KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY

MODELING OF PENSION FUND USING GARCH: CASE STUDY OF SSNIT

By

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Declaration

I hereby declare that this submission is my own work towards the award of the MSc degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the university, except where due acknowledgment had been made in the text.

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Dedication

To God Almighty in whom lies all the treasures of wisdom and Knowledge

Abstract

The introduction of the three-tier contribution scheme under the new pensions Act has brought great reforms in the pensions industry. Currently a total of 18.5% of the contributors basicsalary is deducted for pensions and only 11.0% goes to SSNIT , 2.5% goes to the management of the National Health Insurance Scheme and the remaining 5.0% goes to the second-tier The challenging aspect of the new Pension Act is that , although the contribution level of SSNIT is reduced ,SSNIT is asked to provide monthly pensions for at least fifteen years (additional three years) so far as a contributor lives and also provide forms of invalidity. In view of this, the researcher used the GARCH model in fitting a model for the fund size and predicted the funding level of the Trust, The sustainability ratio of the fund which determines how much sustainable the fund was within this period was also examined The predicted values showed an upwards trend in the fund size and an increase in the sustainability ratio. It was also observed that one major factor that affected the funding level of the trust was indebtedness to the Controller and Accountant Generals Department. It was the institution with the highest indebtedness percentage. The study recommended much lucrative investment opportunities for the Trust to look at, to increase its sustainability level and also measures to put in place to increase its outflow especially its expenses on operations and administration

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CHAPTER 1

Introduction

1.1 Background of the Study

According to Amartey -Vondee (2008) pension is "a series of periodic money payments made to a person who retires from employment because of old age, disability, or the completion of an agreed span of service". Pensions are an attractive component of employee compensation package, and also contain an insurance aspect they will often pay benefits to survivors or disabled beneficiary, while there is an annuity income which insures against risk of longevity.

The periodic money payments made to a person is dependent on the kind of the pension scheme one belongs to. There are two schemes of pension: a defined benefit pension scheme and a defined contribution pension scheme.

In the defined benefit pension scheme, the scheme guarantees a pension based on a set of rules that includes the number of years of contribution, the levels of contribution and a reference salary used for pension's calculation; that is, the benefit or pension is pre-determined with reference to the workers period of contributions and earnings.

In the defined contribution scheme, the pension to be received will depend on the contributions, the returns on investment and the cost of administration. In that, the contribution rate is set in advance and is invested so that the pension payable is whatever the accumulated contributions have produced at the time of the worker's retirement.

Ghana until recently was running three main pension schemes concurrently before the new pension law was passed in December 2008 to bring some reforms into the pensions industry(Agblobi, 2011). The first of these schemes was CAP30: a pension scheme for Pensionable public servants in civil and other public service before 1st January, 1972 and public servants who have exempted by law from participation in the Social security Pension Scheme.

The second of these schemes was the Ghana University Staff Superannuation (GUSS), a scheme established by the British colonial Government to attract and retain high caliber staff in the country's public universities.

The third scheme was the SSNIT scheme which was formed in1965 by the first republic parliament for all workers in both the formal and informal sectors of the economy to join a contributory scheme. This scheme was to help workers enjoy superannuation, invalidity, survivor and other benefits.

The above pension schemes started as defined benefit schemes. However, with the enactment of the Act 766, the new pension law, the three major different schemes listed above have been defined into a 3-tier schemes of which the first tier is a defined scheme while the second and third tiers are defined contribution scheme. The first and second tier schemes are compulsory while the third tier is voluntary for workers to contribute to, for supplement.

1.1.1 What is a Pension Fund?

Investopedia defines a pension fund as "a fund established by an employer to facilitate and organize the investment of employees' retirement funds contributed by the employer and employees" The pension fund is a common asset pool meant to generate stable growth over the long term, and provide pensions for employees when they reach the end of their working years and commence retirement. The major inflows to the Pension Fund managed by the SSNIT is from employees contributions; made by the employees themselves and their employers and the returns on the various investments.

The level of pension fund is a key factor in determining how much reliable and sustainable the fund is to the pensioners, prospective pensioner, governments and a country as a whole. Using quantitative methods such as GARCH modelling helps forecast the level of fund and this makes it important in seeing exactly which mechanisms to put in place in order to maintain or improve the level of fund

1.1.2 Benefits of the CAP 30

The CAP 30 scheme has two benefits for its members, gratuity and pension. To qualify for the CAP 30 benefit, one must first of all be a pensionable officer secondly must be between 45 and 55 years and must have continued in the civil service for at least 10 years without break or have any criminal any criminal charge within the period of service.

The CAP 30 scheme depended heavily on the consolidated fund. The nation's budget, which also draws much of its inflow from the consolidated fund, is always supported by donor agencies hence the inability of the CAP 30 scheme to continue to exist with government support (Yankah, 2008).

1.1.3 Compulsory Savings Scheme

A compulsory saving's scheme was introduced by the government that mandated all workers, who were not apart of the CAP 30 scheme, and employers to contribute into the scheme in 1960. This scheme was created to enable both formal and informal workers to join a social protection scheme. The compulsory scheme was lager turned into a provident fund scheme (Andrews, 2011). Table 2.1 shows the contribution level at the start of the compulsory scheme.

Table 1.1: Contribution Level at start of the compulsory scheme

Employer	Employee	Total
15.%	7.5%	22.5%

source: Government Budget Statements and Statistical Service Monthly Newsletter.

Proceeds from the compulsory savings scheme were used as seed money to finance the Social Security Scheme.

1.2 The Social Security Scheme

The SSNIT Scheme was formed in 1965 by the first Republic parliament when Act 279 was passed. The Act required all workers in both formal and informal sectors of the economy to join a contributory scheme with the following benefits: superannuation, invalidity, survivor and other benefits. The scheme was to pay lump sum benefits to qualified members. The scheme was also to be converted to a pension scheme after five years but this conversion did not take place till 1991. (Andrews, 2011).

A new legislative instrument, the NRCD 127, authorized the establishment of a corporate body, the SSNIT as an independent entity to administer the social security scheme in Ghana in 1972. The legislation mandated all employees with a minimum of five workers to join the scheme excluding the following: the Armed Forces, senior members of Research Institutions, Foreigners in Diplomatic Missions, the Fire Service, the Police Service (Andrews, 2011)

In 1991, a new legislative instrument, PNDCL 247, mandated the conversion of the provident fund (PF) scheme into a pension scheme. The legislation required the SSNIT to: Be responsible for the general administration of the social security law and any regulations under it; Provide partial income replacement for contributors in the event of old age, invalidity of death; Be responsible for the management of the fund; Carry out any other activities incidental to the realization of the objectives.

1.2.1 Financing the Social Security Scheme

The scheme is a partially-funded social insurance scheme. It is financed by contributions from both the employer and the employees and investment income. Originally, the employer pays 15% and the employee pays 7.5% making a total of 22.5% of the employee's basic salary (this is as spelt out in the compulsory savings scheme). However, it was later reduced to a total of 17.5% of the employee's basic salary made up of 12.5% from the employer and 5% from the employee. It is interesting to note that the contribution rate has been changed again with the commencement of the new pensions' law. Table 1.2 shows the contribution level while table 1.3 shows the distribution of the contributions.

Table 1.2: Contribution level of SSNIT

Employer	Employee	Total
12.5%	5.0%	17.5%

source:SSNIT.

Table 1.3: Distribution of Contribution

SSNIT Scheme	NHIS	Total
15.0%	2.5%	17.5%

The SSNIT Scheme used to manage all 17.5% of the contributions till 2004 when the NHIS was introduced and the SSNIT was mandated to pay 2.5% of the contributions to partially finance the NHIS.

1.3 Benefits of the SSNIT Scheme

The SSNIT Scheme has three Benefits, namely;

- Old age pension
- Individual benefit
- Death and survivor's benefit

The old age pension is paid to the policyholder for at least 12 years; invalidity pension is also paid to the in valid policyholder for the life time of the invalid person; finally, the death and survivors' benefit is paid to the nominated dependants of the policyholder.

1.4 Qualifying for the SSNIT Scheme Benefits

You must have contributed for a least 240 months (20 years) in aggregate and must have attained age 60 for full pension or 55 to 59 years for reduced pension. Furthermore, a member of the scheme qualifies for invalidity pension if she/he has worked for at least 12 months in aggregate in the last 3 years, at the time of lodging the claim, and has been confirmed by a medical board appointed by the state that she/he can no longer engage in any employment. This criterion has also changed with the enactment of the new pensions' law Act 766.

1.5 The New Pensions Act, Acts 766(2008)

An Act to provide for pension reform in the country by the introduction of a contributory three -tier pension scheme; the establishment of a National Pensions Regulatory Authority to oversee the administration and management of registered pension schemes and trustees of registered schemes, the establishment of a Social Security and National Insurance Trust to manage the basic national social security scheme to cater for the first tier of the contributory three-tier scheme, and to provide for related matters.

1.6 The Composition of the Three-Tier Pension Scheme Contribution

The contributory three-tier pension scheme comprises of

- a mandatory basic national social security scheme
- a mandatory fully funded and privately managed occupational pension scheme, and
- a voluntary fully funded and privately managed provident fund and personal pension scheme.

1.7 Problem Statement

Ghana as a country run three different pension schemes, namely, the CAP 30, Social Security and National Insurance Trust (SSNIT), the Ghana University Staff Superannuation and other privately managed schemes for about five decades now. With the introduction of the new Pensions Act in December, 2008, the four different schemes have been unified into one 3-tier scheme. The government has also inaugurated the Pension Regulatory Authority to oversee the activities of the Pension Fund Trust, Fund Managers and Insurance Companies in the country.

The key issue is that the contribution levels of the fund have been reduced and the benefits have been increased by the New Act and this has called for an analysis of funding level of the scheme, hence the relevance for the researcher to work in this area.

Based on the enactment of Act 766, the new Pension Law, although the contribution rate from employers and their employees have been increased from 17.5% to 18.5%, the contribution that comes to SSNIT has reduced from 17.5% to 11.0%. The National Health Insurance Scheme gets 2.5% of this deduction for its management and the remaining 5.0% goes to the second tier of pensions managed by the National Pension Regulatory Authority. With this, the lump-sum benefit for pensioners will be paid by the second-tier scheme providers.

Again, the contribution period for enjoying entitlement on the scheme has been reduced from twenty years to fifteen years but the annuity duration has been increased from twelve years to fifteen years (ACT 766).

These disparities outlined above have motivated the researcher to research into this area to determine the impact on the pensions fund level within a time interval. The research problem and question is to determine how sustainable the pension fund level will be over time.

1.8 Research Objective

The main objective of this research is to predict the volatility and sustainability of SSNIT pension fund using GARCH.

1.9 Data, Scope and Limitation of the study

The data for this research is on the pension fund size or level of the Social Security and National Insurance Trust (SSNIT). The time period of data is from the year 2002 to 2010. The annual data on the funding size was obtained for this period and because the nine-year annual data was not sufficient for the model setting, the monthly data for this period were further estimated by the moving average method. This method enabled the researcher to predict the monthly data on the funding size.

The major limitation for this study is the availability of data. It was extremely difficult in acquiring the yearly data from SSNIT, although it is a government institution that should have readily made available data for such studies in order to help in planning and monitoring their activities.

Despite the challenges in the data acquisition, estimates were made in order to construct a good statistical model for analysing and forecasting the fund level.

1.10 Outline of Thesis

The report on this research is in five chapters. The first chapter is the introductory chapter where the terms in the topic are explained in the background, followed by the statement of the problem, the objectives of the study, the data and its scope, the limitations of the study, the data source and the methods employed in the analysis of the data.

The second chapter reviews the relevant literature in relation to the topic under study. Chapter three gives an in-depth discussion on the various theories with regards to the methods employed in the analysis of data. The fourth chapter gives a presentation on the analysis of data and the outcome of the results. The final chapter summarizes the main outcomes and observations of the data analysis. The chapter also discusses key issues from the observation and then makes conclusion and recommendations on the outcomes.

CHAPTER 2

Literature Review

2.1 Introduction

This chapter considers the relevant advances made on the fund level, and review of related works that have been carried out by other researchers on the topical area both theoretical and empirical
Much work has not been done on the position fund in Ghana and in view of this, literature was very scanty. Across the world, several works have been done on stock market returns using the GARCH model in predicting the volatilities. In knowing the volatility of returns one can predict the actual values of returns. The level of pension fund can be measured as a return since several factors come to play before the level is finally reached.
In modern finance theory, Markowitz (1952) used returns volatility as a measure of risk. According to Markowitz (1952) existing literature has supportthat most time series data of financial assest exhibit linear dependence involatility, which is referred to as volatility clustering in econometrics andempirical finance.

Engle (1982) first proposed the ARCH (Autoregressive conditional heteroskedasticity) model, which assumes normal errors for asset returns and successful captures a number of stylized facts of financial assets, such as time - volatility and volatility clustering.

The traditional econometric time series model generally assume a normal distribution of stock returns. However, the financial literature has long been aware that financial returns are non-normal and tend to have leptokurtic and fat - tailed distribution (Mondelbrot, 1963; Fama, 1965) Bali, (2007) modeled the nonlinear dynamics of short-term interest rate volatility with SGED distributions, and conclude that the level - GARCH model that accommodates the tail - thickness of interest rate distribution generates satisfactory volatility forecasts of short - term interest rate.

Xu (1999) and Lee et al. (2001) are two recent papers that estimate volatility for stock markets in China. Xu (1999) modeled volatility for daily spot returns of Shanghai composite stock index from May 21, 1992 to July 14, 1995 and tested the in sample goodness - of - fit of various competing models. The researcher found that the GARCH model is superior to that of either EGARCH or GJR -GARCH models, and indicating that there is almost no leverage effect in the Shanghai stock market since volatility is mainly caused by government policy on stock market under the present financial system.

