MODELLING SUSTAINABLE HARVESTING STRATEGIES OF A FISH POND- A CASE STUDY

BY

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DECARATION

I hereby declare that this submission is my own work towards the MSc. and that, to the best of my knowledge, it contains no material(s) previously published by another person(s) nor material(s), which have been accepted for the award of any other degree of the University, except where the acknowledgement has been made in the text.

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ABSTRACT

In this thesis, we study sustainable harvesting strategies for tilapia fish farming. Modified logistic growth models with constant harvest rate as well as periodic harvesting have all been studied. Modified logistic growth model is applied to fishery systems and animal production where overcrowding and competition of resources are considered. Although, tilapia fish farming is being done on commercial basis in Ghana, there is much less literature in studying sustainable harvesting strategies. Specific objectives of this thesis were to develop a modified logistic growth model to include harvesting rates. Finally, to determine Maximum Sustainable Yield (MSY) of fish that will ensure the tilapia fish supply is continuous and sustainable. Analytical and numerical methods were used to determine the maximum sustainable yield of fish population in a pond. The existence of equilibrium solutions and their stabilities of the modified logistic growth model were theoretically studied. Periodic harvesting strategy is strongly recommended for the selected tilapia fish farm and inland fish harvesters. This thesis will help improve productivity and reduce risk from changes in sale price of tilapia fish.

DEDICATION

This thesis is dedicated to my lovely and supportive wife Mrs.Doris Agudze and our son Collins; and to my extended family, most especially my father; Johnson D. Agudze.

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CHAPTER ONE

INTRODUCTION

Over the last two decades aquaculture have be become an enterprise cherished by the government, urban and rural communities and is gaining ground especially in the Volta Lake. it is an alternate income-generating venture. Fishing in general, provides many benefits to human beings, including food, employment, business opportunities and recreational activities. However, overfishing can reduce the fish stock or biomass of reproductive age below sustainability. It is desirable that in management of a renewable resource, such as fishery a strategy is developed that will allow an optimum harvest rate and yet keep the population above a sustainable level. A modified logistic growth model in terms of harvesting has been developed to help the fish farming sector and the Fisheries Commission to project the population of tilapia fish that is due for harvesting in a given time period.

1.1 BACKGROUND OF THE STUDY

Fish is one of the chief sources of human diet and the main source of protein and fat. Of late, consumers have developed much interest in fish as a healthier alternative meat that prevents the problem of overweight and cardiovascular diseases in human. Fishing industry both marine and aquaculture in Ghana supports the livelihood of about 10% of the population. The importance of the fishing industry stems from the significant contribution of about 60% of the national protein supply and about $87 million exports in 2009.

Historically, marine fishery resources were assumed to be almost limitless, and fishing was taught to have little impact on fish biomass and marine ecosystems. However, in recent years, concern about the condition of fisheries has increased. Therefore, many fisheries experts and fishermen now realize that fishing can have serious effects on marine fish stocks and the ecosystem they inhibit. However, the management of a renewable resource, such as fishery requires a strategy that will allow an optimum harvest rate and yet not extinct the population below a sustainable level. Thus, Fisheries management policies and practices are usually based on catch effort dynamics with little consideration for the ecosystem variations. When fishing effort is increase it has major impact on the short term dynamics and sustainability of the fish population.

According to (Gertjan et al. 2005), tilapia fish farming has been an important source of protein of the world and it is well suited for farming, since they are fast growing and hardy. Tilapia fish also can establish strong population in a very short time if the environment is conducive. Aquaculture is becoming the most preferred option used by agro-based companies to produce tilapia on commercial quantities in the Volta Lake and fish ponds. The sector has commercial hatcheries and various fish farm sizes. The main fish species on fish farms are tilapia (Oreochromis niloticus) and catfish (claria gariepinus). According to Commission of Fisheries 2010, tilapia forms about 80% of aquaculture production in the country due to its fast growth and its resistance against harsh conditions.Various species of tilapia are found in Ghana's rivers, lakes and Lagoons. One of such species of tilapia unique to southern Ghana and south western of Cote d'Ivoire is the tilapia busumana mainly found in Lake Bosutonneswi, the Pra-Offin, Ankobra, Bia and Tano basins. Most Ghanaian tilapia is caught by artisanal fishers and has immense domestic demand for both fresh and locally processed variants.

Lake Volta is the largest man-made Lake; it has a surface area of 8,700 km at the maximum controlled level of 84.7m. The Lake has a total length of 400km, a shoreline length of 5200km, a maximum depth of 90m and an average depth of 18m. Lake Volta and its tributaries drain 70% of the entire area of Ghana (FAO, 2005), covering mostly Northern, Volta, Eastern and Brong Ahafo regions. Lake Volta supports the livelihoods of 300 000 people, of whom nearly 80 000 are fishermen and 20 000 are fish processors or traders. There are 1 000 people involved in the aquaculture subsector, working mainly in pond culture (Mensah *et al.*, 2006). Data obtained during the field survey at various cage farms in Volta Lake revealed that about 10,000 tonnes of tilapia were produced during 2010.

Tropo Farms has been in pond farming for six years and in 2005 developed a pilot-scale cage site on Lake Volta near Akosombo Dam. Tropo farms grow indigenous *O. niloticus* in ponds (breeding and juveniles) and cages (grow-out to market size). Tropo farms for instance, indicated the production of 3000 tonnes of tilapia with about 25 smaller cage farms producing about 100 tonnes each for the same period in consonance with projections made by Blow and Leonard (2007). Figures 1.1 and 1.2 show overview of Volta catch/ Tropo farms cages in Lake Volta and pond respectively.

Figure 1.1 Tropo Farms cages in Lake Volta.

Figure 1.2 Confinement of fish using hapas, in Tropo farms pond.

