Location of another Airport in Ghana Using the Electre Method:

(A Case Study)

By

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A Thesis to Be Submitted to the Department Of Mathamatics, Kwame Nkrumah University of Science and Technology, Kumasi In Partial Fulfilment of the Requirement For The Degree of Master of Science In Industrial Mathematics

IDL

JUNE, 2012

DECLARATION

I hereby declare that this submission is my own work towards the Master of Science and that to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the university, except where due acknowledgement has been made in the text.

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DEDICATION

This work is dedicated to my wife and my lovely children.

ACKNOWLEDGEMENTS

I am sincerely thankful to my supervisor Dr. Francis T. Oduro, for his timeless dedication during the supervision of this work

This work would not be completed without the mention of Mr. Albert Cofie, a lecturer at Sikkim Manipal University and a Data Analyst at Millward Brown for his invaluable contribution to the completion of this thesis.

To many, whose invaluable contributions made this work possible but could not be mentioned I am grateful to you all.

ABSTRACT

The ELECTRE (Elimination et Choix Traduisant La Realité) meaning in English Elimination and Choice Expression Reality evaluation method is widely recognized for high-performance policy analysis involving both qualitative and quantitative criteria. However, a critical advantage of this evaluation method is its capacity to pinpoint the exact needs of a decision maker and suggest an appropriate evaluation approach. The discordance indices of modified ELECTRE evaluation method are used to explain the significance of modified evaluation standards. ELECTRE II evaluation method, applied to case simulation, helps analyze the potential effects triggered by the absolute value of the maximum differentiated performance and the absolute value of the sum of differentiated performance under two discordance index evaluation standards. In general, using the ELECTRE evaluation method in the absence of a differentiation process may produce results opposite to those desired by a decision maker.

The method was used to determine the best out of 5 districts in the Upper West Region of Ghana. Discordance indices were calculated, together with calculations using the adjusted R squared. It was concluded that the best place to cite the airport was in Wa Municipal.

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CHAPTER ONE

INTRODUCTION

1.1 BACKGROUND OF STUDY

With the rapid changes in living environment experienced in recent decades, many cities have placed increasing expectations on land redevelopment to help ease urban planning problems. However, a high priority should be given to a restructuring of the decision-making processes employed during the formulation and execution of redevelopment decisions. These processes demand consideration of complex, multi-faceted issues and, therefore, typically incorporate both qualitative and quantitative evaluation criteria. ELECTRE (Elimination et Choix Traduisant La Realité) is a widely recognized evaluation method with a strong performance track record that can be employed to facilitate decision-making activities which incorporate both qualitative and quantitative criteria. However, ELECTRE evaluation method as currently used uses different evaluation standards for different purposes. Therefore, accurately defining decision maker needs and choosing appropriate evaluation methods represent issues of ongoing concern for scholars involved in the mechanics of this evaluation method. Therefore, in order to make correct, informed decisions, it is important that decision makers have access to complete information and thoroughly understand various alternatives. The discordance index evaluation benchmarks of the modified ELECTRE evaluation method in this study are used to explain the significance of modified evaluation standards. ELECTRE II evaluation method is applied to case simulation. The possible effect triggered by the absolute value of the maximum differentiated performance and the absolute value of the sum of differentiated performance is analyzed under the two discordance index evaluation standards.

The outranking method, part of the Multi-Criteria Decision-Aid (MCDA), was introduced in 1966 when three French scholars (i.e. Benayoun, Roy and Sussmann) initiated ELECTRE evaluation method. Then, scholars and study groups made some important efforts to move this method forward and published worldwide a number of articles involving similar theories and their applications. Firstly, the evaluation method is required to establish preference relation, i.e. outranking relation, and then make consistent exploration and analysis in support of decision makers. ELECTRE I is one of the earliest multi-criteria evaluation method developed among outranking methods. The major purpose of this evaluation method is to select a desirable alternative that meets both the demands of concordance preference above many evaluation benchmarks and of discordance preference under any optional benchmark. ELECTRE I evaluation method generally included three concepts; namely the concordance index, discordance index and threshold value. ELECTRE II evaluation method, developed by scholars Roy and Bertier (1971), represented the improvement and promotion of ELECTRE I. The concordance index and discordance index in ELECTRE II incorporate two extreme opposite relationships, i.e. strong relationship (R_s) and weak relationship (R_w), whereby strong-ranking and weak-ranking are deduced to obtain the final ranking.

The literature confirms that ELECTRE evaluation method is widely considered as an effective and efficient decision aid with a broad range of applications covering policy-making with regard to the use of urban land and planning investments, transport facilities, environmental protection programs, among others. Thanks to the concerted efforts of many scholars in this field, evaluation methods most frequently referred to include ELECTRE I, ELECTRE II, ELECTRE III, ELECTRE IV, ELECTRE IS and ELECTRE A. Despite the numerous versions of the ELECTRE evaluation method, how to select the method most relevant to a particular problem is a key problem for analysts and policy makers. Thus, updating the ELECTRE benchmark to provide the most pertinent response to different issues is crucial. Otherwise, the resulting recommendations are likely to define an objective that falls below policy maker expectations.

As a practical application several alternatives for the distribution facility location were evaluated by the location decision model. The goal of this research is to provide decision makers with a more effective and efficient model for making facility location decision. In this thesis, we show how an integrated decision model can aid location decision by generating a solution that recognizes the practical consideration.

The overall mathematical problem can be formulated as various objective functions but to mention a few below.

1. Minimizing the total set up cost
2. Minimizing the longest distance from the existing facilities
3. Minimizing fixed cost
4. Minimizing total annual operation cost
5. Maximizing service
6. Minimizing average time / distance covered
7. Minimizing Maximum time / distance covered
8. Minimizing the number of located facilities.
9. Maximizing responsiveness. Recently environmental and social objectives based on energy cost, land use and construction cost, congestion noise, quality of life, pollution, fossils fuels crisis and tourism are becoming customary.

1.1.7 Types of airports

There are many types of airports that exist today. These airports range from a single grass airstrip in an agricultural or rural area to the large airports serving major cities. There are seven basic types of airports:

1. Rural airstrip
2. Private airport
3. Military airports
4. Small community airport
5. Regional community airport
6. Regional airport
7. Major city airport

1.1.8 The Importance of Airports

The importance of airports as global business hubs is now being recognized globally. In some parts of the world, entire cities or at least fully-functioning suburbs (with office space, industrial buildings, and residential and retail districts) are being built around airports in a phenomenon known as the aerotropolis.

But even without being an aerotropolis, a city's airport is a first impression — and a last impression. It facilitates face-to-face communication, strikingly important to generating innovation. And airports allow global companies to establish operations in multiple cities and have certain managers and executives moved seamlessly between locations.

1.2 STATEMENT OF THE PROBLEM

Due to the limited number of airports, terminals are hard pressed to handle air traffic congestion and delay in the Ghana Aviation industry. Congestion and delayed have become the order of the day. This had made congestion pricing to be introduced and thus putting more pressure on passengers.

1.3 OBJECTIVES

The objective of this thesis is to use the ELECTRE method to determine the location for citing a new airport in the Upper West region of Ghana.

To rank a set of alternatives, the ELECTRE method as outranking relation theory was used to analyze the data regarding a decision matrix. The concordance and discordance indexes can be viewed as measurements of dissatisfaction that a decision maker uses in choosing one alternative over the other.

We assume *m* alternatives and *n* decision criteria. Each alternative is evaluated with respect to *n* criteria. As result, all the values assigned to the alternatives with respect to each criterion form a decision matrix.

1.4 METHODOLOGY

ELECTRE I is one of the earliest multicriteria evaluation methods, developed among other outranking methods. The major purpose of this method is to select a desirable alternative that meets both the demands of concordance preference above many evaluation benchmarks, and of discordance preference under any optional benchmark. The ELECTRE I generally includes three concepts, namely, the concordance index, discordance index, and the threshold value.

In this study, our model fuzzy ELECTRE along with the opinion of decision makers will be applied by a group decision makers.

The procedure for fuzzy ELECTRE ranking model is given as follows:

Step 1 (determination of the weights of the decision makers). Assume that the decision group contains l decision maker's criteria and gives them designated scores. The importance of the decision makers is, then, considered in linguistic terms (LT). We construct the aggregated decision matrix (ADM) based on the opinions of the decision-makers, and the LT as shown in Table 1.

Step 2 (calculation of TFN's). We set up the TFN's. Each expert makes a pair wise comparison of the decision criteria and gives them relative scores. The aggregated fuzzy importance weight (AFIW) for each criterion can be described as TFN's $\tilde{w}_j = \left(l_j, m_j, u_j\right)$ for $K = 1, 2, \ldots, k,$ and $j = 1, 2, \ldots, n.$ This scale has been employed in the TFN's as proposed by Mikhailov (24).

1.5 JUSTIFICATION OF THE STUDY

The study is significant for the following reasons:

1. It will help to reduce the average travel time and the distance from the farthest customer locations to another airport location.
2. It will help to give an effective co- ordinate for the computation distances and also a central location for the new airport.
3. It will help to reduce fuel consumption since the algorithm gives the average shortest distance for the location of customers.

1.6 LIMITATION OF STUDY

Getting permission to conduct the study in the aviation industry was pretty hectic. There were so many long and avoidable bureautic procedures to be followed which delayed the start and progress of the study due to financial constraints and the short duration within which the study was conducted a larger sample could not be used.

1.7 ORGANIZATION OF STUDY

The study is made up of five main chapters. Chapter 1 deals with the background of the study, statement of the problem, objectives, methodology, justification and limitation of the study.

Chapter 2 deals with the related literature relevant to the study.

Chapter 3 discusses the methodology used for the study.

Chapter 4 presents the results of data analysis and the discussion of the case study.

Chapter 5 deals with the conclusion and recommendation of the study.

CHAPTER TWO

LITERATURE REVIEW

2.1 INTRODUCTION

In this chapter, we review problems and models for undesirable and semi-desirable facility location and their solution methodologies. We mainly focus on the bi-objective location models for a single semi-obnoxious facility with different distance metrics. Also, different approaches on the semi-obnoxious facility location are discussed in the following section.

2.2 Undesirable Facility Location

ReVelle and Eiselt (2005) described the term Location Analysis as siting facilities in some given space that refers to the modeling, formulation, and solution of a class of problems. Also, they introduced four elements that characterize location problems; those are (1) users, who supposed to be already placed at points, (2) type of the facilities that will be located, (3) a space in which users and facilities are located, (4) a metric that indicates distance or times between users and facilities. The facility-users relationship defines the necessary objective function(s) to model the location problem. The majority of the literature on location analysis deals with desirable facility location models. These models are introduced in Francis et al. (1992) and Love et al. (1988) which include many references. On the other hand, both the undesirable and semi-desirable facility location problems have received more recent attention due to the increasing environmental and social awareness.

Erkut and Neuman (1989) introduced the term undesirable facility for noxious (hazardous) and obnoxious (nuisance) facilities. They gave a detailed literature survey and introduced analytical models for undesirable facility location. They suggested using multicriteria tools for the final selection of a place for undesirable facility because of the complexity of the problem.

2.2. OVERVIEW OF EVALUATION METHODS

Some examples of ELECTRE II evaluation methods are introduced below:

A represents the aggregate obtained from n feasible alternatives $(A_1, A_2, ...A_n)$, i.e. $A = \{A_i \mid i = 1, 2, ...n\}$.

