OPTIMAL TREATMENT OF WATER USING LINEAR PROGRAMMING

A CASE STUDY OF SUNYANI WATER TREATMENT PLANT

By

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DECLARATION

I hereby declare that this submission is my own work towards the MSc. And that, to the best of my knowledge, it contains no material previously published by another person now material, which has been accepted for the award of any other degree of the University, except where due acknowledgement has been made in the text.

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ABSTRACT

Primal-dual, one of the interior-point methods was used to reduce treatment cost of water at Sunyani water treatment plant. The 2010 data collected was divided into two to reflect the two major seasons-dry and wet-we have in the country. Each data was modeled into objective function and subject constraints. Matrices generated from each season were run on Matlab 7.10 code. Results that were obtained showed a remarkable reduction in treatment cost compared to actual cost in the same year.

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DEDICATION

I wish to dedicate this thesis to my dear wife, Mrs. Hagar Koufie; my lovely son, Joseph B. K. Koufie and my entire family for their immense contribution in diverse ways during the writing of this thesis.

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CHAPTER 1

INTRODUCTION

1.0 OVERVIEW

With two thirds of the earth's surface covered by water and the human body consisting of 75 percent of water, it is evidently clear that water is one of the prime elements responsible for life on earth.

Water makes up more than two thirds of the weight of the human body, and without it, we would die in a few days. The human brain is made up of 95% water, blood is 82% of water and lungs 90%. A mere 2% drop in our body's water supply can trigger signs of dehydration; fuzzy short-term memory; trouble with basic math; and difficulty focusing on smaller print, such as a computer screen.(Roque, 2008)

According to UN Secretary-General Ban Ki-moon, "Water and sanitation are critical to attaining the Millennium Development Goals (MDGs). Without water there is no life." (Pengfei, 2010)

In Ghana, portable water coverage is very low—about 45% for rural areas and 70% for urban centre. Hence the need to minimize treatment cost of water to ensure its availability and its affordability.

The chapter therefore gives the background to the study, states the problem which the researcher is researching into and the reason for carrying out the research. It also states the justification of the study and the scope of the study.

1.1 BACKGROUND TO THE STUDY

Water is the common name applied to the liquid form (state) of the hydrogen and oxygen compound H_2O. Pure water is an odorless, tasteless, clear liquid. Water is one of nature's most important gifts to mankind. Essential to life, a person's survival depends on drinking water. Water is one of the most essential elements to good health - it is necessary for the digestion and absorption of food; helps maintain proper muscle tone; supplies oxygen and nutrients to the cells; rids the body of wastes; and serves as a natural air conditioning system. Health officials emphasize the importance of drinking at least eight glasses of clean water each and every day to maintain good health.

Water is a key component in determining the quality of our lives. Today, people are concerned about the quality of the water they drink. Although water covers more than 70% of the Earth, only 1% of the Earth's water is available as a source of drinking water. Yet, our society continues to contaminate this precious resource. Water is known as a natural solvent. Before it reaches the consumer's tap, it comes into contact with many different substances, including organic and inorganic matter, chemicals, and other contaminants. Many public water systems treat water with chlorine to destroy disease-producing contaminants that may be present in the water. Although disinfection is an important step in the treatment of potable water, the taste and odor of chlorine is objectionable. And, the disinfectants that are used to prevent diseases, can create byproducts which may pose significant health risks.

Today, drinking water treatment at the point-of-use is no longer a luxury, it is a necessity! Consumers are taking matters into their own hands and are now determining the quality of the water they and their families will drink by installing a drinking water system that will give them clean, refreshing, and healthier water.

Uses of water include agricultural, industrial, household, recreational and environmental activities. Virtually all of these human uses require fresh water.

97% of water on the Earth is salt water, and only 3% is fresh water of which slightly over two thirds is frozen in glaciers and polar ice caps. The remaining unfrozen freshwater is mainly found as groundwater, with only a small fraction present above ground or in the air.

Fresh water is a renewable resource, yet the world's supply of clean, fresh water is steadily decreasing. Water demand already exceeds supply in many parts of the world and as the world population continues to rise, so too does the water demand. Awareness of the global importance of preserving water for the ecosystem has only recently emerged as, more than half the world's wetlands have been lost along with their valuable environmental services.

(Wikipedia, 2010)

It is however sad to note that over 1.1 billion people around the world do not have access to clean water and over 2.4 billion people do not have access to adequate sanitation (W.H.O.,2002).

This limitation puts the world's poorest at risk for contracting waterborne disease such as cholera, dysentery, and typhoid fever. Waterborne infections include diarrhea, which causes 4 percent of the world's deaths and is the leading cause of death among children under the age of five. (http://www.lenntch.com/library/dieases/diseases/waterborne-diseases.htm, 2010)

Lack of clean drinking water and sanitation systems is a severe public health concern in Ghana, contributing to 70 percent of diseases in the country. Consequently, households without access to clean water are forced to use less reliable and unhygienic sources, and often pay more.

It is not cheap to treat water so that it is safe to drink. But it is also mot cheap to treat everyone who becomes ill during outbreak of waterborne diseases.

On the average over 60% of the population who are in the low income segments who live in the rural, the fringes and densely populated areas of the urban centre have least access to water. The overall percentage of the national population of 18.4 million (2001 population figures) with access to safe drinking water supply is low. In 1992 water supply coverage for both rural and urban area of Ghana was 28% and 76% respectively. Water supply to rural communities increased to 41% while urban supply decreased to about 70% in 2001, but this represented over one million additional people having been provided with portable water (Freshwater Country Profile – 2004).

Ghana Water Company Limited (GWCL) is responsible for treatment and distribution of water in Ghana. It is divided into three main divisions and they are:

1. Water supplies
2. Water distributors
3. Administrators.

1.2 PROFILE OF GWCL

The first public water supply system in Ghana, then Gold Coast, was established in Accra just before World War I. Extensions were made to Cape Coast, Winneba and Kumasi in the1920s.

During this period, the water supply systems were managed by the Hydraulic Division of Public Works Department. With time the responsibilities of the Hydraulic Division were widened to include the planning and development of water supply systems in other parts of the country.

In 1948, the Department of Rural Water Development was established to engage in the development and management of rural water supply through the drilling of bore holes and construction of wells for rural communities.

After Ghana's independence in 1957, a Water Supply Division, with headquarters in Kumasi, was set up under the Ministry of Works and Housing with responsibilities for both urban and rural water supplies.

During the dry season of 1959, there was severe water shortage in the country. Following this crisis, an agreement was signed between the Government of Ghana and the World Health Organisation (WHO) for a study to be conducted into the water sector development of the country.

In line with the recommendations of the WHO, the Ghana Water and Sewerage Corporation (GWSC), was established in 1965 under an Act of Parliament (Act 310) as a legal public utility entity. GWSC was to be responsible for:

- water supply and sanitation in rural as well as urban areas.
- the conduct of research on water and sewerage as well as the making of engineering surveys and plans.
- the construction and operation of water and sewerage works,
- the setting of standards and prices and collection of revenues.

In the late 1970s and early 1980s, the operational efficiency of GWSC declined to very low levels mainly as a result of the deterioration in pipe connections and pumping systems. A World Bank report in 1998 states that: "The water supply systems in Ghana deteriorated rapidly during the economic crises of the 1970´s and early 1980´s when Government's ability to adequately operate and maintain essential services was severely constrained.

To reverse the decline in water supply services, interventions in the area of sector reforms and project implementation were made in 1970, 1981 and 1988. Though some gains were derived from these interventions, their general impact on service delivery was very disappointing. Due to the failure of these interventions to achieve the needed results, several efforts were made to improve efficiency within the water supply sector in Ghana especially during the era of the Economic Recovery Programme from 1983 to 1993.

During this period, loans and grants were sought from the World Bank and other donors for the initiation of rehabilitation and expansion programmes, to train personnel and to buy transport and maintenance equipment.

In addition, user fees for water supply were increased and subsidies on water tariffs were gradually removed for GWSC to achieve self-financing.

The government at that time approved a formula for annual tariff adjustments to enable the corporation generate sufficient funds to cover all annual recurrent costs as well as attain some capacity to undertake development projects.

In 1987, a "Five-Year Rehabilitation and Development Plan" for the sector was prepared which resulted in the launching of the Water Sector Restructuring Project (WSRP). The reforms were aimed at reducing unaccounted for water, introducing rationalisation through reduction of the workforce, hiring of professionals and training of the remaining staff.

Accordingly, a number of organisational reforms within the Ghanaian water sector were initiated in the early 1990s. As a first step, responsibilities for sanitation and small towns water supply were decentralized from Ghana Water and Sewerage Corporation to the District Assemblies in 1993.

In 1994 the Environmental Protection Agency (EPA) was established to ensure that water operations did not cause any harm to the environment. The Water Resources Commission (WRC) was founded in 1996 to be in charge of overall regulation and management of water resources utilization. In 1997, the Public Utilities Regulatory Commission (PURC) came into being with the purpose of setting tariffs and quality standards for the operation of public utilities.

With the passage of Act 564 of 1998, Community Water and Sanitation Agency (CWSA) was established to be responsible for management of rural water supply systems, hygiene education and provision of sanitary facilities. After the establishment of CWSA, 120 water supply systems serving small towns and rural communities were transferred to the District Assemblies and Communities to manage under the community-ownership and management scheme.

Finally, pursuant to the Statutory Corporations (Conversion to Companies) Act 461 of 1993 as amended by LI 1648, on 1st July 1999, GWSC was converted into a 100% state owned limited liability, Ghana Water Company Limited, with the responsibility for urban water supply only. (Adombire, 2007)

1.3 PROBLEM STATEMENT

People in Sunyani pay so much for water due to high cost of its treatment by GWC. Those who are unable to pay their bills get their taps disconnected. This leaves such people with no choice but to go in for untreated water from other sources. As such there is prevalence of water borne diseases. Lives are therefore lost because this practice.

The problem at hand is for GWC to optimize (minimize) the cost of treating water with respect to:

- the cost of bags of chemical use in purifying water;
- the cost of fuel used;
- the cost of electricity used

in other to make it more affordable and accessible.

1.4 OBJECTIVES OF THE STUDY

1. Model water treatment cost as a Linear Programming problem.

2. Minimize treatment cost of water at Sunyani water treatment plant.

1.5 METHODOLOGY

The problem of water treatment will be modeled as a linear programming problem.

Primal – Dual, one of the interior point methods will be used to develop the Mathematics Model. The interior point method is preferred over simplex method because interior – point methods approach the boundary of the feasible set in the limit.

Data will be collected from Sunyani water treatment plant at Abesim.

Software program on MATLAB will be developed using the mathematics model to run the data.

1.6 SIGNIFICANCE OF THE STUDY

Tano River is the main source of raw water to the Sunyani water treatment plant. Treated water from this source serves the Sunyani municipality and its environs.

Irregular supply of water year round due to high cost of treatment therefore affects the livelihood of people depending on it.

Application of findings from this research will help reduce cost of water treatment. This means that there will be regular supply of water and so people will not resort to other sources of water.

Quantity of water produced would increase and so could be extended to areas that do not have access to tap water. In effect, it will:

- help reduce the amount of money that government will spend to treat people with water borne diseases.
- reduce the number of lives that are lost out of water borne related diseases
- help increase productivity in other sectors of the economy since manpower needed in those areas will not spend hours they are to use at their work places to treat themselves of water borne diseases.
- serve as a basis for more research to be made on this area.

1.7 THE SCOPE OF THE STUDY

The research aims at developing a water treatment cost model.

The model will be used to determine the optimum cost of treating water at Sunyani headwork.

The work will intend to analyze the cost involved in water treatment and to distribute the same quantity (volume) of water to Sunyani Municipality and its environs at a minimal cost, taking the major seasons in the year into account.