Lee et al. (2001) examined time - series features of stock returns and volatility in four China's stock exchanges. They provided strong evidence of time - varying volatility and indicated that volatility is highly persistent and predictable. Moreover, evidence in support of fat - tailed conditional distribution of returns was found. The papers by Xu (1999) and Lee et al. (2001) just focus on the in - sample goodness - of - fit of volatility models. However, a good starting point to judge competitive models is to assess their out - of - sample forecasting performance. In addition, a leptokurtic and skewed returns distribution should be considered when using emerging market data.

The introduction of GARCH family gives the alternative volatility forecasting models which involve the constant updating of parameter estimates (Poon and Granger, 2003) The forecasting

In another research, Engle (1982) uses ARCH (4) models to estimate the mean and variance of inflation in the UK . The researcher found that the ARCH effect is significant and the estimated variance considerably increase during crisis period. Some prolems with ARCH (q) models led to the more general frame work on GARCH (p.q), proposed by Bollerslev (1986) and Taylor (1986). Bollerslev (1986) focused on the paper of Eagle and Kraft (1983) who attempted to explain the Bollerslev 1986 employed GARCH (1,1) and ARCH (8) in his study and found that GARCH (1,1) outperformed the ARCH (8) in terms of best fir and reasonable lag structure.

McMilan (2000) discussed another aspect of volatility forecasting models which is model evaluation measurement. The evaluation of various GARCH models was comprehensively studies in McMillan (2000) for the case of UK market. The study used ten volatility forecasting models including historical mean, random walk, GARCH and EGARCH, and others to estimate the UK indices from 1984 to 1996 at the daily, weekly and monthly frequencies. The forecast evaluation was performed under both symmetric and asymmetric loss functions. The performance results of these models depend on frequencies, the series and type of loss functions considered. Under symmetric loss, GARCH and moving average model outperformed the other models in forecasting daily volatility whereas, random walk models is superior for monthly volatility forecast.

The forecasting ability of ARCH - type models is comprehensively researched in the study of Hansen and Lunde (2005). Their analysis considers 330 different ARCH - type models to estimate the volatility of DM exchange rate and IBM stock returns. The main findings were that the GARCH

(1,1) was superior in analyzing DM exchange rate rather than any other models, whereas GARCH (1,1) underperformed other models in modeling IBM stock return. The study suggested that good forecasting models required specifications which are able to capture leverage effects.

With respect to forecast series, the choice of data sampling frequency to give the accurate forecast was the focus in various studies. Most of the studies concluded that data frequency is chosen in relation with forecast horizon to improve the forecast accuracy. Generally, for forecasting long horizon, i.e over 10 years, the data sample at low frequency supports to improve the forecast accuracy rather than the high frequency (Figlewski, 1997).

On the other hand, Bollerslev et al (1986) argued that the increase in data frequency improve the performance of forecast models. The complication in choosing data frequency to forecast was partly due to the mean reversion property of volatility series.

In Ghana, not much has been done on the quantitative features of pension fund level. A study on the management of the Social Security was carried out by Kumado and Gockel in the year 2003. Again, Dei also carried out a task on "Public" pension fund management in Ghana". Dei(2001) Fitted a model on the Investment portfolio of the pension Fund. These research above works done on pensions in Ghana did not look very much into the quantitative future of the funding level, although Kumado and Gockel's work recommended reforms in the pension system in Ghana.

Engle (1982) studied the ARCH model and revealed that these models were designed to deal with the assumption of non-stationary found in real life financial data. He based the ARCH model on the idea that a natural way to update a variance forecast was to average the squared deviation of the rate of return from its mean just like the principle used in standard deviation. The ARCH process allowed the conditional variance to change over time as a function of past errors leaving the unconditional variance constant. Empirical evidence revealed that ARCH model required a relatively long lag in the conditional variance equation and so to avoid the problems with negative variance parameters, a fixed lag structure was typically imposed.

Bollerslev (1986) proposed a generalized ARCH (GARCH) to overcome the limitations of the traditional ARCH model of Engle (1982). The GARCH model allowed for both a longer memory and a more flexible lag structure. In the ACRH process, the conditional variance is specified as a linear function of past sample variance only whereas the GARCH process allows lagged conditional variances to enter in the model as well. Both the ARCH and GARCH models of Engle (1982) and Bollerslev (1986) could not tell how the variance of return was influenced differently by positive and negative news. Hence Nelson (1991) extended the ARCH framework in order to better describe the behaviour of return volatilities. His study broke the rigidity of the ARCH and GARCH model specification.

He proposed the Exponential GARCH (EGARCH) model to test the hypothesis that variance of return was influenced differently by positive and negative excess returns. The results revealed that the hypothesis was true and also the excess returns were negatively related to stock market variance. Asri and Mohammad (2011) propose an alternative model for modelling the volatility of the conditional variances: A (Radial Basis Function) RBF-EGARCH Neural Networks Model. Their proposed forecasting model combines a RBF neural network for the conditional mean and a parametric

EGARCH model for the conditional volatility.

They used the regression approach to estimate the weight and the parameters of EGARCH model. They carried out a simulation based on sample of Bank Rakyat Indonesia Tbk stock returns and the results indicated that their proposed model is able to accurately predict 63% upward and downward movements of future predictions. They concluded that the simulation results obtained in the forecasting performances motivates further work, which will involve comparing different method of parameters model estimation.

Kunst (1997) studied the augmented ARCH models which encompasses most linear ARCH-type models. He considered the two basic ARCH variants for auto-correlated series; conditional variance lagged by errors (Engle, 1982) or conditional variance lagged by observations (Weiss, 1984). He evaluated whether the restrictions evolving from these two ARCH variants are valid in practice. Time series of stock market indexes for some major stock exchanges (Standard and Poor 500 index, Stock market index for German, French, British and Japanese) were considered. For the important US Standard & Poor 500 Index and for Japanese and German stock index, the evidence indicated more or less convincingly that fourth-moments structures in financial series may be more complicated than the traditional ARCH model implies.

A non-parametric comparison of sample moments also supported this result. The statistical evidence presented was stronger than the weak evidence on more general structures found by Tsay (1987) in an exchange rate series. For two other countries, France and the United Kingdom, the statistical description achieved by the standard ARCH model appears to be sufficient. Su (2010) employed both GARCH and EGARCH models in studying the financial volatility in China. He applied the daily stock returns data from

January 2000 to April 2010 and split the time series into two parts: before the crisis and during the crisis period.

The empirical results suggested that EGARCH model fits the sample data better than GARCH model in modelling the volatility of Chinese stock returns. The result also showed that long term volatility was more volatile during the crisis period whilst Bad news produced stronger effect than good news for the Chinese stock market during the crisis. Malmsten (2004) used a unified framework for testing the adequacy of an estimated EGARCH model. The tests were Lagrange multiplier type tests and included testing an EGARCH model against a higher-order one and testing parameter constancy. Furthermore, various existing ways of testing the EGARCH model against GARCH models were also investigated as another check of model adequacy. This was done by size and power simulations.

Simulations revealed that the simulated LR test is more powerful than the encompassing test and that the size of the test may be a problem in applying the pseudo-score test. Finally, the simulation results indicate that in practice, the robust versions of their tests should be preferred to non robust ones and they can be recommended as standard tools when it comes to testing the adequacy of an estimated EGARCH (p,q) model. The stylized facts of financial time series using three popular models were studied by Malmsten and Terasvirta (2004). The models used were the GARCH, EGARCH and Autoregressive Stochastic Volatility (ARSV) models and they focused on how well these models are able to reproduce characteristics features (stylized facts) of financial series.

Their study used stock returns as case of the financial series. The results showed that the GARCH model and EGARCH models were at their best when characterizing models based on time series with relatively low kurtosis and high first-order autocorrelation of squares, assuming normality of errors. However the ARSV (1) model is a better option for time series displaying a combination of high kurtosis and high autocorrelations. Blake and Kapetanios (2005) investigated the extent of the effect of neglected nonlinearity on the properties of ARCH testing procedures. They proposed and used a new ARCH testing procedures based on neural networks which are robust to the presence of neglected nonlinearity.

The neural networks were used to purge the residuals of the effects of nonlinearity before applying an ARCH test. Thus they correctly size the ARCH test while retaining good power for the ARCH test. Results based on Monte Carlo simulations showed that the new method alleviated the problem posed by the presence of neglected nonlinearity to a very large extent. Empirical evidence or results based on the application of the new tests procedures to exchange rate data indicated substantial evidence of spurious rejection of the null hypothesis of no ARCH. There was also further evidence that exchange rates exhibited complicated, dynamic behaviour, with important nonlinearity and volatility effects.

Karanasos and Kim (2003) considered the moment structure of the general ARMA (r,s) -EGARCH (p,q) model and compared it with the standard GARCH model and APARCH model. In particular, they derive the autocorrelation function of any positive integer power of the squared errors and also obtained the autocorrelations of the squares of the observed process and cross correlations between the levels and the squares of the observed process assuming that the error term is drawn from either a normal, double exponential or generalised error distributions. Daily data on four East Asia stock indices – Korean Stock price index (KOSPI), Japanese Nikkei index

(Nikkei) and the Taiwanese SE Weighted index (SE) for the period 1980:01 – 1997:04 and the Singaporean Straits Times price index (ST) for the period 1985:01 – 1997:04. They concluded that there were differences in the moment structure between the ARMA (r,s) – EGARCH (p,q) model and the standard GARCH model.

The study also concluded that, to help with model identification, results of the autocorrelations of the squared deviations can be applied to the observed data and its properties compared with the theoretical properties of the models. Based on that, it was observed that the EGARCH model can more accurately reproduce the nature of the sample autocorrelations of squared returns than the GARCH models. Lee and Brorsen (1996) also studied the relative performance of the GARCH model and the EGARCH model by using a Cox-type non-nested test that used the Monte Carlo hypothesis tests.

The approach used by Lee and Brorsen (1996) was similar to the approach used by Pesaran and Pesaran (1993). Whilst the approach of the Pesaran and Pesaran (1993) assumed asymptotic normality, Lee and Brorsen (1996) approach did not assume asymptotic normality. They estimated that the GARCH and EGARCH models of the daily spot prices of Deutsche Mark in terms of the United States dollars using the maximum likelihood procedure. The GARCH model was rejected whilst the EGARCH model was not rejected. The study therefore concluded that the EGARCH models were preferable to the GARCH models in modelling Deutsche mark/dollar exchange rate.

The effects of good and bad news on volatility in the Indian stock markets using asymmetric ARCH models during the global financial crises of 2008-2009 was investigated by Goudarzi and Ramanaraynan (2011). The asymmetric volatility models considered were the EGARCH and TGARCH models and the BSE 500 stock index was used as a proxy to the Indian stock market. The study found out that the BSE 500 return series reacted to good news and bad news asymmetrically. That is the BSE 500 return series reacted differently to good news and to bad news. The EGARCH (1,1) and TGARCH (1,1) models were estimated for the BSE 500 stock returns series using the robust method of Bollerslev-Wooldridge's quasi-maximum likelihood estimation (QMLE) assuming that the Gaussian standard normal distribution.

The results indicated that the conditional means are significant in both estimated models. Hence the SBIC information criterion was applied to select the fittest model to the data. The TGARCH (1,1) model was selected and the study therefore concluded that the TARCH (1,1) model can be possible representative of the asymmetric conditional volatility process for daily return series of BSE 500 as compared to the EGARCH (1,1). Jean-Philippe (2001) examined the forecasting performance of four GARCH-typed models. The comparison was focused on two different aspects; the difference between symmetric GARCH model (traditional GARCH model) and asymmetric models (EGARCH, GJR and APARCH) and the difference between normal tailed symmetric, fat-tailed symmetric and fat tailed asymmetric distributions (i.e. normal distributions against student-t and skewed student-t distributions). The study concluded that noticeable improvements were made when using an asymmetric GARCH in the conditional variance and that the APARCH and GJR outperformed the EGARCH. Furthermore, non-normal distributions provided better in-sample results than Gaussian distributions.

Alberg, Shalit and Yosef (2008) carried a comprehensive empirical analysis of the mean return and conditional variance of Tel Aviv Stock Exchange (TASE) indices is performed using various GARCH models. The prediction performance of these conditional changing variance models were compared to newer asymmetric GJR and APARCH models. The results indicated that the asymmetric GARCH model with fat tailed densities improved overall estimation for measuring conditional variance. The EGARCH model using a skewed student-t distribution is the most successful for forecasting TASE indices as compared to the asymmetric GARCH, GJR and APARCH models.

Angelidies, Benos and Degiannakis (2003) evaluated the performance of an extensive family of ARCH models (GARCH, TARCH and EGARCH) in modelling daily Value-at-Risk (VaR) of perfectly diversified portfolios in five stock indices using a number of distributional assumptions and sample sizes. The five perfectly diversified portfolios were the S&P 500, Nikkei 225, FTSE 100, CAC 40 and DAX 30. The different distributions were normal, student-t and generalised error distribution whilst the sample sizes were 500, 1000, 1500 and 2000. Their results show that under the evaluation framework based on the proposed quartile loss function, there was strong evidence that the combination of the student-t distribution with the simplest EGARCH models produce the most adequate VaR forecasts for the majority of the markets.

Furthermore, the size of the rolling sample used in estimation turned out to be rather important since in simpler models and low confidence levels, a sample size smaller than 2000 improves probability values. In more complex models where leptokurtic distributions are used and when the confidence level is high, a small sample size led to lack of convergence in the estimation algorithms. Finally, there was no consistent relation between the sample sizes and the optimal models as there were significant differences in the VaR forecasts for the same model under the four sample sizes.

Yuksel and Bayram (2005) investigated the stock market volatility in Turkish, Greek and Russian stock markets using the total return indexes based on the domestic currencies of the corresponding countries. The data set covers a period from 1994 - 2004. The study concluded that the GARCH-M (1,1) was the best model for modelling the volatility in the stock markets in Turkey. In the case of the stock markets of Greece, the TARCH (1,2) was the best model whilst the TARCH (1,1) was the best model for the Russian stock markets. Irfan et al (2010) modelled the volatility of short term interest rates in Pakistan and India using the ARCH Family models. The study used the Karachi Inter Bank Offering rate (KIBOR) and Mumbai Inter Bank Offering rate (MIBOR) in Pakistan and India respectively and the various time series models examined included GARCH, EGARCH,TGARCH and PARCH.

