KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI. COLLEGE OF SCIENCE

OPTIMAL PERSONNEL SCHEDULING PROBLEM FOR NURSES; A CASE STUDY OF KOFORIDUA CENTRAL HOSPITAL.

By

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Declaration

I hereby declare that this submission is my own work towards the award of the M.Phil degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the university, except where due acknowledgement had been made in the text.

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Dedication

To GOD Almighty, my parents Mr and Mrs Torto not forgetting my siblings, Christy, Steve and Steph.

Abstract

Nurse Personnel scheduling can be regarded as a partial constraint satisfaction problem in which many techniques have been developed to tackle. In this thesis, we consider the problem of scheduling nurses at the Koforidua Central hospital with the aim of maximizing job satisfaction and reducing the partiality that occurs during the manual timetabling done by the matron. The problem was formulated as an Integer Linear Programming (ILP) problem using the available data from the hospital as at October 2012. This problem was solved with the Branch-and-Bound Method of solving (ILP). It was found that throughout the week the optimal number of nurses for Tuesday and Thursday as well as Wednesday and Friday were 106 and 108 respectively. We concluded that scientific modeling of the scheduling and allocation of nurses was not so different from the traditional nave approach by the matron, except for the recommendation that none of the nurses should go on leave unless their absence has been catered for.

List of abbreviations

NSP	Nures Scheduling Problem
PS	Personnel Scheduling
GI	Gastroenterolgy
NMC	National Midwifery Council
CHNG	Community Health Nurse Group
GHS	Ghana Health Service
GRNA	Ghana Registered Nurses Association
PONGG	Peri-Opeartive Nurses Group Ghana
NCGRNA	National Council of Ghana Registered Nurses Association
LP	Linear Programming
LIP	Linear Integer Programming
B & B	Branch and Bound
B & P	Branch and Cut
B & C	Branch and Price
IP	Integer Programming
NP	Nondeterministic Polynomial

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Chapter 1

Introduction

The Nursing profession affects the health status of patient in several ways as they are the majority of workforce found in hospitals. By their caring professionalism, their experience, academic training, levels of job satisfaction, and their workload they impact on patient outcomes, Aiken et al. (2002). For this reason, one can dispute that there will be no efficient delivery of health care services without a well trained and caring nursing staff. Nonetheless, hospital nursing care accounts for roughly one-quarter to one-third of the hospital operating budget and nearly half of all direct care costs Mark et al. (2004). The increased health care costs have affected all areas of the delivery of health care, including nursing services and nursing labor costs Whelton et al. (2006). As with all types of employees, nurses generally perform better when they are satisfied with their jobs. Research suggests that there are two key job satisfaction factors for a nursing staff which are scheduling and patient-to nurse ratios, which in turn may positively influence patients' hospital length of stay (Chery, 2008).

As in most hospitals in the country, Koforidua Central Hospital is faced with the problem of finding the best nursing personnel schedule. This will help meet the daily fluctuations in patient attendance and care requirements while minimizing nursing labor costs and maximizing job satisfaction. This study seeks to develop an optimal nurses personnel scheduling using linear integer programming in the koforidua central hospital. This chapter takes a look at the background of the study, the general history of nursing and the scope of practice which includes the groups of nurses in Ghana. The problem statement, research questions and objectives, research methodology, justification of the study as well as scope and limitations of the study are also discussed.

1.1 Background of the Study

Personnel scheduling is concerned with the determination of appropriate workforce requirements, workforce allocation and workforce duty assignments for an organization in order to meet internal and external requirements. Providing the right people at the right time at the right cost whilst achieving a high level of employee satisfaction is a critical problem for organizations Ernst et al. (2004). Staff schedules and rostering methods have been applied to various sectors including Transportation systems such as railways, mass transit, airlines and buses. Protection and Emergency services for instance police, ambulance services, fire and security services, air force and navy etc, Call centres, Healthcare systems, Civic services and utilities, Financial services, Venue management, Hospitality and tourism for example hotels, restaurants and retail stores, Manufacturing industries and many more. Employee scheduling and nurse scheduling in particular have been addressed by many scientists for more than 40 years Burke et al. (2004). There are differences in scheduling problems due to various constraints, which leads to the Nurse Scheduling Problem becoming more complex. For example, some hospitals require accommodating schedules to deal with absence of nurses, fluctuations in patient numbers and staff requests so they use non-cyclic schedules. Some hospitals employ a cyclic schedule where for each planning period the same schedule is repeated, and each nurse cycles around a subset of the selected shift patterns. Scheduling employee shifts involves planning in time intervals and estimates of customer demand can vary greatly across time intervals. Employees can be assigned to various shifts specified by length, start time, and the number and length of relief and meal breaks. Thus the general shift scheduling problem involves determining the number of employees to be assigned to each shift and specifying the timing of their relief and meal breaks. It is known that the flexibility provided in shift types (length, start time, and break placements) lower the total staffing costs and the number of required employees Titiyevska (2006). Many organizations have always tried to reduce cost of production and at the same time make an effort to provide efficient service to the customers. Health personnel are exposed to adverse health conditions. These Nurses are more likely to be exposed to human suffering than other health professionals; chronic exposure in this occupational environment has implications for their health and well being Kornhaber (2009). Properly scheduling the nursing staff has a great impact on the quality of health care, the recruitment of nursing personnel, the development of nursing budgets, staff and patient safety, staff and patient satisfaction and administrative workload Gokce (2008). For instance in the health sector,Administrators aim at providing an efficient health delivery at a reduce cost as well as ensuring maximum job satisfaction of their personnel such as nurses, doctors, pharmacists, midwives, etc. Most health institutions have resulted in the use of techniques such as timetabling, sequencing, roistering problem and scheduling of personnel. The Nurse Scheduling Problem (NSP) is a multi-faceted optimization problem of allocating nurses to duty rosters in a particular hospital. All over the world, NSP is increasingly becoming a central point for the reason of cost pressures on hospitals also the declining number of skilful nurses in a particular health sector to help doctors and other health workers to perform efficiently.WHO (2006) showed that 57 countries, including Ghana, faced critical health workforce shortages especially nurses due to poor conditions of work offered them. The scheduling of hospital personnel is particularly challenging because of different staffing needs on different days and shifts. Unlike many other organizations, healthcare institutions need to be staffed 24 hours a day over seven days a week Gokce (2008). Subsequently, in many hospitals, nurses are allowed to request preset shifts, while other nurses are scheduled around these preset shifts.

Since 1950's, Scheduling Problems were being started to be solved by computers, but the computer power was not strong enough for some formulations. Until recently, much more complex scheduling problems were solved using computer solutions. Also, heuristic and metaheuristic algorithms were addressed by many literatures and widely used for optimization problems. In view of the fact that scheduling problem has wide application in real life, a number of studies started to solve the problem in different ways including Nurse Scheduling Wren (1996). Many hospitals are facing the tight spot that they have to provide high-quality services with a relative lower cost, so they identify the resources including nurses for eliminating the sources of waste Burke et al. (2004). In every different case, the fundamental nature of NSP remains the same, which is to assign every nurse in the ward to a suitable set of shifts during the given period. Nonetheless, the objectives of the NSP range from minimizing costs to maximizing the satisfaction of the personnel involved and providing efficient services to patients M.Thompson et al. (2007).

History of Nursing

The word nursing derives its meaning from the Latin word Nutricius which means nourishing. Therefore, in the ancient world nursing was viewed as a role of caring for the family and not a profession. Since the beginning of time, nursing has been shaped by different cultures and societies and by the changes within those societies. When it became apparent that love and nurturing alone were not enough to cure disease, the need for a more educated framework for nurses began to form. Florence Nightingale was perhaps the most centralized figure in the history of nursing. She overcame tremendous social opposition to become a nurse and to care for soldiers during the Crimean War. She was dedicated to her profession through the decreasing death toll among soldiers. Nevertheless, even with these great accomplishments she had not achieved her greatest desire, to open a training school for nurses. In 1860, Florence's dream was finally realized when the Nightingale Training School for Nurses was established. This was the first formal, fully organized training program for nurses. Graduates of the program went into the four corners of the world to teach other nurses and were highly sought by hospitals. While Florence Nightingale did not invent the profession of nursing, she was a living memorial to it and forever will have a place and influence in the history of nursing. Nurses have experienced difficulty with the hierarchy in medicine that has created an impression that nurses' primary purpose is to follow the direction of medics. This tendency is certainly not observed in Nightingale's notes on Nursing, where the doctors are mentioned relatively infrequently and often in critical tones, particularly relating to bedside manner. The modern era has seen the development of nursing degrees and nursing has numerous journals to broaden the knowledge base of the profession. Nurses are often in key management roles within health services and hold research posts at universities. Nursing continues to evolve and be shaped as a body of science and has made great professional strides since its inception. The world history of nursing reveals that nurses, whether formal or informal have long filled a needed place in society.

Scope of Practice

Nurses may be distinguished from other health care providers by their approach to patient care, training, and scope of practice. Many nurses provide care within the ordering scope of physicians, and this traditional role has come to shape the historic public image of nurses as care providers. However, nurses are allowed by most jurisdictions to practice independently in a variety of settings depending on training level. Since postwar period, nurse education has undergone a process of diversification towards advanced and specialized credential and besides, many of the traditional regulations, provider roles and scope of practice are changing. There are several types of nurse in the healthcare organization. Some of these are as follows:

PSYCHIATRIC- Psychiatric nurses provide care for patients and families with psychiatric and mental illnesses. These nurses practice in a variety of settings which include hospitals and institutions.

MIDWIFERY- Midwives are nurses that are specially trained to deal with child-birth and providing prenatal and post-partum care. The midwife is qualified to deliver babies by themselves unless there are extenuating circumstances which require the midwife to consult with a physician.

PAEDIATRIC- Paediatric nurses care for children in all aspects of health care. Paediatric nurses practice in a variety of settings which include hospitals, clinics, and schools and in the homes.

HOLISTIC CARE- Holistic nurses provide medical care for patients while honoring the individual's subjective opinions about health, health beliefs and values. Holistic nursing requires nurses to integrate self-care, self- responsibility, spirituality and reflection into their daily nursing care.

AMBULATORY CARE- Ambulatory care nurses care for patients whose stay in the hospital or other facility will last than 24 hours. Ambulatory care nursing covers a broad range of specialities in the out-patient setting.

ANAESTHESIA - Nurse Anaesthetists work with surgeons, dentists, podiatrists, anaesthesiologists and other doctors to provide anaesthesia to patients before, during and after surgery or childbirth.

EMERGENCY- Emergency nurses assess patients, provide interventions and evaluate care in a time limited and sometimes hectic environments. Emergency nurses work independently and interdependently with various health professionals in an attempt to support patients and their families as they experience illness, injury or crisis.

CARDIAC CARE- the Cardiac Care Nurse works with other members of the medical staff in assessing, intervening and implementing nursing care for the cardiac patient.

HIV/AIDS- HIV/AIDS nurses provide healthcare for patients who are HIV or

AIDS positive. Theses nurses usually have specialized training in HIV/AIDS.

NEONATAL- Neonatal nurses provide care for the newborns by assessing the patient to ensure good health, providing preventative illness and caring for the babies which are sick. The neonatal nurse is responsible for anticipating, preventing, diagnosing and minimizing illness of newborns.

NEUROSCIENCE- Neuroscience nurses care for patients using new therapies and innovative technologies to treat diseases of the nervous system.

ONCOLOGY- Oncology nurses provide care for cancer patients at all stages of treatments and remission.