1.8 LIMITATIONS OF THE STUDY

The major limitations of this study are:

1. The research will not cover all aspects of water treatment and supply.

   Example:

   - Cost of civil structures of the treatment plant.
   - Supply chain

2. The research will not determine the optimal consumption of water used to maintain the plant.

1.9 ORGANIZATION OF THE STUDY

Chapter 1 deals with the general introduction of the study. Chapter 2 gives a review of the existing theoretical and empirical literature. Chapter 3 deals with methodology. Chapter 4 deals with data collection, analysis, estimate and discussion of results. The concluding chapter, Chapter 5 summaries the findings and also provides conclusions and recommendations.

CHAPTER 2

LITERATURE REVIEW

2.0 INTRODUCTION

This chapter reviews the literature on the application of linear programming.

2.1 LITERATURE REVIEW ON LINEAR PROGRAMMING

A study conducted by Kangrang and Compiew (2010) used an allocation LP model that took into account heterogeneity of crop water requirement and crop yield of irrigation area for finding optimal crop pattern. A sensitivity analysis of irrigation efficiency in the modified LP model was accounted for in the study. The proposed model was applied to find the optimal crop pattern of the Huai-Ang Irrigation Project located in the Northeast Region of Thailand during dry-season (Mid December-Mid April). The records of seasonal flow, requested and actual irrigation areas, crop water requirement, crop yield, evaporation and effective rainfall of the project were used for this illustrative application. Results showed that the modified LP model was feasible for finding the optimal crop pattern. Heterogeneous character in allocation LP model provided the crop pattern that was suitable for cultivating crop on the actual land area. The consideration of vary irrigation efficiency in irrigation planning provided the optimal crop pattern that ~~could~~ give the higher gross benefit.

Campbell et al (1992) focused on the application of linear programming (LP) in combining with a geographic information system (GIS) in planning agricultural land-use strategies. The first step of the proposed methodology was to obtain an assessment of the natural resources available to agriculture. The GIS was used to delineate land-use conflicts and provided reliable information on the natural-resource database. This was followed by combining the data on natural resources with other quantifiable information on available labour, market forecasts, technology, and cost information in order to estimate the economic potential of the agricultural sector. Linear Programming was used in this step. Finally, the GIS was applied again to map the crop and land-allocation patterns generated by the Linear Programming model. The results gave concrete suggestions for resource allocation, farm-size mix, policy application, and implementation projects.

Linear programming model was applied by Hassan (2004) to calculate the optimal crop acreage, production and income of the irrigated Punjab. Crops included in the models were wheat, Basmati rice, IRRI rice, cotton, sugarcane, maize, potato, gram and mong / mash. The results showed that the irrigated agriculture in the Punjab was more or less operating at the optimal level. Over all cropped acreage in the Optimal solution decreased by 0.37 % as compared to the existing acreage. However, in the optimal cropping pattern some crops like cotton and pluses gained acreage by 9-10 % each, while maize and Basmati rice remained unchanged. On the other hand crops like wheat, IRRI rice, potato and sugarcane lost acreage by 4-11 %. As a result of optimum croppingpattern income, increased by 1.57 %.

Mullan (2008) used linear programming to solve the ice cream mix calculations and provided a "proof of calculation" showing that the mass of the mix sums to the correct value and all the components e.g. fat also sum to the correct value. During the development of the calculator the researcher produced 11 Excel spreadsheets that covered many of the ice-cream formulation challenges that commercial manufacturers may encounter.

Milind et al (2002) explored a design where, the market signal provided to a supplier was based on the current cost of procurement for the buyer. At the heart of this design lied a fundamental sensitivity analysis of linear programming. Each supplier was required to submit bid proposals that reduced the procurement cost (assuming other suppliers keep their bids unchanged) by some large enough decrement d > a. They showed that, for each supplier, generating a profit maximizing bid that decreased the procurement cost for the buyer by at least d could be done in polynomial time. This implied that in designs where the bids were not common knowledge, each supplier and the buyer could engage in an "algorithmic conversation" to identify such proposals in a polynomial number of steps. In addition, they showed that such a mechanism converged to an "equilibrium solution" where all the suppliers were at their profit maximizing solution given the cost and the required decrement d.

Matthews (2005) evaluated and optimized the utility of the nurse personnel at the Internal Medicine Outpatient Clinic of Wake Forest University Baptist Medical Center. Linear programming (LP) was employed to determine the effective combination of nurses that would allow for all weekly clinic tasks to be covered while providing the lowest possible cost to the department. A specific sensitivity analysis was performed to assess just how sensitive the outcome was to the stress of adding or deleting a nurse to or from the payroll. The nurse employee cost structure in this study consisted of five certified nurse assistants (CNA), three licensed practicing nurses (LPN), and five registered nurses (RN). The LP revealed that the outpatient clinic should staff four RNs, three LPNs, and four CNAs with 95 percent confidence of covering nurse demand on the floor.

Reuter and Deventer (2003) proposed two linear models, the second being a subset of the first, for the simulation of flotation plants by use of linear programming. The first linear model produced the circuit structure, as well as the optimal flow rates of the valuable element between any number of flotation banks incorporating any number of recycle mills. An optimal grade for the valuable element in the concentrate was given by the second model. Operating conditions in the flotation banks and recycle mills were included as bounds in these models, permitting their possible application in expert systems. The simulated circuit structure, concentrate grade and recoveries closely resembled those of similar industrial flotation plants. The only data required by the simulation models were the feed rates of the species of an element, and their separation factors which were estimated from a multiparameter flotation model.

A linear programming model for a river basin was developed by Avdelas et al (1992) to include almost all water-related economic activity both for consumers and producers. The model was so designated that the entire basin or basin sub-division could be analyzed. The model included seven sectors, nine objective function criteria, and three river-flow levels. Economic basis for conflicts among sectors over incidence of cost allocation and level of economic activity were traced to some chosen objective. The disposal of untreated household waste water, particularly from the rural household, directly into the river was consistent with maximizing net benefits and minimizing costs. For each of the three industries analyzed separately, paper, wool and tanning, public treatment of industrial waste water was the optimal treatment process in one or more of the solutions.

Tsakiris and Spiliotis (2004) treated the Systems Analysis formulation problem of water allocation to various users as a linear programming problem with the objective of maximizing the total productivity. This was intended to solve one of the basic problems of Water Resource Management in the allocation of water resources to various users in an optimal and equitable way respecting the constraints imposed by the environment. In this work a fuzzy set representation of the unit revenue of each use together with a fuzzy representation of each set of constraints, were used to expand the capabilities of the linear programming formulation. Numerical examples were presented for illustrative purposes and useful conclusions are derived.

To investigate how farmers could sustain an economically viable agricultural production in salt-affected areas of Oman, Naifer et al (2010), divided a sample of 112 farmers into three groups according to the soil salinity levels, low salinity, medium salinity and high salinity. Linear programming was used to maximize each type of farm's gross margin under water, land and labor constraints. The economic losses incurred by farmers due to salinity were estimated by comparing the profitability of the medium and high salinity farms to the low salinity farm's gross margin. Results showed that when salinity increased from low salinity to medium salinity level the damage was US$ 1,604 ha^{-1} and US$ 2,748 ha^{-1} if it increased from medium salinity to high salinity level. Introduction of salt-tolerant crops in the cropping systems showed that the improvement in gross margin was substantial thus attractive enough for medium salinity farmers to adopt the new crops and/or varieties to mitigate the effect of water salinity.

Khaled (2004) developed four models of optimal water allocation with deficit irrigation in order to determine the optimal cropping plan for a variety of scenarios. The first model (Dynamic programming model (DP)) allocated a given amount of water optimally over the different growth stages to maximize the yield per hectare for a given crop, accounting for the sensitivity of the crop growth stages to water stress. The second model (Single Crop Model) tried to find the best allocation of the available water both in time and space in order to maximize the total expected yield of a given crop. The third model (Multi crop Model) was an optimization model that determined the optimal allocation of land and water for different crops. It showed the importance of several factors in producing an optimal cropping plan. The output of the models was prepared in a readable form to the normal user by the fourth model (Irrigation Schedule Model).

Khan et al (2005) used Linear Programming Model to calculate the crop acreage, production and income of cotton zone. This was carried out in the three districts of the Bahawalpur. These three districts were collected by purposive sampling technique. The study was conducted on 4652 acres of the irrigated areas from the three districts. Crops included in the model were wheat, basmati rice, IRRI rice, cotton and sugar cane. The results showed that the cotton was the only crop, which gained acreage by about 10% at the expense of all other crops. Overall optimal crop acreage decreased by 1.76%, while optimal income was increased by 3.28% as compared to the existing solutions. The study reported that Bahawalpur division was more or less operating at the optimal level.

Chung et al (2008) considered a municipal water supply system over a 15-year planning period with initial infrastructure and possibility of construction and expansion during the first and sixth year on the planning horizon. Correlated uncertainties in water demand and supply were applied on the form of the robust optimization approach of Bertsimas and Sim to design a reliable water supply system. Robust optimization aims to find a solution that remains feasible under data uncertainty. It was found that the robust optimization approach addressed parameter uncertainty without excessively affecting the system. While they applied their methodology to hypothetical conditions, extensions to real-world systems with similar structure were straightforward. Therefore, their study showed that this approach was a useful tool in water supply system design that prevented system failure at a certain level of risk.

Becker (1995) explored the implications of the transformation of the system of water resources allocation to the agricultural sector in Israel from a one in which allotments were allocated to the different users without any permission to trade with water rights. A mathematical planning model was used for the entire Israeli agricultural sector, in which an 'optimal' allocation of the water resources was found and compared to the existing one. The results of the model were used in order to gain insight into the shadow price of the different water bodies in Israel (about eight). These prices could be used to grant property rights to the water users themselves in order to guarantee rational behaviour of water use, since no one could sell their rights at the source itself. From the dual prices of the primal problem they could forecast the equilibrium prices and their implications for the different users. The results showed that there was a potential budgetary benefit of 28 million dollars when capital cost was not included and 64 millions dollars when it was included

Hoesein and Limantara (2010) studied the optimization of water supply for irrigation at Jatimlerek irrigation area of 1236 ha. Jatimlerek irrigation scheme was intended to serve more than one district. The methodology consisted of optimization water supply for irrigation with Linear Programming. Results were used as the guidance in cropping pattern and allocating water supply for irrigation at the area.

A ground water management model based on the linear systems theory and the use of linear programming was formulated and solved by Heidari (1982). The model maximized the total amount of pound water that could be pumped from the system subject to the physical capability of the system and institutional constraints. The results were compared with analytical and numerical solutions. This model was then applied to the Pawnee Valley area of south-central Kansas. The results of this application supported the previous studies about the future ground water resources of the Valley. These results provided a guide for the ground water resources management of the area over the next ten years.

Frizzone et al (1997) developed a separable linear programming model, considering a set of technical factors which might influence the profit of an irrigation project. The model presented an objective function that maximized the net income and specified the range of water availability. It was assumed that yield functions in response to water application were available for different crops and described very well the water-yield relationships. The linear programming model was developed genetically, so that, the rational use of the available water resource could be included in an irrigation project. Specific equations were developed and applied in the irrigation project "Senator Nilo Coelho" (SNCP), located in Petrolina – Brazil. Based on the water-yield functions considered, cultivated land constraints, production costs and products prices, it was concluded that the model was suitable for the management of the SNCP, resulting in optimal cropping patterns.

The study was conducted by Kumar and Khepar (1980) to demonstrate the usefulness of alternative levels of water use over the fixed yield approach when there is a constraint on water. In the multi-crop farm models used, a water production function for each crop was included so that one had the choice of selecting alternative levels of water use depending upon water availability. Water production functions for seven crops, viz. wheat, gram, mustard, berseem, sugarcane, paddy and cotton, based on experimental data from irrigated crops were used. The fixed yield model was modified incorporating the stepwise water production functions using a separable programming technique. The models were applied on a selected canal command area and optimal cropping patterns determined. Sensitivity analysis for land and water resources was also conducted. The water production function approach gave better possibilities of deciding upon land and water resources.

