OPTIMIZATION OF BOARDING FEEDING SYSTEM USING KNAPSACK, A CASE STUDY OF TEMA SENIOR HIGH SCHOOL

By

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DECLARATION

I hereby declare that this submission is my own work towards the MPhil and that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the University, except where due acknowledgment has been made in the text

Dorcas Obeng-Davis

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ABSTRACT

This research seeks to solve the problem of increasing boarding feeding grant each time food price escalates by modelling food items as a knapsack problem and optimizing purchases using knapsack algorithm so as to efficiently purchase food items from suppliers with the same grant. The model used is the bounded knapsack model, which consists of an objective function taking into account the cost of items purchased and the number of times the item is required in the menu.

Secondary data obtained from Tema Senior High School's was coded in Matlab and analysed. The following were the results and conclusions; the model adequately solves the problem and if used, the school would optimize purchases thereby saving an amount of Gh¢68,374.12 on the Government feeding grant per term. The study recommends the following amongst others; First, that Senior High Schools such as Tema Senior High School that provide boarding and feeding should use the model in selecting quantities of items to be purchased in relation to their menu. Second, that the schools practice bulk purchases because prices are beaten down as a result of discount. Finally, that consortiums of school build one warehouse in each Region, so that, perishable and non-perishable food items could be stored for a longer period.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF APPENDICES

LIST OF ABBREVIATIONS

TMA................Tema Metropolitan Assembly

ICT..................Information and Communication Technology

PTA.................Parents and Teachers Association

CMB................Cocoa Marketing Board

CHASS............Heads of Assisted Secondary Schools

GSFP...............Ghana School Feeding Program

WFP................World Food Program

WBG...............World Bank Group

SFP..................School Feeding Program

KP....................Knapsack Problem

KPC................Knapsack Problem With Continues Variables

CKP................Continues Knapsack Problem

TAC................Trading Agent Competition

SCM................Supply Chain Management

MCKP............Multiple Choice Knapsack Problems

BKP................Bounded Knapsack Problem

DEDICATION

This work is dedicated to my Mum and Dad and all members of my family as well as my lovely family in Kumasi, Sister Makafui, Jiro, Joy, and Betha. Thank you all for the love and care. May God richly bless you and keep you for the rest of your life and my family for the love and support they gave me during the writing of my thesis.

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My final appreciation goes to all those who helped me in diverse ways in ensuring the successful completion of the thesis, May God richly bless you.

CHAPTER 1

INTRODUCTION

"In the face of global crises, we must now focus on how school feeding programs can be designed and implemented in a cost-effective and sustainable way to benefit and protect those most in need of help today and in the future." By Robert Zoellick (2009), president of the World Bank Group.

1.1 PROFILE OF TEMA SENIOR HIGH SCHOOL

Tema Senior High School was established and commissioned on 22nd September 1961 with an initial student population of fifty two (52) and six (6) members of staff. Dr F.K Buah was the first Headmaster. It was mainly an Arts and Science Institution. In the same year (1961) the student population increased to a hundred and five (105).

Currently about fifty (50) years down the line the student population is one thousand six hundred and eighty four. Out of this number one thousand two hundred and eighty-four are boarders.

Over the years, this great school has been stewarded by nine seasonal educationalists who believed in academic excellence and total personal development. These great headmasters over the years are as follows from the first to the present. Dr. F. K Boah, Mr. K. A. Yirenkyi, Mr E. L. A. Lamptey, Mr W.K. Agbelie, Mr. A.K. Agyepong, Mr. E. Owusu-Ansah, Mrs. Victoria Opoku, Mrs. Eunice Naadu Quansah and the current is Mrs. Elizabeth Ama Asare

Tema Senior High School has changed with the changing educational reforms. In 1961, it offered subjects in Arts and Science with no business subjects pursued. Also the final examinations were the Ordinary Level and the Advanced Level School Certificate. When the 1987 educational reform was introduced the school considered adding new programs considering the number of streams in the school.

The school now offers four out of five programs in the new educational reform. The programs offered are General Science, General Arts, Business and Vocational Science.

Tema Senior High School has several departments including the ICT department, Science Department, Business Department, Visual Arts department, Mathematics Department, Home Economics Department, Social Sciences Department, Language Department, Physical Education Department, Catering Department, among others. The school has a domestic bursar who works with a team of cooks. Not forgetting a hardworking dining hall master, who addresses complains by students.

1.1.1Catering Department

The school has many departments including the catering department. The catering department has the following number of staff; 4 catering officers or Matrons, 16 cooks and 6 pantry hands. The department has the following infrastructure and equipment

- Dining hall
- Kitchen
- Bread Room

- Grinding Room
- Three Store Rooms
- Two Deep Freezers Fridges

The department has most of the cooking and storage equipment for catering in Second Cycle Schools only with the exception of

- Cold Room
- Bread Cupboard.

The department has been battling with a few problems, the major one being the Government Approved feeding fee per head per day of Gh¢ 1.40 as at 2010-2011 academic years. The daily costing often lies between GH¢1.50 to GH¢2.00. This academic year 2011-2012 the approved feeding fee per head per day has been increased to GH¢1.80 and again food prices has shot up. The department wishes the government find a lasting solution to this problem.

However, they observed that, when they made bulk purchases, it helped them especially as it beats down the prices and helped them achieve their targeted daily costing.

1.1.2Payment of food

Ghanaian students enjoy tuition-free education. A separate fee is however payable by boarding students. This fee covers the cost of boarding, feeding etc. The boarding fees are determined by the government and are revised from time to time. Some students receive grants under various schemes to cover the cost of boarding and any other fees.

Foreign student, however, have to pay tuition, as well as boarding fees, if they are boarders. The boarding fees only cater for the student's feeding. The school may decide to charge for PTA or other charges. I would only concentrate of the boarding feeding fee.

The payment of boarding feeding fee in Tema Senior High School is done in three ways.

- T.M.A Scholarship
- CMB boarding Scholarship
- Companies, Churches, and Organizations scholarship
- Parents or Guardian

The fee is calculated based on the government approved fee per head of Gh¢ 1.80 by the number of days the student would spend in the school totalling an amount of GH¢168.80

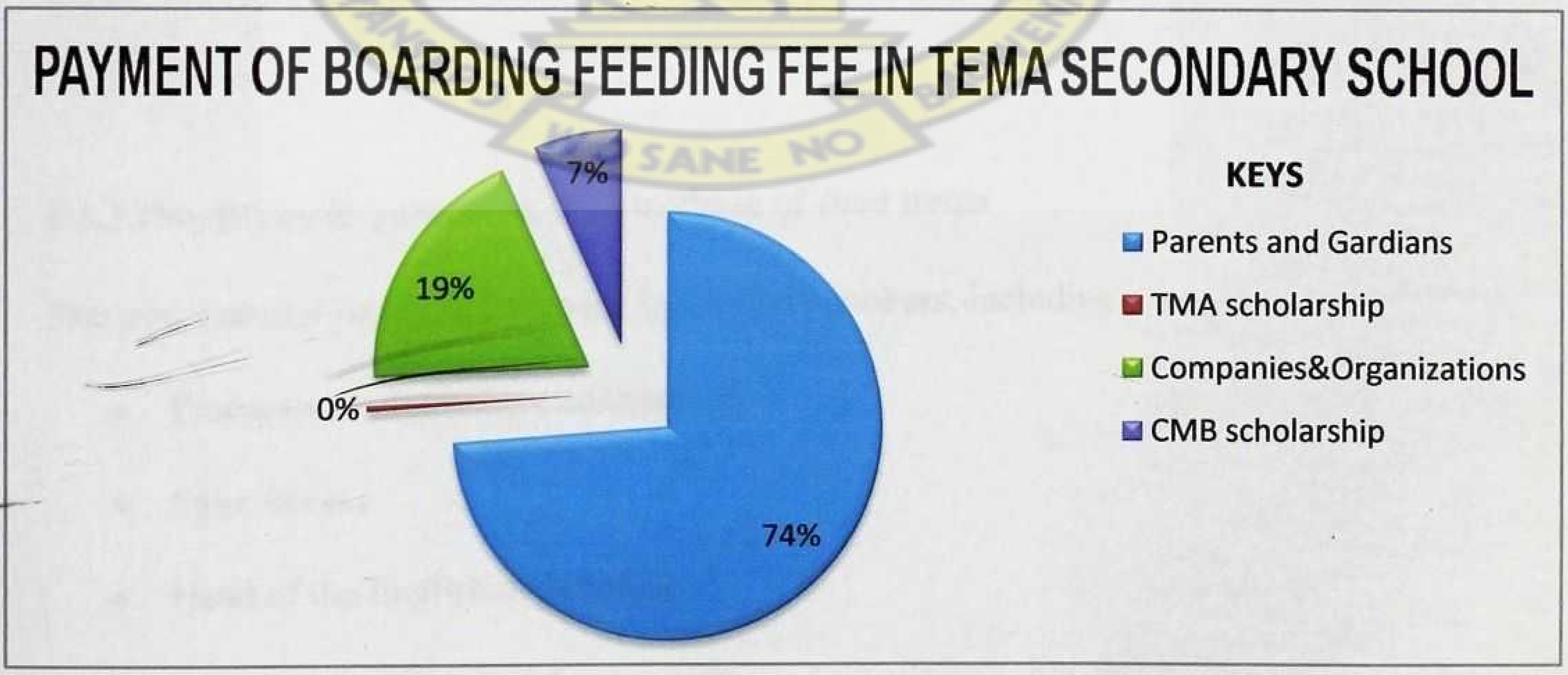


Figure 1.1 Payments of Fees

Government increases the feeding grant each academic year and this has brought untold hardship to most parents since they have to find extra money to pay for their wards fees. This often results in lots of students dropping out of school or becoming day students and attending classes on empty stomachs.

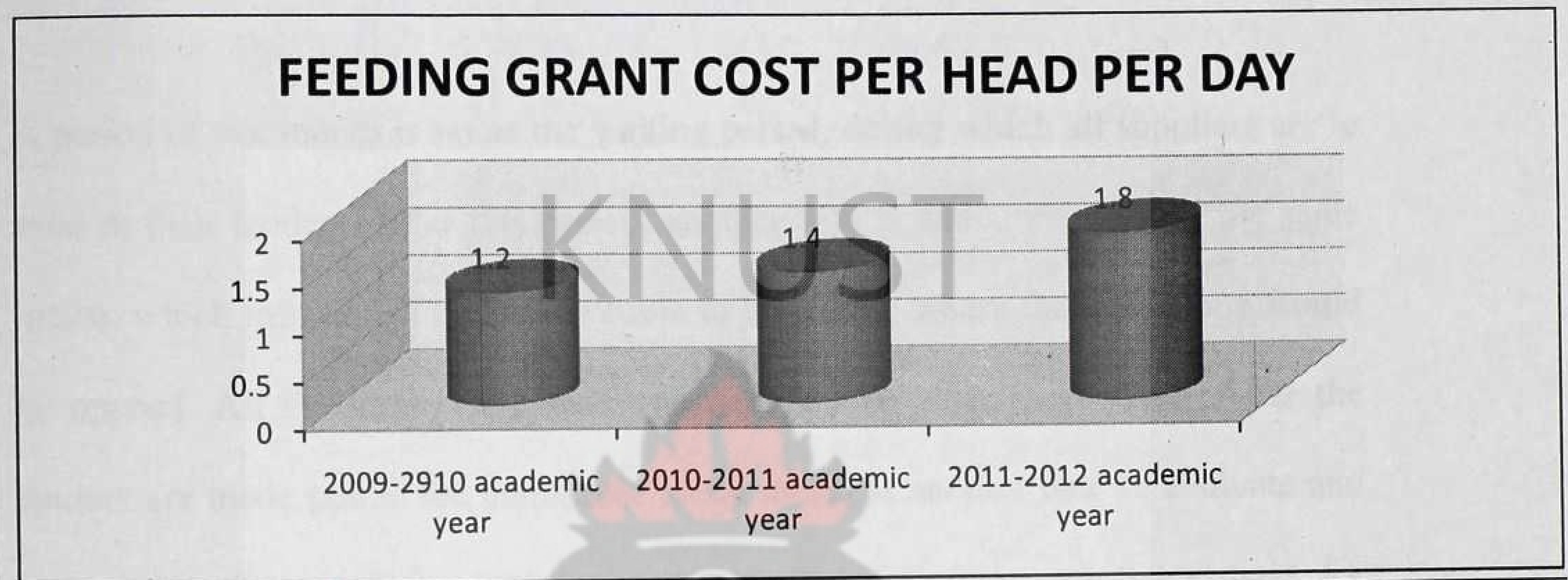


Figure 2.2 Feeding Cost

Hence the big question is that should government increase the grant whenever inflation of food process increases? Or should government maintain the price which also results in food shortages. This is why there is the need to optimize the feeding grants. And this is what the thesis seeks to solve.

1.1.3 Procurement process in the purchase of food items

The procurement committee is made up of five members, including

- Procurement committee chairperson
- Store keeper
- Head of the English department

- Dining hall master (not a permanent member)
- Any teacher could be called upon to join the committee for transparency

During the vacation period, the committee sends an advertisement in the newspapers. Then interested suppliers would bring their price quotations as well as samples of the items they wish to supply. All of which would be sealed either in envelopes or packaged and kept in the tendering box.

A period of one month is set as the waiting period, during which all suppliers are to send in their tenders. After this period, another date is announce through the same media, which informs all tenders to come to a meeting where the tender box would be opened. All the tenders are read out in a fair and transparent way. After the tenders are made public the committee alone meets at another date to evaluate and select the winners of the various contracts. The following are the criteria for selecting the winners;

- Income tax (whether or not the supplier has satisfied his/her tax obligation)
- Vat registration (whether supplier has registered vat)
- Price quotation (this is not to say the lowest price is what is considered over quality)
- Product quality (as to the specification by the committee)

1.1.4 Terms of bridge of contract

A period is allowed for suppliers to prepare to supply. Usually the supply date is normally the start of the academic term. However a contract can be terminated under the following circumstances

- When the supplier is not able to supply during the first week of the academic term.
- When the quality of goods supplied is below the quality of the sample brought during procurement process
- When supplier increases the price.

In this situation, the committee would not just call the next supplier who could have equally won the contract. But another procurement process would have to be done, in order to fill in the vacant position.

1.1.5 History of Problem

In (2008-2009) academic year, the approved feeding grant was 80p. In (2009-2010) academic year, the approved feeding grant was 1.20p. Then in (2010-2011) academic year the grant was increased to 1.40p, and currently (2011-2012) academic year it's now 1.80p. Hence, the government have been increasing the feeding grant each academic year and this affects students who are not benefiting from the grant because they don't hail from the catchment areas. Also heads of institutions running the boarding system have been battling with how to efficiently manage the grant so as to feed large numbers of students on a meagre grant, irrespective of inflation on food prices over the period.