CRITICAL CARE - Critical care nurses provide care for patients and families who are experiencing actual or potential life- threatening illness. More specific fields that fit into the Critical Care category include Cardiac Care, Intensive Care and Neurological and Cardiac Surgical Intensive Care.

FORENSICS- Forensic nurses provide medical care to victims of crime, collect evidence after crimes occur, and provide medical care to patients within the prison system.

GASTROENTEROLOGY- Gastroenterology (GI) nurses provide care to patients with known or suspected gastrointestinal problems who are undergoing diagnostic or therapeutic treatments. GI nurses practice in physicians' offices, inpatient and outpatient endoscopy departments, ambulatory endoscopy centers and inpatients hospital units.

GERIATRICS CARE- geriatrics nurses care for elderly patients in a number of settings which include the patients home, nursing homes and hospitals. Geriatric nurses face constant challenges because their patients are often very ill, very complex and very dependent on the nurse's skills.

(http://wiki.answers.com)

Groups Of Nurses In Ghana

In Ghana there are several groups of nurses and midwives. All nurses and midwives practising in Ghana must be registered with the National Midwifery Council (NMC) and are expected to demonstrate high standards of practice for their profession. The Nursing and Midwifery department works to maintain these high standards and influence change to ensure that all nurses and midwives working in the country continue to adopt the most up-to-date clinical practices. This is done by setting standards for practice, educating and supervision of nurses and midwives.

✓ **General Nurses Group**: The members are clinical nurses based in the hospital and clinics. There was the need to form this group to bring nurses together to educate themselves in current practices to become better nurses.

✓ **Community/Enrolled/Public Health Nurses Group**: The community health nurse group in Ghana comprises of community health from the various health institution. The objective of the group among others is to form formidable group to see to the welfare of it member and discuss issues concerning the carrier progression. Some roles of CHNG

a. Home visit nursing practices.

b. Immunization.

c. School health service.

d. Ante-natal/child welfare clinics.

✓ **The Nurse Educators Group** was started in 1978 at Kumasi. The Group comprises of all Tutors in Nursing and Midwifery as well as Health Assistant Training Institutions in the Ministry of Health. Ghana Health Service as well as those in private institutions in Ghana. The Group is an affiliate of the mother Association (GRNA). The objectives of the Group are as follows:

a. To foster unity among all Tutors in all Nursing, midwifery and post basic training school (government, mission and private schools).

b. To bring all the tutors in the below schools into a unified body to seek their welfare.

c. To promote generally a better understanding and the dignity of members of the group. That is Nursing and Midwifery as well as Health Assistants training institutions in Ghana

d. To promote and support members in matters affecting them.

e. To promote and seek the welfare of members especially during crisis situations.

f. To promote excellent academic performance in the training institutions.

Currently the Group consists of the following Training Institutions:

i Nursing Training Colleges and Schools.

ii Midwifery Training Schools.

iii Public Health Nursing School.

iv Peri-operative/Critical Care.

v Nursing School.

vi Ophthalmology Nursing School.

vii Ear Nose and Throat Nursing School.

viii Health Assistants Training School (Nursing).

✓ **Peri-Operative Nurses Group Ghana (PONGG)**

They are a group of specialized professional registered Nurses who work mainly in the operating theatres. Nurses in this field provide care and support to patients before, during, and after surgery. They are responsible for maintaining a sterile environment in the operating theatre, monitoring the patient during surgery, and coordinating care throughout the process. In assisting the surgeon and the surgical team in their tasks, they are also responsible for making sure the operative team provides the patient with the best care possible. Our motto is: With Surgical Skills We Save.

✓ **The Ophthalmic Nurses Group**

The paramount aim of the group is to promote good eye health for all people living in Ghana.

✓ **Psychiatric/Mental Health Nurses Group**

The Psychiatric Nurses Group was formed and inaugurated on 29 April, 1992. The need for Psychiatric Nurses to come together as a group was necessitated by the following:

i Absence of a National Mental Health Policy

ii Lack or shortage of Trained Mental Health Personnel and Technology

iii Low priority accorded Mental Health and low value attached to Mental Health

iv The Welfare of members

v Promotion of Good Mental Health in the Country.

✓ **Government Registered Midwives Group**

The Government Registered Midwives Group was initially formed under the name Straight Midwives/ Enrolled/Community Midwives Group in about 1974 but remained dormant due to the fact that there were no national executive to steer the affairs of the Group. In May, 2007, the name Straight

Midwives/Enrolled/ Community/Diploma Midwives Group was replaced by the name Government Registered Midwives Group. In June 2009, the Group was inducted into the National Council of the Ghana Registered Nurses Association. In the meantime, the Group is strengthening its educational aspects by establishing Degree Midwifery program to enable Diploma Midwives to specialize in areas of Midwifery. The Group is working hard to promote the practice of Midwifery and to project the profession to meet International Standard.

✓ **Critical Care Nurses Group** Critical care nurses group of Ghana was formed in 2008 with the aim of providing the members with the knowledge and resources necessary to provide optimal care to the critically ill patient. Nurses in acute and critical care settings including neonatal and paediatrics units in the country are also part of this group.

1.2 Statement Of The Problem

The contribution made by nurses in providing health service to their patients cannot be undermined. As administrators and managers of various health sectors seek to minimize cost of production and services rendered, the job satisfaction of the employee should not be sacrificed. Nurses all over the world more especially those in sub-Sahara Africa complain about the nerve-racking nature of their work and poor condition of service as well as the danger on their health in their day to day activities. In this hospital, it takes more or less weeks for the head nurse to build the schedule manually for each month. Fairness is not addressed while making the schedule. For that matter there appears to be inconsistency in the section of day shifts worked. Most at times some say they don't even get rest and break in their days work but in spite of all these the number of patient they attend keeps on increasing as the population of the country increases. The stressful nature coupled with other problems such as finance has resulted in a large number of nurses leaving the country for greener pasture in developed countries. These have been one of the serious problems faced by the Health Ministry in Ghana. There is therefore the need to develop an effective and efficient strategy to ensure that the limited number of nurses left is well scheduled as well as rotated to guarantee maximum job satisfaction of nurses at a minimized cost and that is the problem this research seeks to contribute in solving.

1.3 Objectives of the Research

The objective of the study is

i To formulate an Integer Linear Programming for scheduling of nurses at Koforidua Central hospital.

ii To determine the minimum number of nurses required for each shift in the hospital.

1.4 Research Methodology

Nursing Scheduling Problem consist of assigning a schedule to each worker, which involves building a timetable for a specified period. The timetable should comply with staffing requirements, the rules laid down by the administration and the labour contract clauses. Personnel scheduling problems are highly constrained with both hard and soft ones. In this thesis, one procedure is being presented for solving the Cyclic Nurse Scheduling Problem which involves the construction of duty rosters for nursing staff over a pre-defined period. Nurses are assigned to each shift according to numbers and skill levels required while at the same time balancing the workload among the nurses involved and also considering staff nurses preferences such as providing requested days-off. All of this will be achieved by using software called quantitative management which helps in solving and analysing.

1.5 Justification

Demand for many services varies significantly from hour-to-hour and day-to-day. To achieve productivity goals and provide adequate service, most health service organizations constantly adjust their capacity to match an appropriate fraction of expected demand. Frequently, service capacity is equated with employee staffing levels, which are regulated through employee work schedules. In spite of all these efforts, much needs to be done in developing an efficient roster or scheduling for nurses. The nurse workforce in Sub-Saharan Africa (SSA) is a significant component of its health workforce, perhaps more than other continents. Nurses constitute 45-60 percent of the entire health workforce with nurses responsible for a broad range of services*http://www.icn.ch/global/Issue7SSA.pdf.* (n.d.)

The nurse to patient ratio in Ghana is 1:6705 and is estimated that about 500 skilled nurses leave the country every year*www.ghanadistricts.com/districts* (2012) , due to salary difference. For example purchase parity pay for nurses in Australia or Canada was 14 times that of a Ghanaian nurse, 25 times that of a Zambian nurse, and twice that of a South African nurse Vujicic et al. (2004). This therefore calls for the need to develop an effective schedule to properly manage the nurses remaining in the country to ensure their job satisfaction at a minimum cost.

1.6 Scope Of The Problem.

The research primarily considered this general hospital due to the number of patience that visits the facility in a day. Also it is believed that since there is a nursing training school ~~in the~~ municipality, they do employ some if not most of them in that hospital. Nonetheless, it is due to time and the complex nature of the process in arriving at the best solution.

1.7 Limitations

The data collected was as at October 2012. Since the allocations of nurses were being done monthly, the results will be subject to change. Hence the outcome of the results will change according to the circumstance surrounding the operations in the hospital. It was also anticipated that, the study suffered constraints of time, resource inadequacy thus not all departments were able to provide the needed data and the study has been structured within the confines of the data readily accessed.

1.8 Thesis Organization

The historical background of scheduling, objectives, scope and limitations of the study were covered in chapter one. In chapter two, we shall put forward pertinent literature in the area of distribution. Research methodology is detailed in chapter three. Data collection and analysis are presented in chapter four. Findings, conclusion and recommendation are discussed in chapter five.

1.9 Summary

In this chapter,we presented the background, statement of the problem and objectives of the study. The scope,limitation and organisation of the study were also put forward.In the next chapter, we shall put forward pertinent literature on scheduling.

Chapter 2

Literature Review

2.0 Introduction

In this chapter we shall put forward adequate and relevant literature on scheduling. This is in three sections, Personnel Scheduling, Nurse Personnel Scheduling and Integer Programming

2.1 Personel Scheduling

A meticulous challenge of the staff-scheduling problem is that one constraint might contend with another constraint to the point where a solution is unfeasible. Such constraints compete because the first constraint encourages more consecutive working days and the second constraint encourages less consecutive working days. For example, suppose one constraint tries to put the number of extended weekends (i.e., consecutive days Friday through Monday off) between all employees in equilibrium whiles, the other attempt to have room for individual employee preferences to avoid more than two consecutive working days. Such constraints compete because the former restraint encourages more consecutive working days but the second constraint promotes less consecutive working days. Dean (2008).

Fry et al. (2006) examined the problem of verifying the annual staffing level that decreases total expected costs for a fire department, subject to minimum service-level-based staffing requirements. They developed a quantitative model that permitted for permanent wastage, limited hiring opportunities and stochastic temporary absences which resulted in an exclusive fire fighter work schedule. They developed and put into practice optimal staffing policies for both the continuous and discrete cases, which provide numerical results judge against their optimal solution to several heuristic staffing policies commonly, used in practice and showed the optimal policy can provide considerable savings. Their model was suggestive of traditional news-vendor-type inventory models, but where the uncertainty is dependent on the decision variable.

According to Giovanni and Gentile(2004) solutions have to obey a number of constraints related to workload balancing, shift compatibility and distribution of days off under Integer Programming approach. Formulation of the constraints were common and therefore can be extended to different personnel management problems where staff members must cover shifts and management must assign a fixed number of days off per week.

In organizations where duty is around the clock, seven days a week and every week of the year, timetabling is a very difficult task, managing between the workload and the constraints to be appreciated. Ni and Abeledo (2007), presented a new methodology for solving large-scale employee tour scheduling problems. They proposed an integer programming model where tours are decomposed into shifts and start times. Complex shift and start time rules for both continuous and discontinuous scheduling problems can be modeled by this formulation. A branch-and-price approach is formulated to solve this model efficiently. The methodology was tested on the largest tour scheduling problems found in the open literature. In comparison with an alternative implicit model, their approach showed superior computational performance.

