MODELING LOCATION OF FACILITY AS P-MEDIAN MODEL FOR LOCATION OF SENIOR HIGH SCHOOL AT KASSENA-NANKANA WEST DISTRICT.

A THESIS SUBMITTED TO THE DEPARTMENT OF MATHEMATICS, KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, IN PARTIAL FULFILMENT OF THE REQUIREMENT FOR THE DEGREE OF

MASTER OF SCIENCE IN INDUSTRIAL MATHEMATICS

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DECLARATION

I hereby declare that this submission is my own work towards the award of the MSc. degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the university, except where due acknowledgement had been made in the text.

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I am most grateful to the Almighty God for guiding and sustaining me throughout this course. His everlasting love and Grace has made me see the light of day and subsequent completion of this work.

My profound gratitude to my supervisor, Mr. F.K. Darkwah, whose guidance has enabled me to produce this work.

Special thanks to my parents and my siblings, for their prayers and support throughout my education. While I share the credit of this Masters‟ thesis with all the above mentioned people, responsibility for any errors, shortcomings or omissions in this thesis is solely mine.

DEDICATION

This study is dedicated to my lovely wife Joyce Kuchulah, my three kids Gracious Kuchulah, Esther Kuchulah and Deborah Kuchulah

ABSTRACT

Education is the process of learning geared or directed towards assisting the individual to acquire knowledge, skills and to improve his or her life. The main purpose of this research is solving the problem of locating a Senior High School in the Kassena Nankana district. Ten (10) towns in the district were considered, which are; Nakong, Katiu, Chiana, Kajelo, Kayilo, Paga, Nakolo, Sirigu, Manyoro and Binania. Since the problem is a desirable one, we formulated the problem using the p-median model developed by Hakimi (1964; 1965). Reduction heuristic (RH1, RH2 and RRH) was used to solve the p-median problem. According to the model, the Senior High school should be sited at Chiana with objective value of 56234.

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CHAPTER ONE

INTRODUCTION

Table 4.5 HISTORICAL BACKGROUND OF EDUCATION IN GHANA

Education is the process of learning geared or directed towards assisting the individual to acquire knowledge, skills ad to improve his or her life. Education is not only acquired in schools. It is acquired at home and in a variety of other places outside the school. It is believed that "education" is developed from the Latin words "educare" which means to nurture rear for and "dyco" meaning to grow. The old concept of education is based on the idea that education is a preparation for life. This means that education should be used to refer to all procedures and practices by which individuals are prepared to live their lives.

The new concept of education is based on the idea that education involves procedures and practices that lead to an improvement in the quality of individuals, their living and societal condition as a whole. For instance John Dewey, the great American educator of the early 20^{th} century stated that education is a process of the reconstruction and reconstitution of experiences, giving it a more socialized value through the medium of increased individual efficiency. "The Castle Schools" were type of schools established in the Castles (Cape Coast and Elmina Castles) by the Europeans along the coast of Ghana. These schools were established by the European trading merchants and companies, for example, the Dutch West Indies Company. The Castle Schools were founded long before Britain imposed its colonial rule on Ghana, then Gold Coast.

The first Castle School was founded in 1529 by the Portuguese merchants who built the Elmina Castle. The Dutch also started a school in the same castle in 1637 after seizing it from the Portuguese (Graham, 1978, p. 1). Of all the Castle Schools, the Cape Coast Castle School started in 1694 and revived in 1712 was the most well known. The curriculum of the Castle School included reading, writing and arithmetic which is also known as the 3Rs. Later, religious instruction (scripture) was added. The schools were opened for the "mulato" children and a few of the black children from wealthy African parents. (Graham, 1976).

The Castle Schools also encounted problems. These included financial, low enrolment, interference from Castle authorities and the politically unstable situation in the then Gold Coast. For instance, Philip Quaque's efforts to promote Castle school education. He encountered numerous problems. First, his own salary was in arrears to the tune of £ 369 at the time of his death. Second, he had a very little support from the 'Society for the Propagation of the Gospel' (SPG) (Graham, 1976). There was only one pupil in 1770 and 1771, no pupil in 1772 and two in 1775 (Graham, 1976). The Cape Coast Castle authorities, for instance, interfered with Quaque's work. In 1791, Philip Quaque was suspended for refusing to take up arms and to accompany Governor Field in defense of the fort at Anomabu (Odamtten, 1978, p. 18), Graham (1976), (History of education Ghana, Accra-Tema Ghana publishing corporation, PP 5-26).

All the Christian missions established Primary and Middle Schools (we no longer have middle schools). By 1880, the Basel Missionary established 45 Primary and Middle Schools with total enrollment of over 1,200 (Graham, 1976, p 44). By 1990 the number of their schools had risen to 154 and their pupils were nearly five thousand (Graham, 1976, P 44). By 1990, the Breman Mission had opened twenty primary and middle schools which were being attended by 591 children (Graham, 1976, PP 47-48). By 1880 the Wesleyan Mission also had 83 primary and middle schools. The number of pupils in these schools was 300.

In 20th century a number of educational rules were passed. First in 1902 and then in 1908, for instance, were passed by Governor who stressed 'hand and eye' in education. In other words, he emphasized technical, vocational and agricultural education. However, the most important reform I the colonial era in the twentieth century occurred under Governor Sir Fredrick Gordon Guggisberg (1919-1927). Governor Guggisberg has been praised so much for all that he did for colonial Ghana particularly in the field if education, health and transportation. Gordon Guggisberg came up with "Sixteen Principles of Education" which included

- Primary education has be thorough and be from the bottom to the top
- The provision of secondary schools with standard that will fit young men and women to enter the university.
- Equal opportunities given to boys should be provided for the education of girls
- The sixth principle indicated that staff of teachers must of the highest possible quality

- The sixteenth principle called for “the provision of trade school with technical and elementary education that will fit young men and to become skilled craftsmen and useful citizens”.

(McMillan & Kwamena-Poh, 1975; 57-58)’. To give expression to his sixteenth principles, Governor Guggisberg greatly expanded technical education by opening four Government Trade Schools in 1922 at Yendi, Mampong, Kibi and Asuasi. The Yendi School was later moved to Tamale (McMillan and Kwamena Poh, 1975, P63). There was also the accelerated Development Plan of Education in 1951, the education act of 1966.The pioneered by Dr. Kwame Nkrumah. Then Dzobo education reforms at the Acheampong’s Regime from 1972 to 1978, which helped improve Ghana education up to date. The objectives of the Dzobo Report of 1974 include the following:

1. At the primary level the objectives of the proposed reforms were to promote:
   i. numeracy and literacy
   ii. 4ppropriate4n among children
   iii. inquiry skills:ability to observe, collect information, analyse information, apply principles to new situations etc.
   iv. creative skills among children
   v. the desire for self-improvement, and the desire for truth.
2. At the secondary level, the objectives of the proposed reforms were to:
   i. reinforce the objectives of the primary course
   ii. develop qualities of leadership in students
   iii. equip students with occupational skills

Table 4. At the teacher education level,the objectives aimed to give teachers:

i. A sound basis in the content of the courses they will be teaching

ii. sound professional skills to guide the children/students in the desired direction

iii. manual skills so they can promote similar interest in students

iv. Qualities of leadership

1.1 THE PROPOSALS THAT WERE MADE FOR THE REFORMS OF THE STRUCTURE OF EDUCATION IN GHANA

These include;

- Kindergarten Education -18 to 24 months for age group 4 to 6 years
- Basic first Cycle Education-six years Primary plus three years Junior Secondary .This will be basic, free and compulsory for all.
- Second Cycle Education-From the junior secondary course, there will be selection into the following terminal courses, namely:-
- Senior Secondary Lower courses leading to the GCE 'O' Level: Technical courses ; and Commercial courses.
- Second Cycle Education-Further Courses: this comprises two extra years of Senior secondary Education leading to the GCE 'A' Level; and a Polytechnic Course.
- Students who did not proceed to University from the Secondary Upper Course shall be encouraged to train for middle level professions in institutions available in the system, e.g Polytechnics, Specialist and Teacher Training CollegeS. Dzobo , (1974).

1.2 GHANA EDUCATION THE OBJECTIVES OF THE MINISTRY OF EDUCATION

The development of the school infrastructural in Ghana has been in line with the aims and objectives of the Ministry of Education, and Ghana Education Service. The objectives or aims of education in Ghana include;

- creation of a literate population, if many Ghanaians are able to read, write and compute, many of the social problems such as superstitions, less productive farming method and so o would be solved.