I represents the aggregate obtained from m evaluation criteria $(C_1, C_2, ..., C_m)$ i.e. $I = \{C_j \mid j = 1, 2, ..., m\}$.

Assuming there are identified weights of m evaluation criteria, with W representing the aggregate of m weights $(W_1, W_2, ..., W_m)$ i.e. $W = \{W_j \mid j = 1, 2, ..., m\}$

The performance value of feasible alternative A_i under evaluation criterion C_j is represented by $g_j(A_i)$. With regard to two optional alternatives A_h and A_k, m evaluation criteria are classified into three categories when $g_j(A_k)$ and $g_j(A_k)$ are compared under every criterion C_j:

$$I^+ = \{C_j \mid g_j(A_h) > g_j(A_k)\}$$

$$I^= = \{C_j \mid g_j(A_h) = g_j(A_k)\}$$

$$I^- = \{C_j \mid g_j(A_h) < g_j(A_k)\}$$

I^+ is represented by the aggregate in case the evaluation criterion for the performance value of alternative A_h is better than for alternative A_k, with its weight W^+ shown below:

$$W^+ = \sum_{j \in I^+} W_j$$

$I^=$ is represented by the aggregate in case the evaluation criterion for the performance value of alternative A_h is the same as that for alternative, with its weight shown below:

$$W^= = \sum_{j \in I^=} W_j$$

I^- is represented by the aggregate in case the evaluation criterion for the performance value of alternative is inferior to that for alternative, with its weight shown below:

$$W^- = \sum_{j \in I^-} W_j$$

3.1 Establishment of Concordance Index:

The following models, such as, $W^+, W^+ + W^=$ and $W^+ + \frac{1}{2} W^=$ shall be applied as the numerator in the concordance index. Meanwhile, decision makers may opt to choose the numerator as their evaluation standard. The denominator is $W^+ + W^= + W^-$. In this paper, the model W^+ is used as the numerator. Thus, the evaluation standard for the concordance index shall be based upon the weight ratio (the weight of performance value of alternative A_h better than of A_k, in relation to the weight sum).

Two optional alternatives A_h and A_k are compared with concordance index $C(h,k)$,defined below:

$$C(h,k) = \frac{W^+}{W^+ + W^= + W^-}, \forall h,k; h \neq k$$

To compare alternative A_h with alternative A_k, the concordance index can be calculated by totaling the weight of the performance value of alternative A_h in criterion j better than that of alternative A_k in criterion j. The result is then divided by the weight sum of all criteria. In addition, with respect to $C(h,k) = \sum_{j \in I^+} W_j / \sum_{j \in I} W_j, if \sum_{J \in i} W_j = 1$, the above-specified formula can be represented by, $C(h,k) = \sum_{j \in I^+} W_j$ so $0 \leq C(h,k) \leq 1$

2.3 ESTABLISHMENT OF DISCORDANCE INDEX

Under evaluation criterion C_j, the discordance index $d_j(h,k)$ of feasible alternative A_h is not better than that of alternative A_k, showing that selection of alternative A_h other than better alternative A_k will likely result in the dissatisfaction of decision makers to a considerable extent. In this dissertation, the initial discordance index $d_j(h,k)$ of ELECTRE II evaluation model is defined below:

$$d_j(h,k) = \left| \frac{g_j(A_h) - g_j(A_k)}{\max\limits_{j \in I^-}\left(g_j(A_h), \theta_j\right)} \right|, \forall h,k; h \neq k$$

In the above equation, refers to the differentiation percentage of plans A_h and A_k under evaluation criterion C_j. Besides, θ_j refers to the R-degree parameter used by a decision maker for criteria j to represent the degree of attention paid by the decision maker to criteria j. In other words, decision makers can express their preference for criteria reflecting differing levels of significance. If m criteria are evaluated concurrently, the feasible plan A_h is not superior to plan A_k of the discordance which is defined as the maximum value of discordance index $d_m(h,k)$, under m criteria, the $d_j(h,k)$

Since the denominator of $d_j(h,k)$ is determined by θ_j and $g_j(A_h)$ there is no guarantee that $d_m(h,k)$ is between 0 and 1. Therefore, the following equation does not satisfy:

$$0 \leq d_m(h,k) \leq 1 \qquad \forall h,k; h \neq k$$

In this dissertation $d_s(h,k)$, is used to represent the discordance index with benchmarks of the absolute value of the sum of differentiated performance and modified as below:

$$d_s(h,k) = \sum_{j \in I^-} d_j(h,k)$$

Similarly, there is no guarantee that $d_s(h,k)$,is between 0 and 1, thus, the following equation does not satisfy:

$$0 \le d_s(h,k) \le 1 \quad \forall h,k; h \ne k$$

3.3 Establishment of Strong Outranking and Weak Outranking Relationship:

Two elements are used to establish the outranking relationship value for ELECTRE II : strong outranking relationship R_S and weak outranking relationship R_w. According to the definition of R_S and R_w, policy makers must determine different concordance index levels and discordance index reducible level. It is assumed that p^*, p^0 and p^- are represented by three degressive concordance levels, respectively, and meet the following conditions:

$$0 \le p^- \le p^0 \le p^* \le 1$$

Additionally, q^0 and q^* are represented by two degressive discordance levels, and meet the following conditions:

$$0 < q^0 < q^* < 1$$

After decision maker identify concordance and discordance levels, it is possible to establish value for the strong outranking relationship (R_s) and weak outranking relationship (R_w), calculate strong ranking V' and weak ranking V'' values, and finally, determine the average rankings $\overline{V}$ for final ranking result.

2.4 SEMI-DESIRABLE FACILITY LOCATION WITH NETWORK DISTANCES

Hamacher. et al (2002) deal with multicriteria semi-obnoxious network models with sum and center objectives. They formulate a negatively correlated maxisum minisum semi-desirable facility location model with weighted shortest path network distances. An O(mn.log.n) polynomial time algorithm is developed for this model along with some basic theoretical results. Also, they generalize these results to incorporate maximin and minimax objectives. Their method consists of the direct mapping of the network into criterion space and then calculating the lower envelope that is similar to Hershberger (1989). Hence, this method works for any piecewise linear objective functions. The most encouraging result of this approach is that for bicriterion networks with objective functions in almost opposite directions, a very small proportion of the networks is efficient. Thus, this model is really helpful for decision-maker, since a large part of the network can be out of consideration.

Skriver et al. (2004) proposed a new location model which is a generalization of the biobjective median problem that incorporates both the location and routing aspects in a multiobjective setting. They present two sum objectives and criteria dependent arc lengths. In order to solve this problem, a two-phased method is developed, which can easily be implemented as an interactive method. This method can be considered as a general approach to biobjective combinatorial optimization problems.

2.5 SEMI-DESIRABLE FACILITY LOCATION WITH MIXED PLANAR METRICS

For biobjective location of a semi-obnoxious facility, the majorities of the studies consider both push and pull objectives with the same distance metric. But, this approach may not be appropriate for some cases. For example, while the minisum objective with rectilinear or shortest path network distances realistically models transportation cost, the maximin objective with these distance metrics may or may not model realistically the undesirable effect, depending on the application. However, a combination of different distance metric can help to construct more realistic models (Melachrinoudis, 1999).

Ohsawa and Tamura (2003) deal with a location model for the placement of a semi-obnoxious facility in a continuous plane. In this paper firstly, elliptic maximin and rectilinear minisum biobjective model is formulated. The authors propose a strong property which is about the curvature of the contours of the related push and pull objective: if the curvature of the contour of the push objective is greater than the curvature of the contour of the pull objective for any point in the feasible region, except the singular points, then the efficient set is the subset of the union of these singular points and the boundary of the feasible region. Singular points are defined as the points where the slope of the contour of the objective abruptly changes. They present a unifying technique for finding the efficient set and the trade-off curve associated with such a semi obnoxious facility. This technique is an applicable tool for a variety of bicriteria location models. For example, in this paper, this technique is also applied for an Euclidean maximin and rectilinear minisum location model as well as rectangular maximin and minimax location model and results were compared.

2.6 OTHER APPROACHES ON UNDESIRABLE FACILITY LOCATION

A different class of undesirable facility location problem is the minimal covering problem in which the undesirable effects of a facility are evident only within certain distance from it (circle of influence). This problem was introduced by Drezner and Wesolowsky (1994). They determine the rectangle in addition to the circle that contains the minimum total population. Berman et al. (1996) extended the problem to the network space. Plastria and Carrizosa (1999) solved a very interesting problem with two objectives by considering the radius of the circle as a continuous variable and second objective. They develop polynomial algorithms for producing all efficient discs. The trade-off information of efficient solutions can provide answers to interesting coverage questions, such as finding the facility location that minimizes the population covered within a given radius (previously defined minimal covering problem) or finding the largest circle not covering more than a given total population. They considered a feasible region of any shape in the plane and the results can be extended to a planar network.

Ohsawa et al. (2006) consider a biobjective model to locate a semi-obnoxious facility within a convex polygon where a given number of closest points and farthest points may be neglected in the analyses. It considers simultaneously the resulting push and pull partial covering criteria with Euclidean distances. They developed low complexity algorithms to obtain all efficient solutions and the trade-offs involved by the use of higher-order Voronoi diagrams. Abravaya and Segal (2009) developed low complexity polynomial time algorithms for optimal consumer push-pull partial covering in the Euclidean and rectilinear plane where the bounded region is either a constant size polygon or a rectangle, respectively.

Locating an undesirable facility with expropriation is a more recent approach in this area. The basic idea is that in certain cases there is no point in the decision space that is far enough for all customers to site the undesirable facility. One possibility to resolve this issue is by resettling some of the customers at a given price. Berman et al. (2003) introduced two models for the location problem with expropriation. In the first model, a location on a network is required that maximizes the minimum distance (maximin) from the facility to the non expropriated customer points, subject to a given expropriation budget. In the second model, the expropriation cost is minimized while ensuring that the facility is placed at least certain distance away from all non-expropriated customer points. Berman and Wang (2007) add a second objective to the previous model, the minimization of transportation cost. The two cost objectives are added into one, so the resulting problem is treated as a single objective problem. For a planar feasible region and rectilinear metric they identify a Finite Dominating Set (FDS) that contains the optimal solution.

2.7 PURE LOCATION PROBLEMS

The (discrete demand) Multi Facility Location Problem (MFLOC) is a problem of optimally locating n new service facilities to serve the demand at m existing customer locations or facilities. The general *lp* distance model for this problem can be stated as follows:

Minimize $\sum_{i=l}^{n}\sum_{j=l}^{m} w_{ij}\left(X_i, P_j\right)+\sum_{i=l}^{n-l}\sum_{k=i+l}^{n} v_{ik} l_p\left(X_i, X_k\right)$ where, W_{ij} is a non-negative parameter that represents the annual cost of separating new facility i and existing facility j by a unit distance, v_{ik} is a non-negative parameter that represents the annual cost of separating new facilities i and k by a unit distance by a unit distance,

$x=\left(x_i, y_i\right)$ are the location coordinates of new facility i and $P_j=\left(a_j, b_j\right)$ are the location coordinates of existing facility j.

When $n=1$, the MFLOC becomes a Single Facility Location Problem (SFLOC) (Love and Dowling, 1989). Selecting different distance measures, by substituting different values of p in MFLOC and SFLOC, we obtain different variants of SFLOC and MFLOC.

Furthermore, we can introduce additional variants by restricting new facility locations to specific sites, and by letting demands be either stochastic or distributed over areas. These types of problems are discussed below.