Isken (2004) motivated by difficult staff scheduling problems arising in healthcare institutions, developed an implicit tour scheduling model which includes full and part-time tour types as well as intra-tour start time flexibility. Potential benefits of intra-tour start time flexibility are demonstrated through a computational research. However, in a large tertiary care hospital the model has been embedded in a decision support system and has been used in numerous studies to help estimate staffing needs as well as to analyze the impact of scheduling policies and practices.

Özcan(2007), presented an empirical study on memetic algorithms in two parts. First of all, details of the memetic algorithm experiments with a set of well known benchmark functions were described. In the second part, a heuristic template was introduced for solving timetabling problems. Two adaptive heuristics that utilized the set of constraint-based hill climbers in a co-operative manner are designed based on this template. A hyper-heuristic is a mechanism used for managing a set of low-level heuristics. An appropriate heuristic is chosen and applied to a candidate's solution at each step and both adaptive heuristics can be considered as hyper-heuristics. Memetic algorithms employing each hyper-heuristic separately as a single hill climber are experimented on a set of randomly generated nurse rostering problem instances. Moreover, the standard genetic algorithm and two self-generating multimeme memetic algorithms were compared to the proposed memetic algorithms.

The rotating workforce scheduling problem involves the construction of an efficient sequence of work and rest periods spanning over a number of weeks(Balakrishnan and Wong, 1990) . Thus the schedule must satisfy the workforce requirements during the different shifts of each day and conform to all the other conditions imposed on the work/rest periods and their sequence. They considered the modeling of the rotating workforce scheduling problem as a network flow problem. All the constraints on the problem were incorporated in the network itself, except for the staff-covering constraints that were treated as side constraints. The optimal solution to the problem corresponds to a path in the network and is identified using a dual-based approach. The model dealt with the issues of rest-period identification, work/rest period sequencing and shift scheduling simultaneously and it's designed to handle multiple shift cases with time-varying demands.

Restrepo et al. (2012) reported on real applications in shift scheduling which often required handling rules such as multiple breaks, flexible shift lengths, overtime and multiple activities, among others. Because these rules demanded a high level of flexibility, they modeled the problem as a Multi-Activity Shift Scheduling Problem (MASSP), where multiple activities can be scheduled in a shift. To solve the MASSP, they proposed a column generation-based approach.

Brusco et al. (1995) recognized that service organizations encounter personnel tour-scheduling problems. They focused on the efficient assignment of daily shift schedules and work days to employees across a weekly planning horizon. They acknowledged that the airline industry, for instance, faces a highly complex and difficult tour-scheduling environment for their ground station personnel. The airlines industry have substantially reduced labour cost through efficient tour scheduling methods for workers at plane sides, counters and gates in airline stations. The United Airlines' Pegasys Manpower Planning System used two modules to improve their tour scheduling process. The first module used column generation to improve the selection of employee shifts whereas in the second module, a local search heuristic based on simulated annealing enabled initial feasible tour-scheduling solutions to rapidly improve. It was found that the incorporation of the modules resulted in a potential annual cost savings of more than $8 million hence concluded with a discussion of notable implementation issues and extensions.

A variety of service organizations might make productive use of a scheduling system similar to Pegasys. The environmental conditions described in the Introduction are appropriate to many organizations that provide services in transportation (inter- and intra-city rail, toll-way), communications (telephone operator, telemarketing, messenger service), retailing (department and discount stores, restaurants), public safety (fire, police, emergency), and health care (clinics, hospitals). Effective personnel scheduling in these, and other, high customer contact segments of service organizations should be an important component of ongoing productivity improvement Brusco et al. (1995).

Scheduling and timetabling problems are multi-constrained constraint satisfaction problems that have huge search space (Adamuthe and Bichkar,2012). These problems are Nondeterministic Polynomial (NP) hard. This paper investigates the use of backtracking approaches to laboratory personnel scheduling problem in which the objective is to assign tasks to employees. Their main objective was to search for better solutions than those obtained by authors using genetic algorithmic approach. The performance of backtracking algorithms is tested for different variable orderings, value ordering and consistency enforcement techniques. It is observed that the variable and value ordering backtracking with consistency enforcement techniques gives better results than the chronological backtracking as well as the results reported in the literature.

They perceived that the problem instance under consideration might have even better solutions which can possibly be obtained by suitably modifying the genetic algorithmic approach used earlier by authors or by using other optimization techniques such as simulated annealing or Tabu search. Results attained using backtracking algorithm explains that there are indeed better solutions for the problem under consideration. Moreover, the variable and value ordering have strong effect on the exploration of solutions in backtracking approach.

Both the solution representations produced good solutions. However, the task oriented solution representation was observed to give solutions in lesser time. Also, the outcome of the proposed consistency enforcement approach showed significant reduction in time required to obtain solutions.

2.2 Nurse Personel Scheduling

Constructing duty schedules for nurses at large hospitals is a difficult problem. The objective is usually to ensure that there is always adequate staff on duty, while taking into account individual preferences with respect to work patterns, requests for leave and financial restrictions, in such a way that all employees are treated fairly (Bester et al., 2007).

Bouarab et al. (2010) proposed that nurse scheduling is a complex exercise with multiple and contradictory objectives: minimizing total costs while maximizing the nurses' preferences and requests, and equally distributing workload between nurses. Work constraints imposed by collective agreements and unions as well as contracts have to be respected. They outline the following Constraints in nurse scheduling:

i. Demand for each shift;

ii. Shifts that can be assigned to each particular nurse;

iii. Maximum number of consecutive days of work;

iv. Minimum amount of rest time between two shifts;

v. Isolated days of work or days-off.

Bouarab et al. (2010) thought that scheduling problems are difficult to tackle. When studying practical problems, one can observe that this exercise is often manual, very time consuming and does not always provide the best quality results. The impact of scheduling in the healthcare sector is major in most jurisdictions operating under tight budgets. Meanwhile, resources tend to go in the opposite direction: budget cuts are unavoidable and human resources are increasingly scarce.

Ernst et al. (2004) said the problem of automated staff scheduling has been extensively addressed over the past several decades.Dean (2008) proposed that since hospital nurse care is so important, and since nurse shift scheduling is so complicated, there has been a considerable research effort in automated staff scheduling for hospital nurses.Whelton et al. (2006)also said that nurses are responsible for a huge part of the medical activities, and "account for approximately 25% of the total hospital operating budget and 44% of direct care costs".

Maire-Rothe and Wolfe(1973), stated that nursing salaries make up the largest single element in hospital costs and because of this the effective allocation of available nursing staff to patient care requirements is an important tool in controlling the cost of health care. They used two procedures to demonstrate nursing staff allocations:

i. A cyclical scheduling procedure designed to meet average staffing requirements in a manner consistent with hospital personnel policies and employees' preferences.

ii. A procedure for adjustment of staffing twice daily in accordance with an index of the latest patient care requirements.

Bailyn et al. (2007) discussed that self-scheduling is one aspect of the effort to make the hospital nursing environment more accommodative of nurses lives. It is part of the good employment practices that nurses want and that can help recruitment, retention and possibly patient care.

Fukunaga et al. (2002) proposed that staff scheduling is complicated by the fact that the constraints are usually numerous and varied, and sometimes competing. Even without such complications, with a relatively small number of straight forward constraints, bare-bones staff scheduling is known to be an NP-complete problem.

Grano et al. (2009) mentioned a wide variety of methods that have been used to tackle nurse scheduling: mathematical programming, constraint programming, heuristics and meta-heuristics, hybrid methods as well as simulation. Even creative methods such as auction systems have been applied to tackle nurses' preferences.

Bailyn et al. (2007) proposed that in general, most present literature on self-scheduling agrees to its benefits including some of the following main points:

i. Empowering nursing staff and increasing their control to balance their personal and professional lives, particularly helpful to nurses who have children or part-time schooling.

ii. Increasing predictability and flexibility of the nursing schedule and at the same time, freeing the nursing manager for other tasks.

iii. Enhancing the communication and interaction in the work environment to stimulate cooperative community building.

Ikegami and Niwa (2004), discussed nurse scheduling problems that are challenging in large hospitals and are difficult to resolve fairly. Scheduling objectives might include minimization of remuneration costs or maximization of staff satisfaction associated with a duty schedule. Cost efficient scheduling is very important because nursing salaries typically constitute significance portions of hospital budgets and influence the quality of health care provided.

Yankovic and Green(2011), assumed nursing care is arguably the single biggest factor in the cost of hospital care and patient satisfaction. Inadequate inpatient nursing levels have also been cited as a significant factor in medical errors and emergency room overcrowding. Yet, there is widespread dissatisfaction with the current methods of determining nurse staffing levels, including the most common one of using minimum nurse-to-patient ratios. In their paper, they represented the nursing system as a variable finite-source queuing model.The authors developed a reliable, tractable, easily parametrized two-dimensional model to approximate the actual interdependent dynamics of bed occupancy levels and demands for nursing. They used this model to show how unit size, nursing intensity, occupancy levels and unit length of stay affect the impact of nursing levels on performance and thus how inflexible nurse-to-patient ratios can lead to either under staffing or over staffing. The model is also useful for estimating the impact of nurse staffing

levels on emergency department overcrowding. Computational results using 52 data instances demonstrate the applicability of the proposed approach in solving real-world problems. Component-based approach with evolutionary eliminations for a nurse scheduling problem arising at a major UK hospital. The main idea behind this technique is to decompose a schedule into its components (i.e., the allocated shift pattern of each nurse), to implement two evolutionary elimination strategies mimicking natural selection and the natural mutation process on these components, respectively, and to iteratively deliver better schedules.
The first elimination removes a number of components that are deemed not worthy to stay in the current schedule; the second elimination may also throw out, with a low level of probability, some worthy components and this was proposed by Li et al. (2009).

Burke et al. (2004) also solved a problem of nurse rostering in Belgian hospitals. This is a highly constrained real world problem that was (until the results of this research were applied) tackled manually. The problem basically concerns the assignment of duties to a set of people with different qualifications, work regulations and preferences.
Constraint programming and linear programming techniques can produce feasible solutions for this problem. However, the reality in Belgian hospitals forced them to use heuristics to deal with the over constrained schedules. An important reason for this decision is the calculation time, which the users prefer to reduce. The algorithms presented in this paper are commercial nurse rostering product developed for the Belgian hospital market, entitled Plane.
Isken (2004) offered a practical approach to personnel scheduling problems arising in hospital units with demand that is of an urgent nature, cannot be backlogged, and is highly dependent on the time of day. A simple rounding heuristic is combined with a simulated annealing algorithm to obtain near-optimal solutions to large linear integer programming models of these personnel scheduling problems

in a reasonable amount of time on a personal computer. The models are designed to complement the current state-of-the-art of commercially available hospital staff scheduling systems.

Okada and Okada(1988), presented an approach to the problem of nursing staff scheduling in a hospital. For scheduling nurses, a variety of requirements with varied levels of significance has to be taken into account simultaneously. Because of the nature of the problem, where it is difficult to define what the optimal solution is in a strict sense, they aimed at automating scheduling by following the manual method in a faithful manner. A system for nurse scheduling has been implemented on their personal computer using Prolog. It determines favorable shift assignments on a day-to-day basis, referring to the information accumulated in the data base. In Prolog, various requirements can be expressed with relative ease, and the process of the manual method can be incorporated into the system in a natural way. The computer simulation has been conducted to test the system performance and the obtained results demonstrated the validity of the approach.