- The productions of physically healthy exercises, cleanliness or sanitation are part of this.

- The development of vocational and technical skills is also an important aim of education in Ghana; such skills would be the individual not only to earn living but also to satisfy the manpower requirement of the country.

- Education also has to improve the political awareness of the people. The people must know the rights and responsibilities of the citizens. The importance of voting, obedience to state authority, patriotism and loyalty to Ghana are all necessary for citizen know. Education has to ensure that the people gain this knowledge.

- The development of morality is another aim of education in Ghana. The higher level of dishonesty, corruption, stealing and rape among others indicate that there is something wrong with our education as far as morality is concerned. The development of morally upright individuals should therefore be vigorously pursued.

- The development of critical and logical thinking in the solution everyday problems is also an aim of education in Ghana.

  We have witnessed several changes in our educational system since independence. This is because the practices do not seem to help us to achieve these aims.

  We are therefore still searching for the right methods, content, structure and management practices that would help us best.

1.3 HISTORICAL BACKGROUND OF EDUCATION IN KASSENA - NANKANA WEST DISTRICT

Kassena – Nankana west District with its Administrative Capital Paga forms part of the new districts and municipalities created by the President J. A. Kuffour were inaugurated at their various location on the 29th of February 2008.

The district forms part of the thirteen (13) municipalities and district in the Upper East Region of Ghana. The District shares boundaries with Burkina Faso to the North, Builsa District to the South, Sissala East District in the Upper West Region to the West and Kassena-Nankana East municipal to the East. The population of the District is projected to be 156, 090 for 2010 for the Kassena-Nankana East and West Districts. Efforts are still being made to segregate the population figures for the two Districts. The gender grouping of the population is 75,548 people being male representing 48.4% and the female population of 80, 642 representing 51.6%. The District has a population growth rate of 1%which is below the national rate of 1.1% (2000 PHC).

1.3 STATEMENT OF THE PROBLEM

The researcher has identified the problem and has moved in to find out solutions for the people in the area. Due to the newness of the District there is no Senior High School in the District and because of that pupils who finished J.H.S or B.E.C.E candidates have to travel to other Districts every year to have access to Secondary education which posses a lot of challenges for both parents and wards in the new District and those who could not afford often drop out of school.

1.4 OBJECTIVES OF THE STUDY

The objectives are;

T0o model a location of permanent site for the new Senior High School in Kassena – Nankana West District as a p-median problem.

- To optimally locate permanent Senior High School for Kassena – Nankana West District using repeated reduction heuristic.

1.5 METHODOLOGY

The location of a Senior High School was modeled as a p-median problem. Data on population and road distance were obtained from the district Town and Country planning and the district statistical service respectively. Floyd – warshall algorithm was used to find the shortest path distance matrix connecting the towns and villages selected in the analysis. The reduction heuristics (RH1, RH2 and RRH) were also used to solve the p-median problem.

Table 4.5 JUSTIFICATION OF THE STUDY

The researcher seeks to find out a common location site for the new District Senior High School for the Kassena – Nankana West District which will be closer to all the surrounding towns and villages. This is to help find a central site to locate the new Senior High School in the District which will minimize the distances and cost of travelling to access the facility. P – median and Heuristics RH1, RH2 and RHH will be used to select the best site for the location of the facility.

Table 4.5 THESIS ORGANIZATION

This thesis is organized into five main chapters. Chapter one presents the introduction of the thesis. This consists of the background of the study, the research problem statement, objectives of the research, methodology, and organization of the thesis. Chapter two is the literature review, which looks at briefly work done by other researchers on the topic. Chapter three is the formulation of the mathematical model. Chapter four contains the

Data Analysis and Results. Chapter five looks at Conclusions and Recommendation of the analyzed data.

CHAPTER TWO

LITERATURE REVIEW

2.0 Introduction

The delivery of most public services involves direct contact between the services facility and the target population. Proximity to the target population is very essential in locating a public facility. In location problem, we want to find the right site where or more new facilities should be placed in order to optimize some specified criteria which are usually related to the distance from the facilities to the demand points. This optimization may vary depending on the particular objectives function chosen.

Location problem is concern with the location of one or more facilities in some space, so as to optimize some specified criteria. Often these criteria are linked with distribution costs of providing optimal access for the facilities in question. This does not necessarily follow however when facilities produce some undesirable or obnoxious effect. Here the risk to the local population far outweighs any benefit of close settlement of the facility. This therefore causes the location formulation to change to that of minimizing risk or equivalently maximizing some distances function to the population centers.

The problem of sitting a single facility on a network so as to maximize the minimum Euclidean distance along the arcs of networks, from the nodes present is a trivial use of the obnoxious location question. The problem becomes more difficult, when these distances do not have to lie on the arcs or edges of the network. This allows for the spread of any pollution that is emitted across the plans in which the network lies .The underlying assumption of this formulation lies in the fact that the population decreases with distance uniformly about the facility from which it originates.

In today's modern society, the number of facilities available to the population often defines the quality of life. From dry-cleaners to garages, from fire station to football stadia, all provide a service and so can be considered as a physical entity that provides a service. In other words, a facility can be considered as a physical entity that provides a service. These facilities can be classified into three categories: desirable (non-obnoxious),semi-obnoxious and obnoxious(non-desirable).Most services are provided by desirable or non-obnoxious facilities. There may include super market, warehouses, shops, garages, banks, libraries etc. As the customer needs access of some sort to the facility providing service, it is beneficial if these facilities are sited close to the customers that will be serving. This implies that the customer has better access to the facility. They will use it more often benefiting the facility itself. Darkwah and Amposah, (2007).

To serve a set of communities whose location and demands are known,a number of factors should be considered such as;

(i) The number of the facilities to the demand.

(ii) Size and capacity of facility.

(iii) The allocation of the demand points to open facilities.

iv) Optimizing some objectives location function.

2.1 TYPES OF FACILITIES

In modern society, the number of facilities available to the population often defines the quality of life , hospital schools , free station , police station can so considered as physical entries that provide services. These facilities can be classified into three categories: desirable (non-obnoxious, semi- obnoxious and obnoxious.)

2.1.1 Non- Obnoxious Facilities

Most services are provided by desirable or non-obnoxious facilities. There are facilities that bring comfort to customers and are pleasant in the neighbourhood .They may include supermarkets, warehouse, shops, garages, banks etc. As the customer needs access to the facility providing service. It is beneficial if these facility are sited close to the customer who need their services

2.1.2 Semi –Obnoxious Facilities

Sometime a facility that requires a high degree of accessibility provides a negative or undesirable effect. For example a football stadium provides entertainment and so requires a large amount of access to enable supporters to attend a game. On the other hand, on a match day, Local non-football fans will have to be content to their noise and traffic generated. The generation of traffic and noise will be unpleasant for locals who are not attending the match and who will therefore describe the facility as undesirable. The combination of the two makes this facility semi-obnoxious. Another example is a hospital with an ambulance. Here access is needed for treatment of the local population especially on emergency days. On the other hand the siren of the ambulance may be too noisy to others who might not need its services at the moment in time.

2.1.3 Obnoxious Facilities

An obnoxious facility is one which is useful but has undesirable effect on the inhabitants and users in an area. Examples include equipment which emits pollutants such as noise and radiation or warehouse that contain flammable materials. Other obnoxious facilities are the nuclear power station, installation, although necessary for society. These facility are undesirable and often dangerous to the surrounding inhabitants.

2.2 Some Approaches to Facilities location

Kwarteng et al (2011) considered the problem of locating a semi-obnoxious facility (hospital) as a p-center problem under the condition that some existing facilities are already located in the Amansie-West Berman and Drezner (2008) method was used on a 12-note network which had four existing facilities. The factor rating analysis was use to select Antoakrom and the far that patient to the hospital at the new location (Antoakrom) was determined to be 8km.

In Malczewski and Ogryczak (1990) the location of hospitals is formulated as a multi-objective optimization problem and an interactive approach DIN AS, Dynamimic interactive network analysis system (Ogryczak et al., 1989) based on the so called reference point approach (Wierzbicki,1982) is presented. A real application is presented, considering eight sites for potential location and at least four new hospitals to be built, originating in hundred and sixty three alternative location patterns each of them generating many possible allocation schemes. The authors mention that the system can be used to support a group decision – making process making the final decision less subjective. They also observed that during the interactive process the decision – makers have gradually learned about the set of feasible alternatives and in consequence of this leaning process they have change their preference and priorities.