2.8 RECTILINEAR DISTANCE LOCATION PROBLEMS

The rectilinear distance location problem is a variant of MFLOC in which $p=1$. Among the reasons for considering the rectilinear distance measure is that for a grid of city streets or in a network of aisles in a factory or a warehouse, it is the best applicable distance measure (Francis, 1963). When the norm is used in Model (2.1), the objective function separates into x and y coordinate sub problems, and these can be posed and solved as specially structured linear programming problems.

The Single Facility Rectilinear Distance Location Problem (SFLOC) was first solved by Francis (1963), who characterized a simple median location optimal solution. Later, Francis (1964) solved a special case of the multi facility rectilinear distance location problem, where he dealt with equal weights. In 1970, Cabot *et al*. proposed a network flow solution procedure, while Pritsker and Ghare (1970) suggested a gradient technique.

However, Rao (1973) demonstrated that this latter approach was basically a primal simplex-based linear programming approach, and in the presence of degeneracy, the optimality conditions were not sufficient. Also, Wesolowsky and Love (1971 b) and Morris (1975) showed that the problem with linear locational constraints could be solved by linear programming. A thorough set of necessary and sufficient optimality conditions were finally developed by Juel and Love (1976).

Among other approaches for solving the rectilinear distance Problem MFLOC are the nonlinear approximation method developed by Wesolowsky and Love (1972), where any number of linear and (or) nonlinear constraints defining a convex feasible region can be included, and the hyperboloid approximation procedure for solving the perturbed rectilinear distance MFLOC that was proposed by Eyster et al. (1973). Another specialized simplex based-algorithm is derived by Sherali and Shetty (1977 a). Picard and Ratliff (1978) solved the problem via at most (m-1) minimum cut problems on derived networks containing at most $(n-2)$ vertices. Subsequently, Kolen (1981) exhibited the equivalence of the method of Sherali and Shetty and Picard and Ratliff and that the main difference between these two procedures was principally in the computational implementation. Moreover, this type of approach is known to be the most effective way of solving the rectilinear distance Problem MFLOC. A modified version of the method of Picard and Ratliff (1978) was proposed by

Cheung (1980). Dax (1986 a) gave a new method that, as he stated, handles efficiently the rectilinear distance Problem MFLOC having large clusters, i.e. where several new facilities are located together at one point.

Guccione and Gillen (1991) provided an economic interpretation of a dual problem concerned with maximizing the revenue when using rectilinear distances.

2.9 EUCLIDEAN DISTANCE PROBLEMS

When $p = 2$ in Model (2.1), the resulting problem is called the Euclidean distance multi-facility location problem (EMFL). This problem becomes the Euclidean single facility location problem (ESFL) if $n = 1$. According to Rosen and Xue (1993), there is always an optimal solution for Problem ESFL when the existing facilities are collinear in which the new facility location coincides with one of the existing facilities. Therefore, the literature typically assumes that the existing facilities are non-collinear. For this kind of problem, Weiszfeld (1937) was the first to propose a fixed-point iterative scheme that has come to be known as the Weiszfeld procedure. Also, Cooper (1963) and Kuhn and Kuenne (1962) solved the same problem using this concept. Later, Cooper and Katz (1981) proposed an optimal gradient method with inexact quadratic fit based line-search for Problem ESFL, and showed its superiority to the Weiszfeld algorithm in most cases.

However, Rado (1988) pointed out that when the Weiszfeld procedure is used instead of a gradient-based method, global convergence is achieved and the computation of step-sizes is circumvented. The objective function of the Problem Euclidean Distance Multi-Facility Location Problem ESFL (the Euclidean single facility location) problem is convex and non differentiable. The Non differentiability occurs only at a finite number of points where the location of the new facility matches one of the existing facility locations (Rosen and Xue, 1993). Therefore, the convergence of the Weiszfeld procedure is expected to be very slow when the minimizer coincides with one of the existing facilities (Wang *et al*. 1975). To avoid this behavior, Ostresh (1978) and Balas and Yu (1982) developed a version of the Weiszfeld procedure that contains a step to perturb the solution if it coincides with one of the existing facilities in order to ensure the convergence to an optimum. With this step, the Problem Euclidean Distance Multi-Facility Location Problem (ESFL) can be viewed as a smooth problem (Rosen and Xue, 1993). Eckardt (1980) studied the problem in general spaces. In 1987 and 1989, Xue developed a second-order method to solve the constrained single facility location problem and proved that for any initial point, the algorithm either stops at an optimal solution or generates a sequence that quadratically converges to the optimal solution. Drezner (1985) conducted some sensitivity analyses for the Problem Euclidean Distance Multi-Facility Location Problem (ESFL) and studied the cases where the existing facilities are restricted to small areas and the weights are restricted to given ranges. There is a large amount of literature on the Euclidean Distance Multi-Facility Location Problem (ESFL) problem. For further details on this class of problems, see the books of Francis *et al*.(1991) and of Love *et al*.(1988).

For the multi-facility Problem Euclidean Distance Multi-Facility Location Problem (EMFLP), Francis and Cabot (1972) have proven that the objective function is convex, and it is strictly convex if for $i = 1,...,n$ the set $S_i = \{ j : w_{ij} > 0\}$ is non-empty and the points of *Si* are non-collinear. Furthermore, they have shown that the optimal solution of problem Euclidean Distance Multi-Facility Location Problem (EMFLP) exists and lies in the convex hull of the existing facilities. Several researchers such as Hansen *et al*. (1980), and Juel and Love (1983) have also discussed the existence of the optimal solution in the convex hull of the existing facilities. Miehle (1958) was the first to propose an extension of the Weiszfeld algorithm.

Ostresh (1977) proved that Miehle's algorithm is a descent scheme. Rado (1988) slightly changed Miehle's algorithm and proved that this algorithm always converges to the minimizer of the Problem EMFL for certain well-structured cases in which the objective function is strictly convex. However, Rosen and Xue (1992) constructed a counter example showing that Miehle's algorithm may converge to a non optimal point even for such well-structured problems.

The multi facility problem basically suffers from the non differentiability of the objective function. These points of non differentiability of the objective function occur not only when new and existing facility locations coincide, but also occur on linear subspaces where the new facilities themselves coincide. In addition, since the objective function is not strictly convex, multiple minimizers of the problem are resulted to which iterative schemes might tend to converge.

To overcome the difficulty of having non differentiable objective function in Problem Euclidean Distance Multi-Facility Location Problem (EMFL), Eyster *et al.* (1973) used an extension of the Weiszfeld algorithm. In this procedure, they approximated the objective function by a hyperboloid, which is a smooth function, and derived the associated external equations. This procedure is labeled the Hyperboloid Approximation Procedure (HAP) and is probably the most common procedure for solving the multi facility location problem, using Euclidean distances or even rectilinear distances. In 1977, Ostresh proved that HAP is a descent algorithm under certain conditions. In 1985, Charalambous developed a method to accelerate the rate of convergence of HAP. However, it was only recently that Xue (1991) and Rosen and Xue (1993) proved the global convergence of HAP when applied to the Problems Euclidean Distance Multi-Facility Location Problem (EMFL).

Unfortunately, it is well known that the HAP approach suffers from ill-conditioning effects if the point of convergence is non differentiable (Charalambous, 1985). As a result, other methods that directly focus on this issue of non differentiability have been developed. For example, Calamai and Charalambous (1980) have proposed a pseudo-gradient technique that classifies the new facilities into distinct categories based on their coincidence with other facilities in order to derive a descent method for solving Euclidean Distance Multi-Facility Location Problem (EMFLP). However, Juel (1982) showed that this algorithm could terminate at a suboptimal solution. Chatelon *et al.* (1978) have also approached Euclidean Distance Multi-Facility Location Problem (EMFLP) by using a general - sub gradient method in which search directions are generated based on the sub differential of the objective function over a neighborhood of the current iterate. Sequential unconstrained minimization techniques used by Love (1969) and the Weiszfeld fixed-point iterative method as utilized by Rado (1988), are also among other efforts to solve (EMFLP).

Several second-order methods have also been designed to solve the Problem Euclidean Distance Multi-Facility Location Problem (EMFLP). Calamai and Conn (1980) were the first to propose a projected gradient-based algorithm. Various quadratic convergence approaches have also been developed by Calamai and Conn (1982, 1987) and Overton (1983), in which specialized line-searches are used in conjunction with projected second-order techniques. Rosen and Xue (1992 b) developed an algorithm which, from any initial point, generates a sequence of points that converges to the closed convex set of optimal solutions to the Problem Euclidean Distance Multi-Facility Location Problem (EMFLP).

Since in some multi facility location problems, the optimal solution coincides with one of the existing facilities, researchers have derived necessary and sufficient conditions for optimality to avoid the non differentiability difficulty of the objective function associated with such a coincidence. For a single facility problem with three existing facilities, Juel and Love (1986) proved that it is possible to determine which existing facility is the optimal location by means of a simple geometrical construction. For the multi facility location problem with no constraints on the location of the new facilities, Juel and Love (1980) derived some sufficient conditions for the coincidence of facilities that are valid in a general symmetric metric. These results were later extended by Lefebvre *et al*. (1991) to be applicable to some location problems having certain locational constraints. Examples of other works on this subject are Francis and Cabot (1972), Calamai and Charalambous (1980), Calamai and Conn (1980) and (1982), Dax (1986 b), Overton (1983), Lefebvre *et al*. (1990), and Plastria (1992). Recently, Mazzerella and Pesamosco (1996) have used the optimality conditions of Euclidean Distance Multi-Facility Location Problem (EMFLP) as a tool for obtaining both stopping rules for some computational algorithms such as the projected Newton procedure of Calamai and Conn (1987), and the analytical solution of many simple problems.

Many other contributions for solving this problem have appeared in the literature. Love (1969) applied convex programming to a problem in three dimensions. Recently, Carrizosa *et al*. (1993) derived the geometrical characterizations for the set of efficient, weakly efficient and properly efficient solutions of the Problem Euclidean Distance Multi-Facility Location Problem (EMFLP) when it includes certain convex locational constraints. In addition, Love and Yoeng (1981) explored the bounding method that continuously updates a lower bound on the optimal objective function value during each iteration. This method is based on the idea that the convex hull and the current value of the gradient determine an upper bound on the objective function's improvement. Among other works that have been proposed in deriving such bounds are those due to Elzinga and Hearn (1983), Juel (1984), and Love and Dowling

(1989). Also, Wendell and Peterson (1984) have derived a lower bound from the dual to Euclidean Distance Multi-Facility Location Problem (EMFLP).

Many papers have used the dual as an approach for solving this problem, starting with Witzgall (1964) and Kuhn (1967) who independently addressed the dual problem. Love (1974) developed the dual problem corresponding to a hyperbolic approximation of the constrained multi facility location problem with l_p distances. White (1976) gave a Varignon frame interpretation of the dual problem. Using Sinha's (1966) duality results involving general quadratic forms, Francis and Cabot (1972) derived a differentiable, convex quadratically constrained dual optimization problem, and achieved several useful relationships between the dual and Euclidean Distance Multi-Facility Location Problem (EMFLP). However, they considered the actual use of this dual problem to solve EMFLP as an open problem. More recently, Xue *et al*. (1996) have suggested the use of polynomial-time interior point algorithms to solve this dual problem. Based on this idea, they presented a procedure in which an approximate optimum to Euclidean Distance Multi-Facility Location Problem (EMFLP) can be recovered by solving a sequence of linear equations, each associated with an iterate of the interior point algorithm used to solve the dual problem.