The results from all the ARCH family models indicated that high volatility is present in KIBOR returns while volatility shock is moderately present in MIBOR returns. Also all the ARCH family models were compared using the within sample forecasting performance on basis of root mean squared error (RMSE) and Mean Absolute Error (MAE) and the comparison suggested that MIBOR forecasted better than KIBOR as it had minimum errors. Lastly, the TGARCH was adjudged the best model in both returns because they had all the parameters being significant whilst the PARCH (1, 1) model is selected the second best model based on the criteria of the students t-distribution.

The ARCH-type models were used by Wagala et al (2011) to model the volatility of the Nairobi Stock Exchange weekly returns. The models applied in the study included the ARCH (p), standard GARCH (p,q), IGARCH (p,q) and TGARCH (p,q). The results demonstrated that the ARCH (8) was found to be the most adequate for the NSE index, Bamburi and KQ while ARCH (9) provided the best order for the NBK series. Furthermore four different p and q values were tested for the GARCH (p,q), EGARCH (p,q) and TGARCH (p,q). These were (1,1), (1,2), (2,1) and (2,2). The order (1,1) was the best choice in all cases and it was consistent with results obtained from most GARCH research works.

Comparing the diagnostics and the goodness of fit statistics, the IGARCH (1,1) outperformed the ARCH, EGARCH and TGARCH models due to its stationary in the strong sense. However, because the IGARCH model was unable to capture the asymmetry exhibited by the stock data, the EGARCH (1,1) and the TGARCH (1,1) provided the best options to describe the dependence in the variance for all the four series since they were able to model asymmetry and parsimoniously represent a higher order ARCH (p). Anna (2011) examined the relationship between inflation, inflation uncertainty and output growth with evidence from the G-20 countries using several GARCH and GARCH-M models in order to generate a measure of inflation uncertainty. The study adopted two approaches to test for the impact of inflation uncertainty on inflation and vice versa. The first approach was based on the GARCH-M model that allows for simultaneous feedback between the conditional mean and variance of inflation while the second approach was based on a two-step procedure where Granger methods were employed using the conditional variance of a simple GARCH model.

The results of the study suggested significant positive relationship between inflation uncertainty and inflation in most countries. These results go to support the Cukierman-Matter and Friedman-Ball hypothesis. Also the results of the study provided evidence for the Holland theory; that uncertainty lead to lower and in the case of the effect of inflation uncertainty in output growth, there was little evidence that inflation uncertainty has negative real effects. Chatfield (2000) asserted that the idea behind a GARCH model was similar to that behind the ARMA model with respect to the fact that a higher order AR or MA model may often be approximated by a mixed ARMA model with fewer parameters using a rational polynomial approximation.

He described the GARCH model as an approximation to a higher-order ARCH model. He noted that the GARCH (1, 1) model has become the standard model for describing non constant variance due to its relative simplicity. Empirical evidence has revealed that often ($\alpha + \beta < 1$ so that the stationary condition may be met. However if the $(\alpha + \beta) = 1$, the process ceases to have a finite variance although it can be shown that the squared observations are stationary after taking first differences. In such a situation a better model Integrated GARCH (IGARCH) developed Engle and Bollerslev (1986) by is recommended.

Rafique and Ur-Rehman (2011) compared the volatility behaviour and variance structure of high (daily) and low (weekly, monthly) frequencies of stock returns in Pakistan. The study used data from 1991 to 2008 of the KSE-100 index. By employing the EGARCH model, they found that there are significant asymmetric shocks (leverage effect) to volatility in the three series but the intensity of the shock were not equal for all the series. Furthermore, it was concluded that the variance structure of high frequencies (daily) data is dissimilar from the low frequencies (weekly, monthly) data.

Karanasos et al. (2004) also examined the relationship between inflation and inflation uncertainty in the US using a GARCH model that allows for simultaneous feedback between the conditional mean and variance of inflation. The results showed that there was a strong positive bidirectional relationship between inflation and inflation uncertainty. The results also in agreement with the predictions of economic theory expressed by the Friedman-Ball and Cukierman-Meltzer hypothesis, however, it was in conflict with existing empirical evidence. The study also compared the properties of the observed time series with the theoretical properties of GARCH models to illustrate how theoretical results on correlation structure can facilitate model identification. The results showed that the AR-GARCH-M-L model can approximate reality well. Ling and Li (1997) considered fractionally integrated autoregressive moving average time series models with conditional heteroscedasticity, which combined the popular generalised autoregressive conditional Heteroscedastic (GARCH) and fractional ARIMA models. Drost and Klaassen (1997) constructed adoptive and hence efficient estimators in a general GARCH –M in mean type context including integrated GARCH models.

A time lag between a change in money supply and the inflation rate response was examined by Jehovanes (2007). He employed a modified GARCH model to monthly inflation data for the period 1994 to 2006. The maximum likelihood estimation technique was used to estimate the parameters of the model and to determine significance of the lagged value. Results showed that the GARCH model was a better fit and indicated that a change in supply of money would affect inflation rate considerably in seven months ahead. Brooks (2008) studied the stochastic volatility models and found that most time series models such as GARCH will have forecasts that tend towards the unconditional variance of the series as the prediction horizon increases. This implies that if they are at a low level relative to their historic average they will have a tendency to rise back towards the average and this feature is accounted for in GARCH volatility forecasting models.

Mushtaq et al (2011) examining the relationship between stock exchange market volatility and macroeconomic variables volatility with respect to Pakistan. To measure this time series relationship for Pakistan, exponential generalized autoregressive conditional heteroscedasticity (EGARCH) and lag-augmented vector auto regression (LA-VAR) models were used. It was found that there is a positive relationship of consumer price index (CPI) and foreign direct investment (FDI) with stock market;however, exchange rate (ER) and T-bill rate (TBR) are inversely related to stock market volatility. On the other hand, they found strong evidence that there is a bilateral relationship of FDI and ER with stock prices, while a unidirectional relationship was found between TBR and stock market prices, with the direction from stock prices to treasury bills interest rate. However, a significant causal relationship was not found between CPI and stock prices. The analysis of this study reveals that the stock market of Pakistan is relatively less efficient as compared to US and other developed economies of the world.

Nakajima (2008) proposed the EGARCH model with jumps and heavy-tailed errors, and studied the empirical performance of different models including the stochastic volatility models with leverage, jumps and heavy-tailed errors for daily stock returns. In the framework of a Bayesian inference, the Markov chain Monte Carlo estimation methods for these models were illustrated using a simulation study. The model comparison based on the marginal likelihood estimation was carried out with data on the U.S. stock index. Based on the estimates of the marginal likelihood, the study found that the jumps and heavy-tails raise the marginal likelihood of the EGARCH model. The EGARCH model with jumps and heavy-tails and the SV model with heavy-tails and leverage fit to the data better than other competing models for our dataset.

Ou and Wang (2010) used a probabilistic method called the Relevance Vector Machine (RVM) to predict GARCH, EGARCH and GJR based volatilities of the Hang Seng Index (HSI) for two stage out-of-sample forecasts. The RVM is a powerful tool for prediction problems as it uses a Bayesian approach whose functional form is identical to a well-known Support Vector Machine (SVM).

Their goal was to compare the model with an SVM approach and classical GARCH, EGARCH and GJR models. The experimental results suggested that the proposed models can capture two different asymmetric effects of news impacts, and hence outperform the other models; particularly, the RVM based GJR generated a best ability for first stage forecast and the RVM based EGARCH was superior for the second stage forecast of HSI volatility, in terms of the evaluation metrics: RMSE, MSE, MAD, NMSE, and linear regression R squared.

Duan et al (2006) extended the analytical approach to pricing European options in the GARCH framework developed earlier in Duan, Gauthier and Simonato (1999). They extended the approximation to two other popular GARCH specifications namely the GJR-GARCH and EGARCH using the cumulative asset return as their data set. The study provided the corresponding formula and also examined their numerical performance. In each case, the resulting formula was the Black-Scholes formulae plus adjustment terms accounting for skewness and kurtosis. Also their results suggested that the approximations were adequate, particularly for shorter-maturity options. The results also revealed that their analytical approximation formula can be useful for a large-scale GARCH option pricing model where computation time can be a serious concern. Ramasamy and Munisamy (2012) compared three simulated exchange rates of Malaysian Ringgit with actual exchange rates using GARCH, GJR and EGARCH models. For testing the forecasting effectiveness of GARCH, GJR and EGARCH the daily exchange rates of four currencies - Australian Dollar, Singapore Dollar, Thailand Bhat and Philippine Peso - were used. The forecasted rates, using Gaussian random numbers, were compared with the actual exchange rates of year 2011 to estimate errors. Both the forecasted and actual rates were then plotted to observe the synchronisation and validation. The results showed more volatile exchange rates are predicted well by the GARCH models efficiently than the hard currency exchange rates which are less volatile. Among the three models the effective model was indeterminable as these models forecast the exchange rates in different number of iterations for different currencies. The leverage effect incorporated in GJR and EGARCH models did not improve the results much.

Shamiri and Hassan (2005) examined and estimated the three GARCH(1,1) models (GARCH, EGARCH and GJR-GARCH) using the daily price data of two Asian stock indices, Strait Times Index in Singapore (STI) and Kuala Luampur Composite Index in Malaysia (KLCI) over a 14-years period. The competing models GARCH, EGARCH and GJR-GARCH were developed based on three different distributions, Gaussian normal, Student-t, Generalized Error Distribution. The estimation results showed that the forecasting performance of asymmetric GARCH Models (GJR-GARCH and EGARCH), especially when fat-tailed asymmetric densities are taken into account in the conditional volatility, was better than symmetric GARCH. Moreover, it was found that the AR(1)-GJR model provided the best out-of-sample forecast for the Malaysian stock market, while AR(1)-EGARCH provide a better estimation for the Singaporean stock market.

Jiang (2011) examined the relationship between inflation and inflation uncertainty in China. He believed that it was worthy to investigate the inflation and inflation uncertainty relationship in China as it is commonly believed that one possible channel that inflation imposes significant economic costs is through its effect on inflation uncertainty. Jiang (2011) addressed the relationship of inflation and its uncertainty in China's urban and rural areas separately given the huge urban-rural gaps. The GARCH(1,1) and E-GARCH(1,1) models were used to generate the measure of inflation uncertainty and then Granger causality tests were performed to test for the causality between inflation and inflation uncertainty. GARCH (1, 1)-M models were also employed to further investigate the inflation-uncertainty nexus. The results provided strong statistical supportive evidence that higher inflation raises inflation uncertainty. On the other hand, the evidence on the effect of inflation uncertainty on inflation was mixed and depended on the sample period and areas examined.

Hassan et al (2006) explored the varying volatility dynamic of inflation rates in Malaysia for the period from August 1980 to December 2004. The generalized auto regressive conditional heteroscedasticity (GARCH) and the exponential generalized autoregressive conditional heteroscedasticity (EGARCH) models were used to capture the stochastic variation and asymmetries in the economic instruments. Also, an in-sample evaluation of the sub-periods volatility was done using both models. The results indicated that, the EGARCH model gave better estimates of sub-periods volatility as compared to the GARCH model. Berument et al (2001) used the EGARCH model to model inflation uncertainty in Turkey. Their study used the monthly CPI inflation covering the period from 1986 to 2000. Their study gave further contribution to literature due to the inclusion of seasonal terms in the conditional variance equation.

The results of the study provided evidence to show that in Turkey, the effect on inflation uncertainty of positive shocks to inflation are greater than that of negative shocks to inflation. Also, when monthly dummies were used in modelling both inflation and inflation uncertainty, the effect of lagged inflation on inflation uncertainty disappeared. They concluded that there is no significant lagged effect of inflation on inflation uncertainty. Lastly, there was evidence of significant seasonal effects of inflation on conditional variability. Alam and Rahman (2012) explored the application of GARCH type models such as GARCH; EGARCH; TARCH; and PARCH; to modelling the BDT/USD exchange rate using the daily foreign exchange rate series fixed up by Bangladesh Bank. The BDT/USD time series from July 03, 2006 to April 30, 2012 were used for the study purpose out of which in-sample and out-of-sample date set covered from July 03, 2006 to May 13, 2010 and May 14, 2010 to April 30, 2012 respectively. They benchmarked their results with AR and ARMA models.

They found that all GARCH type models demonstrated that past volatility of exchange rate significantly influenced current volatility. Both the AR and ARMA models were found as the best model as per in-sample statistical performance results, whereas according to out-of-sample, GARCH model was the best model with transaction costs and the TARCH model was nominated as the best model without transaction costs. The EGARCH and TARCH models outperform all the other models as per to in-sample and out-of-sample trading performance outcomes respectively including transaction costs.

CHAPTER 3

Methodology

3.1 Introduction

This chapter deals with the methodology for the study it looks briefly at time series and its basic concepts like stationary. It also considers detail description and explanation of the theory and concept of the ARCH-type models (i.e. ARCH and GARCH models) that would be used in the chapter four to analyse the data.

3.2 Time Series and its Basic Concepts

Chatfield (2004) defines time series as a series or sequence (x_t) of data points measured typically at successive times. The data points are commonly spaced equal in time. Time series analysis comprises methods that attempt to understand the underlying generation process of the data points and construct a mathematical formulae or model to represent the process. The constructed model is then used to forecast future events based on known past events. Time series often makes use of the natural one-way ordering of time so that values in a series for a given time will be expressed as have been derived from past values rather than future values. A time series model usually reflect the fact that observations close together in time domain are more correlated as compared to observations further apart. That is there is "volatility clusters"- small (large) shocks are again followed by small (large) shocks.

