strength at the end-user can be over the signal limit. In this model signal attenuation measured in dB is assumed linearly dependent of the distance with only one attenuation factor for the entire design space. Hence, it is possible to identify the maximal transmission distance and hereby find the legal connections with respect to signal strength.

$$k_{ij} = \begin{cases} 1 & if\,(dist_{ij} \le dist_{max} \wedge LOS\,exist) \\ 0 & else \end{cases} \forall i,j \tag{14}$$

3.9.3 Graph analysis

One analysis of the design space is performed by creating a graph with base station nodes V_{BS} and end-user nodes V_{EU} and legal base station-end-user edges $E_{BS\text{-}EU}$, $G((V_{BS}, V_{EU}), E_{BS\text{-}EU})$. See *Figure 3.2*. The graph is connected if a path connects any pair of nodes in the graph.

If the aim is to connect all end-users to base stations then it is relatively easy to identify base stations which must be in the solution. All base stations that connect to end-users with valency '*1*' need to be in the solution. . 'Valency *1*' means that the end-user in question can only connect to one base station.

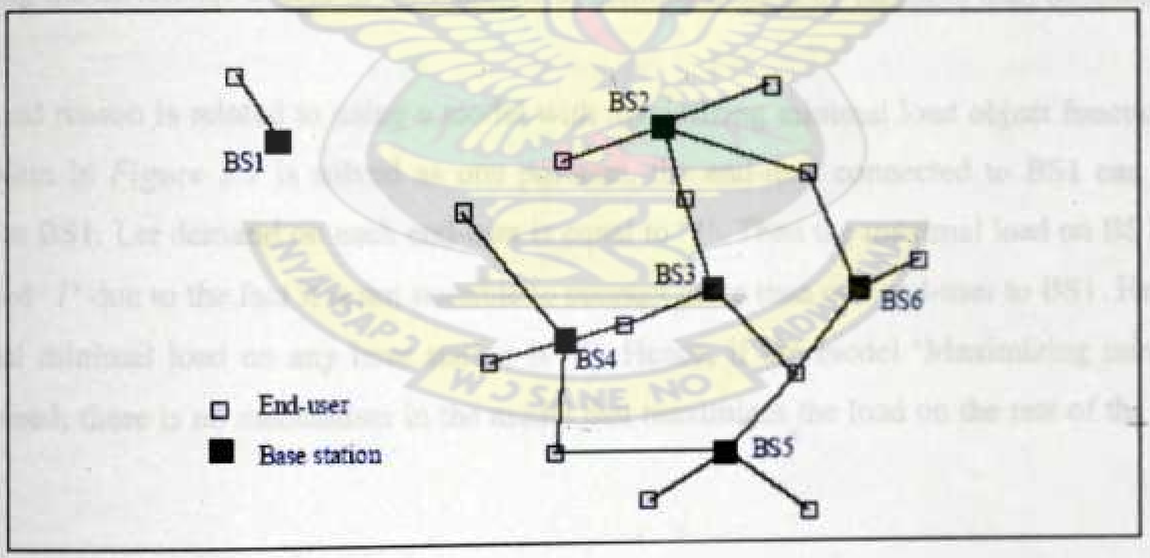

Figure 3.2: Graph Analysis of End-users to Base Station Connection

If the graph is not connected the problem related to each connected component should be solved separately for two reasons:

The first reason is that it makes solving the problem unnecessarily complicated if two separate problems are attempted to be solved as one problem. This is closely related to the use of binary variables to indicate if a base station is active or not. Consider an example with 10 potential base stations that can be divided into two sub-problems with 5 base stations in each. Assume 6 of the 10 base stations can cover all end-users, 3 base stations in each sub-problem. Using the binomial coefficients the number of combinations to be evaluated in each case can be computed.

$$\text{As one problem:} \frac{10!}{6!(10-6)!} = 210$$

$$\text{As two problems:} 2.\frac{5!}{3!(5-3)!} = 20$$

Example 2

As can be seen in *Example 2* the number of combinations is reduced by a factor of *10* when the problem is split-up into smaller problems. Additionally, when problem size is doubled the factor identifying the difference in size in the numbers of combinations is by far more than doubled.

The second reason is related to using a model with maximizing minimal load object function. If the problem in *Figure 3.2* is solved as one problem, the end-user connected to BS1 can only connect to BS1. Let demand on each end-user is equal to '*1*'. Then the maximal load on BS1 can not exceed '*1*' due to the fact it is not possible to connect more than one end-user to BS1. Hereby the global minimal load on any base station is '*1*'. Hence, if the model 'Maximizing minimal load' is used; there is no mechanism in the model that maximizes the load on the rest of the base stations.

Finally, when optimizing it is important to know when it is not possible to improve the best solution found, so far. This is especially important when dealing with problems with discrete variables. The straightforward way of solving problems with discrete variables is to 'relax' the integrality requirement and solve this continuous problem. If this method provides a solution where the relaxed variables are integer values the problem is solved to optimality. Otherwise is it necessary to use other methods to find an optimal integer solution.

If all end-users must be covered, this data analysis can provide an upper bound to the variable L in the Maximizing minimal load problem. In this problem an upper bound for L is the minimal maximal sum of demands that legally can be connected to any base stations that have to be in the solution. As mentioned earlier the base stations that must be in the solution are those connected to end-users with valency '*1*'. When the optimization has reached a solution where all base stations in the solution have a load greater than or equal to the upper bound for L the process can be stopped. At the connected component to the right in *Figure 8* it is easy to see that the base stations BS2, BS4, BS5 and BS6 have to be in a solution. This is due to at least one end-user potentially connected to each of these base stations has valency '*1*'. The minimal maximal potential load on base stations that have to be in the solution is on BS6. The maximal load here is '*3*'. On BS2, BS4 and BS5 the maximal load is '*4*'. Hence, the upper bound for L is '*3*'. Hence, the solver can be stopped when a solution that covers all end-users and has minimal load of '*3*' has been found. Furthermore, this makes it potentially profitable to perform a new optimization using a model where the objective function is minimizing the number of active base stations. The minimum load on each base station L found in the previous optimization is now added as constraint in the model. Due to the L-value being the result of the previous optimization this model is feasible. This second optimization gives the three-criterion solution where all end-users are covered by a minimum number of base stations with the maximal load on each base station.

CHAPTER FOUR
SOLUTION TECHNIQUES

4.0 Solving the Problem

The timeframe for solving the problem should according to the thesis statement be less than the time spent doing this part of the planning by hand. There fore the model must provide a good solution within one hour. The time is measured from when the input data are read from a file where end-users are listed with x- and y-coordinates and demand, and the potential base stations are listed in a file with x- and y-coordinates. The planning is finished when a file with the selected base stations and a file with base station end-user connections have been stored.

Due to the non-linearities mentioned previously, neither *Model 1* nor *Model 2* can necessarily be solved optimally. However, fixing all binary variables makes the models linear. Therefore, one option is to solve the problem by generating all possible combinations of binary variable settings, solve the linear model in each setting and selecting the best solution. The number of linear problems to be solved or possible binary combinations is relatively difficult to find. Fortunately, it is not necessary to find in order to be convinced that this method is not feasible. Consider a situation where the number of base stations in the final solution is known to be 12 out of 50 potential base stations and end-users must connect to the nearest active base station or not connect at all. In this situation the number of possible solutions is the binomial coefficient of 12 out of 50, which is about 1.21E11. Notice that due to the requirement of connecting end-users to base stations in this example, the base station end-user variables is set when the active base stations are given. To be able to evaluate this number of solutions within one hour, the number of solutions evaluated each second must be approximately 34 million. Recall that this situation is when information about the final number of base stations is known and the constraints controlling the connections between end-users and base stations are simple. Even in this simple case it appears to be impossible to evaluate the number of solutions needed within the given timeframe.

The goal is now to find what is the best solution that can be found within the given timeframe.

4.1 Simulated Annealing Algorithm

In an attempt to use a heuristic it is decided to use simulated annealing. This is due to observed ability to produce good solutions in other problems combined with the easy implementation of this heuristic. Besides it is assumed that the evaluation is relatively hard and therefore it is attractive to use a heuristic that require few evaluations for each move. Simulated annealing is a descent-search algorithm. Initialize with a start solution and a start temperature. Find the objective value by evaluating the start solution. Then find a neighborhood solution and evaluate this solution and move to this solution with a probability. If the new objective value is better than the actual value, the probability is '1'. Otherwise the probability is found as a function of the difference between the actual objective value and the new objective value and the temperature. After a number of solutions the temperature is lowered hereby lowering the probability of going to a worse solution. When the heuristic reaches the stop criterion it terminates.

In pseudo code Simulated annealing looks like this:

- Set t = start temperature
- Find a start solution (x)
- Compute the objective value $F(x)$
- Set global best solution $F(x^*) = F(x)$ and $(x^*) = (x)$
- until global stop criterion is reached do
  - until local stop criterion is reached do
    - find a neighborhood solution (x') and compute objective value $F(x')$
    - if $F(x') <= F(x)$
      - set $(x) = (x')$
    - else $F(x') > F(x)$
      - set p = random number $\in [0;1]$
      - if $exp((F(x)-F(x'))/t) < p$
        - set $(x) = (x')$
      - if $F(x') < F(x^*)$
        - set $(x^*) = (x')$
  - reduce temperature t

Before setting up the heuristic the following definitions have to be made.