Erkut and Neuman (1992) present a mixed integer linear model for undesirable facility location. The objectives considered are total cost minimization, total opposition minimization and equity minimization. Caruso et al (1993) present a model for planning an urban solid waste management system. Incineration, composition and recycling are considered for the processing phase and sanitary landfills are considered for the disposal phase. Heuristic techniques (embedded in the reference point approximation) are used to solve the model and, as a consequence, "approximate Pareto solutions" are obtained. By varying the reference point, different solutions can be obtained. The results for a case study (Lombardy region in Italy) are presented and discussed.

Wyman and Kuby (1993, 1995) present a multi-objective mixed integer programming model for the location of hazardous material facilities (including the technologies choice variable) with three objectives functions (cost, risk and equity). Melachrinoudis et al (1995) propose a dynamic multi-period capacitated mixed integer programming model for the location of sanitary landfills.

Fonseca and Captivo (1996; 2006; 2007) study the location of semi obnoxious facilities as a discrete location problem on a network. Several bi-criteria models are presented considering two conflicting objectives, the minimization of obnoxious effect and the maximization of the accessibility of the community to the closest open facility. Each of these objectives is considered in two different ways, trying to optimize its average value over all the communities or trying to optimize its worst value. The Euclidean distance is used to evaluate the obnoxious effect and the shortest path distance is used to evaluate the accessibility. The obnoxious effect is considered inversely proportional to the weighted Euclidean distance between demand points and open facilities, and demand directly proportional to the population in each community. All the models are solved using Chalmet et al (1986), non- interactive algorithm for Bi-criteria Integer Linear Programming modified to an interactive procedure by Ferreira et al (1994). Several equity measures are computed for each non-denominated solution presented to the decision-maker, in order to increase the information available to the decision –maker about the set of possible solutions. Ferreira et al (1996) present a bi-criteria mixed integer linear model for the facility location where the objectives are the minimization of total cost and the minimization of environmental pollution at facility sites. The interactive approach of Ferreira et al (1994) is used to obtain and analyze non-dominated solutions. Giannikos (1998) presents a discrete model for the location of disposal or treatment facilities and transporting hazardous waste through a network linking the population centers that produce the waste and the candidate locations for the treatment facilities method to choose the location for a waste treatment facility in a region of Finland.

Costa et al (2008) develop two bi-criteria models for single allocation hub location problems. In both models the total cost is the first criteria to be minimized. Instead of using capacity constraints to limit the amount of flow that can be received by the hubs, a second objective function is used, trying to minimize the time to process the flow entering the hubs. In the first model, total time is considered as the second criteria and, in the second model, the maximum service time for the hubs are minimized. Non-dominated solutions are generated using an interactive decision-aid approach developed for bi-criteria integer linear programming problems. Both bi-criteria models are tested on a set of instances, analyzing the corresponding non-dominated solutions set and studying the reasonableness of the hubs flow charge for these non-dominated solutions. Ballou (1998) discusses a selected number of facility location methods for strategic planning. He further classifies the more practical methods into a number of categories in the logistics network, which include single–facility location, multi–facility location, dynamic facility location, retail and service location. Christopher and Wills (1972) comprehensively present that whether the problem of depot location is static or dynamic, „Infinite Set" approaches and „Feasible Set" approach can be identified. The infinite set approach assumes that a warehouse is flexible to be located anywhere in a certain area. The feasible set approach assumes that only a finite number of known sites are available as warehouse locations. They believe the centre of gravity method is a sort of infinite set model.

Goldengorin et al, (1999) considered the simple plant location problem. This problem often appears as a sub-problem in other combinatorial problems. Several branch and bound techniques have been developed to solve these problems. The thesis considered new approaches called branch and peg algorithms, where pegging refers to assigning values to variables outside the branching process. An exhaustive computational experiment shows that the new algorithms generate less than 60% of the number of sub-problems generated by branch and bound algorithms, and in certain cases requires less than 10% of the execution times required by branch and bound algorithms. Firstly, for each sub-problem generated in the branch and bound tree, a powerful pegging procedure is applied to reduce the size of the sub-problem. Secondly, the branching function is based on predictions made using the Beresnev function of the sub-problem at hand. They saw that branch and peg algorithms comprehensively out perform branch and bound algorithms using the same bound, taking on the average, less than 10% of the execution time of branch and bound algorithms when the transportation cost matrix is dense. The main recommendation from the results of the experiment is that branch and peg algorithms should be used to solve SPLP instances.

Ballou (1998) states that exact centre of gravity approach is simple and appropriate for locating one depot in a region, since the transportation rate and the point volume are the only location factors. Given a set of points that represent source points and demand points, along with the volumes needed to be moved and the associated transportation rates, an optimal facility location could be found through minimizing total transportation cost. In principle, the total transportation cost is equal to the volume at a point multiplied by the transportation rate to ship to that point multiplied by the distance to that point. Furthermore, Ballou outlines the steps involved in the solution process in order to implement the exact centre of gravity approach properly.

Adjepong framar. Et al in 2009 sought to locate students clinic at a central position among the students halls on the KNUST campus. Floyed warshal local centre and regret analysis we used to find a location to place the students eliminate. Their solution found a location on the road link between republic hall and independence hall at a distance

105m from republic hall. The maximum weighted distance from the facilities to the farthest mode is 1,553,043 mete

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Michael Dzator, Janet A. Dzator in their assertion on Location emergency Facilities: Targeting Efficiency and Cost-Effectiveness that Facility location problems form an important class of industrial optimization problems. These problems typically involve the optimal location of facilities. A facility is just a physical entity that assists with the provision of a service or the production of a product. Examples include: ambulance depot, emergency care centers, fire station, workstation, schools libraries, etc. The objective may involve factors such as cost, distance service utilization. The optimization problems are complicated with the need to meet a number of specified constraints. These constraints may relate to safety, available resources, level of service, time, etc.

The optimization problems are usually grouped into two categories namely service and manufacturing industries. In the service industries, the location of emergency facilities (ambulance, fire station, emergency centers) affects significantly on the safety and well-being of the community. The safety and well-being of the community directly or indirectly on the response time of the emergency facilities. The objective is to minimize the average response time (time between the receipt of a call and the arrival of emergency vehicle). The minimization of the response time measures the of emergency facilities. The performance of these facilities can be improved by either improving the existing location of emergency facilities or increasing the number of facilities. However, increasing the number of facilities is generally limited or impossible due to capital constraints. It is therefore important to locate emergency facilities effectively and efficiently.

The important way to measure the efficiency and the effectiveness of emergency facility is by evaluating the average distance between the customers and the facilities. When the average distance decreases, the accessibility of the facilities increases and this will decrease the average response time. This is known as the p-median problem, which was introduced by Hakimi (1964) and is defined as: determine the location of p facilities to minimize the (total) distance between demands and their closest facility.

The p-median problem is computationally difficult to solve by exact methods because the problem is NP-hard on general networks as shown by Kariv and Hakimi (1979). However, solutions from the p-median models are considered efficient since they bring the facility locations into closer proximity of the users. The difficulty of solving the p-median problem by exact method has led researchers to consider sub optimal solutions generated by heuristic approaches. Heuristics for solving the p-media problem have been discussed in Daskin (1995), Maranzana (1964), Teitz and Bart(1968) and Denshan and Rushton (1992).

This paper discusses three new heuristic methods solving the p-median problem. These methods are motivated by the desire to eliminate outliers from having strong influence over the final solution given by the heuristics. These heuristics will also improve the delivery of emergency medical care by properly locating emergency facilities in an area. In these heuristics, the facility location problem can be formulated as a network optimization problem as follows. The geographical region is partitioned into a number of sub-regions and a corresponding graph is constructed, each node of this graph represents a sub-region and each link of the graph represents the fact that the corresponding regions share a boundary. This gives us a structural model. Non-structural information is added as weights on the nodes (reflected expected demand in region) and the links (reflect travel time). Usually the nodes of the network represent possible location of facilities. An efficient reduction method is then used to address the problem of outliers.

Computational results, based on 400 random uniformly generated problems, show that the heuristics gives a good performance when compared with the optimal. Motivated by their performance the best heuristic is further compared with the 400 random problems and the well-known p-median heuristics giving better solution in most cases.