Petrovic and Berghe(2008), mentioned that despite decades of research into automated methods for nurse rostering and some academic successes, one may notice that there is no consistency in the knowledge that has been built up over the years and that many healthcare institutions still resort to manual practices. One of the possible reasons for this gap between the nurse rostering theory and practice is that often the academic community focuses on the development of new techniques rather than developing systems for healthcare institutions. In addition, methods suitable for one problem are usually not easily transferable to other problems. In real-world healthcare environments, a personnel manager cannot afford to model a problem and construct a roster using available approaches in order to quantitatively determine which one suits best. There is a lack of criteria for the comparison of approaches to provide a clear picture about their advantages and disadvantages and therefore their suitability to a problem in hand. This paper introduces seven criteria: expressive power, flexibility, algorithmic power, learning capabilities, maintenance, rescheduling capabilities, and parameter tuning, that may offer guidance to researchers and developers of systems for nurse rostering. Two approaches to nurse rostering, which are of very different nature, are evaluated and compared against the introduced criteria. One approach is based on meta-heuristics, while the other employs case-based reasoning.

Dowsland (1998) addressed the problem of producing rosters for nursing staff in a large general hospital by using tabu search with strategic oscillation. The objective is to ensure that enough nurses are on duty at all times while taking account of individual preferences and requests for days off in a way that is seen to treat all employees fairly. This is achieved using a variant of tabu search which repeatedly oscillates between finding a feasible cover, and improving it in terms of preference costs. Within each phase the search is controlled by a combination of different neighborhoods and candidate lists designed to aggressively seek out local optima and then to react to the problems encountered on arrival. The result is a robust and effective method which is able to match the quality of solutions produced by a human expert.

2.3 Integer Programming.

Glover and MacMillan (1986), extended the standard shift scheduling problem by discarding key limitations such as employee homogeneity and the absence of connections across time period blocks. The resulting increased generality yields a scheduling model that applies to real world problems confronted in a wide variety of areas. The price of the increased generality is a marked increase in size and complexity over related models reported in their literature. The integer programming formulation for the general employee scheduling problem, arising in typical real world settings, contains from one million to over four million zero-one variables.

By contrast, studies of special cases reported over the past decade have focused on problems involving between 100 and 500 variables. They characterized the relationship between the general employee scheduling problem and related problems, reporting computational results for a procedure that solves these more complex problems within 98 - 99% optimality and runs on a microcomputer. The authors viewed their approach as an integration of management science and artificial intelligence techniques.

The benefits of such an integration are suggested by the fact that other zero-one scheduling implementations reported in their literature, including the one awarded, the Lancaster Prize in 1984, have obtained comparable approximations of optimality only for problems from two to three orders of magnitude smaller, and then only by the use of large mainframe computers.

Hojati and Patil (2011), anticipated that scheduling of heterogeneous, part-time, service employees with limited availability is especially challenging because employees have different availability and skills, and work different total work hours in a planning period, e.g., a week. The constraints typically are to meet employee requirements during each hour in a planning period with shifts which have a minimum & maximum length, and do not exceed 5 work days per week for each employee. The objectives typically are to minimize over staffing and to meet the target total work hours for each employee during the planning period. They decompose this problem into (a) determining good shifts and then (b) assigning the good shifts to employees, and use a set of small integer linear programs to solve each part. They apply this method to the data given in a reference paper and compare their results. Also, several random problems are generated and solved to verify the robustness of their solution method.

Ezik et al. (2001) studied a workforce planning and scheduling problem in which weekly tours of agents must be designed. Their motivation for this study comes from a call center application where agents serve customers in response to incoming phone calls. Similar to many other applications in the services industry, the demand for service in call centers varies significantly within a day and among days of the week. In their model, a weekly tour of an agent consisting of five daily shifts and two days off, where daily shifts within a tour may be different from each other. The starting times of any two consecutive shifts, however, may not differ by more than a specified bound. Furthermore, a tour must also satisfy constraints regarding the days off, for example, it may be required that one of the days off is on a weekend day. The objective is to determine a collection of weekly tours that satisfy the demand for agents' services, while minimizing the total labor cost of the workforce. We describe an integer programming model where a weekly tour is obtained by combining seven daily shift scheduling models and days-off constraints in a network flow framework. The model is flexible and can accommodate different daily models with varying levels of detail. It readily handles different days-off rules and constraints regarding start time differentials in consecutive days. Computational results are also presented.

Brunner and Edenharter (2011), presents a strategic model to solve the long-term staffing problem of physicians in hospitals using flexible shifts. The objective is to minimize the total number of staff subject to several labor agreements. A wide range of legal restrictions and facility-specific staffing policies are considered. In general, the model is capable to incorporate different experience levels. In the simplest version the model decides about the number of staff for two experience levels, i.e. the number of residents (low experience) versus specialists (high experience). Shifts are constructed implicitly by the model and may have different starting times and several lengths. This allows more flexibility in the scheduling process. We formulate the problem as a mixed-integer program and solve it applying a column generation based heuristic. Using data provided by an anesthesia department of 1100-bed hospital, computational results demonstrate the usage of the model as decision supporting tool when staffing decision are made by hospital management.

Chapter 3

Methodolgy

Introduction

This chapter discusses the fundamental theory of Linear Programming with regards to its definition and formulation, component, objectives and the method of analysis of the current data to arrive at the objective.

3.1 Linear Programming

A linear programming problem is a problem in which a linear function is to be maximized (or minimized), subject to a finite number of linear constraints. It is also a Mathematical technique used in computer modeling to find the best possible solution in allocating limited resources (energy, machines, materials, money, personnel, space, time, etc.) to achieve maximum profit or minimum cost. However, it is applicable only where all relationships are linear and can accommodate only a limited class of cost functions. Linear programming cannot handle arbitrary restrictions and this have to be linear. This means that a linear function of the decision variables must be related to a constant, where related can mean less than or equal to, greater than or equal to, or strictly equal to.

3.1.1 Terminologies

Every linear model consists of a set of decision variable which represents the decisions to be made. This is in contrast to a problem data, which are values that are either given or can be simply calculated from what is given.

An objective function is a function which is to be either minimized or maximized. In the case of a maximization (minimization) problem, if arbitrarily large (small) values of the objective function can be achieved, then the linear program is said to be unbounded. A feasible maximum (minimum) problem is said to be unbounded if the objective function can assume arbitrarily large positive (negative) values at feasible vectors; otherwise, it is said to be bounded.

A linear programming problem is said to be feasible if the constraint set is not empty; otherwise it is said to be infeasible. Thus there are three possibilities for a linear programming problem. It may be bounded feasible, it may be unbounded feasible, and it may be infeasible. The value of a bounded feasible maximum (minimum) problem is the maximum (minimum) value of the objective function as the variables range over the constraint set.

More precisely, the maximization (minimization) problem is unbounded if for all $M \epsilon R$ there exists a feasible point x with objective function value greater than (less than) M. It has to be linear in the decision variables, which means it must be the sum of constants times the decision variables and a set of constraints which are limiting feasible decisions.

A feasible solution or feasible point is a point that satisfies all of the constraints. If such a point exists, the problem is feasible; otherwise, it is infeasible. The set of all feasible points is called the feasible region or feasible set.

3.2 The Fundamental Assumptions of Linear Programming

A problem can be realistically represented as a linear program if the following assumptions hold:

i. The constraints and objective function are linear.

   a. This requires that the value of the objective function and the response of each resource expressed by the constraints are proportional to the level of each activity expressed in the variables.

   b. Linearity also requires that the effects of the value of each variable on the values of the objective function and the constraints are additive. In other words, there can be no interactions between the effects of different activities; i.e., the level of activity X_1 should not affect the costs or benefits associated with the level of activity X_2.

ii. Divisibility: the values of decision variables can be fractions. Sometimes these values only make sense if they are integers; then we need an extension of linear programming called integer programming.

iii. Certainty: the model assumes that the responses to the values of the variables are exactly equal to the responses represented by the coefficients.

iv. Data: formulating a linear program to solve a problem assumes that data are available to specify the problem.

3.3 Fundamental theorem of linear programming

If L is a linear system in standard form, then there are just three possibilities:

i. L has an optimal solution with a finite objective function.

ii. L is unbounded (in which case we can return a solution, though the notion of optimal is undefined).

iii. L is infeasible (no solution satisfies all its conditions).

3.4 The Importance of Linear Programming

i. Real static problems lend themselves to linear programming formulations.

ii. Many real life problems can be approximated by linear models.

iii. Many outputs generated by linear programs provides useful "what's best" and "what-if" information.

3.5 The Simplex Method

The Simplex method is the name given to the solution algorithm for solving linear programming problems developed by George Dantzig in 1947. An example is an n-dimensional convex figure that has exactly $n+1$ extreme point. For example, a Simplex in two dimensions is triangle, and in three dimension it is a tetrahedron. The Simplex method refers to the idea of moving from one extreme point to another on the convex set that is formed by the constraint set and non-negativity conditions of the linear programming problem. By solution algorithm we refer to an iterative procedure having fixed computational rules that leads to a solution to the problem in a finite number of steps (i.e., converges to an answer). The Simplex method is algebraic in nature and is based upon the Gauss-Jordan elimination procedure. Consequently, in practice, the algorithm is usually programmed and executed on a digital computer. Many computer codes embodying the essence of the Simplex method are in existence. The widespread development and use of these codes attests to the importance of linear programming in decision making. A fundamental understanding of the linear programming can best be gained by manually working through linear programming problems using the Simplex method.

3.5.1 The Simplex Algorithm

The Simplex algorithm is an iterative procedure that provides a structured method for moving from one basic feasible solution to another, always maintaining or improving the objective function until an optimal solution is obtained. To solve a Simplex algorithm problem algebraically, we must be able to sequentially generate a set m of basic feasible solutions that correspond to the extreme points of the feasible solution space. Naturally, we first must determine an initial basic feasible solution. Recall that a basic solution to a set of m equations in n variable $(n > m)$ is obtained by setting $(n - m)$ variables equal to zero and solving the resulting system of m equations in m variables. The m variables are referred to as the "basic" variables or as the variables "in the basis". The variables are referred to as the non basic variables or as the variables "not in the basis". A basic feasible solution is defined as being a basic solution where all in of the basic variables are non-negative (≥ 0) . A non-degenerate basic feasible solution is defined as being a basic solution, where all m of the basic variables are greater than zero(≥ 0) .

The major steps in the simplex algorithm are as follows:

Step 1: Given the problem formulation with m equalities in n unknowns. Select set of m variables that yield an initial basic feasible solution.

Step 2: Analyse the objective function to see if there is a non-basic variable that is equal to zero in the initial basic feasible solution, but that would improve the value of the objective function if made positive. If no such variable can be found, then the current basic feasible solution is optimal, and the Simplex algorithm stops. If however, such a variable can be found, the Simplex algorithm continues to step 2.

Step 3: Using the non-basic variable selected in step 2. Determine how large it can become before one of the m variables in the current basic feasible solution becomes zero. Eliminate (drive to zero) this current basic variable and replace (increase to the maximum permissible value) the non-basic variable selected in step 2.

Step 4: Solve the problem using the Gauss-Jordan elimination procedure. For the current m variables. Return to step 2.

Given that a feasible solution exists and that the optimal value of the objective function is finite, the Simplex algorithm, as outlined in the preceding steps, will lead to an optimal solution in a finite number of iterations.

