KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI

INSTITUTE OF DISTANCE LEARNING

A CONDITIONAL P-CENTER LOCATION PROBLEM OF THREE WAREHOUSES FOR UNILEVER GHANA LIMITED: A CASE STUDY OF THE ASHANTI REGION OF GHANA

By

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A Thesis submitted to the Institute of Distance Learning, Kwame Nkrumah University of Science and Technology in partial fulfillment of the requirements for the award of the Master of Science degree in industrial mathematics

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DECLARATION

I hereby declare that this submission is my own work towards the MSc and that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the University, except where due acknowledgement has been made in the text.

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ABSTRACT

Location problems deal with finding the right site where one or more new facilities should be placed in order to optimize some criteria, such as, minimizing cost or travel distance from demand points and so on.

A location problem is considered as a conditional p-centre problem when we are given the location of existing facilities and we are to locate p additional facility or facilities so as to minimize the maximum travel distance between the demand points, each to its nearest facility whether existing or new.

This thesis considers the problem of locating three additional warehouses for Unilever Ghana Limited as a p-center problem given that some existing warehouses are already located in the Ashanti Region of Ghana. The Berman and Drezner (2008) method was used on an 18-node network which had three existing warehouses and three closest nodes to these facilities. Three additional facility locations were added. Three sites, namely Konongo, Ejisu and Suame were determined by the method.

Factor rating method was used to determine that Konongo was located first, followed by Ejisu and Suame.

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DEDICATION

This research work is dedicated to the almighty God for his divine grace, guidance and protection that has brought me this far.

I would also like to dedicate this work to my parents, Mr. Anyabilla Amalis of blessed memories and Madam Ndebugri Comfort A. and all my siblings whose spiritual and moral support has sustained me through.

Finally, to my wife, Mrs. Debora A. Anyabilla ,my lovely children Dora A. Awinbun and Irene A. Azabugwin without whose moral and spiritual support in addition to their encouragement l could not have come this far.

TABLE OF CONTENTS

Contents **Page**

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

1.0 INTRODUCTION

Facility location problems have occupied an important place in operation research since the early 1960's. Almost every public and private sector enterprise that we can think of has been faced with the problem of locating facilities. Government agencies need to determine locations of offices and other public services such as schools, hospitals, fire stations, ambulance buses etc. Industrial firms must determine locations for fabrication and assembly plants as well as warehouses. But it must be noted that the success or failure of facilities depends partly on the locations chosen for those facilities. A location problem establishes a set of facilities to minimize the cost of satisfying a set of demands (customers) with respect to a set of constraints. There are four components that describe location problems; customers, who are assumed to be already located at points or on routes, facilities that will be located, space in which customers and facilities are located and a metric that indicates geographical and chronological distances between customers and facilities. Generally, location problems deals with finding the right site where one or more new facilities should be placed in order to optimize some criteria, which are usually related to the distance from the facilities to the demand points.

Facilities are assets that are built, installed or establish to enhance the quality or facilitate the use of land for a particular purpose.

Location basically is the act of putting something in place or position where it can be identified.

Location models are used in a variety of applications such as locating warehouses within supply chain to minimize average time to market, locating automatic teller machines to best serve the banks customers, locating noxious materials (e.g. waste dumps) to maximize its distance to the public etc. (Hale and Moberg 2003).

A warehouse is a commercial building for storage of goods or products. Warehouses are used by manufactures like Unilever Ghana Limited, importers, exporters, wholesalers, transport businesses, customs etc. They are usually large buildings in industrial areas of cities (like Kumasi), towns and villages. They load and unload goods /products from trucks.

This thesis aims to locate three additional warehouses for Unilever Ghana Limited in Ashanti Region.

A location problem is considered as a conditional p-centre problem when we are given the locations of existing facilities and we are to locate p additional facility or facilities so as to minimize the maximum distance between the demand points , each to its nearest facility whether existing or new. The conditional location problem then is to locate new facilities to serve a set of demand points given that facilities are already located. When q is equal to zero (q = 0), the problem is unconditional. In conditional p – centre problems, once the new locations are determined, a demand can be supplied either by one of the existing or by one of the new facilities whichever is the closest facility to the demand (Berman and Drezner, 2008)

1.1 HISTORICAL BACKGROUND OF UNILEVER GHANA LIMITED

Unilever Ghana Limited is Ghana's leading manufacturer of fast moving consumer goods and is one of Ghana's leading co-operate citizens. It all began in 1787 when two European trading firms, Swanzy and King arrived in the Gold Coast. They later merged in 1931 to form the nucleus of the United Africa Company of Gold Coast (UAC) with a strong commitment to the socio-economic development of the nation – trading manufacturing, agriculture and real estate. In 1963, UAC established Lever Brothers, a consumer goods manufacturing business.

Unilever Ghana came into being on July 14, 1992 when two significant and complimentary Unilever PLC subsidiaries, UAC Ghana Limited, reputed for excellence in marketing and distribution and Lever Brothers Ghana Limited which is strong in manufacturing merged to form Unilever Ghana Limited.

Unilever Ghana Limited has been successful over the years in continually seeking to improve and satisfy the everyday needs of all consumers with superior quality, competitively priced branded products and services.

Following the company nationalization programme in 1997/98 to concentrate on its core activities, Unilever Ghana today manufactures and markets three (3) broad categories of products under Food (e.g. Blue Band Margarine, Lipton, Royco, and Annapuma Salt) Home care (e.g. Keysoap , Omo and Sunlight) and personal care (e.g. Pepsodent, Close up , Lux , Geisha etc.) categories.

Unilever Ghana Limited has made a big difference in the people's lives by reaching to its numerous consumers with quality products that care for their families and help them get more out of life. The company achieves this by maintaining the highest standard of corporate behavior towards its employees, consumers, customers, shareholders and indeed their operating environment. It is therefore not surprising to find Unilever products in every home in Ghana.

Creating goodwill: As a good and long-standing corporate citizen, Unilever Ghana has over the years, strongly contributed to the realization of the nation's socio-economic goals. The company has contributed immensely to employment rural wealth and poverty alleviation, education, health, sports and environmental programmes and activities. Unilever Ghana has indeed managed its commitment to its social obligations with the same attention and professionalism that it has applied to its business activities.

1.2 BACKGROUND STUDY OF UNILEVER GHANA LIMITED IN ASHANTI REGION

The region is centrally located in the middle belt of Ghana. It lies between longitude 0.15 W and 2.25W and latitude 5.50N and 7.46N. The region shares boundaries with four of the ten political regions, Brong Ahafo Region in the North, Eastern Region in the east, Central Region in the south and Western Region in the west.

The Ashanti Region occupies a total land area of 24,389 square kilometre representing 10.2 per cent of the total land area of Ghana. It is the third largest region after Northern and Brong Ahafo regions.

The region is the most populous and one of the most rapidly growing regions in the country. It has a population of 4,725,046 representing 19.5 percent of the country's population. The growth rate is 3.4 percent and second after Greater Accra and has a population density of 148.1 percent per square kilometer. The percentage increase in population in 2010 was 30.8 percent and females outnumber males. There are twenty-one administrative district (but before eighteen) in the region including Kumasi Metropolis.

A study conducted identified the following:

1. High rate of youth unemployment
2. Inadequate warehouses (distribution centres) for Unilever Ghana Limited in the region.
3. Low levels of income
4. Missing links in road network.
5. Lack of adequate information on trade, industry and tourism in the metropolis.
6. High rate of population growth.
7. Population growth rate outstrips growth of housing supply.

The region has only three (3) Unilever warehouses managed by O.A.B Limited, NAN Limited and A.P.M Limited. The twenty-one districts in the region, therefore , gets supply from three warehouses .This means that over the thousands and hundreds of distribution points, shops , stores , kiosks and open market table-top customers , all depend on these three warehouses for Unilever food products, personal and home care products.

Again, generally, the longer distance business men and women travel to and sell goods /products the higher the prices of those goods and products. This could deny many households of the quality food, personal and homecare products of Unilever.

1.3 PROBLEM STATEMENT

Unilever Ghana Limited has only three (3) warehouses in the Ashanti region supplying twenty-one districts including Kumasi Metropolis and over thousands and hundreds of distributions points, shops, stores, kiosks and open market table top customers all over the major towns and villages throughout the region resulting in a long distance travel cost, inadequate supply and high prices for Unilever products for the ever increasing population and its health and unemployment situation.

This work, therefore seeks to find the optimal sites for additional three warehouses in the region using the p – centre model.

1.4 OBJECTIVE OF THE STUDY

1. To locate three additional warehouses for Unilever Ghana Limited in Ashanti Region as a p – centre problem.
2. To solve the conditional p – centre problem using Berman and Drezner algorithm.
3. To use the Factor Rating Method to determine the first, second and third locations.

1.5 METHODOLOGY

This study is to locate three warehouses in Ashanti Region using the conditional P – centre model.

Data on road distance between selected towns in the region were collected and used.

Floyd's algorithm was used to find the distance matrix, d $(\boldsymbol{i}, \boldsymbol{j})$ for all pairs shortest path.

And then a modified shortest distance matrix [y,TC,X]=sproject(p) defined in Matlab, thus applying Berman and Drezner's algorithm, was used to locate three warehouses for Unilever Ghana Limited in the Ashanti Region.

A factor rating method was used to find aggregate scores to determine the first, second and final optimal locations.

1.6 JUSTIFICATION

Additional warehouses in the region could contribute to;

1. Minimum travel cost and low prices for Unilever products.
2. Easy access to quality food and personal as well as home care product of the company.
3. Improve the health status and quality of life of families in the region and the country as a whole.
4. Employment, rural wealth and poverty alleviation, education and goodwill to the region and government as a whole.

It is hoped that the results of this study would help to inform Unilever Ghana Limited about the right sites to locate three warehouses in the region.

1.7 LIMITATION OF THE STUDY

Due to difficulties in getting all the data for the study, because of some towns not having direct road links, the study did not consider all towns but some selected ones in the region.

Also, because of financial constraints, the study did not cover the entire region which is one of the largest in the country.

1.8 THESIS ORGANIZATION

Chapter one gives the introduction, background, problem statement, objective, justification, limitation and organization of the study.

The second chapter deals with the literature review.

Chapter three also presents the research Methodology.

Data collection, analysis and discussion are considered in chapter four.