2.3 THE P-MEDIAN MODEL AND EMERGENCY FACILITY.

The criterion for finding a good location for emergency facilities requires the improvement of the response times to the emergency calls. The response time depend on the distance between the emergency facilities and the emergency sites. Thus, the aim of locating emergency facilities is to locate these facilities such that the average (total) distance travelled by those who visit or use these facilities is minimized. This measures the effectiveness and efficiency of the emergency facilities. Thus, the utility derived from using those facilities increases as the distance between them decreases. That is as travel distances increases, facility accessibility decreases and the effectiveness of the facility located decreases giving rise to increase response time. The p-median problem measures this effectiveness. It is clear that people tend to travel to the closest facility regardless of the distance or time travelled. A good way to achieve that is by the application of the p-median problem.

The p-median problem consists of determining the location for p emergency facilities to minimize the weighted distance between emergency (demand) points and their closest new emergency facility. The following authors such as Serra and Marinov, 1998; Marchandani, 1980; Berlin et al., 1976; Paluzzi, 2004; Carson and Batta; 1990 etc. Use p-median problem to locate the emergency facilities. We present the mathematical model for the p-median problem by defining the following notations as follows:-

$I = \{1, \ldots, m\}$, the set of demand locations,

$J = \{1, \ldots, n\}$, candidates site for facilities,

d_{ij} = the shortest distant between location i and location j,

X_{ij} = 1 if the customer at location *i* is allocated to facility at location *j*, 0 otherwise,

y_j = 1 if a facility is established at location *j*, 0 otherwise,

p = the number of facilities to be established,

a_i = the population at the demand node *i*.

The mathematical method of a *p*-median problem can specified as follows,

$$\text{Min} \sum_i \sum_j a_i \, d_{ij} X_{ij} \tag{1}$$

Subject to

$$\sum_{j \in J} x_{ij} = 1, \quad \forall i \in I \tag{2}$$

$$\sum_{j \in J} y_j = p \tag{3}$$

$$x_{ij} \leq y_j \qquad \forall i \in I, \ \forall j \in J \tag{4}$$

$$y_j \in \{0,1\}, x_{ij} \in \{1,0\} \tag{5}$$

The objective (1) is to minimize the total distance from customers or clients to their nearest facility. Constraint (2) shows that the demand of each customer or client must be met. From constraint (3), the number of facilities to be located is *p*. Constraint (4) shows that customers must be supplied from open facility, and constraint (4) shows that customers must be supplied from an open facility, and constraint (5) restricts the variables to 0,1 values.

Several extensions have been proposed for the p-median based models to improve their efficiency (Daskin *et al*., 1988). Extensions to the *p*-median problem that account for its stochastic has been given by Fitzsimmons (1973), Weaver and Church (1985) and Swoveland *et al*.(1973).

CHAPTER THREE

METHODOLOGY

3.1 INTRODUCTION

Facility location represents the process of identifying the best location for a service, commodity or production facility. Facility location models can be classified into three broad categories. These are p-median, p- centre, and the covering problem (maximal covering model and set covering models). The location models may have different approaches, especially when considering their objective functions. Some models seek to minimize location costs, while others try to minimize distances, and others are interested in demand coverage. The location models are explained below:

- Set Covering Model: Minimization of the location cost of the facilities needed to cover the total demand.
- Maximal Covering Model: Maximization of the total covered demand.
- *P*-Median Model: Minimization of the total demand-weighted distance or the average distance between nodes and facilities.
- *P*-Center Model: Minimization of the maximum distance between a demand node and its closest facility

We now look at the various models discuss in this section and then propose one for the study.

3.2 THE COVERING PROBLEM

Unlike the *p*-median problem which seeks to minimize the total travel distance, covering models are based on the concept of acceptable proximity. The objective of covering models is to provide "coverage" to demand points. A demand point is considered as covered only if a facility is available to service the demand point within a distance limit. Covering models can be classified according to several criteria. One of such criteria is the type of objective, which allows us to distinguish between two types of formulations. The first type is set covering model and the second type is maximal covering model.

3.2.1 SET COVERING MODEL

The objective of this model is to locate the minimum number of facilities required to "cover" all of the demand nodes (Toregas et al., 1971). The model is described below:

$$Min \sum_{i} c_i X_i \qquad (3.10)$$

Subject to:

$$\sum_{i \in Nj} X_i \geq 1, \quad \forall j \qquad (3.11)$$

$$X_i \in \{0,1\}, \quad \forall i \qquad (3.12)$$

Where

$$X_i = \begin{cases} 1, & \textit{if a facility is activated at candidate site } i \\ 0, & \textit{otherwise} \end{cases}$$

$c_i =$ Location cost of a facility at node i

$S =$ Maximum coverage distance

$N_j =$ Set of all candidate sites which can cover demand node $j(\mathrm{i.e.}\ \mathrm{N}_j = \{\mathrm{i}/\,\mathrm{d}_{ij} \le \mathrm{S}\});$

In this model, the objective function (3.10) minimizes the location cost of the facilities needed to cover all demands. Constraint (3.11) stipulates that each demand node must be covered. Constraints (3.12) are the integrality constraints

3.2.2 Maximal covering model

The objective of the Maximal covering location problem (MCLP) is to locate a predetermined number of facilities, p, in such a way as to maximize the demand that is covered. Thus, the MCLP assumes that there may not be enough facilities to cover all of the demand nodes. If all nodes cannot be covered, then the model seeks the siting scheme that covers the most demand (Church and ReVelle, 1974). The model is described below:

$$Max \sum_j D_j Z_j \text{..}(3.13)$$

$$Z_j \le \sum_{i \in N_j} X_i,\ \ \forall j \text{...}(3.14)$$

$$\sum_i X_i \le p \text{...}(3.15)$$

$$\mathrm{X}_i \in \{0,1\},\ \ \forall i \text{...}(3,16)$$

$$\mathrm{Z}_j \in \{0,1\},\ \ \forall j \text{..}(3.17)$$

Where

$D_j =$ Demand at node j;

$$Z_j = \begin{cases} 1 & \textit{if node } j \textit{ is covered} \\ 0, & \textit{otherwise} \end{cases}$$

$$X_i = \begin{cases} 1 & \textit{if a facility is activated at candidate site } i \\ 0 & \textit{otherwise} \end{cases}$$

$N_j =$ Set of all candidate sites which can cover demand node j $(\mathrm{N}_j = \{\mathrm{i} / \mathrm{d}_{ij} \leq \mathrm{S}\})$

The objective function (3.13) maximizes the total covered demand. Constraints (3.14) link the location and coverage variables, Constraint (3.15) states that at most p facilities are to be located. Constraints (3.16) and (3.17) are integrality constraints

3.3 THE P-CENTER MODEL

The p-center problem (Hakimi, 1964;1965) addresses the problem of minimizing the maximum distance that demand is from its closet facility given that we are siting a pre-determined number of facilities. The centre problem is a minimax problem. The 1-center problem is a classical optimization problem that looks at the location of a single facility such that all the demand nodes are covered. Under the 1-center problem, we have the vertex centre problem, which seeks to locate the facilities on the nodes of a network. There is also the “absolute” p-center problem that permits the facilities to be anywhere along the arcs or the network. Both versions are examined in weighted and un-weighted situations. In the un-weighted problem, all demand nodes are treated equally. In the weighted model, the distances between demand nodes and facilities are multiplied by a weight associated with the demand node. For example, this weight might represent a nodes importance or, more commonly, the level of its demand. The weights may have different interpretations such as time per unit distance, cost per unit distance or loss per unit distance. The model is described below:

Minimize W ……………………………………………….(3.18)

Subject to

$$\sum_j Y_{ij} = 1 \;\; \forall j \text{..}(3.19)$$
$$\sum_i X_i = p \text{..}(3.20)$$
$$Y_{ij} \le X_i \;\; \forall i, j \text{..}(3.21)$$
$$\mathrm{W} \ge \sum_i a_{ij} Y_{ij} \;\; \forall j \text{...}(3.22)$$
$$X_j = 0,1 \;\; \forall j \text{..}(3.23)$$
$$\mathrm{Y}_{ij} \in \{0,1\} \;\; \forall i, j \text{..}(3.24)$$

For the weighted p-center model constraint (3.22) becomes $\mathrm{W} \ge h_i \sum_j a_{ij} Y_{ij} \;\; \forall j$

Where

$a_{ij} =$ Distance from node i to facility j

p = Number of facilities to locate

$$x_j = \begin{cases} 1, & \text{if we locate at candidate site } i \\ 0, & \text{if not} \end{cases}$$

$$Y_{ij} = \begin{cases} 1, & \text{if demand node } j \text{ is assigned to facility } i \\ 0, & \text{otherwise} \end{cases}$$

$W =$ Maximum distance between a demand node and the facility to which it is assigned

The objective function (3.18) minimizes the maximum distance between a demand node and the closest facility to the node. Constraints (3.19) state that all of the demand at node *i* must be assigned to a facility at some node *j* for all nodes *i*. Constraint (3.20) stipulates that P facilities are located. Constraint (3.21) state that demands at node *i* cannot be assigned to a facility at node *j* unless a facility is located at node *j*. Constraint (3.22) state that the maximum distance between a demand node and the nearest facility to the node (W) must be greater than the distance between any demand node *i* and the facility *j* to which it is assigned. Constraint (3.23) and (3.24) are the integrality constraints.