Banks and Fleck (2010) applied Linear programming techniques to ground-water- flow model in order to determine optimal pumping scenarios for 14 extraction wells located downgradient of a landfill and upgradient of an estuary. The model was used to simulate flow as well as the effects of a pump-and-treat remediation system designed to capture contaminated ground water from the water-table aquifer before it reached the adjacent estuary. The objective function involved varying pumping rates and frequencies to maximize capture of ground water from the water-table aquifer. At the same time, the amount of water extracted and needing treatment was minimized. The constraints placed on the system insured that only ground water from the landfill was extracted and treated. To do this, a downward gradient from the disposal area toward the extraction wells was maintained.

Mousavi et al (2004) presented a long-term planning model for optimizing the operation of Iranian Karoon-Dez reservoir system using an interior-point algorithm. The system is the largest multi-purpose reservoir system in Iran with hydropower generation, water supply, and environmental objectives. The focus was on resolving the dimensionality of the problem of optimization of a multi-reservoir system operation while considering hydropower generation and water supply objectives. The weighting and constraints methods of multi-objective programming were used to assess the trade-off between water supply and hydropower objectives so as to find noninferior solutions. The computational efficiency of the proposed approach was demonstrated using historical data taken from Karoon-Dez reservoir system.

A groundwater management problem in a coastal karstic aquifer in Crere, Greece subject to environmental criteria was studied by Karterakis et al (2007) using classical linear programming and heuristic optimization methodologies. A numerical simulation model of the unconfined coastal aquifer was first developed to represent the complex non-linear physical system. Then the classical linear programming optimization algorithm of the Simplex method was used to solve the groundwater management problem where the main objective was the hydraulic control of the saltwater intrusion. A piecewise linearization of the non-linear optimization problem was obtained by sequential implementation of the Simplex algorithm and a convergence to the optimal solution was achieved. The solution of the non-linear management problem was also obtained using a heuristic algorithm. A Differential Evolution (DE) algorithm that emulates some of the principles of evolution was used. A comparison of the results obtained by the two different optimization approaches was then presented. Finally, a sensitivity analysis was employed in order to examine the influence of the active pumping wells in the evolution of the seawater intrusion front along the coastline.

Heidari (2007) formulated and solved ground water management model based on the linear systems theory using linear programming. The model maximized the total amount of pound water that could be pumped from the system subject to the physical capability of the system and institutional constraints. The results were compared with analytical and numerical solutions. Then, this model was applied to the Pawnee Valley area of south-central Kansas. The results of this application supported the previous studies about the future ground water resources of the Valley. These results provided a guide for the ground water resources management of the area over the next ten years.

Turgeon (1986) developed a parametric mixed-integer linear programming (MILP) method for selecting the sites on the river where reservoirs and hydroelectric power plants were to be built and then determining the type and size of the projected installations. The solution depended on the amount of money the utility was willing to invest, which itself was a function of what the new installations would produce. This method was used based on the fact that the branch-and-bound algorithm for selecting the sites to be developed (and consuming most of the computer time) was solved a minimum number of times. Between the points where the MILP problem was solved, LP parametric analysis was applied.

Isa (1990) used of Linear Programming (LP) and other mathematical procedures to evaluate watershed and perpetuity constraints on forest land use for a selected scenario in Terengganu, Peninsular Malaysia. The LP model provided a range of feasible solutions for decision making. Equations were derived for the model to show interaction of sedimentation due to road construction, timber harvesting, and other related forest management activities. Sensitivity analysis was used to test model behavior. Results indicated the constraining effects of sedimentation upon forest revenues when sedimentation was allowed to vary within the feasible region of the model (i.e., from 600,000 m3/decade up to 1,150,000 m3/decade.

CHAPTER 3

METHODOLOGY

3.0 INTRODUCTION

This chapter presents the methodology used for developing water treatment cost model. The first part of this chapter defines some terminologies, the linear programming model, theoretical methods used in solving it (the simplex algorithm and duality) and software for solving linear programming. The next part is the data collected.

3.1.0 LINEAR PROGRAMMING

Linear programming (LP) is a mathematical method for determining a way to achieve the best outcome (such as maximum profit or lowest cost) in a given mathematical model for some list of requirements represented as linear equations.

More formally, linear programming is a technique for the optimization of a linear objective function, subject to linear equality and linear inequality constraints. Given a polytope and a real-valued affine function defined on this polytope, a linear programming method will find a point on the polytope where this function has the smallest (or largest) value if such point exists, by searching through the polytope vertices.

Linear programs are problems that can be expressed in the form:

$$\text{maximize} \quad c^{\top} x$$
$$\text{subject to} \quad Ax \leq b$$

where **x** represents the vector of variables (to be determined), **c** and **b** are vectors of (known) coefficients and A is a (known) matrix of coefficients. The expression to be maximized or minimized is called the objective function ($\mathbf{c}^T\mathbf{x}$ in this case). The equations $A\mathbf{x} \le \mathbf{b}$ are the constraints which specify a convex polytope over which the objective function is to be optimized. (In this context, two vectors are comparable when every entry in one is less-than or equal-to the corresponding entry in the other. Otherwise, they are incomparable.)

Linear programming can be applied to various fields of study. It is used most extensively in business and economics, but can also be utilized for some engineering problems. Industries that use linear programming models include transportation, energy, telecommunications, and manufacturing. It has proved useful in modeling diverse types of problems in planning, routing, scheduling, assignment, and design.

3.1.1 STANDARD FORM

Standard form is the usual and most intuitive form of describing a linear programming problem. It consists of the following four parts:

- A **linear function**
- **Problem constraints**
- **Non-negative variables**
- **Non-negative right hand side constants**

Given an m - vector, $\boldsymbol{b}$ = $(b_1,\dots,b_m)^T$, an n - vector, $\boldsymbol{c}$ = $(c_1,\dots,c_n)^T$, and an $m\times x$ matrix,

$$A = \begin{pmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & . & a_{2n} \\ . & . & . & & & . \\ . & . & & . & & . \\ . & . & & & . & . \\ a_{m1} & a_{m2} & . & . & . & a_{mn} \end{pmatrix}$$

of real numbers.

3.1.2 THE STANDARD MAXIMUM PROBLEM

Find an n - vector $\boldsymbol{x} = (x_1, ..., x_n)^T$, to maximize

$$c^T x = c_1 x_1 + ... + c_n x_n$$

subject to the constraints

$$a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n \leq b_1$$

$$a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n \leq b_2$$

$$.$$

$$.$$

$$.$$

$$a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n \leq b_m$$

And

$$x_1 \geq 0, x_2 \geq 0, ..., x_n \geq 0 .$$

3.1.3 THE STANDARD MINIMUM PROBLEM

Find an m – vector $y = (y_1, \ldots, y_m)^T$, to minimize

$$y^T b = y_1 b_1 + \ldots + y_m b_m$$

subject to the constraints

$$y_1 a_{11} + y_2 a_{12} + \ldots + y_m a_{1m} \geq c_1$$

$$y_1 a_{12} + y_2 a_{22} + \ldots + y_m a_{m2} \geq c_2$$

$$\vdots$$

$$y_1 a_{1n} + y_2 a_{2n} + \ldots + y_m a_{mn} \geq c_n$$

and

$$y_1 \geq 0, y_2 \geq 0, \ldots, y_m \geq 0.$$

3.1.4 MODEL

A model is a mathematical representation of an ideal or a real life situation in which all factors contributing to the idea or situation are represented by variables and sometimes, constant figures.

3.1.5 SYSTEM OF EQUATIONS / INEQUALITIES

This is a collection of linear equations or inequalities involving the same set of variables. A solution to a linear system is to assign numbers to the variables such that all the equations or inequalities are simultaneously satisfied.

3.1.6 BOUNDARY LINE

It is a line or border around outside of a shape. The boundary line defines the space or area.

3.1.7 CORNER POINTS

These are a set of points in the feasible region, which are the intersection of two or more boundary lines. The optimal solution of an objective function if it exist, occurs at the corner point of the feasible region.

3.1.8 BOUNDED SOLUTION

A feasible solution is said to be bounded if it can be contained in a closed figure. If the solution cannot be contained in a closed figure, it is said to be unbounded. Under such condition, the optimal solution to the objective function may not exist.

3.1.9 OPTIMAL SOLUTION

This is the set of points of all feasible regions that produces the optimal value (maximum or minimum) of the objective function.

3.1.10 GRAPHICAL REPRESENTATION OF SYSTEMS OF EQUATIONS.

A graphical representation is only possible if the decision variable is either one or two. All the variables must have a power of one and they are either added or subtracted. The constraint must be of the following forms $(\leq, \geq, \text{or} =)$ and the objective must either be a maximization or minimization. One variable is represented by the horizontal axis called the x – axis and the other by the vertical axis called the y – axis of the graph. Negative values are not allowed since they make no sense in real life.

A linear equation with two variables is shown as a straight line on a graph. It is easier and faster to draw the line using its intercept(s) on the axis.

Suppose a situation gives the following objective function and constraints

Maximize: $5x_1 + 3x_2$

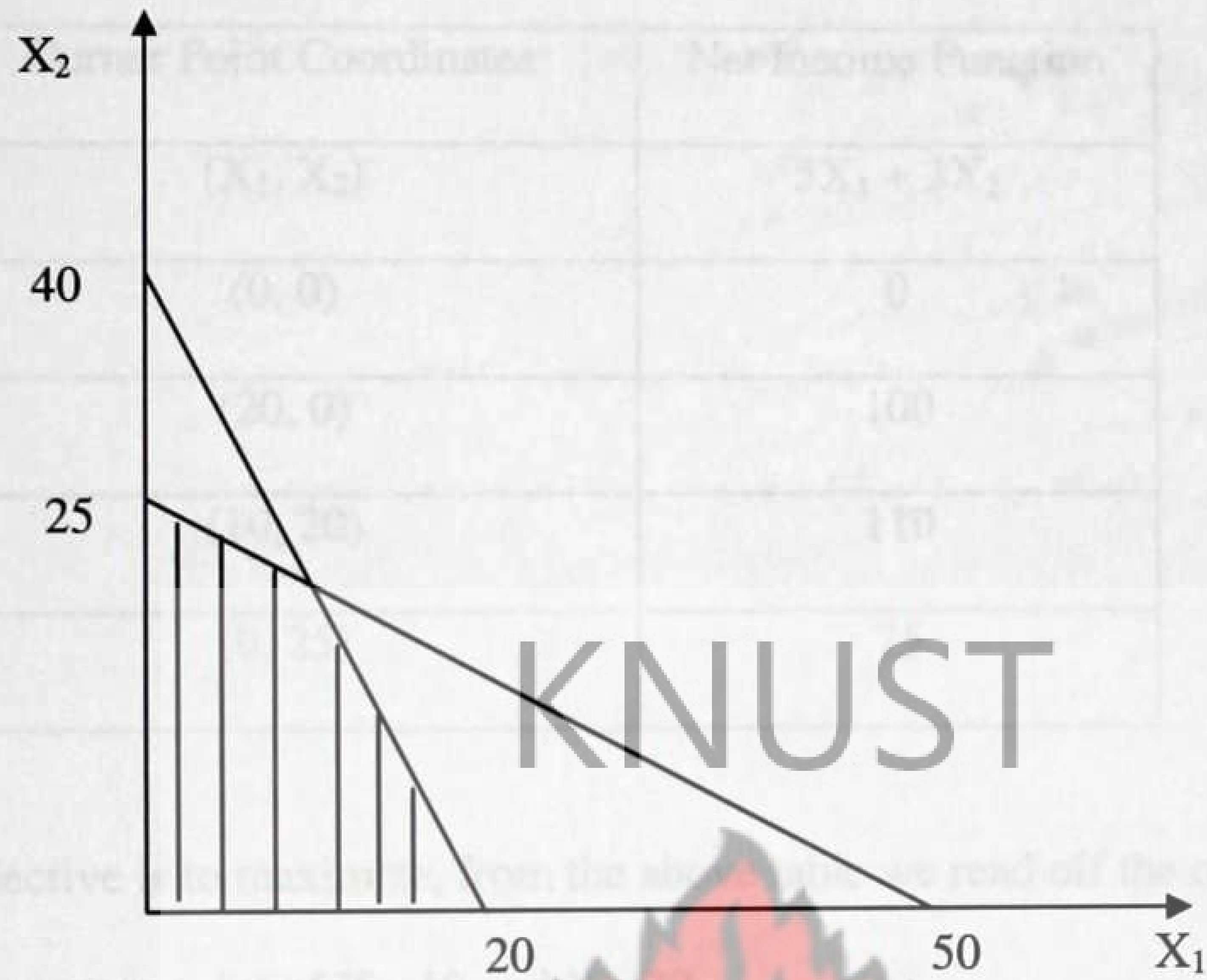
Subject to: $2x_1 + x_2 \leq 40$

$x_1 + 2x_2 \leq 50$

$x_1, x_2 \geq 0$

The purpose of the representation is to find the set of all points (x_1, x_2) all the inequalities in the system simultaneously. To represent an inequality of the form $aX_1 + bX_2 \leq c$ or $aX_1 + bX_2 \geq c$, draw on a number plane the line $aX_1 + bX_2 = c$. Pick a point in either side of the line and pluck its coordinates into the constraint. If it satisfies the condition, this side is feasible; otherwise the other side is feasible. Using this idea the system of inequalities are all represented on the same number line.