We will now proceed to develop a tabular approach for the Simplex algorithm. The advantage of using this tableau Format is that it provides a more structured method for moving from one basic feasible solution to another and allows us to avoid the task of constantly rewriting all of the variables and equations of the problem.

3.5.2 Setting Up Initial Simplex Tableau

In developing a tabular approach for the simplex algorithm, we will attempt to use an instructive and consistent set of notations that will enhance our understanding of the process. The term that are used in the initial Simplex tableau are defined as follows:

C_j = objective function coefficients for variable j

b_i = right-hand-side coefficients (value) for constraint$i$

a_{ij} = coefficients of variable jth in constraint i

C_b = objective function coefficients of the basic variables.

Notation that we will be using extensively in the following development of the Simplex method is as follows:

C Row: the row of objective function coefficients.

b Column: the column of right-hand-side values of the constraint equations.

$[A]$ matrix the matrix (with m rows and n columns) of the coefficients of the variables in the constraint equations.

GENERAL FORM - INITIAL SIMPLEX TABLEAU												
		Decision variables					Slack Variables					
C_j		c_1	c_2	...	c_n		0	0	...	0	**Solution**	**(objective function coeffi- cients)**
C_B	**Basic Vari- able**	x_1	x_2	...	x_n		1		...	s_m		**(Headings**
0	s_1	a_{11}	a_{12}	...	a_{1n}		1	0	...	0		**Constrain coeffi- cients**
...	s_2	a_{21}	a_{22}	...	a_{2n}		0	1	...	0		
0	...	...	...	...	...		...	...	...	...		
	s_m	a_{m1}	a_{m2}	...	a_{mn}		0	0	...	1		
		Z_1	Z_2	...	Z_{mn}	...	Z_{11}	Z_{12}	...	Z_{1m}	**Current value of Ob- jective func- tion**	
	$c_j - Z_j$	$c_1 - Z_1$	$c_2 - Z_2$	...	$c_{mn} - Z_{mn}$	...	$c_{I_2} - Z_{I_1}$	$c_{I_2} - Z_{I_2}$	...	$c_{I_n} - Z_{I_n}$		**Reduced cost (Ne contribu- tion/unit)**

Table 3.1: Initial Simplex Tableau

The initial Simplex tableau is a representation of the standard linear programming form, with some supplemental rows and columns. The top row of the table presents the C ; the objective function coefficients. The next row is the headings for the various columns. Then, there are $c_j - Z_j$ in rows, which represent the constraint coefficients. Following is the Z_j row. Finally, is the row, which is the net contribution per unit of the *jth* variable.

The leftmost column in the tableau indicates the values of the objective function coefficients associated with the basic variables, with a set of constraints, the initial basis will consist of i_n slack variables with C_B values equal to zero. The next column is headed "Basic Variables" and simply lists those i_n variables that are currently basic, or in the solution. The next $i_n \pm n$ columns contain the constraint coefficients; the final (rightmost) column displays the solution values of the basic variables.

3.5.3 Solving Minimization Linear Programming Problems

There is a number of resource allocation problems involved in the minimization of a function, such as costs, rather than the maximization of a function such as profits. Fortunately, the Simplex algorithm applies just as readily to minimization problems as to maximization problems. First, it should be apparent that any minimization problem can be easily converted into an equivalent maximization problem. Thus: Minimizing

$$Z = \sum_{j=1}^{n} c_j x_j \tag{3.1}$$

Is equivalent to Maximizing

$$Z' = \sum_{j=1}^{n} (-c_j) x_j \tag{3.2}$$

for example, Minimize Z = $5x_1$+$2x_2$

Is equivalent to Maximize Z = - $5x_1$- $2x_2$ If we do not choose to employ this conversion, the only difference in the application of the simplex algorithm is that in solving minimization problems we select the non-basic variable with the smallest (i.e., most negative) $c_j - Z_j$ value to enter the basis. The variable removal criterion and the pivoting computations are the same for both maximization and minimization problems. In minimization problems, we know that optimality has been reached when all $c_j - Z_j$ values are zero or positive.

3.6 Sensitivity Analysis

Sensitivity analysis is designed to study the effect of changes in the parameters of the Linear Programming (LP) model on the optimal solution. Such analysis is regarded as an integral part of the (extended) solution of any LP problem. It gives the model a dynamic characteristic that allows the analyst to study the manners of the optimal solution as a result of making changes in the model's parameters. The ultimate objective of the analysis is to obtain information about possible new optimum solutions (corresponding to changes in the parameters) with minimal additional computations.

Sensitivity Problem 1: How much change is allowed in the objective function coefficients?

A change in the objective function coefficients can affect only the gradient of the straight line representing it. The optimum corner point of a given solution space depends totally on the gradient of the objective function.

Sensitivity Problem 2: How much is the worth of a resource unit?

This problem deals with the study of the sensitivity of the optimum solution to changes in the right-hand side of the constraints. If the constraints represent a limited resource, the problem reduces to studying the effect of changing the availability of the resource. The specific goal of this sensitivity problem is to determine the effect of changes in the right-hand side of constraints on the optimum objective value. In essence, the results are given as predetermined ranges of the right-hand side within which the objective optimum value will change (increase or decrease) at a given constant rate.

3.7 Duality and Integer Programming

3.7.1 Duality

Subsequent to any given linear programming problem, called the Primal Problem, there is another linear programming problem called the Dual Problem. Since a given linear programming problem can be stated in several forms (standard form, canonical form, general form etc), it follows that the form of the dual problem will depend on the form of the primal problem.

Dual's General LP ie Converting a primal to a dual or the reverse.

i. Multiply the objective function by -1 and change "max" to "min" or "min" to "max".

ii. Multiply an inequality constraint by -1 to change the direction of the inequality.

iii. Replace an equality constraint

$$\sum_{j=1}^{n} a_{ij}x_j = b_i \tag{3.3}$$

with two inequality constraints

$$\sum_{j=1}^{n} a_{ij}x_j \leq b_i \tag{3.4}$$

$$-\sum_{j=1}^{n} a_{ij}x_j = -b_i \tag{3.5}$$

iv. Replace a variable that is nonpositive with a variable that is its negative. For example, if x_j is specified to be nonpositive by $x_j \leq 0$, replace every occurrence of x_j with $-\hat{x}_j$ and require $-\hat{x}_j \geq 0$

v. Replace a variable that is unrestricted in sign with the difference of two nonnegative variables. For example, if x_j is unrestricted (sometimes called free), replace every occurrence of x_j with $x_j^+ - x_j^-$ require that $x_j^+ - x_j^-$ be nonnegative variables.

Using these transformations, every LP can be converted into an equivalent one in standard form.In this case equivalent means that an optimal solution to the original problem can be obtained from an optimal solution to the new problem.

3.7.2 Integer Programming

Integer Linear Program is a linear program with the additional requirement that some or all of the decision variables must be integer. An integer linear program is said to be an all-integer linear program if all of the variables are required to be integer. An example of two-variables all-integer linear programming model is stated below.

Max $2x_1 + 3x_2$

subject to $2x_1 + 3x_2 \leq 4$

$\frac{2}{3}x_1 + 3x_2$

$x_1 + 2x_2 \leq 6$

x_1,$x_2 \geq 0$ and integer

When the phrase "and integer" is dropped from the above model, we will be left with a two-variable linear program. The linear program that results from dropping the integer requirements for decision variables is referred to as LP Relaxation of the Integer Linear Program.

An integer linear in which some but not all of the decision variables are required to be integer is called a mixed-integer program. When some or all the integer variables are only permitted to assume the values zero and one, then we have binary or 0-1 integer linear program. Capital budgeting and bank location problems are applications of 0-1 integer linear program.

Exact methods for solving linear integer programming optimization (LIP) problems

The development of exact optimization methods for LIP optimization problems during the last 50 years had been very successful. There are at least three different approaches for solving integer programming problems, although they are frequently combined into "hybrid" solution procedures in computational practice. They are considered briefly as follows.

1. Cutting planes algorithms based on polyhedral combinatorics.

2. Enumerative approaches and Branch and Bound, Branch and Cut and Branch and Price methods.

3. Relaxation and decomposition techniques.

Cutting Plane algorithms based on polyhedral combinatorics.

The underlying idea of polyhedral combinatorics is to replace the constraint set of an integer programming problem by an alternative convexification of the feasible points and extreme rays of the problem. Both the size and the complexity of the problems solved have been increased considerably when polyhedral theory was applied to numerical problem solving.

The general cutting plane approach relaxes initially the integrality restrictions on the variables and solves the resulting linear program over the constraint system. In case the linear program is unbounded or infeasible, the same is valid for the integer program. In case the solution to the linear program is integer, this is the optimal solution to the integer program. When the linear program has a not integer optimal solution, then a facet-identification problem has to be solved. Here the objective is to find a linear inequality that "cuts off" the fractional linear programming solution while assuring that all feasible integer points satisfy the inequality, that is, an inequality that "separates" the fractional point from the polyhedron. The terminating conditions for this algorithm are as follows;

i. An integer solution is found (the LIP problem is successfully solved).

ii. The linear program is infeasible and therefore the integer problem is infeasible.

iii. No cut is identified by the facet-identification procedures either because a full description of the facial structure is not known, or because the facet-identification procedures are inexact, that is, there is no possibility for algorithmically generating cuts of a known form.

In case the cutting plane procedure is terminated because of the third possibility, then, in general, the search process has "tightened" the linear programming formulation so that the resulting linear programming solution value is much closer to the integer solution value. Another strategy for cutting-plane algorithms is to maintain integrality and dual feasibility and then to use cuts to obtain primal feasibility.

Enumerative approaches

These approaches are known under different names. The most popular of them are Branch and Bound, implicit enumeration and divide and conquer. The explicit enumeration is the simplest approach to solving a pure integer programming problem by means of enumeration of all possibilities, which are finite in number. However, due to the "combinatorial explosion" of number of these possibilities resulting from the parameter "size", only instances having relative small size could be solved by such an approach within a reasonable computational time limit. Sometimes many possibilities can be implicitly eliminated by domination or feasibility arguments. Besides straight forward or implicit enumeration, the most commonly used enumerative approach is called Branch and Bound (B&B), where the "branching" refers to the enumeration part of the solution technique and "bounding" refers to the fathoming of possible solutions by comparison to a known upper or lower bound on the solution value. A variety of strategies that have been used within the general Branch and Bound framework are being described as follows;

Branch and Cut.

The bounds obtained from the LP-relaxations are often weak, which may cause standard B&B algorithms to fail in practice. It is therefore of crucial importance to tighten the formulation of the problem to be solved. The idea of dynamically adding the cutting planes to the problem is one way of obtaining stronger bounds. Combining cutting plane algorithm with B&B results in a very powerful class of Branch and Cut (B&C) algorithms. The idea is to generate cutting planes throughout the B&B tree of a standard B&B algorithm, in order to get tight bounds at each node. The B&C algorithm consists of following major components:

a. Automatic reformulation procedures.

b. Heuristics which provide "good" feasible integer solutions.

c. Cutting plane procedures which tighten the linear programming relaxation to the linear integer problem under consideration.

These components are embedded into a tree-search framework as in the B&B approach to integer programming; whenever possible, there is used a fourth component:

d. the procedure permanently fixes variables (by reduced cost implications and logical implications) and does comparable conditional fixing throughout the search-tree.

These four components are combined so as to guarantee optimality of the solution obtained at the end of the calculation.