The last chapter five gives the conclusion and recommendation of the study.

CHAPTER 2

LITERATURE REVIEW

2.0 INTRODUCTION

Facility location problem is the process of identifying the best location for a warehouse, commodity, production facility, service or set of facilities so as to optimize a given function subject to a set of constraints.

In looking out for the best way to serve a set of communities whose location and demands are known, we need to consider among others;

a. The place and number of facilities (warehouses) to serve the demand.
b. The size and capacity of each facility.
c. The number and existing location of each facility.
d. Population and markets
e. Optimizing some criteria, usually related to distance between facilities size and demand points.
f. Availability of space (land).

This optimization may vary depending on the particular objective function chosen. The function could be either to; minimize travel time or cost, minimize average response time, minimize maximum travel time or cost and or maximize net income (Amponsah , 2007).

Achievement of significant cost savings and improvements in profitability requires a typical retail company to make long term decisions regarding the structure of its supply chain network and bringing its facilities (warehouses), distributors and customers closer together under the strategic supply chain planning (Shapiro, 2005).

Generally location problems investigate where to physically locate a set of facilities to optimize a given object function. However most location models deals with desirable facilities such as warehouse , emergency service (e .g ambulance, fire), service and transportation centre (e.g. automatic teller machine for banks) etc., which interacts with the customers where distance travel is involved. Therefore, typical criteria for such decisions include minimizing some function of the distance between facilities and or customers (i.e. average travel time, average response time, cost function of travel or response time, maximum travel time or cost etc.)

But, during the last two decades, these responsible for the overall development of the area, where the new facility is going to locate (i.e. central government, local authorities) as well as those living in the area (the population), show an increased in preserving the area's quality of life. Hence the introduction of new terms in location theory as: noxious, obnoxious, semi-obnoxious, hazardous and so on.

Example of undesirable facilities include; warehouses containing flammable materials, equipment emitting particular smell or noise, nuclear or military installation etc.

The study of facility decisions has a long history in literature (reviews). A typical facility location problem involves optimal placement of facilities by minimizing the cost associated with or maximizing the desirability gained by the placement. Moreover, certain in reciprocal- relationship points mostly in the form of supply and demand Points, are involved in these kinds of decisions making problems. Locating warehouses, distribution points and manufacturing sites are classic examples. Other terms such as location analysis, site sustainability analysis and land use suitability analysis are also terms used for locations studies in which proper placement is the whole or a major component.

From the point of view of operations researchers, the inception of location theory dates back to 1929 when Alfred Weber published his book titled “theory of the location of industries” (Weber, 1929). These initiatives formed the basis of descriptive and normative location theories respectively. Up to now, several researchers and others have developed the topic, and many handbooks and scientific papers have been published on this subject.

Drezner (1995) categorized various objective functions in three main types based on their type of influence: pushing, it pushing the facilities away from undesirable points; pulling, i.e. pulling the facilities towards demand, or balancing types of objectives. As listed by Farah Ain et al., (2010), various objective functions have been used in the literature to address various types of requirements. Examples include minimizing total set up cost, longest distance from the existing facilities, average time or distance travelled and maximum distance or times travelled.

2.1 OTHER APPROACHES TO FACILITY LOCATION PROBLEMS

Christopher and Wills (1972) comprehensively present that whether the problem of depot location is static or dynamic, 'infinite set' approaches and feasible set approach can be identified. The infinite set approach assumes that a warehouse is flexible to be located anywhere in a certain area. The feasibility set approach assumes that only a finite number of known sites are available as warehouse locations. They believe the centre of gravity method is a part of infinite set model.

Berkhuizen et al, (1988) proposed conducting location and feasibility analysis at national, regional and local levels to determine the most suitable area for the location of large wind energy project in the Netherlands. In cases dealing with undesirable facilities, Erkut and Neuman (1989) suggested using a two-stage decision process that includes site generations to identify potential locations and site selection to select final location.

Whether a facility to be placed is noxious or not communities are expected to response to or influenced the positive or the negative risks involved in the placement. "Respond" and "influences" are considered formative elements of social implications of location decisions. Reams and Templet (1988) found that political ideological, social, demographic, physical and economic factors are influential in communities' responses in the case of locating incinerators.

Fonseca and Captivo (1996; 2006; 2007) study the location of semi-obnoxious facilities as a discrete location problem a network. Several bi-criteria models are presented considering two conflicting objectives, the maximization of the accessibility of the community to the closest open facility. Each of those objectives is considered in two different ways, trying to optimize its average value over all the communities or trying to optimize its worst value. The Euclidean distance is used to educate the obnoxious effect and the shortest path distance is used to evaluate the accessibility. The obnoxious effect is considered inversely proportional to the weighted Euclidean distance between demand points and open facilities, and demand directly proportional to the population in each community.

In Malczewki and Ogryczak (1990) the location of hospitals is formulated as a multi-objective optimization problem and an interactive approach DINAS, dynamic interactive network analysis system (Ogryczak et al, 1989) based on the so called reference point approach (Wierzbicki, 1982) is presented .a real application is presented considering eight sites for potential locations and at least four new hospitals to be built , origination in hundred and sixty-three alternative location pattern each of them generating many possible allocation schemes. The authors mention that the system can be used to support a group decision-making process making the final decision less subjective. They also observed that during the interactive process the decision-makers have gradually learned about the set of feasible alternatives and in consequence of this learning process they have charge their preference and priorities.

As acknowledged by Dehghanian and Mansour (2009) integrating social dimensions in operation research (OR) models have been underutilized. “The social sustainability dimension has largely been absent in OR contributions, with the exception of some areas investigating the health impacts of institutional operation”. Since the majority of analytical location models have been developed in OR, there is an overall underrepresentation of social dimensions in facility location models.

The social dimension of the location problem can also be viewed from the social responsibility (SR) point of view. Although various definitions have been proposed for this concept, a large degree of congruity is seen among the proposed definitions (Dahlsrud, 2008).

In a nutshell, SR characterizes the efforts that are being made in a socially responsible manner. In this sense, the concept of SR can be extended to the scope of the current research to specify the requirements of location decisions that are made in a socially responsible manner.

Giannikos (1998) presents a discrete model for the location of disposal or treatment facilities and transporting hazardous waste through a network linking the population centres that produce the waste and the candidate locations for the treatment facilities method to choose the location for a waste treatment facility in a region of Finland.

2.2 THE P-CENTER LOCATION PROBLEM

The primary objective of this study is to locate three warehouses in addition to the existing ones to cover as much of the potential customer demand as possible.

One of the many location models to consider in this regard is the conditional location problem to locate **p** new facilities to serve a set of demand points given that **q** facilities are already located. In the conditional P-Center location problem, once the new **p** location(s) is/are determined a demand can be supplied either by one of the existing or by one of the new facilities whichever is the closest facility to the demand (Berman, 2008).The P-Center location problem determines the location of **p** facilities.

Each demand point is supplied from the closest facility, new or existing. The objective is to minimize the maximum distance for all demand points. The p-Center problem seeks to choose **p** facilities from among a set of **M** possible locations and assigning **N** customers to them so as to minimize the maximum distance between a customer and the warehouse to which it is allocated.

As Marianov and Serra (2002) pointed out, both the p-median and covering problems can be considered benchmarks in the development of facility location models. While the p-center is also an important location model, the location set covering problem can be used as a sub problem in solving the classical p-center problem (Handler and Mirchandani, 1979; Handler, 1990). Daskin (2000) showed how the maximal covering model can be used effectively in place of the location set covering model as a sub-problem in solving the unweight vertex p-center problem.

Goldman (1969), Hakimi and Maheshwari (1972) and Wendell and Hurter (1973) studied a multi-commodity facility location problem, in which the nodes of the network include sources (supply points) and destinations (demand points). There are multiple types of demands (commodities) between source-destination points and each commodity may require several stages of processing. The objective is to minimize the total transportation cost through locating **n**center facilities. However, they assumed that each facility is capable of performing any of the processing stages on all commodities. In other words, only a single type of facility is located on the network.

Another version of multi-commodity facility location problem (Geoffrion and Graves, 1974) is an extension of the incapacitated facility location problem. There are several commodities produced at several plants and there is a known demand for each commodity at each demand node. The objective is to minimize the total distribution cost and facility cost through locating distribution centre facilities.

Drezner and Wesolowsky (2001) and Berman, Drezner and Wesolowsky (2002) considered the collection of Depots location problem in the plane and on the network, respectively. In this problem, a single facility needs to be located to serve a set of customers. Also consider the situation where there are two different types of facilities denoted by **type-x** and **type-y** facilities, in the system. Customer demand occurs only at the nodes of the network.

G= (N, L), where **N** is the set of nodes with **|N| = n** and **L** is the set of Links. Customers in the system have to travel to a facility to obtain a service e.g. shopping. There are three user groups in the system: **type-x (type-y)** customers who only need service from **type-x (type-y)** facilities and **type-xy** customers who need service from both types of facilities. The aim here is to minimize the total travel distance of the system through locating **m type-x** facilities and **p type-y** facilities on the network.

Krumke 1995 considered the generalization of the P-Center problem, which is called the α-Neighbour P-center problem (P-CENTER $^{(\alpha)}$). Given a complete edge-weighted network, the goal is to minimize the maximum distance of a customer to itsα nearest neighbor in the set of P Centers. In general, he shows that finding an O ($2^{\text{Poly(IVI)}}$) – approximation for P-CENTER $^{(\alpha)}$ is NP-hard (Garey and Johnson, 1979), where IVI denotes the number of nodes in the network. If the distances are required to satisfy the triangle inequality, there can be no polynomial time approximation algorithm with a (2 – E) performance guarantee for any fixed E > O and any fixed α≤ P, unless P=NP. Here he presented a simple yet efficient algorithm that provides a 4-approximation for α≥ 2.

The selected formulation for this study, the conditional p-median and p-center problems

Berman and Drezner (2007) discuss the conditional p-median and p-center problems on a network. Demand points are supplied by the closest warehouse whether existing or new. To avoid creating a new location for an artificial warehouse and force the algorithm to locate a new warehouse thereby creating an artificial demand point, the distance matrix was just modified. They suggested solving both conditional problems by defining a modified shortest distance matrix D. The formulation they presented in this paper provided better results than those obtained by the best known formulations. The work presented in this thesis is based on this paper.