Original time series data are made up of various patterns that derived on casual factors which are identified by time series analysis methods. The four patterns that characterize economic and business series are the long-run development known as the trend, cyclical or periodic component, seasonal component and the error or residual component. The trend component deals with the general and overall pattern of the time series; the cyclical component refers to the variation in the series which arise out of the phenomenon of business cycles. It usually spans within periods of more than one year. The seasonal variations refers to the periodic and repetitive ups and downs in the series that occur within a year and lastly the error term is the component that contains all moments which neither belong to the trend nor to the cycle nor to the seasonal component.

The models for time series data can have many forms and represents different stochastic process which could be linear or non-linear. Among the linear models include autoregressive (AR) model of order (p), moving average (MA) of order (q) and autoregressive moving average (ARMA) model of order (p,q). A combination of the above models produce the autoregressive integrated moving average (ARIMA) model with a generalized model known as the autoregressive fractionally integrated moving average (ARFIMA) model.

The non-linear time series model represent or reflect the changes of variance along with time known as heteroscedasticity. With these models, changes in variability are related to and/or predicted by recent past values of the observed series. The wide variety of non-linear models include the symmetric models such autoregressive conditional Heteroscedastic (ARCH) model with order (p) and Generalized ARCH (GARCH) model with order (p,q). Other asymmetric models are the Power ARCH (PARCH), Threshold GARCH (TGARCH), Exponential GARCH (EGACRH), Integrated GARCH (IGARCH), etc. All these asymmetric models have order (p,q).The above mentioned non-linear models form part of a large family of the ARCH-type models. In this study, three of such models – ARCH, GARCH and EGARCH- would be fitted to the data set. The theory and concepts of these models are explained in detail in later sections of this chapter. Other forms of non-linear models include the bilinear model, threshold autoregressive (TAR), state-dependent model, markov switching models, etc.

3.3 Stationary and Non Stationary Processes

The foundation of time series analysis is stationarity. That is before time series analysis is carried out, one need to verify whether the series is stationary or otherwise. However, an assumption of stationarity is usually made. In this section, we define and describe stationarity (non-stationarity). A series is said to be stationary if the mean and auto covariances of the series do not depend on time. There are two forms of stationarity- strict stationarity and weak stationarity. Under strict stationarity, the common distribution function of the stochastic process does not change by a shift in time. That is a time series x_t is said to be strictly stationary if the joint distribution of $(x_1, ..., x_k)$ is identical to that of $(x_{(1+t)}, ..., x_{(k+t)})$ for all t, where k is an arbitrary positive integer and (1,...,k) is a collection of k positive integers. The shifting of the time origin by t has no effect on the joint distribution which depends only on the intervals between the two set of points given by t which is called a lag.

The concept of strict stationary is difficult to apply in practice and hence weak stationary or stationary in the second moment is often assumed. A time series x_t is weakly stationary if both the mean of x_t and the covariance between x_t and x_s are time-invariant. More specifically,x_t is weakly stationary if:

- E $(x_t) = \mu$, which is a constant, and
- cov $(x_t, x_s) = \gamma$ which is only a function of the time distance between the two random variables and does not depend on the actual point in time t.

3.4 Volatility

According to (investopedia, 2011) , volatility refers to the amount of uncertainly or risk about the size of changes in a security's value A higher volatility means that a security's value can potentially be spread out over a larger range of values. This means that the price of the security can change dramatically over a short time period in either direction. A lower volatility means that a security's value does not fluctuate dramatically, but changes in value at a steady pace over a specified period of time.

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2. $cov(x_t, x_s) = \gamma$ which is only a function of the time distance between the two random variables and does not depend on the actual point in time t.

3.4.1 Characteristics of a Financial Time Series

Financial time series analysis is concerned with the theory and practice of asset valuation over time. It is a highly empirical discipline, but like other scientific fields theory forms the foundation for making inference. There is, however, a key feature that distinguishes financial time series analysis from other time series analysis. Both financial theory and its empirical time series contain an element of uncertainty. For example, there are various definitions of asset volatility, and for a stock return series, the volatility is now directly observable. As a result of the added uncertainly, statistical theory and methods play an important role in financial time series analysis.

Campbell et.al, (1997) argued that it isboth logically inconsistent and statistically inefficient to use volatilitymeasures that are based on the assumption of constant volatility over someperiod when the resulting series moves through time "In the case of financialdata, for example, large and small errors tend to occur in clusters, i.e., largereturns are followed by more large returns, and small returns by more small returns. This suggests that returns are serially correlated.

3.5 Description of the GARCH Model

While conventional time series and econometric models operate under an assumption of constant variance, the Autoregressive Conditional

Heteroskedastic (ARCH) process introduced by Engle (1982) allows conditional variance to change over time as a function of past errors leaving the unconditional variance constant. This model has proved very useful in the modelling of several different economic phenomena (Bollersely, 1986)

The ARCH model had an arbitrary linear declining lag structure in the conditional variance equation to take account of the long memory typically found in empirical work, since estimating a totally free lag distribution often will lead to violation of the non-negativity constraints (Bollerslev, 1986)

The Generalized ARCH (GARCH), introduced and developed by Bollerslev (1986), allows for both a longer memory and a much more flexible lag structure. It is argued that a simple GARCH model provides a marginally better fit and more plausible learning mechanism that ARCH model with an eight -order linear declining lag structure as in Engle and Kraft (1983). The GARCH model explains variance by two distributed lags, one on past residuals to capture high frequency effects, and the second on lagged values of the variance itself to capture longer term influence.
Let ϵ_i denote a real-valued discrete-time stochastic process and λ_i the information set $(\sigma - field)$all information through time t.

$$\epsilon_i|\lambda_{t-1} \sim N(0, \sigma_t^2), \quad , \tag{3.1}$$

where

$$E(\epsilon_i|\lambda_{t-1}) = 0 \tag{3.2}$$

and

$$var(\epsilon_i|\lambda_{t-1}) = \sigma_t^2 \tag{3.3}$$

with

$$\sigma_t^2 = \omega + \sum_{i=1}^{\mu} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{v} \beta_j \sigma_{t-j}^2 \tag{3.4}$$

where $\mu > 0, v \geq 0, \omega > 0, \alpha \geq 0, \quad for$

$i = 1, ..., \mu \quad \beta_j \geq 0$, j = 1,..., v Then the stochastic process ϵ_t is a GARCH (u,v) process.

If v=0 then the process ϵ_t,reduces to the ACRH(u) process, and for $v = u = 0 \quad \{\epsilon_t\}$ is simple white noise.

In GACRH (u) process the conditional variance is specified as a linear function of past sample variance only, whereas the GARCH process allows lagged conditional variance to enter as well. For a strong GARCH " it is usually assumed that $\epsilon_t|\sqrt{\sigma^2} \sim iidN(0,1)$

GARCH has its simple form when v=u=1, i.e GARCH(1, 1) and this is the most popularly GARCH model. Where v=u=1

$$\sigma^2 = \omega + \sum_{i=1}^{\mu} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{v} \beta_j \sigma_{t-j}^2 \tag{3.5}$$

becomes

$$\sigma^2 = \omega + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{3.6}$$

where $\omega > 0, 0 \leq \alpha_1, \beta_1 \leq 1, \alpha_1 + \beta_1 < 1$

According to Figlewski (2004), the intuitive forecasting strategy of the GARCH (1,1) model is that the estimated volatility at a given date is a combination of the long run variance and the variance expected for last period, adjusted to incorporate the size of the last period's observed shock Figlevski, 2004) To forecast the volatility of a GARCH model, considering the GARCH (1,1) and assuming the forecast origin is, t, then

$$\alpha^2 = \omega + \alpha_1 \epsilon_{t-1}^2 + \beta_t \sigma_{t-1}^2 \tag{3.7}$$

For a 1-step ahead forecast we have

$$\sigma^2 = \omega + \alpha_1 \epsilon_{t-1}^2 + \beta_t \sigma_t^2 \tag{3.8}$$

Where σ_t^2 is the volatility at time index t.

3.6 ARCH (m) Model

An ARCH process is a mechanism that includes past variance in the explanation of future variances (Engle, 2004). The ARCH model was developed by Engle (1982) and it provides a systematic framework for volatility modelling. ARCH models specifically take the dependence of the conditional second moments in consideration when modelling.

Let x_t be the mean-corrected return, ϵ_t be the Gaussian white noise with zero mean and unit variance and I_t be the information set at time t given by $I_t = x_1, x_2, ..., x_(t-1)$. Then the ARCH (m) model is specified as:

$$x_t = \sigma_t \epsilon_t \tag{3.9}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 x_{t-1}^2 + ... + \alpha_m x_{t-m}^2 \tag{3.10}$$

where $\alpha_0 > 0, \alpha_i \geq 0, 1 = 1, ..., m$

and

$$E(x_t|I_t) = E[E(X_t|I_t)] = E[\sigma_t E(\epsilon_t)] = 0 \tag{3.11}$$

$$V(x_t|I_t) = E(x_t^2) = \sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i x_{t-1}^2 \tag{3.12}$$

and the error term ϵ_t is such that

$$E(\epsilon_t|I_t) = 0 \tag{3.13}$$

$$V(\epsilon_t|I_t) = 1 \tag{3.14}$$

From equations (3.13) and (3.14), it can be seen that the error term ϵ_t is a conditional standardised martingale difference. A stochastic series x_t is said to be a martingale difference if its expectation with respect to past values of another stochastic series y_i is zero.
That is

$$E(x_{(t+i)}|y_{(i,)}y_{(i-1,)}...) = 0, for, i = 1, 2, ... \tag{3.15}$$

From the structure of the model, it can be seen that the dependence of the present volatility x_t is a simple quadratic function of its lagged values. The coefficients α_i ,i=0,. . . ,m can consistently be estimated by regressing on x_t^2 on $x_(^2t-1)$, $x_(^2t-2)$, ..., $x_(^2t-m)$. To ensure that the conditional variance σ_t^2 is always positive for all t, it is required that $\alpha_0 > 0$ and $\alpha_i \geq 0$, i=1,...,m. From equations (3.13) and (3.14) that large past squared values $x_(^2t-i)$, i=1,...,m imply a large conditional variance σ_t^2 for the present volatility x_t. Consequently, x_t tends to assume a large value in absolute value. Hence under the ARCH framework, large shocks tend to be followed by another large shock. We would take a particular case of the ARCH (m) model where m =1, ARCH(1) to help understand the ARCH(m) better.

3.6.1 ARCH(1) Model

The ARCH (1) model is a special case of the general ARCH (m) model. Let x_t be the mean-corrected return, ϵ_t be the Gaussian white noise with zero mean and unit variance. If I_t is the information set available at time t given by

$I_t = x_1, x_2, ..., x_(t-1)$, then the process x_t is ARCH (1) where m = 1, if

$$x_t = \sigma_t \epsilon_t \tag{3.16}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 x_{t-1}^2 \tag{3.17}$$

where α_0 and α_1 are unknown parameters. The process x_t can be stated conditionally in terms of I_t similar to the variance σ_t^2 under the normality assumption of the error term ϵ_t. Again to ensure that the conditional variance is always positive, the constraints $\alpha_0 > 0$ and $\alpha_i \geq 0$, i=1,...,m is required. Since the ARCH (1) is a special case of ARCH (m), whatever applies to the ARCH (m) model also applies for the ARCH (1). Hence it can be concluded from equations (3.17) and (3.18) that a large past squared mean-corrected return $x_(^2 t-i)$, i=1,...,m implies a large conditional variance (σ_t^2), resulting in x_t being large in absolute value. For the ARCH (m) models to be valid, the presence of ARCH effects should be statistically significant and hence the presence of the ARCH effects should be tested for.

3.7 GARCH (m,s) Model

The Generalized ARCH (GARCH) model was developed by Bollerslev (1986) as an extension of the ARCH model in the same way the ARMA process is an extension of the AR process. The principle of parsimony may be violated when a model has a large number of parameters resulting in difficulties in using the model to adequately describe the data. In particular, although the ARCH model is simple, it may require many parameters as there might be a need for a large value of lag q and hence the principle of parsimony would be violated in such a case. An ARMA model may have fewer parameters compared to the AR model and similarly, a GARCH model may contain fewer parameters when compared to an ARCH model. Thus a GARCH model may be preferred to an ARCH model using the principle of parsimony.

Let $x_t = r_t - u_t$ be the mean corrected return, where r_t is the return of an asset, u_t is the conditional mean of x_t. Then the x_t follows a GARCH (m,s) model if

$$x_t = \sigma_t \epsilon_t \tag{3.18}$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i x_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{j-1}^2 \tag{3.19}$$

Where ϵ is a sequence of independent, identically distributed random variable with mean zero and unit variance and parameters of the model are α_i, i=0,...,m and β_i, j=1,...,s such that $\alpha_i \leq 0$ and $\beta_j \geq 0$; $\sum_{v}^{i=1}(\alpha_i + \beta_i) < 1$, where v =max(m,s) and $\alpha_i = 0$ for $i > m$ and $\beta_j = 0$ for $j > s$. The constraints on $\alpha_i + \beta_i$ implies that the unconditional variance of x_t is finite, whereas its conditional variance σ_t^2, evolves overtime. From equation (3.25) and (3.26), it is seen that the GARCH(m,s) model employs the same equation (3.16) for the mean corrected return x_t as in the ARCH (m) but the equation for the volatility includes s new terms. Therefore equations (3.25) and (3.26) reduces to a pure ARCH (m) model if s = 0. Thus the GARCH model generalizes the ARCH model by introducing values of $\sigma_{t-1}^2, \sigma_{t-2}^2, ...,$. The parameters α_i and β_j are respectively referred to as the ARCH and GARCH parameters.