Crystal Lake Fish Ltd was established in the late 1990s in the Asuogyaman District of Ghana's Eastern Region. Crystal Lake Fish Ltd grows indigenous tilapia (*O. niloticus*) in ponds and concrete tanks (breeding and juveniles) and cages (grow-out to market size). The farm has 24 circular (8 m diameter each) tanks for hatchery (8) and nursing (16) purposes. There are about 8 cages installed in circa 25 m deep water at Crystal Lake, each with a diameter of 15 m and a depth of 4m. Each cage is stocked with 50 000 fingerlings of *O. niloticus* at 30 g that are cultured for six months. Crystal Lake Fish Ltd.'s annual production is around 340 tonnes of whole fish. Figures 1.3 shows Feeding time at Crystal Lake Fish Ltd., Lake Volta

Figure 1.3 Feeding time at Crystal Lake Fish Ltd., Lake Volta

The West African Fish Limited also build up tilapia fish farm in Volta Lake about 10 miles away from Asikuma in Eastern region of Ghana in 2008. It has a capacity of producing about 2000 tonnes per year. The tilapia is kept in quality cages with the best knowhow. West African Fish ltd. is currently selling all of the fish they produce at domestic markets. The fish are sold in fresh condition, to customers such as wholesalers and retailers, as well as directly to restaurants, hotels and the like.

In 2008, the Fishery Commission conducted a regional basis survey and found that the Western region has the largest number of fish farmers and ponds followed by the Brong Ahafo and Ashanti region as shown in table 1.1 below.

Table 1.1 Fish Farm Data (2008)

	No. of fish farmers	No. of ponds	No. of functional ponds	Total surface area (ha)
Ashanti	304	746	746	118.71
Brong Ahafo	333	761	761	138.63
Central	253	633	610	39.91
Eastern	107	311	311	20.35
Greater Accra	64	233	207	39.5
Volta	143	308	254	67.35
Western	1650	2550	2550	59.1
Upper East	15	25	25	7.52

Source: Fisheries Commission

The management of a renewable resource, such as fishery requires a strategy that will allow an optimum harvest rate and yet not extinct the population below a sustainable level. Thus, Fisheries management policies and practices are usually based on catch effort dynamics with little consideration for the ecosystem variations. When fishing effort is increase it has major impact on the short term dynamics and sustainability of the fish population.

Mathematical model is a mathematical description(often by means of a function or an equation) of a real-world phenomenon such as size of a population, the demand for a product, the speed of a falling objects, the life expectancy of person at birth and many instances.

Mathematical models have been used widely to estimate the population dynamics of animals for so many years as well as human population dynamics. Recently, the use of mathematical models has been extended to agriculture sector especially in cattle farming to predict continuous and optimum supply. The logistic growth model in terms of harvesting has been studied to help the fish farming sector and the Fisheries

Commission to project the population of tilapia fish for a given time period. This will enable them to be prepared with effective solutions to ensure that the tilapia fish supply can meet the consumer demand.

1.2 PROBLEM STATEMENT

Fisheries management is a complex process and requires the integration of resources in biology and ecology as well as socioeconomic and institutional factors that affect the harvesting in policy- making. According to the Food and Agriculture Organization of the United Nations (FOA, 2007), in the year 2005, about 50% of the fish stock under observation experienced overexploitation or depletion.

Even though tilapia fish farming has been commercialized, there is much less literature available in studying fish harvesting strategies in Ghana. Thus, mathematical models do not widely feature in studying fish harvesting management strategies in Ghana. Hence, the need to use a mathematical model to estimate fish harvesting strategies that will ensure the tilapia fish supply is continuous and not gets to extinction.

1.3 OBJECTIVES OF THE THESIS

The main objective of the thesis is to study two harvesting management strategies (constant and seasonal harvesting models) of tilapia fish population in a pond in the Eastern region of Ghana. The specific objectives of the thesis include:

1. To develop a modified logistic growth model to include harvesting rates.
2. To determine maximum sustainable yield (MSY) of tilapia fish population in a pond.

1.4 JUSTIFICATION

Fishing industry in Ghana supports the livelihood of about 10% of the population and according to the Ghana Statistical Service fishing contributed 2.4% of Gross Domestic Product (GDP) in 2010. The importance of the fishing industry stems from the significant contribution of about 60% of the national protein supply and about $89 million exports in 2010. Due to this significant contribution of fishing industry in the economy of Ghana, aquaculture is becoming the most preferred option used by agro-based companies to produce tilapia on commercial quantities in the Volta Lake and fish pond. Since tilapia fish has been an important source of protein in some areas of the world and it is well suited for farming, because they are fast growing and hardy.

Determining socially acceptable harvesting strategy is undoubtedly one of the most challenging and most controversial problems in the management of the renewable resources such as fishery. Sustainable harvest of a renewable resource occurs if we remove the resource from the population at a rate that can be compensated by the growth of the population and in this situation we can effectively remove individuals from the population while allowing the population to exists. Often sustainable practices such as harvesting strategies are not carried out in fishing industry. The use of mathematical models in determining sustainable fish harvesting strategy has not been widely applied in Ghanaian fishing industry and this has necessitated this study.

1.5 METHODOLOGY

In this study, a modified logistic model that includes harvesting is developed. The non-linear autonomous and non autonomous differential equations that model the harvesting management strategies are qualitatively and numerically analyzed. The data for the study is obtained from the following sources; Fisheries commission, Tropo Farms, Crystal Lake Fish Ltd and West African Fish Ltd.

1.6 SCPOE OF THE THESIS

The study covers some selected tilapia fish ponds and tilapia fish cages in the Eastern region of Ghana.

The study will enable fishermen to use a mathematical model to predict the tilapia fish at maturity and use appropriate harvesting management strategy that can ensure that tilapia fish supply is continuous throughout the year. A modified logistic model which includes harvesting fish at constant and periodic rates will be implemented.

1.7 ORGANISATION OF THE THESIS

The main body of the thesis includes, Introduction; Literature review; the Model development; Findings; Discussions; Conclusions and Recommendations; References and Appendices.

Chapter one covers introduction, background, problem statement, objectives of the thesis, justification, methodology, scope of the thesis and organization of the thesis.

In chapter two we shall review the overview of aquaculture and tilapia fish production in Ghana. We will also review some related literatures about growth models and harvesting management strategies models. Chapter three focuses on the model development and the analysis of non-linear logistic growth with harvesting models. In chapter four, we shall deal with model implementation and analysis of results. Finally, the conclusions and recommendations of the thesis will be presented in chapter five.

In the next chapter, we shall review the origin of fishing, overview of aquaculture in Ghana and some mathematical models in fish population growth and harvesting strategies.

CHAPTER TWO

REVIEW OF FUNDAMENTALS

In this chapter, we would outline the origin of fishing and overview of aquaculture of tilapia fish production in Ghana. The population harvesting and application of mathematical models for fish population growth and harvesting strategies would also be reviewed.