Also, other papers dealing with duality of various constrained versions of this problem have appeared, such as the paper of Idrissi *et al*. (1989) where a primal-dual method to solve the constrained multi-facility location problem with mixed norms was presented, and the paper of Love and Kraemer (1973) where a dual decomposition method for solving the constrained Euclidean Distance Multi-Facility Location Problem (EMFLP) was given.

Extensions of results and algorithms for the Euclidean distance problem to the general l_p distance problem have also been studied. In 1993, Brimberg and Love utilized the Weiszfeld algorithm to solve the single facility problem with l_p distance measures. Other examples of such extensions are Drezner and Wesolowsky (1978), Morris and Verdini (1979), and Juel and Love (1981). For the constrained multifacility location problem with l_p distance, Love (1974) developed the dual problem corresponding to a hyperbolic approximation of the objective function.

2.10 SQUARED-EUCLIDEAN DISTANCE PROBLEMS

If we define the facility-customer separation penalty to be proportional to the square of the Euclidean distance, then the resulting problem is called a squared-Euclidean distance problem. This problem is separable in the x and y variables. Eyster and White (1973) cite some special applications for this class of problems. In this problem, it is obvious that the function that is to be minimized is strictly convex, and unlike the Euclidean distance case, it has continuous first partial derivatives with respect to x and y. Consequently, the optimal solution of both the single and the multi facility problems is unique and can be obtained by simple calculus techniques. However, for the multi facility case, one needs to solve two systems of n linear equations in n variables (Francis *et al*., 1992).

Consequently, the solution of the squared-Euclidean distance problem has been used to obtain a good starting solution for the corresponding EMFLP (Francis *et al*., 1992).

2.11 VARIANTS AND EXTENSIONS

The literature on the pure location problem is quite extensive and several variants and extensions have been proposed. One type of variant enforces a separation of facilities by adding a set of metric side-constraints to the problem, as in Schaefer and Hurter (1974). In another variant, the components of the location problem are considered to be stochastic, as in the papers by Cooper (1974) and Katz and Cooper (1974, 1976), where the coordinates of the destination locations are specified by probability distributions. A version of this problem in which the transportation costs are random variables was considered by Seppala (1975) and Drezner and Wesolowsky (1981). For the multi facility case, Aly and White (1978) solved the Euclidean distance problem where the existing facilities and the interactions between the facilities are random variables. Rao and Varma (1985) also studied this same problem, while Wesolowsky (1971 b) considered randomness in a single facility location problem using rectilinear distances instead of Euclidean distances.

Models where the destinations are permitted to be uniformly distributed over regions have also been considered in the literature. Wesolowsky and Love (1971a) solved such a rectilinear distance multi facility location problem by developing a gradient search algorithm. For the single facility case of the former problem, Love (1972) used a nonlinear optimization technique. Bennet and Mirakhor (1974) solved an approximation of the same problem by replacing the areas with their center of gravity. The extension of this problem to a similar problem, but with general $l_p$distances and general demand area shapes, was considered by Drenzner and Wesolowsky (1980). In their paper, they explored a two-stage iterative procedure, based on the Weiszfeld procedure, to solve the single facility problem. Drezner (1986) considers the problem EMFLP and the squared-Euclidean distance location problem when both new and existing facilities have circular shapes and demand and service has a uniform probability density inside each shapes.

Cavalier and Sherali (1986a) also consider Euclidean problems having uniform demand distributions over convex polygons.

Another type of variant includes models that require the relocation of the sources over a Multi period horizon due to the changes in demand weights or locations of the destinations. Among the contributions to this variant is the dynamic approach for solving the rectilinear distance single facility location problem by Wesolowsky (1973), and for solving the multi-facility case by Wesolowsky and Truscott (1976), Scott (1971), and Cavalier and Sherali (1985). Several papers such as Katz and Cooper (1981), Aneja and Parlar (1991) and Butt and Cavalier (1996) have considered a non convex variant of the problem ESFL when there are forbidden regions present in the plane.

A new variant of the location problem that removes the restriction of having non-negative demand weights has also been treated more recently by Tellier and Polanski (1989) and Drezner and Wesolowsky (1990). In addition, Tuy *et al*.(1995) developed an algorithm to solve the single facility problem that is based on a representation of the objective function as a difference of two convex functions.

In another variant, different spaces where the location can take place, other than Euclidean plane, have also been considered by some researchers. Love (1969) has extended the location problem to three dimensions. Other researchers have considered the spherical distance in treating large region location problems because the surface of the earth is geodesic, rather than Euclidean. Aly *et al*. (1979), Dhar and Rao (1982), Plastria (1987), and Aykin and Babu (1987) are among those who have studied this problem.

There are still further variants of the location problem, such as those which involve different types of distance norms, as in Ward and Wendell (1980) and Wu (1994), others that require the transportation costs to be increasing and continuous functions of distance as in Hansen *et al*. (1985). Klincewicz *et al*.(1986) proposed optimal and heuristic algorithms for a variant of location problems in which customers require several different products. Abdelmalek (1985) developed a method to optimally position a single facility that provides a service to a set of moving facilities over a time horizon. Sherali and Kim (1992) considered a more general problem of determining the optimal paths of a service facility that moves through a region containing some existing facilities, while Kim *et al*. (1992) studied the same problem but where the existing facilities are distributed over a network. For a review of the formulation and solutions of several classes of location problems see Francis *et al*. (1983).

2.12 LOCATION AND LOCATION-ALLOCATION PROBLEMS ON NETWORKS

Consider a network *G*(*N*, *A*) having *n* nodes *vi*, *i*=1,..., *n*, each with a demand of *hi i* □*N*, and having links a_j, *j A*, each containing a uniform spread of demand of total weight w_j, *j* □1,..., *n*. Let the locations, which are to be determined on *G*, of some *p* facilities be represented by *X* □{*x*1, *x*2,..., , x_p)}, and let *d*(*v*, *X*) min {*d*(*v*, *x*1), *d*(*v*, *x*2),..., *d*(*v*, x_p)} represent the shortest path distance on *G* from any point *v* in *G* to a facility location.

2.13 *P*-MEDIAN PROBLEMS

The *p*-median Problem (*p*-M) is to locate *p* new facilities, called medians, on the network *G* in order to minimize the sum of the weighted distances from each node to its nearest new facility (Francis *et al*., 1992). This problem can be mathematically stated as follows:

Minimize $f(X) = \sum_{i=l}^{n} h_i d(v_i, X)$ X on G

If $p = 2$, then this problem can be viewed as a location-allocation problem (LAP). This is because the location of the new facilities will determine the allocation of their service in order to best satisfy the nodal demands. Hakimi (1964) proved that in networks, a set of optimal locations will always coincide with the vertices. He also proposed an enumerative-graph theoretic approach for the problem. Revelle and Swain (1970) proposed other procedures to solve this problem after reformulating it as an integer programming (IP) problem. Jarvinen *et al*. (1972) also used this IP formulation and proposed a branch-and-bound algorithm for this problem. Due to the NP-hardness of the problem, several heuristic procedures have been developed, such as those of Maranzana (1964) and Teitz and Bart (1968). Beasly (1993) has also developed Lagrangian heuristics for this *p*-median location problem, based on Lagrangian relaxation and sub gradient optimization concepts.

Several variants and extensions of the *p*-median problem have been addressed in the literature. One type of variant, studied by Pesamosca (1991), considers the interaction weights between the new facilities as well as the connection scheme as a tree. This case was treated as a problem EMFLP on a tree and its optimality conditions were then obtained using the optimality conditions of *p* problems of the type ESFL. Accordingly, for solving the problem EMFLP, a fixed point algorithm was developed to iteratively solve problem ESFL using the Weiszfeld algorithm if differentiability is met, and otherwise, the algorithm switches over to Miehle's algorithm. Another type of variant involves placing the capacity restrictions on the facilities to be located. When the capacity is finite, the resulting problem is called a capacitated problem; otherwise the problem is incapacitated.

Cavalier and Sherali (1986b) presented exact algorithms to solve the *p*-median problem on a chain graph and the 2-median problem on a tree graph, where the demand density functions are assumed to be piecewise uniform. For the incapacitated *p*-median problem, Chiu (1987) addressed the 1- median problem on a general network as well as on a tree network. Dynamic location considerations on networks are addressed by Sherali (1991).

Recently, Francis *et al*. (1993) developed a median-row-column aggregation algorithm to solve large-scale rectilinear distance *p*-median problems. On the other hand, Sherali and Nordai (1988) gave certain localization results and algorithms for solving the capacitated *p*-median problem on a chain graph and the 2- median problem on a tree graph. Another variant involves the treatment of a continuous demand over the network, which arises in some situations such as the location of public service facilities or in probabilistic distributions of demand. Among the contributions on this .variant are Minieka (1978), Handler and Mirchandani (1979), Chiu (1987) and Derardo *et al*. (1982). Combining the two last variants, Sherali and Rizzo (1991) solved an unbalanced, capacitated *p-median* problem on a chain graph with a continuum of link demands. For solving this problem, they considered two unbalanced cases, the deficit and over-capacitated cases, provided a first-order characterization of optimality for these two problems and developed an enumerative algorithm based on a partitioning of the dual space. There are still further variants that include capacity restrictions on links, probabilistic travel times on links, and maximum distance constraints. For surveys of research done on location problems on networks, see Handler and Mirchandani (1979), Kariv and Hakimi (1979 b), and Tansel *et al*. (1983).

CHAPTER THREE

METHODOLOGY

3.0 INTRODUCTION

3.1 Background

Bernard Roy conceived the ELECTRE method in 1966 in response to the deficiencies of the existing decision-making methods. This method has evolved through a number of versions (I through IV); all are based on the same principles but are operationally somewhat different.

The ELECTRE method may not always be the best decision aid; however, it is a proven approach. It has several unique features not found in other methods, including the concepts of outranking, as well as indifference and preference thresholds. Additionally, the ELECTRE method can especially be used to convert qualitative data to quantitative.

With its dynamic characteristics, this method may be applied successfully to many problems. However, to obtain reliable results from this method, a decision maker must define the problem broadly, identify the main constraints, and most importantly identify the primary objective. Next, a decision maker should establish an interdisciplinary committee to address the problem, and the members of this committee should be experts in various fields related to the problem. This committee must have experience and the ability to handle the problem with an interdisciplinary approach. They must be unbiased. Likewise, the decision maker must eschew personal bias that could deviate the interdisciplinary committee from the primary objective.

Using the ELECTRE method,

- Identify the alternative locations,
- Identify the important criteria of the problem,

- Assess each alternative location according to each criterion
- Employ the ELECTRE method to solve the problem.

The simplest method of the ELECTRE family is ELECTRE I.

The ELECTRE methodology is based on the concordance and discordance indices defined as follows. We start from the data of the decision matrix, and assume here that the sum of the weights of all criteria equals to 1. For an ordered pair of alternatives (A_j, A_k) the concordance index c_{jk} is the sum of all the weights for those criteria where the performance score of A_j is least as high as that of A_k, i.e.

$$c_{jk} = \sum_{i:a_{ij} \geq a_{ik}} w_i ,$$

Clearly, the concordance index lies between 0 and 1.

The computation of the discordance index d_{jk} is a bit more complicated: $d_{jk} = 0$, if $a_{ij} > a_{ik}$ i=1,...,m, i.e. the discordance index is zero if Aj performs better than Ak on all criteria,. Otherwise, i.e. for each criterion where Ak outperforms Aj, the ratio is calculated between the difference in performance level between Ak and Aj and the maximum difference in score on the criterion concerned between any pair of alternatives. The maximum of these ratios (which must lie between 0 and 1) is the discordance index.