The increasing empirical evidence indicates that both pure and mixed integer programming problems can be solved to proven optimality in economically feasible computation times by methods based on the polyhedral structure of integer programs.

Branch and Price

The philosophy of Branch and Price (B&P) is similar to the one of Branch and Cut. Indeed, the pricing and the cutting are procedures for tightening the LP-relaxation of the problem. In Branch and Price, the concept of column generation is combined with a Branch and Bound algorithm. The simplex algorithm arises at the origin from the column generation concept, where only variables with negative reduced costs are allowed to enter the basis at each iteration. Given an

LP model with a huge number of variables, possibly depending exponentially on the instance size, it would be efficient to consider only the variables potentially improving the objective function. The main idea of column generation is to efficiently determine a variable with negative reduced costs to enter the basis, add it to the problem, resolve it and iteratively repeat this process until no variable with negative reduced costs exists any more.

In general, the method is often used for obtaining LP/LIP models with an exponential number of variables, which provide tighter bounds than the original compact LP/LIP pair. Since column generation is an algorithm for solving LPs, it has to be combined with another method in order to solve LIPs to optimality. The B&P algorithm is the result of combining column generation with B&B problems. Routing and scheduling are the most suitable areas for application of Branch and Price methods.

From a theoretical point of view, B&C and B&P are closely related, since column generation in the primal problem corresponds to cut generation in the dual and vice-versa. Furthermore, B&C and B&P can be combined in the so called Branch and Cut and Price algorithms, where both cuts and variables are dynamically generated.

Relaxation and Decomposition Method

There are three wide spread approaches for relaxation of the general LIP problem, which are designed to find an upper bound of the optimal value for the maximizing LIP problem: Linear Programming (LP) relaxation, Combinatorial relaxation and Lagrangian relaxation. The first two approaches extend the feasible domain without any change in the objective function of the problem. The third approach provides another maximizing objective function, which has the same or greater value in a fixed feasible domain.

The LP relaxation for the Integer Programming model is obtained by dropping the integrality constraints on the variables. For realization of the combinatorial relaxation there are at least two approaches exploiting the combinatorial structure of the problem. The first approach is based on the concept of valuated matroids, introduced by Dress and Wenzel. The other approach, which is called the structural approach, utilizes algorithms to compute an upper bound on the objective function and is often based on a graph-theoretic method.

Considering LP relaxation, it was mentioned that relaxing the integrality restriction is one approach to solution of linear integer programming problems. But, this is not the only approach to relaxing the problem. The idea of dropping constraints can be embedded into a more general framework, called Lagrangian relaxation. This is an alternative approach, where a set of "complicating" constraints is included into the objective function in a Lagrangian fashion (with fixed multipliers that are iteratively changed). The complicating constraints are removed from the constraint set. In this way the resulting sub-problem could be solved considerably easier.

To realize a Lagrangian relaxation it is necessary that the structure of the problem being solved is clear in order to relax then the constraints that are "complicating". A related approach which attempts to strengthen the bounds of Lagrangian relaxation is called Lagrangian decomposition. This approach consists of isolating sets of constraints. In this way are obtained separately, easy problems to solve over each of the subsets. The dimension of the problem is increased by creating linking variables which link the subsets. All Lagrangian approaches are problem dependent. There is developed no general theory applicable to say, in arbitrary zero-one or LIP problems underlying polyhedral structure of these problems. Thus, in order to use this approach, one must be able to both identify specific mathematical structures inherent in the problem and then study the polyhedron associated with that structure.

3.8 Branch and Bound Method

There are several ways of solving Pure Integer and Mixed Integer Programming problems. One of the best known and widely used ones is the Branch and Bound method. The Branch and Bound method is the basic workhorse technique for solving integer and discrete programming problems. The method is based on the observation that the enumeration of integer solutions has a tree structure.

The name of the method comes from the branching that happens when a node is selected for further growth and the next generation of children of that node is created. The bounding comes in when the bound on the best value attained by growing a node is estimated.

Now the main idea in branch and bound is to avoid growing the whole tree as much as possible, because the entire tree is just too big in any real problem. Instead Branch and Bound grows the tree in stages, and grows only the most promising nodes at any stage. It determines which node is the most promising by estimating a bound on the best value of the objective function that can be obtained by growing that node to later stages.

Another important aspect of the method is pruning, in which you cut off and permanently discard nodes when you can show that it, or any its descendants, will never be either feasible or optimal. The name derives from gardening, in which pruning means to clip off branches on a tree. Pruning is one of the most important aspects of branch and bound since it is precisely what prevents the search tree from growing too much. To describe branch and bound in detail, we first need to introduce some terminologies. Also to completely specify how the process is to proceed, we need to define policies concerning selection of the next node, selection of the next variable, how to prune, and when to stop.

Terminologies

i. Node is any partial or complete solution. For example, a node that is two levels down in a 5-variable problem might represent the partial solution 3-17-?-?-? in which the first variable has a value of 3 and the second variable has a value of 17. The values of the last three variables are not yet set.

ii. Leaf (leaf node) is a complete solution in which all of the variable values are known.

iii. Bud (bud node) is a partial solution, either feasible or infeasible. Think of it as a node that might yet grow further, just as on a real tree.

iv. Bounding function is the method of estimating the best value of the objective function obtainable by growing a bud node further. Only bud nodes have associated bounding function values. Leaf nodes have objective function values, which are actual values and not estimates. It is important that the bounding function be an optimistic estimator. In other words, if you are minimizing, it must underestimate the actual best achievable objective function value; if maximizing it must overestimate the best achievable objective function value.

v. Branching, growing, or expanding a node is the process of creating the child nodes for a bud node. One child node is created for each possible value of the next variable. For example, if the next variable is binary, there will be one child node associated with the value zero and one child node associated with the value one.

vi. Incumbent is the best complete feasible solution found so far. There may not be an incumbent when the solution process begins. In that case, the first complete feasible solution found during the solution process becomes the first incumbent.

The three popular policies for node selection are;

a. Best-first or global-best node selection: choose the bud node that has the best value of the bounding function anywhere on the branch and bound tree. If we are minimizing, this means choosing the bud node with the smallest value of the bounding function; if maximizing choose the bud node with the largest value of the bounding function.

b. Depth-first: choose only from among the set of bud nodes just created. Choose the bud node with the best value of the bounding function. Depth-first node selection takes you one step deeper into the branch and bound tree at each iteration, so it reaches the leaf nodes quickly. This is one way of achieving an early incumbent solution. If you cannot proceed any deeper into the tree, back up one level and choose another child node from that level.

c. Breadth-first: expand bud nodes in the same order in which they were created.

How to Prune

As mentioned above, there are two main reasons to prune a bud node.

1.0 Showing that no descendent will be feasible.

Methods for showing that no descendant can ever be feasible vary with the specific problem. In problems that include standard arithmetic constraints, it is sometimes easy to detect this condition. For example, consider a partial solution $(x_1, x_2, x_3, x_4) = (1,1,?,?)$ in an all-binary problem which has the constraint $-10x_1 - 5x_2 + 6x_3 + 4x_4 \geq 0$. Now that x_1 and x_2 are both set to 1, there are no possible settings of x_3 and x_4 which will satisfy the constraint. Hence we can deduce that all of the descendants of this bud node will be infeasible, so the node is pruned.

2.0 Showing that no descendent will be optimal.

The method for showing that no descendant will be optimal is standard. If the bud node bounding function value is worse than the objective function value for the incumbent, then the bud node can be pruned. This is because the bounding function is an optimistic estimator. Suppose we are maximizing and the incumbent solution has an objective function value of 87. If the optimistic bounding function overestimates for the bud node having a value of only 79, then we know that no descendent of the bud node will ever exceed 79, let alone 87, and so none of the descendants can ever be optimal. So the bud node is pruned. The same reasoning applies in reverse if minimizing.

Finally, we need a terminating rule to tell us when to stop expanding the branch and bound tree. To guarantee that we have reached optimality, we stop when the incumbent solution's objective function value is better than or equal to the bounding function value associated with all of the bud nodes.

Chapter 4

Data Collection and Analysis

4.1 Introduction

Profile Of the Hospital

The Eastern Regional Hospital was established in 1962, and doubles as the Municipal Hospital for the New Juaben Municipality with its 155000 inhabitants. There have been two major structural additions to it since its establishment. The first was in 1972 when an Adult Out Patient Department (OPD) with Medical Records, Internal Medicine, Kids, Surgical and Maternity Wards with Theatre were added.

In 1998 additional Kitchen, Main Theatre, X-ray Department, Laundry and Mortuary Departments were included. There has been no major rehabilitation or additional structures since then.

The 323-bed hospital compared to 314 in 2004 now serve as a referral point for about sixteen(16) district hospitals in the Eastern Region. The hospital offers the following services: Internal Medicine including Anti Retro viral Therapy, Paediatrics, Surgery, ~~Medicine~~, Dental, Ophthalmology, Physiotherapy, Ear, Nose and Throat, Pharmacy, Laboratory, X-ray, Ultrasound, Catering and Hospitality, Laundry, Mortuary and Primary Healthcare Services.

The Regional hospital is a Ghana Health Service facility- A secondary referral level-not for profit healthcare organization. Their intention is to satisfy their numerous customers, be they: patients, visitors to patients, inpatients, corporations, supplier's maintenance team, staff and employers to Ghana Health Service and Ministry of Health.

They also strive to imbue in their employees the spirit of service to their customers/clients to achieve a much satisfied clientle.

The total nurses in the various departments are being determined by the Matron. The Matron ensures, that the assigned number of nurses comprises of the various ranks that they have available. The hospital operates within 24 hrs. and this is broken down into the three shifts except certain departments. The morning shift runs from (7am - 3pm) whiles afternoon shift (3pm-11pm) and night shift (11pm-7am). These departments are the Dot Centre, Diabetes Centre, Antenatal Care, Anti-Retro viral Therapy, Ear, Nose and Throat, that operates within 8am-5pm. Currently, the hospital's scheduling is Cyclical. This means that, the same schedule is repeated as long as the requirements do not change. These kind of scheduling are easy to build but may be very rigid and may adapt difficulty to change. When it comes to the matron's realization that the number of patients who visits the hospital on certain days is not equal, she makes sure that more nurses are put on shift than any other day in order to prevent long queues in the hospital and these days are termed as their peak days.

Table 4.1: Rank of nurses in Eastern Regional Hospital

Rank of Nurses	Total
Deputy Director of Nursing Service(DDNS)	1
Principle Nursing Officer(PNO)	13
Senior Nursing Officer(SNO)	20
Nursing Officer(N/O)	21
Senior Staff Nursing Midwifery(SSNM)	2
Senior Staff Nurse(SSN)	11
Staff Nurse(SN)	34
Principal Midwifery Officers(PMO)	10
Senior Midwifery Officers(SMO)	8
Midwifery officer(M/O)	11
Senior Staff Midwives(SSM)	13
Staff Midwife(SM)	12
Enrolled Nurse Superintendent(EN/SUPDT)	73
Principle Enrolled Nurse(PEN)	7
Senior Enrolled Nurse(SEN)	6
Health Assistant Clinical(HAC)	14
Community Health Nurse(CHN)	4

Table 4.2: Allocation of Nurses in the department/ward.