2.1 ORIGIN OF FISHING

According to Graham (1956), fisheries science took its characteristic form from around 1890 onwards with a blend of zoology and statistics but each with a new form and function. He further said 'the form was knowledge like that of a fisherman, and function was guidance to better use of the stocks of fish'. Indications of fishing have been found in archeological sites as early as the Late Paleolithic period, some 50000 years ago, revealing a long history of the use of fish by humans. More recently, fish and fishing have been depicted in rock carvings in southern Africa and southern Europe dating from 25000 years ago. Based on this, Sahrhage and Lundbeck noted that fishing is one of the oldest professions, along with hunting. Unlike hunting, however, fishing continued to be an important occupation even to modern times, and fishing methods have been repeatedly improved over the millennia.

The development of fishing on all continents and in most cultures can be more clearly traced since the early Mesolithic period, 10 000 BC, and can be seen in archaeological artefacts such as kitchen middens, paintings and fishing gear. Forn example, in the port city of Ginnosar on Lake Kinneret, where St Peter lived, excavations have revealed consistent occupation and evidence of fishing since the Bronze Age. By 4000 BC in some areas the archaeological record is complete enough to reveal the evolution of fishing gear. For example, the evolution from simple to compound fish hooks has been demonstrated for the cultures on Lake Baikal in this period.

Anon (1921) revealed that in Asia the importance of fishing can be traced back for thousands of years, especially in the Yellow River in China and in the Inland Sea off Japan. Fishing in China was primarily in fresh water, and tended towards the development of fish farming rather than catching technology. However, illustrations of Chinese fishing methods showed them frequently to be unique: for example multiple lines of hooks in complex arrays and trained cormorants with neck rings. Chinese methods were subsequently used in Japan, initially in the Sea of Japan. In the Americas fishing appears to have been imported with the colonizing peoples, with rock paintings of fish and fishing as early as 10000 BC in Patagonia. In the northern regions sealing and whaling developed, along with more traditional fishing, at least as early as 2000BC. The practice of aquaculture started in Asia, Ancient Egypt and in Central Europe. In Asia, it was around 500 BC by a Chinese politician (Ling, 1977). In Egypt (Africa), tilapia as a native, was raised in ponds around 2500 BC.

(Ling, 1977) have found that, the earliest species of fish cultured was the common carp (*Cyprinus carpio*), by a native of China. In addition, Indian carp culture existed in the 11th Century AD (Pillay, 1990). Similarly, aquaculture started in Europe from the middle Ages with the introduction of common carp culture in monastic ponds. Subsequently, during the 14th century, the propagation of trout was introduced in France and the monk Don Pinchot and, discovered in the same period; the method of artificial impregnation of trout eggs (Davies, 1956). Furthermore, commercial trout culture in freshwater was developed in France, Denmark, Japan, Italy and Norway (Pillay, 1990).

Specifically, the British introduced trout as sport fisheries in their Asian and African Colonies. Moreover, the development of fish culture in North America became possible through the propagation of trout Salmon and Black bass. In the Czech Republic, these fishes were cultured in large ponds which were built from around 1650 and are still in use (Wikipedia, 2009).

(Hecht, 2006; Lazard *et al*. 1991) have pointed out that aquaculture started in Sub-Saharan Africa in the 1950s with the main objectives of food security, income and creation of jobs for the rural poor families. Eventually, it began to drop after 4 decades as compared with Asia. The proof was that, Africa realized a sum of US $72.5 million from 1978 to 1984 while Asia and the Pacific recovered US $171.3 million (Lazard *et al*. 1991).

However, (Ridler and Hishamunda, 2001) also discovered that the African continent is environmentally friendly with the farming of tilapia, African catfish and carps. Despite the potential, the Region contributes less than 1% to world aquaculture production. Consequently, this has caused a high pressure on capture fishery due to the growing population of Africa that depends on fish protein.

According to Asmah (2008), an increment of fish supply, from 6.2 to 9.3 million tonnes per year will help to reduce the pressure and Muir (2005) further explained that more than 8.3% of the total tonnage is needed from aquaculture on annual average productions in 2010 in Sub- Saharan Africa alone. In support of this, FAO, UNDP, World Bank and France funded projects in countries like Cameroon, Cote d‘voire, Kenya, Madagascar and Zambia (Lazard *et al*. 1991).

In West Africa, the Gambia started aquaculture in the 1970’s in the form of trials using tilapia culture in rice fields (Jawo, 2007; Jallow, 2009). Later on, in 1982, a company known as West African Aquaculture limited started the culture of *Peneaus monodon* in the coastal region (Jallow, 2009). This company became well established in The Gambia in 2000. Similarly in 1988, two fish farms were operated in Western Region by Scan Gambia limited (Jallow, 2009).

2.2 OVERVIEW OF AQUACULTURE IN GHANA

Fish farming have begun in Ghana in 1953 by the then Department of Fisheries. Thus, it served as hatcheries to support the then culture-based reservoir fishery development programme of the colonial administration. In 1957, the government of Ghana adopted a policy to develop fish ponds for farming within all irrigation schemes in the country (FAO, 2000, 2009). Aquaculture also called fish farming was taken up enthusiastically in the late 1970's by the Accra Metropolitan Assembly (AMA) as an alternate income-generating venture. Efforts were made to develop fish farms on suitable lands near urban centres, where water is readily available. Few fish farmers were successful, but most of them faced management problems due to inadequate training and information. Thus, the fish farming program meant to reduce poverty in the urban towns failed. There was a boost in early 1980s, following a nation-wide campaign by then military government. Subsequently, the first experimental fish farm was established in the Upper West Region in 1985. During the period of 1982 to 1985, the number of fish ponds increased from 578 to 1,390. Gradually, the number rose to 1,400 in 1986; covering an average surface area of 685 m square (Amisah and Quagrainie, 2007).

In order to increase further, research collaboration between International Centre for Living Aquatic Resources Management (ICLARM) and the Institute of Aquatic Biology (IAB), Accra, Ghana, began in 1991 to investigate the development of aquaculture on small holder farms (Pullin and Prein, 1994). In the period between 1990 and 2004, the technology of fingerlings production improved tremendously but there were neither marine nor brackish water aquaculture establishment in the country. The major species grown were *Oreochromis niloticus, Clarias gariepinus and Heterotis niloticus.* The majority of farmers were small-scale operators using extensive fish farming systems (FAO, 2000, 2009).