CHAPTER 3

METHODOLOGY

3.0 NETWORK LOCATION MODELS

The network location problems are concerned with finding the right places to locate one or more facilities in a network of demand point, i.e. Customer locations represented by nodes in the network, that optimizes a certain objective function related to the distance between the facilities and the demand points.

3.1 BASIC FACILITY LOCATION MODELS

This section presents models classified according to their consideration of distance. The total (or average) distance models and maximum distance models.

3.1.1 TOTAL OR AVERAGE DISTANCE MODELS

A crucial measure and input into the location model is the distance between the nearest service centre (warehouse) and the demand point. Most facility location planning situations in the public and private sectors are concerned with the total travel distance between facilities and demand nodes. For example, in the private sector, the concern might be the location of production facilities that receive their inputs from established sources by truckload deliveries. And in the case of the public sector, one might want to locate a network of service providers such as automated teller machines by Banks in such a way as to minimize the total distance that customers must travel to reach their closest facility. This approach may be referred to as an “efficiency” objective against the “equity” objective of minimizing the maximum distance, which is mentioned in other models.

1. **The P-Median Problem**: The P-median model (Hakimi, 1964, 1965) finds the locations of P facilities to minimize the demand-weighted total distance between demand nodes and the facilities to which they are assigned.

2. **The maximum location problem**: The maximum location problem seeks the location of p facilities such that the total demand-weighted distance between demand nodes and the facilities to which they are assigned is maximized.

3.1.2 MAXIMUM DISTANCE MODEL

In considering the location problem, the determination of measurement of the distance is also important. The two major distance measurements used in many location studies are the rectilinear distance and the Euclidean distance. The rectilinear distance is the most popular one in urban area model in which the allowable orientations are orthogonal ones. In the Euclidean distance, there is no restriction of orientations to travels. But neither of the measurements necessarily gives the good approximation of distance in the urban travel distance cases.

In the facility location problems, an acceptable distance is set a priority, also known as a "covering" distances. Demand within the covering distance of its closest warehouse is considered "covered". An underlying assumption of this measure of covering distance is that demand is fully satisfied if the nearest facility is within the coverage distance and is not satisfied if the closest warehouse is beyond that distance.

1. **Set covering location model**: The objective of this model is to locate the minimum number of facilities required to "cover" all of the demand nodes (Toregan et al, 1971).

2. **The maximal covering location problem**:

The maximal covering location problem (MCLP) is also set out to locate a predetermined number of facilities (warehouses), P, in such a way as to maximize the demand that is covered. Thus, the MCLP assumes that there may not be enough warehouses to cover all the demand nodes. If all nodes cannot be covered, the MCLP model seeks the sitting scheme that covers the most demand (Church and ReVelle, 1974).

3. The P-dispersion problem:

The P-dispersion problem (PDP) is the problem of locating p warehouses at some of n predefined locations such that the distance sum between the p warehouses is maximized. The problem has applications in telecommunication (where it is desirable to disperse the transceivers in order to minimize interference problem), and in location of shops and service-stations (where the mutual competition should be minimized).

4. P-Center problem:

The p-center problem seeks the location of p facilities (warehouses). Each demand point receives its service from the closest warehouse. The objective is to minimize the maximal distance for all demand points. There are several possible variations of the basic model. The "vertex" p-center problem restricts the set of candidate facility/warehouse sites to the nodes of the network while the "absolute" p-center problem permits the warehouses to be anywhere along the arcs or the network. Both versions can be either weighted or unweight. In the unweight problem, all demand nodes are treated equally. In the weighted model, the distances between demand nodes and warehouses are multiplied by a weight associated with the demand node. That is, this weight might represent a node's importance or, more commonly, the level of its demand.

3.2 THE P-CENTER PROBLEM

This is the problem of locating p (warehouses) in order to minimize the maximum response time, that is, the time between a demand site and the nearest warehouse, using a given number of P.

By their definition and the decision variable;

$W =$

$\textit{The maximum distance between a demand node and the warehouse to which it is assigned.}$

$$X_{ij} = \begin{cases} \mathbf{1} \text{ if the demand node } \boldsymbol{j} \text{ is assigned to a warehouse at node } \boldsymbol{i} \\ \mathbf{0} \text{ if not} \end{cases}$$

The P-centre problem can then be formulated as follows:

```
function [y,TC,X] = sproject(p)
% [y,TC,X] = pcenter(p,C)
%          = pcenter(p,C,dodisp),  display intermediate results
%     p = scalar number of NFs to locate
%     C = n x m variable cost matrix,
%         where C(i,j) is the cost of serving EF j from NF i
%       dodisp = true, default
%     y = p-element NF site index vector
%    TC = total cost
%       = sum(sum(C(X)))
%     X = n x m logical matrix, where X(i,j) = 1 if EF j allocated to NF i
% Matlog Version 15 07-Jan-2013 (http://www.ise.ncsu.edu/kay/matlog)
clc, close all hidden
dataR = load('data12.mat');
C = dataR.data;
% C(:,[1:2,10]) = [];
% C([1:2,10],:) = [];
[y,TC,X] = pcenter(p,C,false);
function [y,TC,X] = pcenter(p,C,dodisp)
% Input Error Checking ***********************************************
narginchk(2,3);
[n,m] = size(C);
```

```
ifnargin< 3 || isempty(dodisp), dodisp = true; end

if ~isscalar(p) || p > n || p < 1

error('"p" must be between 1 and "n".')

elseif ~isscalar(dodisp) || ~islogical(dodisp)

error('"dodisp" must be a logical scalar.')

end

% End (Input Error Checking) ************************************************

[y,TC] = ufladd(0,C,[],p); if dodisp, fprintf('  Add: %f\n',TC), end

[y,TC] = uflxchg(0,C,y); if dodisp, fprintf(' Xchg: %f\n',TC), end

[y1,TC1] = ufldrop(0,C,[],p); if dodisp, fprintf(' Drop: %f\n',TC1), end

[y1,TC1] = uflxchg(0,C,y1); if dodisp, fprintf(' Xchg: %f\n',TC1), end

if TC1 < TC, TC = TC1; y = y1; end

ifdodisp, fprintf('Final: %f\n',TC), end

y = sort(y);

X = logical(sparse(y(argmin(C(y,:),1)),1:m,1,n,m));

function [y,TC,X] = ufladd(k,C,y,p)

% Input Error Checking

****************************************************

narginchk(2,4);

C = C';  % Make column-based to speed up minimization

[m,n] = size(C);

ifisscalar(k), k = repmat(k,1,n); else k = k(:)'; end

ifnargin< 3, y = []; else y = y(:)'; end
```

```
ifnargin< 4, p = []; end
if length(k) ~= n || any(k < 0)
error('"k" must be a non-negative n-element vector.')
elseif ~isempty(y) && (any(y < 1) || any(y > n) || ...
length(y) ~= length(unique(y)))
error('"y" must contain integers between 1 and n.')
elseif ~isempty(p) && (~isscalar(p) || p > n || p < 1)
error('"p" must be between 1 and "n".')
elseif ~isempty(p) && ~isempty(y) && length(y) < p
error('"p" cannot be greater than length of "y"')
end
% End (Input Error Checking) ************************************************
ifisempty(y)
  [TC,y] = min(sum(C,1) + k);
else
TC = sum(k(y)) + sum(min(C(:,y),[],2));
end
ny = 1:n; ny(y) = [];
TC1 = Inf;
done = false;
while ~done
  c1 = min(C(:,y),[],2);
  k1 = sum(k(y));
```

```
for i = 1:length(ny)
TCi = k1 + k(ny(i)) + sum(min(c1,C(:,ny(i))));
ifTCi< TC1
        [i1,TC1] = deal(i,TCi);
end
end
if (isempty(p) && TC1 < TC) || (~isempty(p) && length(y) < p)
      y = [y ny(i1)];
ny(i1) = [];
      TC = TC1;
if ~isempty(p), TC1 = Inf; end
else
done = true;
end
end
y = sort(y);
X=logical(sparse(y(argmin(C(:,y),2)),1:m,1,n,m));
```

Maximize

W…………………………………..(1)

Subject to:

$\sum_{j\in J} x_j = P$……………………………………….(2)

$\sum_{j\in J} y_{ij} = 1 \quad \forall_i \in I$ …………………..(3)

$y_{ij} - x_j \leq 0 \quad \forall_i \in I,\ j \in J$ ………..(4)

$W - \sum_{j\in J} h_i d_{ij} y_{ij} \geq 0 \quad \forall_i \in I$ ………..(5)

$x_j \in \{0 , 1) \quad \forall_i \in I$ ………………...(6)

$y_{ij} \in \{0 , 1\} \quad \forall_i \in I,\ j \in J$ …………(7)

The objective function (1) minimizes the maximum demand-weighted distance between each demand node and its closest open warehouse. Constraint (2) makes sure that P warehouses are to be located. Constraint set (3) requires that each demand node be assigned to exactly one warehouse. Constraint set (4) restricts demand node assignments only to open warehouses. Constraint (5) defines the lower bound on the maximum demand-weighted distance, which is being minimized. Constraint (6) established the sitting decision variable as binary. Constraint set (7) requires the demand at a node to be assigned to one warehouse only. Constraint set (7) can be replaced by $y_{ij} \geq 0 \quad \forall_i \in I; j \in J$ because constraint set (4) guarantees that $y_{ij} \geq 1$. If some y_{ij} are fractional, we simply assign node *i* to its closest open warehouse facility (Current et al, 2001).

3.3 THE CONDITIONAL P-CENTER PROBLEM

The conditional p-center location problem is to locate p new warehouses to serve a set of demand points given that q warehouses are already located. When q=o, the problem is unconditional. With this conditional p-center problem, once the new p locations are determined, a demand can be served either by one of the existing or by one of the new warehouses whichever is the nearest warehouse to the demand..

Consider the network G= (N, L) where;

N= the set of nodes; |N| =n

L= the set of links

Let d(x, y) be the shortest distance between any x, y ∈ G. Suppose that there is a set Q(|Q|)=q of existing warehouses. Let $Y=(Y_{1,\dots,} Y_q)$and $X=(X_1,X_2,\dots, X_p)$ be vectors of size q and p respectively, where Y_i is the location of existing warehouse *i* and x_i is the location of new warehouse *i*. Without any loss of generality we do not need to assume that $Y_i \in N$. The conditional P-center location problem is to;

$$\text{Min}\ [g(x) = \max_{i=1,\ldots,n}\ \text{Min}\ \{d(X, i)\ ,\ d(Y\ , i)\}]$$

Where d(x, *i*) and d(Y, *i*) is the shortest distance from the nearest warehouse in X and Y respectively to the node *i*, (Berman and Simchi-Levi, 1990).