- Problem representation
- Start solution
- Neighborhood
- Solution evaluation
- Cooling scheme
- Stop criterion

4.2 Problem representation

The design of the problem representation is made with focus on ease of computing. The straightforward method is to use the model concept from the mathematical models. In the model the number of variables is in the worst case roughly the number of base stations multiplied by the number of end-users. A large number of these variable combinations are illegal. For instance, each end-user can only be connected to one base station and base stations cannot cover end-users that are farther away from the base station than the maximal transmission distance of five kilometers (5km). When generating a random start the risk of generating an illegal solution is large. Hence, there is a risk of wasting time on evaluating illegal solutions. Alternatively, building up a solution while performing simultaneous check of the legality can be computationally demanding. Therefore the result may be that the number of legal solutions evaluated is the same as if allowing generation of illegal solutions. Hence, it seems potentially profitable to consider the model of the network aiming for a solution representation that is easy to evaluate.

4.2.1 Graph models

Inspired by article [17] the legal connections between base stations and end-users can be described as a network *Figure 4.1*. This network is a bipartite graph with base stations on one side and end-users on the other. Edges between base stations and end-users are legal connections i.e. an edge connects an end-user with a base station if it is possible to get sufficient signal strength and LOS at the end-user from the base station despite the signal attenuation. This simple network model only describes potential connections. However, the final FWA-network can be considered as a flow network where each end-user has a demand and where base stations have a capacity.

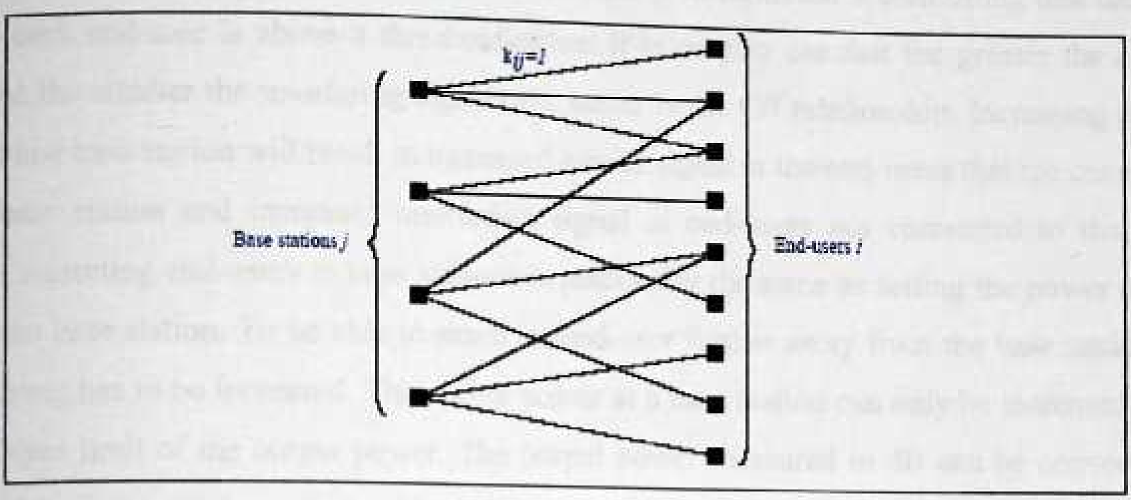

Figure 4.1: Graph Model 1

Taking the demand and capacity parameters into account, the flow network can be modeled as in *Figure 4.2*. When finding the maximal flow from the super source through the network to the super sink, the load on each base station and on each link between base station and end-user is given. The disadvantages with this model are e.g. the end-users can be serviced by more than one base station at the same time and this model does not strive toward a maximization of the load on the active base stations thereby minimizing the number of active base stations.

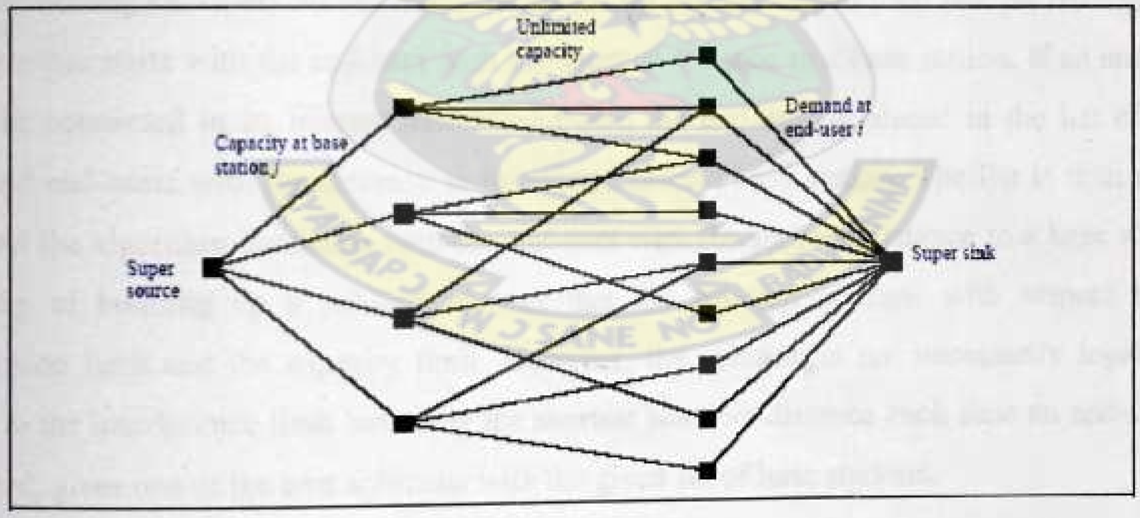

Figure 4.2: Graph Model 2

Both *Figure 4.1* and *Figure 4.2* give inspirations to formulate the algorithm that connects end-users to base stations. Consider the non-linear constraint in *Equation 9* controlling that the C/I-value at each end-user is above a threshold value. It is easy to see that the greater the carrier signal and the smaller the interfering signals the better is the C/I relationship. Increasing output power at one base station will result in increased carrier signal at the end-users that are connected to that base station and increased interfering signal at end-users not connected to that base station. Connecting end-users to base stations is practically the same as setting the power output level at the base station. To be able to reach an end-user further away from the base station the output power has to be increased. The output power at a base station can only be increased up to the hardware limit of the output power. The output power measured in dB can be converted to transmission distance using a linear function.

In the attempt to create a way of generating solutions, end-users are connected to the nearest base station if this base station has sufficient capacity available. Otherwise it is attempted to connect the end-user to the second nearest base station if possible else the third and so on until the end-user is either connected or the distance to the nearest base station with available capacity is exceeding the maximal transmission limit. If the distance to the nearest base station with vacant capacity is exceeding the maximal transmission distance the end-user is not connected at all. When connecting end-users all end-users are sorted by the distance to their nearest base station. The algorithm starts with the end-user with the shortest distance to a base station. If an end-user cannot be connected to its nearest base station then the end-user is placed in the list of non-connected end-users with the distance to its second nearest base station. The list is then sorted again and the algorithm continues with the end-user with the shortest distance to a base station. This way of building up a solution ensures that the solution is legal with respect to the transmission limit and the capacity limit. However, the solution is not necessarily legal with respect to the interference limit but using the shortest possible distance each time an end-user is connected, gives one of the best solutions with the given set of base stations.

Finally, due to the unambiguous way of connecting end-users to base stations the entire solution is given when a set of active base ~~stations is~~ selected.

4.3 Start solution

The initial solution is the starting point for the search for good solutions. In this problem the initial solution can be given as a set of active base stations. Depending on the target for the optimization the minimum number of active base stations can be pre-computed. If all end-users must be connected, the minimum number of active base station is the total demand divided by the capacity of the base station rounded up to nearest integer. However, it is not certain that all end-user can be connected using any set of base stations of this size or any other size due to interference constraints.

The initial solution for this problem is found by a random selection of base stations. End-users are connected to the selected base stations as indicated in the previous section.
The solution is constructed in the following way

- find a set of base stations
- find nearest active base station for each end-user
- Connect end-users to base stations starting with the shortest end-user base station connection then the second shortest and so on.

Before each end-user is connected a number of checks have to be performed to ensure that the solution still is legal when the actual end-users have been connected. The checks performed are the following:

- Max. transmission distance.
- Max. base station capacity.
- Maximal simple overlap between the actual base station and all neighboring base stations.
- Maximal tapering difference between the actual base station and all neighboring base stations.

If the end-user exceeds the limits in either one of the checks the end-user is not connected at all. The program then proceeds to the next end-user.

4.4 Neighborhood

The neighborhood is defined as the solutions that can be reached by one move from the actual solution.

In this problem the neighborhood is defined as follows:

- One active base station is turned off
- One inactive base station is turned on.