In the last decade, fish farming or aquaculture became an enterprise cherished by the government, urban and rural communities and gaining ground especially in the Volta Lake. On regional basis, a survey by the Commission in 2008 revealed that the Western region has the largest number of fish farmers and ponds followed by Brong Ahafo and Ashanti regions. Recently, aquaculture is becoming the most preferred option used by agro-based companies to produce tilapia on commercial lines in the Volta Lake.

According to (FAO, 2006), Aquaculture provides 50% of the world fish production and is an alternate seafood to wild fisheries and generate income and employment. Traditionally, there are three forms of aquaculture in Ghana, namely acadjas or brush-parks in lagoons and reservoirs; hatsis (fish holes) and whedos(mini-dams) in the coastal lagoons; and freshwater clams (Egeria radiata) in the lower Volta, young clams are collected and "planted" in "Owned areas of the river(Perin and Ofori, 1996). Intensive system of culture is used by the major farms having cage culture technology. Dams, ponds and small reservoirs are fished out and stocked regularly in the extensive system of aquaculture.

According to (Awity, 2005), a single commercial cage farm contributed about 21% (200 tons of 950 tons) to total aquaculture production in 2004.

(Ofori et al. 2010), suggested that if cage farmers in Ghana can produce yields of 50-150kg/m^3 per 9 months as done elsewhere in Africa, less than 100 hectares of fish cages can produce yields matching the current capture fisheries production of 90000 metric tons. Braimah (1995) addressed fisheries of Lake Volta. Estimated yield was 42-52 kg/ha/year based on catch statistics, and 12 kg/ha/year based on the morphedaphic index, MEI, (Ryder *et al.*, 1974). Tilapia is a major component of the harvest, with catches influenced by water level (higher catches when water level is low). During reservoir drawdown, standing timber is harvested for firewood and to facilitate beach seining. However, standing timber in the reservoir basin is important for periphyton production.

Braimah (1995) estimated that 52% of the fish caught were dependant on invertebrates exploiting this periphyton. Removal of standing timber, in conjunction with overfishing, has negatively impacted the fish stocks. He noted that Tilapia fish farming has been an important source of income in some areas of the world and it is well suited for farming, since they are fast and hardy. Tilapia fish also can establish strong population in very short time duration if the environment is right (Gertjan, et al. 2005). This has made tilapia fish a very important protein source.

According to (Thomas & Michael,1999), the period of maturity for the tilapia fish is 6 months and estimates that 80% of will survive to maturity. Department of fisheries in Ghana's survey revealed that tilapia forms 80% of aquaculture production. Most Ghanaian fish is caught by artisanal fishers and most of the catch is salted and dried or smoked and it heads to the domestic market.

In 2010, tilapia production from fish farms was about 10000 tons. This figure was gotten during the field survey at various cage farms in the Volta Lake. Tropo farms for instance, produce 3000 tons of tilapia in 2010, with about 25 smaller cage farms producing about 100 tons each for the same year in consonance with projections made by Blow and Leonard (2007).

Capture fishery may produce an average of 100 tons per year from major landing sites of Dzemeni, Abotaose, Kpando, Kete Krachi, Yeji, Kpong, and Asutuare.

According to(Blow & Leonard, 2007), the capture of other desirable species such as catfishes can also be expanded through cage aquaculture in addition to Nile tilapia(Oreochiromis Niloticus) which is currently the only species cultured in Ghana.

The West African Fish Limited has tilapia farm in Lake Volta near Asikuma (10 miles), started to build up the facility in 2008 and has a capacity of producing about 2000 tons per year. The farm's tilapia is kept in quality cages with best knowhow. Tilapia is a good fish for warm water aquaculture. They are easily spawned, use a wide variety of natural foods as well as artificial feeds, tolerate poor water quality, and grow rapidly at warm temperatures. These attributes, along with relatively low input costs, have made tilapia the most widely cultured freshwater fish in tropical and subtropical countries. Consumers like tilapia firm flesh and mild flavor, so markets have expanded rapidly in the U.S. during the last 10 years, mostly based on foreign imports.

A flurry of media activity has centered on fisheries issues in the past year prompted by the release of several studies and reports that point to growing crises and controversy in both wild fisheries and aquaculture. A recent report from a panel of fishermen, scientists, business leaders, and government officials pointed to overfished and depleted stocks in U.S. waters, along with severe habitat degradation (Pew Oceans Commission 2003). The report argued that the restoration of U.S. fisheries requires a major overhaul of policy, including the introduction of ecosystem-based management and stronger regulations.

(Myers and Worm 2003) did a much-publicized study in Nature and have reported that the population of large predatory marine fish has been reduced by 90 percent since preindustrial times. Another recent study by (Watson and Pauly 2001) argued that correcting reported Chinese fisheries statistics to levels that better fit estimates of biophysical potential renders global catch trends far less favorable (Watson and Pauly 2001). The Food and Agriculture Organization of the United Nations (FAO), particularly in its State of World Fisheries and Aquaculture publications, has consistently sounded the alarm over threatened stocks of wild fish (FAO 1995, 1998, and 2000a).

The rapidly growing field of aquaculture, which now accounts for 30 percent of the world's food fish, has also pushed its way into the media spotlight. For some years now, aquaculture has been seen as a possible savior for the overburdened wild fisheries sector, and an important new source of food fish for the poor (FAO 1995; Williams 1996). However, there are some problems with the industry. A recent report from the World Wildlife Fund argued that some forms of aquaculture place pressure on wild fisheries through demand for wild-caught fish as feed (Tuominen and Esmark 2003).

2.3 POPULATION HARVESTING

Miner & Wicklin, (1996) have defined population harvesting as the removal of constant number of individual from a population during each time period.

According to Idels and Wang (2008) constant harvesting is where a fixed number of fish were removed each year, while periodic harvesting is usually thought of as a sequence of periodic closure and openings of different fishing grounds. Advocates of population harvesting have pointed out that stable populations of deer, fish, and other game animals, harvesting can be used to reduce the number of animals who needlessly die from starvation or other natural causes. On the other hand, unregulated harvesting can lead a population to the brink of extinction, as is evidenced by well-known examples such as the North American Bison (Bison bison) and several populations of whales. Harvesting policy has been used to stabilize population in an environment with limited resources or carrying capacity.

Aanes et al. (2002) siad the most important for successful management of harvested population is that, harvesting strategies are sustainable, not leading to instabilities or extinctions and produces great results for the year with little variation between the years. Thus, it can supply the market demand throughout the year. Harvesting has been an area under discussion in population as well as in community dynamics (Murray, 1993).