The government feeding grant is calculated as per head per day of all meals taken by each student as well as catering for the purchases of cooking utensils, fuel, food items, among others. This is a major challenge facing all government second cycle institutions across the country. This is due to the fact that within the academic year, we have both the harvesting season and the lean seasons. In each of these seasons food prices differ, hence it becomes very difficult to manage the same resource throughout the term.

According to Daily Guide issue on the 18th July 2009, states that there has been food shortages in School Campuses which compelled Heads of Institutions to push forward their vacation dates to 22nd July, 2009 and 28th July, 2009 instead of the first week in August.

Some students who were interviewed at the Labone Senior High School expressed their displeasure at the meals served during that term admitting they never enjoyed a balanced diet throughout the term as compared with the previous terms were they were served sumptuous meals. They again complained that they were to vacate in the first week in August only to be told a few days ago that they are going home on July 28.

The headmaster of Presbyterian Boys School (PRESEC) admitted that the school fell short food items but its suppliers gave them the items on credit while payments were made later on. This has sustained the school up till now but by the daily costing estimates the school ~~spent Gh¢~~ 1.40 per day on each student which is far above the government's quota.

At the Bolgatanga Girls School, first year students were made to go home two weeks before the actual date. This brought serious consequences on student as well as their parents.

The head mistress of Aburi Girls Senior High School was of the view that students could no longer feed on the meagre 80Gp because prices of food prices keep escalating. Her hopes were that Government finds a lasting solution to the problem.

During the meeting between CHASS and GES, CHASS demanded an increase of GH¢1.20 per student but GES stood at GH¢1.00. Hence to break this impasse, the Ministry of Finance and Economic Planning (MFEP) approved an upward adjustment of GH¢1.20 for all public Schools with effect from 2009-2010 academic year.

Following this, the Ministry of Education approved another upward adjustment of Gh¢1.40 effective at the start of the 2010-2011 academic years. Currently, in the 2011-2012 academic year, the Ghana Education Service (GES) has approved a new feeding fee of GH¢1.80 per day for boarding students in schools, following a request by the Conference of Heads of Assisted Secondary Schools (CHASS) for an upward review of the old fee of GH¢1.40 per student per day.

1.2 PROBLEM STATEMENT

Over the years, authorities running boarding schools in Ghana including Tema Senior High School claim that the feeding grant is not sufficient. Hence the study investigates the feeding schedule in Tema Senior High School.

1.3 OBJECTIVES

The objectives of the study are:

i. model food item purchases as a knapsack problem.

ii. to optimize purchases using Knapsack algorithm.

1.4 METHODOLOGY

In this study, we shall use the bounded knapsack model which is made up of an objective function taking into account the cost of items purchased and the number of times that items is required in the menu. Data used in the study is secondary data. The daily costing for a period of one month are obtained from the domestic bursar, the procurement documents are obtained from the chief account officer. The solution algorithm would be programmed using mat lab. Other sources of information are the library and internet.

1.5 JUSTIFICATION

The Senior High Boarding System improves the academic performance of students in most senior high school and is of great benefit and to the Ministry of Education and the Ghana Educational System.

However, there is the need to find a solution, which can help Heads of the various institutions to manage the approved feeding grant by the government to cater for inflation of food prices over the period.

However, without any adequate scientific method of selection the quantities of food items to be bought, the maximum benefit, which could have been derived from the same budget, would not have been achieved. The problem of selecting food items to be bought can be modelled by a bounded knapsack problem and an appropriate program written based on the algorithm such that with just a click of a button, the maximum benefit is achieved under the budget.

Economically, if the school decides to budget for the year, the results would be that, food staffs are purchased during the harvest season and stored in a warehouse so that during the drought season when prices of food is escalating the school would not be affected. If this is done, demands for increases in feeding grant due to escalating food prices would be a thing of the past.

Socially, the research ensures that students are feed regularly throughout the academic year and ~~this eliminates~~ deviant behaviours which is normally associated with poor diet (research by University of South California 2008). The social problem of school drop outs due to increases in fees would be reduced.

1.6 THESIS ORGANIZATION

In this thesis, we shall explore ways to solve the problem of food shortages in our Senior High School campuses as well as the influx of Government interventions in the increasing feeding grant annually, which results in untold hardship to parents who are the majority stakeholders.

The thesis is organized in five chapters. Chapter one deals with the background of sturdy area, then an overview of the Catering department, which is the most essential department in this research. Then how payment of food is done, and who are the majority stakeholders. Followed by the procurement procedure in Tema Senior High School, the problem at hand, the essence of the study, the method in which the problem can be solved and the justification of the thesis.

In Chapter two, we shall review pertinent literature on existing models, which will be useful in the study, the areas are the Ghana school feeding program models, boarding feeding systems in Ghana, and projects using knapsack model in inventory and all other application of the knapsack problem.

In chapter three we shall put forward the methodology. Chapter four deals with data collection and analysis, while Chapter five presents the conclusions and recommendations of the study.

CHAPTER 2

LITERATURE REVIEW

2.0 INTRODUCTION

This chapter focuses on the review of important literature on the core aspects of the topic under study. There has been some extensive research in the area of school feeding programs and policy but I have found no study on optimization of boarding feeding grant using knapsack model. Hence we shall review works on three areas;

i. School Feeding Program models

ii. Knapsack models

iii. Knapsack inventory models

In this chapter we shall present a review of literature on feeding policies as well as abstracts on the applications of knapsack models.

2.1 SCHOOL FEEDING PROGRAM MODELS

Dank et al. (2008) studied, the school feeding program in Ghana, they observed that, the Ghana School Feeding Program (GSFP) has been in full operation for just over one year. The GSFP has pursued an aggressive expansion policy with the aim of immediately reaching as many hungry children as possible while simultaneously building the market for small farmers by procuring locally produced foods. The initial success of the GSFP is evidenced by the fact that it now serves 975 Schools and approximately 400,000 children. Additionally, school enrolment and attendance in GASP schools throughout Ghana is increasing. At the same time, as the program grows it faces many challenges inherent to organizational expansion. The GSFP, accordingly, continues to Refine and improve its strategies in an attempt to create a sustainable and self-sufficient model that is necessary for long-term success.

Bundy et al. (2009) did a study on rethinking School feeding. This project was financed jointly the World Food Programme (WFP) and the World Bank Group (WBG), to develop a guidance on how to develop and implement an effective school feeding program which would be both a productive safe net as well as a fiscally sustainable investment in human capital which would be a part of a long term global effort to achieve education for all and social protection for the poor. There have been demands from low-income countries affected by social shocks from the current global crises for ways to enhance the school feeding program so as to provide a social safety net for them. This analysis was initiated in response. There are numerous benefits in implementing the school feeding program but the problem in most countries is the sustainability of the program.

This review highlights three main findings. Firstly, school feeding programs in low-income countries exhibit large variation in cost, with concomitant opportunities for cost containment. Second, as countries get richer, school feeding costs become a much smaller proportion of the investment in education. For example, in Zambia the cost of school feeding is about 50 percent of annual per capita costs for primary education; in Ireland it is only 10 percent. Thirdly, the main preconditions for the transition to sustainable national programs are mainstreaming school feeding in national policies and plans, especially education sector plans; identifying national sources of financing; and expanding national implementation capacity.

However, the project concluded that the effectiveness of school feeding programs is dependent upon several factors, including the selection of modality (in-school meals, fortified biscuits, take-home rations, or some combination of these); the effectiveness of targeting; and the associated costs. The overall conclusion is that the global food, fuel, and financial crises and the refocusing of government efforts on school feeding that has followed, provide an important new opportunity to help children today and to revisit national policies and planning for long-term sustainability tomorrow. Taking full advantage of this opportunity will require a more systematic and policy-driven approach to school feeding by both governments and development partners.

Mohammed (2009) focused on the impact and challenges associated with the introduction of FCUBE with emphasis on the Capitation grant and the school feeding programmes in the Ashiedu Keteke Sub-Mero of the Greater Accra Region of Ghana. Data was collected from Six Basic Schools in the Sub- Metro under Circuits 12 and 13. His study reviled that even though the introduction of the policy has improved enrolment, the quality of education is faced with a lot of obstacles and challenges such as access to school, shortage of teachers, economic and social cultural practices etc. It seems evident from the analysis in this study and observations that despite the achievements of government, there still are a number of children out of school in Ghana and being denied the right to education. He therefore concluded that the goals of universal access to primary quality education cannot be achieved through the linear expansion of existing public schools system alone. One major limitation he faced was that the sample was quite small due to limited time and resources. The study contributes to the understanding of what the various education policies say and what really happens on the ground.

Tineke (2007) did a study to determine the nutrients intake from school meals as well as out of school meals in primary school children. He also determined the impact on the demand for locally produced foods. This was his mode of data collection; data was collected in four primary schools in four different districts in Central Region in Ghana in the period of February to April 2007. The study population consisted of 129, class three children aged between (7 to 16 years). An anthropometric measurement was taken to determine their nutritional status. Data collection on nutrient intake from school meals was done using 1-day weighed dietary records and weighing the portion sizes of the selected 3rd grade children.

After the sturdy the following was his findings and conclusions. The Ghana SFP succeeded in increasing the dietary diversity of the diet of the school children in the selected schools. This may reflect in the nutritional adequacy and nutritional status of the primary school children, but no internationally agreed upon cut-off points are available to use as a reference. The Ghana SFP meets its own recommendations for energy intake and protein intake. Vitamin A intake is probably sufficient, but the iron intake remains low, which raises concern. The impact of the Ghana SFP on the local demand for staple foods at district level seems limited.

Earle (2001)analysed the interaction between water, food and trade in Botswana, Namibia, South Africa and Zimbabwe. His hypothesis was that a country with well-developed social resources can overcome physical resource scarcity and continue developing in a sustainable manner. In terms of the international grain trade this means that countries with higher levels of social resources will be able to import grain, which provides the bulk of the per capita calorie intake in the region. Each tonne of grain imported represents over a tonne of water saved locally. This water, used in the manufacture of grain, is called virtual water. The level of reliance of each of the four countries on virtual water is assessed and compared to the state of food security in the country. On an international level, the factors which have a large effect on the viability of a virtual water import policy are the level of agricultural assistance and trade barriers in developed countries. The possible implications of a change in world terms of trade are investigated. He used the Heckscher-Ohlin model and applied it to the sixty-three countries' grain trade in relation to water resources. He concluded that the model is found not to predict the reality well, yet the four countries studied do show a positive correlation between virtual water and food security. This leads to the recommendation that for such a study to be successful it has to use the relative endowments of soil water between the countries as a factor of production.

Nyamekye (2008) examined the role of financial management in the smooth administration of some selected SHS within Ashanti and Central Regions of Ghana. The main sources of funds were GOG (Government of Ghana) and service Grants, School Fees (IGF), GET iWl/HIPC, Scholarships/Bursary Grant and other Non-Formal income (PTA dues, extra classes etc). The study revealed the following; the results of the study indicated among others that Ashanti Region recorded the highest source of income because it has the highest students' population due to the Computerized Secondary School Placement System (CSSPS). Arrears of the GOG Grant and GET Fund/HIPC might have been paid in support for the maintenance of essential academic facilities, developmental and infrastructural projects. The GET Fund/HIPC is disbursed from the GETFUND/HIPC Secretariat in accordance with the student population. The government gives GOG according to the cost of living, prices of commodities/inflation and the living standard of people. Also Internally Generated Fund (IGF) proved to be high. A total of Gh¢ 940,136.35 was recorded as surplus which was added to general reserves. In addition, the Non-Formal income could drain the SHS accounts because it is shared as incentive to teachers. It must therefore be incorporated into the main stream Accounts of the Institutions. Ashanti Region again recorded the highest student debtors. This may be due to the fact that some students, when they verify from the internet that they have not performed well, used their fees to register November/December Examination, and don't even come to the school for their certificates.

Punt (2009) presented a situational analysis on the Ghana School Feeding Programme (GSFP), which is a combined initiative from the New Partnership for Africa's Development (NEPAD), the Government of Ghana (GOG) and the Government of the Netherlands as a part of Ghana's measures to reach the Millennium Development Goals (MDG). Her findings were;

1) In the GSFP pupils in selected public primary schools get a healthy lunch every school-going day to increase enrolment, retention and attendance, and to increase the health of the children.

2) It also boosted domestic food production, as well as the reduction of poverty by the home-grown component of the GSFP.

3) It also provided ready-market for the local farmers in the communities of the beneficiary programme schools.

4) Since the pilot in 2005 the programme has grown rapidly and now covers about 600.000 children at some 1000 schools. On the one hand, enrolment and attendance are increasing as a result of the implementation of the GFSP. This study aims at obtaining a comprehensive picture of the situation at local level, including both the GSFP schools and the farmers. To put the local situation in the proper context, a more general evaluation is executed as well. Consequently, opportunities for the strengthening of the relation between the GSFP caterers and local farmers can be identified in order to change the GSFP into a sustainable and endogenous structure of the local market mechanisms in Ghana.

2.2 KNAPSACK MODELS

Knapsack problems have been studied intensively in the past decade attracting both theorist and practitioners. The theoretical interest arises mainly from their simple structure which both allows exploitation of a number of combinational properties and permits more complex optimization problems to be solved through a series of knapsack type. From a practical point of view, these problems can model many industrial applications, the most classical applications being capital budgets, cargo loading and cutting stock. In this section we shall discuss abstracts on knapsack models

Jacko and Mora (2007), were motivated by a food promotion problem, we introduce the Knapsack Problem for Perishable Items (KPPI) to address a dynamic problem of optimally filling a knapsack with items that disappear randomly. The KPPI naturally bridges the gap and elucidates the relation between the space-hard restless bandit problem and the np-hard knapsack problem. Our main result is a problem decomposition method resulting in an approximate transformation of the KPPI into an associated 0-1knapsack problem. The approach is based on calculating explicitly the marginal productivity indices in a generic finite-horizon restless bandit subproblem.

Büthe and Briskorn (2012) stated that the 0-1 knapsack problem with a single continuous variable (KPC) is a natural extension of the binary knapsack problem (KP), where the capacity is not any longer fixed but can be extended which is expressed by a continuous variable. This variable might be unbounded or restricted by a lower or upper bound, respectively. This paper concerns techniques in order to reduce several variants of KPC to KP, which enables the authors to employ approaches for KP. The authors proposed both, an equivalent reformulation and a heuristic one bringing along less computational effort. The authors concluded and recommendations that showed that the heuristic reformulation can be customized in order to provide solutions having an objective value arbitrarily close to the one of the original problem.