These two operations change the final number of base stations. In some cases it is desirable to be able to control the final number of base stations. Hence, these two operations can be set inactive. If they are active a random process controls which one of all three operations that is used in a given iteration.

4.5 Solution evaluation

For a given solution it is necessary to be able to evaluate if the solution is better or worse than the previous solution.

In the heuristic the objective function evaluation is initially based on the number of not connected end-users. The function for finding the number of not connected end-users is

$$\text{Not Connect end-users} = \sum_{i}\left(1-\sum_{j} c_{ij}\right)\cdots i \in I, j \in J \tag{15}$$

The aim of this optimization is to find a solution where a minimal number of end-users are not connected.

The heuristic also considered some additional options for the optimization. These options are minimal sum of not connected demand and minimal sum of either not connected demand or end-users weighted by a density measuring. In *Equation 16* is the general equation for the objective function. If neither demand, nor weight are desired in the solution the respective variables, d_i and w_i, are set equal to 1.

$$weighted\_demand = \sum_{i}\left(1-\sum_{j} c_{ij}\right).d_i.w_i \cdots \forall i \in I, j \in J \tag{16}$$

The density weight is made the following way. Design space is tiled into a 20 times 20 grids that makes the boundaries for the density measuring. The demand or number of end-users in each tile is summed. These 400 values are normalized by dividing the each value by the largest value and added a constant of '*0.5*'. The result is a value in the interval [*0.5; 1.5*] for each of the 400 tiles. When computing the objective value each not connected end-user or demand is multiplied by the weight of the tile where the end-user is located. The advantages using these weights in the objective function is that the heuristic strives toward a solution where covering densely populated areas have a higher priority than covering sparsely populated areas. This is in line with radio cellular network operators.

4.5.1 Circle overlap

The total circle overlap is computed as the sum of area of design space that is covered by both base stations in any pair of active base stations. The formula for computing this area is given by

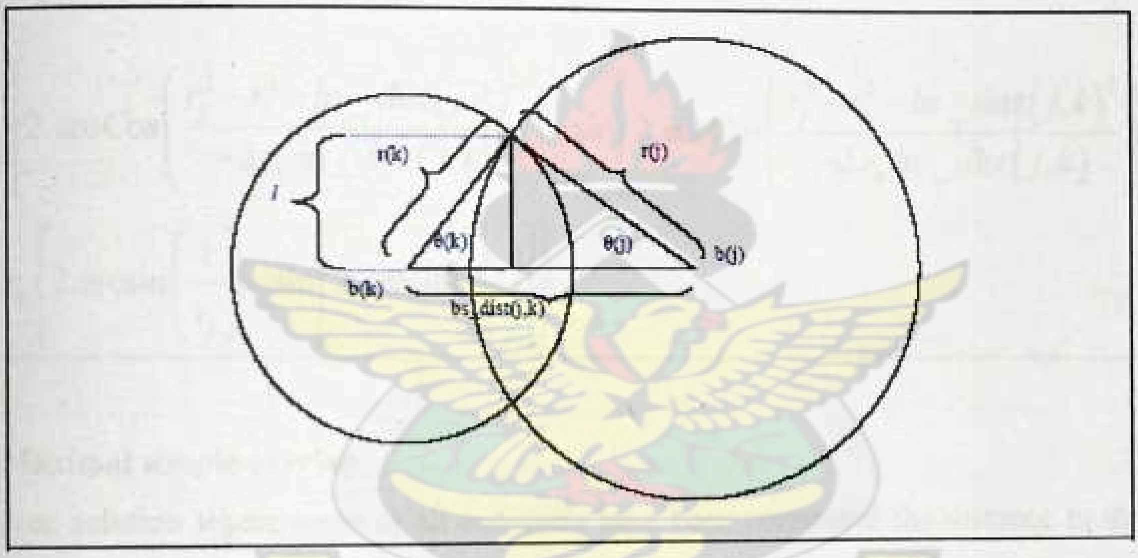

Figure 4.3: Circle Overlap Diagram

$$\cos(\theta_k) = \frac{r_j^2 - r_k^2 - bs\_dist(j,k)^2}{-2.r_k.bs\_dist(j,k)}$$

$$\Downarrow$$

$$l = \sqrt{\left\{\left[r_k\left(\frac{r_j^2 - r_k^2 - bs\_dist(j,k)^2}{-2.r_k.bs\_dist(j,k)}\right)\right]^2 + r_k^2\right\}}$$

$$\Downarrow$$

$$\sin(\theta_j) = \frac{1}{r_j}$$

$$\Downarrow$$

$$bs\_ol(j,k) =$$

$$\Downarrow$$

$$\frac{1}{2}.r_j\left\{2.\mathrm{arc}Cos\left(\frac{r_j^2 - r_k^2 - bs\_dist(j,k)^2}{-2.r_k.bs\_dist(j,k)}\right) - \sin\left[2.\mathrm{arc}Cos\left(\frac{r_j^2 - r_k^2 - bs\_dist(j,k)^2}{-2.r_k.bs\_dist(j,k)}\right)\right]\right\}$$

$$+\frac{1}{2}.r_k\left\{2.\arcsin\left(\frac{1}{r_j}\right) - \sin\left[2.\arcsin\left(\frac{1}{r_j}\right)\right]\right\}$$

4.5.2 Maximal simple overlap

For a given solution where some or all end-users have been connected the distance to the most distant end-user at each base station identifies the current radius of the coverage circle. The distance between any two base stations must not be less than a given percentage of the sum of the covering radii of the base stations. See the example in *Figure 4.3*. Here the sum of r_k and r_j multiplied by a percentage, K, must be less than the distance between the base stations.

$$(r_j + r_k).K \prec bs\_dist_{i,j} \cdots \forall j,k \in J; k \in [0,1] \quad \textbf{(17)}$$

4.5.3 Load deviation

The load deviation is the sum of difference between utilized capacity and a given minimum load on base stations where the utilized capacity is less than the minimum load 'L'. Load deviation is computed using *Equation 18.*

$$Load\_deviation = \sum_j \left(L - \sum_i c_{ij} d_i \right) b_j \cdots j \in J \left| \left(\sum_i c_{ij} d_i \leq L \right) \cdots i \in I \right. \quad \textbf{(18)}$$

4.5.4 Tapering

For any base station the distance to the most distant end-user identifies the radius of the coverage circle. Tapering constraint is formulated as a bound in a way where the latest end-user-base station connection is legal if the following apply. For any pair of the actual connecting base station and all other base stations, the smallest radius is greater than the largest radius multiplied by a constant.

$$\min\left(r_j, r_k\right) \succ K.\max\left(r_j, r_k\right) \cdots \forall j,k \in J \left| \max\left(r_j, r_k\right) \succ \frac{bs\_dist_{i,j}}{2}; K \in [0,1] \right. \quad \textbf{(19)}$$

The tapering constraint does not apply for any pair of base stations when the algorithm starts. If end-users were evenly distributed it would cause no problem to connect end-users. This is due to the fact that coverage circles then would 'grow' at almost the same rate. At the point when two circles starts to overlap they are almost the same size. In the actual problem end-users can be distributed in any fashion. Consider a situation where the border between a densely populated area and a sparsely populated area is narrow *see Figure 4.4*. Because the end-user that is about to be connected is the one with the shortest distance to a base station the cover circles at base station j and k will have approximately the same size when base station k have no more vacant capacity. As the process of connecting end-users continues the size of the coverage radius at base station j grows. Naturally no end-user located on the right side of the center normal can be connected to base station j due to the fact that end-users must only connect to the nearest base station. Therefore no end-user located at the right side of the center normal can induce a cover circle on base station j exceeding the center normal. End-users located on the left side of the center normal but more distant from the base station than the center normal, can be connected to base station j. Hereby this or these end-users induce a cover circle that also cover parts of the area to the right of the center normal. If the check of tapering is made only when circles overlap then the solution is already illegal. Hence, it is decided to make the tapering check when at least one of the cover radii exceeds the center normal.

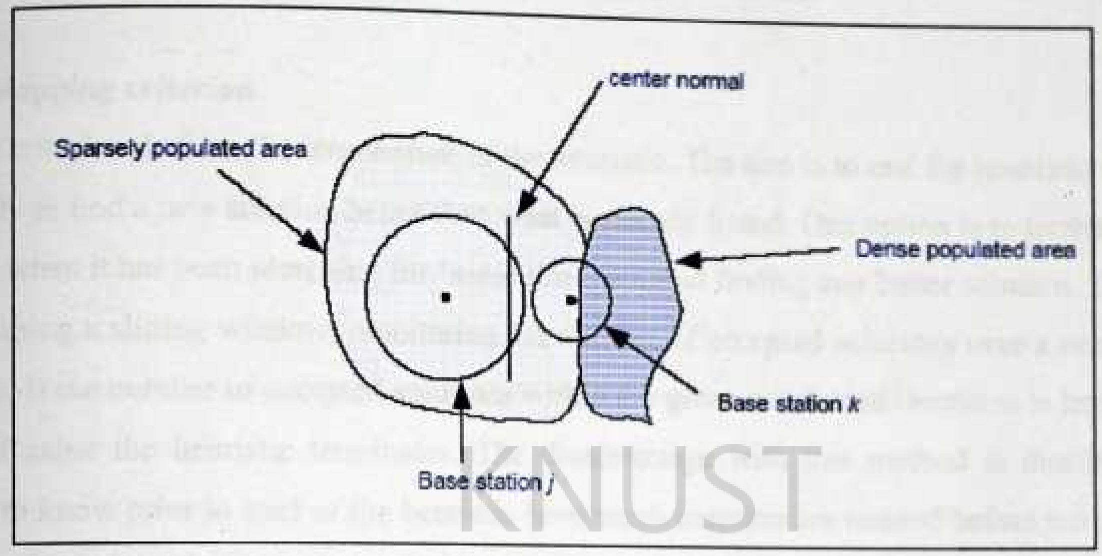

Figure 4.4: Population Distribution Diagram

4.6 Cooling scheme

The cooling scheme indicates how the temperature is going to be reduced over time and hereby reducing the probability of going to a solution worse than the actual solution.