	Rank of Nurses																
DEPARTMENT	DDNS	PNO	SNO	N/O	SSNM	SSN	SN	PMO	SMO	M/O	SSM	SM	EN/SUPT	PEN	SEN	HAC	CH
Surgical	-	-	2	-	-	3	5	-	-	-	-	-	4	1	-	5	-
Recovery	-	1	1	1	-	1	1	-	-	-	-	-	3	-	-	3	-
Lying-In	-	-	2	-	-	1	-	1	-	2	2	3	-	-	-	2	-
Labour	-	-	-	1	-	-	-	2	-	2	2	3	-	-	-	2	-
Psychiatry	-	-	-	-	-	2	9	-	-	-	-	-	1	-	-	-	-
Female	-	-	1	-	-	2	5	-	-	-	-	-	1	-	-	-	-
Male	-	1	-	-	-	-	5	-	-	-	-	-	5	-	-	-	-
Prenatal Gynaec	-	-	-	1	-	-	-	3	3	4	3	2	-	-	-	-	-
Adult OPD	-	1	-	2	-	2	1	-	1	-	-	-	1	1	-	7	-
Theatre	-	-	2	3	-	1	3	-	-	-	-	-	3	-	-	3	-
Special baby care	-	-	1	-	-	-	1	-	-	3	-	-	-	2	-	2	-
Kids	-	-	-	2	-	-	5	-	-	-	-	-	4	-	-	-	-

4.2 Assumptions

The following assumptions are made for the modelling of the optimal scheduling of the nurses, which is based on the requirements made by the hospital in preparing their schedule. These are as follows;

i. There must be at least one Principal Nurse Officer among the total number of nurses assigned to each department.

ii. Every nurse must work at most 8hrs per shift.

iii. It is assumed that if one works on a night shift, he/she is entitled to a day-off.

4.3 Formulation of decision variables

In modelling the problem to find the optimal solution to the nurse scheduling for each day, the following variables will be used.

Notations

X_1 is the number of nurses assigned to morning shift.

X_2 is the number of nurses assigned to afternoon shift.

X_3 is the number of nurses assigned to night shift.

X_4 is the number of nurses on leave.

q is the minimum number of nurses available for morning shift.

r is the minimum number of nurses available for afternoon shift.

s is the minimum number of nurses available for night shift.

t is the minimum number of nurses on leave.

4.4 Formulation of objective function

To find the minimum number of nurses that would be needed to optimize the quality of health service delivery at the central hospital; the objective function is stated as

$MinimizeR = X_1 + X_2 + X_3 + X_4$, where R is number of nurses required for each day of the week.

4.5 Formulation of constraints

q,r and s are the constants representing the minimum number for morning shift, afternoon shift and night shift constraints respectively and t is the maximum number of nurses for off duty and leave respectively. The constraints on the nurses are as follows:

i. Here there should not be a case whereby those on leave will exceed those on the morning, afternoon or night shift.

ii. There should be at least one nurse for each shift.

iii. The number of nurses on morning shift and afternoon shifts must be greater than those on night shift.

These constraints are mathematically stated as follows:

1. $X_1 \geq q$ for morning shift

2. $X_2 \geq r$ for afternoon shift

3. $X_3 \geq s$ for night shift

4. $X_4 \leq t$ for those on leave

5. $X_1 + X_2 + X_3 \geq X_4$

6. $X_1 \geq X_4$

7. $X_2 \geq X_4$

8. $X_3 \geq X_4$

9. $X_1 + X_2 \geq X_3$

$X_1, X_2, X_3, X_4 \geq 1$ and integer.

Monday

$X_1 \geq 56$ for morning shift

$X_2 \geq 33$ for afternoon shift

$X_3 \geq 22$ for night shift

$X_4 \leq 16$ for those on leave

$X_1 \geq X_4$

$X_2 \geq X_4$

$X_3 \geq X_4$

$X_1 + X_2 \geq X_3$

$X_1 + X_2 + X_3 \geq X_4$

where $X_1, X_2, X_3, X_4 \geq 1$ and integer

Tuesday

$X_1 \geq 54$ for morning shift

$X_2 \geq 31$ for afternoon shift

$X_3 \geq 21$ for night shift

$X_4 \leq 15$ for those on leave

$X_1 \geq X_4$

$X_2 \geq X_4$

$X_3 \geq X_4$

$X_1 + X_2 \geq X_3$

$X_1 + X_2 + X_3 \geq X_4$

where $X_1, X_2, X_3, X_4 \geq 1$ and integer

Wednesday

$X_1 \geq 47$ for morning shift

$X_2 \geq 37$ for afternoon shift

$X_3 \geq 24$ for night shift

$X_4 \leq 6$ for those on leave

$X_1 \geq X_4$

$X_2 \geq X_4$

$X_3 \geq X_4$

$X_1 + X_2 \geq X_3$

$X_1 + X_2 + X_3 \geq X_4$

where $X_1, X_2, X_3, X_4 \geq 1$ and integer

Thursday

$X_1 \geq 52$ for morning shift

$X_2 \geq 32$ for afternoon shift

$X_3 \geq 22$ for night shift

$X_4 \leq 10$ for those on leave

$X_1 \geq X_4$

$X_2 \geq X_4$

$X_3 \geq X_4$

$X_1 + X_2 \geq X_3$

$X_1 + X_2 + X_3 \geq X_4$

where $X_1, X_2, X_3, X_4 \geq 1$ and integer

Friday

$X_1 \geq 53$ for morning shift

$X_2 \geq 31$ for afternoon shift

$X_3 \geq 24$ for night shift

$X_4 \leq 14$ for those on leave

$X_1 \geq X_4$

$X_2 \geq X_4$

$X_3 \geq X_4$

$X_1 + X_2 \geq X_3$

$X_1 + X_2 + X_3 \geq X_4$

where $X_1, X_2, X_3, X_4 \geq 1$ and integer

Saturday

$X_1 \geq 37$ for morning shift

$X_2 \geq 28$ for afternoon shift

$X_3 \geq 23$ for night shift

$X_4 \leq 10$ for those on leave

$X_1 \geq X_4$

$X_2 \geq X_4$

$X_3 \geq X_4$

$X_1 + X_2 \geq X_3$

$X_1 + X_2 + X_3 \geq X_4$

where $X_1, X_2, X_3, X_4 \geq 1$ and integer

Sunday

$X_1 \geq 31$ for morning shift

$X_2 \geq 27$ for afternoon shift

$X_3 \geq 23$ for night shift

$X_4 \leq 12$ for those on leave

$X_1 \geq X_4$

$X_2 \geq X_4$

$X_3 \geq X_4$

$X_1 + X_2 \geq X_3$

$X_1 + X_2 + X_3 \geq X_4$

where $X_1, X_2, X_3, X_4 \geq 1$ and integer

4.6 Results and Discussion

The data collected and formulated was solved with QM for Windows.

The data collected was formulated as an integer linear program problem and solved with Quantitative Management software called QM for Windows. This software is used for quantitative analysis by management in their decision making.

Generally, our interest is to find the minimum number of nurses required by the hospital for each day of the week in order to ensure efficient service delivery. Solutions obtained showed that nurses must be made available to the Hospital for each of the shifts whereas no allocations were made for those to be on leave/ off duty throughout the week. Below are the detailed results.

Table 4.3: The optimal nurse scheduling for Mondays

X_1	56
X_2	33
X_3	22
X_4	0

Initially the total nurses needed for Monday was 127 but after solving the problem it reduced to 111. The 111 nurses are allocated to morning shift, afternoon shift and night. The remaining of 16 nurses could be shared accordingly as to those who will be on leave or off duty.

The optimal nurse scheduling for Tuesdays The minimum number of nurses required for Tuesday is 106. This number is allocated into the minimum number required for Morning, afternoon and evening shifts as shown in the table above. The remaining of 15 nurses could be shared accordingly to any job assignment at

Table 4.4: The optimal nurse scheduling for Tuesday

X_1	54
X_2	31
X_3	21
X_4	0

the judgement of the Head nurse or hospital authorities.

Table 4.5: The optimal nurse scheduling for Wednesdays

X_1	47
X_2	37
X_3	24
X_4	0

A minimum number of nurses needed on Wednesday is 108. This is further broken into 47 nurses being allocated to morning shift, 37 nurses to afternoon shift and 24 to night shift. Therefore no allocation is made for off duty/ leave. The remaining 6 would have to be on standby in case off any emergency.

Table 4.6: The optimal nurse scheduling for Thursday

X_1	52
X_2	32
X_3	22
X_4	0

The minimum number of nurses required for Thursday is 106. This allocates nurses to morning shift, afternoon shift and night as shown in the table above. The remaining 10 nurses could be added to periods where the Leadership deems necessary in order to reduce pressure on the nurses at each session at the hospitals own discretion.

The minimum number of nurses required for Friday is 108. This is the number of nurses to be allocated to morning shift, afternoon shift and night as shown in the table above. The remaining of 14 nurses could be shared accordingly as to

Table 4.7: The optimal nurse scheduling for Friday

X_1	53
X_2	31
X_3	24
X_4	0

those who will be on leave or off duty at the hospital's Head nurse. Originally the

Table 4.8: The optimal nurse scheduling for Saturday

X_1	37
X_2	28
X_3	23
X_4	0

total nurses needed for Saturday was 98 but after solving the problem it reduced to 88. The 88 nurses are allocated to morning shift, afternoon shift and night. The remaining of 10 nurses could be shared accordingly as to when there is more patience than the accustomed.

Table 4.9: The optimal nurse scheduling for Sunday

X_1	31
X_2	27
X_3	23
X_4	0

A minimum number of nurses needed on Sunday is 81. This is further broken into 31 nurses being allocated to morning shift, 27 nurses to afternoon shift and 23 to night shift. Therefore no allocation is made for off duty/ leave. The remaining 12 would have to be on standby in case off any emergency.

4.7 Sensitivity analysis on the entire solution

Sensitivity analysis is often used for integer linear programming problem than the LP problem. That is a very small change in one of the coefficients in the constraints can cause a relatively large change in the optimal value. In our case, any time there is a change in the total number of nurses on the various days then the integer linear program problem has to be resolved with slight variation in the coefficients before an optimal solution is chosen for implementation.

Chapter 5

Conclusion and Recommendation

5.1 Conclusion

Throughout the week, the optimal number of nurses for Tuesdays and Thursdays as well as Wednesdays and Fridays were 106 and 108 respectively. The mean number of nurses needed to be at work for the entire week is 103 whereas those on morning shift, afternoon shift and night shift of the mean number are 47, 31 and 23 nurses respectively at all times.
The optimal solution for Monday is 111 nurses. Among these 56 nurses represent morning shift, 33 nurses for afternoon shift and 22 nurses for night shift.
Tuesday's optimal solution is 106 nurses. Among these 54 nurses is for morning shift, 31 for afternoon shift and 21 for night shift.
The optimal solution for Wednesday is 108 nurses. These include 47 nurses stands for morning shift, 33 nurses for afternoon shift and 22 nurses for night shift.
The optimal solution for Thursday is 106 nurses. These include 52 nurses standing for morning shift, 32 nurses for afternoon shift and 22 nurses for night shift.
Friday has 53 nurses on morning shift, 31 nurses on afternoon shift and 24 nurses on night shift. Hence the optimal solution obtained is 108.
The optimal solution for Saturday is 98 nurses. Amongst this 98, 37 nurses stands for morning shift, 28 nurses for afternoon shift and 23 nurses for night shift.

The optimal solution for Sunday is 81 nurses. 31 nurses representing morning shift, 27 nurses for afternoon shift and 23 nurses for night shift.