The GARCH (m,s) model can be stated differently. Let $\eta_t = x_t^2 = \sigma_t^2$ so that $\sigma_t^2 = x_t^2 - \eta_t$, By substituting $\sigma_{t-i}^2 = x_{t-i}^2 - \eta_{t-i}$ (i=0,...,m) into equation (3.25) the GARCH (m,s) can be written as

$$x_t^2 = \alpha_0 + \sum_{i=1}^{v} (\alpha_i + \beta_i) x_{t-i}^2 + \eta_t - \sum_{j=1}^{s} \beta_j \eta_{t-j} \tag{3.20}$$

where v=max(m,s), $\alpha_i = 0$ for $i > m$ and $\beta_j = 0$ for $j > s$

Thus the equation of σ_t^2 has ARIMA(m,s) representation and it can be seen that η_t is a martingale difference series (i.e E(η_t)=0 and $cov(\eta_t, \eta_{t-1})$ for $j \leq 1$). However the η_t is not an independent, identically distributed random sequence. In order to find the GARCH(m,s) process, we solve for α_0 in equation (3.27) by letting the variance of x_t be σ_t^2. This yields

$$\alpha_0 = \sigma_t^2(1 - \sum_{i=1}^{m} \sigma_i - \sum_{j=1}^{s} \beta_j) \tag{3.21}$$

And substituting the value of α_0 as given by equation (3.28) into equation (3.27) gives

$$\begin{aligned} x_t^2 &= \sigma_t^2[1 - \textstyle\sum_{i,j=1}^{v}(\alpha_i + \beta_j)] + [\textstyle\sum_{i,j=1}^{v}(\alpha_i + \beta_j)]x_{t-i}^2 - \textstyle\sum_{j=i}^{s}\beta_j\eta_{t-j} + \eta_t \\ &= \sigma_t^2 + \sum_{i,j=1}^{v}(\alpha_i + \beta_j)(x_{t-i}^2 - \sigma_t^2) - \sum_{j=1}^{s}\beta_j\eta_{t-j} + \eta_t \end{aligned} \tag{3.22}$$

Therefore

$$x_t^2 - \sigma_t^2 = \sigma_{i,j=1}^{v}(\alpha_i + \beta_j)(x_{t-1}^2 - \sigma_t^2) - \sum_{j=1}^{s}\beta_j\eta_{t-j} + \eta_t \tag{3.23}$$

Multiplying both sides by $(x_{t-k}^2 - \sigma_t^2)$ results in

$$(x_{t-k}^2-\sigma_t^2)(x_t^2-\sigma_t^2) = \sum_{i,j=1}^{v}(\alpha_i+\beta_j)(x_{t-i}^2-\sigma_t^2)(x_{t-k}^2-\sigma_t^2)-\sum_{j=1}^{s}\beta_j\eta_{t-j}(x_{t-k}^2-\sigma_t^2)+\eta_t(x_{t-k}^2-\sigma_t^2) \tag{3.24}$$

And taking expectations, we have

$$E[(x_{t-k}^2-\sigma_t^2)(x_t^2-\sigma_t^2)] = E[\sum_{i,j=1}^{v}(\alpha_i+\beta_j)(x_{t-i}^2-\sigma_t^2)(x_{t-k}^2-\sigma_t^2)] - E[\sum_{j=1}^{s}\beta_j\eta_{t-j}(x_{t-k}^2-\sigma_t^2)] + E[\eta_t(x_{t-k}^2-\sigma_t^2)] \tag{3.25}$$

But $E[\eta_t(x_{t-k}^2 - \sigma_t^2)] = (x_{t-k}^2 - \sigma_t^2)E(\eta_t|x_t) = 0$ since η is a martingale difference and also

$$E[\beta_j\eta_{t-j}(x_{t-k}^2 - \sigma_t^2)] = E[(x_{t-k}^2 - \sigma_t^2)E(\eta_{t-j}|x_{t-j})] = 0, for, k < j$$

Thus the auto covariance of the squared returns for the GARCH (m,s) model is given by

$$\begin{aligned} cov(x_t^2, x_{t-k}^2) &= E[\textstyle\sum_{i,j=1}^{v}(\alpha_i + \beta_j)(x_{t-i}^2 - \sigma_t^2)x_{t-k}^2 - \sigma_t^2] \\ &= \sum_{i,j=1}^{v}(\alpha_i + \beta_j)cov(x_t^2, x_{t-k+i}^2) \end{aligned} \tag{3.26}$$

Dividing both sides by σ_t^2 gives the autocorrelation function at lag k as

$$\rho_k = \sum_{i,j=1}^{v} (\alpha_i + \beta_j)\rho_{k-i}, for, k \leq (m+1) \tag{3.27}$$

This result is analogous to the Yuler-Walker equations for an AR process. Hence the autocorrelation function (ACF) and the partial autocorrelation function (PACF) of the squared returns in a GARCH process has the same pattern as those of an ARMA process. The ACF and PACF are useful in determining the orders m and s of the GARCH (m,s) process. Also the ACF is used in checking model accuracy; in which case, the ACF's of the residuals indicates the presence of a white noise if the model is adequate.

The parameters $\alpha_0, \alpha_1, ..., \alpha_m; \beta_1, \beta_2, ..., \beta_s$ affects the autocorrelation but given the $\rho_k, ..., \rho_{m+1} - v$, the autocorrelation at higher lags are determine uniquely by the expression in equation (3.32). (Bollerslev,1986) as cited in Ngailo (2011). Denoting the $v^t h$ partial autocorrelation for x_t^2 by Φ_{vv}

$$\rho_k = \sum_{i,j=1}^{v} \Phi_{vv}\rho_{k-i}, k = 1, ..., v = max(m, s) \tag{3.28}$$

It can be seen from equation (3.32) that cuts off after lag m for an ARCH (m) process such that$\Phi_{vv} \neq 0$ for $k \leq m$ and $\Phi_{vv} = 0$ for $k > m$ and its similar to the AR (m) process and decays exponentially (Bollerslev, 1986). To understand the theory and concepts of the GARCH model, we would focus on the special case of the GARCH (1, 1) model.

3.7.1 GARCH (1, 1) Model

The GARCH (1,1) model is a particular case of the GARCH (1,1) model where the orders m and s are both equal to one (i.e. m = s = 1). Let x_t be the mean corrected return, ϵ_t be a Gaussian white noise with mean zero and unit variance. If I_t is the information set available at time t given by $I_t = \{x_1, x_2, ..., x_{t-1}; \sigma_1^2, \sigma_2^1, ..., \sigma_{t-1}^2\}$. Then the process x_t follows a GARCH(1,1) model if

$$x_t = \sigma_t \epsilon_t \tag{3.29}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 x_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{3.30}$$

Where α_0, α_1 and β_1 are the parameters of the model such that $\alpha_0 \leq 0, \alpha_1 \leq 0, \beta_1 \leq 0$ and $(\alpha_i + \beta_i < 1)$ The constraints on the parameters are to ensure that the conditional variance σ_t^2 is positive. Clearly from (3.36) and (3.37) shows that large past mean corrected return σ_t^2 or past conditional variance σ_{t-1}^2 give rise to large values of σ_t^2 (Tsay,2002). It can be seen that x_t is martingale difference as the conditional mean is zero (i.e. $E(x_t|I) = 0$).
Taking $\eta_t = x_t^2 - \sigma_t^t$ so that $\sigma_t^2 = x_t^2 - \eta_t$, the GARCH(1,1) can be presented differently. By substituting $\sigma_{t-1}^2 = x_{t-1}^2 - \eta_t$ into equation (3.37), the GARCH(1,1) can be written as

$$x_t^2 = \alpha_0 + (\alpha_1 + \beta_1)x_{t-1}^2 + \eta_t - \beta_1 \eta_{t-1}$$

$$x_t^2 = \alpha_0 + \alpha_1 x_{t-1}^2 + \beta_1 (x_{t-1}^2 - \eta_{t-1}) + \eta_t \tag{3.31}$$

Again it can be seen that η is a martingale difference series as $\mathrm{E}(\eta_t|I_t) = 0$ (i.e. $\mathrm{E}(\eta_t)$=0 and $\mathrm{cov}(\eta_t, \eta_{t-j})$ for j ≥ 1) and η_t is an uncorrelated sequence. This implies from equation (3.38) that

$$E(x_t^2) = \sigma_t^2 = \alpha_0 + (\alpha_1 + \beta_1)E(x_{t-1}^2)$$

$$\Longrightarrow \sigma_t^2 = \alpha_0 + (\alpha_1 + \beta_1)E(\sigma_t^2 \epsilon_t^2)$$

$$\Longrightarrow \sigma_t^2 = \alpha_0 + (\alpha_1 + \beta_1)\sigma_t^2 E(\epsilon_t^2)$$

$$\Longrightarrow \sigma_t^2 = \alpha_0 + (\alpha_1 + \beta_1)\sigma_t^2, Since, E(\epsilon_t^2) = Var(\epsilon_t^2) = 1$$

$$\Longrightarrow \alpha_0 = (1 - \alpha_1 - \beta_1)\sigma_t^t$$

$$\Longrightarrow \sigma_t^2 = \frac{\alpha_0}{(1 - (\alpha_1 + \beta_1))}, \tag{3.32}$$

provided $|\alpha_1 + \beta_1| < 1$

3.7.2 Estimation of the GARCH (1,1) Model

In estimating the GARCH F(1, 1) model, we consider the maximum likelihood estimation often parameters as shown by Bollerslev. The joint probability density function often stochastic observations, $\epsilon_t = t = 1, ..., N$ can be written as the product of the conditional densities conditioning on the previous observations.

$$F_{\epsilon_1,\epsilon_2,...,\epsilon_N}(\epsilon_1, \epsilon_2, ..., \epsilon_N) = \{\prod_{t=2}^{N} f_{\epsilon_1|\epsilon_1,\epsilon_2,...,\epsilon_{N-1}}(\epsilon_1|\epsilon_1, \epsilon_2, ..., \epsilon_{t-1})\} f_{\epsilon_1}(\epsilon_1) \quad (3.33)$$

On the conditional normal assumption, the conditional density of $\epsilon_m, m = 2, ..., N$ conditioning on $\epsilon_1, ..., \epsilon_{m-1}$ is given by

$$\prod_{t=2}^{N} f_{\epsilon_1|\epsilon_1,\epsilon_2,...,\epsilon_{N-1}}(\epsilon_1|\epsilon_1, \epsilon_2, ..., \epsilon_{t-1}) = \frac{1}{\sqrt{2\pi\sigma_m^2}} exp - \{\frac{\epsilon_m^2}{2\sigma_m^2}\} \quad (3.34)$$

$$\sigma^2_{\epsilon_m|\epsilon_1,\epsilon_2,...,\epsilon_(m-1)} = 1 \quad (3.35)$$

and $\mu = 0$ and the conditional likelihood function, given ϵ_t σ_1^2 is

$$L(\omega, \alpha_1, \beta_1) = f_{s_2,...,s_m|s_1\sigma_1^2}(\epsilon_2, ..., \epsilon_m|\epsilon_1, \sigma_1^2) = \prod_{i=2}^{N} \frac{1}{\sqrt{2\pi\sigma_m^2}} exp - \{\frac{\epsilon_m^2}{2\sigma_m^2}\} \quad (3.36)$$

Where $\sigma_1^2 = \omega + \alpha_1\epsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2$

3.7.3 Forecasting with GARCH (m,s) Model

Forecasting of a GARCH model can be obtained using methods similar to those of an ARMA model. Thus the conditional variance of x_t is obtained by taking the conditional expectation of the squared mean corrected returns. Consider the GARCH (m,s) model as stated in equations (3.36) and (3.37). Assuming a forecasting origin of t, then the ς-step ahead volatility forecast is given by

$$x_t^2(\varsigma) = E(x_{t+\varsigma}^2|x_t)$$

$$x_t^2(\varsigma) = \alpha_0 + \sum_{i=1}^{m}(\alpha_i + \beta_i)E(x_{t+\varsigma-i}^2|x_t) + \eta_t - \sum_{j=1}^{s}\beta_j E(\eta_{t+\varsigma}|x_t)$$

$$x_t^2(\varsigma) = \sigma_t^2(\varsigma) \quad (3.37)$$

Where $x_t^2, ..., x_{t+1-m}^2; \sigma_t^2, ..., \sigma_{t+1-s}^2$ are assumed known at time t and the true parameters $\alpha_i; (i = 1, ..., m)$ and $\beta_j; (j = 1, ..., s)$ values are replaced by their estimates.

Considering the special case of GARCH (1, 1) model in equations (3.36) and (3.37) and assuming that the forecast origin of t, the 1-step ahead volatility forecast is given by

$$\sigma_t^2(1) = x_t^2 = \alpha_0 + \alpha_1 x_2^t + \beta_1\sigma_t^2 \quad (3.38)$$

where x_t and σ_t^2 are known at the time index t.