The easiest cooling scheme is to reduce the temperature by it with a constant after a specified number of moves. The result of this is a logarithmic temperature curve. The optimization is then done by performing a fixed number of iterations at each temperature level until the stop criterion is reached. However, when the temperature is high the algorithm accepts more or less all neighborhood solutions and performs no actual descent search. Hence, in some cases it can be desirable to reduce the temperature at a faster rate when the temperature is high. Using both 'number of accepted solutions' and 'a fixed number of evaluated solutions' as criterion reducing the temperature can do this. The accepted number of solutions is naturally lower than the fixed number of solutions.

At the end of the optimization it is essential that the temperature is not too low before the stop criterion is reached. If the temperature is too low the algorithm will not accept any 'bad' solutions at all and hereby making further search futile. One option is to 'reheat' and hereby make it possible for the algorithm to accept 'bad' solutions again. In the actual case it is decided to use a simple cooling scheme. After evaluating a fixed number of neighborhood solutions the temperature is lowered by multiplying the temperature by a constant.

4.7 Stopping criterion

The stop criterion decides the termination of the heuristic. The aim is to end the heuristic when it is unlikely to find a new solution better than what is already found. One option is to terminate the heuristic when it has been searching for 'some time' without finding any better solution. This can be done using a sliding window, monitoring the number of accepted solutions over a number of iterations. If the number of accepted solutions within the given number of iterations is less than a threshold value the heuristic terminates. The disadvantage with this method is that it is not possible to know prior to start of the heuristic how much iteration are needed before termination and hereby determine how much time the optimization will take.

Another stop criterion is to terminate the heuristic after a fixed number of iterations. This method requires a number of test runs in order to determine how many iterations must be performed until the heuristic does not find any new better solutions within a given 'window'. Naturally, the point when the heuristic does not find any better solutions is in some way dependent on the temperature. If the temperature is low and the heuristic has been searching the local area for some time there is a good chance hat it has found the local minimum and cannot escape from there and therefore cannot get to another local minimum.

Due to the fact that in the current case time is a critical parameter, it is decided to use a stop criterion that terminates the heuristic after a fixed number of iterations. Hereby is it possible prior to optimization to compute how long it takes until the heuristic terminates.

4.8 Parameter setting

The following parameters must be set prior to perform the test run:

- Start temperature
- End temperature
- Cooling rate
- Number of local iterations
- Number of global iterations

All five parameters are in some way related to each other. In the current case the optimization must be performed within the given timeframe of one hour. When the time/iteration is known it is simple to determine how much iteration can be performed within this given timeframe. Let '*TI*' indicate the total number of iterations that can be performed. After *TI* iterations the temperature must be reduced from the start temperature to the end temperature.

Start and end temperatures are relatively easy to find using the following procedures. In order to make a good search of the solution space the start temperature must be set at a level where the heuristic accepts many of the generated solutions. This regardless that for each pair of new and actual solutions the new solution can be worse than the actual one. In literature about simulated annealing [8] accept rates at 50-70% of all generated solutions are considered to be sufficient for a good search. For finding the start temperature a first guess of the temperature is made, then 10 test runs with 100 solutions generated in each is made and the average number of accepted solutions in those 10 times 100 solutions are computed. Then the temperature is adjusted and 10 more test runs are made. This is continued until the number of accepts is satisfactory.

When setting the end temperature the task is to ensure that time is not wasted by performing search for solutions when the end temperature is too low to accept any worse solutions at all. However, the end temperature must not be set so high that the heuristic does not perform a thorough search of the neighborhood before terminating. The way of finding the end temperature is to perform a test run where the end temperature is set very low. The setting of the end temperature is then found by a visual inspection of a plot of the actual objective value at each iteration. The end temperature is the temperature at the time where the actual objective value has not changed for 'some time'.

Note that the start and end temperatures are dependent on the numerical size of the difference between the actual and the new objective value. Hence, if the evaluation function is changed it is important to check that the start and end temperatures still are right. Local iteration is the number of iterations that are performed at each temperature level. Global iteration is the number of times the temperature must be reduced in order to go from start temperature to end temperature. See *Equation 20*. Hence, global iteration multiplied by local iteration must be equal to *TI*.

$$Start\_temperature.cooling\_rate^{global\_iteration} = End\_temperature \tag{20}$$

The combination of cooling rate, global iteration and local iteration can be anything from *TI* local iterations at start temperature and only one temperature drop to end temperature, to one local iteration at each temperature level and reducing the temperature *TI* times. This is possible as long as the product of global and local iterations is equal to *TI*.

The relation between start temperature, end temperature, cooling rate and global iteration can be expressed by *Equation 21*.

$$global\_iteration = \frac{In\left(\dfrac{End\_temperature}{Start\_temperature}\right)}{In(cool\_rate)} \tag{21}$$

Setting the size of local iteration must be done with some respect to the size of the neighborhood. Prior to termination the heuristic must be allowed a good probability that the algorithm finds the minimal solution in the neighborhood. The number of solutions in the neighborhood in the actual case is the number of active base stations multiplied by the number of inactive base stations. Hence, identifying the number of iterations that are sufficient for a good probability must be done by performing a number of test runs where different settings of global and local iterations, combined with the appropriate cooling rates are tested.

4.8.1 Cooling rate

The typical value for cooling rate is between '*0.94*' and '*0.98*' and initially the value '*0.96*' is selected.

4.8.2 Global iteration

When start temperature, end temperature and cool rate are decided, global iteration is simple to compute using *Equation 21*:

4.8.3 Local iteration

The local iteration is set in accordance with the results of the test runs, mentioned earlier.

4.8.4 Tapering constant

The tapering constant for this heuristics is taken be 0.70

4.8.5 Overlap constant

The overlap constant for this heuristics is also taken be 0.70

CHAPTER FIVE

COMPUTATIONAL EXPERIMENTS /RESULTS

5.1 Testing the Metaheuristic

The Metaheuristic is implemented in C. The visualization of the result is made using functions developed in the program Matlab. All data used in these tests are from a 20 times 20-kilometer square proposed design area called "Fielmuo". The data simulating end-users are randomly chosen among companies and household within the proposed design space, with between 25 and 500 employees or inmates. For every 5 employees or inmates the company/household demands 64 kbps. The data simulating potential base stations are randomly chosen among all companies and household within the proposed design space. The test data set for the end-users and base station can be seen in *appendix 1*.

5.2 Results

The result of the first test run with 7 base stations plotted can be seen in *Figure 5.1*. Details about the settings of the heuristic can be seen in *appendix 2*. The number of not connected end-users in this solution is 272 and the minimal load on a base station is 64.512 kbps. A visual inspection of the result unveils that this solution is sufficiently good. This is due to the non violation of the two constraints, tapering and overlap.