The system above can be represented by:



The shaded region represents all set of points which satisfy all the constraints.

Its corresponding corner points are:

A (0, 0)

B (20, 0)

C (10, 20)

D (0, 25)

Evaluating the objective function at each of the corner point yields the result shown on ~~the table~~ below.

Table 3.1 Objective Function Value at Each Corner Point

Corner Point Coordinates	Net Income Function
(X_1, X_2)	$5X_1 + 3X_2$
(0, 0)	0
(20, 0)	100
(10, 20)	110
(0, 25)	75

Since the objective is to maximize, from the above table we read off the optimal value to be 110 with a corner point of $X_1=10$ and $X_2=20$.

3.2.0 THE SIMPLEX METHOD

It is a systematic way of examining the vertices of the feasible region to determine the optimal value of the objective function.

Simplex usually starts at the corner that represents doing nothing. It moves to the neighbouring corner that best improves the solution. It does this over and over again, making the greatest possible improvement each time. When no more improvements can be made, the most attractive corner corresponding to the optimal solution has been found.

3.2.1 THE STANDARD MAXIMUM FORM FOR A LINEAR PROGRAM

A standard maximum problem is a linear program in which the objective is to maximize an objective function of the form:

$$Z = C_1X_1 + C_2X_2 + ... + C_nX_n$$

subject to:

$$a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n \leq b_1$$

$$a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n \leq b_2$$

.

.

.

$$a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n \leq b_m$$

where $x_1, x_2, ..., x_n \geq 0$

and $b_j \geq 0$ for $j = 1, 2, ..., m$

3.2.2 THE SIMPLEX TABLEAU

To set up the simplex tableau for a given objective function and its constraints, add none negative slack variable s_i to the constraints. This is to convert the constraints into equations. The constraints therefore become:

$$a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = b_2$$

.

.

$$a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n = b_m$$

where $x_i \geq 0$ for $i = 1, 2, ..., n$

Table 3.2; shows table for formulating simplex tableau

	c_j	c_1	c_2	...	c_n	0	0	...	0	
C_B	B. V.	x_1	x_2	...	x_n	s_1	s_2	...	s_n	RHS
0	s_1	a_{11}	a_{12}	...	a_{1n}	1	0	...	0	b_1
0	s_2	a_{21}	a_{22}	...	a_{2n}	0	1	...	0	b_2
.	.	.	.		.	.	.		.	.
.	.	.	.		.	.	.		.	.
.	.	.	.		.	.	.		.	.
0	s_m	a_{m1}	a_{m2}	...	a_{mn}	0	0	...	1	b_m
	z_j	0	0	...	0	0	0	...	0	0
	$c_j - z_j$	c_1	c_2	...	c_n	0	0	0	0	

C_B is the objective function coefficients for each of the basic variables.

Z_j is the decrease in the value of the objective function that will result if one unit of the variable corresponding to the j^{th} column of the matrix formed from the coefficients of the variables in the constraints is brought into the basis (thus if the variable is made a basic variable with a value of one)

$C_j - Z_j$ called the Net Evaluation Row, is the net change in the value of the objective function if one unit of the variable corresponding to the j^{th} column of the matrix (formed from the coefficient of the variables in the constraints), is brought into solution.

From the $C_j - Z_j$ row we locate the column that contains the largest positive number and this becomes the Pivot Column. In each row we now divide the value in the RHS by the positive entry in the pivot column (ignoring all zero or negative entries) and the smallest one of these ratios gives the pivot row. The number at the intersection of the pivot column and the pivot row is called the PIVOT.

We then divide the entries of that row in the matrix by the pivot and use row operation to reduce all other entries in the pivot column, apart from the pivot, to zero.

3.2.3 THE STOPPING CRITERION

The optimal solution to the linear program problem is reached when all the entries in the net evaluation row, that is, $C_j - Z_j$, are all negative or zero.

3.2.4 MINIMIZING THE OBJECTIVE FUNCTION

Standard form of LP problem consists of a maximizing objective function. Simplex method is described based on the standard form of LP problems. If the problem is a minimization type, the objective function is multiplied through by – 1 so that the problem becomes maximization one.

$$\text{Min } F = -\text{ Max } F$$

3.2.5 CONSTRAINTS OF THE ≥ TYPE

The LP problem with 'greater-than-equal-to' (≥) constraint is transformed to its standard form by subtracting a non negative surplus variable from it:

$$a_i x \geq b_i$$

is equivalent to

$$a_i x - s_i = b_i \text{ and } s_i \geq 0 .$$

3.2.6 CONSTRAINTS WITH NEGATIVE RIGHT HAND SIDE CONSTANTS

Multiply both side of the constraint by – 1 and add either an artificial variable or a surplus and artificial variable as required. Assuming we have the constraint:

$$-2x_1 + 7x_2 \leq -10$$

Multiplying both sides by a negative gives:

$$2x_1 - 7x_2 \geq 10 .$$

To convert the new constraint into equality, we add both a surplus and artificial variable as follow:

$$2x_1 - 7x_2 - 1s_1 + 1A_1 = 10$$

where s_1 and A_1 are surplus and artificial variables respectively.

3.2.7 EQUALITY CONSTRAINT

Situation where any of the constraints is of the linear programming is of the form:

$$a_1x_1 + a_2x_2 + ... + a_nx_n = b ,$$

The single constraint is replaced with the following two constraints:

$$a_1x_1 + a_2x_2 + ... + a_nx_n \le b \text{ and } a_1x_1 + a_2x_2 + ... + a_nx_n = b.$$

The usual procedure is then applied.

3.2.8 UNCONSTRAINED VARIABLES

If some variable x_j is unrestricted in sign, replace it everywhere in the formulation by $x'_j - x''_j$, where $x'_j \ge 0$ and $x''_j \ge 0$.

Example

Maximize $Z = 3x_1 + 4x_2$

Subject to:

$$2x_1 + 4x_2 \le 120$$

$$2x_1 + 2x_2 \le 80$$

where $x_1 \ge 0$, $x_2 \ge 0$.

The equations then become:

Maximize:

$$Z = 3x_1 + 4x_2 + 0S_1 + 0S_2$$

Subject to:

$$2x_1 + 4x_2 + S_1 = 120$$

$$2x_1 + 2x_2 + S_2 = 80$$

Where $x_1 \ge 0$, $x_2 \ge 0$, $s_1 \ge 0$, $s_2 \ge 0$.

3.2.9 INITIAL SIMPLEX TABLEAU

This is the first table that is generated from the coefficients of the objective function and its constraint variables.

Table 3.3; shows the initial simplex tableau

	C_j	3	4	0	0	
C_B	Basic Variables	X_1	X_2	S_1	S_2	Solution
0	S_1	2	4	1	0	120
0	S_2	2	2	0	1	80
	Z_j	0	0	0	0	0
	$C_j - Z_j$	3	4	0	0	

3.2.10 PIVOTING

To pivot a matrix about a given element, called the pivot element, is to apply row operation so that the pivot element is replaced by 1 and all other entries in the same column become 0. We apply Gaus – Jordan elimination procedure to do that.

3.2.11 THE PIVOT ELEMENT

It is selected by applying the following rules

i) Select the pivot column by locating the most positive value on the $C_j - Z_j$ row.

ii) Divide each entry in the last column by the corresponding entry (from the same row) in the pivot column. The row in which the smallest positive ratio is obtained is the pivot row.

From the above $\frac{120}{4} = 30$ and $\frac{80}{2} = 40$.

The intersection of the pivot column and the pivot row gives the pivot element.

The variable entry

This is the column variable at the pivot element. From the above the variable is X_2. It now goes to the basis.

3.2.12 THE VARIABLE EXIT

This is the horizontal variable at the pivot element. The variable is S_1. This variable now leaves the basis.

The second simplex tableau now becomes:

Table 3.4; shows the second simplex tableau

	C_j	3	4	0	0	
C_B	Basic variables	X_1	X_2	S_1	S_2	Solution
4	X_2	1/2	1	¼	0	30
0	S_2	1	0	- 1/2	1	20
	Z_j	2	4	1	0	120
	C_j - Z_j	1	0	- 1	0	

There is still a $C_j - Z_j = 1$ (positive), so optimality has yet not been attained.

Following the same procedure as above gives the third simplex tableau. Since the result from the table optimizes the objective function, it is called the final simplex tableau.

Table 3.4; shows the final simplex tableau

	C_j	3	4	0	0	
C_B	Basic variables	X_1	X_2	S_1	S_2	Solution
4	X_2	0	1	½	- ½	20
3	X_1	1	0	- 1/2	1	20
	Z_j	3	4	½	1	140
	C_j - Z_j	0	0	- 1/2	- 1	

Since $C_j - Z_j$ contains zeros and negatives, it implies that optimality has been reached.

The objective function will therefore have its maximum when $X_1 = 20$ and $X_2 = 20$.

3.3.0 THE INTERIOR - POINT METHOD

Interior point methods are certain class of algorithms used to solve linear and nonlinear convex optimization problems. They follow a path through the interior of the feasible region until the final solution is attained.

All interior point algorithms are based on the general framework which is summarized below:

3.3.1 GENERAL OPTIMIZATION ALGORITHM

(1) Given an iterate x^k, find the search direction Δx by solving the linear system

$$\nabla f(x^k)\Delta x = -f(x^k).$$

(2) Find the step size α_k.

(3) Update x^k to $x^{k+1} = x^k + \alpha_k \Delta x$.

The symbol ∇f represents the derivative, gradient, or Jacobian of the function f depending on the definition of the function f.

3.3.2 STARTING POINT

The choice of starting point depends on two requirements: the centrality of the point and the magnitude of the corresponding infeasibility. These conditions are met by solving two least squares problems which aim to satisfy the primal and dual constraints:

$$\min_x x^T x \quad s.t \quad Ax = b$$

$$\min_{(y,s)} s^T s \quad s.t \quad A^T y + s = c$$

These problems have solution:

$$\tilde{x} = A^T (AA^T)^{-1} b, \qquad \tilde{y} = (AA^T)^{-1} Ac, \qquad \tilde{s} = c - A^T \tilde{y}$$

The solution is further shifted inside the positive orthant to obtain the starting point as:

$$w^o = \left(\tilde{x} + \delta_x e, \tilde{y}, \tilde{s} + \delta_s e\right),$$

where δ_x and δ_s are positive quantities.

3.3.3 SEARCH DIRECTION

It is $(\Delta x, \Delta y, \Delta z)$. It is obtained by solving the Newton's equation:

$$\nabla f(x, y, s) \begin{bmatrix} \Delta y \\ \Delta x \\ \Delta s \end{bmatrix} = -f(x^k, y^k, s^k)$$

3.3.4 STEP SIZE

The choice of step-size is essential in proving good convergence properties of interior point methods.