For a multi-step ahead forecast, we use $x_t^2 = \sigma_t^2\epsilon_t^2$ and rewrite the volatility equation in equation(3.36) as

$$\sigma_{t+1}^2 = \alpha_0 + (\alpha_1 + \beta_1)\sigma_t^2 + \alpha_1\sigma_t^2(\epsilon_t^2 - 1) \quad (3.39)$$

Where t=h+1, the above equation becomes

$$\sigma_{h+2}^2 = \alpha_0 + (\alpha_1 + \beta_1)\sigma_{h+1}^2 + \alpha_1\sigma_{h+1}(\epsilon_{h+1}^2 - 1) \quad (3.40)$$

A 2-step ahead volatility forecast at the forecast origin t is given as

$$\sigma_t^2(2) = x_t^2(2) = \alpha_0 + (\alpha_1 + \beta_1)\sigma_t^2(1) \quad (3.41)$$

and in general, the ς-step ahead forecast is given as

$$\sigma_t^2(\varsigma) = x_t^2(\varsigma) = \alpha_0 + (\alpha_1 + \beta_1)\sigma_t^2(\varsigma - 1), \varsigma > 1 \quad (3.42)$$

This result is exactly the same as that of an ARMA (1, 1) model with AR polynomial 1-$(\alpha_1 + \beta_1)$. By repeated substitutions in equation (3.37), the ς-

step ahead volatility forecast can be written as

$$\sigma_t^2(\varsigma) = \frac{\alpha_0[1-(\alpha_1+\beta_1)^{\varsigma-1}]}{1-(\alpha_1+\beta_1)} + (\alpha_1+\beta_1)^{\varsigma-1}\sigma_t^2(1) \tag{3.43}$$

Therefore

$$\sigma_t^2(\varsigma) \longrightarrow \frac{\alpha_0}{(1-(\alpha_1+\beta_1))}, as, \varsigma \longrightarrow \infty$$

Provided that $(\alpha_1 + \beta_1) < 1$

Consequently, the multi-step ahead volatility forecast of a GARCH (1, 1) model converges to the unconditional variance of x_t, as the forecast horizon increase to infinity provided that the variance of x_t (σ_t^2) exists (Tsay,2002). Despite the added advantage that the GARCH model brought to the ARCH – type models, the GARCH model had the same weakness as the ARCH model. It also assumes that the return volatilities (conditional variance) respond equally to positive and negative shocks. That is the GARCH model is a symmetric model and does not capture the asymmetry effect that is inherent in most real life financial data (Frimpong and Oteng - Abayie, 2006). To circumvent this problem of asymmetric effects on the conditional variance, Nelson (1991) extended the ARCH framework by proposing the Exponential GARCH (EGARCH) model.

3.8 Model Selection Criteria

The ACF and PACF assist in determining the order of the model but this is just a suggestion of where the model can be built from an d it is imperative to build the model around the suggested model order (Aidoo,2010). Several models with different order can be considered and the ultimate (most suitable) model be selected from the family of candidate models that characterize the ordering data. The information criteria have been widely used in time series analysis to determine the appropriate order of a model. The idea behind the information criteria is to provide a measure of information in terms of the order of the model, which strikes a balance between the measure of goodness of fit and parsimonious specification of the model. The information criteria make use of the Kullback-Leibler effect in determining the suitable model. The Kullback-Leibler quantity of information contained in a model is the distance from the 'true' model and is measured by the log likelihood function (Aidoo, 2010). Several selection criteria have been proposed to aid in selecting the most appropriate model. Among others, we have the Akaike Information criteria (AIC) by Akaike (1974), Bayesian Information criterion (BIC) by Schwartz (1974), Hannan-Quinn (HQ) by Hannan and Quinn (1979), the coefficient of determination (R^2), etc.

The several competing models are ranked according to their AIC, BIC or HQ values with the model having the lowest information criterion value being the best. If two or more competing models have the same or similar AIC, BIC or HQ values, then the principle of parsimony is applied to select the most appropriate model. The principle of parsimony states that a model with fewer parameters is usually better than a complex model. Alternatively to the use of the principle of parsimony, forecast accuracy test between the competing models can be used (Aidoo, 2010). In general, the model selected as the most appropriate model by two different criteria may differ and thus it should be noted that the selection of an ARCH-type model depends on the selection criteria used (Talke, 2003).

3.8.1 Akaike Information Criterion (AIC)

The Akaike Information Criterion (AIC) was introduced by Hirotogu Akaike in 1973. It was the first model selection criterion to gain widespread acceptance. The AIC was an extension to the maximum principle and consequently the maximum likelihood principle is applied to estimate the parameters of the model once the structure of the model has been specified. The AIC is defined as

$$AIC = 2(N) - 2(loglikelihood) \tag{3.44}$$

where N denotes the number of parameters in the model. Given a family of competing models of various structures, the maximum likelihood estimation is used to fit the model and the AIC is computed based on each model fit. The selection of the most appropriate model is then made by considering the model with the minimum AIC. Akaike's idea was to combine estimation and structural determination into a single procedure. The first term of the AIC in equation (3.29) measures the goodness of fit of the model whereas the second term is called the penalty function of the criterion since it penalizes a candidate model by the number of parameters used.

The advantages of the AIC is that it is useful for both in-sample and out-of-sample forecasting performance of a model. In-sample forecasting indicates how the chosen model fits the data in a given sample while out-of-sample forecasting is concerned with determining how a fitted model forecast future values of the regressed given the values of the regressors. Secondly, the AIC is useful for both nested and non-nested models.

Despite the advantages of the AIC such as mentioned above, the AIC has been criticised because of its inconsistency and tendency to over-fit a model. This inconsistency was shown by Shibatal (1976) for autoregressive models (AR) and Hannan (1982) for ARMA models as cited in Shittu and Asemota, 2009. To overcome this problem especially that of inconsistency, Schwartz (1978) proposed the Bayesian Information Criterion.

3.8.2 Bayesian Information Criterion

The Bayesian Information criterion (BIC) is related to the Bayes factor and is useful for selecting the most appropriate model out of a candidate of families of models. The BIC is obtained by replacing the non-negative factor 2(N) in equation (3.29) by kln(n). Hence, the BIC is defined as

$$BIC = kln(n) - 2(loglikelihood) \tag{3.45}$$

Where k denotes the number of parameters in the model, n is the length of the time series or the sample size. Again, the maximum likelihood estimation is used to fit the model and the BIC is computed for each of the models in a family of competing models and the fitted model with the minimum BIC is considered to be most appropriate model. Comparing equations (3.29) and (3.30), it is can be seen that the BIC imposes a harsher penalty than AIC especially for models with many parameters (i.e. complex models).

The advantages of the Bayesian information criterion is that for a wide range of statistical problems, it is order consistent (i.e. when the sample size goes to infinity, the probability of choosing the right modal converges to unity) leading to more parsimonious model. Also, like the AIC, the BIC can be used to compare in-sample or out-of-sample forecasting performance of a model.

3.9 Model Diagnostic Checks and Adequacy

The model diagnostic checks are performed to determine the adequacy or goodness of fit of a chosen model. The model diagnostic checks are performed on residuals and more specifically on the standardized residuals (Talke, 2003). The residuals are assumed to be independently and identically distributed following a normal distribution (Tsay, 2002). Plots of the residuals such as the histogram, the normal probability plot and the time plot of residuals can be used.

If the model fits the data well the histogram of residuals should be approximately symmetric. The normal probability plot should be a straight line while the time plot should exhibit random variation. The ACF and the PACF of the standardized residuals are used for checking the adequacy of the conditional variance model. The Lagrange multiplier and the Ljung Box Q-test are used to check the validity of the ARCH effects as well as test for autocorrelation in the data. To test the presence of ARCH effects, the null hypothesis of no ARCH effects is rejected if the probability value (p-value) is less than specified level of significance.

In case of testing for the presence of autocorrelation, the null hypothesis of no autocorrelation is rejected if the Ljung –Box (Q) statistics of some of the lags are significant. Thus if the probability value of Ljung –Box (Q) statistics of some of the lags are less than the specified level of significance, then the null hypothesis of no autocorrelation is rejected. Once the estimated model satisfies all these model assumptions, it can be seen as an appropriate representation of the data. Having established that the model fits the data well, the model can then be used to compute forecasts of the series under consideration.

3.10 Sustainability Ratio

The sustainability ratio of the fund, S, which is a ratio of the investment income,I_{income} and the total expenditure, E_{total}, measures how sustainable the Fund is. thus,

$$S_T = \frac{I_{income}}{E_{total}}$$

Where $E_{total} \neq 0$ and $E_{total} = AO_{expenses} + B_{gold}$ where $AO_{expenses}$ and

B_{gold} represents administrative and operational expenses and benefits paid respectively.

How great S, is determines how sustainable and impressive the Fund has performed.

CHAPTER 4

Analysis and Findings

4.1 Summary Statistics and Data Description

Descriptive statistics associated with the fund size of the SSNIT pension fund and statistics for the test procedures are presented in Table 5 below. There is no evidence of fat tails, since the Kurtosis is less than 3, the normal value, and the evidence of positive skewness, which means that the data is tailed to the right.

Table 4.1: Descriptive of the SSNIT Fund level from the year 2002 to 2010

STATISTICS	VALUES
No: of observation	120
Mean	128.455
Variance	4730.800
Minimum	21.640
Maximum	259.200
Skewness	0.127
Kurtosis	1.642

According to the values 5, the 120 number of observation for the period under review had its mean fund level to be 128.455 (GHC'm) with 21.640 (GHC'm) being the minimum level and 259.200 (GHC'm) as the maximum level. The unconditional sample skewness measure is close to zero (0.127), as is the case with normal distribution (Elyasiani, Mansur, 1998).

A time series plot of the data and also help check if there were outliers in the data. From the figure 4.1, it can be observed that the first two years and the early part of the third year had an upward trend along the same gradient.

The sharpness of the gradient dropped at the early part of the third year to the end of the same year and dropped further in the fifth year of the period under study. The period afterwards recorded an increase in the steepness of the gradient for two years and dropped again till the later part of the period under review where there was sudden increase. On the whole, there was an upward trend prevailing in the data although there were times when the fund level for proceeding periods fell below its level for earlier period.

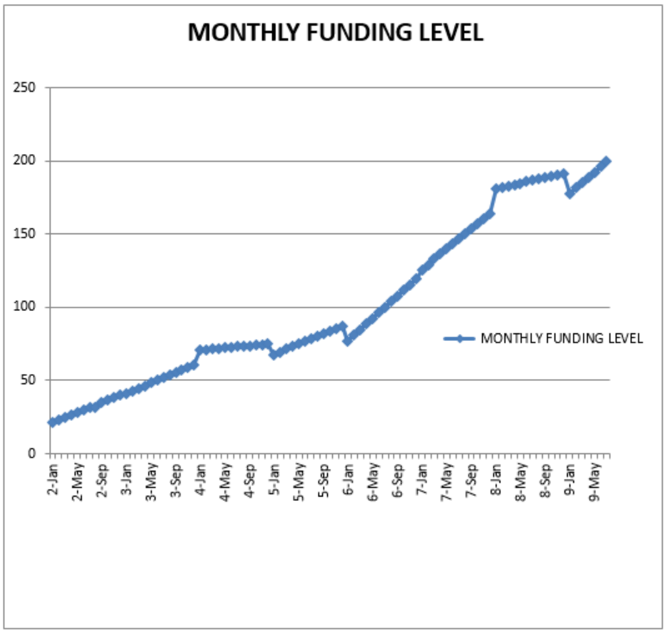

Figure 4.1: Time series plot of the SSNIT pension fund level from 2002 to 2010

Table 4.2: Box-Pierce-Ljung Statistic at a lag of n.

Ljung-Box-Pierce	Degrees of Freedom	Critical Value	Q Statistics	p Value	Hypothesis
Q(w)	10	26.2	106.95	0	1
Q(24)	22	43.0	157.78	0	1
Q(36)	34	58.6	177.12	0	1

The Box - pierce - Ljung statistics text conducted on the data lags of 12^o, 24^o and 36^o at, 22 and 34 degrees of freedom respectively gave critical values of 26.22, 43.0 and 58.6 respectively,details on this is summarized in table 6.

The ACF of the SSNIT fund level as displayed in figure 2 was very high and decrease gradually, an indication of tailing off to zero. This shows the plot is non-stationary but the PACF plot as shown in figure 3 remained constant from zero till the fifteenth lag where it decrease and then rose sharply to its highest level. The level decreased again till the twentieth lag where it rose gently.

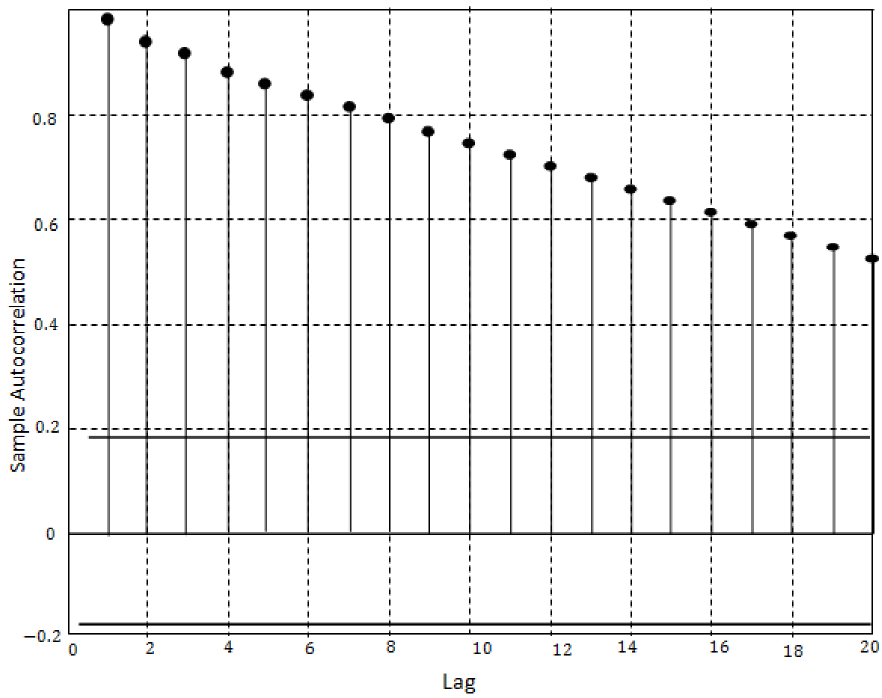

Figure 4.2: An ACF plot of the SSNIT Pensions Fund
Sample Autocorrelation Function (ACF) of the SSNIT PENSIONS FUND