A concordance threshold c* and discordance threshold d* are then defined such that 0<d*<c*<1.

Then, Aj outranks Ak if the cjk>c* and djk<d*, i.e. the concordance index is above and the discordance index is below its threshold, respectively.

This outranking defines a partial ranking on the set of alternatives. Consider the set of all alternatives that outrank at least one other alternative and are themselves not outranked. This set contains the promising alternatives for this decision problem. Interactively changing the level thresholds, we also can change the size of this set.

The ELECTRE I method is used to construct a partial ranking and choose a set of promising alternatives. ELECTRE II is used for ranking the alternatives. In ELECTRE III an outranking degree is established, representing an outranking creditability between two alternatives which makes this method more sophisticated (and, of course, more complicated and difficult to interpret), Figueira et al (2004).

ELECTRE evaluation method allows qualitative and quantitative criteria to be handled simultaneously. The existing discordance index evaluation benchmark always uses the absolute value of the maximum differentiated performance as the evaluation standard. This study proposes another benchmark, namely the absolute value of the sum of differentiated performance as the evaluation standard. In fact, those two evaluation standards represent different decision-making approaches, based on whether a decision maker focuses on discrepancies in the most important criteria or in the overall criteria, respectively. In general, using of ELECTRE evaluation methods, without the differentiation process, may produce a result that is opposite that targeted by the decision maker. The following explains this concept.

To rank a set of alternatives, the ELECTRE method as outranking relation theory was used to analyze the data regarding a decision matrix. The concordance and discordance indexes can be viewed as measurements of dissatisfaction that a decision maker uses in choosing one alternative over the other.

We assume m alternatives and n decision criteria. Each alternative is evaluated with respect to n criteria. As result, all the values assigned to the alternatives with respect to each criterion form a decision matrix.

Let $W = (w_1, w_2, ..., w_n)$ be the relative weight vector of the criteria, satisfying

$$\sum_{j=1}^{n} w_j = 1$$

Then, the ELECTRE method can be summarized as follows:

Normalization of decision matrix $X = \left(x_{ij}\right)_{mxn}$ is carried out by calculating $r_{\hat{y}}$ which represents the normalization of criteria value.

Let $i = 1, 2, \ldots m \text{ and } j = 1, 2, \ldots, n.$

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}$$

The weighted normalization of decision matrix is calculated with the following formula:

$$V = \left(v_{ij}\right)_{mxn} v_{ij} = r_{ij}.w_{ij}, \sum_{j=1}^{n} w_j = 1$$

After calculating weight normalization of the decision matrix, concordance and discordance sets are applied. The set of criteria is divided into two different subsets. Let $A = [a_1, a_2, a_3]$ denote a finite set of alternatives. In the following formulation, we divide data into two different sets of concordance and discordance. If the alternative Aa_1 is preferred over alternative Aa_2 for all the criteria, then the concordance set is composed.

The concordance set is composed as follows: $C\left(a_1, a_2\right) = \left\{ j \mid v_{a1j} > v_{a2j} \right\}$,

$\left(a_1, a_2 = 1, 2, \ldots, m, \text{ and } a_1 \neq a_2\right)$

$C\left(a_1, a_2\right)$ is the collection of attributes where Aa_1 is better than, or equal, to Aa_2.

On completing of $C_{a_1 a_2}$, apply the following discordance set: $D\left(a_1, a_2\right) = \left\{ j \mid v_{a_2 j} < v_{a_1 j} \right\}$

The concordance index of $\left(a_1, a_2\right)$ is defined as follows $C_{a_1 a_2} = \sum_{j*} w_{j*}$,

$j*$ are the attributed contained in the concordance set $C\left(a_1, a_2\right)$. The discordance index $D\left(a_1, a_2\right)$ represents the degree of disagreement in $A_{a_1} \rightarrow A_{a_2}$ in the following way:

$$D_{a_1a_2} = \frac{\sum_{j*}\left|v_{a_1j*} - v_{a_2bj*}\right|}{\sum_j\left|v_{a_1j} - v_{b_2j}\right|}$$

The decision maker's opinion and the LT's

LT's	Scale
Extremely Good(EG)	9
Very Good(VG)	7
Good(G)	5
Medium Bad(MB)	3
Bad(B)	2
Very Bad(VB)	1

$j*$ are the attributes contained in the discordance set $D(a_1, a_2)$ and v_{ij} is the weighted normalization criterion of the alternative i on criterion j.

This method implies that Aa_1 outranks Aa_2 when $C_{a_1a_2} \geq \bar{C}$ and $D_{a_1a_2} \leq \bar{D}$.

$\bar{C}$: The averages of $C(a_1, a_2)$

$\bar{D}$: The averages of $D(a_1, a_2)$

3. The Fuzzy ELECTRE Method

ELECTRE I is one of the earliest multicriteria evaluation methods, developed among other outranking methods. The major purpose of this method is to select a desirable alternative that meets both the demands of concordance preference above many evaluation benchmarks, and of discordance preference under any optional benchmark. The ELECTRE I generally includes three concepts, namely, the concordance index, discordance index, and the threshold value.

In this study, our model fuzzy ELECTRE along with the opinion of decision makers will be applied by a group decision makers.

The procedure for fuzzy ELECTRE ranking model has been given as follows:

Step 1 (determination of the weights of the decision makers). Assume that the decision group contains l decision maker's criteria and gives them designated scores. The importance of the decision makers is, then, considered in linguistic terms (LT). We construct the aggregated decision matrix (ADM) based on the opinions of the decision-makers, and the LT as shown in Table 1.

Step 2 (calculation of Triangular Fuzzy Numbers (TFNs)). We set up the TFN's. Each expert makes a pair wise comparison of the decision criteria and gives them relative scores. The aggregated fuzzy importance weight (AFIW) for each criterion can be described as TFN's $\tilde{w}_j = \left(I_j, m_j, u_j\right)$ for $K = 1, 2, \ldots, k$ and $j = 1, 2, \ldots, n$.

This scale has been employed in the TFN's as proposed by Mikhailov (2003) and shown in Table 2.

Now, the TFN's are set up based on the FN's and assigned relative scores:

$$l_j = \left(l_{j1} \otimes l_{j2} \otimes \ldots \otimes l_{jk}\right)^{1/k},\ j = 1, 2, \ldots k,$$

$$m_j = \left(m_{j1} \otimes m_{j2} \otimes \ldots \otimes m_{jk}\right)^{1/k},\ j = 1, 2, \ldots k,$$

$$u_j = \left(u_{j1} \otimes u_{j2} \otimes \ldots \otimes u_{jk}\right)^{1/k},\ j = 1, 2, \ldots k,$$

Importance Intensity	Triangular Fuzzy Scale
1	(1,1,1)
2	(1.6,2.0,2.4)
3	(2.4,3.0,3.6)
5	(4.0,5.0,6.0)
7	(5.6,7.0,8.4)
9	(7.2,9.0,10.8)

Then the AFIW for each criterion is normalized as follows:

$\tilde{w}_j = \left(w_{j1}, w_{j2}, w_{j3}\right)$ where

$$\tilde{G}_T = \left(\sum_{j=1}^{k} l_j, \sum_{j=1}^{k} m_j, \sum_{j=1}^{k} u_j \right)$$

The fuzzy geometric mean of the fuzzy priority value is calculated with normalization priorities for factors using the following:

$$\tilde{w}_i = \frac{\tilde{G}}{\tilde{G}_T} = \frac{\left(l_j, m_j, u_j\right)}{\left(\sum_{j=1}^{k} l_j, \sum_{j=1}^{k} m_j, \sum_{j=1}^{k} u_j \right)} = \left(\frac{l_j}{\sum_{j=1}^{k} u_j}, \frac{m_j}{\sum_{j=1}^{k} m_j}, \frac{u_j}{\sum_{j=1}^{k} l_j} \right)$$

At a later stage, the normalized AFIW matrix is constructed as follows: $W = \left[\tilde{w}_1, \tilde{w}_2, \ldots \tilde{w}_n\right]$

Step 3(Calculation of the decision matrix)

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{21} \\ \cdots & \cdots & \cdots & \cdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix}$$

Step 4. Calculation of the normalized decision matrix and the weighted normalized decision matrix.

The normalized decision matrix is calculated in the following way,

$$r_{ij} = \frac{1/x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \text{ for minimization}$$

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \text{ for maximization} \quad i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n,$$

$$r_{ij}=\begin{bmatrix} r_{11} & r_{12} & \dots & r_{1n} \\ r_{21} & r_{22} & \dots & r_{21} \\ \dots & \dots & \dots & \dots \\ r_{m1} & r_{m2} & \dots & r_{mn} \end{bmatrix}$$

Thus, the weighted normalized decision matrix based on the normalized matrix is constructed as follows: $\tilde{V}=\left[\tilde{v}_{ij}\right]_{mxn}$, where $\tilde{v}_{ij}$:normalized positive triangular FN's.

Step 5 (calculation of concordance and discordance indexes). These indexes are measured for different weights of each criterion $\left(w_{j1},w_{j2},w_{j3}\right)$. The concordance index $C_{a_1a_2}$ represents the degree of confidence in pairwise judgments $\left(A_{a_1}\rightarrow A_{a_2}\right)$ accordingly, the concordance index to satisfy the measured problem can be written with the following formula:

$C^1_{a_1a_2}=\sum_{j*}w_{j1}$ $\quad C^2_{a_1a_2}=\sum_{j*}w_{j2}$, $C^3_{a_1a_2}=\sum_{j*}w_{j3}$ where $j*$ the attributes are contributes contained in the concordance set $C\left(a_1,a_2\right)$.

On the other hand, the preference of the dissatisfaction can be measured by discordance index. $D\left(a_1,a_2\right)$, which represents the degree of disagreement in $\left(A_{a_1}\rightarrow A_{a_2}\right)$ as follows:

$$D^1_{a_1a_2}=\frac{\sum_{j+}\left|v^1_{a_{1j+}}-v^1_{a_{2j+}}\right|}{\sum_j\left|v^1_{a_{1j}}-v^1_{a_{2j}}\right|},\quad D^2_{a_1a_2}=\frac{\sum_{j+}\left|v^2_{a_{1j+}}-v^2_{a_{2j+}}\right|}{\sum_j\left|v^2_{a_{1j}}-v^2_{a_{2j}}\right|},\quad D^3_{a_1a_2}=\frac{\sum_{j+}\left|v^3_{a_{1j+}}-v^3_{a_{2j+}}\right|}{\sum_j\left|v^3_{a_{1j}}-v^3_{a_{2j}}\right|}$$

j^+ are the attributes contained in the discordance set $D\left(a_1,a_2\right)$, and v_{ij} is the weighted normalized evaluation of the alternative i on the criterion j.

DMU	Criteria	DM1	DM2	DM3	DM4
P1	C1	VG	VG	G	G
	C2	G	VG	MB	MB
	C3	VG	G	B	VG
	C4	VG	VG	G	G
P2	C1	G	VG	MB	B
	C2	VG	VG	G	MB
	C3	VG	VG	B	B
	C4	VG	VG	MB	G
P3	C1	VG	VG	G	VG
	C2	VG	G	G	VG
	C3	VG	G	VG	G
	C4	VG	VG	VG	VG

Step 6 (calculating the concordance and discordance indexes). This final step deals with determining in the concordance and discordance indexes in other words, the defuzzification process using the following formula:

$$C^*_{a_1a_2} = \sqrt[z]{\prod_{z=1}^{Z} C^z_{a_1a_2}} \qquad D^*_{a_1a_2} = \sqrt[z]{\prod_{z=1}^{Z} D^z_{a_1a_2}} \quad \text{where } Z = 3$$

The dominance of the A_{a_1} over the A_{a_2} becomes stronger with a larger final concordance index $C_{a_1a_2}$ and a smaller final discordance index $D_{a_1a_2}$.