3.4 BERMAN AND SIMCHI-LEVI ALGORITHM

Berman and Simchi-Levi (1990) suggest to solve the conditional P-center problem on a network by an algorithm that requires one-time solution of an unconditional (P + 1) - center problem.

Step 1: Let D be a distance matrix with rows corresponding to demands and columns corresponding to potential locations. For the p-center problem the columns of D correspond to the set of local centers C. The idea is to create a new potential location representing all existing facilities. If a demand point is utilizing the services of an existing warehouse, it will use the services of the closest existing warehouse. Therefore, the distance between a demand point and the new location is the minimum distance calculated for all existing facilities.

Step 2: To force the creation of a facility (warehouse) at the new location, a new demand point is created with a distance of zero to the new potential location and a large distance to all other potential locations. The new distance matrix D is constructed by adding a new location a_0(a mew column) to D so that the new columns represent the Q existing location and a new demand point V_0 with an arbitrary positive weight. For each demand point (node) *i*, d (*i*, a0) = min $_{k\in Q}$ {d*i*k} and d ($V_{0,}$ a_0) = 0. For each potential location (node) *i*,d (V_0 , j) = M (m is a large number). Again the nodes in Q and Potential locations Q are removed.

Step 3: Find the optimal new location using the distance matrix D for the network with the objective function.

To illustrate the approach, we consider the network of figures 3.1; the numbers next to the links are lengths. Suppose that existing set of warehouses are nodes 2 and 3, and only one warehouse is to be located.

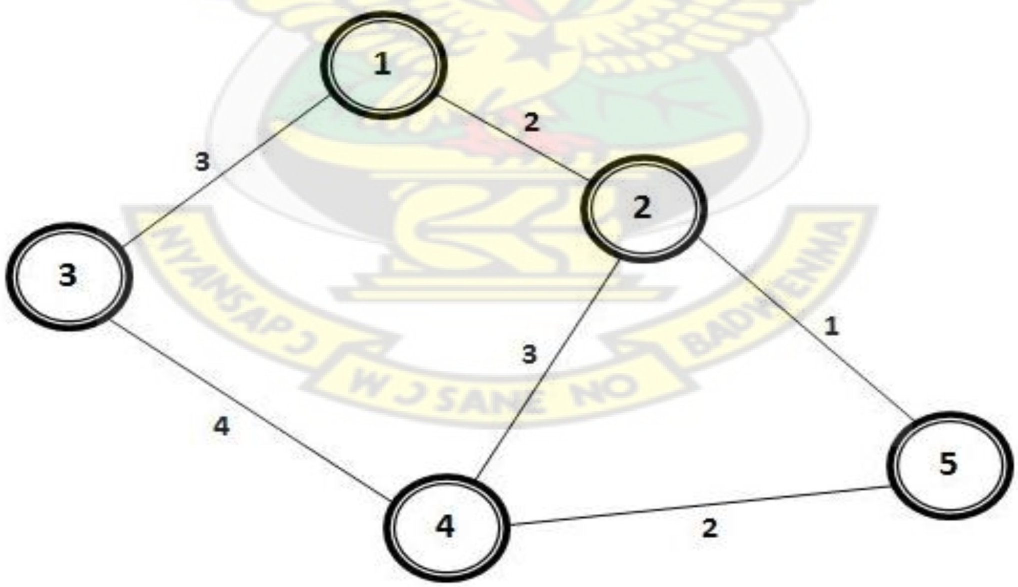

Figure 3.1: Sample network for p-center problem

Step 1.By using the Floyd's algorithm, we obtain the all pair shortest path distance matrix for the above network as shown in Table 3.1.

In table 3.1, column 1 and row 1 represents the demand nodes and potential location respectively, and all other rows represent the interconnected distances.

Table 3.1: All pair shortest path distance matrix, D.

Demand Nodes	Potential Location				
	1	2	3	4	5
1	0	2	3	5	3
2	2	0	5	3	1
3	3	5	0	4	6
4	5	3	4	0	2
5	3	1	6	2	0

Step 2: In step 2, determine a modified shortest distance matrix, D by adding a new location a_0 (that is a new column) to D and adding a new demand point V_0 (also a new row) with an arbitrary positive weight to the rows. For each demand point (node) i, d (i, a_0) = min $_{k \in Q}$ {d i k} and d ($V_{0,}$ a_0) = 0. For each potential location (node) i, d (V_{0} , j) = M (m is a large number), as shown in Table 3.2

Table 3.2a The modified Distance Matrix, D

Demand Nodes	Potential Location					
	1	2	3	4	5	a_0
1	0	2	3	5	3	2
2	2	0	5	3	1	0
3	3	5	0	4	6	0
4	5	3	4	0	2	3
5	3	1	6	2	0	1
V_0	M	m	m	m	m	0

Table 3.2b: The modified Distance Matrix $\hat{D}$ with nodes 2 and 3 removed

Demand Nodes	Potential Location			
	1	4	5	a_0
1	0	5	3	2
4	5	0	2	3
5	3	2	0	1
V_0	M	m	m	0

Step 3. Here we find the optimal new location using the distance matrices, $\hat{D}$ and the objective function.

$$\text{Minimize } [g(x) = \max_{i=1,\ldots,n} \ \min \{d(x,i) , d(Y , i)\}]$$

Taking the Distance Matrix, $\hat{D}$

$$\text{Minimize } [g(x) = \max_{i=1,\ldots,n} \min \{d(x, i), d(Y, i)\}]$$

$x = \{1, 4, 5, a_0\}$ $\quad\quad$ $Y = \{2, 3\}$

At x= 1

i = 1, d (1, 1), d (2, 1), d (3, 1)

0, 2, 3

Min = 0

i = 2, d (1, 2), d (2, 2), d (3, 2)

0, 2, 3

Min = 0

i = 3, d (1, 3), d (2, 3), d (3, 3)

3, 5, 0

Min = 0

i = 4, d (1, 4), d (2, 4), d (3, 4)

5, 3, 4

Min = 3

i = 5, d (1, 5), d (2, 5), d (3, 5)

3, 1, 6

Min = 1

Therefore at x=1, the maximum = 3, at node 4. The results are then summarized and shown below in Table 3.3, with column 5 representing the maximum distance between demand nodes and rows represent the maximum interconnected distances.

Table 3.3: Optimal location Min (g(x)) using $\hat{\hat{D}}$

Demand Notes	1	4	5	Maximum
1	0	3	1	3
4	2	0	1	2
5	2	2	0	2
Minimum			→	2

From Table 3.3, it is easy to verify that, the optimal new location using $\hat{\hat{D}}$ is node 5 with an objective function value of 2.

3.5 BERMAN AND DREZNER'S ALGORITHM

Berman and Drezner (2008) discuss a very simple algorithm that solves the conditional P-center problem on a network. The algorithm requires one-time solution of an unconditional p-center problem using an appropriate shortest distance matrix. Rather than creating a new location for an artificial warehouse and force the algorithm to locate a new warehouse thereby creating an artificial demand point, they just modify the distance matrix.

Step 1 : Let D be a distance matrix with rows corresponding to potential locations and columns corresponding to demands.

Step 2 : Solve the conditional p-center problem by defining a modified shortest distance matrix [y,TC,X]=sproject(p) using Matlab codes.

The unconditional p-center problem using the appropriate D solves the conditional p-center problem. This is, since if the shortest distance from node *i* to the new p warehouses are larger than Min $_{k \in Q}$ {d i k}**,** then the shortest distance to the existing q warehouses is utilized. Notice that the size of D is n x |c| for the conditional p-center.

Step 3 : Use the factor rating method to find the aggregate scores to determine which warehouse to locate first, second and third.

To demonstrate the algorithm, a 5 node network depicted in figure 3.1 is considered where the numbers next to the links are lengths. We solve the p-center problem. Suppose that the existing set of warehouses is Q= {2, 3} and P=2, the new nodes (towns) to locate two warehouses are demonstrated below:

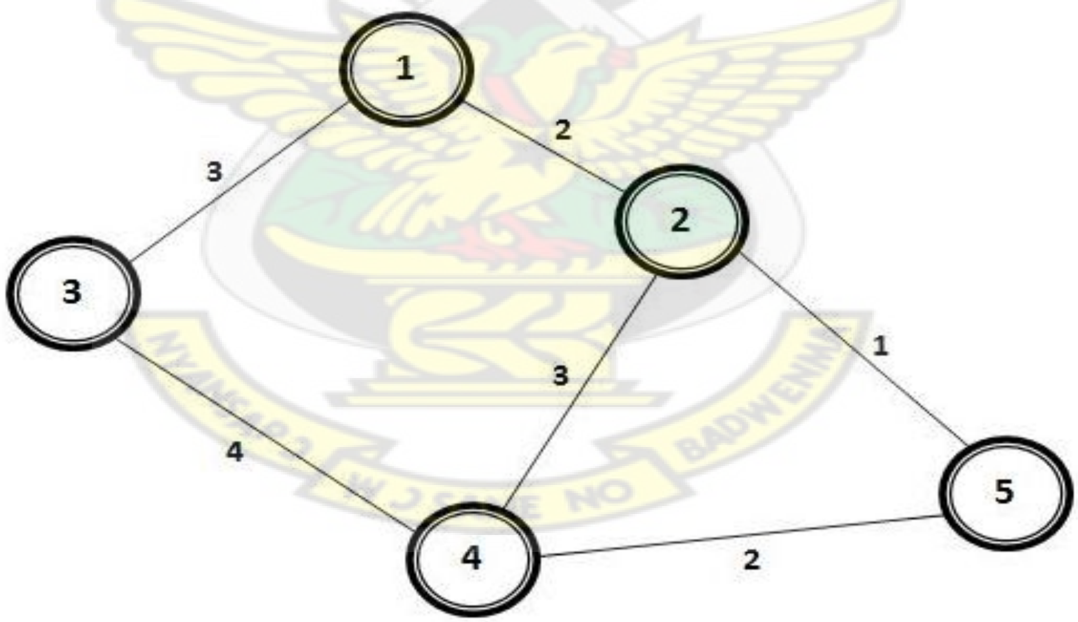

Figure 3.1: Sample network for P-center problem.