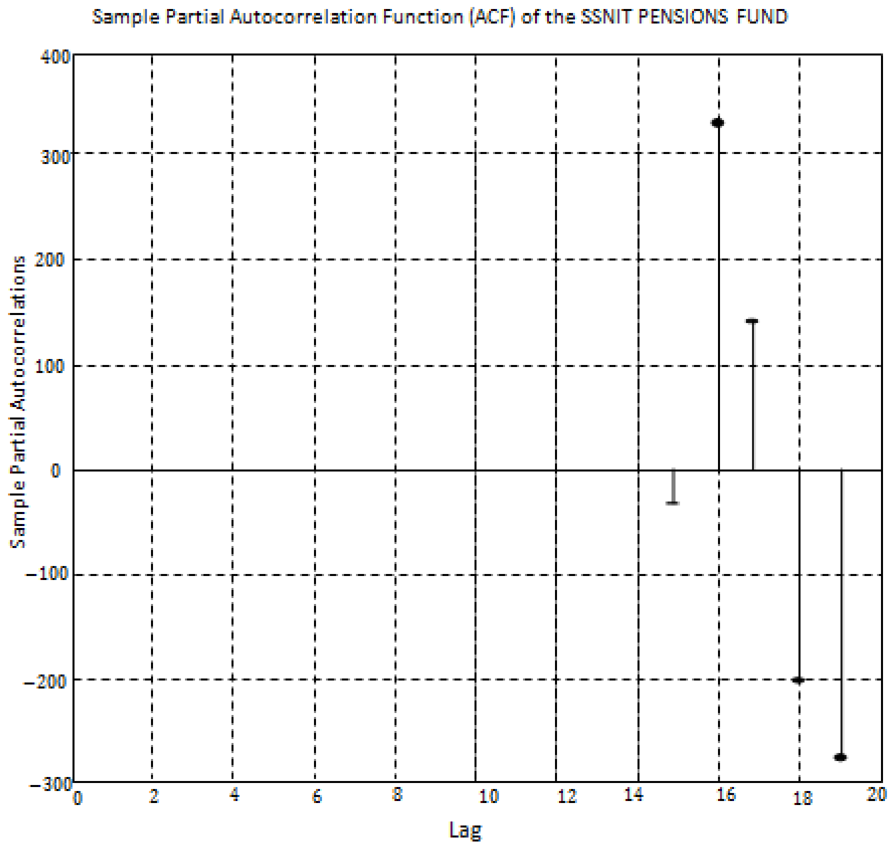

Figure 4.3: PACF plot of the SSNIT Pensions Fund
Sample partial Autocorrelation Function of the SSNIT PENSIONS FUND

4.2 Estimation of the GARCH model parameters and the fitting of the GARCH model

The parameters of the GARCH model could be estimated by the garch fit command in matlab and table 4.3 gives the output for this command on the data. From the output, we can fit the model for volatility from

$$\sigma_t^2 = \omega + \alpha 1 \varepsilon_t^2 - 1 + \beta_1 \sigma_{t-1}^2 \tag{4.1}$$

as

$$\sigma_t^2 = 0.5302 + 0(\sigma_{t-1}^2) + 1(\varepsilon_{t-1}^2) \tag{4.2}$$

Which simplifies as

$$\sigma_t^2 = 0.5302 + \varepsilon_{t-1}^2 \tag{4.3}$$

Since the value of $\beta_1 = 0$ and $\alpha_1 = 1$, it means the volatility at time t, σ_t^2 , is not correlation with the immediate past volatility or the lagged condition variance , σ_{t-1}^2, it is highly correlated with the immediate past squared residual or the lagged of the squared residual, ε_{t-1}^2. Again, in the absence of immediate past volatility or the lagged conditional variance, σ_{t-1}^2 and immediate past squared residual or the lagged squared residual, ε_{t-1}^2, the expected volatility at time t, σ_t^2, is 0.5302.

The above submission means, the volatility of the SSNIT pensions fund at anytime is not dependent on the immediate past volatility but it is strongly influenced by the lagged of the squared residual and when there is no history on the volatility of the pension fund, the expected volatility at any time is 0.53052.

The fund level. Y_t, can also be estimated by the relation $Y_t = c + \varepsilon_t$

where c is a constant estimated from the empirical data. From the estimated in Table 7, c is 72.264 so the fund level can be modeled as:

$$Y_t = 72.264 + \varepsilon_t$$

but,

$$\varepsilon_t = \sqrt{\sigma^2_{t+1} - 0.53052}$$

So,

$$Y_t = 72.264 + \sqrt{\sigma^2_{t+1} - 0.53052}$$

This means in the absence of any information with regards to the SSNIT pension fund, the level of the fund is 72.264.

Table 4.3: Estimates of the parameters of the GARCH Model

Parameter	Value	Std. Error	T-Stats
C	72.264	0.35511	203.5001
ω	0.53052	0.52812	1.0045
GARCH (1) or β_1	0	0.60273	0.0000
ARCH (1) or α_1	1	0.52382	1.9090

4.3 Prediction

4.3.1 Prediction beyond data

A thirty - six period forecast on the fund level beyond the data was conducted and the results are recorded in Table 8. From Table 8 and Table 9, there is an upward trend in the monthly and yearly funding level. Figure 5 shows the graphical display of the prediction.

Table 4.4: A 36 - monthly forecast of the SSNIT PENSION Fund level beyond the data

TIME IN MONTHS	PREDICTION	TIME IN MONTHS	PREDICTION
Jan - 11	262.07	Jul - 12	305.65
Feb - 11	264.83	Aug - 12	307.92
Mar - 11	267.52	Sep - 12	310.18
Apr - 11	270.14	Oct - 12	312.44
May - 11	272.70	Nov - 12	314.70
Jun - 11	275.21	Dec - 12	316.95
Jul - 11	277.68	Jan - 13	319.21
Aug - 11	280.11	Feb - 13	321.46
Sep - 11	282.51	Mar - 13	323.71
Oct - 11	284.89	Apr - 13	325.96
Nov - 11	287.25	May - 13	328.21
Dec - 11	289.59	Jun - 13	330.45
Jan - 12	291.91	Jul - 13	332.70
Feb - 12	294.22	Aug - 13	334.95
Mar - 12	296.52	Sep - 13	337.19
Apr - 12	298.82	Oct - 13	339.44
May - 12	301.10	Nov - 13	341.68
Jun - 12	303.38	Dec - 13	343.92

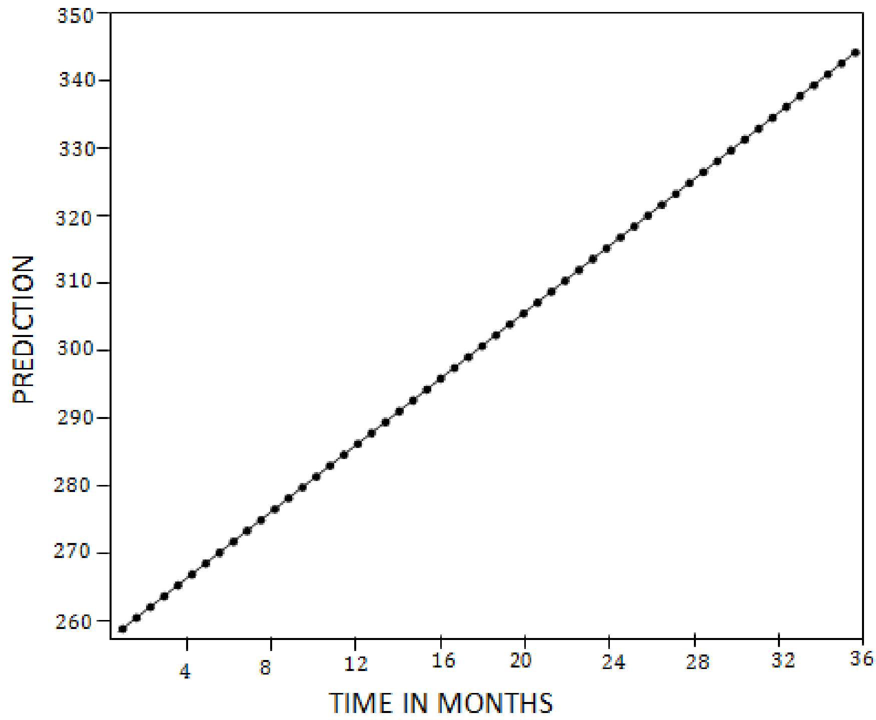

Figure 4.4: Time Series Plot of Prediction

Table 4.5: Yearly forecast of the SSNIT PENTION Fund Level beyond the data

TIME IN YEARS	PREDICTED FUND SIZE (IN GHANA CEDIS)
2011	3045.31
2012	3360.47
2013	3675.64
2014	3990.8
2015	4305.96
2016	4621.12
2017	4936.28
2018	5251.45
2019	5566.61
2020	5881.77
2021	6196.93
2022	6512.09
2023	6827.26
2024	7142.42
2025	7457.58
2026	8087.9
2028	8403.07
2029	8718.23
2030	9033.39

4.3.2 Prediction within data

A twelve - period forecast on the fund level within the data was conducted and the results are recorded in Table 10 below. From Table 10, there is an upward trend in the predicted funding level. In comparing the predicted level to the actual level, the predicted values were closer to the actual values at the earlier period of the prediction. The yearly predictions for the data are also shown in table 11 and figure 6. The data shows an upward trend in data.

Table 4.6: A twelve - period forecast of the SNNIT PENSION Fund Level within the data

TIME IN MONTHS	ACTUAL	PREDICTION
Jan -10	219.2	220.52
Feb - 10	222.83	223.05
Mar - 10	226.47	225.48
Apr - 10	230.11	227.86
May - 10	233.75	230.18
Jun - 10	237.38	232.45
Jul - 10	241.02	234.69
Aug - 10	244.66	236.9
Sep - 10	248.66	239.08
Oct - 10	251.23	241.26
Nov - 10	255.57	243.41
Dec - 10	259.2	245.55

Table 4.7: Yearly prediction of the SSNIT PENSION Fund level within the data

YEAR	ACTUAL VALUE	PREDICTION VALUE
2002	370.02	369.01
2003	621.5	603.24
2004	875.1	895.52
2005	929.2	1182.95
2006	1182	1356.06
2007	1737.7	1620.55
2008	2234.6	2049.35
2009	2374.8	2464.6
2010	2900.6	2696.35

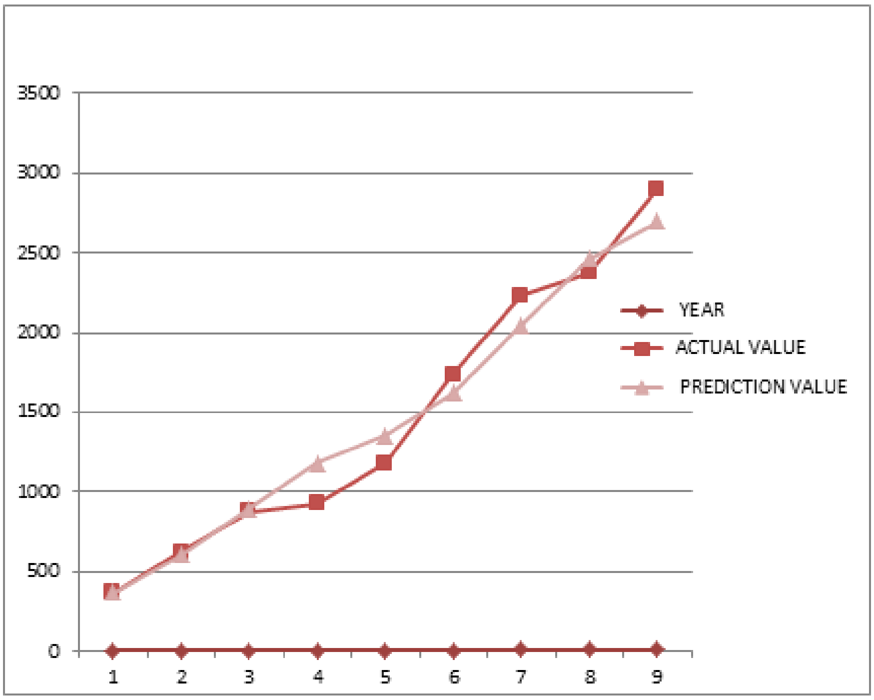

Figure 4.5: A yearly forecast of the SSNIT PENSION Fund Level within the dat

4.4 Diagnostic Analysis

The procedure includes observing residual plot and its ACF and PACF diagram. If the ACF and PACF of the model residuals show no significant lags, the selected model is appropriate. The residual plot displays cluster of volatility so the ARCH/GARCH should be used to model the volatility of the series to reflect more recent changes and fluctuations.