Consequently, the best alternative is yielded, where $C(a_1,a_2) \geq \bar{C}$, $D(a_1,a_2) \geq \bar{D}$

$\bar{C}$: The averages of $C_{a_1a_2}$

$\bar{D}$: The averages of $D_{a_1a_2}$

This example involves six-criterion evaluation analysis. The normalization performance of three alternatives is shown as follows:

Table 3.1: Data Sheet of Simple Case Normalization Performance

	$c1$	$c2$	$c3$	$c4$	$c5$	$c6$
$a1$	2	2	2	2	2	4
$a2$	3	3	3	3	3	1
$a3$	3	7	5	1	5	6

It is hypothesized that all three alternatives exceed the concordance index threshold and the denominators of discordance index $d_j(h,k)$ are the same (it is assumed that the decision maker preference value in the denominator of individual criteria is 1; In other words, $\theta_j = 1$ is the value of the denominator in each criterion). The absolute value of the maximum differentiated performance and the sum of differentiated performance are used to calculate the discordance index. The results are as follows:

$$a12 = \max\left(|2-3|,|2-3|,|2-3|,|2-3|,|2-3|,|2-3|\right) = \max\left(1,1,1,1,1\right) = 1$$

$$a21 = \max\left(|1-4|\right) = \max\left(3\right) = 3$$

$$a13 = \max\left(|2-3|,|2-7|,|2-5|,|2-5|,|4-6|\right) = \max\left(1,5,3,3,2\right) = 5$$

$$a31 = \max\left(|1-2|\right) = \max\left(1\right) = 1$$

$$a23 = \max\left(|3-7|,|3-5|,|3-5|,|1-6|\right) = \max\left(4,2,2,5\right) = 5$$

$$a32 = \max\left(|1-3|\right) = \max\left(2\right) = 2$$

Whereas, a12 and a21 are used as examples to explain the relationship based on a1 and a2. The result shows that a12< a21. In the screening process for discordance indices, a1 is superior to a2 (according to the screening principle of ELECTRE evaluation method, an alternative with smaller discordance index is more likely to become preferred alternative). Similarly, other results are compared, and the following conclusion is reached:

Whereas a3> a1> a2

Use of the absolute value of the sum of differentiated performance:

$$a12 = \left(|2-3|+|2-3|+|2-3|+|2-3|+|2-3|+|2-3|\right) = \left(1+1+1+1+1\right) = 5$$

$$a21 = \left(|1-4|\right) = \left(3\right) = 3$$

$$a13 = \left(|2-3|+|2-7|+|2-5|+|4-6|\right) = \left(1+5+3+3+2\right) = 14$$

$$a31 = \left(|1-2|\right) = \left(1\right) = 1$$

$$a23 = \left(|3-7|+|3-5|+|3-5|+|1-6|\right) = \left(4+2+2+5\right) = 13$$

$$a32 = \left(|1-3|\right) = \left(2\right) = 2$$

Similarly, other results are compared, and the following conclusion is reached:

Whereas, a3> a2> a1

The above result shows that the ranking result obtained by using the absolute value of the maximum differentiated performance is (a3> a1> a2), However, using the absolute value of the sum of differentiated performance gives a ranking result of (a3> a2> a1). The rankings for alternatives a1 and a2 trade places in the two results. Alternative a3 appears to be the optimal alternative under both evaluation benchmarks, yet, the relative difference between a3 and a1, as well as a3 and a2, has changed significantly. Using a1 and a3 as an illustrative example, the relative discrepancy in discordance indices for the two alternatives has increased from 4(a13 - a31 = 5 - 1= 4) to 13 (a13 - a31 = 14 - 1= 13). As seen, the discrepancy between the two evaluation benchmarks is significant. The discrepancy becomes even more significant with an increase in the number of evaluation criteria. In fact, the two evaluation benchmarks represent different meanings. The use of the absolute value of the maximum differentiated performance indicates the attention of decision maker is focused on the greatest utility discrepancy of performance on criterion, while the use of the absolute value of the sum of differentiated performance indicates he or she is focused on the utility accumulative discrepancy of performance on criterion. This study aims to explain the difference through actual simulation and clarify the concept through application and analysis of ELECTRE II evaluation method. The following are provided as illustrative examples.

ELECTRE evaluation method allows qualitative and quantitative criteria to be handled simultaneously. The existing discordance index evaluation benchmark always uses the absolute value of the maximum differentiated performance as the evaluation standard. This study proposes another benchmark, namely the absolute value of the sum of differentiated performance as the evaluation standard. In fact, those two evaluation standards represent different decision-making approaches, based on whether a decision maker focuses on discrepancies in the most important criteria or in the overall criteria, respectively. In general, using of ELECTRE evaluation methods, without the differentiation process, may produce a result that is opposite that targeted by the decision maker. The following explains this concept.

This example involves six-criteria evaluation analysis. The normalization performance of three alternatives is shown as follows:

Table 3.2: Data Sheet of Simple Case Normalization Performance

	$c1$	$c2$	$c3$	$c4$	$c5$	$c6$
$a1$	2	2	2	2	2	4
$a2$	3	3	3	3	3	1
$a3$	3	7	5	1	5	6

It is hypothesized that all three alternatives exceed the concordance index threshold and the denominators of discordance index $d_j(h,k)$ are the same (it is assumed that the decision maker preference value in the denominator of individual criteria is 1; In other words, $\theta_j = 1$ is the value of the denominator in each criterion). The absolute value of the maximum differentiated performance and the sum of differentiated performance are used to calculate the discordance index. The results are as follows:

$$a12 = \max\left(|2-3|,|2-3|,|2-3|,|2-3|,|2-3|,|2-3|\right) = \max\left(1,1,1,1,1\right) = 1$$

$$a21 = \max\left(|1-4|\right) = \max\left(3\right) = 3$$

$$a13 = \max\left(|2-3|,|2-7|,|2-5|,|2-5|,|4-6|\right) = \max\left(1,5,3,3,2\right) = 5$$

$$a31 = \max\left(|1-2|\right) = \max\left(1\right) = 1$$

$$a23 = \max\left(|3-7|,|3-5|,|3-5|,|1-6|\right) = \max\left(4,2,2,5\right) = 5$$

$$a32 = \max\left(|1-3|\right) = \max\left(2\right) = 2$$

Whereas, a12 and a21 are used as examples to explain the relationship based on a1 and a2. The result shows that a12< a21. In the screening process for discordance indices, a1 is superior to a2 (according to the screening principle of ELECTRE evaluation method, an alternative with smaller discordance index is more likely to become preferred alternative). Similarly, other results are compared, and the following conclusion is reached:

Whereas a3> a1> a2

Use of the absolute value of the sum of differentiated performance:

$$a12 = \left(|2-3|+|2-3|+|2-3|+|2-3|+|2-3|+|2-3|\right) = \left(1+1+1+1+1\right) = 5$$

$$a21 = \left(|1-4|\right) = \left(3\right) = 3$$

$$a13 = \left(|2-3| + |2-7| + |2-5| + |4-6| \right) = \left(1+5+3+3+2 \right) = 14$$

$$a31 = \left(|1-2| \right) = \left(1 \right) = 1$$

$$a23 = \left(|3-7| + |3-5| + |3-5| + |1-6| \right) = \left(4+2+2+5 \right) = 13$$

$$a32 = \left(|1-3| \right) = \left(2 \right) = 2$$

Similarly, other results are compared, and the following conclusion is reached:

Whereas, a3> a2> a1

The above result shows that the ranking result obtained by using the absolute value of the maximum differentiated performance is (a3> a1> a2), However, using the absolute value of the sum of differentiated performance gives a ranking result of (a3> a2> a1). The rankings for alternatives a1 and a2 trade places in the two results. Alternative a3 appears to be the optimal alternative under both evaluation benchmarks, yet, the relative difference between a3 and a1, as well as a3 and a2, has changed significantly. Using a1 and a3 as an illustrative example, the relative discrepancy in discordance indices for the two alternatives has increased from 4(a13 - a31 = 5 - 1= 4) to 13 (a13 - a31 = 14 - 1= 13). As seen, the discrepancy between the two evaluation benchmarks is significant. The discrepancy becomes even more significant with an increase in the number of evaluation criteria. In fact, the two evaluation benchmarks represent different meanings. The use of the absolute value of the maximum differentiated performance indicates the attention of decision maker is focused on the greatest utility discrepancy of performance on criterion, while the use of the absolute value of the sum of differentiated performance indicates he or she is focused on the utility accumulative discrepancy of performance on criterion. This study aims to explain the difference through actual simulation and clarify the concept through application and analysis of ELECTRE II evaluation method.

3.2 THE FACTOR RATING METHOD

The factor rating method is a method used to find a suitable location for a facility considering a number of factors.

The factors include: labour cost (wages, unionization, and productivity), labour availability, proximity to raw materials and supplier, proximity to markets, state and local government fiscal policies, environmental regulations, utilities, site cost, transportation, and quality of life issues within the community, foreign exchange and quality of government. When using the factor rating method, the following six steps must be followed strictly.

These are:

- Develop a list of relevant factors.
- Assign a weight to each factor to reflect its relative importance in management's objective.
- Develop a scale for each factor (for example, 1 to 10 or 1 to 100)
- Have management or related people score each relevant factor, using the scale developed above.
- multiply the score by the weight assigned to each factor and total the score for each location.
- Make a recommendation based on the maximum point score; considering the result of quantitative approaches as well.

Table (3.3) below gives an example of the map coordinates and shipping loads for a set of cities that we wish to connect trough a central 'hub'.

Factor	**Factor No**	**Rating Weight**	**Ratio Of Rate**	**Location A**	**Location B**	**Location C**
1	Proximity to port facilities	5	0.25	25	20	20
2	Power source availability and cost	3	0.15	12	10.5	15
3	Work force attitude and cost	4	0.2	6	12	14
4	Distance from Tema	2	0.1	1	8	6
5	Community Desirability	2	0.1	9	6	8
6	Equipment supply in area	3	0.15	7.5	9	13.5
7	Economic activity	1	0.05	4.5	3	3
				65	**68.5**	**79.5**

CHAPTER FOUR

ANALYSIS OF DATA

4.0 INTRODUCTION

Depending on the number of alternative locations, finding the best location can take a tremendous amount of time. Processing huge amounts of data requires decision makers to employ some of the site selection models in the large-scale problems. There are a number of site selection models to assist in analyzing various site selection scenarios.