Benisch et al. (2005) examined the problem of choosing discriminatory prices for customers with probabilistic valuations and a seller with indistinguishable copies of a good. They showed that under certain assumptions this problem can be reduced to the continuous knapsack problem (CKP). The authors presented a new fast epsilon-optimal algorithm for solving CKP instances with asymmetric concave reward functions. They also showed that their algorithm can be extended beyond the CKP setting to handle pricing problems with overlapping goods (e.g. goods with common components or common resource requirements), Rather than indistinguishable goods. They provided a framework for learning distributions over customer valuations from historical data that are accurate and compatible with their CKP algorithm, and validated their techniques with experiments on pricing instances derived from the Trading Agent Competition in Supply Chain Management (TAC SCM). Their results confirmed that their algorithm converges to an epsilon-optimal solution more quickly in practice than an adaptation of a previously proposed greedy heuristic.

Zhong and Young (2009) described the use of an integer programming tool, Multiple Choice Knapsack Problem (MCKP), to provide optimal solutions to transportation programming problems in cases where alternative versions of projects are under consideration. Optimization methods for use in the transportation programming process were compared and then the process of building and solving the optimization problems discussed. The concepts about the use of MCKP were presented and a real-world transportation programming example at various budget levels were provided. The authors illustrated how the use of MCKP addresses the modern complexities and provides timely solutions in transportation programming practice.

Martello and Toth (1998) presented a new algorithm for the optimal solution of the 0-1 Knapsack problem in 1998, which is particularly effective for large-size problems. The algorithm is based on determination of an appropriate small subset of items and the solution of the corresponding "core problem": from this they derived a heuristic solution for the original problem which, with high probability, can be proved to be optimal. The algorithm incorporates a new method of computation of upper bounds and efficient implementations of reduction procedures. They also reported computational experiments on small-size and large-size random problems, comparing the proposed code with all those available in the literature

Akinc (2006) addressed the formulation and solution of a variation of the classical binary knapsack problem. The variation that was addressed is termed the fixed-charge knapsack problem, in which sub-sets of variables (activities) are associated with fixed costs. These costs may represent certain set-ups and/or preparations required for the associated sub-set of activities to be scheduled. Several potential real-world applications as well as problem extensions/generalizations were discussed. The efficient solution of the problem is facilitated by a standard branch-and-bound algorithm based on (1) a non-iterative, polynomial algorithm to solve the LP relaxation,(2) various heuristic procedures to obtain good candidate solutions by adjusting the LP solution, and(3) powerful rules to peg the variables. Computational experience shows that the suggested branch-and-bound algorithm shows excellent potential in the solution of a wide variety of large fixed-charge knapsack problems.

Pisinger (2005) proposed a specialized algorithm that solves an expanding core problem through dynamic programming such that the number of enumerated item types is minimal. Sorting and reduction is done by need, resulting in very little effort for the pre-processing. Compared to other algorithms for Bounded Knapsack Problem (BKP), the presented algorithm uses tighter reductions and enumerates considerably less item types. Computational experiments were presented, showed that the presented algorithm outperforms all previously published algorithms for BKP. He concluded that several types of large-sized 0-1 Knapsack Problems (KP) may be easily solved, but in such cases most of the computational effort is used for sorting and reduction. In order to avoid this problem it has been proposed to solve the so-called core of the problem: a Knapsack Problem defined on a small subset of the variables. The exact core cannot, however, be identified before KP is solved to optimality, thus, previous algorithms had to rely on approximate core sizes.

Pisinger (1997) presented an algorithm for Knapsack Problems, where the enumerated core size is minimal, and the computational effort for sorting and reduction also is limited according to a hierarchy. The algorithm is based on a dynamic programming approach, where the core size is extended by need, and the sorting and reduction is performed in a similar "lazy" way. Computational experiments were presented for several commonly occurring types of data instances. Experience from these tests indicated that the presented approach outperforms any known algorithm for KP, having very stable solution times. The multidimensional 0–1 knapsack problem, defined as a knapsack with multiple resource constraints, is well known to be much more difficult than the single constraint version.

Douglas (2009) modelled the problem of how long a bus should be on the road before it's replaced. The aim of the thesis is therefore to determine a schedule of disposals and replacements of the higher bus, taking into account the revenue generated, operating cost and the salvage values, such that the total cost of these activities is minimized. Data was collected from the State Transport Company Office in Kumasi on the revenue generated, operating cost, and the salvage values on the bus with time. The problem was solved by using dynamic programming and knapsack. He concluded that the company should always dispose its buses when they are two years old.

Mosche et al. (2005) reviewed knapsack applications to many situations such as; hiring workers, scheduling jobs, and bidding in sponsored search auctions. They stated that online knapsack problem is inapproximable, so they made the assumption that elements arrive in a random order. Hence our problem can be thought of as a weighted version of the classical secretary problem, which they called the knapsack secretary problem. In the authors' methodology he used the random-order assumption; they designed a constant-competitive algorithm for arbitrary weights and values, as well as a e-competitive algorithm for the special case when all weights are equal. They concluded that, in contrast to previous work on online knapsack problems, they do not assume any knowledge regarding the distribution of weights and values beyond the fact that the order is random.

Peasah (2009) explored ways of effectively and efficiently selects commercials from a pile of commercials within a fixed time to achieve optimal use of air time. in order to maximize space and airtime in the FM stations in Ghana, he used the method of knapsack algorithm and also to develop a software for the knapsack algorithm using Visual Basic dot NET, which can be used by any researcher or radio station. The software could also be modelled to solve many industrial problems: capital budgeting, cargo loading, cutting stock, to mention the most classical applications. At the end he gave the following conclusions and recommendations; There are situations where too many adverts also spoil the beauty of a program and makes it boring. In this case, it is considered as more than one constraint (i.e. both adverts limit and time limit, where the adverts number and limited time are not related), we get the multiple-constrained knapsack problem. We recommend that in future such a situation should be considered.

Myers (2009) focused on some inventory management policies for substitutable and perishable items under demand uncertainty. A set of perishable products with fixed shelf lives is considered under an (R,Si) system of inventory control where demand for a preferred product can be satisfied by a substitute product with a known probability, in the event of a stock out of the preferred product. While taking demand substitution and product expiration into account, the retailer is faced with the decision of determining the order-up-to level, Si, for each product i which maximizes expected total profit, given a common review period, R, determined exogenously.

Under demand uncertainty, the problem detailed in his sturdy involves stochastic optimization. An exact closed form expression, however, for expected profits becomes difficult for certain parameter values involving product shelf-life, product substitution, and lead time. As an alternative approach, order replenishment, demand consumption, substitution, and product expiration can be effectively modelled using discrete-event simulation. Through a discrete-event simulation model, each realization of the profit function can be evaluated for a selected value of Si, and a mean profit value can be estimated after a number of replications of a simulation run. In order to find the best Si solution, the technique of simulation-optimization is used.

Realff et al. (2004) investigated the use of branch-and-bound for solution of the classical knapsack problem. It was shown that the best configuration of the algorithm could be the data dependent. ~~So~~ an „intelligent optimization system needs to configure itself automatically with the control knowledge appropriate to the problem the user is solving.

Broekmeulen and van Donselaar (2007) introduced a new replenishment policy for fixed perishable inventory under a periodic review system for a single product under stochastic, none-stationary demand. The review system is based on an (R,s nQ) system where every R units of time, nQ units of inventory are ordered if the inventory position drops below s, where Q is a fixed batch quantity of the single product, and n is an integer multiple. Daily product demand is assumed to follow a gamma distribution, but the expected daily demand varies in a weekly cycle. A base (R,s,nQ) policy is derived along with the proposed heuristic referred to as the EAR policy which takes into account the age of the inventory in the system. The EAR policy also factors in whether the issuing policy is FIFO or LIFO. The EAR heuristic is tested against a selected Base policy in a factorial design simulation study. Overall, the EAR policy outperforms the base policy in 96% of the experiments with FIFO issuing and more than 99% of the time, with LIFO issuing.

Balachandar et al.(2008) presented a heuristic to solve the 0-1 multi-constrained Knapsack problem, which is NP-hard. In the study, they exploited the dominance property of the constraints to reduce the search space to near optimal solutions of 0-1 multi- constrained knapsack problem. They also presented the space and computational complexity of solving 0-1 multi-constrained knapsack problems using that approach. The results ~~from~~ relative large size test problems showed that the heuristic can successfully be used for finding good solutions for highly constrained NP-hard problems.

Jarugumilli (2012) put forward a nonlinear 0-1 knapsack problem arising in the context of a multi-product network design model considering lead time and safety stocks presented in Sourirajan, Uzsoy and Ozsen(2011). The knapsack problem of interest in this thesis arises as a sub-problem within a Lagrangian heuristic for the network design model. The objective function of the knapsack problem is neither convex, nor concave and is non-separable, which precludes conventional dynamic programming methods. We develop a branch and bound algorithm to obtain exact solution to the knapsack problem, and a genetic algorithm to obtain approximate solutions. A computational experiment is conducted to evaluate the performance of the the procedure.

Abboud et al. (1997) presented an interactive procedure for the multi-objective multidimensional 0-1 knapsack problem that takes into consideration the incorporation of fuzzy goals of the decision maker. It is easy to use since it requires from the decision maker to handle only one parameter, the aspiration level of each objective. Additionally, it is fast and can treat such problem as 0-1 knapsack problem using already available software, the primal effective gradient method, which is meant for solving the large-scale cases.

Freville et al. (2004) came out with an efficient pre-processing procedure for large-scale instances. The algorithm provided sharp lower and upper bounds on the optimal value. It is also a tighter equivalent representation by reducing the continuous feasible set and by eliminating constraints and variables. The scheme was proved to be very effective through a lot of computational experiments with test problems of the literature and large-scale randomly generated instances.

Aissi et al. (2007), studied the approximation of min-max. The authors reviewed versions of classical problems like shortest path, minimum spinning tree, and knapsack. For a constant number of scenarios, they established fully polynomial-time approximation schemes for the mini- max versions of these problems, using relationships between multi-objective and mini-max optimization. They used the dynamic programming and classical trimming techniques to construct a fully polynomial-time approximation scheme for min-max regret shortest path. Additionally, they established a fully polynomial-time approximation scheme for min-max regret spinning tree and proved that min-max regret knapsack is not at all approximable. For a non-constant number of scenarios, in which case, min-max and min-max regret versions of the polynomial-time 35 solvable problems usually become strongly NP-hard, non-approximability result were provided for min-max (regret) versions of shortest path and spinning tree.

Gomes et al. (2007) put forward the problem of inaccuracy of the solution generated by meta-heuristic approaches for combinatorial optimisation bi-criteria 0-1 knapsack problems. The authors proposed a hybrid approach that combines systematic and heuristic searches to reduce that inaccuracy in the context of a scatter search method. They also presented the comparisons with small and medium size instances solved by the exact methods. Large size instances were also considered and the quantity of the approximation was evaluated by taken into account the proximity to the upper frontier, devised by the linear relaxation, and the diversity of the solutions. They also compared the approach with other two well-known meta-heuristic approaches. The results showed the effectiveness of the proposed approach for small, medium and large size instances.

Tsesmetzis et al. (2008) received multiple concurrent requests for services demonstrating different QoS properties. The authors introduced the "Selective Multiple Choice Knapsack Problem" that aims at identifying the services, which should be delivered in order to maximise the profit providers, subject to maximum bandwidth constraints. The problem was solved by a proposed algorithm that had been empirically evaluated via numerous experiments.

Beasley (2002) discussed the basic features of population heuristic and provided practical advice about their effective use for solving operations research problems including knapsack.

The Knapsack problem model is a general resource allocation model in which a single resource is assigned to a number of alternatives with the objective of maximizing the total return.

Owoloko ~~et~~ al. (2010) ~~used the~~ application of the knapsack problem model to the placement of advert slots in the media. The aim was to optimize the capital allocated for advert placements. The general practice is that funds are allocated by trial and error and at the discretions of persons. This approach most times do not yield maximum results, lesser audience are reached. But when the scientific Knapsack problem model was applied to industry data, a better result was achieved, wider audience and minimal cost was attained.

Kwarteng (2011) modelled the TV 3 adverts selection problems as 0 – 1 single knapsack problem so as to maximize the returns from their commercials. In this work, the author obtained the data on TV 3 adverts from the following zones: A1: TV 3 News 360 (19:00 hours GMT), A4: Music – Music (20: 30 – 21:30 GMT) every Saturday. A9: Mid Day Live (12: 00 – 12: 30 GMT) Dynamic programming algorithm was used to solve the problem. To carry out the computations, the computer software, matlab was used to analyse the problem. The computational results showed that the optimal incomes of adverts from TV 3 News 360, Music – Music and Mid – Day Live programmes are Gh ¢30,005.00, Gh ¢15,696.00 and Gh ¢ 4,675.20 respective.

Balachandar and Kannan (2011) presented a heuristic approach to solve the 0/1 multidimensional knapsack/covering problem (MKCP) which is NP-hard. The intercept matrix of the constraints, used to find optimal or near optimal solution of the MMKP is proposed. This heuristic approach is tested for benchmark problems of sizes up to (500x30), taken from or-library and the results are compared with optimum solutions. Space and computational complexity of solving MKCP using this approach are also presented. The performance of our heuristic is compared with the best state-of-the-art heuristic algorithms with respect to quality of the solutions found and the corresponding execution time. The encouraging results especially for relatively large size test problems indicate that this heuristic approach can successfully be used for finding good solutions for highly constrained NP-hard problems.

O'Leary (1995) determined what subset of items provides the greatest return. Typically, it is used in situations such as budgeting, where there are only enough funds to sponsor a subset of projects. This paper provides results that allow us to determine when one project "dominates" another that is when some project is always preferable to another. Those results are useful since they allow us the ability to reduce the number of variables and the overall budget constraint. This leads to a smaller, more tightly constrained problem. In some cases, establishing domination results leads to a complete solution of knapsack problems.

Amponsah et al. (2011) put forward the Optimal Selection, case study Ghana Television (GTV). The Knapsack Problems are among the simplest integer programs, which are NP-hard. Problems in this class are typically concerned with selecting from a set of given items, each with a specified weight and value, a subset of items whose weight sum does not exceed a prescribed capacity and whose value is maximum. The specific problem that arises depends on the number of knapsacks (single or multiple) to be filled and on the number of available items of each type (bounded or unbounded). In this research paper, we shall consider the application of classical 0-1 knapsack problem with a single constraint to selection of television advertisements at critical periods such as prime time news, news adjacencies, Break in News and peak times using the simple heuristic algorithm.