Finally the ACT and PACF of squared residuals will help confirm if the residuals(noise term) are not independent and can be predicted. White noise cannot be predicted either linearly or nonlinearly. If the residuals are strict white noise, they are independent with zero mean, normally distributed and ACF & PACF of squared residuals displays no significant lags. The diagrams are shown below

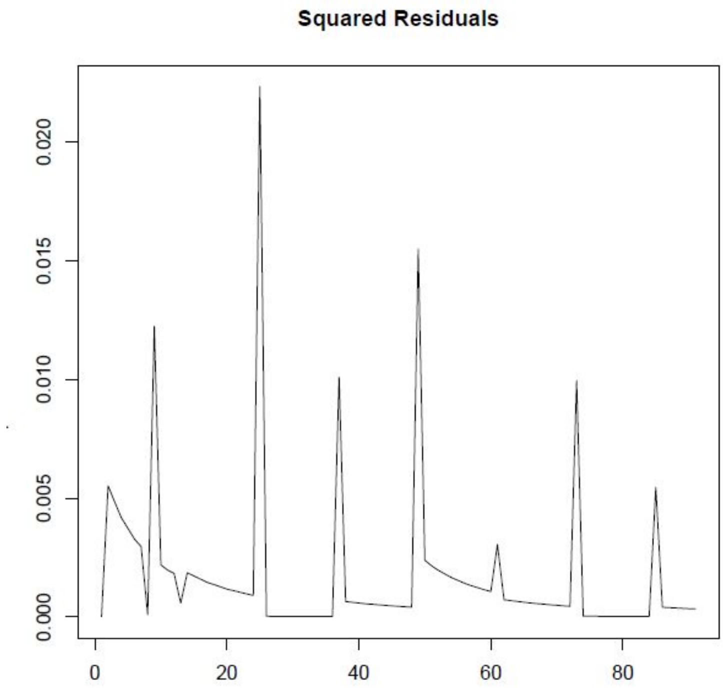

Figure 4.6: Plot of Squared Residuals

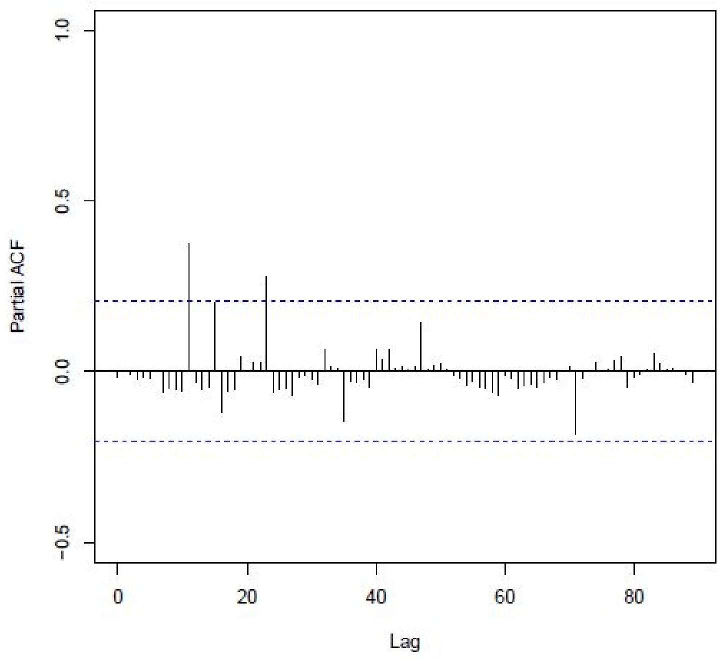

Figure 4.7: PACF of Squared Residuals

The squared residual plot shows cluster of volatility at some point in time. the residuals therefore shows patterns that might be modeled. The PACF cuts off after some lags even though some lags are significant. The final check on the model is to look at Q-Q plot of the residuals of the ARIMA-ARCH model, which is Residual/sqrt(Conditional variance). we compute directly from R and the Q-Q graph to check the normality of the residuals. below are the Q-Q plots

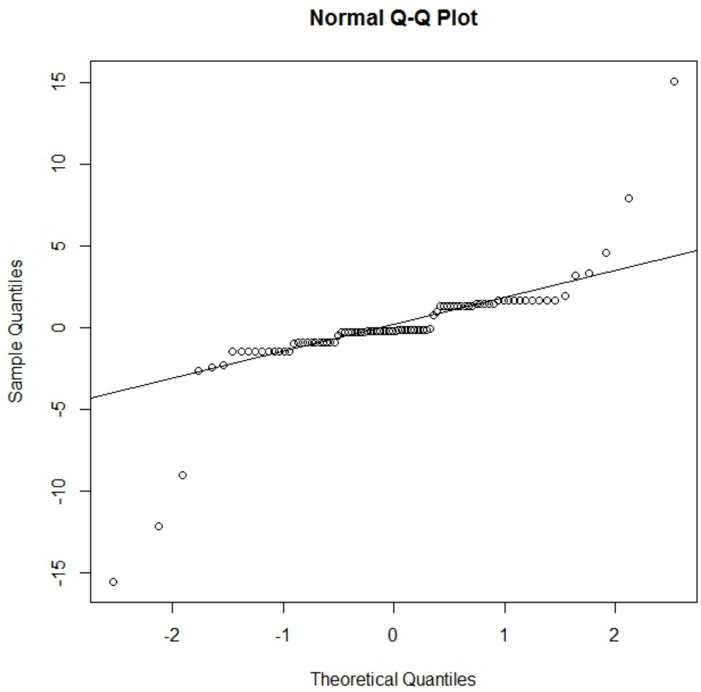

Figure 4.8: ARIMA-ARCH Residuals

The plots shows that residuals seem to be roughly normally distributed although some points remain of the line. Since model residuals are normally distributed hence residuals are independent and can be predicted

CHAPTER 5

Conclusion and Recommendations

5.1 Introduction

This chapter discusses the summary of results of the data analysis, observations, conclusions and recommendations.

5.2 Summary

In all 120 data records were analyzed on the SSNIT Pension Fund level from the year 2002 to 2010 with the mean and variance begging constant. The fund level recorded an upward trend, an indication of how well the fund is doing.
The Generalised Autoregressive Conditional Heteroskedastic GARCH (1, 1) was the model used for the analysis and the fitting of the model since the data on the SSNIT Funding level was a financial time series data that exhibit some level of volatilities. The fitted GARCH (1, 1) for the data is:

$$\sigma_t^2 = 0.53052 + \varepsilon_{t-1}^2$$

and

$$Y_t = 72.264 + \sqrt{\sigma_{t+1}^2 - 0.53052}$$

5.3 Findings of the Study

5.3.1 Inflow on the SSNIT Pension Fund Levels

It was observed during the investigation of this research that, the major inflows of the funds to the SSNIT Pension fund were the contribution from contributors and the returns from the various investment packages t he trust engages in. A five-year trend analysis of contributions collected for the period (2006-2010) shows that, the percentage change of contribution has been positive from the year 2006-2009 (indicating an increase in contribution size) with the year 2008-2009 recording the highest percentage change of 59.43%. The percentage change year 2009-2010 was -13.60% an indication of a decrease in contribution size; this may be as the results of the implementation of the new Pensions Act, which has reduced the percentage of contribution from contributors to SSNIT from 18.5% to 13.5% o f which 2.5% is used to finance the National Health Insurance Scheme. The difference of 5.0% currently is sued to fiancé the second tier schemes of mandatory occupational pensions scheme.

Again on the inflow to the Pension Funds the returns on the investment income, I_{income} of SSNIT was also critically examined form the year 2006-2010 and it was observed that, the investment returns in the year 2006(125.62) did better than in 2007(105.84), 2008 (148.74) and 2009(116.28). The year 2010 recorded a higher investment income (205.25) than the preceding year. all the investment returns figures are in Million Ghana Cedis, from the values of the investment returns, it can be observed that all even years do better than its consecutive odd year for a reasons that could not be substantiated. The reason may be probably due to the fact that the investment packages SSNIT invest in performs better in even years than odd years.

5.3.2 Outflows on the SSNIT Pension Fund Levels

Now to the outflows of the Fund, it was observed that the major outflows to the SSNIT Fund were the trust's Administrative and Operational Expenses, and Benefits paid, . From the year 2006 to 2009, there was an increasing trend in the trust Administrative and Operational expenses from 23.45 to

79.91 (Amount is in Million Ghana Cedis). The dries may be as a result of the increasing trend in the cost of living. There was a reduction in the Administrative and Operational Expenses from 79.91 to 50.72 (Million Ghana Cedis) a decrease of 36.52%.
The major outflow of the Fund is the Benefits paid to beneficiaries of the Trust. The benefits paid are in the form of Old-Age Lumpsum refund, Old-Age Invalidity and Survivors benefits. Form the year 2006 up to 2010, the benefits paid have been 79.87, 117.08, 163.43, 223.24 and 310.73 respectively with the values in Million Ghana Cedis, an increasing trend of 31.78% 28.36%, 26.79% and 28.16% respectively.

5.3.3 Sustainability Ratio

Observing the data on the sustainability ration from 2006 to 2010, the year 2006 showed a very impressive performance of 1.22 time, with 2007 dropping to 0.72 times, 2008 increasing to 0.75 times, 2009 decreasing to 0.38 times and 2010 increasing to 0.57 times. Again, as was in the case with investment return, the sustainability ratio of the years that are even did better than its consecutive odd year. this may be as a result of the fact that the sustainability ratio is a functions of investment returns and has a positive correlation with the investment returns.

5.3.4 Indebtedness

Another factor that affects the funding level is indebtedness. From the annual reports of SSNIT for the year 2010, among the list of institutions indebted to the scheme is the Controller and Accountant General's Department who contributes 71.4% to the indebtedness of the scheme, private establishments contributes 23.29% and subverted establishment contributes the remaining 4.7%. Analyzing the annual indebtedness from 2002 to 2010, the year 2011 recorded the highest percentage of 11.3%, following by the year 2010, with a percentage of 8.0. The years 2003 and 2007 recorded the least indebtedness percentage of 3.1. The years 2002, 2006 and 2008 recorded the respective percentages of 5.4, 3.6 and 3.2. The years 2004 and 2005 each recorded an indebtedness percentage of 3.5

5.4 Discussions

From the observation the data analysis, investment returns incomes were doing very well in years that are even than in the consecutive odd year. no special reason could be accounted for this behaviour in trend. This characteristic in trend behaviour was also seen with the sustainability ratio. Although the sustainability ratio is a function of the investment returns, one would think the behaviour of the total expenditure may have neutralized this property but this was not so. From the analysis of the administrative and operational cost, it was observed that the trend in expenses was an increasing one. This may be probably due to the rise in the cost of them (inflation) and in the standard of living. The year 2010 observed a decrease in the trend of administrative and operational cost. The reason to this cannot clearly be stated but it appears due to the awareness of the implementation of the new Pensions Act. The trust was very circumspective in its way of spending in order to probably increase their sustainability ratio. One other factor that affected the fund level is indebtedness. The percentage of indebtedness was low until 2009 when it rose. The measure that kept it low should be enforced to keep the percentage to minimal. The institution with the highest indebtedness percentage is the controller and Accountant's General Department, a governmental institution that handless the payroll f most governmental institution that handles the payroll of most governmental institutions. The deduction of the government workers' social security is done by the Controller and Accountant's General Department before the workers even receive their salaries, so there should not be any account where they should owe the payment of government workers social security contributions to SSNIT. This raises several taught on who keeps the deductions for what reasons before it is paid to SSNIT and who pays the interest on the deductions from the time it was deducted till the time it is paid to SSNIT.

The sustainability ratio for the year 2009 was not encouraging at all and this may be attributed to probably the change in government. The value rose steadily in the year 2010. The value is expected to increase in the year 2011 all things being equal. The year 2012 is also expected to perform creditably well although it is an election year; this is on the basis that the year 2008 did better than its preceding year despite the fact that the year was also an election year. Again, for the year 2012, because of the implementation of the Single Spine salary Structure, most contributors enjoyed an increase an increase in salary which has a high positive correlation on the contribution to the fund. Arrears were also paid to contributors and this will really influence the sustainability ratio for the years 2011 and 2012.

5.5 Conclusion

The model was a good predictor of the fund level as it was able to adequately predict the past outcomes. Although the New Pensions Act affected the fund level of the Trust, the Trust was able to increase its investment income and in view of that increased its sustainability. Though the sustainability ratios had been increased, the ratio has been less than one and this is not a good indication on how well the fund is doing.

5.6 Recommendation

Due to the reduction on the percentage on contribution that goes to SSNIT, the Trust should seek for much more lucrative investment opportunities and engage in in-order to sustain the fund. Some of these ventures are loaning service, estate development for mortgage facilities for pensioners. The Trust should reduce its administrative and operational expenses in other to sustain the fund since not much can be done to benefits paid to beneficiaries. This will go a long way to at least increase the sustainability ratio. Stringent measures such as prosecution, enforcement of compliance activities, negotiations and the payment of interest on any defaulting amount owned by institutions and employers in order to discourage institutions nor employers who default to the Trust. There is also the need for more research to be carried out to determine the sustainability of the pension fund.

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Appendix

DATE	SNNIT MONTHLY FUNDING LEVEL (AMOUNT IN MILLION GHANA CEDIS)
Jan-02	21.64
Fed-02	23.31
Mar-02	24.99
Apr-02	26.66
May-02	28.34
Jun-02	30.01
Jul-02	31.69
Aug-02	31.37
Sep-02	35.04
Oct-02	36.72
Nov-02	38.39
Dec-02	40.07
Jan-03	41.05
Feb-03	42.86
Mar-03	44.68
Apr-03	46.5
May-03	48.31
Jun-03	50.13
Jul-03	51.95
Aug-03	53.76
Sep-03	55.58
Oct-03	57.4
Nov-03	59.22
Dec-03	61.03
Jan-04	70.87
Feb-04	71.25
Mar-04	71.62
Apr-04	71.99
May-04	72.37
Jun-04	72.74

DATE	SNNIT MONTHLY FUNDING LEVEL (AMOUNT IN MILLION GHANA CEDIS)
Jul-04	73.11
Aug-04	73.49
Sep-04	73.86
Oct-04	74.24
Nov-04	74.61
Dec-04	74.98
Jan-05	67.81
Feb-05	69.56
Mar-05	71.31
Apr-05	73.06
May-05	74.8
Jun-05	76.55
Jul-05	78.3
Aug-05	80.05
Sep-05	81.8
Oct-05	83.55
Nov-05	85.3
Dec-05	87.05
Jan-06	76.86
Feb-06	80.71
Mar-06	84.55
Apr-06	88.39
May-06	92.24
Jun-06	96.08
Jul-06	99.93
Aug-06	103.77
Sep-06	107.61
Oct-06	111.46
Nov-06	115.3
Dec-06	119.14
Jan-07	125.91
Feb-07	129.34
Mar-07	132.78
Apr-07	136.22
May-07	139.66
Jun-07	143.09

DATE	SNNIT MONTHLY FUNDING LEVEL (AMOUNT IN MILLION GHANA CEDIS)
Jul-07	146.53
Aug-07	149.97
Sep-07	153.4
Oct-07	156.84
Nov-07	160.28
Dec-07	163.71
Jan-08	180.89
Feb-08	181.86
Mar-08	182.83
Apr-08	183.8
May-08	184.77
Jun-08	185.74
Jul-08	186.71
Aug-08	187.68
Sep-08	188.64
Oct-08	189.61
Nov-08	190.58
Dec-08	191.55
Jan-09	177.9
Feb-09	181.53
Mar-09	185.17
Apr-09	188.81
May-09	192.45
Jun-09	196.08
Jul-09	199.72