C.W. Clark, et al. 2005 and many other authors stress that optimal management of renewable resources has an important relationship to long term sustainability. In addition, they have extensively studied the optimal harvesting policies for harvesting that is constant or proportional to the resource abundance and have proposed that continuous proportional harvest is optimal when compared to other forms of harvesting, or that their optimal harvesting policy is nearly that of a continuous harvest policy.

In (C.S. Lee and P. Ang, Jr., 1991), Lee and Ang investigated a logistic type seaweed growth model in which the growth and death rates of seaweed are known periodic functions of time, and they deduced an optimal periodic harvesting strategy which maximizes the average accumulated yield of an unspecified but periodic seaweed biomass having the same period as that of the growth and death rates. They noted, however, that in practice, a constant harvest rate is more practical, and hence, preferable to a time-varying one.

M.S. Boyce et al. (2005), have considered five different exploitation strategies on a single population that grows logistically with a seasonal carrying capacity. They conclude that the optimal harvest should be timed during the period of maximal decline in the carrying capacity. They investigate the maximum annual yield and population persistence as a function of both the seasonality and the intrinsic rate of increase. They compare these harvest strategies under certain combinations of environmental variability and intrinsic growth rate values. Among the five different harvesting strategies considered (constant exploitation, linear exploitation, 6-month open/closed harvest, time dependent harvest and pulse harvest), pulse harvesting was found to be optimal (in the sense of maximum annual yield) in all situations where they varied the intrinsic rate of increase and environment.

The Harvest Control Rule is a variable over which the management strategy has some direct control and describes how the harvest is intended to be controlled by management in relation to the state of some indicator of stock status. For example, a harvest control rule can describe the various values of fishing mortality which will be aimed at for various values of the stock abundance.Harvest tactics are the regulatory tools (e.g., quotas, seasons, gear restrictions) used to implement a harvest strategy. Harvest tactics are quite diverse, and almost all fisheries employ gear restrictions, area restrictions, and some limitation of seasons. Quotas are increasingly employed in large-scale commercial fisheries, whereas closed seasons and closed areas are common in recreational fisheries. Management procedures represent the combination of data collection, assessment procedure, harvest strategy, and harvest tactics (Washington, D.C. 2001). Harvesting has been considered a factor of stabilization, destabilization, improvement of mean population levels, induced fluctuations, and control of non-native predators (Michel, 2007).

Recently, Braverman and Mamadani (2008) have considered both autonomous and non autonomous population models and found that constant harvesting is always superior to impulsive harvesting even though impulsive harvesting can sometimes do as good as constant harvesting. But Ludwig D, (1980) studied models with random fluctuations and found that constant effort harvesting does worse than other harvesting strategies.

Constant rate depletion on the discrete Ricker model was studied by Sinha.S,and Parthasarathy. S (1996), where it was numerically shown, that populations exhibiting chaotic oscillations are not necessarily vulnerable to extinction.

Xu et al. (2005) have investigated harvesting in seasonal environments of a population with logistic growth and found that pulse harvesting is usually the dominant strategy and that the yield depends dramatically on the intrinsic growth rate of population and the magnitude of seasonality. Furthermore, for large intrinsic growth rate and small environmental variability, several strategies such as constant exploitation rate, pulse harvest, linear exploitation rate, and time-dependent harvest are quite effective and have comparable maximum sustainable yields. However, for populations with small intrinsic growth rate but subject to large seasonality, none of these strategies is particularly effective, but still pulse harvesting provides the best maximum sustainable yield.

According to European Union (2006), Maximum Sustainable Yield (MSY) is theoretically, the largest catch that can be taken from a fishery stock over an indefinite period. Under the assumption of logistic growth, the MSY will be exactly at half the carrying capacity of a species, as this is the stage at when population growth is highest. In fisheries terms, maximum sustainable yield (MSY) is the largest average catch that can be captured from a stock under existing environmental conditions (National

Research Council (NRC). 1998). MSY aims at a balance between too much and too little harvest to keep the population at some intermediate abundance with a maximum replacement rate.

Fisheries Act (1996) studied maximum sustainable yield' in relation to any stock, means the greatest yield that can be achieved over time while maintaining the stock's productive capacity, having regard to the population dynamics of the stock and any environmental factors that influence the stock

However, the MSY has been widely criticized as ignoring several key factors involved in fisheries management and has led to the devastating collapse of many fisheries. As a simple calculation, it ignores the size and age of the animal being taken, its reproductive status, and it focuses solely on the species in question, ignoring the damage to the ecosystem caused by the designated level of exploitation and the issue of by-catch.

2.4 HISTORICAL PERSPECTIVE OF LOGISTIC GROWTH MODEL

Robert Malthus was the first to formulate theoretical treatment of population dynamics in 1798 and P.F Verhulst formed the Malthus theory into a mathematical model called logistic equation that led to non-linear differential equation in 1838 (Alan, 1992). However, Verhulst's work went unappreciated until in 1920, when Raymond Pearl and Lowell Reed rediscovered the logistic equation and made it famous (F. Brauer, et al. 2001). This model was used to estimate population of humans, animal and fish production. The logistic growth model accounts for the limitations of resources having an impact on the growth of populations. These limitations are expressed as a saturation level or population carrying capacity that exists as populations get larger. The carrying capacity represents the population size that available resources can continue to support and is unique for different species. The following are the assumptions for the logistic growth model. We assume a closed population of a single species in a specified region, where there are no time lags, nor stochastic or chance events. We also assume that population growth rates respond instantaneously to any changes and this rate is only influenced by the population abundance. In addition, the population abundance at which the per capita growth rate is zero, occurring at the carrying capacity, does not vary in time. The model should be regarded as a metaphor for populations that have a tendency to grow from zero up to some carrying capacity N. Originally a much stricter interpretation was proposed, and the model was argued to be a universal law of growth (Pearl, 1927).

The logistic equation was tested in laboratory experiments in which colonies of bacteria, yeast or other simple organisms were grown in conditions of constant climate, food supply, and the absence of predators. These experiments often yielded sigmoid (S-shaped) growth curves, in some senses with an impressive match to the logistic equation. On the other hand, the agreement was much worse for fruit flies, flour beetles, and other organisms that have complex life cycles, involving eggs, larvae, pupae, and adults. In these organisms, the predicted asymptotic approach to a steady carrying capacity was never observed—instead the populations exhibited large, persistent fluctuations after an initial period of logistic growth.