Feng (2001) presented the rain fade compensation problem for downlink transmission in the Ka-band satellite by dynamic resource allocation. We formulate the problem mathematically in the framework of Knapsack Problems (KP). In particular, we show the resource allocation problem is equivalent to a Multi-choice Multiple Knapsack Problem (MCMKP), which, in general, is very hard to solve in a reasonable time. By introducing the seeding theory into the antenna scheduling, we decompose the original MCMCP into a sequence of Multiple-choice Knapsack Problems (MCKP), which are easier to solve.

CHAPTER 3

METHODOLOGY

3.1 INTRODUCTION

The knapsack problem has been studied for more than a century, with early works dating as far back as 1897. It is not known how the name "knapsack problem" originated, though the problem was referred to as such in the early works of mathematician Tobias Dantzig (1884–1956. suggesting that the name could have existed in folklore before a mathematical problem had been fully defined.

What is the knapsack problem all about? Let's consider the scenario. Suppose a hiker has to fill up his knapsack by selecting from among various possible objects those which will give him maximum comfort. This can be mathematically formulated by numbering the objects from 1to n and introducing a vector of binary variables x_j such that

Figure 3.1

An Illustration of the Knapsack Problem

j= {1,2, 3... n} where x_j means

$$x_i = \begin{cases} 1. \ if the i^{th} item has been selected \\ 0. \quad otherwise \end{cases}$$

Then, if p_j is a measure of the comfort of the hiker by choosing object j, whilst w_j is the objects weight and C the weight of the bag he can carry. The selection of the object is the solution to the problem. The selection is from among all binary vectors x satisfying the constraint C. $\sum_i^n w_i x_i \le C$

The equation which maximizes the objective function is

$$Z = \sum_i^n v_i x_i$$

Well if you are reading this work today you might not be interested in a hiker problem. You should not worry, knapsack can be applied suppose one wants to invest all or part-capital of C dollars and considering n possible investments. Let p_j be the profit expected form the investment j. W_j is the amount of dollars it requires. It is self-evident that the optimal solution of the knapsack will indicate the best possible choice of investments.

3.2 THE 1-0 KNAPSACK PROBLEM

The 0-1 knapsack problem is a problem of choosing a subset of the x_i items such that the corresponding value sum is maximized without having the sum of the weight w to exceed the capacity

We assume $v_1, v_2, v_3, \ldots, where\ V_i$ are strictly positive integers. Define Z the optimal value as the maximum value that can be attained with weight less than or equal to C using items x_i where i={1,2,3,......,n}

We can define $Z(v, x)$recursively as follows:

max $\quad z = \sum_i^n v_i x_i$ 3.1

Subject to: $\sum_i^n w_i x_i \leq C$ 3.2

$$x_i = \begin{cases} 1, & if the i^{th} item has been selected \\ 0, & otherwise\ has\ not\ been\ selected \end{cases}$$

Since the value and weights are positive values, it would be supposed without loss of generality, that

$$\sum_{i-1}^{n} w_i x_i \leq C$$

The constant C represents the maximum weight that the knapsack is permitted to hold.

3.1.1 Examples of Real World Applications of Knapsack Problems and their Formulations

Diet Problem

There are n different food items to choose form. Also there are m different elements (Vitamins A, B,...,magnesium,..., Calories and so on). Where each element has a lower and an upper limit for intake. There is also a cost constraint since each food item has its own cost. The problem we may like to solve would be how to minimize cost and adhere to the nutritional requirements. This situation can be mathematically formulated using MKP algorithm.

An optimized solution is achieved when the objective function in equation (3.1) subject to equation (3.2) are satisfied for all $x_j = \{0,1\}, for all i = 1,2,3, \dots, n. and j = 1,2,3, \dots, n$

Selection of projects to fund

This algorithm can also be used to solve problems in the governmental and non-governmental organizations as well as companies that deal with project selection and funding. In selection of projects to fund there may be n different projects to select from. And each project may have duration let's say m years. However there is an agreed budget for each year which needs not to be exceeded. And each project would serve a purpose or yield profit when completed, hence we value each project. However, the objective is to maximize the profit and not exceed the yearly budgets. Hence to achieve this we formulate the problem mathematically using the MKP as done previously. An optimized solution is achieved when the objective function in equation (3.1) subject to equation (3.2) are satisfied for all $x_j = \{0,1\}, for all i = 1,2,3, \dots, n. and j = 1,2,3, \dots, n$

Household Expenditure

In a household of n different individuals of different ages typically. And m different items of needs (sugar, meat, t-roll, soap,..). Each item has a lower and upper limit for intake. Also, each food item has its cost. The problem we may like to solve would be how to minimize cost and adhere to the intake requirements so as to prevent shortages. This situation can be mathematically formulated using MKP algorithm. An optimized solution is achieved when the objective function in equation (3.1) subject to equation (3.2) are satisfied for all $x_j = \{0,1\}, for all i = 1,2,3, \dots, n. and j = 1,2,3, \dots, n$

Knapsack problems have been extensively studied in the last decade attracting both theorist and practitioners. The theoretical side of interest arise due to its simple structure which allows exploitation of a number of combinatorial properties. Also, complex optimization problems can be solved through a series of knapsack-type sub-problems. However, on the practical point of view, the problem can model many industrial situations and many other areas also.

3.1 Illustrative Example

Let's look at an example to explain these steps. Assuming a traveller has a travelling bag (knapsack) that takes a maximum capacity of C=10kg. The traveller has $\boldsymbol{x_i}$ items (1,2,3,…,n). The items weight and are of value to the traveller. How many items can the traveller put in the knapsack so as not to exceed C as well as maximize the total value sum?

Table 3.1 NUMBER OF ITEMS, WEIGHT OF ITEM AND THE VALUE OF ITEM

TYPE OF ITEMS	N0 OF ITEMS	WEIGHT OF ITEM (w_i)	VALUE OF ITEM (v_i)
1	4	1	2
2	3	3	8
3	2	4	11

4	2	7	20
TOTAL		15	41

The knapsack problem can be modelled as follows

If an item is included, then

$$x_i = 1$$

And if an item is not included, then

$$x_i = 0$$

The maximization function is given as

$$z = \sum_i^n v_i x_i \ldots\ldots.. (3.3)$$

Subject to

$$\sum_i^n w_i x_i \leq 10 \ldots\ldots (3.4)$$

of table 3.1 the model becomes Max

$$\{2(x_1 + x_2 + x_3 + x_4) + 8(x_5 + x_6 + x_7) + 11(x_8 + x_9) + 20(x_{10} + x_{11})\}$$

Subject to $\{1(x_1 + x_2 + x_3 + x_4) + 3(x_5 + x_6 + x_7) + 4(x_8 + x_9) + 7(x_{10} + x_{11})\}$

Data structure $x = [\{0,1\}, \{0,1\}, \{0,1\}, \ldots, \{0,1\}] = \{0,1\}^n$

Assuming $s_1 = \{1,1,0,0,1,0,0,0,1,0,1\}$ is a solution, we check if it's feasible. By substituting into equation 3.4 we shall have

$$\{1(1+1+0+0)+3(1+0+0)+4(0+1)+7(0+1)\}$$

$$= 2+3+4+7 \leq 10$$

$$= 16 \leq 10 \text{thisstatementisfalsehence} S_1 \text{isnotfeasible.}$$

We define a simple flip operation by changing zeros to ones and vice versa. When I flipped I had S2={1,1,0,0,1,0,0,0,1,0,0} assuming its a solution we check whether its feasible by substituting into equation (3.4)

We obtain

$$\{1(1+1+0+0)+3(1+0+0)+4(0+1)+7(0+0)\}$$

$$= 2+3+4 \leq 10$$

$$= 9 \leq 10 \text{thisstatementistruehence} S_2 \text{isafeasiblesolution.}$$

The objective function value corresponds to S_2 is

$$f(S_2) = 2(1+1+0+0)+8(1+0+0)+11(0+1)+20(0+0)$$

$$f(S_2) = 23$$

Hence we find other feasible solution S_3, S_4, and so on. Since we want to optimize the objective function, our optimal solution is the one that gives the highest objective function value as our optimal solution.

Next, we shall discuss the different knapsack models available. Knapsack Problem is a well-known problem and several exact and heuristic algorithms have been proposed for its solution. The exact algorithm can be subdivided into two classes: 1-0 knapsack problem and the fractional knapsack problem. For the purpose of the thesis we shall only concentrate on the 1-0 knapsack problem and its generalized models.

3.3 MULTIPLE-CHOICE KNAPSACK PROBLEM

Now let's consider the generalization arising when the item set is partitioned into subsets and additional constraint imposed and that one item per subset is selected. This is referred to as the Multiple-Choice knapsack problem.

3.4 BOUNDED KNAPSACK PROBLEM

Now further generalization of the knapsack problem is by assuming that for all j such that$j = \{1,2,3,\dots,n\}$, than bj items of profit p_j and weight w_j $(b_j \leq \frac{c}{w_j})$ is known as the bounded knapsack problem. Hence the bounded knapsack problem is defined by

$$maximize \sum_{j=1}^{n} p_j x_j = Z$$

$$\sum_{j=1}^{n} w_j x_j \leq C, \qquad 0 \leq x_j \leq b_j$$

{j=1,2,3,...,n} and $x_j's, areintegers$

3.5 THE UNBOUNDED KNAPSACK PROBLEM

The unbounded is the generalization of the bounded knapsack, the case were $b_j=+\infty$ the upper bound is infinity and j={1,......,n} formulated as

$$maximize \sum_{j=1}^{n} p_j x_j = Z$$

$$\sum_{j=1}^{n} w_j x_j \leq C, \qquad 0 \leq x_j \leq +\infty s$$

{j=1,2,3,...,n} and $x_j's, areintegers$

3.6 THE SUBSET-SUM PROBLEM

This is a case of knapsack problem arising when $V_j = W_j$ where {j=1,2,3... n}

Formulated as

$$maximize \sum_{j=1}^{n} w_j x_j = Z$$

Subject to $\sum_{j=1}^{n} w_j x_j \leq C, \qquad forx\begin{cases}1, & ifincluded \\ 0, & otherwise\end{cases}$

And J={1,......,n}

3.7 THE CHANGE-MAKING PROBLEM

A particular bounded knapsack problem which arises when $p_j=1$ and j= {1,.....,n} and in the capacity constraint. We impose equality instead of inequality. Hence formulated as

$$maximize \sum_{j=1}^{n} x_j = Z$$

$$\sum_{j=1}^{n} w_j x_j = C, \qquad 0 \le x_j \le b_j$$

Where x_j is an integer and j={1,....,n}

Such a bounded knapsack problem is referred to as change-making problem. This can be used to model a cashier, having to assemble a given change C using the maximum and minimum number of coins. However the same change-making problem can be changed into UNBOUNDED CHANGE-MAKING PROBLEM by making $b_j = +\infty$

3.8 MULTIPLE CONSTRAINT KNAPSACK PROBLEMS

The multi-constraint knapsack problem is a generalization of the 0/1 knapsack problem. The multi-constraint knapsack problem has *m* constraints and one objective function to be maximized while all the *m* constraints are satisfied.

In this type of problem, each solution variable, X_{ij}, is restricted to only binary value, thus either Xij=1 or $X_{ij}=0$

$$X_{ij} = \begin{cases} 1 & \text{if item j i} \in \text{i} \\ 0, & \text{if otherwise} \end{cases}$$

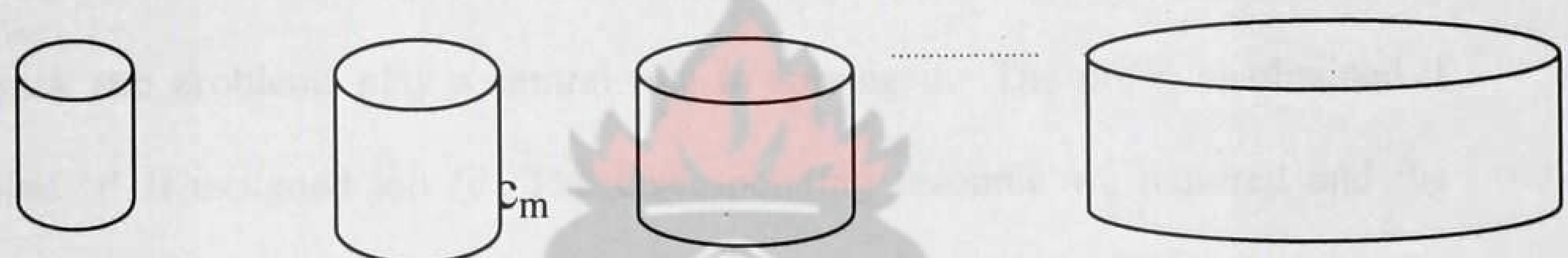
$$Maximixe \qquad Z = \sum_{i=1}^{n}\sum_{j=1}^{n}(vx_{ij}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.31)$$

$$subject\ to \qquad \sum_{j=1}^{n}(w_j x_{ij}) \le C_i \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.32)$$

$$and \qquad \sum_{i=1}^{n} x_{ij} \le 1 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.33)$$

$$for\ x_j = \{0,1\}, for\ all\ i = 1,2,3, \ldots, n. and\ j = 1,2,3, \ldots, n.$$

In this type of algorithm there are *m* knapsacks of different capacities $C_1, C_2, C_3, \ldots, C_m$



C_1 C_2 C_3 C_m

There are also n numbers of objects to fill each with different value. Additionally each object has m possible weights w_{ij}. For example an object i has weight $w_{ij}=1$ when considered for inclusion in the j^{th} knapsack.

$$X_{ij} = \begin{cases} 1 \text{ if item j is selected form the container i} \\ \qquad 0\text{, if otherwise} \end{cases}$$

The objective of this algorithm is to find a vector $X=(x_1, \ldots. x_n)$ that would guarantee that no knapsacks are not overfilled so that yields are maximized.

In other words, the objective is to pack in the knapsack that is classified into multiple mutually exclusive classes. Within each class, there are several different items. The problem is to select some items from each class so as to minimize the total cost while the total size of the items does not exceed the limited capacity of the knapsack.