3.4 THE P-MEDIAN MODEL

The p-median model (Hakimi, 1964; 1965) finds the locations of p facilities to minimize the demand-weighted total distance between demand nodes and the facilities to which they are assigned. The p-median problem may be formulated as follows

Minimize

$$\sum_i \sum_j h_i d_{ij} Y_{ij} \quad \text{.......(3.25)}$$

Subject to

$$\sum_j Y_{ij} = 1 \ \forall i \quad \text{.......(3.26)}$$

$$\sum_j X_j = P \quad \text{.......(3.27)}$$

$$Y_{ij} \leq X_i \ \forall i, j \quad \text{.......(3.28)}$$

$$X_j \in \{0,1\} \ \forall j \quad \text{.......(3.29)}$$

$$\mathrm{Y}_{ij} \in \{0,1\} \ \forall j \quad \text{.......(3.30)}$$

Where

h_i = Demand at node i

d_{ij} = Distance between customer i and candidate facility j

$$x_j = \begin{cases} 1, & \text{if we locate at candidate site } j \\ 0, & \text{if not} \end{cases}$$

$$Y_{ij} = \begin{cases} 1, & \text{if customer } i \text{ is served by facility } j \\ 0, & \text{otherwise} \end{cases}$$

The objective function (3.25) minimizes the total demand – weighted distance between each demand node. The constraints insure that the various properties of the problem are enforced. Specifically: Constraint (3.26) requires that, each demand node i be assigned to exactly one facility j. Constraint (3.27) requires that exactly P facilities are located. Constraint (3.28) links the location variables, and the allocation variables. Constraints (3.29) and (3.30) insure that the location variables (X) and the allocation variable (Y) are binary. The median formulation given above assumes that facilities are located on the nodes of the network. Because of the binary constraints (3.29) and (3.30), the p – median formulation above cannot be solved with standard linear programming technique.

3.5 SOLUTION METHODS FOR THE *P*-MEDIAN PROBLEM

A number of heuristic algorithms have been proposed to solve the p-median problem. These types of heuristics can be classified into what Golden et al (1980) calls construction algorithms and improvement algorithms. Daskin (1995) discusses three heuristics: a myopic algorithm, an exchange heuristic and a neighbourhood search algorithm.

3.5.1The Myopic Algorithm

This algorithm “constructs” a solution by locating the first facility at the one location that minimizes demand weighted total distance. This objective is calculated through total enumeration of the possible solutions. Subsequent facilities are located in a similar fashion, while holding the previously located facilities constant. The myopic heuristic is simple and thus to understand and apply. The main problem with this approach is that once a facility is selected it stays in all subsequent solutions. Consequently, the final solution attained may be far from optimal. The algorithm is given by;

$$Min \sum h_i d_{ij}$$

Where

$h_i =$ Population of each suburb in the first column

$d_{ij} =$ the distance matrix

ALGORITHM STEPS

1. Compute the total demand weighted-distance $(h_i \times d_{ij})$ for each row

2. Compute the sum $\sum h_i d_{ij}$ for each column

We locate the facility at node or column with $Min \sum h_i d_{ij}$.

3.5.2 ILLUSTRATIVE EXAMPLE

We consider the data in Table 3.1 to illustrate the myopic algorithm.

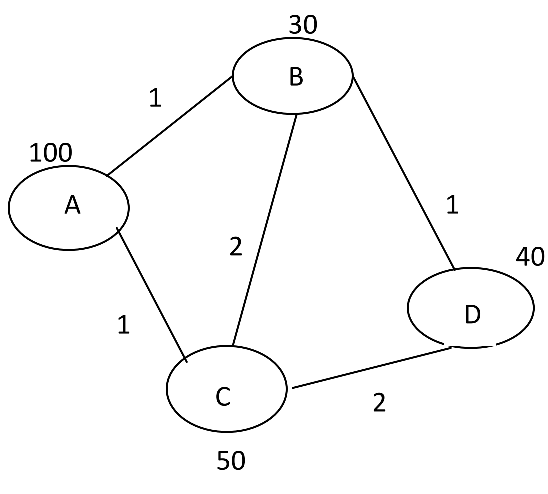

Figure 3.1: Road network with population

The road network is shown in Table 3.1

Table 3.1: Road networks of the four locations

d_{ij}	A	B	C	D
A	0	1	1	∞
B	1	0	2	1
C	1	2	0	2
D	∞	1	2	0

By using the Floyd's algorithm, we obtain the shortest path distance matrix for the above network. This is shown in Table 3.2

Table 3.2: The shortest path distance matrix

d_{ij}	A	B	C	D
A	0	1	1	2
B	1	0	2	1
C	1	2	0	2
D	2	1	2	0

Table 3.3: Shortest path distance matrix with demand (h_i)

h_i	d_{ij}	A	B	C	D
100	A	0	1	3	5
30	B	1	0	2	4
50	C	3	2	0	2
40	D	5	4	2	0

We find $[h_i \times d_{ij}]$ and sum the entries in the various columns. The column with the least value gives solution to the p-median problem. This is shown in Table 3.4

Table 3.4: The first myopic solution to the p-problem

$[\mathbf{h_i} \times \mathbf{d_{ij}}]$	**A**	**B**	**C**	**D**
A	0	100	300	500
B	30	0	60	120
C	150	100	0	100
D	200	160	80	0
Total	380	360	440	720

From Table 3.4 we locate the facility at node B since it has the least optimal value of 360. To locate a second facility, we compute$[h_i \times \max\{d(j,i), d(i,B)\}]$for each node location pair (i,j). Hence we adjust the distance matrix and the results is shown in Table 3.5

Table 3.5: The second myopic solution to the p-problem

$[\mathbf{h_i} \times \mathbf{max}\{\mathbf{d}(\mathbf{j},\mathbf{i}), \mathbf{d}(\mathbf{i},\mathbf{B})\}]$	**A**	**B**	**C**	**D**
A	0	100	100	100
B	0	0	0	0
C	100	100	0	100
D	160	160	80	0
Total	260	360	180	200

From Table 3.5 node C has the optimal value of 180. Hence we locate the second facility at node C.

3.5.3 Neighborhood Search Algorithm

One of the earliest improvement heuristics is the neighborhood search algorithm (Maranzana, 1964). In this method, we begin with any feasible solution or specifically a set of p facility sites. Demand nodes are then assigned to their nearest facility. The set of nodes assigned to a facility constitutes a “neighborhood” around that facility. Within each neighborhood, the 1-median problem can be solved optimally by simply evaluating each potential site in the neighborhood and selecting the best. The facilities are then relocated to the optimal 1-median locations within each neighborhood. Then, if any facility sites are relocated, new neighborhoods can be defined and the algorithm is repeated. This cycle continues until there are no further changes in the facility sites or neighborhoods.

3.5.4 Exchange Heuristic:

The most widely known improvement method was introduced by Teitz and Bart (1968). The basic idea is to move a facility from the location it occupies in the current solution to an unused site. Each unused location is tried in turn and when a move produces a better objective function value, then that relocation is accepted and we have a new (improved) solution. When an improved solution is obtained, the search process is repeated on the new solution. The procedure stops when no better solution can be found via this method. Although commonly used as a p-median problem, this approach has been found useful in innumerable facility location models. While seemingly straightforward in concept, the exchange heuristic has a number of alternative approaches that can be used in implementing it. One, of course, is the process described above, where every time an exchange is found that yields a better solution, the search process is restarted and applied to improve this new solution. Alternatively, we could select the best osolution after considering all possible moves for a given facility site, or even choose the best after all possible exchanges for all sites are examined. There are many other variations possible, and these often influence the computational speed of the heuristic. The most efficient implementation of the exchange algorithm was presented by Whitaker (1983). His “Fast Interchange” method is described in detail in Mladenovic and Hansen (1997). One issue in using improvement heuristics is to decide how the initial solution is generated. An obvious choice is to use the result of another heuristic, such as one of the greedy heuristics mentioned earlier. However, since the interchange heuristic is relatively fast, many analysts have applied it to a series of randomly generated solutions, selecting the best solution among all of the local optima found as the one to be implemented.