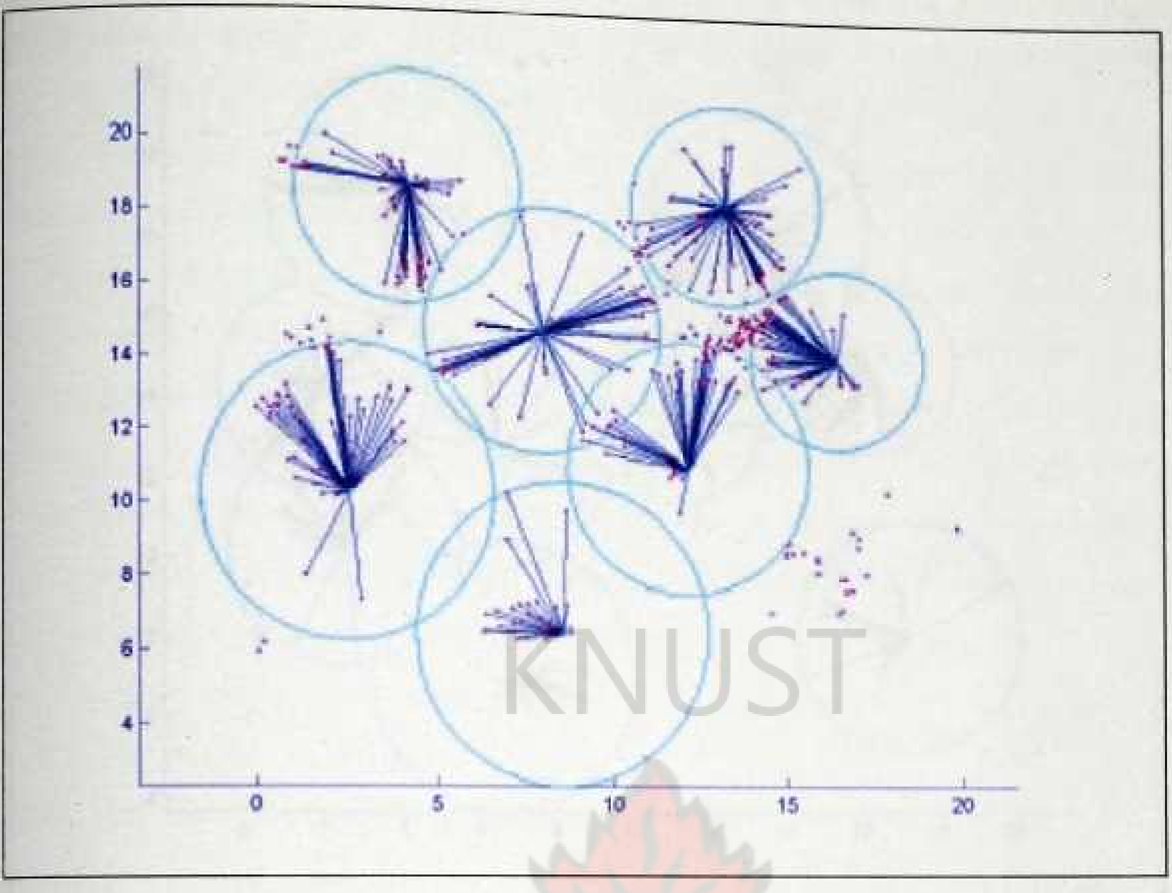

Figure 5.1: Pilot of First Test Run Results

A plot of the result of the test run in the case with 9 base stations can be seen in *Figure 5.2*. Details about the settings of the heuristic can be seen in *appendix 3*. Here the number of not connected end-users is 198 and the minimal load on a base station is 40.832 kbps.

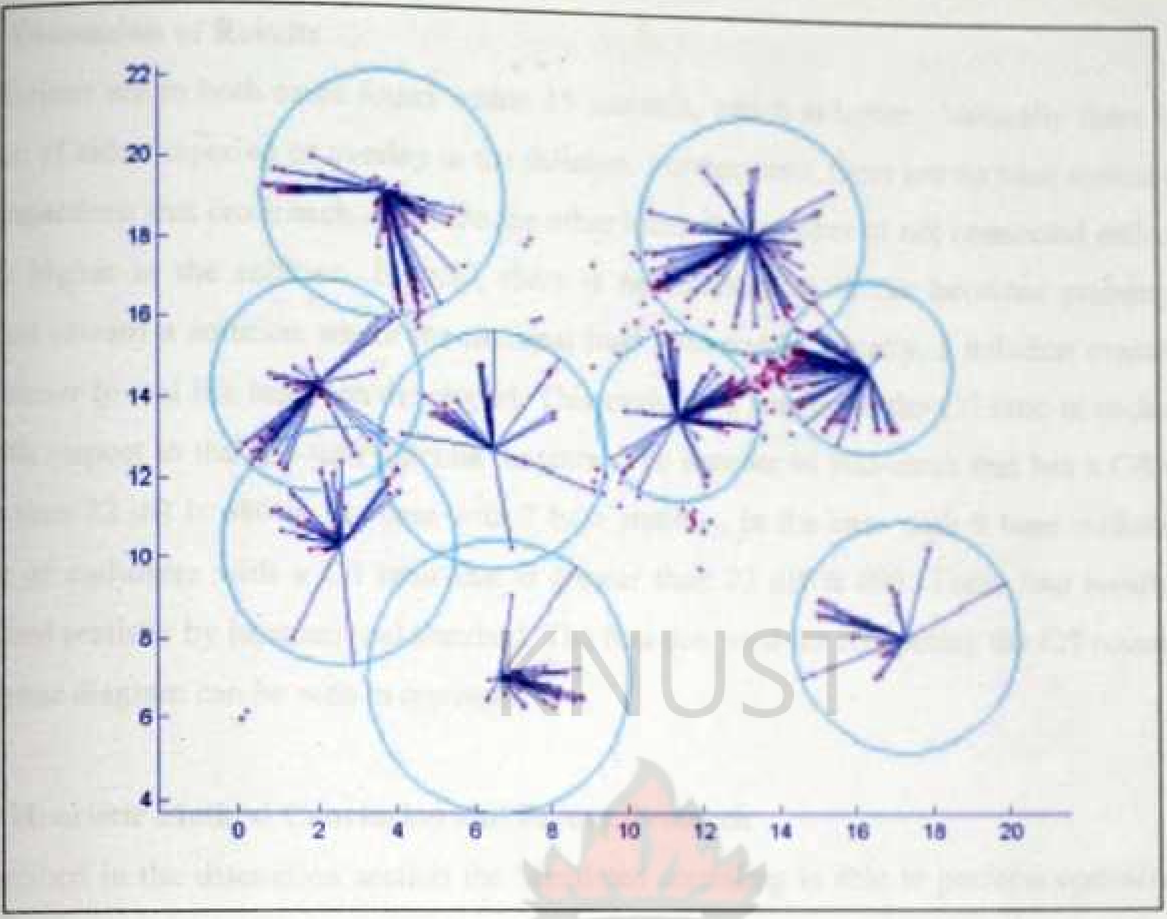

Figure 5.2: Pilot of Second Test Run Results

Initial tests were performed on three different computers in order to measure the performance in run time. The results of the evaluations/second test are the following:

Table 5.1: Performance of Three Different Computers on Run Time

Processor	Operation System	Evaluations/second
Pentium II , 500MHz	Win 98	48
Pentium III, 800 MHz	Win 2000	61
Pentium IV, 1.70 GHz	Win XP	220

Using the Pentium IV, 1.70 MHz computer it is possible to perform an evaluation of approximately 660,000 solutions within the given timeframe of one hour for the given problem size.

5.3 Discussion of Results

The solutions are in both cases found within 15 seconds, which is better. Naturally there is no violation of either tapering or overlap in the solution. Furthermore, there are no base station end-user connections that cross each other. On the other hand, the number of not connected end-users is a bit higher in the solution. Equally, there is no mechanism in the heuristic pushing the algorithm toward a solution where the minimal load is maximal. Finally, a solution evaluation that is closer to real life has been developed. This evaluation computes the C/I ratio at each end-user with respect to the end-user antenna diagram. The number of end-users that has a C/I ratio greater than 22 dB is 680 in the case with 7 base stations. In the case with 9 base stations the number of end-users with a C/I ratio that is greater than 22 dB is 693. These two results are considered realistic by international standard. The function used for computing the C/I ration and the antenna diagram can be seen in *appendix 4*.

5.4 Heuristic Method Conclusion and Future Research

As described in the discussion section the Simulated annealing is able to perform optimization using the number of not connected end-users as optimization parameter with success within the given time frame. That is the heuristic method is able to perform the location of base stations and connecting end-users in less than 60 seconds. Issues interesting for future testing could include the feature that provides the option to weight the end-users according to the density of the area where they are located. The heuristic focus has only been on using the number of not connected end-users as objective value. However, consider the situation where setting the number of base stations is a part of the optimization. Here, other parameters must be included in the objective function in order to make the heuristic able to weight the marginal improvement of the end-user coverage when adding or removing base stations.

The feature of optional optimizing the maximal minimal load on active base stations is not in the heuristic. However, it appears to be worth considered a function that makes it possible to optimize the network using the minimal load on a base station as objective function. Another option is to use a linear sum of connected end-users and the minimal load on a base station as an objective function. Implementation of these functions is assumed to improve the commercial usability of the heuristic.

5.5 General Conclusions

According to the section 'Heuristic Method Conclusion', a model able to place base stations and connect end-users to base stations, spending less than one hour, has been developed and implemented successfully.