The optimized solution lies closely to the boundary of the feasible region.

$$Maximixe \qquad Z = \sum_{i=1}^{n}\sum_{j=1}^{n}(vx_{ij}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.31)$$

$$subject\ to \qquad \sum_{j=1}^{n}(w_j x_{ij}) \leq C_i \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.32)$$

$$for\ x_j = \{0,1\}, for\ all\ i = 1,2,3, \ldots, n.\ and\ j = 1,2,3, \ldots, n.$$

The multiple knapsack model is a generalization of the assignment problem. Strictly speaking it should not be included as a knapsack problem but it's included because knapsack sub problems play a central role in solving it. The profit v_{ij} obtained if machine 'i' is assigned job 'j'. The corresponding resource w_{ij} required and the amount C_j of resources available to machine j.

3.4 THE GREEDY KNAPSACK ALGORITHM

George Dantzig proposed a greedy approximation algorithm to solve the unbounded knapsack problem. His version sorts the items in decreasing order of value per unit of weight, v_i/w_i. It then proceeds to insert them into the sack, starting with as many copies as possible of the first kind of item until there is no longer space in the sack for more. Provided that there is an unlimited supply of each kind of item, if m is the maximum value of items that fit into the sack, then the greedy algorithm is guaranteed to achieve at least a value of m/2. However, for the bounded problem, where the supply of each kind of item is limited, the algorithm may be far from optimal.

3.2 DYNAMIC PROGRAMMING ALGORITHM

$\text{Let} i = \{1,2,3, \ldots, n\}$

$w_i = \text{weightofthei}^{\text{th}}\text{item}$

$v_i = \text{thevalueoftheithitem}$

$W = \text{themaximumweighttheknapsackcantake}$

Step 1: Initialization

For w = 0 to W and i = 1 to n

Let B[0,w] = 0 and B[i, 0] = 0

Step 2: For i = 1 to n and w = 0 to W

Compute B[i,w] = v_i+B[i-1,w-w_j];

$$\text{if} B[i, w] > B[i-1, w] \text{and} w_i \leq w,$$

Then the item, i can be part of the solution in the table.

Otherwise go to step 3

Step 3: if $w_i \leq w \text{but} B[i, w] \leq B[i-1, w]$,

$$\text{computethefollowing} B[i, w] = B[i-1, w]$$

Step 4: if $w_i > w$, we compute B [i,w] = B [i-1,w]

Repeat the process until all the data are considered.

Step 5: Select the maximum number from the solution table as the optimum solution:

Let i=n and k=W

$\text{If} B[i, k] \neq B[i-1, k] \text{then, marktheithitemasintheknapsack}$

i = i-1, k = k-w, else

set i = i-1

Let's take an example

3.5.1 Example

Let the number of items, i = {1,2,3,4,...} the maximum weight of the knapsack, W= 5kg. From the table below, find the optimum solution and set of items that give the optimal solution.

Table 3.2Table of items, weights and values

Item,(i)	Weight, (wi)	Value, (vi)
1	2	3
2	3	4
3	4	5
4	5	6

Elements (weight, value) = {(2,3), (3,4), (4,5), (5,6)}.

From the above, $n = 4$,

Step 1: initialization,

$W = 0$ to W, *i.e* $w = \{0, 1, 2, 3, 4, 5\}$; $i = \{1, 2, 3, 4\}$

compute

B[0, w] = 0; and B[i, 0] = 0, as shown in the table below

i/w	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0					
2	0					
3	0					
4	0					

Using the first item, (2,3), compute for row 3 as shown below

$i=1$, $v_i = 3$, $w_i = 2$, $w = 1$;

Since $w_i > w$ go to step 4

Step 4: compute $B[I,w]=B[i-1,w]$

$B[1,1]= B[1-1,1]= B[0,1]=0$

Step 2:

For $B[1, 2]$; $i=1$, $w = 2$; since $wi \leq w$,

Compute $B[i, w] = vi + B[i-1,w - wi]$

$B[1, 2] = 3 + B[0,0] = 3$

For $B[1, 3]$; $i = 1$, $w = 3$; since $wi \leq w$,

$B[1, 3] = 3 + B[1-1,3-2] = 3 + B[0, 1] = 3$

For $B[1, 4] = 3$; and

$B[1, 5] = 3$

i/w	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	3	3	3	3
2	0					
3	0					
4	0					

Using the second item, (3, 4), compute the values for row 4 as shown in table below

$wi = 3$, $vi = 4$

For B[2, 1]; $i = 2$, $w = 1$, since $wi > w$ .compute B[i, w] = B[i-1, w]

B[2, 1] = B[2-1, 1] = B[1, 1] = 0

For B[2, 2], $wi > w$ then B[2, 2] = B[1, 2] = 3

For B[2,3], $wi \leq w$; B[i, w] = vi + B[i-1, w-wi]

B[2, 3] = 4 + B[1, 0] = 4 + 0 = 4

B[2, 4] = 4 + B[1, 1] = 4 + 0 = 4

B[2, 5] = 4 + B[1, 2] = 4 + 3 = 7

$i \backslash w$	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	3	3	3	3
2	0	0	3	4	4	7
3	0					
4	0					

Using item (4, 5), compute the values of row 5 as shown below

$wi = 4$, $vi = 5$

For B[3, 1], since $wi > w$, *go to step 4*

Compute B[*i*, *w*] = B[*i*-1, *w*]

B[3, 1] = B[2, 1] = 0

B[3, 2] = B[2, 2] = 3

B[3, 3] = B[2, 3] = 4

B[3, 4] , $wi \leq w$, *compute* B[*i*, *w*] = *vi* + B[*i* – 1, *w* - *wi*]

B[3, 4] = 5 + B[2, 0] = 5 + 0 = 5

B[3, 5] = 5 + B[2, 1] = 5 + 0 = 5,

Since B[3, 5] < B[2, 5] , go step 3; (i.e. B[*i*, *w*] = B[*i*-1, w])

B[3, 5] = B[2, 5] = 7

i\w	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	3	3	3	3
2	0	0	3	4	4	7
3	0	0	3	4	5	7
4	0					

Use the last item, (5, 6) to compute the entries in row 6 as shown below: w_i= 5, *vi* = 6

For B[4, 1], since *wi* > *w* , *go to step 4*

Compute B[*i*, *w*] = B[*i*-1, *w*]

B[4, 1] = B[4-1, 1] = B[3, 1] = 0

B[4, 2] = B[3, 2] = 3

B[4, 3] = B[3, 3] = 4

$B[4, 4] = B[3, 4] = 5$

$B[4, 5] = B[3, 5] = 7$

i\w	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	3	3	3	3
2	0	0	3	4	4	7
3	0	0	3	4	5	7
4	0	0	3	4	5	7

Step 5: From the last table, the maximum number is 7. Therefore, the optimum solution is 7

Step 6: Finding the particular items to be included in the knapsack.

Let $i = n$ and $k = w$

if $B[i, k] \neq B[i\text{-}1, k]$ then, mark the i^{th} item as in the knapsack

$i = i\text{-}1$, $k = k - w$, else

$i = i - 1$

From the complete solution table, we conclude that

i\w	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	3	3	3	3
2	0	0	3	4	4	7
3	0	0	3	4	5	7
4	0	0	3	4	5	7

Let $i = 4$, $k = 5$, $vi = 6$, $wi = 5$

$B[i,k] = 7$ and $B[i-1, k] = 7$

$B[i, k] \neq B[i - 1]$,

so item 4 should not be included in the knapsack.

Consider, $i=3$, $k = 5$, $vi = 6$ and $w_i = 5$

$i\backslash w$	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	3	3	3	3
2	0	0	3	4	4	7
3	0	0	3	4	5	7
4	0	0	3	4	5	7

$B[i, k] = 7$ and $B[i-1] = 7$

$B[i, k] \neq B[i-1, k]$, so item 3 cannot be part of the knapsack

Consider, $i = 2$, $k = 5$, $v_i = 4$, and $w_i = 3$

$i\backslash w$	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	3	3	3	3
2	0	0	3	4	4	7
3	0	0	3	4	5	7
4	0	0	3	4	5	7

$B[i, k] = 7$ and $B[i-1, k] = 3$. Since $B[i, k] \neq B[i-1,k]$, then the item 2 should be included in the

Knapsack $k - w_i = 2$

Also consider, $i = 1$, $k = 2$, $v_i = 3$ and $w_i = 2$

$i\backslash w$	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	3	3	3	3
2	0	0	3	4	4	7
3	0	0	3	4	5	7
4	0	0	3	4	5	7

$B[i, k] = 3$, $B[i\text{-}1, k] = 0$, $k - wi = 0$

Since $B[i, k] \neq B[i\text{-}1, k]$, then item 1 can be part of the knapsack.

Again, consider $i = 0$, and $k = 0$

$i\backslash w$	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	3	3	3	3
2	0	0	3	4	4	7
3	0	0	3	4	5	7
4	0	0	3	4	5	7

Therefore, the optimal solution is 7 which could be obtained by selecting items with number, $i = \{1, 2\}$

CHAPTER 4

INTRODUCTION

4.1 DATA COLLECTION

Data was taken from Tema Senior High School's Catering department, Accounting department and Bookstore. Mrs Patience Anann, the second Matron, provided the schools daily costing for the first term 2011/2012 academic year of which the costing (prices) for 10^{th} -16^{th} October was used. The table of costing are shown in Appendix A1-A7. Table 4.1shows the weekly menu for Tema Secondary school, this shows the different meals served in the dining hall each week.

Table 3.1: TEMA SENIOR HIGH SCHOOL WEEKLY MENU

MEALS	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY	SUNDAY
BREAK FAST	Hausa Koko with Milk And bread	Rice Water With milk and bread	Tom brown With Bread	Oblayo with groundnut	Hausakoko and bread	Oblayo With bread	Tea with bread
LUNCH	Gari and beans and fried plantain	Kenkey With fish and Pepper	Wakye with Shito with fish	Gari and Beans	Wakye with shito and fish	Gari and Beans	Kenkey with peper and tin fish / egg
SUPPER	Rice with Palaver source and fish	Rice and Groundnut Soup with Fish	Kenkey With fish and pepper	Rice with Kontomere Stew	Kenkey with tin fish and pepper	Rice with kontom-ere stew	Jollof with chicken

From the menu the matron enters daily costing (prices) of items for each meal for a week. After entering daily ~~cost, she~~ calculates the total cost per week. Average cost per week and daily cost. Then she deduces the average cost per head for the week.

Next we take a look at the purchase orders made at the beginning of the third term. I realised that the school buys items as when it's needed. Appendix C Purchase Order shows the table of purchases made on the 27th May 2012. Also the procurement data is displayed in Appendix D shows the procurement purchases for the first month of the term. These food items were supplied by the suppliers at the commencement of the third term. But the store keeper noted that more supplies would be made as when they run out of stalks.

Therefore, taking the data of daily costing in Appendix A1-A7, all the ingredients in the menu was listed and grouped into perishable and non-perishable. Table 4.2 shows the list of non-perishable food items, column 3 is the list of unit prices of each item in Ghana cedi. The weekly cost in column 4 is calculated by multiplying the unit cost by column 5 thus quantity of items needed each week. Column 6 is the number of times the ingredients are used in the menu weekly.

The quantity of the item needed per term is calculated by multiplying column 4 that is the quantity of items needed each week by the number of weeks in the term (14) this is what is seen in column 7. In column 8, number of times that ingredient is used in preparing the menu.

Table 4.2COST OF NON-PERISHABLE FOOD ITEMS

No	Non-Perishable items	Unit Cost	Quantity of item per week	Weekly Cost	Quantity of Item per term (14wks)	Total cost per term	Number of times used in menu
1	Sugar	91	2.5	227.5	35	2912	7
2	beans	250	3.5	875	49	12250	5
3	Milk	18.5	6	111	84	1554	2
4	Millet	140	1	140	14	1960	2
5	Groundnuts	360	1	360	14	5040	4
6	Richoco	85	3	255	42	3150	1
7	Maize	145	7.5	1087.5	105	15225	6
8	Rice	78	22	1716	308	24024	7
9	Salt	45	1	45	14	630	18
10	Gari	145	3	435	42	6090	3
11	tomatoes puree	40	5.5	220	77	3080	11
12	palm oil	8.6	20	172	280	2408	5
13	cooking oil	74	5	370	70	5180	6
14	canned fish	54	8	432	112	6048	2
15	magi	5	11	55	154	770	11
16	wakye leaves	2.5	4	10	56	140	2
	Total cost (Gh¢)			6511		90461	

Table 4.3 also shows the list of perishable food items similar to Table 4.2.

Table 4.3 COST OF PERISHABLE FOOD ITEMS

N0	Perishable Item	Unit cost	Quantity of the item need each week	Weekly cost	Number of times needed Per week	Quantity of the Item needed per term (14 weeks)	Total cost per term
1	Egg	8	20	160	1	280	2240
2	Fresh tomatoes	240	3	720	14	42	10080
3	Salted fish	2	5	10	5	70	140
4	Frozen fish	95	4	380	1	56	5320
5	Frozen tuna	6	74	444	5	888	6216
6	Garlic & ginger	5	4	20	7	56	280
7	Chicken	38	9	342	1	126	4788
8	Kontomire	30	4	120	2	56	1680
9	Ripe plantain	120	1	120	1	14	1680
10	Dry herring	25	6	150	2	84	2100
11	Shrimps	35	4	140	2	56	1960
12	Agushie	12	4	48	2	56	672
13	Onions	7.14	1	100	14	14	1400
	Total cost Gh¢			2754			38556

Next, we formulate the model as follows, from Table 4.3(the cost of non-perishable food items), the budget for purchasing all the non-perishable items for a week is C_1=¢10,000. The school would like to purchase some items in bulk as to get some discount on the prices, but would not want to buy quantities that would exceed what is needed for 3 weeks. We represent this arbitrary value less than a month with the letter b=3. However, the school would like to buy what the menu requires for at least one week and this represented as a=1.

Table 4.2 contains the data of non-perishable food items. From the list, the weekly cost is represented in column 3 as weight (W_i). The number of times the items are used in a week is considered as our value (V_i) as shown in column 4 of the table below.

Table 4.4 Data for Non-Perishable Food items

NAMES		W_i	V_i
X_1	Sugar	227.5	7
X_2	Beans	875	5
X_3	Milk	111	2
X_4	Millet	140	2
X_5	Groundnuts	360	4
X_6	Richoco	255	1
X_7	Maize	1087.5	6
X_8	Rice	1716	7
X_9	Salt	45	18
X_{10}	Gari	435	3
X_{11}	Tomato paste	220	11
X_{12}	palm oil	172	5
X_{13}	cooking oil	370	6
X_{14}	canned fish	432	2
X_{15}	Magi	55	11
X_{16}	wakye leaves	10	2

From Table 4.5 the cost of perishable food items, we formulate our model with the following parameters, the budget for purchasing all the perishable items for a week is C=¢2,800. The school would like to purchase some items in bulk as to get some discount on the prices, but would not want to buy quantities that would exceed what is needed for 3 weeks. We represent this arbitrary value less than a month with the letter b=3. However, the school would like to buy what the menu requires for at least one week and this represented as a=1.