3.5.4 Reduction Heuristics (RH1, RH2 AND RRH):

Myopic algorithm for the *p*-median problem uses all the values of the distance matrix without any modification to solve the problem of extreme values (outliers). With reduction heuristics, we tried to eliminate the problem of outliers by using a reduction technique. Outliers can have a strong influence over the final solution. We also eliminate the uncertainty of choosing a good initial solution in the case of the Neighborhood search and Exchange heuristics by using a specific and efficient way of selecting the initial solution for the three new heuristics RH1, RH2 and RRH. In this study we proposed the reduction heuristic for the p-median problem.

3.5.4.1 Reduction Heuristics (RH1, RH2 and RRH):

The aim of the heuristics is to eliminate the outliers before using the data. This will enhance a facility to be located at nodes that are not far away from all customers, so the cost of using these facilities is minimized. We obtained the initial solution set for the heuristics by first eliminating the outliers and then sum the columns. We then choose the nodes corresponding to the first *p* nodes of the totals arrange in ascending order. The initial set is the first *p* nodes corresponding to the first *p* total, which is arranged in ascending order. We use the initial solution to reduce the distance matrix by setting the nodes that corresponding to the initial set for both rows and columns to zero. This is done with the assumption that customers at those nodes are not charged to uses the facilities. For *RH1*, the columns of the resulting distance matrix are added and the minimum value is chosen for substituting into the initial solution. We finally choose the set with the minimum objective value. In the case of *RH2*, all the nodes not in the initial solution are exchanged one-by-one for the nodes in the initial solution. We then choose the facility set with the minimum objective value as the final solution. However, for both heuristics, we choose the initial set as the final solution if there is no improvement in the objective value after the swapping procedure. Motivated by the performance of the two new heuristics (*RH1* and *RH2*), we extend *RH2* and propose a new heuristic, which we call Repeated Reduction Heuristic (*RRH*). The process of reducing the matrix is similar to *RH2* but, in this case, the reduction is done repeatedly until there is no improvement in the final solution. We describe the three new reduction heuristics for the p-median problem below.

3.5.4.1.1 REDUCTION HEURISTIC ONE (RH1)

- **Step 1:** Set the number of nodes and facilities to be equal to n and p respectively.
- **Step 2:** Arrange the n values for each column in ascending order and delete the last α number of values from each column. Next, let the resulting number of nodes be equal to n' (i.e. $n' = n - \alpha$ where α is p for less than twenty nodes, $2p$ for less than thirty nodes, $3p$ for less than forty nodes etc.)
- **Step 3:** Sum the first n' values for each column, arrange the values in ascending order, and choose the first p nodes as the initial set.

- **Step 4:** Set the columns and rows corresponding to the initial set to zero and sum the columns of the resulting distance matrix.
- **Step 5:** Choose the node or nodes corresponding to the minimum value and substitute for the nodes in the initial set.
- **Step 6:** Choose the set corresponding to the minimum objective value after the substitution procedure reaches the final solution. Otherwise, go to step 3 and choose the initial set as the final solution if that value is lower.

3.5.4.1.2 REDUCTION HEURISTIC TWO (RH2)

For ***RH2,*** Steps 1 to 4 is the same as ***RH1*** and the remaining steps are outlined below.

- **Step 5:** Substitute all the nodes not in the initial set with the nodes in the initial set.
- **Step 6:** Choose the set corresponding to the minimum value as the final solution. Otherwise, we choose the initial set as the final solution if that is lower We note that the different swapping procedure lead to an improved final solution as compared with *RH1* (Section 3.3.1.1).

3.5.4.1.3 REPEATED REDUCTION HEURISTIC (RRH)

In this heuristic, we repeatedly use the final solution of *RH2* as the initial set and use step 4 of RH1, and steps 5 and 6 of *RH2*. We continue this until there is no improvement in the final solution. We note that the repeated reduction incorporated in *RRH* has increased its performance as compared with *RH2*. The proposed heuristics are unique in three different ways. First, the methodology is simple and tractable. Second, the elimination of outliers gives a good initial solution. Third, the determination of swapping a node or nodes and the swapping procedure gives a good final solution. We also note that an improvement procedure can be further introduced to reduce the response time.

3.5.4.2 ILLUSTRATIVE EXAMPLE

We consider the data in Table 3.6 to illustrate the three heuristics (RH1, RH2 and RRH).

Table 3.6: illustrative example

NODES	A	B	C	D	E
A	0	57	62	34	52
B	57	0	41	93	18
C	62	41	0	19	22
D	34	93	19	0	43
E	52	18	22	43	0

3.5.3.1 Solution by RH1

To locate one facility we eliminate one greatest value in each column. Hence, we eliminate 62 in column 1 (node A), 93 in column 2 (node B), 62 in column 3 (node C), 93 in column 4 (node D) and 52 in column 5 (node E). This is shown in Table 3.2

Table 3.2: Elimination of outliers

NODES	A	B	C	D	E
A	0	57	0	34	0
B	57	0	41	0	18
C	0	41	0	19	22
D	34	0	19	0	43
E	52	18	22	43	0
Totals	**143**	**116**	**82**	**96**	**83**

From Table 3.2 we choose node C as an initial solution for RHl, RH2 and RRH. We then set row and column of node C of the data as shown in Table 3.1 to zero. The result is shown in Table 3.3.

Table 3.3: Setting row and column of node C to zero

NODES	A	B	C	D	E
A	0	57	0	34	52
B	57	0	0	93	18
C	0	0	0	0	0
D	34	93	0	0	43
E	52	18	0	43	0
Totals	**143**	**168**	**-**	**127**	**165**

From Table 3.3 summing the non-zero columns the resulting totals gives node D with the minimum value. So for RH1 we compare node C with node D.

Table 3.4: Solution by RH1

	A	B	C	D	E	Total
{C,D}	34	41	0	0	22	97

Choose the minimum value of C and D in comparison; 82, 127, and 97. The minimum value for the three is 82. Hence the final solution for RH1 is 82 which is node C.

3.5.3.2 Solution by RH2

In the case of RH2 we use all the nodes not in the initial solution C for comparing for nodes in the initial solution. The result is shown in Table 3.5

Table 3.5: Solution by RH2

	A	B	C	D	E	Total
{A,C}	0	41	0	19	22	74
{B, C}	57	0	0	19	22	98
{C, D}	34	41	0	0	22	97
{C, E}	52	18	0	19	0	89

Choose node A since it gives an optimal solution of 74.

3.5.3.3 Solution by RRH

Use A as an initial solution for RRH. Set row and column of A (in Table 3.1) to zero. The result is shown in Table 3.6.

Table 3.6: Setting row and column of node A to zero

NODES	A	B	C	D	E
A	0	0	0	0	0
B	0	0	41	93	18
C	0	41	0	19	22
D	0	93	19	0	43
E	0	18	22	43	0
Totals	**-**	**152**	**82**	**155**	**83**

From Table 3.6, summing the non-zero columns, node C gives the minimum value. Compare node C with node A and all nodes not in the initial solution A. The result is shown in Table 3.7

	A	B	C	D	E	Total
{C, A}	0	41	0	19	22	82
{C, B}	57	0	0	19	18	94
{C, D}	34	41	0	0	22	97
{C, E}	52	18	0	19	0	89

Choose the minimum value of A and C in comparison; 74, and 82. The minimum value is 74. Hence the final solution for RRH is 74 which is node A.

CHAPTER FOUR

DATA ANALYSIS AND RESULTS

4.0 INTRODUCTION

In this chapter we propose and use the p-median model and reduction heuristic to solve the problem of locating a Senior High School in the Kassena Nankena district. There are major ten (10) towns in the district. Secondary data was obtained which were 2010 population and housing census data from the Municipal statistical service department and road distances data from town and country planning. The towns and their respective population are shown in Table 4.1.