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Appendix 1

Table 4: End-users Coordinates and Demand

eu_test

	COORDINATES		
ID	**X**	**Y**	**DEMAND**
1	6	3	2104
2	5	3	2240
3	10	4	2162
4	8	6	2090
5	8	14	2123
6	4	14	2122
7	12	12	2291
8	13	10	2362
9	12	8	2123
10	9	11	2094
11	7	10	1472
12	9	8	704
13	10	6	904
14	6	5	1000
15	13	8	2403
16	16	4	903
17	14	2	2081
18	16	7	2402
19	13	4	2403
20	11	10	2044
21	8	7	2345
22	12	4	2245
23	13	6	1100
24	10	5	2134
25	15	6	2311
26	6	12	2050
27	9	10	405
28	12	4	606
29	10	16	907
30	14	16	905
31	14	12	908
32	15	9	901
33	19	10	302
34	20	8	607
35	18	12	409
36	16	12	910
37	10	12	790
38	11	2	1500

39	15	8	1105
40	16	5	1200
41	11	7	1203
42	12	17	2031
43	5	17	2114
44	12	5	2215
45	9	4	1451
46	13	7	409
47	16	11	1409
48	6	8	2105
49	14	7	2504
50	6	9	505
51	15	14	2256
52	13	14	2507
53	18	8	2107
54	17	6	1107
55	20	6	1900
56	17	8	2001
57	11	3	2080
58	7	16	2101
59	6	15	2003
60	5	18	2102
61	8	17	2104
62	9	17	2041
63	15	15	2004
64	13	11	2211
65	19	14	2003
66	17	14	2002
67	12	13	2004
68	10	11	2041
69	16	14	801
70	13	18	802
71	11	18	803
72	16	18	803
73	15	17	845
74	13	16	865
75	9	15	832
76	7	14	402
77	8	19	903
78	6	20	304
79	15	19	904
80	19	17	900
81	15	4	670
82	11	9	2087

83	11	15	987
84	8	5	786
85	8	1	776
86	14	1	2456
87	16	2	965
88	18	3	475
89	18	6	543
90	15	9	923
91	17	12	443
92	5	7	921
93	3	16	2365
94	2	19	1109
95	11	17	1120
96	6	13	712
97	14	20	1012
98	18	18	512
99	10	20	1260
100	2	17	328
101	6	19	681
102	14	17	1254
103	14	13	2433
104	16	13	2317
105	5	15	504
106	13	15	540
107	17	17	590
108	17	20	563
109	7	5	543
110	9	3	543
111	15	3	542
112	5	6	541
113	4	6	508
114	8	13	506
115	3	14	576
116	10	15	489
117	7	2	450
118	13	3	567
119	4	3	573
120	19	4	571
121	17	1	560
122	3	6	523
123	2	4	546
124	14	18	543
125	4	10	500
126	1	1	600

127	1	2	2300
128	1	3	603
129	1	4	605
130	1	5	745
131	1	6	764
132	1	7	764
133	1	8	563
134	1	9	786
135	1	10	721
136	1	12	532
137	1	13	743
138	1	14	721
139	1	15	643
140	1	16	314
141	1	17	700
142	1	18	701
143	1	19	1020
144	1	20	703
145	2	1	704
146	2	3	305
147	2	5	605
148	2	7	704
149	2	8	608
150	2	10	875
151	2	12	607
152	2	14	543
153	2	15	600
154	2	18	606
155	2	20	673
156	3	20	729
157	3	19	756
158	3	18	434
159	3	17	790
160	3	15	780
161	3	13	760
162	3	12	740
163	3	11	730
164	3	10	720
165	3	9	710
166	3	8	1009
167	3	7	1106
168	3	5	400
169	3	4	1000
170	3	3	2000

171	3	2	1003
172	3	1	1005
173	4	1	1000
174	4	2	1004
175	4	5	1004
176	4	7	1003
177	4	9	1049
178	4	11	1190
179	4	15	1130
180	4	17	620
181	4	18	310
182	4	20	450
183	5	20	410
184	5	19	1070
185	5	16	1180
186	5	15	510
187	5	14	1507
188	5	12	890
189	5	11	880
190	5	10	570
191	5	9	860
192	5	8	860
193	5	5	850
194	5	4	540
195	5	1	830
196	6	1	820
197	6	2	421
198	6	4	832
199	6	7	834
200	6	11	834
201	6	17	810
202	6	18	823
203	7	10	850
204	7	17	360
205	7	15	864
206	7	13	662
207	7	12	864
208	7	11	368
209	7	9	870
210	7	8	872
211	7	7	574
212	7	6	876
213	7	4	878
214	7	3	880

215	7	1	482
216	8	2	884
217	8	4	886
218	8	8	808
219	8	10	390
220	8	11	692
221	8	16	894
222	8	18	896
223	8	20	2098
224	11	11	902
225	9	1	905
226	9	2	506
227	9	6	907
228	9	7	605
229	9	9	908
230	9	12	910
231	9	13	912
232	9	14	2014
233	9	16	913
234	9	18	915
235	9	19	916
236	9	20	317
237	10	19	518
238	10	17	920
239	10	11	902
240	10	9	433
241	10	8	924
242	10	3	926
243	10	1	528
244	11	1	929
245	11	4	330
246	11	5	932
247	11	6	934
248	11	8	736
249	11	13	938
250	11	14	965
251	11	16	934
252	11	19	954
253	11	20	932
254	12	20	931
255	12	15	732
256	12	11	1320
257	12	10	432
258	12	7	340

259	12	3	1350
260	12	2	1360
261	12	1	1364
262	13	1	1342
263	13	5	1269
264	13	9	1350
265	13	13	357
266	13	17	1356
267	13	19	1720
268	13	20	1380
269	14	19	1909
270	14	15	1236
271	14	11	1250
272	14	18	2136
273	14	6	540
274	14	5	945
275	14	3	1236
276	15	1	467
277	15	2	1290
278	15	5	1350
279	15	7	1000
280	15	10	1908
281	15	12	1180
282	15	13	660
283	15	16	2334
284	15	20	300
285	16	20	520
286	16	19	530
287	16	17	524
288	16	15	530
289	16	9	524
290	16	3	324
291	16	1	528
292	17	3	430
293	17	4	521
294	17	5	523
295	17	7	567
296	17	9	369
297	17	10	567
298	17	11	568
299	18	1	564
300	18	2	2032
301	18	5	543
302	18	7	532

303	18	9	380
304	18	11	582
305	18	14	584
306	18	15	586
307	18	16	2808
308	18	19	590
309	18	20	2090
310	17	13	592
311	17	15	593
312	17	16	1948
313	17	19	595
314	19	20	594
315	19	19	592
316	19	18	590
317	19	16	598
318	19	13	596
319	19	12	590
320	19	11	1008
321	19	9	1602
322	19	8	1604
323	20	1	1060
324	19	3	1608
325	19	2	1600
326	19	1	1619
327	20	2	1613
328	20	3	1150
329	20	4	1617
330	20	5	1190
331	20	7	621
332	20	9	1622
333	20	10	623
334	20	11	1630
335	20	12	1624
336	20	13	1659
337	20	14	1623
338	20	15	1626
339	20	16	1648
340	20	17	1247
341	20	18	1635
342	20	19	628
343	20	20	634
344	7	20	390
345	7	19	1340
346	1	11	1505

347	4	13	1349
348	1.5	2	1472
349	1.5	3	1342
350	1.5	4	1673
351	1.5	5	521
352	1.5	6	1347
353	1.5	7	1384
354	1.5	8	1498
355	1.5	9	1346
356	1.5	10	1344
357	1.5	11	1343
358	1.5	12	450
359	1.5	13	1350
360	1.5	14	1500
361	1.5	15	1520
362	1.5	16	1354
363	1.5	1	356
364	1.5	17	1358
365	1.5	18	2330
366	1.5	19	2340
367	1.5	20	1578
368	2.5	1	1234
369	2.5	2	1234
370	2.5	3	440
371	2.5	4	448
372	2.5	5	449
373	2.5	6	447
374	2.5	7	445
375	2.5	8	446
376	2.5	9	465
377	2.5	10	432
378	2.5	11	478
379	2.5	12	498
380	2.5	13	450
381	2.5	14	499
382	2.5	15	400
383	2.5	16	490
384	2.5	17	452
385	2.5	18	430
386	2.5	19	421
387	2.5	20	437
388	3.5	1	459
389	3.5	2	460
390	19	7	458

391	19	5	1809
392	4	16.5	1820
393	5	16.5	1804
394	6	16.5	1047
395	7	16.5	1804
396	8	16.5	1804
397	9	16.5	806
398	10	17.5	1868
399	11	17.5	808
400	3.5	3	1889
401	3.5	4	1802
402	3.5	5	801
403	3.5	6	1805
404	3.5	7	806
405	3.5	8	806
406	3.5	9	1806
407	3.5	10	750
408	3.5	11	450
409	3.5	12	754
410	3.5	13	754
411	3.5	14	753
412	3.5	15	552
413	3.5	16	753
414	3.5	17	752
415	3.5	18	556
416	3.5	19	756
417	3.5	20	759
418	4.5	1	745
419	4.5	2	751
420	4.5	3	758
421	4.5	4	758
422	4.5	5	1703
423	4.5	6	1747
424	4.5	7	1737
425	4.5	8	1746
426	4.5	9	750
427	4.5	10	1752
428	4.5	11	1750
429	4.5	12	1708
430	4.5	13	750
431	4.5	14	1700
432	4.5	15	1400
433	4.5	16	750
434	4.5	17	1740