2.5 SOME HARVESTING MODELS

There are many other mathematical models that were used to model fish population that is undergoing harvesting. The following sections describe some of the models.

2.5.1 GOMPERTZ GROWTH MODEL

Similar to the logistic growth model, is the Gompertz growth model introduced by Benjamin Gompertz in 1825. It has been used to describe the growth of many tumors, as well as biological and economical growth (Kot M.et al .2001).

The model can be described with the ordinary differential equation and initial condition

$$\frac{dN}{dt} = rN\, ln\left(\frac{N}{K}\right) \tag{2.01}$$

$$N(0) = N_0$$

The solution to this initial value problem can be solved as a first order separable differential equation and can be written as

$$N(t) = K\left(\frac{N_0}{K}\right)e^{-rt} \tag{2.02}$$

The pattern of the solution is similar to that of logistic growth model, however, in this model the inflection point is smaller than that of the logistic equation and occurs at $\frac{K}{e}$. One difference between the two models occurs when we consider the per capita growth rates. In the case of the logistic growth model, the per capita growth of the population decreases linearly as a function of population abundance whereas in the case of Gompertz growth, this rate decreases exponentially.

In a closed population, there can be many forms of mortality to individuals in the population, one such example being harvesting. The effects of harvesting usually pose negative impacts to target and non-target populations. There have been many examples of species that have gone extinct or been near to going extinct partly or wholly as a direct/indirect consequence of harvesting. To avoid disastrous consequences onto the population, it is natural to devise a management plan that can allow the population to be sustained while harvesting occurs. There are many forms of harvesting that can be imposed on a population. Gompertz modified his model to include harvesting rate and this is governed by the differential equation with initial condition

$$\frac{dN}{dt} = g\big(N(t)\big) - h\big(N(t)\big) \tag{2.03}$$

$$N(0) = N_0$$

where $g(N(t))$ describes the growth function for the population and $h(N(t))$ describes the mortality due to harvesting. Two specific harvesting functions, $h(N(t)) = D$ and $h(N(t)) = EN(t),$ with effort E, represent constant and proportional harvesting, respectively. In the simplest cases, E is constant. Models with these forms of harvesting can be found in numerous references including (F. Brauer, et al. 2001).

2.5.2 THE RICKER'S GROWTH MODEL

W. E. Ricker (1958), model fish population without harvesting and the model is used to predict the number of fish that will be present in a fishery. The Ricker model is a classic discrete population model which gives the expected number (or density) of individuals N_{t+1} in generation $t + 1$ as a function of the number of individuals in the previous generation,

The model assumptions are as follows:

1. Valuable fish are those above a certain age, so we exclude juvenile fish,
2. Salmon spawn once per year,
3. Upon spawning, the adult fish die,
4. Valuable fish must replace their parents in order to maintain a viable.

The model can be described with the ordinary differential equation (2.04).

$$\frac{dN}{dt} = \beta_0 N e^{-\alpha N} - \mu N \qquad (2.04)$$

where β_0 is the original number of juvenile fish, μ is the mortality rate of the fish and N is the population of the fish, $-\alpha$ is the rate of cannibalism per adult.

The Ricker added harvesting component to his model as a factor affecting fish population, based on the claim that;

1. Harvesting is periodic,

2. Harvesting does not occur during mating season.

Hence, he modeled the fish harvesting with an oscillatory sine function.

$$H(t) = a(1 + \sin bt)$$

In this function, a is a harvesting rate limit, b modifies the frequency of fishing cycles per unit time. Ricker added 1 to insure a positive quantity for the sine term:

Thus, the final factor affecting fish population is this harvesting function, so our model becomes:

$$\frac{dN}{dt} = \beta_0 N e^{-\alpha N} - \mu N - a(1 + \sin bt) \qquad (2.05)$$

2.5.2 THE SAECHER'S SURPLUS PRODUCTION MODEL

The surplus production model was used to estimate the sustainable yield of sergestid shrimp in the southwestern Taiwan based on an assumption of a unit stock.

A discrete deterministic form of stock dynamics is expressed as:

$$B_{t+1} = B_t + rB_t\left(1 - \frac{B_t}{K}\right) \qquad (3.06)$$

where B_t is the biomass for year t, r is intrinsic population growth rate, K is carrying capacity(Saecher M.B.1954). Extending the model to include catch becomes Schaefer's model:

$$B_{t+1} = B_t + rB_t\left(1 - \frac{B_t}{K}\right) - C_t \qquad (3.07)$$

where C_t is the catch during year t.

Schaefer model can be connected to the catch rates to the stock biomass and hatchability coefficient (q).

$$I_t = qB_t = \frac{C_t}{E_t}$$

where I_t is the index catch per unit effort of the relative abundance for year t, C_t is the catch during year t, E_t is the fishing effort during year t (Haddon, M., 2001)

2.5.3 YIELD MODELS

From a global perspective, large river ecosystems are the critical lotic resources with respect to Fisheries (Dodge, 1989).

Welcomme (1985) developed yield models for large rivers relating river basin area and length of the main channel to catches. For river basin area the relationship is:

$$C = 0.03A^{0.97} \qquad (3.08a)$$

Where $C =$ annual yield in tons, and $A =$ river basin area in km^2.

For length of the main channel, the relationship is:

$$C = 0.0032L^{1.98} \qquad (3.08b)$$

where C = annual yield in tons, and L = channel length in km.

Welcomme (1985) estimated that yield potentials from African river and floodplain fisheries ranged from 5 to 143 kg/ha/year.

The general model estimates that fishery yield for a $25km$ segment is $1\,875.5\ kg$/year. However, considering cumulative influences upstream to downstream and using the model developed by Welcomme (1985), we note that at a distance of $50\ km$ from the river's source, a $25km$ section of river yields $9\,113\ kg$/year and at a distance of $250\ km$ downstream from the source, a $25km$ section of the river yields $37\,197\ kg$/year. If dams were constructed at a distance of $400\ km$ from the river's source, and resulted in loss of a $25km$ section of the river at that point, the reservoir would need to compensate for $57\,925\ kg$/year. This could be accomplished, for example, with tropical reservoirs having mean depth of $5m$, TDS of $100\ mg/$l and a surface area of somewhat more than $1\,000\ ha$. Temperate reservoirs with the same mean depth and TDS could compensate for this loss of river fishery yield with a surface area of $4\,728\ ha$.