Table 4.1: Population and nodes of the ten suburbs of Kassena Nankana district

NODE	TOWN	Population
A	Nakong	3450
B	Katiu	3415
C	Chiana	11700
D	Kajelo	2713
E	Kayilo	3563
F	Paga	12195
G	Nakolo	4172
H	Sirigu	7495
I	Manyoro	3959
J	Binania	3210

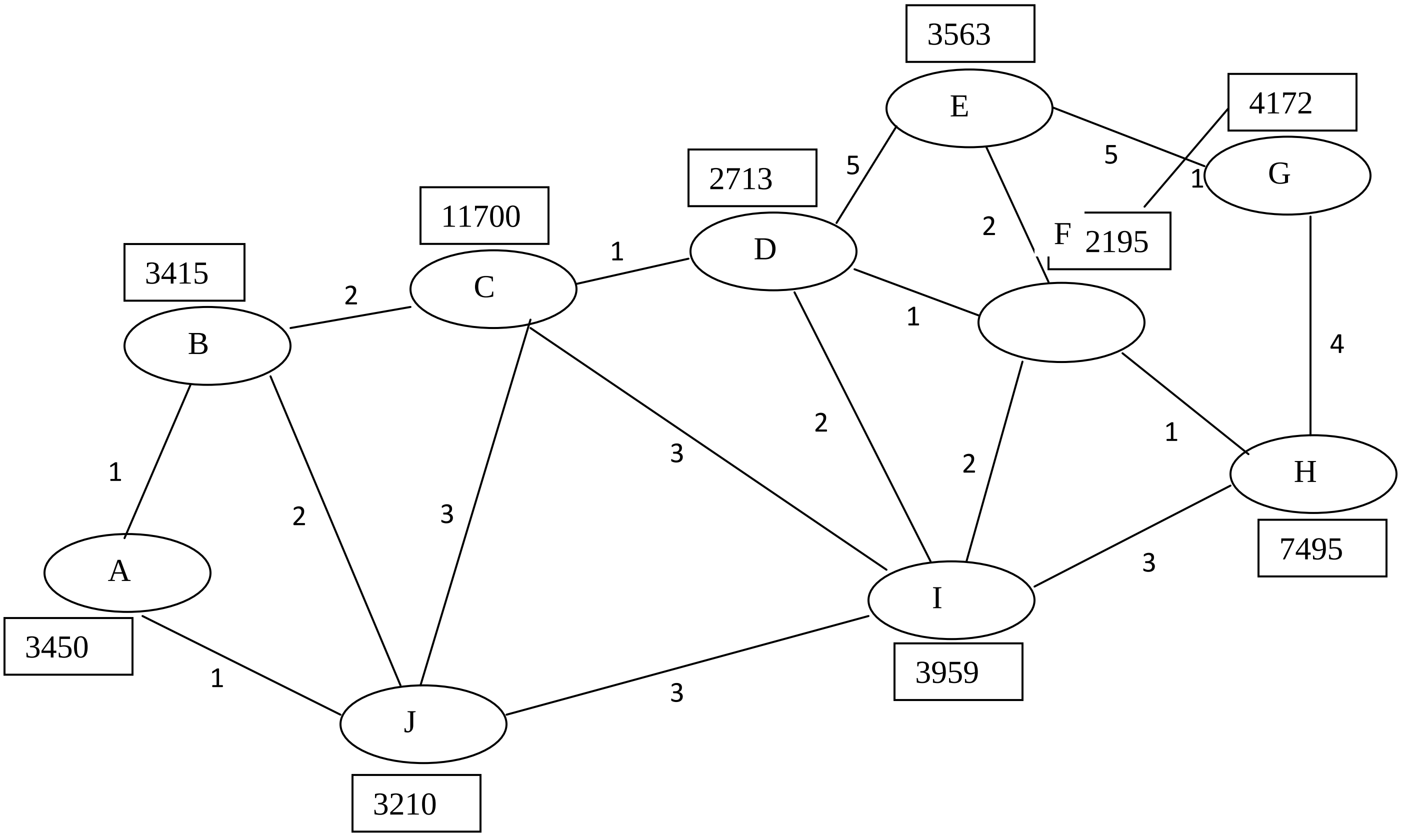

Figure 4.1: Populations and their nodes

We find the shortest path distance matrix using Floyd Warshall algorithm. This is shown in Table 4.2.

Table 4.2: Shortest path distance matrix

d_{ij}	A	B	C	D	E	F	G	H	I	J
A	0	1	3	4	9	5	14	6	6	1
B	1	0	2	3	10	6	15	7	5	2
C	3	2	0	1	6	2	3	6	3	3
D	4	3	1	0	5	1	4	7	2	4
E	9	10	6	5	0	2	5	3	4	9
F	5	6	2	1	2	0	1	1	2	5
G	14	15	3	4	5	1	0	4	3	6
H	6	7	6	7	3	1	4	0	3	6
I	6	5	3	2	4	2	3	3	0	3
J	1	2	3	4	9	5	6	6	3	0

Table 4.3 shows the demand node (h_i) and the shortest path distance matrix (d_{ij}). The demand node (h_i) is displayed in the first column and the rest of the column displays the shortest path distance matrix.

Table 4.3: The shortest path distance matrix (d_{ij}) and demand node (h_i)

h_i	d_{ij}	A	B	C	D	E	F	G	H	I	J
3450	A	0	1	3	4	9	5	14	6	6	1
3415	B	1	0	2	3	10	6	15	7	5	2
11700	C	3	2	0	1	6	2	3	6	3	3
2713	D	4	3	1	0	5	1	4	7	2	4
3563	E	9	10	6	5	0	2	5	3	4	9
12195	F	5	6	2	1	2	0	1	1	2	5
4172	G	14	15	3	4	5	1	0	4	3	6
7495	H	6	7	6	7	3	1	4	0	3	6
3959	I	6	5	3	2	4	2	3	3	0	3
3210	J	1	2	3	4	9	5	6	6	3	0

4.2 The p-median problem

Minimize

$$\sum_{i}\sum_{j} h_i d_{ij} Y_{ij} \qquad (4.10)$$

Subject to

$$\sum_{j} Y_{ij} = 1 \ \forall i \qquad (4.11)$$

$$\sum_{j} X_j = P \qquad (4.12)$$

$$Y_{ij} \le X_i \ \forall i, j \qquad (4.13)$$

$$X_j \in \{0,1\} \ \forall j \qquad (4.14)$$

$$\mathrm{Y}_{ij} \in \{0,1\} \ \forall j \qquad (4.15)$$

Where

h_i = Demand at node i

$d_{ij} =$ Distance between customer i and candidate facility j

$$x_j = \begin{cases} 1, & \text{if we locate at candidate site } j \\ 0, & \text{if not} \end{cases}$$

$$Y_{ij} = \begin{cases} 1, & \text{if customer } i \text{ is served by facility } j \\ 0, & \text{otherwise} \end{cases}$$

The values of $h_i, \mathrm{d}_{ij}, Y_{ij}$ are found in Table 4.3

P=1, P is the number of facilities to be located.

Calculate the weighted distance ($h_i \times d_{ij}$). The result is shown in Table 4.4

Table 4.4: The weighted distance

$h_i \times d_{ij}$	A	B	C	D	E	F	G	H	I	J
A	0	3450	10350	13800	31050	17250	48300	20700	20700	3450
B	3415	0	6830	10245	34150	20490	51225	23905	17075	6830
C	35100	23400	0	11700	70200	23400	35100	70200	35100	35100
D	10852	8139	2713	0	13565	2713	10852	18991	5426	10852
E	32067	35630	21378	17815	0	7126	17815	10689	14252	32067
F	60975	73170	24390	12195	24390	0	12195	12195	24390	60975
G	58408	62580	12516	16688	20860	4172	0	16688	12516	20860
H	44970	52465	44970	52465	22485	7495	29980	0	22485	44970
I	23754	19795	11877	7918	15836	7918	11877	11877	0	11877
J	3210	6420	9630	12840	28890	16050	19260	19260	9630	0

4.3 SOLUTION BY REDUCTION HEURISTICS

We solve the p-median problem above using reduction heuristic (RH1, RH2 and RRH). Manual solution for the three heuristics is obtained as follows;

4.3.1 Solution by RH1

The solution of RH1 follows the following steps:

Step 1: To locate one facility, we eliminate one greatest value in each column of Table 4.4. Hence, we eliminate 60975 in column 1 (node A), 73170 in column 2 (node B), 44970 in column 3 (node C), 52465 in column 4 (node D), 70200 in column 5 (node E), 23400 in column 6 (node F), 51225 in column 7 (node G), 70200 in column 8 (node H), 35100 in column 9 (node I) and 60975 in column 10 (node J). The result is shown in Table 4.5.