The step size is chosen so that the positivity of x and s are preserved when updated. $\alpha^{\max}$ is the maximum step size that is chosen until one of the variables becomes zero (0).

$\alpha^{\max}$ is calculated as follows:

$$\alpha_P^{\max} = \min\left\{-\frac{x_i}{(dx)_i} : (dx)_i < 0, i = 1,\dots,n\right\},$$

$$\alpha_D^{\max} = \min\left\{-\frac{s_i}{(ds)_i} : (ds)_i < 0, i = 1,\dots,n\right\},$$

$$\alpha^{\max} = \min\left\{\alpha_P^{\max}, \alpha_D^{\max}\right\}.$$

Since none of the variables is allowed to be zero (0), $\alpha = \min\{1, \theta\alpha^{\max}\}$ is taken, where $\theta \in (0,1)$. The usual choice is $\theta = 0.9$ or $\theta = 0.95$.

3.3.4 TERMINATION CRITERIA

Due to the presence of the barrier term that keeps the iterates away from the boundary, they can never produce and exact solution. Feasibility and complementarity can therefore be attained only within a certain level of accuracy.

For these reasons, termination criteria for the algorithm to be used has to be decided on.

Some common termination criteria used in practice are as follows:

$$\frac{\|Ax-b\|}{1+\|x\|_\infty} \le 10^{-p}, \quad \frac{\|A^T y + s - c\|}{1+\|s\|_\infty} \le 10^{-p}, \quad \frac{|c^T x - b^T y|}{1+|b^T y|} \le 10^{-q}.$$

The values of p and q required depend on the specific application.

3.4.0 PRIMAL DUAL METHODS

It is one of the three main categories of the interior point methods. The primal dual algorithm operates simultaneously on the primal and the dual linear programming. They find the solutions (x^*, y^*, s^*) of

$$\begin{bmatrix} A & 0 & 0 \\ 0 & A^T & I \\ S^k & 0 & A^k \end{bmatrix} \begin{bmatrix} d_x \\ d_y \\ d_s \end{bmatrix} = \begin{bmatrix} r_P^k \\ r_D^k \\ -X^k s^k + \gamma\mu_k e \end{bmatrix}$$

by applying variants of Newton's method to the above and modifying the search directions and the step lengths so that inequalities $(x, s) \geq 0$ are satisfied strictly at every iteration. $X, S \in R^{n \times n}$ are diagonal matrices of x_i, s_i respectively and $e \in R^n$ is a vector of ones.

3.4.1 THE PRIMAL PROBLEM

Given the linear programming problem in the standard form:

(P) minimize $c^T x$

subject to $Ax = b$, $x \geq 0$

where $c \in R^n$, $A \in R^{m \times n}$ and $b \in R^m$ are given data, and $x \in R^n$ is the decision variable.

The dual (D) to the primal (P) can be written as:

(D) maximize $b^T y$

subject to $A^T y + s = c$, $s \geq 0$

with variables $y \in R^m$ and $s \in R^n$.

The centering parameter (σ)

It balances the movement towards the central path against the movement toward optimal solutions. If $\sigma = 1$, then the updates move towards the center of the feasible region. If $\sigma = 0$, then the update step is in the direction of the optimal solution.

The duality Gap (μ)

It is the difference between the primal and dual objective functions. Theoretically, these two quantities are equal and so give a result of zero (0) at optimality. In practice however, the algorithm drives the result down to a small amount. This is given by the equation:

$$\mu \equiv \frac{1}{n}(x^T s) = c^T x - b^T y$$

While $\mu \geq \varepsilon$, Newton's method is applied until $\mu \leq \varepsilon$ when the algorithm terminates. ε is a positive fixed number.

The general standard minimum problem and the dual standard maximum problem may be together illustrated as:

Table 3.6; shows standard minimum and dual maximum constraints

	x_1	x_2	$\cdots$	x_n	
y_1	a_{11}	a_{12}	$\cdots$	a_{1n}	$\geq b_1$
y_2	a_{21}	a_{22}	$\cdots$	a_{2n}	$\geq b_2$
.	.	.		.	.
.	.	.		.	.
.	.	.		.	.
y_n	a_{m1}	a_{m2}	$\cdots$	a_{mn}	$\geq b_m$
	$\leq c_1$	$\leq c_2$	$\cdots$	$\leq c_n$	

3.4.2 FORMULATION OF THE DUAL FROM THE PRIMAL OF A LINEAR PROGRAMMING PROBLEM

To illustrate this, let us consider the problem:

Minimize $x_1 + x_2$

subject to $x_1 + 2x_2 \geq 4$

$4x_1 + 2x_2 \geq 12$

$-x_1 + x_2 \geq 1$

$x_1 \geq 0, x_2 \geq 0.$

As a first step, a matrix A is formed from the coefficients of the primal objective function and its constraints as:

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 4 & 2 & 12 \\ -1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

A second matrix B is formed from the transpose of A. That is:

$$B = \begin{bmatrix} 1 & 4 & -1 & 1 \\ 2 & 2 & 1 & 1 \\ 4 & 12 & 1 & 0 \end{bmatrix}$$

The dual problem is formulated as follows:

Maximize $4y_1 + 12y_2 + y_3$

subject to $y_1 + 4y_2 - y_3 \le 1$

$2y_1 + 2y_2 + y_3 \le 1$

$y_1 \ge 0, y_2 \ge 0$ and $y_3 \ge 0$.

It must be noted that if the objective function in the primal is to be minimize, then its dual objective function will maximize and vice versa.

3.4.3 THE PRIMAL-DUAL ALGORITHM

Initialization

1. Choose $\beta, \gamma \in (0,1)$ and $(\varepsilon_P, \varepsilon_D, \varepsilon_G) > 0$.

Choose (x^0, y^0, s^0) such that $(x^0, s^0) > 0$ and $\|X^0 s^0 - \mu_0 e\| \le \beta\mu_0$

where $\mu_0 = \dfrac{(x^0)^T s^0}{n}$.

2 Set $k = 0$

3. Set $r_p^k = b - Ax^k$, $r_D^k = c - Ak^T y^k - s^k$, $\mu_k = \dfrac{(x^k)^T s^T}{n}$

4. Check the termination. If $\left\| r_P^k \right\| \le \varepsilon_P$, $\left\| r_D^k \right\| \le \varepsilon_D$, $(x^k)^T s^k \le \varepsilon_G$, then terminate.

5. Compute the direction by solving the system

$$\begin{bmatrix} A & 0 & 0 \\ 0 & A^T & I \\ S^k & 0 & A^k \end{bmatrix} \begin{bmatrix} d_x \\ d_y \\ d_s \end{bmatrix} = \begin{bmatrix} r_P^k \\ r_D^k \\ -X^k s^k + \gamma \mu_k e \end{bmatrix}$$

6. Compute the step size

$$\alpha = \max\{\alpha' : \left\| X(\alpha)s(\alpha) - \mu(\alpha)e \right\| \le \beta(\alpha), \forall \alpha \in [0, \alpha']\},\ \text{where}$$

$$x(\alpha) = x^k + \alpha d_x,\ s(\alpha) = s^k + \alpha d_s \ \text{and}\ \mu(\alpha) = \frac{x^T(\alpha)s(\alpha)}{n}.$$

7. Update $x^{k+1} = x^k + \alpha_k d_x$, $y^{k+1} = y^k + \alpha d_y$, $s^{k+1} = s^k d_s$

8. Set $k = k+1$, and go to step 3.

3.4.4 NUMERICAL EXAMPLE

Minimize $2x_1 + 1.5x_2$

Subject to $12x_1 + 24x_2 \ge 120$

$16x_1 + 16x_2 \ge 120$

$30x_1 + 12x_2 \ge 120$

$x_1 \le 15$

$x_2 \le 15$

$x_1 \ge 0, x_2 \ge 0$

Standard non-negative equations

Minimize $2x_1 + 1.5x_2$

$$\begin{aligned}
\text{Subject to}\quad & 12x_1 + 24x_2 - x_3 &&= 120\\
& 16x_1 + 16x_2 \quad - x_4 &&= 120\\
& 30x_1 + 12x_2 \quad\quad - x_5 &&= 120\\
& x_1 \quad\quad\quad\quad + x_6 &&= 15\\
& x_2 \quad\quad\quad\quad\quad + x_7 &&= 15\\
& x_1, \dots, x_7 \geq 0
\end{aligned}$$

Substituting $(\tilde{x}_1, \tilde{x}_2) = (10,10)$ into the equations above and solving for the slack variables $(\tilde{x}_3, \dots, \tilde{x}_7)$ give:

$$\tilde{x} = \begin{bmatrix} 10 \\ 10 \\ 240 \\ 200 \\ 300 \\ 5 \\ 5 \end{bmatrix} > 0$$

For an initial dual iterate, the algorithm requires a $\tilde{y}$ such that $\tilde{s} = c - A^T y > 0$. Writing these explicitly give:

$$\tilde{s}_1 = 2 - 12\tilde{y}_1 - 16\tilde{y}_2 - 30\tilde{y}_3 - 1\tilde{y}_4 \quad > 0$$

$$\tilde{s}_2 = 1.5 - 24\tilde{y}_1 - 16\tilde{y}_2 - 12\tilde{y}_3 \quad -1\tilde{y}_5 > 0$$

$$\tilde{s}_3 = 0 + 1\tilde{y}_1 \qquad\qquad\qquad\qquad\qquad > 0$$

$$\tilde{s}_4 = 0 \qquad +1\tilde{y}_2 \qquad\qquad\qquad\qquad > 0$$

$$\tilde{s}_5 = 0 \qquad\qquad +1\tilde{y}_3 \qquad\qquad\qquad > 0$$

$$\tilde{s}_6 = 0 \qquad\qquad\qquad -\tilde{y}_4 \qquad\qquad > 0$$

$$\tilde{s}_y = 0 \qquad\qquad\qquad\qquad -1\tilde{y}_5 > 0$$

The above inequalities are satisfied by putting in $\tilde{y}_1 = \tilde{y}_2 = \tilde{y}_3 = 1$ and $\tilde{y}_4 = \tilde{y}_5 = -60$.