Step 1: By using the Floyd's algorithm, we obtain all pair shortest path distance matrix for the network above. Column 1 and row 1 represents the demand node and potential location respectively, and all other rows represent the interconnected distances.

The table for all pairs shortest path distance matrix D, is as below:

Table 3.1 All pair shortest distance matrix D

Demand Nodes	Potential Location				
	1	2	3	4	5
1	0	2	3	5	3
2	2	0	5	3	1
3	3	5	0	4	6
4	5	3	4	0	2
5	3	1	6	2	0

Step 2: Determine a modified shortest distance matrix by:

[y,TC,X]=sproject(2)

Y =

2 5

TC=

15

X=

(2, 1) 1

(2, 2)   1

(2, 3)   1

(5, 4)   1

(5, 5)   1

The results mean that nodes 2 and 5 are located with a functional value of 1. The results are displayed in a matrix form for further analysis by typing:

Full (x).

The table below summarizes the results of the Modified shortest distance matrix with existing facility nodes.

Table 3.4 The modified Shortest Distance Matrix [y, TC, X]=sproject with Q

Demand Nodes	Potential Location				
	1	2	3	4	5
1	0	0	0	0	0
2	1	1	1	0	0
3	0	0	0	0	0
4	0	0	0	0	0
5	0	0	0	1	1

Column one and row one representing demand node and potential location respectively and X($\boldsymbol{i}$, $\boldsymbol{j}$) =1 means the row is located.

Step 2 The existing warehouse nodes, Q= {2, 3} are removed from the five node shortest distance matrix D and this is shown in Table 3.4b below.

Table 3.5 Data 3 with existing warehouses removed.

Demand Nodes	Potential Location		
	1	4	5
1	0	5	3
4	5	0	2
5	3	2	0

Step 3.Then the optimal new location is found using the modified shortest path distance matrix[y,TC,x] = sproject (2), where P=2,rows representing potential location and columns representing demand.

$X(\boldsymbol{i}, \boldsymbol{j}) = 1$ means is locate and

$X(\boldsymbol{i}, \boldsymbol{j}) = 0$ means is not located

Example; [y,TC,X]=sproject(2);

Y=

1 5

TC=

20

X=

(1, 1) 1

(5, 4) 1

(5, 5) 1

This means nodes 1 and 5 are located with a functional value of 1 and node 5 will serve demand node 4 as well. The result is displayed in matrix form by typing;

Full(x) >>

Table 3.6 Modified shortest distance matrix with nodes 2 and 3 removed

Demand Nodes	Potential Location		
	1	4	5
1	1	0	0
4	0	0	0
5	0	1	1

From the table above, it can be verified that node 4 is not located, that is, X($\boldsymbol{i}$, $\boldsymbol{j}$)=0. Node 1 is located and will serve only that community. Node 5 is located and will serve nodes 4 as well because that community (node4) is closer to node5 than node1.

3.6 FACTOR RATING METHOD

To determine which town to locate first as both cannot be located at the same time, the factor rating method is considered. By this method, the following steps should be taken:

1. Develop a list of relevant factors.

2. Assign a weight to each factor to reflect the views of the community.

3. Develop a scale for each factor.

4. Have related people to score each relevant factor using the scale developed in 3 above.

5. Multiply the scores by the weight assigned to each factor and find the aggregate scores for each location.

6. Make a recommendation based on the highest aggregate score.

(Amponsah, 2007).

Table 3.8 illustrates an example of the factors rating method used to decide among the two sites, where to locate the first warehouse. Five relevant factors and rating weights assigned to each as shown in the two tables below are used to demonstrate the method;

Table 3.7 gives the results that opinion leaders, business men and women and other related people score each factor ranging from one to hundred for both locations.

Table 3.7 Location rate on a 1 to 100 basis

Factor	Location A	Location B
1	59	51
2	80	80
3	30	50
4	70	60
5	60	70

Table 3.8 Relevant scores on factors for location of a Warehouse.

Factor	Factor Name	Rating weight	Ratio of rate	Location A	Location B
1	Population and market size	5	0.25	14.75	12.75
2	Access to public/private transport	3	0.15	12	12
3	Land acquisition	6	0.30	9	15
4	Power-source &water availability	5	0.20	14	12
5	Cost of labour, security etc.	2	0.10	6	7
	TOTAL	**20**		**55.75**	**58.75**

Clearly from their respective aggregate scores shown in table 3.8 above, location B is recommended to be located first since it has the higher aggregate score of 58.75.

CHAPTER 4

DATA COLLECTION, ANALYSIS AND RESULTS

4.1 DATA COLLECTION AND ANALYSIS

The length of shortest path and cost between connecting towns is of interest in this study. For this reason the set of distances of roads linking selected towns was obtained from the Kumasi Metropolitan Assembly Urban Roads (K.M.A Urban Roads) Ashanti Region. To ensure that the location decision resulting from the model are not only profitable but equitable and sustainable, there was the need to develop an eighteen node network, thus taking into consideration the eighteen selected towns in the Ashanti region. Below are the eighteen selected towns and their respective assigned nodes, shown in table 4.1

Table 4.1 some selected towns in Ashanti Region

Town	Node	Town	Node
Kejetia	1	Ash-Town	10
Adum	2	Suame	11
Asokwa	3	Kronum	12
Atonsu	4	Pankrono	13
Santasi	5	Ejisu	14
Danyame	6	Konongo	15
Nkawie	7	Agogo	16
Abuakwa	8	Agona	17
Bantama	9	Offinso	18

With existing warehouses at Kejetia, Adum and Asokwa, communities closest to existing warehouses include Ash-Town, Bantama and Danyame. These communities form part of the set of existing nodes; that is, node 1, node 2, node 3, node 6, node 9, and node 10. The above data is then developed into a network of figure 4.1 below. Numbers in the circles are the nodes representing the eighteen selected towns and the numbers next to the links are the various road distances in kilometres.

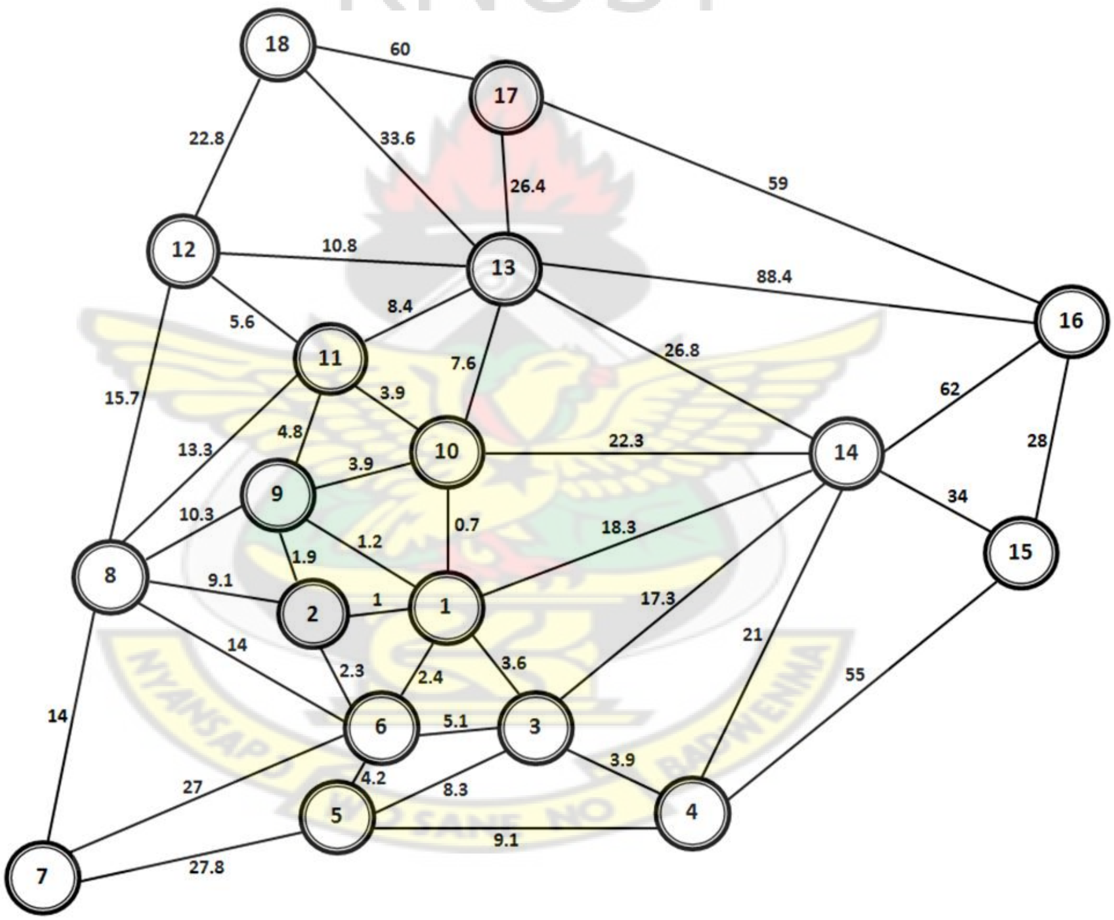

Figure 4.1 Network of selected towns in Ashanti Region

From the network in figure 4.1 above, an all pair shortest path distance matrix, D, is found and shown in Table 4.2a by using the Floyd's algorithm. Colum one and row one represent the demand node and potential location respectively, the other rows also represent the interconnecting road distances.