435	4.5	18	1738
436	4.5	19	1509
437	4.5	20	1760
438	5.5	1	760
439	5.5	2	1780
440	5.5	3	1760
441	5.5	4	1390
442	5.5	5	760
443	5.5	6	750
444	5.5	7	769
445	5.5	8	786
446	5.5	9	465
447	5.5	10	743
448	5.5	11	790
449	5.5	12	780
450	5.5	13	780
451	5.5	14	286
452	5.5	15	2090
453	5.5	16	569
454	5.5	17	2360
455	5.5	18	334
456	5.5	19	2427
457	5.5	20	670
458	6.5	1	1780
459	6.5	2	830
460	6.5	3	580
461	6.5	4	340
462	6.5	5	2169
463	6.5	6	560
464	6.5	7	1290
465	6.5	8	348
466	6.5	9	670
467	6.5	10	780
468	6.5	11	560
469	6.5	12	896
470	6.5	13	480
471	6.5	14	1267
472	6.5	15	1790
473	6.5	16	1299
474	6.5	17	2266
475	6.5	18	1885
476	6.5	19	2044
477	6.5	20	899
478	7.5	1	877

479	7.5	2	866
480	7.5	3	855
481	7.5	4	677
482	7.5	5	900
483	7.5	6	399
484	7.5	7	366
485	7.5	8	1388
486	7.5	9	1666
487	7.5	10	490
488	7.5	11	990
489	7.5	12	690
490	7.5	13	340
491	7.5	14	1687
492	7.5	15	1490
493	7.5	16	390
494	7.5	17	569
495	7.5	18	989
496	7.5	19	808
497	7.5	20	2310
498	8.5	1	1909
499	8.5	2	1910
500	8.5	3	1819
501	8.5	4	1773
502	8.5	5	1785
503	8.5	6	543
504	8.5	7	730
505	8.5	8	760
506	8.5	9	459
507	8.5	10	780
508	8.5	11	768
509	8.5	12	345
510	8.5	13	456
511	8.5	14	308
512	8.5	15	907
513	8.5	16	505
514	8.5	17	910
515	8.5	18	912
516	8.5	19	516
517	8.5	20	323
518	9.5	1	420
519	9.5	2	2022
520	9.5	3	934
521	9.5	4	930
522	9.5	5	523

523	9.5	6	932
524	9.5	7	934
525	9.5	8	536
526	9.5	9	438
527	9.5	10	340
528	9.5	11	384
529	9.5	12	1472
530	9.5	13	704
531	9.5	14	705
532	9.5	15	404
533	9.5	16	706
534	9.5	17	707
535	9.5	18	385
536	9.5	19	1205
537	9.5	20	2004
538	10.5	1	2205
539	10.5	2	710
540	10.5	3	408
541	10.5	4	809
542	10.5	5	503
543	10.5	6	909
544	10.5	7	912
545	10.5	8	1149
546	10.5	9	915
547	10.5	10	920
548	10.5	11	922
549	10.5	12	923
550	10.5	13	2101
551	10.5	14	380
552	10.5	15	375
553	10.5	16	301
554	10.5	17	801
555	10.5	18	971
556	10.5	19	822
557	10.5	20	824
558	12	17.5	240
559	11.5	1	1232
560	11.5	2	1330
561	11.5	3	1232
562	11.5	4	1350
563	11.5	5	1234
564	11.5	6	1256
565	11.5	7	345
566	11.5	8	2045

567	11.5	9	245
568	11.5	10	1451
569	11.5	11	371
570	11.5	12	2074
571	11.5	13	2341
572	11.5	14	623
573	11.5	15	1723
574	11.5	16	823
575	11.5	17	865
576	11.5	18	1346
577	11.5	19	960
578	11.5	20	965
579	12.5	1	963
580	12.5	2	976
581	12.5	3	167
582	12.5	4	976
583	12.5	5	1230
584	12.5	6	870
585	12.5	7	891
586	12.5	8	897
587	12.5	9	896
588	12.5	10	678
589	12.5	11	675
590	12.5	12	675
591	12.5	13	987
592	12.5	14	657
593	12.5	15	876
594	12.5	16	2100
595	12.5	17	1234
596	12.5	18	235
597	12.5	19	2317
598	12.5	20	2109
599	13.5	1	2089
600	13.5	2	1685
601	13.5	3	1567
602	13.5	4	1569
603	13.5	5	1580
604	13.5	6	582
605	13.5	7	2034
606	13.5	8	2208
607	13.5	9	2129
608	13.5	10	1470
609	13.5	11	1256
610	13.5	12	1400

611	13.5	13	1500
612	13.5	14	1600
613	13.5	15	700
614	13.5	16	1800
615	13.5	17	900
616	13.5	18	860
617	13.5	19	1780
618	13.5	20	1150
619	14.5	1	548
620	14.5	2	1330
621	14.5	3	1240
622	14.5	4	2150
623	14.5	5	2260
624	14.5	6	1234
625	14.5	7	2100
626	14.5	8	200
627	14.5	9	2200
628	14.5	10	400
629	14.5	11	2110
630	14.5	12	1901
631	14.5	13	1950
632	14.5	14	1400
633	14.5	15	400
634	14.5	16	300
635	14.5	17	1290
636	14.5	18	1239
637	14.5	19	1239
638	14.5	20	670
639	15.5	1	679
640	15.5	2	1678
641	15.5	3	674
642	15.5	4	680
643	15.5	5	1682
644	15.5	6	682
645	15.5	7	684
646	15.5	8	1684
647	15.5	9	686
648	15.5	10	688
649	15.5	11	1681
650	15.5	12	142
651	15.5	13	1422
652	15.5	14	423
653	15.5	15	1426
654	15.5	16	1480

655	15.5	17	1422
656	15.5	18	429
657	15.5	19	1426
658	15.5	20	470
659	16.5	1	2016
660	16.5	2	2430
661	16.5	3	1260
662	16.5	4	567
663	16.5	5	908
664	16.5	6	955
665	16.5	7	995
666	16.5	8	877
667	16.5	9	340
668	16.5	10	550
669	16.5	11	2400
670	16.5	12	1580
671	16.5	13	2025
672	16.5	14	337
673	16.5	15	345
674	16.5	16	880
675	16.5	17	556
676	16.5	18	1230
677	16.5	19	699
678	16.5	20	557
679	17.5	1	1008
680	17.5	2	337
681	17.5	3	880
682	17.5	4	1885
683	17.5	5	2066
684	17.5	6	944
685	17.5	7	955
686	17.5	8	800
687	17.5	9	800
688	17.5	10	908
689	17.5	11	550
690	17.5	12	2090
691	17.5	13	370
692	17.5	14	2080
693	17.5	15	669
694	17.5	16	390
695	17.5	17	390
696	17.5	18	1690
697	17.5	19	700
698	17.5	20	600

699	18.5	1	1230
700	18.5	2	1790
701	18.5	3	2370
702	18.5	4	700
703	18.5	5	800
704	18.5	6	600
705	18.5	7	1356
706	18.5	8	2290
707	18.5	9	439
708	18.5	10	890
709	18.5	11	665
710	18.5	12	872
711	18.5	13	870
712	18.5	14	879
713	18.5	15	872
714	18.5	16	584
715	18.5	17	659
716	18.5	18	329
717	18.5	19	452
718	18.5	20	880
719	19.5	1	883
720	19.5	2	990
721	19.5	3	660
722	19.5	4	1602
723	19.5	5	882
724	19.5	6	1222
725	19.5	7	890
726	19.5	8	490
727	19.5	9	890
728	19.5	10	670
729	19.5	11	679
730	19.5	12	349
731	19.5	13	2050
732	19.5	14	2300
733	19.5	15	1300
734	19.5	16	600
735	19.5	17	1800
736	19.5	18	1000
737	19.5	19	1100
738	19.5	20	2203
739	1	1.5	337
740	2	1.5	777
741	3	1.5	888
742	4	1.5	499

743	5	1.5	887
744	6	1.5	398
745	7	1.5	556
746	8	1.5	776
747	9	1.5	668
748	10	1.5	777
749	11	1.5	666
750	12	1.5	555
751	13	1.5	999
752	14	1.5	776
753	15	1.5	1449
754	16	1.5	550
755	16	17.5	1500
756	17	1.5	448
757	18	1.5	559
758	19	1.5	558
759	20	1.5	1590
760	1	2.5	1937
761	2	2.5	2341
762	3	2.5	2000
763	4	2.5	2010
764	5	2.5	1000
765	6	2.5	1000
766	7	2.5	1500
767	8	2.5	1006
768	9	2.5	1200
769	10	2.5	1300
770	11	2.5	1900
771	12	2.5	908
772	13	2.5	780
773	14	2.5	890
774	15	2.5	679
775	16	2.5	800
776	17	2.5	808
777	18	2.5	806
778	19	2.5	805
779	20	2.5	609
780	1	3.5	560
781	2	3.5	1800
782	3	3.5	2190
783	4	3.5	1800
784	5	3.5	1900
785	6	3.5	679
786	7	3.5	2098