Table 4.7 is the data used. From Table 4.5 we list the perishable food items. The weekly cost is represented in column 3 as weight (W_i). The number of times the items are used in a week is considered as our value (V_i), shown in column 4.

Table 4.5Data for Perishable Food Items

NAMES		W_i	V_i
X_1	Egg	160	1
X_2	Fresh tomatoes	720	14
X_3	Saltedfish(kobi)	10	5
X_4	Frozen fish	380	1
X_5	Frozen tuna	444	5
X_6	Garlic & ginger	20	7
X_7	Chicken	342	1
X_8	Kontomire	120	2
X_9	Ripe plantain	120	1
X_{10}	Dry herring	150	2
X_{11}	Shrimps	140	2
X_{12}	Agushie	48	2
X_{13}	Onions	100	14

The budget capacity used by the school for weekly purchases of non-perishable food items is C_1=Gh¢10,000. And the budget capacity for weekly purchases of Perishable food items is C_2=Gh¢2,800.

The following items were however not included in the menu, bread because the ingredients were not listed in the daily costing. However soya beans and pepper were also not included because the school purchase them in bulk for the term. The other items are fuel and soap shown in Appendix E.

4.2 MODEL FORMULATION

n_i: the total number of items/ ingredients needed for the term as obtained in the first column

w_i :the price of the ingredient representing the weight of the item.

v_i : the value of the ingredient representing the number of times that ingredient is needed per week

a: the lower bound or the lowest quantity to be purchase.

b: upper bound or the highest quantity to be purchased

X_i: the quantities selected by the software to be bought under a maximum capacity W

We listed the major ingredients in the schools menu based on the daily costing as displayed in the Appendix A1-A7. Also we grouped the ingredients into perishable and none-perishable as displayed below.

4.3 BOUNDED KNAPSACK ALGORITHM

We formulate this model based on the bounded with lower and upper limits.

$$maximize \quad \sum_{i=1}^{n} v_i x_i$$

$$subject\ to \sum_{i=1}^{n} w_i x_i \leq C, \quad a \leq x_i \leq b \qquad i = 1, \dots\dots, n$$

Model Instance for Non-Perishable Food Items

Maximize $Z = \sum_{i=1}^{n} v_i x_i$

$=7(x_1)+5(x_2)+2(x_3)+2(x_5)+4(x_6)+1(x_7)+6(x_9)+7(x_{10})+18(x_{11})+3(x_{12})+11(x_{13})+5(x_{14})+ \quad 6(x_{15})+11(x_{16})+2(x_{17})$

Subject to $\sum_{i=1}^{n} w_i x_i \leq 10,000$

$= 22.7(x_1)+875(x_2)+111(x_3)+140(x_4)+952(x_5)+360(x_6)+254.58(x_7)+$

$1087.5(x_8)+1716(x_9)+45(x_{10})+435(x_{11})+220(x_{12})+172(x_{13})+370(x_{14})+$

$432(x_{15})+55(x_{16})+10(x_{17})$

Where $1 \leq x_i \geq 3$

Coefficient of objective function was obtained from the forth column in Table 4.7.

And the confident of the constraint function was obtained from the third column in Table 4.7.

Model Instance for Perishable Items

Maximize $Z=\sum_{i=1}^{n} v_i x_i$

$=1(x_1)+14(x_2)+5(x_3)+1(x_4)+5(x_5)+7(x_6)+1(x_7)+2(x_8)+1(x_9)+2(x_{10})+2(x_{11})$
$+2(x_{12})+$

$14(x_{13})$.

Subject to $\sum_{i=1}^{n} w_i x_i \le 2800$

$=160(x_1)+720(x_2)+10(x_3)+380(x_4)+444(x_5)+820(x_6)+342(x_7)+120(x_8)+120$
$(x_9)+150(x_{10})+140(x_{11})+48(x_{12})+100(x_{13})$

Where $1 \le x_i \ge 3$

Coefficient of objective function was obtained from the forth column in Table 4.8.

And the confident of the constraint function was obtained from the third column in

Table 4.8.

4.3.1 A STEP BY STEP ALGORITHM FOR BOUNDED KNAPSACK PROBLEM

Step 1: Select one choice of each item randomly between the lower bound and the upper bound.

Step 2: Check to see if selected choices satisfies all constraints.

Step 3: Calculate the optimal value.

Step 4: Go back to step 1 and step 2 and perform 2^n times.

Step 5 Select the maximum optimized value as the final result.

4.4 COMPUTATIONAL PROCEDURE (COST OPTIMIZER)

A programming code Cost Optimizer in Appendix F is the Matlab code for the knapsack problem.

Computer specifications are

- Processor speed: 1.60GHz and 1.60 Ghz
- Memory (RAM): 1.00 GB
- System Type 32-bit operating system

The main features of the code are;

1. The number of iterations per trial is 65536
2. The number of trials 3
3. The input data is provided below

Non-Perishable Items:

w=[227.5,875,111,140,2,360,255,1087.5,1716,45,435,220,172,370,432,55,10]

v=[7,5,2,2,4,1,6,7,18,3,11,5,6,2,11,2]

l=16 (number of items)

a= 1

b=3

Perishable Items:

w=[160,720,10,380,444,20,342,120,120,150,140,48,100]

v=[1,14,5,1,5,7,1,2,1,2,2,2,14]

l=13(number of items)

a=1

b=3

4.7 RESULTS

4.7.1 RESULTS FOR NON-PERISHABLE FOOD ITEMS

Table 4.6, illustrates the results for a weekly budget of C=Gh¢10,000 for non-perishable food items. Column 5 shows the cost optimizer results for the weekly budget. Column 6 represent the value for each item selected. The total number of times each is used for the corresponding number of weeks. Z=221 is the total value shown in the last row.

Table 4.6 COMPUTERIZED RESULTS FOR NON-PERISHABLE ITEMS

				Weekly 10,000	Optimal values
NAMES		W	V	W=¢9,933	
X_1	Sugar	227.5	7	2	14
X_2	Beans	875	5	1	5
X_3	Milk	111	2	3	6
X_4	Millet	140	2	2	4
X_5	Groundnuts	360	4	3	12
X_6	Richoco	255	1	1	1
X_7	Maize	1087.5	6	1	6
X_8	Rice	1716	7	1	7
X_9	Salt	45	18	3	54
X_{10}	Gari	435	3	1	3
X_{11}	Tomatoepaste	220	11	3	33
X_{12}	palm oil	172	5	3	15
X_{13}	cooking oil	370	6	3	18
X_{14}	canned fish	432	2	2	4
X_{15}	Magi	55	11	3	33
X_{16}	wakye leaves	10	2	3	6
Z					**221**

We observe from column 5 of Table 4.6 that, with a budget of approximately ¢10000, the school can purchase all non-perishable items for one week. The amount used was ¢9933 and the minimum quantity purchased for each item can satisfy the menu for one week, whilst the maximum quantity can satisfy the menu for 3 weeks as observed in column 5.

4.7.2RESULTS FOR PERISHABLE FOOD ITEMS

From Table 4.7 below, the computer analysed and gave the optimized values for non-perishable food items for weekly budget. The optimal Z value is shown in the last row. Column 6 represent the value for each item selected. It is the total number of times each item selected is used for the number of corresponding number of weeks. Z=74 is the total sum of values.

Table 4.7 COMPUTERIZED RESULTS FOR PERISHABLE ITEMS

				Weekly	Optimal values
NAMES		w_i	V_i	W=¢2794	
X_1	Egg	160	1	1	1
X_2	Fresh tomatoes	720	14	1	14
X_3	Saltedfish(kobi)	10	5	3	15
X_4	Frozen fish	380	1	1	1
X_5	Frozen tuna	444	5	1	5
X_6	Garlic & ginger	20	7	2	14
X_7	Chicken	342	1	1	1
X_8	Kontomire	120	2	1	2
X_9	Ripe plantain	120	1	1	1
X_{10}	Dry herring	150	2	1	2
X_{11}	Shrimps	140	2	1	2
X_{12}	Agushie	48	2	1	2
X_{13}	Onions	100	14	1	14
Z					**74**

We observe from column 5 of Table 4.7 that with a budget of approximately ¢2800, the school can purchase all perishable items for one week. Except salted fish and Garlic which can be purchased for 3 and 2 weeks respectively. The amount used by the model is Gh¢2794

4.8 DISCUSSIONS

Based on the results, if the model is used for both perishable and non-perishable food items, the school would purchase all non-perishable items needed by the menu in a week for Gh¢ 6,511. The knapsack model suggested that, with a budget of Gh¢10,000 these quantities can be purchased as shown in column 4. These quantities can satisfy the menu more than a week and a maximum of three weeks. The savings from knapsack each week is ¢3489 is displayed in column 5 as shown in Tables 4.8.

Table 4.8 WEEKLY SAVINGS FOR NON-PERISHABLE FOOD ITEMS

NAMES		Actual Weekly Purchases	Knapsack purchases	Savings For each Week	Number of weeks saved
X_1	Sugar	2.5 bags	5bags	2.5 bags	1
X_2	Beans	3.5 bags	3.5bags	0	0
X_3	Milk	6 boxes	12boxes	6 boxes	2
X_4	Millet	1 bag	2bags	1bag	1
X_5	Groundnuts	1bag	3bags	2 bag	2
X_6	Richoco	3 boxes	3boxes	0	0
X_7	Maize	7.5 bags	7.5bags	0	0
X_8	Rice	22 bags	22bags	0	0
X_9	Salt	1bag	3bags	2 bags	2
X_{10}	Gari	3 bags	3bags	0	0
X_{11}	Tomatoepaste	5.5 cartons	16.5	11 cartons	2
X_{12}	palm oil	20 gallons	60gallons	40 gallons	2
X_{13}	cooking oil	5 Jeri cans	15 Jeri-cans	10 Jeri-cans	2
X_{14}	canned fish	8 cartons	16cartons	8 cartons	1
X_{15}	Magi	11 packs	33 packs	22 packs	2
X_{16}	wakye leaves	4 bundles	12 bundles	8 bundles	2
		Gh¢6511		Gh¢**3489**	

With good storage facilities and with a budget of ¢10,000, the school can purchase all the ingredients needed for a week with an amount of ¢6511 shown in the last row in column 3. The extra quantities that can be purchased for storage is recorded in column 5 the total savings made in a week is ¢3543.

Table 4.9 Projected Results for Term Purchases for Non-Perishable Food Items

NAMES		Term purchase	Knapsack purchases	Savings for each term	Number of weeks saved
X_1	Sugar	35 bags	70bags	35bags	14
X_2	Beans	49 bags	49bags	0	0
X_3	Milk	84 boxes	162boxes	78 boxes	28
X_4	Millet	14 bags	28bags	14 bags	14
X_5	Groundnuts	14 bags	42bags	28 bags	28
X_6	Richoco	42 boxes	84boxes	42 boxes	0
X_7	Maize	105bags	105bags	0	0
X_8	Rice	308bags	308bags	0	0
X_9	Salt	14 bags	42bags	28 bags	28
X_{10}	Gari	42 bags	42bags	0	0
X_{11}	Tomato paste	77cartons	231cartons	154 cartons	28
X_{12}	palm oil	280gallons	840gallons	560 gallons	28
X_{13}	cooking oil	70 Jeri cans	210Jeri cans	140 Jeri cans	28
X_{14}	canned fish	112cartons	224cartons	112 cartons	14
X_{15}	Magi	154 packs	462packs	308parks	28
X_{16}	wakye leaves	56 bundles	168bundles	112 bundles	28
		Gh¢91154		Gh¢48846	

If the model is used throughout the term, the school can purchase all the ingredients need for the term with an amount of Gh¢91154 as shown in the last row in column 5. Column 6 is the savings the school would make after they have purchased all that is needed for the term. The school can store these food items up to the tune of Gh¢48846.

The school purchases perishable food items in the open market each week to supplement the menu. These purchases are done each week, because the shelf life of the items is short and the school does not have enough storage facilities to save surpluses. Therefore the budget allocated for purchases of perishable food items is Gh¢2800.

Based on the results in Table 4.10, for perishable food items, the school would purchase the perishable items needed by the menu in a week for Gh¢2754. The knapsack model suggested that, with a budget of GH¢2800 these quantities shown in column 4 can be purchased. The savings the school would be making on perishable food items each week from the budget is ¢46 as shown in Tables 4.10. The amount the model uses to purchase these items each week is ¢2794 is displayed in column 4 as shown in Tables 4.8.

Table 4.10 WEEKLY SAVINGS FOR PERISHABLE FOOD ITEMS

NAMES		Weekly Purchases	Knapsack purchases	Savings For each Week	Number of weeks saved
X_1	Egg	20 creates	20 creates	0	0
X_2	Fresh tomatoes	3 boxes	3 boxes	0	0
X_3	Salted fish (kobi)	5 bags	15bags	10 bags	2
X_4	Frozen fish	4 cartons	4 cartons	0	0
X_5	Frozen tuna	74 cartons	74 cartons	0	0
X_6	Garlic & ginger	4 tins	8 tins	4 tins	1
X_7	Chicken	9 cartons	9 cartons	0	0
X_8	Kontomire	4 bags	4 bags	0	0
X_9	Ripe plantain	1 set	1 set	0	0
X_{10}	Dry herring	6 tins	6 tins	0	0
X_{11}	Shrimps	4 tins	4 tins	0	0
X_{12}	Agushie	4 tins	8tins	0	0
X_{13}	Onions	1 bag	3bags	0	0
		Gh¢2754	Gh¢2794	Gh¢40	

However with good storage facilities and with a budget of Gh¢2800, the school can purchase all the ingredients needed for a week with an amount of Gh¢2754 shown in the last row in column 3. The extra quantities, purchased for storage is recorded in column 5. By subtracting the quantities needed in column 3 from the knapsack purchases in column 4 we have the savings made in a week in column 5 amounting to Gh¢40.

Now we project the weekly results in to the term. The Table below shows the projected term purchases for Perishable food items. Columns 1 and 2 are the variable name and item name respectively. Column 3 is the actual quantities used in the term, while column 4 shows the quantities the model suggested to be purchased with the budget. Because it's optimized, there are extra quantities that can be saved and this is shown in column 5.