This gives:

$$\tilde{y} = [1 \quad 1 \quad 1 \quad -60 \quad -60],\ \tilde{s} = [4 \quad 9.5 \quad 1 \quad 1 \quad 1 \quad 60 \quad 60] > 0.$$

The matrices generated are as follows:

$$A = \begin{bmatrix} 12 & 24 & -1 & 0 & 0 & 0 & 0 \\ 16 & 16 & 0 & -1 & 0 & 0 & 0 \\ 30 & 12 & 0 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},\quad B = \begin{bmatrix} 120 \\ 120 \\ 120 \\ 15 \\ 15 \end{bmatrix} \text{ and } C = [2 \quad 1.5 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0].$$

With $\alpha = 0.995$ and complementarity tolerance $\varepsilon = 0.00001$, the algorithm will stop when all $x_j s_j < 0.00001$.

$$
\left[\begin{array}{ccccccc|ccccc}
-0.4000 & 0 & 0 & 0 & 0 & 0 & 0 & 12 & 16 & 30 & 1 & 0 \\
0 & -0.9500 & 0 & 0 & 0 & 0 & 0 & 24 & 16 & 12 & 0 & 1 \\
0 & 0 & -0.0042 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -0.0050 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.0033 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -12 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -12 & 0 & 0 & 0 & 0 & 1 \\
\hline
12 & 24 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
16 & 16 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
30 & 12 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
\left[\begin{array}{c}
\Delta x_1 \\ \Delta x_2 \\ \Delta_3 \\ \Delta x_4 \\ \Delta x_5 \\ \Delta_6 \\ \Delta_7 \\ \hline \Delta y_1 \\ \Delta y_2 \\ \Delta y_3 \\ \Delta y_4 \\ \Delta y_5
\end{array}\right]
=
\left[\begin{array}{c}
4 \\ 9.5 \\ 1 \\ 1 \\ 1 \\ 60 \\ 60 \\ \hline 0 \\ 0 \\ 0 \\ 0 \\ 0
\end{array}\right]
$$

Solving the system yields:

$$
\Delta x = \begin{bmatrix} -0.1017 \\ -0.0658 \\ -2.7997 \\ -2.6803 \\ -3.8414 \\ 0.1017 \\ 0.0658 \end{bmatrix}
\quad \text{and} \quad
\Delta y = \begin{bmatrix} -0.9883 \\ -0.9866 \\ -0.9872 \\ 61.2208 \\ 60.7895 \end{bmatrix}
$$

Δs is obtained by setting:

$$
\Delta s = -\tilde{s} - \tilde{x}^{-1}\tilde{\Sigma}\Delta x
$$

$$
= \begin{bmatrix} 4 \\ 9.5 \\ 1 \\ 1 \\ 1 \\ 60 \\ 60 \end{bmatrix}
- \begin{bmatrix}
0.4000 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.9500 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.0042 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.0050 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.0033 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 12 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 12
\end{bmatrix}
\begin{bmatrix} -0.1017 \\ -0.0658 \\ -2.7997 \\ -2.6803 \\ -3.8414 \\ 0.1017 \\ 0.0658 \end{bmatrix}
$$

$$
= \begin{bmatrix} -3.9593 \\ -9.4375 \\ -0.9883 \\ -0.9866 \\ -0.9872 \\ -61.2208 \\ -60.7895 \end{bmatrix}
$$

The ratio $\tilde{x}_j/(-\Delta x_j)$ is computed for each of the five $\Delta x_j < 0$, and θ_x is set to the smallest:

$$\Delta x_1 < 0 : x_1/(-\Delta x_1) = 10/0.1017 = 98.3284$$

$$\Delta x_2 < 0 : x_2/(-\Delta x_2) = 10/0.0658 = 152.0034$$

$$\Delta x_3 < 0 : x_3/(-\Delta x_3) = 240/2.7997 = 85.7242$$

$$\Delta x_4 < 0 : x_4/(-\Delta x_4) = 200/2.6803 = 74.6187 = \theta_x$$

$$\Delta x_5 < 0 : x_5/(-\Delta x_5) = 300/3.8414 = 78.0972$$

The ratios $\tilde{s}_j/(-\Delta s_j)$ are computed in the same way to determine θ_s:

$$\Delta s_1 < 0 : \tilde{s}_1/(-\Delta s_1) = 4/3.9593 = 1.0103$$

$$\Delta s_2 < 0 : \tilde{s}_2/(-\Delta s_2) = 9.5/9.4375 = 1.0066$$

$$\Delta s_3 < 0 : \tilde{s}_3/(-\Delta s_3) = 1/0.9883 = 1.0118$$

$$\Delta s_4 < 0 : \tilde{s}_4/(-\Delta s_4) = 1/0.9866 = 1.0136$$

$$\Delta s_5 < 0 : \tilde{s}_5/(-\Delta s_5) = 1/0.9872 = 1.0130$$

$$\Delta s_6 < 0 : \tilde{s}_6/(-\Delta s_6) = 60/61.2208 = 0.9801 = \theta_s$$

$$\Delta s_7 < 0 : \tilde{s}_7/(-\Delta s_7) = 60/60.7895 = 0.9870$$

The step length is given by:

$$\begin{aligned} \theta &= \min(1, \alpha\theta_x, \alpha\theta_s) \\ &= \min(1, 0.995 \cdot 74.6187, 0.995 \cdot 0.9801) = 0.975159 \end{aligned}$$

The iteration ends with the computation of the next iterate as:

$$\tilde{x} = \tilde{x} + \theta(\Delta x) = \begin{bmatrix} 10 \\ 10 \\ 240 \\ 200 \\ 300 \\ 5 \\ 5 \end{bmatrix} + 0.97519 \begin{bmatrix} -0.1017 \\ -0.0658 \\ -2.7979 \\ -2.6803 \\ -3.8414 \\ 0.1017 \\ 0.0658 \end{bmatrix} = \begin{bmatrix} 9.9008 \\ 9.9358 \\ 237.2699 \\ 197.3863 \\ 296.2541 \\ 5.0992 \\ 5.0642 \end{bmatrix}$$

The algorithm carries out a total of 9 iterations before reaching a solution that satisfies the stopping condition.

<u>TABLE SHOWING ITERATIONS OF FIRST TWO VARIABLES OUT OF SEVEN IN THE PROBLEM</u>

iter	x_1	x_2	θ	$\max x_j s_j$
0	10.0000	10.0000	---	300.000000
1	9.9008	9.9358	0.975159	11.058316
2	6.9891	9.2249	0.423990	6.728827
3	3.2420	8.5423	0.527256	2.878729

4	1.9835	6.6197	0.697264	1.156341
5	2.0266	5.4789	0.693037	0.486016
6	1.8769	5.6231	0.581321	0.189301
7	1.7204	5.7796	0.841193	0.027134
8	1.6683	5.8317	0.979129	0.000836
9	1.6667	5.8333	0.994501	0.000004

The optimal solution for the first two variables are $x_1 = 1.6667$ and $x_2 = 5.8333$ with stopping condition of $0.000004 < 0.00001$

CHAPTER 4

DATA ANALYSIS AND RESULTS

4.1 DATA COLLECTION

The data was collected from Sunyani water treatment plant. They consist of cost and quantity of chemical, electricity and fuel that were used to treat water from January to December, 2010.

The data was categorized into two aspects to reflect the two major seasons in the year. These are the dry and wet seasons. The dry season is from November to March and the wet season is from April to October.

Dry season

Table 4.1 TABLE SHOWING COST OF CHEMICAL, ELECTRICITY AND FUEL

Month	Chemical(GH¢)	Electricity	Fuel
November	13514.00	1778.57	375.10
December	11758.00	2123.81	403.23
January	8249.15	1979.66	413.15
February	9199.00	1966.92	397.20
March	12202.00	9171.27	367.50

Table 4.2 TABLE SHOWING QUANTITY OF CHEMICAL, ELECTRICITY AND FUEL

Month	Chemical(Bags)	Electricity(kw/h)	Fuel(Litre)
November	1969.5	15621	321.75
December	1709.5	16378	345.87
January	1214.5	15001	345.38
February	1254.5	14249	340.70
March	1707.25	14922	315.23

Wet season

4.3 TABLE SHOWING COST OF CHEMICAL, ELECTRICITY AND FUEL

Month	Chemical (GH¢)	Electricity(GH¢)	Fuel(GH¢)
April	15152.00	1995.49	393.75
May	18171.00	2040.53	376.20
June	15642.00	2063.07	329.35
July	15820.00	2028.85	450.00
August	12589.00	2018.68	378.10
Sept	13791.00	2023.04	452.44
October	15774.00	2116.76	412.51

Table 4.4 TABLE SHOWING QUANTITY OF CHEMICAL ELECTRICITY AND FUEL

Month	Chemical(Bag)	Electricity(kw/h)	Fuel(Litre)
April	2133.75	15160	337.74
May	2582.25	15580	322.69
June	2324.25	15802	282.50
July	2337	15471	386.19
August	1889	15374	324.32
September	2014	15414	388.08
October	2332	16307	353.83

4.2 CALCULATION OF PARAMETERS

The objective function is formulated based on the following factors: total monthly chemical cost, total monthly electricity cost and total monthly fuel cost (include transport and other lubricants).

Cost = Price x Quantity

Hence Total Treatment Cost = $\sum_{i=1}^{n} a_i x_i = a_1x_1 + a_2x_2 + a_3x_3$.

The coefficients (a_1, a_2, a_3) of the objective function are the unit prices of chemical electricity and fuel respectively.

$$a_1 = \frac{\text{average monthly price of bag of chemical}}{\text{Average monthly bags of chemical}} = \frac{13488.43}{1955.46} = 6.90$$

$$a_2 = \frac{\text{average monthly price of electricity}}{\text{Average monthly unit of electricity}} = \frac{20092.25}{154399.17} = 0.13$$

$$a_3 = \frac{\text{average monthly price of litre of fuel}}{\text{Average monthly litre of fuel}} = \frac{395.73}{339.44} = 1.17$$

4.2.1 CONSTRAINTS COEFFICIENT

The seasonal average cost, average quantities and usage ratio of chemical, electricity and fuel are presented in table. The seasonal average cost and quantity were obtained by using the general formula below for both seasons (Ref. pages 63 and 64):

Dry season

$$\text{Seasonal average cost/quantity} = \frac{\text{Total monthly cost/quantity}}{\text{Number of months}}$$

The usage ratios were calculated by using the formula:

$$\text{Usage Ratio} = \frac{\text{Seasonal cost}}{\text{Seasonal quantity}}$$

4.2.2 MODEL FOR DRY SEASON

Minimize Total Cost $= 6.9x_1 + 0.13x_2 + 1.17x_3$

Subject to $\quad 7.99x_1 + 33.49x_3 \geq 80620$

$$0.12x_2 + 32.05x_3 \geq 11015$$

$$6.25x_3 \geq 1000$$

$$x_1, x_2 x_3 \geq 0$$

4.2.3 MODEL FOR WET SEASON

Minimize Total Cost = $6.9x_1 + 0.13x_2 + 1.17x_3$

Subject to $\quad 6.39x_1 + 39.05x_3 \geq 97123$

$$0.12x_2 + 30.03x_3 \geq 13016$$

$$5.59x_3 \geq 1320$$

$$x_1, x_2 x_3 \geq 0$$

4.3 COMPUTATIONAL PROCEDURE

The coefficients of the cost functions, left-hand side constraint inequalities and right-hand side constants were written in matrices form. Matlab program software was used for coding the primal-dual algorithm.

The matrices were inputted in the Matlab program code and ran on AMD Anthlon™ 64×2 Dual-Core processor TK-57, 32-bit operating system, 1.90GHz speed, Windows Vista Dell laptop computer.

The code ran successfully on five trials with nineteen iterations for each trial.

4.4 MATRICES FORMULATION

Using A, B and C for the matrices of left-hand side inequalities, right-hand side constants and cost functions respectively, then:

Dry Season

$$A=\begin{bmatrix}7.99 & 0 & 33.49\\ 0 & 0.12 & 32.05\\ 0 & 0 & 0.25\end{bmatrix},\quad B=\begin{bmatrix}80620\\ 11015\\ 1000\end{bmatrix}\quad \text{and}\quad C=\begin{bmatrix}6.9\\ 0.13\\ 1.17\end{bmatrix}.$$

Wet Season

$$A=\begin{bmatrix}6.39 & 0 & 39.05\\ 0 & 0.12 & 30.03\\ 0 & 0 & 5.59\end{bmatrix},\quad B=\begin{bmatrix}97123\\ 13016\\ 1320\end{bmatrix}\quad \text{and}\quad C=\begin{bmatrix}6.9\\ 0.13\\ 1.17\end{bmatrix}.$$

4.5 RESULTS

Results of final test run for TOTAL WATER TREATMENT COST IN DRY SEASON after 19 iterations:

$$1.0e+04 \qquad\qquad 1.0e-10$$

$$x=\begin{bmatrix}0.9419\\ 4.9058\\ 0.1600\end{bmatrix},\quad s=\begin{bmatrix}0.0135\\ 0.0026\\ 0.7926\end{bmatrix},\quad y=\begin{bmatrix}1.8636\\ 1.0833\\ -9.9955\end{bmatrix}\quad \text{and}\quad f=7.1559e+04$$

Results of final test run for TOTAL WATER TREATMENT COST IN WET SEASON after 19 iterations:

$$x = 1.0e+04 \begin{bmatrix} 1.3756 \\ 4.9374 \\ 0.0236 \end{bmatrix}, \quad s = 1.0e-09 \begin{bmatrix} 0.0039 \\ 0.0011 \\ 0.2280 \end{bmatrix}, \quad y = \begin{bmatrix} 1.0798 \\ 1.0833 \\ -13.1537 \end{bmatrix} \quad \text{and } f = 1.0161e+005$$

4.6 SUMMARY OF RESULTS

During the dry season, it will take 9,419 bags of chemical, 49,058 kw/h of electricity and 1600 litres of fuel for treatment of water to cost GH¢71,559.00.