Table 4.5 Elimination of outliers

$h_i \times d_{ij}$	A	B	C	D	E	F	G	H	I	J
A	0	3450	10350	13800	31050	17250	48300	20700	20700	3450
B	3415	0	6830	10245	34150	20490	0	23905	17075	6830
C	35100	23400	0	11700	0	0	35100	0	0	35100
D	10852	8139	2713	0	13565	2713	10852	18991	5426	10852
E	32067	35630	21378	17815	0	7126	17815	10689	14252	32067
F	0	0	24390	12195	24390	0	12195	12195	24390	0
G	58408	62580	12516	16688	20860	4172	0	16688	12516	20860
9H	44970	52465	0	0	22485	7495	29980	0	22485	44970
I	23754	19795	11877	7918	15836	7918	11877	11877	0	11877
J	3210	6420	9630	12840	28890	16050	19260	19260	9630	0
Total	**211,776**	**211,879**	**99,684**	**103,201**	**191,226**	**83,214**	**185,379**	**134,305**	**126,474**	**166,006**

Step 2: Compute the column totals in Table 4.5, and then choose the column with the minimum as an initial solution for RH1. From Table 4.5, choose node F (83,214) as an initial solution for RHl.

Step 3: Solution F to zero and sum the columns and rows corresponding to the initial set to zero and sum the columns of the resulting distance matrix. Here, set rows and columns of node F of the data as shown in Table 4.4 to zero. The result is shown in Table 4.6.

Table 4.6: Setting rows and columns of node F to zero

$h_i \times d_{ij}$	A	B	C	D	E	F	G	H	I	J
A	0	3450	10350	13800	31050	0	48300	20700	20700	3450
B	3415	0	6830	10245	34150	0	51225	23905	17075	6830
C	35100	23400	0	11700	70200	0	35100	70200	35100	35100
D	10852	8139	2713	0	13565	0	10852	18991	5426	10852
E	32067	35630	21378	17815	0	0	17815	10689	14252	32067
F	0	0	0	0	0	0	0	0	0	0
G	58408	62580	12516	16688	20860	0	0	16688	12516	20860
H	44970	52465	44970	52465	22485	0	29980	0	22485	44970
I	23754	19795	11877	7918	15836	0	11877	11877	0	11877
J	3210	6420	9630	12840	28890	0	19260	19260	9630	0
TOTAL	**211776**	**211879**	**144,654**	**143471**	**237036**	**-**	**224409**	**172284**	**137184**	**166006**

Step 4: From Table 4.6 summing the non-zero columns the resulting totals gives node I with the minimum value of 137,184.

- refer to Table 4.4
- Compare elements of same rows and columns F and I
- Pick minimum value of each row of the two columns

Table 4.7: Solution by RH1

	A	B	C	D	E	F	G	H	I	J	**Total**
{F,I}	17250	17075	23400	2713	7126	0	4172	7495	0	9630	**88861**

Step 5: Choose the minimum value of F and I in comparison; 83214, 137184, 88861. The minimum value for the three is 83214. Hence the final solution for RH1 is 83214 which is node F.

4.3.2 Solution by RH2:

For RH2, Steps 1 to 3 is the same as RH1 and the remaining steps are outlined below

Step 4: compare all the nodes not in the initial solution F and selecting the minimum value in each column. In this case {A, B, C, D, E, G, H, J}, for comparing for node in the initial solution which is F. This gives

Table 4.8: Solution by RH2

	A	B	C	D	E	F	G	H	I	J	Total
{A, F}	0	3415	23400	2713	7126	0	4172	7495	7918	3210	59449
{B, F}	3450	0	23400	2713	7126	0	4172	7495	7918	6420	62694
{C, F}	10350	6830	0	2713	7126	0	4172	7495	7918	9630	56234
{D, F}	13800	10245	11700	0	7126	0	4172	7495	7918	12840	75296
{E, F}	17250	20490	23400	2713	0	0	4172	7495	7918	16050	99488
{G, F}	17250	20490	23400	2713	7126	0	0	7495	7918	16050	102442
{H, F}	17250	20490	23400	2713	7126	0	4172	0	7918	16050	99119
{J, F}	3450	6830	23400	2713	7126	0	4172	7495	7918	0	63104

Step 5: Compare the values of nodes and rows and pick the value which is lower; the lower value is 56234 which is node C which gives an improved optimal solution.

4.3.3 Solution by Repeated Reduction Heuristic (RRH)

In this heuristic, use the final solution of *RH2* as the initial set and use step 3 of *RH1*, and steps 4 and 5 of *RH2*. We continue this until there is no improvement in the final solution. In this case we choose C as the initial solution for RRH. We therefore set rows and columns of C (in Table 4.4) to zero. The result is shown in Table 4.9.

Table 4.9: Setting rows and columns of C to zero

$h_i \times d_{ij}$	A	B	C	D	E	F	G	H	I	J
A	0	3450	0	13800	31050	17250	48300	20700	20700	3450
B	3415	0	0	10245	34150	20490	51225	23905	17075	6830
C	0	0	0	0	0	0	0	0	0	0
D	10852	8139	0	0	13565	2713	10852	18991	5426	10852
E	32067	35630	0	17815	0	7126	17815	10689	14252	32067
F	60975	73170	0	12195	24390	0	12195	12195	24390	60975
G	58408	62580	0	16688	20860	4172	0	16688	12516	20860
H	44970	52465	0	52465	22485	7495	29980	0	22485	44970
I	23754	19795	0	7918	15836	7918	11877	11877	0	11877
J	3210	64200	0	12840	28890	16050	19260	19260	9630	0
Total	**237651**	**242809**	**-**	**117971**	**135786**	**83214**	**170886**	**101410**	**81384**	**191881**

From Table 4.9, summing the non-zero columns, node I gives the minimum value of 81384. We compare nodes C with minimum value of node I and all nodes not in the initial solution. This gives a possible solution set of {C, I}, {C, A}, {C, B}, {C, D}, {C, E}, {C, F}, {C, G}, {C, H}, {C, J}

Table 4.10: solution by RRH

	A	B	C	D	E	F	G	H	I	J	Total
{C, I}	10350	6830	0	2713	14252	24390	12516	22485	0	9630	103166
{C, A}	0	3415	0	2713	21378	24390	12516	44970	11877	3210	140964
{C, B}	3450	0	0	2713	21378	24390	12516	44970	11877	6420	144174
{C, D}	10350	6830	0	2713	17815	12195	12516	44970	7918	9630	124937
{C, E}	10350	6830	0	2713	0	24390	12516	22485	11877	9630	100791
{C, F}	10350	6830	0	2713	7126	0	4172	7495	7918	9630	56234
{C, G}	10350	6830	0	2713	17815	12195	0	29980	11877	9630	101390
{C, H}	10350	6830	0	2713	12195	12195	12516	0	11877	9630	78306
{C, J}	3450	6830	0	2713	21378	24390	12516	44970	11877	0	128124

The value for node C is minimum value of 56234 it repeat the same value for node C in Table 4.8. We choose node C as the final solution since it gives an optimal solution of 56234.

4.4 DISCUSSION OF RESULTS

The solution for RH1 was 83214. Then this was improved in RH2 to 56234. On applying RRH there was improvement found in node C, so node C was selected as an optimal solution with the value 56234.

CHAPTER FIVE

CONCLUSSION AND RECOMMENDATION

5.1 CONCLUSSION

The problem of locating Senior High School was formulated using the p-median problem. The problem was then solved using the reduction heuristic algorithm to determine the optimal solution of locating one Senior High School in the Kassena Nankana district. Ten (10) communities in the district were considered, taken into consideration the population of the district and road distances. According to the model, the Senior High school should be sited at Chiana with objective value of 56234.

5.2 RECOMMENDATION

In view of the result obtained in this study, the following recommendations are made:

- Corporate bodies such as the Kassena Nankana Municipal Assembly, Ghana Education Service as well as private individuals, who want to establish Senior high school in the Kesena Nankana Municipality, should site it at Chiana.

- In this thesis we proposed reduction heuristic algorithm to solve a p-median problem of locating Senior High School in the Kassena Nankana district, researchers can also use greedy add heuristic also known as the myopic algorithm to study the p-median problem discuss in this thesis.

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