Table 4.2a All pairs shortest distance matrix D

node	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1		1	3.6	7.5	6.6	2.4	24.1	10.1	1.2	0.7	4.6	10.2	8.3	18.3	52.3	80.3	34.7	33
2	1		4.6	8.5	6.5	2.3	23.1	9.1	1.9	1.7	5.8	11.2	9.3	19.3	53.3	81.3	35.7	34
3	3.6	5		3.9	8.3	5.1	32.1	3.7	4.8	4.3	8.2	13.8	11.9	17.3	51.3	79.3	38.3	36.6
4	7.5	9	3.9		9.1	9	36	17.6	8.7	8.2	12.1	17.7	15.8	21	55.3	83	42.2	40.5
5	6.6	7	8.3	9.1		4.2	27.8	15.6	17.8	7.3	11.2	16.8	14.9	24.9	58.9	86.9	41.3	39.6
6	2.4	2	5.1	9	4.2		27	14	3.6	3.1	7	12.6	10.7	20.7	54.7	84.4	37.1	35.4
7	24.1	23	32.1	36	27.8	27		14	24.3	24.8	27.3	29.7	32.4	2.4	76.4	104	58.8	52.5
8	10.1	9	13.7	17.6	15.6	14	14		10.3	10.8	13.3	15.7	18.4	28.4	62.4	90.4	44.8	38.5
9	1.2	2	4.8	8.7	7.8	3.6	24.3	10.3		1.9	4.8	10.4	9.5	19.5	53.5	81.5	35.9	33.2
10	0.7	2	4.3	8.2	7.3	3.1	24.8	10.8	1.9		3.9	9.5	7.6	19	55	81	34	32.3
11	4.6	6	8.2	12.1	11.2	7	27.5	13.3	4.8	3.9		5.6	8.4	22.9	56.9	84.9	34.8	28.4
12	10.2	11	13.8	17.7	16.8	12.6	29.7	15.7	10.4	9.5	5.6		10.8	28.5	62.5	90.5	37.2	22.8
13	8.3	9	11.9	15.8	14.9	10.7	32.3	18.4	9.5	7.6	8.4	10.8		26.6	60.6	58.4	26.4	33.6
14	18.3	19	17.3	21	24.9	20.7	42.4	28.4	19.5	19	22.9	28.5	26.6		34	62	53	51.3
15	52.3	3	51.3	55	58.9	54.7	76.4	62.4	53.5	53	56.9	62.5	60.6	34		28	87	85.3
16	8.6	81	71.3	83	86.9	84.4	104	90.4	81.5	81	84.9	90.5	88.4	62	28		59	113.3
17	34.7	36	38.3	42.2	41.3	37.1	58.8	44.8	35.9	34	34.8	34.2	26.4	53	87	59		60
18	33	34	36.6	40.5	39.6	35.4	52.5	38.5	33.2	32.3	28.4	22.8	33	51.3	85.3	113.3	60	

After using Floyd's algorithm to obtain the above table, it is then put into word excel and imbedded in matlab which can then be used for the calculation. When this is done, we obtain the table below.

Table 4.2b Data 18 set. Mat

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
	DATA IN (km)	KEJETIA	ADUM	ASOKW A	ATONS U	SANTAS I	DANYA ME	NKAWIE	ABUAKWA	BANTAMA	ASHTOWN	SUAME	KRONUM	PANKRONO	EJISU	KONONGO	AGOGO	AGONA	OFFINSO
1	KEJETIA		1	3.6	7.5	6.6	2.4	24.1	10.1	1.2	0.7	4.6	10.2	8.3	18.3	52.3	80.3	34.7	33
2	ADUM	1		4.6	8.5	6.5	2.3	23.1	9.1	1.9	1.7	5.6	11.2	9.3	19.3	53.3	81.3	35.7	34
3	ASOKW A	3.6	4.6		3.9	8.3	5.1	32.1	13.7	4.8	4.3	8.2	13.8	11.9	17.3	51.3	79.3	38.3	36.6
4	ATONS U	7.5	8.5	3.9		9.1	9	36	17.6	8.7	8.2	12.1	17.7	15.8	21	55	83	42.2	40.5
5	SANTAS I	6.6	6.5	8.3	9.1		4.2	27.8	15.6	7.8	7.3	11.2	16.8	14.9	24.9	58.9	86.9	41.3	39.6
6	DANYA ME	2.4	2.3	5.1	9	4.2		27	14	3.6	3.1	7	12.6	10.7	20.7	54.7	84.4	37.1	35.4
7	NKAWIE	24.1	23.1	32.1	36	27.8	27		14	24.3	24.8	27.3	29.7	32.4	42.4	76.4	104.4	58.8	52.5
8	ABUAKWA	10.1	9.1	13.7	17.6	15.6	14	14		10.3	10.8	13.3	15.7	18.4	28.4	62.4	90.4	44.8	38.5
9	BANTAMA	1.2	1.9	4.8	8.7	7.8	3.6	24.3	10.3		1.9	4.8	10.4	9.5	19.5	53.5	81.5	35.9	33.2
10	ASHTOWN	0.7	1.7	4.3	8.2	7.3	3.1	24.8	10.8	1.9		3.9	9.5	7.6	19	53	81	34	32.3
11	SUAME	4.6	5.6	8.2	12.1	11.2	7	27.3	13.3	4.8	3.9		5.6	8.4	22.9	56.9	84.9	34.8	28.4
12	KRONUM	10.2	11.2	13.8	17.7	16.8	12.6	29.7	15.7	10.4	9.5	5.6		10.8	28.5	62.5	90.5	37.2	22.8
13	PANKRONO	8.3	9.3	11.9	15.8	14.9	10.7	32.4	18.4	9.5	7.6	8.4	10.8		26.6	60.6	88.4	26.4	33.6
14	EJISU	18.3	19.3	17.3	21	24.9	20.7	42.4	28.4	19.5	19	22.9	28.5	26.6		34	62	53	51.3
15	KONONGO	52.3	53.3	51.3	55	58.9	54.7	76.4	62.4	53.5	53	56.9	62.5	60.6	34		28	87	85.3
16	AGOGO	80.3	81.3	79.3	83	86.9	84.4	104.4	90.4	81.5	81	84.9	90.5	88.4	62	28		59	113.3
17	AGONA	34.7	35.7	38.3	42.2	41.3	37.1	58.8	44.8	35.9	34	34.8	37.2	26.4	53	87	59		60
18	OFFINSO	33	34	36.6	40.5	39.6	35.4	52.5	38.5	33.2	32.3	28.4	22.8	33.6	51.3	85.3	113.3	60	

The table above is the all pair shortest distance matrix D for all 18-nodes in word-excel.

4.2 BERMAN AND DREZNER'S ALGORITHM

Now at this stage, we use the Berman and Drezner's algorithm (2008) to solve the problem. We start by formulating the conditional p-center problem as

Min [(g{x) = min {d(x,,d(y,$\boldsymbol{i}$)}]

$\boldsymbol{i}$ =1,…, n

Let d(x, y) be the shortest distance between any x, y ∈ G. Suppose that there is n set Q (IQI=q) of existing facilities. Let Y=(Y1,….Yq) and X=(X_1, X_2,……..Xp) be vectors of size q and p respectively, where Y$\boldsymbol{i}$ is the location of existing facility $\boldsymbol{i}$ and X$\boldsymbol{i}$ is the location of new facility $\boldsymbol{i}$. Where (x, $\boldsymbol{i}$) and (Y, $\boldsymbol{i}$) is the shortest distance from the closest facility in X and Y respectively to the node $\boldsymbol{i}$, (Berman and Simchi-Levi, 1990).

Considering figure 4.1 (data 18set.mat) the set of location of new facilities X={4,5,7,8,11,12,13,14,15,16,17,18} and the set of location of existing facilities Y={1,2,3,6,9,10} then the conditional P-center problem is to take away the set of nodes with existing warehouses and use the remaining nodes to find three new locations using the modified shortest distance matrix [Y, TC, X]=sproject(3).

$$\text{Minimize g(x) = \{min} \begin{pmatrix} d(4,i),(l,i)\ldots\ldots\ldots\ldots d(10,i) \\ ,, \\ ,, \\ d(18,i),d(l,i)\ldots\ldots\ldots\ldots d(10,i) \end{pmatrix}$$

Where $\boldsymbol{i}$ = {1, 2, 3, ………………………, 18}

After taking out the set of locations of existing warehouses, we have below the towns to consider for new location in Table 4.3a

Table 4.3a: All pairs shortest path distance matrix of towns considered for new location.

Demand Node	Potential location										
	1	2	3	4	5	6	7	8	9	10	11	12
1	-	9.1	36	17.6	12.1	17.7	15.8	21	55	83	42.2	40.5
2	9.1	-	27.8	15.6	11.2	16.8	14.9	24.9	58.9	86.9	41.3	39.6
3	36	27.8	-	14	27.3	29.7	32.4	42.4	76.4	104.4	58.8	52.5
4	17.6	15.6	14	-	13.3	15.7	18.4	28.4	62.4	90.4	44.8	38.5
5	12.1	11.2	27.3	13.3	-	5.6	8.4	22.9	56.9	84.9	34.8	28.4
6	17.7	16.8	29.7	15.7	5.6	-	10.8	28.5	62.5	90.5	37.2	22.8
7	15.8	14.9	32.4	18.4	8.4	10.8	-	26.6	60.6	88.4	26.4	33.6
8	21	24.9	42.4	28.4	22.9	28.5	26.6	-	34	62	53	51.3
9	55	58.9	76.4	62.4	56.9	62.5	60.6	34	-	28	87	85.3
10	83	86.9	104.4	90.4	84.9	90.5	88.4	62	28	-	60	113.3
11	42.2	39.6	58.8	44.8	34.8	37.2	26.4	53	87	59	-	60
12	40.5	39.6	52.5	38.5	28.4	22.8	33.6	51.3	85.3	113.3	60	-

To do the actual calculation using the modified shortest distance matrix [y, TC, X] = sproject(p), the above table is put in word-excel which is then imbedded in matlab and named data12.mat. as below.

Table 4.3b Data12.mat: The set of towns considered for new locations.