For wet season, it will take 13,756 bags of chemical, 49,374 kw/h of electricity and 2,280 litres of fuel for the treatment of water to cost GH¢108,410.00

CHAPTER 5

CONCLUSION AND RECOMMENDATION

5.1 Conclusion

The treatment cost of water at Sunyani treatment plant was modeled into a Linear Programming problems to reflect on the two major seasons we have in Ghana-dry and wet. The analysis done in chapter four using primal-dual interior point algorithm showed that water treatment cost of GH¢73,898.00 and GH¢124,028.00 for dry and wet seasons respectively could be optimized at GH¢71,559.00 in the dry season and GH¢101,610.00 in the wet season to treat the same quantity of water. Cost of treating water can therefore be reduced with respect to the factors that influence the treatment cost.

5.2 Recommendations

Based on the study, the following recommendations are made.

This work should serve as basis for further in reducing treatment cost of water. Finally, with Sunyani water treatment plant was used as case study, the researcher recommends that Ghana Water Company Ltd adopt the project code to help determine the right quantities of chemicals, electricity and fuel to be used at their various pumping stations in order to reduce their treatment cost.

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APPENDICES

Appendix 1

Tables of cost, quantities and allocations of chemical, electricity and fuel for dry season

TABLE SHOWING COST OF CHEMICAL, ELECTRICITY AND FUEL

Month	Chemical(GH¢)	Electricity	Fuel
November	13514.00	1778.57	375.10
December	11758.00	2123.81	403.23
January	8249.15	1979.66	413.15
February	9199.00	1966.92	397.20
March	12202.00	9171.27	367.50

TABLE SHOWING QUANTITY OF CHEMICAL, ELECTRICITY AND FUE

Month	Chemical(Bags)	Electricity(kw/h)	Fuel(Litre)
November	1969.5	15621	321.75
December	1709.5	16378	345.87
January	1214.5	15001	345.38
February	1254.5	14249	340.70
March	1707.25	14922	315.23

TABLE SHOWING USAGE RATIO OF CHEMICAL, ELECTRICITY AND FUEL

	Chemical	Electricity	Fuel
Average Cost	10984.43	19646.45	391.24
Average Quantity	1374.77	159727	335.59
Ratio/Unit	7.9918	0.1230	1.1658

TABLE SHOWING COST/QUANTITY AND THE USAGE RATIO OF FUEL

Chemical House		Pump House		Transportation	
Cost(GH¢)	Quantity	Cost(GH¢)	Quantity	Cost(GH¢)	Quantity
~~528.75~~	15.79	602.15	18.79	750.02	120.00
33.49		32.05		6.25	

TABLE SHOWING ALLOCATION OF CHEMICAL, ELECTRICITY AND FUEL

Dry Season	Chemical House	Pump House	Transportation
Chemical Cost	80,500.00	0	0
Electricity Cost	0	10865.00	0
Fuel Cost	120.00	150.00	1000.00
Total	80,620.00	11015.00	1000.00

Appendix 2

Tables of cost, quantities and allocations of chemical, electricity and fuel for wet season

TABLE SHOWING COST OF CHEMICAL, ELECTRICITY AND FUEL

Month	Chemical (GH¢)	Electricity(GH¢)	Fuel(GH¢)
April	15152.00	1995.49	393.75
May	18171.00	2040.53	376.20
June	15642.00	2063.07	329.35
July	15820.00	2028.85	450.00
August	12589.00	2018.68	378.10
Sept	13791.00	2023.04	452.44
October	15774.00	2116.76	412.51

TABLE SHOWING QUANTITY OF CHEMICAL ELECTRICITY AND FUEL

Month	Chemical(Bag)	Electricity(kw/h)	Fuel(Litre)
April	2133.75	15160	337.74
May	2582.25	15580	322.69
June	2324.25	15802	282.50
July	2337	15471	386.19
August	1889	15374	324.32
September	2014	15414	388.08
October	2332	16307	353.83

TABLE SHOWING USAGE RATIO OF CHEMICAL, ELECTRICITY AND FUEL

	Chemical	Electricity	Fuel
Average Cost	17277	20410.68	398.93
Average Quantity	2703.25	16879.1	342.19
Ratio/Unit	6.3912	0.1209	1.1658

TABLE SHOWING THE COST/QUANTITY OF FUEL

Chemical House		Pump House		Transportation	
Cost(GH¢)	Quantity	Cost(GH¢)	Quantity	Cost(GH¢)	Quantity
742.35	19.01	869.42	28.95	952.58	170.41
39.05		30.03		5.59	

TABLE SHOWING ALLOCATION OF CHEMICAL, ELECTRICITY AND FUEL

	Chemical House	Pump House	Transportation
Chemical Cost	97,000.00	0	0
Electricity Cost	0	12854.00	0
Fuel Cost	123.00	162.00	1320.00
Total	97,123.00	13016.00	1320.00

APPENDIX 3

Matlab Code for the Algorithm

```
function [x,y,s,f] = pdip(A,b,p)
% primal-dual interior-point method for problem
%
% min p'x s.t. Ax=b, x>=0,
%
% whose dual is
%
% max b'y  s.t. A'y+s=p,  s>=0.
%
% calling sequence:
%
% [x,y,s,f] = pdip(A,b,p)
%
% input: A is an m x n SPARSE constraint matrix.
%    b is an m x 1 right-hand side vector
%    p is an n x 1 cost vector.
%
% output: x is the  n x 1 solution of the primal problem
%      y is the m x 1 dual solution
%      s is the n x 1 vector of "dual slacks"
%      f is the optimal objective value

if nargin ~= 3
  error('must have three input arguments');
end

if ~issparse(A)
  error('first input argument A must be a SPARSE matrix; possibly use sparse() to
convert');
end

t0=cputime;
[m,n] = size(A);
if m <= 0 or n <= 0
  error('input matrix A must be nontrivial');
end

if n ~= length(p)
  error('size of vector p must match number of columns in A');
end
if m ~= length(b)
  error('size of vector b must match number of rows in A');
```

```
end

% set initial point, based on largest element in (A,b,p)
bigM = max(max(abs(A)));
bigM = max([norm(b,inf), norm(p,inf), bigM]);
x = 100*bigM*ones(n,1); s = x; y = zeros(m,1);

% find row/column ordering that gives a sparse Cholesky
% factorization of ADA'
ordering = symmmd(A*A');
bc = 1+max([norm(b), norm(p)]);

for iter=1:100

% compute residuals
  Rd = A'*y+s-p;
  Rp = A*x-b;
  Rc = x.*s;
  mu = mean(Rc);
  relResidual = norm([Rd;Rp;Rc])/bc;
%  fprintf('iter %2i: mu = %9.2e, resid = %9.2e\n', iter, mu, relResidual);
  fprintf('iter %2i: mu = %9.2e, resid = %9.2e\n', iter, full(mu), ...
          full(relResidual));
  if(relResidual <= 1.e-7 & mu <= 1.e-7) break; end;
  Rc = Rc - min(0.1,100*mu)*mu;

  % set up the scaling matrix, and form the coefficient matrix for
  % the linear system
  d = min(5.e+15, x./s);
  B = A*sparse(1:n,1:n,d)*A';
  % use the form of the Cholesky routine "cholinc" that's best
  % suited to interior-point methods
  R = cholinc(B(ordering,ordering),'inf');

  % set up the right-hand side
  t1 = x.*Rd-Rc;
  t2 = -(Rp+A*(t1./s));

  % solve it and recover the other step components
  dy = zeros(m,1);
  dy(ordering) = R\(R'\t2(ordering));
  dx = (x.*(A'*dy)+t1)./s;
  ds = -(s.*dx+Rc)./x;

  tau = max(.9995,1-mu);
  ap = -1/min(min(dx./x),-1);
  ad = -1/min(min(ds./s),-1);
```

```
  ap = tau*ap;
  ad = tau*ad;
  x = x + ap*dx;
  s = s + ad*ds;
  y = y + ad*dy;
end

f = p'*x;

% convert x,y,s to full data structures
x=full(x); s=full(s); y=full(y);

fprintf('Done!\t[m n] = [%g %g]\tCPU = %g\n', m, n, cputime-t0);
return;
```

APPENDIX 4

Result of Programme run for Dry Season

Iteration 1

$$1.0e+04 \qquad\qquad 1.0e+06 \qquad\qquad 1.0e+08$$

$$x = \begin{bmatrix} 1.3446 \\ 5.3065 \\ 0.4191 \end{bmatrix}, \qquad s = \begin{bmatrix} 8.8584 \\ 8.8188 \\ 8.87676 \end{bmatrix} \qquad \text{and} \qquad y = \begin{bmatrix} -0.0111 \\ -0.7346 \\ 3.8120 \end{bmatrix}$$

Iteration 2

$$1.0e+04 \qquad\qquad 1.0e+07 \qquad\qquad 1.0e+07$$

$$x = \begin{bmatrix} 0.9421 \\ 4.9060 \\ 0.0162 \end{bmatrix}, \qquad s = \begin{bmatrix} 0.4202 \\ 0.1062 \\ 1.3492 \end{bmatrix}, \qquad \text{and} \qquad y = \begin{bmatrix} -0.0526 \\ -0.8849 \\ 4.6035 \end{bmatrix}$$

Iteration 3

$$1.0e+04 \qquad\qquad 1.0e+07 \qquad\qquad 1.0e+05$$

$$x = \begin{bmatrix} 0.9419 \\ 4.9058 \\ 0.0160 \end{bmatrix}, \qquad s = \begin{bmatrix} 0.0335 \\ 0.0064 \\ 1.9479 \end{bmatrix}, \qquad \text{and} \qquad y = \begin{bmatrix} -0.4192 \\ -5.3603 \\ -1.4319 \end{bmatrix}$$

Iteration 4

$$1.0e+04 \qquad\qquad 1.0e+06 \qquad\qquad 1.0e+04$$

$$x = \begin{bmatrix} 0.9419 \\ 4.9058 \\ 0.0160 \end{bmatrix}, \qquad s = \begin{bmatrix} 0.0335 \\ 0.0064 \\ 1.9729 \end{bmatrix}, \qquad \text{and} \qquad y = \begin{bmatrix} -0.4193 \\ -5.3619 \\ -1.8232 \end{bmatrix}$$

Iteration 5

$$x = 1.0e+04 \begin{bmatrix} 0.9419 \\ 4.9058 \\ 0.0160 \end{bmatrix}, \quad s = 1.0e+05 \begin{bmatrix} 0.0337 \\ 0.0065 \\ 1.9818 \end{bmatrix} \quad \text{and} \quad y = 1.0e+03 \begin{bmatrix} -0.4204 \\ -5.3857 \\ -1.8406 \end{bmatrix}$$

Iteration 6

$$x = 1.0e+04 \begin{bmatrix} 0.9419 \\ 4.9058 \\ 0.0160 \end{bmatrix}, \quad s = 1.0e+04 \begin{bmatrix} 0.0338 \\ 0.0065 \\ 1.9907 \end{bmatrix} \quad \text{and} \quad y = \begin{bmatrix} -41.4569 \\ -539.9578 \\ -193.8755 \end{bmatrix}$$

Iteration 7

$$x = 1.0e+04 \begin{bmatrix} 0.9419 \\ 4.9058 \\ 0.0160 \end{bmatrix}, \quad s = 1.0e+03 \begin{bmatrix} 0.0340 \\ 0.0065 \\ 1.9997 \end{bmatrix} \quad \text{and} \quad y = \begin{bmatrix} -3.3875 \\ -53.2642 \\ -28.4663 \end{bmatrix}$$