MAP OF GHANA

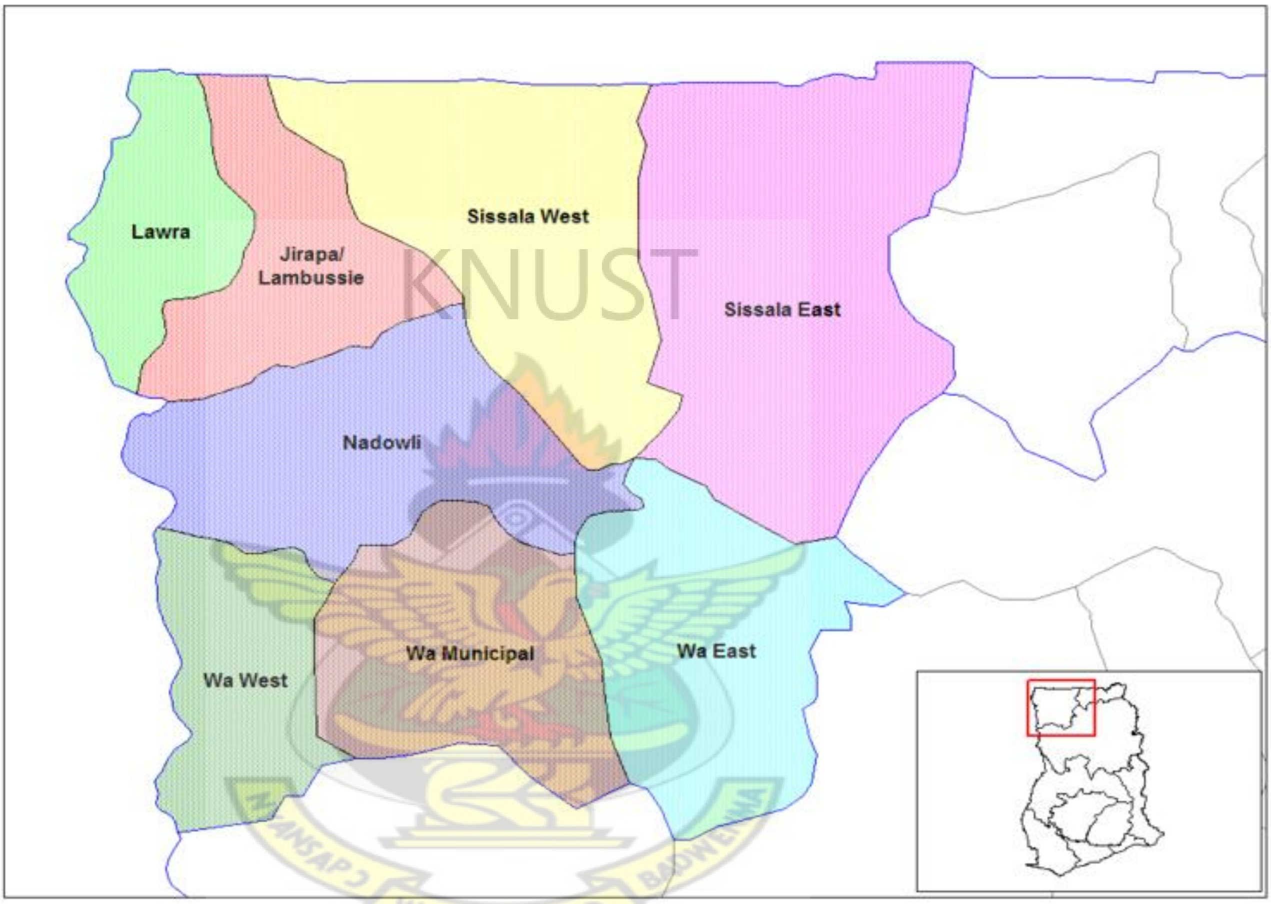
Figure 4.1 Showing the Map of Ghana

DISTRICT MAP OF UPPER EAST REGION OF GHANA

Figure 4: 2 District Map of the Upper East Region of Ghana

The Upper West Region of Ghana contains the following 8 districts:

- Jirapa/Lambussie District



- Lawra District
- Nadowli District
- Sissala East District
- Sissala West District
- Wa East District
- Wa Municipal District
- Wa West District

The Upper West Region of Ghana is located in the northwestern corner of the country and is bordered by Burkina Faso to the north. It has 8 districts and a population of 576,583(2000 census).The capital and largest city is Wa. Other towns include Nandom, Daffiema, Jirapa, Kaleo, Nadowli, Lawra and Tumu. Wa is the largest predominantly Islamic city in Ghana. It is in this region, to the south of Wa, tourists can find the Wechiau Hippopotamus Sanctuary.

The major economic activity of the region is agriculture. Crops grown include corn, millet, groundnuts, okro, shea butter, and rice. Sheep, goats, chickens, pigs and guinea fowl are raised for meat and eggs. Because the region is poor and the dry season is long, extending roughly from October to May, many people leave the area to work in the southern part of Ghana for at least part of the year. The major ethnic groups are the Dagaba, Sisaala and Wala. The Dagaba live in the western part of the region, the Sisaala live in the eastern areas, and the Wala live in Wa and a few of the nearby villages. The Sisaala and Dagaba are mostly Christian and animist, while most Wala are Muslim; Wa is the largest predominantly Islamic city in Ghana. Waali, the language of the Wala, and the Dagaare language are mutually intelligible. It is in this region to the south of Wa, tourist can find the Wechiau Hippopotamus Sanctuary.

FACTORS TO BE USED FOR SITING OF AIRPORT IN THE UPPER WEST REGION

Based on a scale of 1 to 10, the following criteria will be used to determine the best location in the Upper West Region to site the domestic airport:

FACTORS

1. Transportation options

2. Work force availability,

3. Proximity to raw material sources, ease of contracting and acquiring costs,

4. Quality of life

5. Social life, entertainment opportunities, sports, shopping opportunities,

6. Living costs (rent, shopping) compared to wage rates,

7. Sewage and garbage service and facilities (for the industrial facilities)

8. Topography of the building site (flat, hill, etc.),

9. Access to water,

10. Energy availability

Table 4.1Factors and ratings for the 8 district in the upper west region

Factor	Jirapa	Lawra	Nadowli	Sissala East	Sissala West	Wa East	Wa Municipal	Wa West
1	2.5	2.5	2.5	0.0	2.5	2.5	2.5	2.5
2	0.0	2.5	5.0	0.0	2.5	5.0	5.0	5.0
3	5.0	7.5	2.5	0.0	2.5	2.5	5.0	2.5
4	7.5	2.5	0.0	2.5	5.0	2.5	7.5	0.0
5	2.5	2.5	0.0	2.5	7.5	7.5	0.0	0.0
6	5.0	2.5	0.0	5.0	2.5	5.0	2.5	2.5
7	2.5	0.0	2.5	2.5	2.5	0.0	5.0	2.5
8	5.0	0.0..	2.5	5.0	2.5	0.0	5.0	2.5
9	2.5	2.5	2.5	2.5	2.5	2.5	5.0	0.0
10	2.5	2.5	22.5	0.0	5.0	2.5	7.5	0.0

Using SPSS dataanalysis, the location with the highest score, will be considered the best place to cite the airport. A sample size of fifty was used in administering questionnaires to determine the best out of the 8 districts so site a domestic airport.

Factors

1 Transportation options

2 Work force availability

3 Proximity to raw material sources, ease of contracting and acquiring costs

4 Quality of life

5 Social life, entertainment opportunities, sports, shopping opportunities

6 Living costs (rent, shopping) compared to wage rates

7 Sewage and garbage service and facilities (for the industrial facilities)

8 Topography of the building site (flat, hill, etc.)

9 Access to water

10 Energy availability

Table 4.2Results of SPSS Data analysis

	Jirapa	Lawra	Nadowli	Sissala East	Sissala West	Wa East	Wa Municipal	Wa West
Factor	Mean	Mean	Mean	Mean	Mean	Mean	Mean	Mean
1	2	3	2	2.5	2	3.5	1.5	2.5
2	0	.3	4.4	1	2.5	4	5.5	4
3	5	7.5	2.5	3	2	2.5	5	1.5
4	6.4	2.5	1	2.5	5	3	6.5	1
5	2.5	2.5	1	2.5	7.5	6.5	4	0
6	4	2	0.5	5	2	4	2.5	2.5
7	2.5	1.5	2.5	2.5	2	1.5	5.5	3.5
8	4.5	0	2.5	5	2.5	0.5	5.5	2.5
9	2	2	2	2.5	2.5	2.5	5	0
10	2.5	3	3.5	0.5	3	2.5	7.5	0.5
	3.1	2.7	2.2	2.7	3.1	3.1	4.9	1.8

From Table 4.2 above, it can be seen that the best place to site the domestic airport is Wa Municipal. This is because it has the highest average rating of all the 10 factors used for the study.

To further test that Wa Municipal is the optimal location for siting of the airport, we run a regression to ascertain the fact that Wa Municipal is the location of choice to site the airport, based on the 10 factors.

Table 4.3:The general regression model for the 8 districts

District	r^2	Adjusted r^2	Winning District
Jirapa	.048	.002	
Lawra	0.148.	0.130	
Nadowli	0.00	-0.21	
Sissala East	0.02	-0.19	
Sissala West	0.01	-0.020	
Wa East	0.080	0.060	
Wa Municipal	0.155	0.137	Wa Municipal
Wa West	0.097	0.078	

Table 4.3 shows the values for r^2 and adjusted r^2.It shows that the winning district is Wa Municipal, i.e the preferred location for citing of the airport.

The adjusted r^2 shows the strength of the relationship factor and the choice of district. This means that 13.7% of the variation between the factor and the choice of district can be explained by the strength of the relationship between factor and choice of district.

The district with the least strength of relationship is the Nadowli district, which is -0.21

The Electré method will now be applied to this site selection problem in selecting the location of an airport in the Upper West Region of Ghana. Alternative locations (options) and criteria will be shown with symbols. The following steps are used to solve the problem.

a. Step1: Identifying the ….Options

There will be eight alternative locations in this example problem. These alternatives (options) are represented by the following symbols: A, B, C, D, E, F G and H.

b. Step 2: Identifying the Criteria

The criterion list can be expanded according to the particular characteristics of the problem.

c. Step 3: Weighing the Criteria

Weighing the criteria is one of the most vital points of this method.Sort the criteria by their levels of importance and score each criterion by considering the primary objective.

d. Step 4: Determining Scales

Instead of using numerical grades to evaluate the options according to the criteria, the options must be evaluated with the qualitative measures, such as: “Very good, good, not bad, bad, and very bad.” Next, these qualitative results will be converted to numerical values according to the predetermined scales. The upper and lower limits of the scales will match the “very good” and “very bad,” and the intermediate values (good, not bad, bad) will be calculated with the interpolation. For example, a scale from 10 to 0 can be represented as below:

- Very good : 10.0
- Good : 7.5
- Not bad : 5.0

- Bad : 2.5
- Very bad : 0.0

The following requirements about the scales were adhered to:

- There must be as many different scales as the number of different weights, which are determined during Step 3.
- The scale range of the highest weight must be the largest, and the scale range for the lowest weight must be the narrowest. Accordingly, the scale range of lower weights must be a subset of the scale ranges of higher weights. For example, assume that very important criterion “a” has a weight of 4, less important criteria “b and d” has a weight of 2, and the least important criteria “c and e” has a weight of 1. In this example, the scale ranges are chosen as 0 to 10 for “a,” 2 to 8 for “b and d,” and 3 to 7 for “c and e”. The ranges can be chosen differently as long as the range of less important criteria is a subset of the higher important criteria. For example, when an option is evaluated as Very Good for two different criteria that have different weights (importance level), this option will get a higher score from the more important criterion and a lower score from the less important criterion. The number of different scales with different ranges is equal to the number of different weights used in the problem.