Table 4.11 Projected Results for Term Purchases for Perishable Food Items

	NAMES	Actual Term quantities	Knapsack purchases	Savings for each term	Number of weeks saved
X_1	Egg	280 creates	280 creates	0	0
X_2	Fresh tomatoes	42 boxes	42 boxes	0	0
X_3	Saltedfish(kobi)	70 boxes	210 boxes	140 bags	28
X_4	Frozen fish	56 cartons	56 cartons	0	0
X_5	Frozen tuna	1036 cartons	1036 cartons	0	0
X_6	Garlic & ginger	56 tins	112 tins	56 tins	14
X_7	Chicken	126 cartons	126 cartons	0	0
X_8	Kontomire	56 bags	56 bags	0	0
X_9	Ripe plantain	14 bags	14 bags	0	0
X_{10}	Dry herring	84 tins	84 tins	0	0
X_{11}	Shrimps	56 tins	56 tins	0	0
X_{12}	Agushie	56 tins	56 tins	0	0
X_{13}	Onions	14 bag	14 bag	0	0
TOTAL		Gh¢38,544	Gh¢39116	Gh¢560	

If the model is used throughout the term, the school can purchase all the ingredients need for the term with an amount of Gh¢38,544 as shown in the last row in column 3. The total savings the school will make at the end of the term after all purchases is

Gh¢560 shown in the last row in column 5.The school can store these food items in column 5 for use in successive terms.

However, for both perishable and non-perishable food items, the upper bound b=3 was used because with increments in upper bound to weeks beyond three, the optimal solution was the same.

Furthermore, we realise that, the reason the school uses these budget C_1=¢10,000 and C_2=¢2800 is as a result of

1. Firstly, the school buys perishable items in cartons and boxes according to how the items are packaged. For example, in a week the school needs 70 tin of milk but they buy 13 crates. Each creates contains 24 tins hence they buy 312 tins of milk each week. Since this quantity is bought each week the rest may be pilfered.
2. Also because the school does not have a good storage facility, if items are bought to prevent spoilage the excess may also be pilfered which may result in losses.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

5.1 CONCLUSIONS

- The first objective of this study, to model food purchases as a knapsack problem, has been adequately satisfied. We have been able to model this problem as a bounded knapsack problem with lower and upper bound and a programming code was written in Matlab language to solve the problem based on the model.
- Secondly, the second objective of this study, to optimize purchases using knapsack algorithm has also been met. The budget total for both perishable and non-perishable food item is Gh¢12,800. With this budget total the total savings the school can make each week for both perishable and non-perishable food items is Gh¢3,583.
- Thirdly, the projected budget for non-perishable food items for a term of 14 weeks is Gh¢140,000 and that for perishable food items is Gh¢39200. Hence for both perishable and non-perishable food items the projected term budget total is 1Gh¢179,200. With this budget total the savings the school makes on this budget each term is Gh¢49,490.

5.2 OTHER FINDINGS

i. The cost of bread and fuel as well as other ingredients which was not included in the model shown in Appendix E is Gh¢9,487.88.

ii. If we add the cost (I.) above to the total actual amount used for both perishable and non-perishable we have ¢9,487 +¢129,710= Gh¢139,197

iii. Also if we subtract the total budget and the bread cost other expenditure in (II) from the government grant we have ¢202,560-¢139,197= Gh¢63,363. This amount can be used to cater for other expenditure not covered by the research for example water and electricity.

iv. With the model the actual cost of purchases all items in the menu thus both perishable and non-perishable is Gh¢91154+ Gh¢38,556= Gh¢129,710

v. The total amount that can be used to purchase items that could be stored for use in successive terms is ¢47908+¢560= Gh¢489,468.

vi. The savings from the budget after all purchases has been made is Gh¢938+Gh¢84= Gh¢1,022

5.3 RECOMMENDATIONS

We recommend that Tema Senior High School as well as all Boarding Second Cycle Schools use the model Bounded Knapsack Problems (BKP) and the Software in selecting items to buy for the menu.

Savings from the total budget should be used to buy more food items for storage to be used in the next term. We also suggest that the school buys in bulk at the beginning of the term since bulk purchases beats down prices due to discounts.

Furthermore, we recommend that the school finds suppliers for the perishable food items instead of the open market as shown in appendix C. In that case a more competitive price would be obtained when the procurement process is followed.

I also recommend that purchases of food items in second cycle institutions be computerized and staff be trained in the use of the mat lab program so as to effectively use the model.

I also recommend that a consortium of Secondary schools build one warehouse in each region, to store their food items. So that both perishable and non-perishable food items could be stored for a long period. In this way it would encourage bulk purchases which ensure stabilization of prices.

There should be strict measures by the school to check the problem of pilfering. The excesses that would be purchased for storage should be monitored to prevent pilfering by the catering staff so as to meet the target and realise the benefit of the study.

Major limitation to this study discovered is that, the model was tested with only one school with boarding facility in the Greater Accra Region of Ghana. If this was applied to two more schools randomly selected form all ten regions, a more universal solution could be achieved which could be modelled to all schools to solve the problem.

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Appendix A1-A7 TEMA SENIOR HIGH SCHOOL COSTING SHEET

TERM: 1ST DATE: 10-10-2011 DAY: MONDAY NO. OF STDS: 1008 TOTAL COST PER HEAD PER DAY: Gh ₵ 1.1

BREAK FAST (HOUSA KOOKO & BREAD)					LUNCH (BEANS STEW AND GARI)				SUPPER (RICE AND BROWN STEW)			
S/N	ITEM	QTY	UNIT COST(₵)	TOTAL COST(₵	ITEM	QTY	UNIT COST	TOTAL COST(₵)	ITEM	QTY	UNIT COST	TOTAL COST(₵
1	Sugar	6tins	5.75	34.50	Beans	24tins	4.25	102.00	Rice	4 bags	70.00	280.00
2	bread	13	1.70	221.00	Gari	1bag		85.00	Flour	2 tin	3.08	6.16
3	millet	18tins	2.13	38.34	Tin tomato	1 tin		5.95	Tin tomato	3 tins	5.95	17.85
5	fuel			5.00	Fresh tomato			51.43	Fresh tomato			51.43
					Onions			12.86	onions			12.86
					Pepper	½ tin	6.7	3.38	salt			2.00
					Palm Oil	4 gal	8.57	34.28	Magi			5.00
					Kobi			2.00	Salad oil	15ltr	2.68	40.2
					Salt			2.00	Garlic&ginger			3.00
					Soap			2.78	Pepper	½tin	6.75	3.38
					Fuel			20.00	Soap	1bar	2.78	2.78
					Magi			5	Fuel			20.00
Total +10% overhead charges				=298.84 29.88 =Gh₵328.72	Total +10% overhead charges			=326.68 32.66 =Gh₵359.34	Total +10% overhead charges			=444.66 44.46 =Gh₵489.12
Cost Per Head Per Meal				0.33	Cost Per Head Per Meal			0.36	Cost Per Head Per Meal			0.49

Appendix A2 TEMA SENIOR HIGH SCHOOL COSTING SHEET

TERM: 1ST DATE: 11-10-2011 DAY: TUESDAY NO. OF STDS: 1008 TOTAL COST PER HEAD PER DAY: Gh ₵ 1.79

BREAK FAST (RICE PORIDGE & BREAD)					LUNCH KENKEY, FRIED FISH AND HOT PEPPER				SUPPER (RICE AND GROUNDNUT SOUP)			
S/N	**ITEM**	**QTY**	**UNIT COST(Gh₵)**	**TOTAL COST(Gh₵)**	**ITEM**	**QTY**	**UNIT COST**	**TOTAL COST(Gh₵)**	**ITEM**	**QTY**	**UNIT COST**	**TOTAL COST(Gh₵)**
1	Sugar	6tins	5.75	34.50	Maize	2bags	90.00	180.00	Rice	3 bags	70.00	210.00
2	Bread	13	1.70	221.00					Frozen tuna	18 pcs	6.00	108.00
3	Milk	70tins	0.88	61.60	Corn husk	1 bag	13.00	13.00	Tin tomato	2 tins	5.95	11.90
4	Rice	1 bag		70.00	Fresh tomato			68.58	Fresh tomato			34.29
5	Fuel			5.00	onions			12.86	onions			12.86
6					Pepper	2 tin	6.75	13.50	Pepper	½tin	6.75	3.38
7					Salad Oil	18 litters	2.68	48.24	Magi			5.00
8					Frozen fish	4 carton	85.00	340.00	Garlic &ginger			3.00
9					salt			3.00	salt			2.00
10					soap	1 bar		2.78	Soap	1bar	2.78	2.78
11					fuel			20.00	Fuel			20.00
12									groundnut	20tins	6.75	125.00
Total **+10% overhead charges**				=393.10 39.31 =Gh₵ 432.41	Total +10% overhead charges			=701.96 70.19 =Gh₵772.15	Total +10% overhead charges			=538.21 53.82 =Gh₵592.03
Cost Per Head Per Meal				**0.43p**	**Cost Per Head Per Meal**			**0.77**	**Cost Per Head Per Meal**			**0.59**

Appendix A5 TEMA SENIOR HIGH SCHOOL COSTING SHEET

TERM: 1ST DATE: 12-10-2011 DAY: WEDNESDAY NO. OF STDS: 1008 TOTAL COST PER HEAD PER DAY: Gh ¢ 1.7

	BREAK FAST (TOM BROWN & BREAD)				LUNCH (WAAKYE WITH SHITO AND TUNA)				SUPPER (KENKEY, SHITO &CANNED FISH)			
S/N	ITEM	QTY	UNIT COST(¢)	TOTAL COST(¢)	ITEM	QTY	UNIT COST	TOTAL COST(¢)	ITEM	QTY	UNIT COST	TOTAL COST¢
1	Sugar	6tins	5.75	34.50	Rice	3 bags	70.00	210.00	Maize	2bags	90.00	180.00
2	Bread	13	1.70	221.00	Frozen tuna	20 pcs	6.00	108.00	Canned fish	4 carton	48.00	192.00
3	Soybeans	2 tins	2.38	4.76	Beans	20tins	4.25	85.00	Corn husk	1 bag	13.00	13.00
4	Maize	½ bag	90.00	45.00	Tin tomato	4 tins	5.95	23.80	Freshtomato			68.58
5	groundnut	4 tins	6.25	25.00	Fresh tomato			34.29	onions			8.57
6	Salt			1.00	Salad oil	25litres	2.68	67.00	Pepper	2 tin	6.75	13.50
7	Fuel			5.00	Onions			34.28				
8					Pepper	4tin	6.75	27.00				
9					Magi			5.00				
10					Garlic ginger			8.00				
11					salt			2.00	Salt			3.00
12					Soap	1bar	2.78	2.78	Soap	1 bar		2.78
					Fuel			20.00	Fuel			20.00
					Wakye leaves	2bundle	2.50	5.00				
					Shrips & herings	2:3	30:20	60+60=120				
Total +10% overhead charges				=336.26 33.62 =Gh¢ 369.88	Total +10% overhead charges			789.91 78.9 =Gh¢868.06	Total +10% overhead charges			501.43 50.14 Gh¢551.57
Cost Per Head Per Meal				0.35p	Cost Per Head Per Meal			0.83p	Cost Per Head Per Meal			0.53p

Appendix A4 TEMA SENIOR HIGH SCHOOL COSTING SHEET

TERM: 1ST DATE: 13-10-2011 DAY: THURSDAY NO. OF STDS: 1050 TOTAL COST PER HEAD PER DAY: Gh ¢1.2

BREAK FAST (OBRAYO AND GROUNDNUT)					**LUNCH (BEANS STEW AND GARI & Fruits)**				**SUPPER (RICE AND PALAVER SAUCE)**			
S/N	**ITEM**	**QTY**	**UNIT COST(Gh¢)**	**TOTAL COST(Gh¢)**	**ITEM**	**QTY**	**UNIT COST**	**TOTAL COST(¢)**	**ITEM**	**QTY**	**UNIT COST**	**TOTAL COST(¢)**
1	Sugar	6tins	5.75	34.50	Beans	24tins	4.25	102.00	Rice	4 bags	70.00	280.00
2	Maize	½ bag	90.00	45.00	Gari	1bag		85.00	kobi			2.00
3	groundnut	20tins	6.25	38.34	Tin tomato	1 tin		5.95	Tin tomato	1tin	5.95	5.95
5	Salt			1.00	Fresh tomato			51.43	Fresh tomato			51.43
6	Fuel			5.00	onions			12.86	onions			12.86
					Pepper	½ tin	6.7	3.38	salt			2.00
					Palm Oil	4 gal	8.57	34.28	Magi			5.00
					kobi			2.00	Palm oil	4 gal	8.57	34.28
									Frozen tuna	8pck	6.00	48.00
					salt			2.00	Garlic ginger			2.00
					soap			2.78	Pepper	½tin	6.75	3.38
					fuel			20.00	Soap	1bar	2.78	2.78
					magi			5	Fuel			20.00
					orange	1200pc		84.00	agushie	2tins	12.00	24.00
									Soya beans	1 tin	2.38	2.38
									kontomire	2bags	30.00	60.00
Total +10% overhead charges				=210.50 21.05 =Gh¢231.55	Total +10% overhead charges			=410.68 41.06 =Gh¢451.74	Total +10% overhead charges			=556.06 55.60 =Gh¢611.66
Cost Per Head Per Meal				**0.22p**	**Cost Per Head Per Meal**			**0.43p**	**Cost Per Head Per Meal**			**0.58p**

Appendix ASTEMA SENIOR HIGH SCHOOL COSTING SHEET

TERM: 1ST DATE: 14-10-2011 DAY: FRIDAY NO. OF STDS: 1050 TOTAL COST PER HEAD PER DAY: Gh ¢1.62

BREAK FAST (HOUSA KOOKO &BREAD)					LUNCH (WAAKYE WITH SHITO AND TUNA)				SUPPER (KENKEY WITH SHITO &CANNED FISH)			
S/N	**ITEM**	**QTY**	**UNIT COST(Gh¢)**	**TOTAL COST(Gh¢)**	**ITEM**	**QTY**	**UNIT COST**	**TOTAL COST(Gh¢)**	**ITEM**	**QTY**	**UNIT COST**	**TOTAL COST(Gh¢)**
1	Sugar	6tins	5.75	34.50	Rice	3 bags	70.00	210.00	Maize	2bags	90.00	180.00
2	Bread	13	1.70	221.00	Frozen tuna	20 pcs	6.00	120.00				
3	Millet	18tins	2.13	38.34	Beans	20tins	4.25	85.00				
4	Fuel			5.00	Tin tomato	4 tins	5.95	23.80	Cornhusk	1 bag	13.00	13.00
5					Fresh tomato			34.29	Fresh tomato			68.58
6					Salad oil	25litres	2.68	67.00				
7					onions			17.14	Onions			8.57
8					Pepper	4tin	6.75	27.00	Pepper	2 tin	6.75	13.50
9					Magi			5.00				
10					Garlic ginger			8.00	Canned fish	4 ctn	48.00	192.00
11					salt			2.00	Salt			3.00
12					Soap	1bar	2.78	2.78	Soap	1 bar		2.78
13					Fuel			20.00	Fuel			20.00
14					Wakye leaves	2bundle	2.50	5.00				
15					Shrips & herings	2:3	30:20	60+60=120.00				
Total +10% overhead charges				=298.84 29.88 =Gh¢328.72	Total +10% overhead charges			747.01 74.7 =Gh¢821.71	Total +10% overhead charges			=501.43 50.14 =Gh¢551.57.03
Cost Per Head Per Meal				**0.33**	**Cost Per Head Per Meal**			**0.78p**	**Cost Per Head Per Meal**			**0.53p**