787	8	3.5	2320
788	9	3.5	1209
789	10	3.5	1200
790	11	3.5	1005
791	12	3.5	900
792	13	3.5	1002
793	14	3.5	490
794	15	3.5	2386
795	16	3.5	2078
796	17	3.5	456
797	18	3.5	423
798	19	3.5	2089
799	20	3.5	459
800	1	4.5	456
801	2	4.5	2342
802	3	4.5	2079
803	4	4.5	432
804	5	4.5	480
805	6	4.5	482
806	7	4.5	483
807	8	4.5	485
808	9	4.5	486
809	10	4.5	2228
810	11	4.5	2378
811	12	4.5	1832
812	13	4.5	2230
813	14	4.5	478
814	15	4.5	983
815	16	4.5	680
816	17	4.5	789
817	18	4.5	676
818	19	4.5	1200
819	20	4.5	450
820	1	5.5	866
821	2	5.5	2100
822	3	5.5	798
823	4	5.5	1001
824	5	5.5	1100
825	6	5.5	567
826	7	5.5	1280
827	8	5.5	2060
828	9	5.5	2170
829	10	5.5	1779
830	11	5.5	1770

831	12	5.5	888
832	13	5.5	1450
833	14	5.5	399
834	15	5.5	590
835	16	5.5	488
836	17	5.5	2299
837	18	5.5	590
838	19	5.5	668
839	20	5.5	449
840	1	6.5	660
841	2	6.5	880
842	3	6.5	490
843	4	6.5	499
844	5	6.5	1980
845	6	6.5	2309
846	7	6.5	2008
847	8	6.5	2000
848	9	6.5	667
849	10	6.5	369
850	11	6.5	2040
851	12	6.5	990
852	13	6.5	2180
853	14	6.5	2088
854	15	6.5	2088
855	16	6.5	888
856	17	6.5	556
857	18	6.5	325
858	19	6.5	2209
859	20	6.5	678
860	1	7.5	2099
861	2	7.5	2409
862	3	7.5	1230
863	4	7.5	1230
864	5	7.5	1234
865	6	7.5	367
866	7	7.5	789
867	8	7.5	987
868	9	7.5	679
869	10	7.5	1784
870	11	7.5	809
871	12	7.5	1436
872	13	7.5	767
873	14	7.5	871
874	15	7.5	762

875	16	7.5	870
876	17	7.5	1152
877	18	7.5	434
878	19	7.5	632
879	20	7.5	769
880	1	8.5	879
881	2	8.5	470
882	3	8.5	789
883	4	8.5	467
884	5	8.5	888
885	6	8.5	306
886	7	8.5	689
887	8	8.5	2038
888	9	8.5	689
889	10	8.5	2332
890	11	8.5	325
891	12	8.5	990
892	13	8.5	370
893	14	8.5	554
894	15	8.5	2080
895	16	8.5	2045
896	17	8.5	1088
897	18	8.5	447
898	19	8.5	998
899	20	8.5	887
900	6	9.5	2395
901	7	9.5	1220
902	8	9.5	644
903	9	9.5	340
904	10	9.5	578
905	11	9.5	327
906	12	9.5	398
907	13	9.5	1070
908	14	9.5	332
909	15	9.5	2116
910	16	9.5	2126
911	17	9.5	399
912	18	9.5	588
913	19	9.5	557
914	20	9.5	2203
915	6	10.5	2010
916	7	10.5	2106
917	8	10.5	2094
918	9	10.5	2202

919	10	10.5	2010
920	11	10.5	1085
921	12	10.5	775
922	13	10.5	887
923	14	10.5	907
924	15	10.5	766
925	16	10.5	2364
926	17	10.5	1978
927	18	10.5	654
928	19	10.5	764
929	20	10.5	887
930	6	11.5	543
931	7	11.5	1350
932	8	11.5	310
933	9	11.5	654
934	10	11.5	2473
935	11	11.5	504
936	12	11.5	399
937	13	11.5	2204
938	14	11.5	347
939	15	11.5	2468
940	16	11.5	500
941	17	11.5	900
942	18	11.5	910
943	19	11.5	904
944	20	11.5	405
945	6	12.5	606
946	7	12.5	904
947	8	12.5	2104
948	9	12.5	370
949	10	12.5	975
950	11	12.5	960
951	12	12.5	1600
952	13	12.5	943
953	14	12.5	1454
954	15	12.5	960
955	16	12.5	2326
956	17	12.5	954
957	18	12.5	323
958	19	12.5	521
959	20	12.5	332
960	6	13.5	945
961	7	13.5	718
962	8	13.5	918

963	9	13.5	551
964	10	13.5	943
965	11	13.5	953
966	12	13.5	623
967	13	13.5	956
968	14	13.5	2201
969	15	13.5	2306
970	16	13.5	601
971	17	13.5	2412
972	18	13.5	411
973	19	13.5	900
974	20	13.5	599
975	6	14.5	2430
976	7	14.5	2141
977	8	14.5	2400
978	9	14.5	2051
979	10	14.5	751
980	11	14.5	1646
981	12	14.5	541
982	13	14.5	901
983	14	14.5	908
984	15	14.5	909
985	16	14.5	2039
986	17	15.5	1201
987	18	14.5	905
988	19	14.5	806
989	20	14.5	2045
990	1	15.5	2446
991	2	15.5	2428
992	3	15.5	2043
993	4	15.5	2407
994	5	15.5	2041
995	6	15.5	1509
996	7	15.5	1199
997	8	15.5	1798
998	9	15.5	990
999	10	15.5	2196
1000	11	15.5	2088

Table 5: Base Stations Coordinates and Capacity

bs_test

	COORDINATE		
ID	**X**	**Y**	**CAPACITY**
1	10	10	120,000
2	14	14	120,000
3	6	6	120,000
4	16	8	120,000
5	6	14	120,000
6	10	18	120,000
7	10	14	120,000
8	4	16	120,000
9	12	6	120,000
10	10	7	120,000
11	16	16	120,000
12	4	12	120,000
13	8	12	120,000
14	14	10	120,000
15	12	11	120,000
16	2	13	120,000
17	8	3	120,000
18	14	4	120,000
19	12	16	120,000
20	4	8	120,000
21	16	12	120,000
22	6	10	120,000
23	13	12	120,000
24	19	6	120,000
25	4	4	120,000
26	13	2	120,000
27	19	15	120,000
28	8	9	120,000
29	18	10	120,000
30	18	4	120,000
31	18	13	120,000
32	2	11	120,000
33	8	15	120,000
34	4	19	120,000
35	7	18	120,000
36	15	18	120,000
37	2	16	120,000
38	17	2	120,000
39	9	5	120,000
40	5	2	120,000

41	2	6	120,000
42	2	9	120,000
43	18	17	120,000
44	12	19	120,000
45	2	2	120,000
46	11	12	120,000
47	14	7	120,000
48	6	16	120,000
49	10	2	120,000
50	16	6	120,000

Appendix 2

Out_loop	0
Global iteration	108
Lokal iteration	18
T_start	80.00
Cool rate	0.96
Tapering constant	0.70
Overlap constant	0.70
Validate by density	0
Validate by demand	1
Val. obj by demand	0
Number of base stations	7
Seed	0
Number of not connected end_user	272
Number of End-user hit rate	0.73
End-user demand hit rate	0.75
Best objective value	272.00
Number of end_users with C/I < 22	48
Share of end_users with C/I < 22	0.07
Total number of not connected end_user	320

Fixed base station -1

Base station	Load	x_coordinate	y_coordinate	Radius
10	120000	12.174	11.052	3.475
13	119744	4.198	18.625	3.165
22	120000	2.546	10.299	4.055
23	119808	13.286	18.309	2.736
36	100544	7.999	14.774	3.378
39	64512	8.572	6.475	4.164
48	119936	16.453	14.077	2.473

Appendix 3

Out_loop	0
Global iteration	108
Lokal iteration	18
T_start	80.00
Cool rate	0.96
Tapering constant	0.70
Overlap constant	0.70
Validate by density	0
Validate by demand	1
Val. obj by demand	0
Number of base stations	9
Seed	0
Number of not connected end_user	198
Number of End-user hit rate	0.80
End-user demand hit rate	0.81
Best objective value	198.00
Number of end_users with C/I < 22	109
Share of end_users with C/I < 22	0.14
Total number of not connected end_user	307

Fixed base station -1

Base station	Load	x_coordinate	y_coordinate	Radius
5	40832	17.289	8.160	2.932
12	119744	16.318	14.981	2.197
15	119808	3.689	19.106	3.096
18	58944	6.636	7.080	3.412
22	61056	2.546	10.299	2.948
23	119808	13.286	18.309	2.958
29	55872	6.485	12.730	2.961
42	118144	1.995	14.317	2.642
46	119936	11.341	13.668	2.113

Appendix 4

Computing C/I

The function for computing C/I values at each end-user is the following:

C/I measured for end-user i assigned to base station k

$$C = 10^{z}$$

$$P = \underset{j \neq k}{\ } 10^{z}$$

$$z = \frac{(27 - (92.5 + 20 * \log10(dist_{ij}) + 20 \cdot \log10(F)) + gain(angle\_diff) + 33.8)}{10}$$

$$C/I = C/\log10(P)$$

Where

F = transmission frequency measured in GHz

Angle_diff = the angular difference between the direction toward the assigned base station and the actual interfering base station.

The gain diagram is based on the following values at the given angles. The values at angles between the given ones are computed using simple linear interpolation between the adjacent values.

Angle	0	2	8	30	90	100	180
Gain	0	0	-17	-22	-30	-35	-40