		1	2	3	4	5	6	7	8	9	10	11	12
	DATA IN (km)	ATONS U	SANTAS I	NKAWIE	ABUAKWA	SUAME	KRONUM	PANKRONO	EJISU	KONONGO	AGOGO	AGONA	OFFINSO
1	ATONS U	0	9.1	36	17.6	12.1	17.7	15.8	21	55	83	42.2	40.5
2	SANTAS I	9.1	0	27.8	15.6	11.2	16.8	14.9	24.9	58.9	86.9	41.3	39.6
3	NKAWIE	36	27.8	0	14	27.3	29.7	32.4	42.4	76.4	104.4	58.8	52.5
4	ABUAKWA	17.6	15.6	14	0	13.3	15.7	18.4	28.4	62.4	90.4	44.8	38.5
5	SUAME	12.1	11.2	27.3	13.3	0	5.6	8.4	22.9	56.9	84.9	34.8	28.4
6	KRONUM	17.7	16.8	29.7	15.7	5.6	0	10.8	28.5	62.5	90.5	37.2	22.8
7	PANKRONO	15.8	14.9	32.4	18.4	8.4	10.8	0	26.6	60.6	88.4	26.4	33.6
8	EJISU	21	24.9	42.4	28.4	22.9	28.5	26.6	0	34	62	53	51.3
9	KONONGO	55	58.9	76.4	62.4	56.9	62.5	60.6	34	0	28	87	85.3
10	AGOGO	83	86.9	104.4	90.4	84.9	90.5	88.4	62	28	0	59	113.3
11	AGONA	42.2	41.3	58.8	44.8	34.8	37.2	26.4	53	87	59	0	60
12	OFFINSO	40.5	39.6	52.5	38.5	28.4	22.8	33.6	51.3	85.3	113.3	60	0

4.2.1 THE ALGORITHM

Steps

1. Let D be a distance matrix with rows corresponding to potential locations and columns corresponding to demands.
2. Solve the conditional p-Center problem by defining a modified shortest distance matrix in matlab as follows;

   function [y, TC, X] = sproject (P)

   [y,TC,X] = p center (P,C)

   = p center (P, C dodisp), display intermediate results.

   P= scalar number of NFs to locate

C= n x m variable cost matrix where c($\boldsymbol{i}$, $\boldsymbol{j}$) is the cost of serving EF$\boldsymbol{j}$ from NF$\boldsymbol{i}$

Dodisp=true, default

Y=p element NF site index vector

TC=total cost

= sum (sum (c (xj))

X= nxm logical matrix, where x($\boldsymbol{i}$, $\boldsymbol{j}$)= 1 ie EF$\boldsymbol{j}$ allocated to Nf$\boldsymbol{i}$

3. Find the new towns to locate the warehouses using

   [y,TC,x] = sproject (p), where

   P=1, 2, 3…

   rows represent potential locations and columns represents demands nodes.

   X($\boldsymbol{i}$, $\boldsymbol{j}$) = 1 means is locate and

   X($\boldsymbol{i}$, $\boldsymbol{j}$) = 0 means is not located

4. Display results in matrix form using full (x)

4.3 BERMAN AND DREZNER'S SOLUTION

By considering the all pair shortest path distance matrix for the eighteen node network of some selected towns in the Ashanti Region in table 4.2a above, a new shortest path distance matrix [y, TC, X]=sproject(p) is formulated and used for the calculation given tables 4.4 & 4.5.

4.3.1 THE MODIFIED SHORTEST DISTANCE MATRIX (sproject)

By defining a modified shortest distance matrix, sproject in MATLAB; thus

[y,TC,X] =sproject (P), $\forall_i \in$ NF, j $\in$ EF, the location problem is solved as follows:

Considering all eighteen selected towns (table 4.2a), to locate facilities (warehouses) in three towns including towns with existing facilities, we put p=3

Thus [y,TC, X) = Project (3)

The results are displayed below:

Y=

1	15	17

TC=

153

X =

(1, 1)	1
(1, 2)	1
(1, 3)	1
(1, 4)	1
(1, 5)	1

(1, 6)	1
(1, 7)	1
(1, 8)	1
(1, 9)	1
(1, 10)	1
(1, 11)	1
(1, 12)	1
(1, 13)	1
(1, 14)	1
(15, 15)	1
(15, 16)	1
(17, 17)	1
(1, 18)	1

The results mean that nodes I, 15 and 17 are located. The total cost is 153 and X($\boldsymbol{i}$, $\boldsymbol{j}$) =1 means all rows with functional value 1 are located. The results are displayed in a matrix form for further analysis by typing.

Full (x) >>.

Table 4.4 summarizes the results of table 4.2b into the modified shortest path distance matrix [y, TC, X] =sproject(3).

Full (x) >>

Ans =

Columns 1 through 11

Table 4.4 A modified shortest distance matrix [y, TC, X] =sproject(3) with Q

1	1	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

Columns 12 through 18

Table 4.4 A modified shortest distance matrix [y, TC, X] =sproject(3) with Q continued.

1	1	1	0	0	0	1
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	1	1	0	0
0	0	0	0	0	0	0
0	0	0	0	0	1	0
0	0	0	0	0	0	0

Note that the results of the modified shortest distance matrix with existing set of nodes (Q= {1, 2, 3, 6, 9, 10}), node 1 is located again. Furthermore, node 1 alone is serving fourteen (14) different towns which are densely populated (see Appendix B). In this situation, demand will outweigh supply. Hence the set of demand nodes and potential location of the existing facilities (Q) is removed given Table 4.3a which is put into excel, that is, table 4.3b and imbedded in matlab. The modified shortest distance matrix [y, TC, X] = sproject(p) is used to find the three new locations of twelve remaining towns using table 4.3b.

Thus we put p = 3 into the model since we wish to find three new locations.

[y, TC, X] =sproject (3)

In column form the results is displayed below.

Y =

5	8	9

TC =

156.7

X =

(8, 1)	1
(5, 2)	1
(5, 3)	1
(5, 4)	1
(5, 5)	1
(5, 6)	1
(5, 7)	1
(8, 8)	1
(9, 9)	1
(9, 10)	1

(5, 11) 1

(5, 12) 1

To display these results in matrix form for further and better analysis and discussion, type full(x) and is displayed as below:

Full (x) >>

Ans =

Column 1 through 11

Table 4.5 Modified shortest distance matrix [y, TC, X] =sproject(3) with set Q removed.

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	0	0	0	1
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	1	1	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

Table 4.5 continued.

Column 12

0
0
0
0
1
0
0
0
0
0
0
0

From the table above, nodes 5, 8 and 9 are located. That is Suame, Ejisu and Konongo respectively. To decide among the three, where to locate the first warehouse, the factor rating method is used.

4.4 FACTOR RATING METHOD

From the Berman and Drezner's algorithm, three different locations were found to be optimal, thus suame (models 5), Ejisu (node 8) and Konongo (node 9). To decide among the three communities, which should be located first, the factor rating method is used.

Determining the first location of the warehouses, five relevant factors listed below is noted as shown in Table 4.6 with the respective rating weight attached to each factor.

Table 4.6: Relevant Factors and Rating Weights

Factor	Factor Name	Rating weight
1	Population and market size.	5
2	Access to public, private and other means of transportation.	3
3	Land acquisition (space)	6
4	Availability of water, power-source and cost.	4
5	Labour cost, security and other supporting services	2

Table 4.7 below summarizes the result that opinion leaders, business men and women and other related people score each factor, ranging from one up to hundred (1 to 100) for all three locations.

Table 4.7 Location Rate on a 1 to 100 basis

Factor	Suame	Ejisu	Konongo
1	59	51	83
2	80	80	70
3	30	50	90
4	70	60	70
5	60	70	80

Now, to determine where to locate first, the ratio of rating weight is multiply by the rating scores of the town for each particular factor. The result are then shown in table 4.8

Table 4.8: Rating Scores of Locations

Factor	Rating weight	Ratio of rate	Suame	Ejisu	Konongo
1	5	0.25	14.75	12.75	20.75
2	3	0.15	12	12	10.5
3	6	0.30	9	15	27
4	5	0.20	14	12	14
5	2	0.10	6	7	8
TOTAL	**20**		**55.75**	**58.75**	**80.25**

From their respective aggregate scores, Konongo is recommended to be located first since it has the highest aggregate score of 80.25, followed by Ejisu and Suame.

4.5 ANALYSIS AND DISCUSSION

With the algorithm demonstrated above, and considering the 18–node network depicted in Figure 4.1, and solving the conditional p–center problem with Q=(1,2,3,6,9,10) and P =3, gives the results in table 4.4. When Q is taken out of the 18 node distance matrix D, and we wish to find three (3) new locations for the warehouses, we use table 4.3b (ie data12) and put p =3 into [y, TC, X] = sproject (3). It is easy to verity, that, without Q, the results are also shown in Table 4.5. Thus the optimal new locations are found using the modified shortest distance matrix, [y,TC,X]= sproject(p). From the two tables, that is 4.4 and 4.5, X($\boldsymbol{i}$, $\boldsymbol{j}$) = 1 means that, that row is located and X($\boldsymbol{i}$, $\boldsymbol{j}$) = 0 means is not located. Thus rows 1, 15 and 17 are located. As can be seen from table 4.4, kejetias (1) is located again and to serve about fourteen towns which are densely populated with high demand. Konongo (15) is located and to serve only one town, Agogo (16) and Agona (17) is located but can only serve Agona township because is not closer to any of the selected towns. Moreover its population is far less. Hence Q is removed and table 4.3b (date 12) is used with the modified shortest distance matrix to find the optimal new locations as shown in Table 4.5. In this final table 4.5, nodes 5, 8 and 9, thus suame, Ejisu and Konongo respectively are located.

This will cover a total distance of 156.7km. Now X = (8, 1) means row 8 is located and will supply column 1 (ie. demand node 1). Thus a warehouse located in Ejisu(8) will supply Ejisu and Atonsu(1) communities and so on. Full (X) means, display X in matrix form. Hence from Table 4.5, the optimal new locations using the modified shortest distance, [y, TC, X] = sproject(p), thus by applying Berman and Drezner's algorithm, the three new warehouse can be located at node 5 (Suame), node 8 (Ejisu) and node 9 (Konongo) with the functional value X ($\boldsymbol{i}$, $\boldsymbol{j}$) = 1.

Furthermore, to decide among the three, where to locate first since the three locations may not be done at the same time due to financial constraints, the factor rating method was used (refer to table 4.6, 4.7 and 4.8). By this method, the ratio of rating weight is multiplied by the rating scores of each town for the particular factor. From Table 4.8, it is clear that Konongo (node 9) has the highest aggregate score of 80.25, Ejisu (nodes 8) 58.75 and suame (node 5) 55.75. It, therefore, implies that, Konongo is located first, followed by Ejisu and Suame in that order. Hence it is recommended for Unilever Ghana Limited to locate the first warehouse at Konongo, second at Ejisu and the third at Suame in the Ashanti Region.

CHAPTER 5

CONCLUSION AND RECOMMENDATION

5.1 CONCLUSION

The main purpose of this study was to use the conditional p-center model to locate three additional warehouses for Unilever Ghana Limited in the Ashanti Region.