Iteration 8

$$x = 1.0e+04 \begin{bmatrix} 0.9419 \\ 4.9058 \\ 0.0160 \end{bmatrix}, \quad s = \begin{bmatrix} 3.4119 \\ 0.6551 \\ 200.8650 \end{bmatrix} \quad \text{and} \quad y = \begin{bmatrix} 0.4366 \\ -4.3759 \\ -11.8509 \end{bmatrix}$$

Iteration 9

$1.0e+04$

$$x=\begin{bmatrix}0.9419\\4.9058\\0.0160\end{bmatrix}, \qquad s=\begin{bmatrix}0.3427\\0.0658\\20.1769\end{bmatrix} \quad \text{and} \quad y=\begin{bmatrix}0.8207\\0.05350\\-10.1819\end{bmatrix}$$

Iteration 10

$1.0e+04$

$$x=\begin{bmatrix}0.9419\\4.9058\\0.0160\end{bmatrix}, \qquad s=\begin{bmatrix}0.0344\\0.0066\\2.0268\end{bmatrix} \quad \text{and} \quad y=\begin{bmatrix}0.8593\\1.0282\\-10.0143\end{bmatrix}$$

Iteration 11

$1.0e+04$

$$x=\begin{bmatrix}0.9419\\4.9058\\0.0160\end{bmatrix}, \qquad s=\begin{bmatrix}0.0035\\0.0007\\0.2036\end{bmatrix} \quad \text{and} \quad y=\begin{bmatrix}0.8631\\1.0778\\-9.9974\end{bmatrix}$$

Iteration 12

$1.0e+04$

$$x=\begin{bmatrix}0.9419\\4.9058\\0.0160\end{bmatrix}, \qquad s=\begin{bmatrix}0.0003\\0.0001\\0.0205\end{bmatrix} \quad \text{and} \quad y=\begin{bmatrix}0.8635\\1.0828\\-9.9957\end{bmatrix}$$

Iteration 13

$$1.0e+04$$

$$x=\begin{bmatrix}0.9419\\4.9058\\0.0160\end{bmatrix},\qquad s=\begin{bmatrix}0.0000\\0.0000\\0.0021\end{bmatrix}\qquad \text{and}\qquad y=\begin{bmatrix}0.8636\\1.0833\\-9.9956\end{bmatrix}$$

Iteration 14

$$1.0e+04 \qquad\qquad 1.0e-03$$

$$x=\begin{bmatrix}0.9419\\4.9058\\0.0160\end{bmatrix},\qquad s=\begin{bmatrix}0.0035\\0.0007\\0.2063\end{bmatrix}\qquad \text{and}\qquad y=\begin{bmatrix}0.8636\\1.0833\\-9.9955\end{bmatrix}$$

Iteration 15

$$1.0e+04 \qquad\qquad 1.0e+04$$

$$x=\begin{bmatrix}0.9419\\4.9058\\0.0160\end{bmatrix},\qquad s=\begin{bmatrix}0.0035\\0.0007\\0.2073\end{bmatrix}\qquad \text{and}\qquad y=\begin{bmatrix}0.8636\\1.0833\\-9.9957\end{bmatrix}$$

Iteration 16

$$1.0e+04$$

$$x=\begin{bmatrix}0.9419\\4.9058\\0.0160\end{bmatrix},\qquad s=\begin{bmatrix}0.0035\\0.0007\\0.2082\end{bmatrix}\qquad \text{and}\qquad y=\begin{bmatrix}0.8636\\1.0833\\-9.9955\end{bmatrix}$$

Iteration 17

$1.0e+04 \qquad\qquad 1.0e-07$

$$x = \begin{bmatrix} 0.9419 \\ 4.9058 \\ 0.0160 \end{bmatrix}, \qquad s = \begin{bmatrix} 0.0119 \\ 0.0023 \\ 0.7003 \end{bmatrix} \qquad \text{and} \qquad y = \begin{bmatrix} 0.8636 \\ 1.0833 \\ -9.9955 \end{bmatrix}$$

Iteration 18

$1.0e+04 \qquad\qquad 1.0e-10$

$$x = \begin{bmatrix} 0.9419 \\ 4.9058 \\ 0.0160 \end{bmatrix}, \qquad s = \begin{bmatrix} 0.0135 \\ 0.0026 \\ 0.7926 \end{bmatrix} \qquad \text{and} \qquad y = \begin{bmatrix} 0.8636 \\ 1.0833 \\ -9.9955 \end{bmatrix}$$

Iteration 19

$f = 7.1559e+004$ ans: $1.0e+04$

$$x = \begin{bmatrix} 0.9419 \\ 4.9058 \\ 0.0160 \end{bmatrix}$$

APPENDIX 5

Result of Programme Run for Wet Season

Iteration 1

$$x = 1.0e+04\begin{bmatrix}1.8605\\5.4205\\0.5092\end{bmatrix}, \qquad s = 1.0e+07\begin{bmatrix}1.0669\\1.0634\\1.0683\end{bmatrix} \quad \text{and} \quad y = 1.0e+08\begin{bmatrix}-0.0167\\-0.8857\\4.8557\end{bmatrix}$$

Iteration 2

$$x = 1.0e+04\begin{bmatrix}1.3759\\4.9376\\0.0239\end{bmatrix}, \qquad s = 1.0e+07\begin{bmatrix}0.4270\\0.1462\\1.5614\end{bmatrix} \quad \text{and} \quad y = 1.0e+07\begin{bmatrix}-0.0668\\-1.2186\\6.7342\end{bmatrix}$$

Iteration 3

$$x = 1.0e+04\begin{bmatrix}1.3756\\4.9374\\0.0236\end{bmatrix}, \qquad s = 1.0e+07\begin{bmatrix}0.0329\\0.0092\\1.8975\end{bmatrix} \quad \text{and} \quad y = 1.0e+06\begin{bmatrix}-0.0515\\-0.7640\\1.0693\end{bmatrix}$$

Iteration 4

$$x = 1.0e+04\begin{bmatrix}1.3756\\4.9374\\0.0236\end{bmatrix}, \qquad s = 1.0e+06\begin{bmatrix}0.0329\\0.0092\\1.9190\end{bmatrix} \quad \text{and} \quad y = 1.0e+05\begin{bmatrix}-0.0515\\-0.7648\\1.0358\end{bmatrix}$$

Iteration 5

$$x = 1.0e+04 \begin{bmatrix} 1.3756 \\ 4.9374 \\ 0.0236 \end{bmatrix}, \quad s = 1.0e+05 \begin{bmatrix} 0.0331 \\ 0.0092 \\ 1.9276 \end{bmatrix} \quad \text{and} \quad y = 1.0e+04 \begin{bmatrix} -0.0517 \\ -0.7681 \\ 1.0392 \end{bmatrix}$$

Iteration 6

$$x = 1.0e+04 \begin{bmatrix} 1.3756 \\ 4.9374 \\ 0.0236 \end{bmatrix}, \quad s = 1.0e+04 \begin{bmatrix} 0.0332 \\ 0.0093 \\ 1.9363 \end{bmatrix} \quad \text{and} \quad y = 1.0e+03 \begin{bmatrix} -0.0509 \\ -0.7706 \\ 1.0321 \end{bmatrix}$$

Iteration 7

$$x = 1.0e+04 \begin{bmatrix} 1.3756 \\ 4.9374 \\ 0.0236 \end{bmatrix}, \quad s = 1.0e+04 \begin{bmatrix} 0.0334 \\ 0.9344 \\ 1.9450 \end{bmatrix} \quad \text{and} \quad y = \begin{bmatrix} 0.5550 \\ -6.7034 \\ -2.6071 \end{bmatrix}$$

Iteration 8

$$x = 1.0e+04 \begin{bmatrix} 1.3756 \\ 4.9374 \\ 0.0236 \end{bmatrix}, \quad s = \begin{bmatrix} 3.3538 \\ 0.9344 \\ 195.3755 \end{bmatrix} \quad \text{and} \quad y = \begin{bmatrix} 0.5550 \\ -6.7034 \\ -2.6071 \end{bmatrix}$$

Iteration 9

$1.0e+04$

$$x=\begin{bmatrix}1.3756\\4.9374\\0.0236\end{bmatrix},\qquad s=\begin{bmatrix}0.3369\\0.0939\\19.6255\end{bmatrix}\qquad \text{and}\qquad y=\begin{bmatrix}1.0271\\0.3012\\-12.0943\end{bmatrix}$$

Iteration 10

$1.0e+04$

$$x=\begin{bmatrix}1.3756\\4.9374\\0.0236\end{bmatrix},\qquad s=\begin{bmatrix}0.0338\\0.0094\\1.9714\end{bmatrix}\qquad \text{and}\qquad y=\begin{bmatrix}1.0745\\1.0094\\1.9714\end{bmatrix}$$

Iteration 11

$1.0e+04$

$$x=\begin{bmatrix}1.3756\\4.9374\\0.0236\end{bmatrix},\qquad s=\begin{bmatrix}0.0034\\0.0009\\0.1980\end{bmatrix}\qquad \text{and}\qquad y=\begin{bmatrix}1.0793\\1.0754\\-13.1430\end{bmatrix}$$

Iteration 12

$1.0e+04$

$$x=\begin{bmatrix}1.3756\\4.9374\\0.0236\end{bmatrix},\qquad s=\begin{bmatrix}0.0003\\0.0001\\0.0199\end{bmatrix}\qquad \text{and}\qquad y=\begin{bmatrix}1.0798\\1.0825\\-13.1526\end{bmatrix}$$

Iteration 13

$$1.0e+04$$

$$x=\begin{bmatrix}1.3756\\4.9374\\0.0236\end{bmatrix},\qquad s=\begin{bmatrix}0.0000\\0.0000\\0.0020\end{bmatrix}\qquad\text{and}\qquad y=\begin{bmatrix}1.0798\\1.0833\\-13.1536\end{bmatrix}$$

Iteration 14

$$1.0e+04$$

$$x=\begin{bmatrix}1.3756\\4.9374\\0.0236\end{bmatrix},\qquad s=\begin{bmatrix}0.0034\\0.0010\\0.2007\end{bmatrix}\qquad\text{and}\qquad y=\begin{bmatrix}1.0798\\1.0833\\-13.1537\end{bmatrix}$$

Iteration 15

$$1.0e+04 \qquad\qquad 1.0e-04$$

$$x=\begin{bmatrix}1.3756\\4.9374\\0.0236\end{bmatrix},\qquad s=\begin{bmatrix}0.0035\\0.0010\\0.2016\end{bmatrix}\qquad\text{and}\qquad y=\begin{bmatrix}1.0798\\1.0833\\-13.1537\end{bmatrix}$$

Iteration 16

$$1.0e+04 \qquad\qquad 1.0e-05$$

$$x=\begin{bmatrix}1.3756\\4.9374\\0.0236\end{bmatrix},\qquad s=\begin{bmatrix}0.0035\\0.0010\\0.2025\end{bmatrix}\qquad\text{and}\qquad y=\begin{bmatrix}1.0798\\1.0833\\-13.1537\end{bmatrix}$$

Iteration 17

$$1.0e+04 \qquad 1.0e-07$$

$$x=\begin{bmatrix}1.3756\\4.9374\\0.0236\end{bmatrix}, \qquad s=\begin{bmatrix}0.0168\\0.0047\\0.9777\end{bmatrix} \qquad \text{and} \qquad y=\begin{bmatrix}1.0798\\1.0833\\-13.1537\end{bmatrix}$$

Iteration 18

$$1.0e+04 \qquad 1.0e-09$$

$$x=\begin{bmatrix}1.3756\\4.9374\\0.0236\end{bmatrix}, \qquad s=\begin{bmatrix}0.0039\\0.0011\\0.2280\end{bmatrix} \qquad \text{and} \qquad y=\begin{bmatrix}1.0798\\1.0833\\-13.1537\end{bmatrix}$$

Iteration 19

$f = 10161e+05$ ans: $1.0e+04$

$$x=\begin{bmatrix}1.3756\\4.9374\\0.0236\end{bmatrix}$$