Although the distance compensating model for this exercise was developed for African rivers, it has been used successfully for rivers in other regions (e.g. the Mekong, Danube and Magdalena rivers) (Welcomme, 1985).

2.5.4 THE BEVERTON–HOLT GROWTH MODEL

The Beverton – Holt (1957) model, introduced in the context of fisheries in 1957, is a classic discrete-time population model which gives the expected number Y_{n+1} (or density) of individuals in generation t + 1 as a function of the number of individuals in the previous generation.

The Beverton–Holt is given as

$$Y_{n+1} = \frac{\mu k_n Y_n}{k_n + (\mu - 1)Y_n} \quad (3.09)$$

where $\mu > 1$ is the population growth rate and k_n is the population carrying capacity at time n.

Equation (3.09) was studied under periodic and conditional harvesting and has found that in a constant capacity environment, constant rate harvesting is the optimal strategy.

In the next chapter, sustainable harvesting models are developed and their equilibrium solutions studied.

CHAPTER THREE

MODEL DEVELOPMENT

In this chapter, we would derive population dynamic models for fishery system that will include harvesting management strategies. An exponential population growth is outlined first. Next, a modified logistic growth model of recruitment and spawning stock biomass that takes account of the limiting factors of fish population growth is described. This is followed by a description of harvesting strategies such as constant harvesting and seasonal harvesting models.

3.1 POPULATION DYNAMICS MODEL

In fishery management, it is important to determine the maximum rate at which fish can be harvested given a certain fish population. So given a fish population $Y_{0,}$ then we want to find the maximum harvest rate that does not kill off the population. We first develop a differential equation that model the natural fish population growing at an indefinite exponential rate. This differential equation will later be modified to include factors limiting the extent to which fish population can grow, hence the derivation of a modified Logistic growth model. We would extend the modified logistic growth model by incorporating constant harvesting model and seasonal (periodic) harvesting model and we will study how the demise (fishing) of certain number of fish will affect the fish supply. Our models would allow fish harvesters to insert the parameters specific to their fish population to determine what frequency of harvesting their fish population can tolerate and yet not gets to extinction. To examine systematically the consequences of these harvesting strategies, we shall use numerical simulations and qualitative methods in analyzing the constant and seasonal harvesting models.

Fish population can be model using the Balance Law. The Balance Law states that the change in the amount of substance in a compartment can be determined by the difference between the rates at which that substance is leaving the compartment, and the rate at which the substance is entering the compartment. We can express this relationship mathematically as follows:

$$Net\ rate\ of\ change\ =\ Rate\ in\ -\ Rate\ out$$

which is called the balance law.

Applying the balance law to model fish populations, the compartment represents the Lake or the pond and the birth and mortality rates correspond to the "rate in" and the "rate out." In view of the above, the balance law in fish population becomes:

$$\textit{The rate of change in the fish population} = Birth\ Rate\ -\ Death\ Rate\ -\ Harvest\ Rate$$

where the death rate specifically implies "natural" causes of death, and the harvest rate is the amount of fish caught by humans.

Biology tells us that birth rates and death rates in a fish population are proportional to the population's size. If we let the population of fish in the lake or pond at any time *t*, be represented by the function *y(t)*, then the birth rate can be described as $b(y(t))$ where, *b* is a proportionality constant.

The death rate in a fish population is affected by two factors, which are

1. Fish dying of "old age", and
2. Fish dying due to a scarcity of food, oxygen and other resources required for survival.

The second of these two factors is referred to as overcrowding and it is proportional to the population of the fish.

3. 2 EXPONENTIAL GROWTH MODEL.

We first develop a standard exponential growth/decay model that describes quite well the population of species becoming extinct or the short-term behavior of a population growing in an unchecked fashion. The differential equation that models the natural fish population growth is based on the following assumptions.

3.2.1 MODEL ASSUMPTIONS.

The following are the assumptions of the exponential growth model:

1. The rate of change of population is directly proportional to the current population size.
2. For small populations the growth rate is positive.

The parameters involved are t, Y, and r , where

t : time of harvest in months.

Y: fish population in tons

r : proportionality constant or growth-rate coefficient.

3.2.2 MODEL FORMULATION

Following the assumption that the rate of change of population is directly proportional to the current population size, a model for a single population that obeys Malthusian theory of growth or Gompertz growth in the absence of harvesting is developed as follows.

$$\frac{dY}{dt} \propto Y$$

leading to the following differential equation model

$$\frac{dY}{dt} = rY, \qquad r > 0 \qquad (3.01)$$

where r is constant.

Solving the differential equation (3.01) by the method of separation of variable and integration, we have

$$\frac{\quad Y}{Y} = rdt$$

$$\int \frac{dY}{Y} = r\int dt$$

$$lnY = rt + c$$

$$\Rightarrow Y = c_1e^{rt}$$

where $c_1 = e^c$

with initial condition

$$Y(0) = y_0$$

the solution of the equation (3.01) is given as

$$Y(t) = y_0e^{rt} \qquad (3.02)$$

where $y_0$is the population at time t = 0.

Equation (3.02) predicts that the fish population would grow exponentially for $t > 0$, and it is known as Malthusian model of population growth.

3.2.3 ANALYSIS OF EXPONENTIAL GROWTH MODEL

The solution pattern of the equation (3.02) is dependent on the various values of growth rate r.

For $r = 0$,

the total population remain constant over time, as shown in figure 3.1

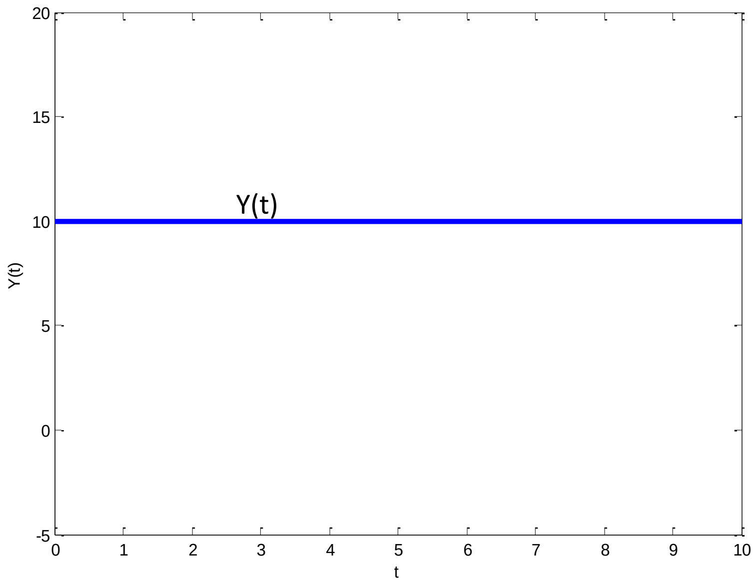

Figure 3.1 Exponential growth with growth rate r = 0

Figure 3.1, shows that when growth rate is zero, the population remains at the same level throughout the period.