Table 4.4 showing the scale rating

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	Criterion a	Criterion b – d	Criterion c – e
Very Good	10	8	7
Good	7.5	6.5	6.0
Not Bad	5.0	5.0	5.0
Bad	2.5	3.5	4.0
Very Bad	0.00	2.5	3.0

Determining Scales

E. Step 5: Evaluating Options Regarding Criteria.

Each option is evaluated regarding all the criteria. In the example problem, the options "A, B, C, D, E" are evaluated regarding to criteria "a, b, c, d, e"

Table 4.5 showing criteria and options

		OPTIONS						
CRITERIA		A	B	C	D	E	Weight	Scale
	A	5	5	2.5	7.5	2.5	4	0 – 10
	B	2	3.5	6.5	8.0	5	2	2 – 8
	C	7	3	5	6	7	1	3 – 7
	D	6.5	8	6.5	2	5	2	2 – 8
	E	4	6	5	4	6	1	3 – 7

The complete concordance matrix is presented below:

Table 4.6 Showing Concordance Matrix

	A	B	C	D	E
A	*	0.90	0.50	0.70	0.40
B	0.5	*	0.30	0.70	0.40
C	0.7	0.7	*	0.70	0.60
D	0.4	0.3	0.30	*	0.40
E	0.7	0.7	0.8	0.60	*

Concordance Matrix

In the example problem, the discordance matrix value of “C outranks A” assumption is calculated as below.

• Compare column C values of Table 3.4 with column A values.

• Find the criteria that the score of option C are less than the score of option A [using Table 3.4, the score of option C is less than option A for the criterion a (2.5<5) and criterion c (5<7)].

• Subtract the scores of option A from the scores of option C and determine the greatest deviation among these pairs of scores [(5-2.5=) 2.5 and (7-5=) 2; therefore the greatest deviation is 2.5 (2.5>2)].

• Divide the greatest deviation by the largest scale range (10 – 0 = 10). This value is the discordance indicator for the C outranks A assumption and will be inserted in the intersection cell of row A and column C (0.25=2.5/10).

• Note this value (0.25) in the intersection cell of Row B and Column E on the concordance matrix. The outcome is called the first discordance matrix (s=1) (“s” is the discordance parameter).

The completed first discordance matrix is presented below:

Table 4.7 Showing Completed First C[illegible]ance Matrix

	A	B	C	D	E
A	*	0.40	0.25	0.45	0.25
B	0.2	*	0.25	0.60	0.30
C	0.45	0.3	*	0.45	0.15
D	0.60	0.45	0.50	*	0.50
E	0.30	0.40	0.20	0.30	*

First Discordance Matrix

If one option is much worse than another option (high discordance), then the outranking assumption between these two options will automatically be penalized. However, this may not be a favorable situation because the discordance indicator can meet the second condition when the second discordance matrix (s=2) is formed. The second discordance matrix helps to control the results of the first discordance matrix. The following steps are used to calculate the second discordance matrix (s=2)

• Follow the first two steps in the concordance procedure above.

• Subtract the scores of option A from the scores of option C and determine the second greatest deviation among these pairs of scores. If the previous deviation (greatest deviation = 2.5) is the only deviation between the grades of the option pair, then the discordance indicator will be zero (0). Similarly, if the previous deviation (2.5) is equal to the second greatest deviation between the grades of the option pair, this deviation will be used to calculate the discordance indicator again. For the example problem, (5-2.5=) 2.5 and (7-5=) 2; therefore the second greatest deviation is 2.

Divide this value by the largest scale range (10 – 0 = 10). This value will be presented in the intersection cell of row A and column C of second discordance matrix (s=2). For the example problem, this value is 0.2 (2/10).

The completed second discordance ma 56 =2) is presented below:

Table 4.8 Showing Completed Second Concordance Matrix

	A	B	C	D	E
A	*	0.00	0.20	0.10	0.15
B	0.15	*	0.15	0.20	0.25
C	0.10	0.20	*	0.10	0.15
D	0.25	0.30	0.15	*	0.30

E	0.20	0.15	0.10	0.20	*

g. Step 7: Electing and Decision

Before determining the best option, the threshold and nucleus concepts must be explained. There are two kinds of thresholds: the preference threshold (p) and the indifference threshold (q). The decision maker specifies the indifference thresholds. The choice of appropriate thresholds is not easy, but realistically, non-zero values should be chosen for p and q. While the introduction of this threshold goes some way toward incorporating how a decision maker actually feels about realistic comparisons, a problem remains. There is a point at which the decision maker changes from indifference to strict preference. Conceptually, there is good reason to introduce a buffer zone between indifference and strict preference, an intermediary zone where the decision maker hesitates between preference and indifference. This zone of hesitation is referred to as "weak preference", and is modeled by introducing a preference threshold, p. Thus, the ELECTRE Method is proposed as a double threshold model.

Using the thresholds, the following preference relation can be defined for A outranks B assumption: If the concordance indicator (value in the concordance matrix) of this option pair is greater than or equal to "p" and the discordance indicator is less than or equal to "q," then A is preferred to B. These two conditions are reviewed for all pairs and the results will beshown in the solution figure. In this figure, each option is represented with a node and each option pair is connected with an arrow that points from the outranking (preferred) option to the other one. Thus, there will be two types of nodes in the chart, one of which is the node with at least one arrow-entering and the other type is a no-arrow-entering node. No-arrow entering nodes are called nucleus. If a node is not connected to any other nodes, this node can also be an element of the nucleus. However, this situation can be risky and misleading and should be resolved.

The actual solution process begins with the first concordance and discordance matrices (s=1) and the beginning threshold values are chosen. In the example problem, the beginning thresholds are chosen as p=0.7 and q=0.45 (notation: 0.7/0.45/1). If the discordance indicator is equal to 0.45 or less in the first discordance matrix, and the concordance indicator in the corresponding cell of the concordance matrix is also equal to 0.7 or greater, then this cell willbe marked with "*" in the solution figure. This comparison is repeated for the entire concordance and discordance matrices. The results of the example problem are shown in Figure 3.1. B and D options (alternatives) are the two nuclei of the first iteration because no arrow is entering these nodes. Since there must be only one nucleus node, the indifference threshold (q) is increased to 0.6 in this example (notation: 0.7/0.6/1). This situation is shown in the second part of Figure 3.1. After this iteration, the only nucleus will be the option D.

The second discordance matrix (s=2) is used to control this result. When the discordance parameter is increased from s=1 to s=2, then the values of the discordance indicators in the discordance matrix will decrease. Therefore, decreasing the value of the indifference threshold q is wise. The indifference threshold is chosen as q=0.15 and preference threshold is not changed (p=0.7) (notation: 0.7/0.15/2). If the discordance indicator is equal to 0.15 or less in the second discordance matrix and also the concordance indicator in the corresponding cell of the concordance matrix is equal to 0.7 or greater, then this cell will be marked with "*" in the third part of Figure 3.1. The B and D options (alternatives) are again the two nuclei of the third iteration. Since there must be only one nucleus node, the indifference threshold (q) is increased to 0.2 in thisexample (notation: 0.7/0.2/1). This situation is shown in the last part of the Figure 3.1. After this iteration, the only nucleus will be the option D again. This result is consistent with the first finding. Therefore the final location should be option D which represents Wa Municipal, representing the optimal location to cite an airport.

	A	B	C	D	E
A		*		*	
B					
C	*	*		*	
D					
E	*	*	*		

(0.7/0.45/1)

Figure 4.3 Showing preference and indifference threshold of 0.7 and 0.45 respectively

	A	B	C	D	E
A		*		*	
B				*	
C	*	*		*	
D					
E	*	*	*		

(0.7/0.6/1)

Figure 4.4 Showing preference and indifference threshold of 0.7 and 0.6 respectively

	A	B	C	D	E
A		*		*	
B					
C	*			*	
D					
E		*	*		

(0.7/0.15/2)

Figure 4.5 Showing preference and indifference threshold of 0.7 and 0.15 respectively

	A	B	C	D	E
A		*		*	
B				*	
C	*	*		*	
D					
E	*	*	*		

(0.7/0.2/2)

Figure 4.6Showing preference and indifference threshold of 0.7 and 0.2respectively

CHAPTER FIVE

SUMMARY OF RESULTS, RECOMMENDATIONS AND CONCLUSIONS

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The purpose of this research was to investigate the feasibility or possibility of citing an airport in the Upper West Region of Ghana. Several different factors were considered, some of which included the following:

- Transportation options
- Work force availability
- Proximity to raw material sources, ease of contracting and acquiring costs
- Quality of life, etc

Based on a rating scale from 1 to 10, 50 respondents gave their views as to where they think will be suitable for sighting a domestic airport based on 10 factors.

A combination of the Factor Rating method and the Electre method was used and it was recommended that the new airport be cited in Wa Municipal.

The conclusion of the research is based on the assumptions made during the analysis.

The main assumption was the precision of the survey data collected from the participants. Since the sample sizes for both surveys were relatively small, the surveys can be conducted by extending the participation and this will increase the precision of the survey data used to make a site selection decision.

CONCLUSION

- TheELECTREMethod is a very 61 facility location technique that can be adopted locally to site facilities across the country. I admonish our scientists and engineers to look to utilizing some of these techniques to help develop the country.
- Since this Method is very practical and easy to understand, I suggest that it be taught at the lower levels so that students can appreciate it better when they reach the tertiary level.
- Facilities like roads are needed to be built in several places to facilitate movement of goods and services to and from the domestic airport.

The government should do well by locating a domestic airport in the Upper West Region of Ghana. This will open up the region into tourism and other business opportunities.

RECOMMENDATIONS

Since the world is transforming to a digital environment, organizations use information technology (IT) solutions greatly to achieve their goals. The government must monitor the external environment closely and adapt quickly to compete with the changing requirements. The transformation and technological improvements for this airport will be difficult as well as the high operating costs are extremely costly. Therefore, the government should determine its strategic logistical needs to compete with the current technological developments, increase the level of customer service and perform its mission in the most cost effective way.

LIMITATIONS

The insufficiency of available data for the research cannot be overemphasized.

Actually, other costs must also be included in a further analysis. Since the data was not available, the transportation costs savings could be underestimated. For example, the transportation costs from other customer locations all over to the proposed airport as well as to Accra were not included in this research. With this additional data, the savings or losses in the transportation costs of the other customers could be calculated more accurately.

TOPICS FOR FURTHER RESEARCH

The following research topics warrant further study:

This analysis can include the personnel, process, and overhead savings, etc.

• The analysis of the actual location of a domestic terminal. This analysis can include the initial planning, timing of the relocation, the cost of moving, etc.

In this thesis, we developed the Euclidean maximin with the weighted network minisum biobjective location model for citing a semi-desirable facility on a transportation network with mixed distance metrics. Although the planar and the network model seem different in structure, they are modeled to solve the same real-life problem. In many cases, it is more realistic to use the combination of the two models. For example, while the dispersion of airborne pollution such as dust, gases, noise and odor makes more sense to model in the plane with Euclidean distances, the network model would be more appropriate to model transportation cost. Therefore, we used a mixed distance metric model in which linearly approximated road network is embedded on the plane, so that each point on the network corresponds to a point in the plane.

Instead of using a traditional solution approach, we investigated the properties of our biobjective problem in both decision and criterion space and used Hakimi's (1964) vertex optimality property for evaluating the minisum objective as well as we introduced some properties that reduce the number of candidate edge maximin points. We developed two powerful fathoming procedures to eliminate inefficient edges and edge segments and used them as early as possible in the algorithm. Unfathomed edges and edge segments are mapped into the objective space and 2-dimensional search is applied to construct the nondominated set, whose inverse image yields the efficient set.

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