Appendix A6TEMA SENIOR HIGH SCHOOL COSTING SHEET

TERM: 1ST DATE: 15-10-2011 DAY: SATURDAY NO. OF STDS: 1120 TOTAL COST PER HEAD PERDAY: Gh ¢ 1.08

BREAK FAST (OBRAYO AND GROUNDNUT)					LUNCH (BEANS STEW AND GARI)				SUPPER (RICE AND PALAVER SAUCE)			
S/N	ITEM	QTY	UNIT COST(Gh¢)	TOTAL COST(¢)	ITEM	QTY	UNIT COST	TOTAL COST(Gh¢)	ITEM	QTY	UNIT COST	TOTAL COST(Gh¢)
1	Sugar	6tins	5.75	34.50	Beans	24olonka	4.25	102.00	Rice	4bags	70.00	280.00
2	Maize	½ bag	90.00	45.00	Gari	1bag		85.00	Kobi			2.00
3	groundnut	20tins	6.25	38.34	Tin tomato	1 tin	5.95	5.95	Tin tomato	1tin	5.95	5.95
5	Salt			1.00	Fresh tomato			51.43	Fresh tomato			51.43
6	Fuel			5.00	onions			8.57	Onions			12.86
					salt			2.00	Salt			2.00
					Pepper	½ tin	6.75	3.38	Magi			5.00
					Palm Oil	4 gal	8.57	34.28	Palm oil	4 gal	8.57	34.28
					kobi			2.00	Frozen tuna	8pck	6.00	48.00
					soap			2.78	Garlic&ginger			2.00
					fuel			20.00	Pepper	½tin	6.75	3.38
					magi			5	Soap	1bar	2.78	2.78
									Fuel			20.00
									Agushie	2tins	12.00	24.00
									Soybeans	1 tin	2.38	2.38
Total +10% overhead charges				=210.50 21.05 =Gh¢231.55	Total +10% overhead charges			=322.39 32.24 =Gh¢354.62	Total +10% overhead charges			=556.06 55.60 =Gh¢611.66
Cost Per Head Per Meal				**0.33**	**Cost Per Head Per Meal**			**0.32p**	**Cost Per Head Per Meal**			**0.58**

Appendix A7 **TEMA SENIOR HIGH SCHOOL COSTING SHEET**

TERM: 1ST DATE: 16-10-2011 DAY: SUNDAY NO. OF STDS: 1120 TOTAL COST PER HEAD PER DAY: Go¢ 1.77

BREAK FAST (TEA & BREAD)					LUNCH (EGG STEW AND KENKEY)				SUPPER (JOLLOF RICE AND CHICKEN)			
S/N	ITEM	QTY	UNIT COST(Gh¢)	TOTAL COST(Gh¢)	ITEM	QTY	UNIT COST	TOTAL COST	ITEM	QTY	UNIT COST	TOTAL COST(Gh₵)
1	Sugar	4tins	5.75	23.00	eggs	20crates	7.00	140.00	Rice	4 bags	70.00	280.00
2	Bread	13	1.70	221.00	maize	2 bags	90.00	180.00				
3	Richoco	36	6.17	222.12	Tin tomatoes	4 tins	5.95	23.80	Tin tomato	8 tins	5.95	47.60
5	Milk	70tins	0.88	61.60	Fresh tomato			51.43	Fresh tomato			51.43
	Fuel			5.00	onions			12.86	Onions			12.86
					Pepper	½ tin	6.7	3.38	Salt			2.00
					Salad oil	9 liters	2.68	24.12	Magi			5.00
					Corn husk	1 bag		13.00	Salad oil	15ltr	2.68	40.20
					salt			2.00	Chicken	9cart	35.00	315.00
					Garlic &ginger			8.00	Garlic &ginger			7.00
					soap			2.78	Pepper	½tin	6.75	3.38
					fuel			20.00	Soap	1bar	2.78	2.78
					magi			5	Fuel			20.00
Total +10% overhead charges				=532.72 53.27 =Gh¢585.99	Total +10% overhead charges			=485.37 48.53 =¢533.90	Total +10% overhead charges			=444.66 44.46 =Gh¢489.12
Cost Per Head Per Meal				**0.52p**	**Cost Per Head Per Meal**			**0.48p**	**Cost Per Head Per Meal**			**0.49**

Appendix BLISTS OF FOOD ITEMS, CONSUMPTION, UNIT AND WEEKLY PRICES FOR A TERM

ITEMS	CONSUMPTION	Type of measure	PRICE		WEEKS IN A TERM	TOTAL COST PER TERM
			unit	weekly		
Rice	27	Bags	78	2106	14	29,484
Gari	3	Bags	145	435	14	6,090
Sugar	3	Bags	91	273	14	3,822
Flour	10	Bags	83	830	14	11,620
Margarine	2	Buckets	33	66	14	924
Tomato paste	5.5	Cartons	40	220	14	3,640
Milk	13	Crates	18.5	55.5	14	3,367
Richoco	3	Boxes	85	255	14	3,570
Cooking oil	5	Jeri cans	74	370	14	5,180
Shrimps cube	2	Tins	52	104	14	1,456
Omo multiactive	0.5	Box	45	22.5	14	315
Key soap	12	Bars	44	44	14	462
Tin fish	9	Cartons	54	486	14	6,804
Maize	9	Bags	145	1305	14	18,270
Beans	3.5	Bags	250	875	14	12,250
Millet	1	Bag	140	140	14	1,960
Soya beans	2.5	Bags	125	0	Per term	312
Pepper	3.5	Bags	350	0	Per term	1,225
Groundnut	1	Bag	360	360	14	5,040
Eggs	20	Crates	8	160	14	2,240
Salt	1	Bag	45	45	14	630
Fish	4	Cartons	95	380	14	5,320
Onion	1	Bag	100	100	14	1,400
Dried shrimps	4	Tins	35	140	14	1,960
Herrings	6	Tins	25	150	14	2,100
Ginger	4	Tins	5	20	14	280
Kontomire	4	Bags	30	120	14	1,680
Wakye leaves			10	10	14	140
Salted fish			20	20	14	280
Fruits			84	84	14	1,176
Agushie	6	Tins	15	90	14	1,260
Fresh tomatoes	3	Box	350	1050	14	14,700
TOTAL			**3034.5**	**10316**		**148,957**

Appendix C THE PURCHASE ORDER FOR THE BEGINNING OF THE THIRD TERM

PURCHASE ORDER						
No	DESCRIPTION	QUANTITY	UNIT OF MEASURE	UNIT PRICE	TOTAL PRICE	NAME OF ADDRESS OF SUPPLIER
1	Ripe Plantain				120	Open market
2	Egg	20	crates	8	160	Open market
3	Spices				40	Open market
4	Fuel& gas				140	Open market
5	Dry shrimps	3	tins	35	105	Open market
6	Dry henries	5	tins	25	125	Open market
7	Fresh tomatoes	3	box		1050	Open market
8	Frozen chicken	8	cartons	38	304	Open market
9	Kontomire	3	bags	40	120	Open market
10	Spices/poly bag				40	Open market
11	Polishing of Maize				12	Open market
12	Frozen fish	3	cartons	95	285	Open market
13	Local salt	1	bag	45	45	Open market
14	Agushie	4	tins	15	60	Open market
15	Oranges	1000	Per 100 pcs	7	70	Open market
16	Wakye leaves&kobi				20	Open market
17	Maize	9	bags	145	1305	Open market
18	Beans	3.5	bags	250	875	Open market
19	Millet	1	bag	140	140	Open market
20	Groundnut	1	bag	360	360	Open market
21	Fresh fish	4	cartons	95	380	Open market
22	Pepper	3.5	bags	350	1225	Open market
23	Soya beans	2.5	bags	125	312.5	Open market
24	Ginger	4	Tins	5	20	Open market
25	Palm oil	20	Gallons	8.6	172	Open market
26	Margarine	2	buckets	33	66	Open market
27	Flour	10	bags	83	830	Open market
	TOTAL				¢8382	

Appendix D PROCUREMENT PURCHASES FOR THE FIRST MONTH OF THE TERM

DETAILS OF PROCUREMENTS						
No	**DESCRIPTION**	**QUAN TITY**	**UNIT OF MEASURE**	**UNIT PRICE**	**TOTAL PRICE**	**NAME OF ADDRESS OF SUPPLIER**
1	Cooking oil	30	Jeri cans	74	2350	TWO THOUSAND LTD
2	Richocho	30	cartons	92	2760	SDTM-GH LTD
3	Fresh tuna	1	ton	3200	3200	AFKO FISHERIES
4	Flour for bread	40	50kg bag	82.35	3294	IRANI BROTHERS
5	Margarine	20	buckets	32.64	652.8	KEYSONS LTD
6	Key bar Soap	7	Carton	43.25	302.75	KEYSONS LTD
7	Omo multiactive	6	Carton	44.88	269.28	KEYSONS LTD
8	Magi shrimps	5	Carton	52.03	260.15	KEYSONS LTD
9	Gino tomato paste	30	carton	39.79	1193.7	KEYSONS LTD
10	Vietnam Rice	100	bags	77.52	7.752	KEYSONS LTD
11	Sugar	30	bags	90.79	2723.7	KEYSONS LTD
12	Sultana Rice	5	bags	76	380.01	KEYSONS LTD
13	Mackerel (tin fish)	27	cartons	54	1458	Open Market
	TOTAL				**¢18852.142**	

Appendix E OTHER ITEMS NOT INCLUDED

No	Perishable Item	Unit cost	Quantity of the item need each week	Weekly cost	Quantity of the Item needed per term (14 weeks)	Total cost per term
1	Soy beans	2.38	2.5 bags	125	35	312
2	Pepper	6.75	3.5	350	49	1,225
3	Bread	2.00	65	130	910	1820
4	Soap	2.78	14	38.92	196	544.88
5	Fuel	45	7	315	98	4410
6	Fruits	84	1	84	14	1176
	Total cost in (Gh¢)			1042.92		9,487.88

All the ingredients are listed in Table 4.4 and 4.5, however items that was listed in the daily costing in appendix A1-A7 but not included in the model are listed in the table below. Soy beans and pepper was not included because the school buys these items in bulk for the term as shown in appendix B. Also in the daily costing did not include ingredients for making bread.

From Table 4.6 we observe that total amount needed to purchase these items for a term is Gh¢9,487.88.

Appendix F MATLAB CODE (COST OPTIMIZER)

```
%THE COST OPTIMIZER
clc % this is to clear the sheet for new calculations
disp('THIS IS A PROGRAM FOR OPTIMIZING BOARDING FEEDING COST')
disp('This is specific for the Tema High Schools menu')
disp('enter the total budget(W) you would not like to exceed')
W=input('W=');% here the user enters the total budget for the school
w=input('w='); %enter the vector of prices of the items
v=input('V=');%the user should input a vector of values
n=input('n=');% enter the quantities available
%next we put these vectors in a matrices
disp('the solutions is given below')%the knapsack program is
displayed as
q=65536;
T=[];
X=[];
K=[0,0];
z=0;
y=0;

for p=1:q
   [r,l]=size(n);
for h=1:l;
        s(h,:)=(n(:,h)-1)*rand();
end
   S=round(s);
      M=[w;v]';
              T(:,1)=M(:,1).*S;
              T(:,2)=M(:,2).*S;
     K=sum(T(:,[1 2]));
if K(:,1)>W
% reject the results hence go back to next results

         K(:,1)=0;%to delete the value
elseif K(:,2)<y
              K(:,2)=y;
elseif K(:,2)>y
         y=K(:,2);
         z=K(:,1);
         X=S;
         disp(y)
         disp(X)

 end
 end


 end

 end
     disp('the optimal solution is')
           max(y)
      disp(X)
```

Appendix G Manual Solution to the Optimized Knapsack Model

The optimized knapsack solution for non-perishable food item is shown below

Maximize $Z = \sum_{i=1}^{n} v_i x_i$

=7 (8)+5 (8)+2 (9) +2 (6) +4 (8)+1 (9) +6 (4) +7 (4)+18 (8)+3 (8)+11 (8)+5 (8)+6 (8)+2 (6) +11 (6) +2 (8)= 221

Subject to $\sum_{i=1}^{n} w_i x_i \leq 10000$

=22.7 (8)+857 (8)+111 (9) +140 (6) +360 (8)+254.58 (9) +1087.5 (4)+1716 (4) +45 (8) +435 (8)+220 (8)+172 (8)+370 (8)+432 (6) +55 (6) +10 (8).

= 9933

The optimized knapsack solution for Perishable food items is shown below

Maximize $Z = \sum_{i=1}^{n} v_i x_i$

=1 (6) +14 (6)+5 (9) +1 (5)+5 (5)+7 (9) +1 (5)+2 (9) +1 (7)+2 (6)+2 (5)+2 (7) +14 (9)

=74

Subject to $\sum_{i=1}^{n} w_i x_i \leq 2800$

=160 (6)+1050 (6)+10 (9) +380 (5)+444 (5)+820 (9) +342 (5)+120 (9) +120 (7)+150 (6)+140 (5)+60 (7)+100 (9)

=2794

Appendix G:AUTHENTICATING LETTER

Tema Secondary School

NIL NISI OPTIMA

P.O. Box 300, Tema.
Our Ref:____________

2nd August, 2012

PROCUREMENT OFFICER

This is to certify that on the average Tema Secondary School uses GH¢2,800 for the purchase of perishable food items on the open market each week. Also on the average, the school uses GH¢10,000 for the purchase of non-perishable food items for a week.

Sincerely,

....

(Mrs. Evelyn Ansah)