So considering the objective function and the formulation [y, TC, X] = sproject (p), as shown in page 52, thus applying the Berman and Drezner algorithm, the three warehouses can be located at these nodes; node 5 (Suame), node 8 (Ejisu) and node 9 (Konongo).

Furthermore, by identifying relevant factors for the location of warehouses, determining rating weights and analysis of scores, the first warehouse to be located among the three sites/communities is Konongo which had the highest aggregate score of 80.25. The second and third to be located are Ejisu and Suame with aggregate scores of 58.75 and 55.75 respectively by the factor rating method (see table 4.8). The total distance covered was 156.7km.This implies a total distance of 156.7km more will be covered for these three new locations and the other communities allocated to them as demand nodes.

5.2 RECOMMENDATIONS

By the results obtained in this study, the following recommendations are made;

1. Unilever Ghana Limited is advised to locate the three warehouses at Suame, Ejisu and Konongo in the Ashanti Region.

2 It is also recommended that the first warehouse should be located at Konongo, second at Ejisu and the last one at Suame.

3 Whenever the company is considering other regions for more locations, the model used in this study could be considered. But other researchers who will like to do further work could consider Z, Drezner's algorithm that requires solving 0(log n) unconditional p-center problem.

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APPENDIX A

```
  function [y,TC,X] = sproject(p)

% [y,TC,X] = pcenter(p,C)

%         = pcenter(p,C,dodisp),  display intermediate results

%     p = scalar number of NFs to locate

%     C = n x m variable cost matrix,

%         where C (i,j) is the cost of serving EF j from NF i

%dodisp = true, default

%     y = p-element NF site index vector

%    TC = total cost

%       = sum(sum(C(X)))

%     X = n x m logical matrix, where X(i,j) = 1 if EF j allocated to NF i

% Matlog Version 15 07-Jan-2013 (http://www.ise.ncsu.edu/kay/matlog)
```

```
clc, close all hidden

dataR = load('data12.mat');

C = dataR.data;

% C(:,[1:2,10]) = [];

% C([1:2,10],:) = [];

[y,TC,X] = pcenter(p,C,false);

function [y,TC,X] = pcenter(p,C,dodisp)

% Input Error Checking
*****************************************************

narginchk(2,3);

[n,m] = size(C);

if nargin < 3 || isempty(dodisp), dodisp = true; end

if ~isscalar(p) || p > n || p < 1

   error('"p" must be between 1 and "n".')

elseif ~isscalar(dodisp) || ~islogical(dodisp)

   error('"dodisp" must be a logical scalar.')

end
```

```
% End (Input Error Checking) **********************************************

 [y,TC] = ufladd(0,C,[],p); if dodisp, fprintf('  Add: %f\n',TC), end

[y,TC] = uflxchg(0,C,y); if dodisp, fprintf(' Xchg: %f\n',TC), end

 [y1,TC1] = ufldrop(0,C,[],p); if dodisp, fprintf(' Drop: %f\n',TC1), end

[y1,TC1] = uflxchg(0,C,y1); if dodisp, fprintf(' Xchg: %f\n',TC1), end

 if TC1 < TC, TC = TC1; y = y1; end

if dodisp, fprintf('Final: %f\n',TC), end

 y = sort(y);

X = logical(sparse(y(argmin(C(y,:),1)),1:m,1,n,m));

 function [y,TC,X] = ufladd(k,C,y,p)

 % Input Error Checking
**************************************************

narginchk(2,4);

 C = C'; % Make column-based to speed up minimization

[m,n] = size(C);

if isscalar(k), k = repmat(k,1,n); else k = k(:)'; end

 if nargin < 3, y = []; else y = y(:)'; end
```

```
if nargin < 4, p = []; end
 if length(k) ~= n || any(k < 0)
   error('"k" must be a non-negative n-element vector.')
elseif ~isempty(y) && (any(y < 1) || any(y > n) || ...
      length(y) ~= length(unique(y)))
   error('"y" must contain integers between 1 and n.')
elseif ~isempty(p) && (~isscalar(p) || p > n || p < 1)
   error('"p" must be between 1 and "n".')
elseif ~isempty(p) && ~isempty(y) && length(y) < p
   error('"p" cannot be greater than length of "y"')
end
% End (Input Error Checking) ************************************************
 if isempty(y)
   [TC,y] = min(sum(C,1) + k);
else
   TC = sum(k(y)) + sum(min(C(:,y),[],2));
end
```

```
ny = 1:n; ny(y) = [];
TC1 = Inf;
done = false;
while ~done
  c1 = min(C(:,y),[],2);
  k1 = sum(k(y));
  for i = 1:length(ny)
    TCi = k1 + k(ny(i)) + sum(min(c1,C(:,ny(i))));
    if TCi < TC1
      [i1,TC1] = deal(i,TCi);
    end
  end
  if (isempty(p) && TC1 < TC) || (~isempty(p) && length(y) < p)
    y = [y ny(i1)];
    ny(i1) = [];
    TC = TC1;
    if ~isempty(p), TC1 = Inf; end
```

```
  else
    done = true;
  end
end
 y = sort(y);
X = logical(sparse(y(argmin(C(:,y),2)),1:m,1,n,m));
 function [y,TC,X] = uflxchg(k,C,y)
 % Input Error Checking
****************************************************
narginchk(3,3);
 C = C';  % Make column-based to speed up NF indexing
[m,n] = size(C);
if isscalar(k), k = repmat(k,1,n); else k = k(:)'; end
y = y(:)';
 if length(k) ~= n || any(k < 0)
  error('"k" must be a non-negative n-element vector.')
elseif ~isempty(y) && (any(y < 1) || any(y > n) || ...
```

```
    length(y) ~= length(unique(y)))
  error('"y" must contain integers between 1 and n.')
end
% End (Input Error Checking) ***********************************************
 TC = sum(k(y)) + sum(min(C(:,y),[],2));
 TC1 = Inf;
if length(y) > 1, done = false; else done = true; end
while ~done
  [c1,idx] = min(C(:,y),[],2);
  k1 = sum(k(y));
  ny = 1:n; ny(y) = [];
  for i = 1:length(y)
    is = i == idx;
    c1i = c1; c1i(is) = min(C(is,y([1:i-1 i+1:end])),[],2);
    for j = 1:length(ny)
      TCij = k1 - k(y(i)) + k(ny(j)) + sum(min(c1i,C(:,ny(j))));
      if TCij < TC1
```

```
          [i1,j1,TC1] = deal(i,j,TCij);
        end
      end
    end
    if TC1 < TC
      [ny(j1),y(i1)] = deal(y(i1),ny(j1));
      TC = TC1;
    else
      done = true;
    end
end
 y = sort(y);
X = logical(sparse(y(argmin(C(:,y),2)),1:m,1,n,m));
 function [y,TC,X] = ufldrop(k,C,y,p)
 % Input Error Checking
*******************************************************
narginchk(2,4);
```

```
C = C';  % Make column-based to speed up NF indexing
[m,n] = size(C);
if isscalar(k), k = repmat(k,1,n); else k = k(:)'; end
if nargin < 3 || isempty(y), y = (1:n)'; else y = y(:)'; end
if nargin < 4, p = []; end
if length(k) ~= n || any(k < 0)
  error('"k" must be a non-negative n-element vector.')
elseif ~isempty(y) && (any(y < 1) || any(y > n) || ...
     length(y) ~= length(unique(y)))
  error('"y" must contain integers between 1 and n.')
elseif ~isempty(p) && (~isscalar(p) || p > n || p < 1)
  error('"p" must be between 1 and "n".')
elseif ~isempty(p) && ~isempty(y) && length(y) < p
  error('"p" cannot be greater than length of "y"')
end
% End (Input Error Checking) ************************************************
TC = sum(k(y)) + sum(min(C(:,y),[],2));
```

```
 TC1 = Inf;
if length(y) > 1, done = false; else done = true; end
while ~done
  [c1,idx] = min(C(:,y),[],2);
  k1 = sum(k(y));
  for i = 1:length(y)
    is = i == idx;
    TCi = k1 - k(y(i)) + sum(c1(~is)) + ...
      sum(min(C(is,y([1:i-1 i+1:end])),[],2));
    if TCi < TC1
      [i1,TC1] = deal(i,TCi);
    end
  end
  if (isempty(p) && TC1 < TC) || (~isempty(p) && length(y) > p)
    y(i1) = [];
    TC = TC1;
    if ~isempty(p), TC1 = Inf; end
```

```
  else
     done = true;
  end
end
 y = sort(y(:)');
X = logical(sparse(y(argmin(C(:,y),2)),1:m,1,n,m));
 function [i,j,y] = argmin(X,DIM)
 % Input Error Checking
*****************************************************************
if nargin > 1 && nargout > 1
  error('DIM can only be used with a single output argument.')
end
% End (Input Error Checking) ***********************************************
 if nargin < 2
  [y,i] = min(X);
else
  [y,i] = min(X,[],DIM);
```

```
end

if nargout > 1

  if size(X,1) == 1

    j = i; i = 1;

  elseif size(X,2) == 1

    j = 1;

  else

    [y,j] = min(y); i = i(j);

  end

end
```

APPENDIX B

2000 POPULATION AND HOUSING CENSUS			
LOCALITY	**TOTAL**	**MALE**	**FEMALE**
ADUM	*8,016*	*4,042*	*3,974*
ASOKWA	*18,747*	*9,800*	*8,947*
ATONSU-AGOGO	*45,778*	*22,851*	*22,927*
SANTASI	*9,421*	*4,616*	*4,805*
DANYAME	*5,340*	*2,732*	*2,608*
DANYAME	*2,951*	*1,534*	*1,417*
BANTAMA	*22,060*	*10,495*	*11,565*
ASHANTI NEWTOWN (ODUMASI)	*20,031*	*10,214*	*9,817*
OLD SUAME	*15,392*	*7,756*	*7,636*
PANKRONO	*36,683*	*18,383*	*18,300*
KRONOM	*13,988*	*7,194*	*6,794*
OFFINSO	12,327	5,759	6,568
ABUAKWA	16,582	8,270	8,312
NKAWIE-KUMA	4,836	2,468	2,368
NKAWIE-PANYIN	1,519	791	728
KONONGO	26,735	13,519	13,216
AGOGO	28,271	13,650	14,621
EJISU	10,923	5,215	5,708
AGONA	9,321	4,660	4,661