Nevertheless, the model as a result provided the minimum number of nurses required for each day and recorded zero for off duty and leave. It was also noticed that all the optimal number of nurses for the various days fell within the overall number of nurses that the hospital have employed. This means that the hospital is not under staffed in respect of nurses.

5.2 Recommendations

1. It is essential for the Nursing profession to be on off duty or Leave, therefore necessary considerations could be made by management as to those to be on leave or off duty whiles meeting the minimum requirements of nurses for each day by ensuring that there are standby nurses to replace anybody who intend to go on any form of leave.

2. In order to reduce partiality and discrepancies in the scheduling for each department, we recommend that the hospital uses a scientific approach like the integer linear program on a long term.

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Table A.2: Nurses available on Tuesdays

Department/Ward	Morning shift	Afternoon shift	Night shift	On Leave/off duty
Surgical	5	3	3	2
Recovery	5	4	2	-
Lying-In	4	2	2	1
Labour	3	2	3	3
Psychiatry	7	3	2	-
Female	4	3	2	3
Male	5	4	1	1
Prenatal Gynaec	6	4	2	3
Adult OPD	5	2	-	-
Theatre	2	-	-	-
Special baby care	3	2	2	1
Kids	5	2	2	1
Total	54	31	21	15

Table A.3: Nurses available on Wednesdays

Department/Ward	Morning shift	Afternoon shift	Night shift	On Leave/off duty
Surgical	6	3	3	1
Recovery	6	5	2	-
Lying-In	3	2	2	1
Labour	3	3	3	-
Psychiatry	6	4	2	2
Female	4	3	2	-
Male	5	4	1	-
Prenatal Gynaec	5	4	2	2
Adult OPD	2	2	2	-
Theatre	-	2	1	-
Special baby care	2	2	2	-
Kids	5	3	2	1
Total	47	37	24	6

Appendix A

Table A.1: Nurses available on Mondays

Department/Ward	Morning shift	Afternoon shift	Night shift	On Leave/off duty
Surgical	5	2	3	2
Recovery	5	4	2	-
Lying-In	4	2	2	1
Labour	3	3	3	1
Psychiatry	8	3	2	-
Female	4	3	2	3
Male	5	3	1	1
Prenatal Gynaec	5	5	2	3
Adult OPD	6	2	-	-
Theatre	2	1	-	2
Special baby care	3	2	2	-
Kids	6	3	3	3
Total	56	33	22	16

Table A.4: Nurses available on Thursdays

Department/Ward	Morning shift	Afternoon shift	Night shift	On Leave/off duty
Surgical	5	3	3	3
Recovery	5	3	2	1
Lying-In	4	2	2	-
Labour	3	2	3	2
Psychiatry	10	4	2	-
Female	4	3	2	-
Male	5	3	1	1
Prenatal Gynaec	4	5	2	-
Adult OPD	5	1	-	-
Theatre	1	1	1	2
Special baby care	2	2	2	2
Kids	4	3	2	-
Total	52	32	22	10

Table A.5: Nurses available on Fridays

Department/Ward	Morning shift	Afternoon shift	Night shift	On Leave/off duty
Surgical	6	3	3	-
Recovery	5	3	2	3
Lying-In	3	2	2	1
Labour	3	2	3	3
Psychiatry	8	5	2	-
Female	5	3	2	3
Male	3	2	2	1
Prenatal Gynaec	4	4	3	1
Adult OPD	6	2	-	-
Theatre	2	-	1	2
Special baby care	3	2	2	-
Kids	5	3	2	-
Total	53	31	24	14

Table A.6: Nurses available on Saturdays

Department/Ward	Morning shift	Afternoon shift	Night shift	On Leave/off duty
Surgical	4	4	3	-
Recovery	3	3	2	2
Lying-In	3	2	2	1
Labour	3	3	3	2
Psychiatry	4	3	2	-
Female	3	2	2	1
Male	3	3	2	1
Prenatal Gynaec	5	2	2	-
Adult OPD	3	1	-	-
Theatre	-	-	1	2
Special baby care	2	2	2	1
Kids	4	3	2	-
Total	37	28	23	10

Table A.7: Nurses available on Sundays

Department/Ward	Morning shift	Afternoon shift	Night shift	On Leave/off duty
Surgical	4	3	3	2
Recovery	3	3	2	2
Lying-In	3	2	2	1
Labour	2	2	3	1
Psychiatry	3	3	2	-
Female	3	2	2	-
Male	4	2	2	1
Prenatal Gynaec	4	4	2	-
Adult OPD	-	-	-	-
Theatre	-	1	1	2
Special baby care	2	2	2	1
Kids	3	3	2	2
Total	31	27	23	12

Optimal number of nurses for Monday's solution						
	morning shift	afternoon shift	night shift	off duty/leave		RHS
Minimize	1.	1.	1.	1.		
morning Constraint	1.	0.	0.	0.	>=	56.
afternoon Constraint	0.	1.	0.	0.	>=	33.
night Constraint	0.	0.	1.	0.	>=	22.
off duty/ leave Constraint	0.	0.	0.	1.	<=	16.
morning and leave/ off duty Constraint	1.	0.	0.	-1.	>=	0.
afternoon and leave/ off duty Constraint	0.	1.	0.	-1.	>=	0.
Morning, afternoon and leave/ off duty Constraint	0.	0.	1.	-1.	>=	0.
Morning, afternoon and leave/ off duty Constraint	1.	1.	-1.	0.	>=	0.
Morning, afternoon, night and leave/ off duty Constraint	1.	1.	1.	-1.	>=	0.
Solution->	56.	33.	22.	0.	Optimal Z->	111.

Table A.8: Optimal number nurse for Monday's solution

Optimal number of nurses for Tuesday's solution						
	morning shift	afternoon shift	night shift	off duty/leave		RHS
Minimize	1.	1.	1.	1.		
morning Constraint	1.	0.	0.	0.	>=	54.
afternoon Constraint	0.	1.	0.	0.	>=	31.
night Constraint	0.	0.	1.	0.	>=	21.
off duty/leave Constraint	0.	0.	0.	1.	<=	15.
morning and leave/ off duty Constraint	1.	0.	0.	-1.	>=	0.
afternoon and leave/ off duty Constraint	0.	1.	0.	-1.	>=	0.
night and leave/ off duty Constraint	0.	0.	1.	-1.	>=	0.
morning afternoon and leave/ off duty Constraint	1.	1.	-1.	0.	>=	0.
morning afternoon night and leave/ off duty Constraint	1.	1.	1.	-1.	>=	0.
Solution->	54.	31.	21.	0.	Optimal Z->	106.

Table A.9: Optimal number nurse for Tuesday's solution

Optimal number of nurses for Wednesday's solution						
	morning shift	afternoon shift	night shift	off duty/leave		RHS
Minimize	1.	1.	1.	1.		
morning Constraint	1.	0.	0.	0.	>=	47.
afternoon Constraint	0.	1.	0.	0.	>=	37.
night Constraint	0.	0.	1.	0.	>=	24.
off duty/ leave Constraint	0.	0.	0.	1.	<=	6.
morning and leave/ off duty Constraint	1.	0.	0.	-1.	>=	0.
afternoon and leave/ off duty Constraint	0.	1.	0.	-1.	>=	0.
night and leave/ off duty Constraint	0.	0.	1.	-1.	>=	0.
morning afternoon and leave/ off duty Constraint	1.	1.	-1.	0.	>=	0.
morning afternoon night and leave/ off duty Constraint	1.	1.	1.	-1.	>=	0.
Solution->	47.	37.	24.	0.	Optimal Z->	108.

Table A.10: Optimal number nurse for Wednesday's solution

Optimal number of nurses for Thursday's solution						
	morning shift	afternoon shift	night shift	off duty/leave		RHS
Minimize	1.	1.	1.	1.		
morning Constraint	1.	0.	0.	0.	>=	52.
afternoon Constraint	0.	1.	0.	0.	>=	32.
night Constraint	0.	0.	1.	0.	>=	22.
off duty/ leave Constraint	0.	0.	0.	1.	<=	10.
morning and leave/ off duty Constraint	1.	0.	0.	-1.	>=	0.
afternoon and leave/ off duty Constraint	0.	1.	0.	-1.	>=	0.
night and leave/ off duty Constraint	0.	0.	1.	-1.	>=	0.
morning afternoon and leave/ off duty Constraint	1.	1.	-1.	0.	>=	0.
morning afternoon night and leave/ off duty Constraint	1.	1.	1.	-1.	>=	0.
Solution->	52.	32.	22.	0.	Optimal Z->	106.

Table A.11: Optimal number nurse for Thursday's solution

Optimal number of nurses for Friday's solution						
	morning shift	afternoon shift	night shift	off duty/leave		RHS
Minimize	1.	1.	1.	1.		
morning Constraint	1.	0.	0.	0.	>=	53.
afternoon Constraint	0.	1.	0.	0.	>=	31.
night Constraint	0.	0.	1.	0.	>=	24.
off duty/ leave Constraint	0.	0.	0.	1.	<=	14.
morning and leave/ off duty Constraint	1.	0.	0.	-1.	>=	0.
afternoon and leave/ off duty Constraint	0.	1.	0.	-1.	>=	0.
night and leave/ off duty Constraint	0.	0.	1.	-1.	>=	0.
morning afternoon and leave/ off duty Constraint	1.	1.	-1.	0.	>=	0.
morning afternoon night and leave/ off duty Constraint	1.	1.	1.	-1.	>=	0.
Solution->	53.	31.	24.	0.	Optimal Z->	108.

Table A.12: Optimal number nurse for Friday's solution

Optimal number of nurses for Saturday's solution						
	morning shift	afternoon shift	night shift	off duty/leave		RHS
Minimize	1.	1.	1.	1.		
morning Constraint	1.	0.	0.	0.	>=	37.
afternoon Constraint	0.	1.	0.	0.	>=	28.
night Constraint	0.	0.	1.	0.	>=	23.
off duty/ leave Constraint	0.	0.	0.	1.	<=	10.
morning and leave/ off duty Constraint	1.	0.	0.	-1.	>=	0.
afternoon and leave/ off duty Constraint	0.	1.	0.	-1.	>=	0.
night and leave/ off duty Constraint	0.	0.	1.	-1.	>=	0.
morning afternoon and leave/ off duty Constraint	1.	1.	-1.	0.	>=	0.
morning afternoon night and leave/ off duty Constraint	1.	1.	1.	-1.	>=	0.
Solution->	37.	28.	23.	0.	Optimal Z->	88.

Table A.13: Optimal number nurse for Saturday's solution

Optimal number of nurses for Sunday's solution						
	morning shift	afternoon shift	night shift	off duty/leave		RHS
Minimize	1.	1.	1.	1.		
morning Constraint	1.	0.	0.	0.	>=	31.
afternoon Constraint	0.	1.	0.	0.	>=	27.
night Constraint	0.	0.	1.	0.	>=	23.
off duty/leave Constraint	0.	0.	0.	1.	<=	12.
morning and leave/ off duty Constraint	1.	0.	0.	-1.	>=	0.
afternoon and leave/ off duty Constraint	0.	1.	0.	-1.	>=	0.
night and leave/ off duty Constraint	0.	0.	1.	-1.	>=	0.
morning afternoon and leave/ off duty Constraint	1.	1.	-1.	0.	>=	0.
morning afternoon night and leave/ off duty Constraint	1.	1.	1.	-1.	>=	0.
Solution->	31.	27.	23.	0.	Optimal Z->	81.

Table A.14: Optimal number nurse for Sunday's solution