For $r < 0$,

the entire population will tend to extinction at an exponential rate since individuals cannot replace themselves due to some environmental factors.

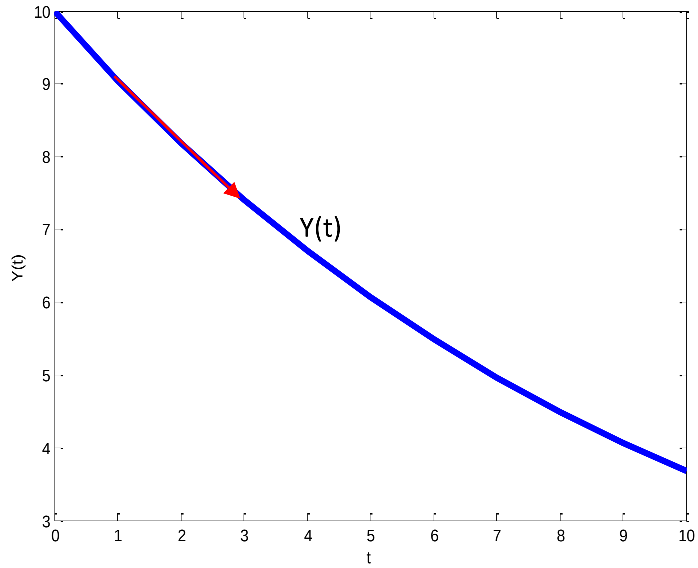

Figure 3.2 Exponential growth with growth rate r < 0

From figure 3.2, we can see that when the growth rate is negative, the population Y(t) decreases as time t increases. Negative growth rate may be as a result of diseases, lack of food. This is not physically meaningful because negative population or decreasing fish population is not beneficial to fishermen.

For $r > 0$,

individuals are able to replace themselves each generation and as a result the entire population will grow indefinitely at an exponential rate. At this point, there are no mechanisms limiting the population from growing so it will continue to grow (figure 3.3).

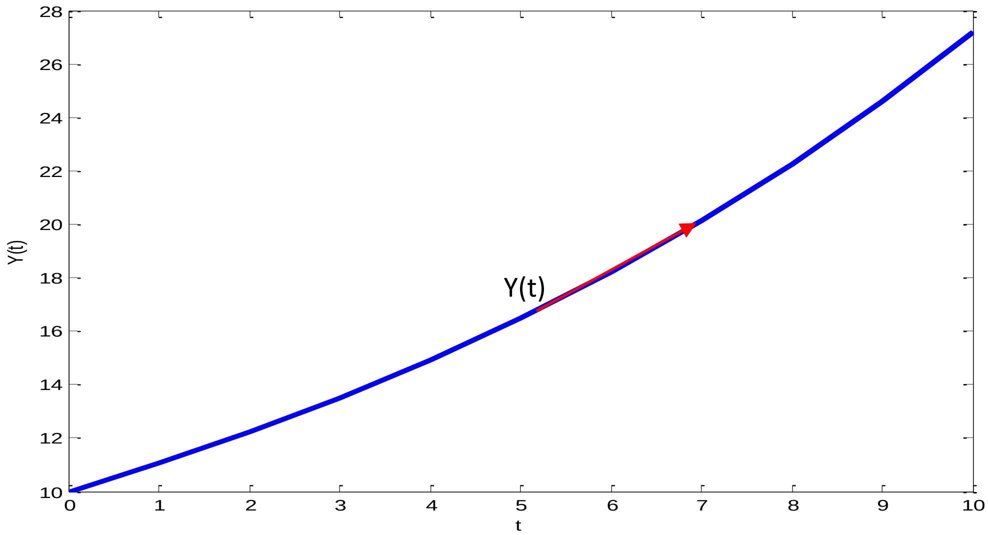

Figure 3.3 exponential growth with growth rate $r > 0$

Figure 3.3 shows that as time t increases fish population Y(t) also increases indefinitely without bound. This indefinite growth is not feasible since the environment and other resources are not unlimited in the real world. Hence, the differential equation (3.01) does not provide a very accurate model for fish population when the population is very large. In view of this a modified logistic growth model would be developed in the next section.

3.3 A MODIFIED LOGISTIC GROWTH MODEL

In real life, some populations do grow exponentially provided that the population is small. But in most large populations individual members eventually do compete with each other for food, living space, air and other basic natural resources needed for growth. When, populations start by increasing in an exponential manner, the population levels off when it approaches its carrying capacity (living space) or decreases towards the carrying capacity.

3.3.1 MODEL ASSUMPTIONS

In general, the following are the assumptions of our modified logistic growth model.

1. The supply of resources such as food, oxygen and space are limited.

2. Growth rate decreases as the population is sufficiently large.

3. Growth rate increases as the population is sufficiently small.

3.3.2 MODEL FORMULATION

Let us consider that the proportionality factor r, measuring the population growth rate in equation (3.01), is now a function of the population f(Y). As the population increases and gets closer to carrying capacity C, the rate r decreases. One simple sub model for r is the linear

$$f(Y) = \left(1 - \frac{Y}{C}\right)$$

Substituting this function into equation (3.01) leads to the modified logistic growth model

$$\frac{dY}{dt} = rY\left(1 - \frac{Y}{C}\right) \qquad (3.03)$$

where, the variable Y can be interpreted as the size of the fish population in tonnes.

C is referred to as the carrying capacity of the environment and the parameter r is called the growth rate.

Equation (3.03) is a more realistic model that describes the growth of species subject to constraints of space, food supply, and competitors/predators. This equation is called the modified logistic population growth model by P.F Verhulst.

Figure 3.4 is the graph of the modified logistic growth model (equation 3.03).

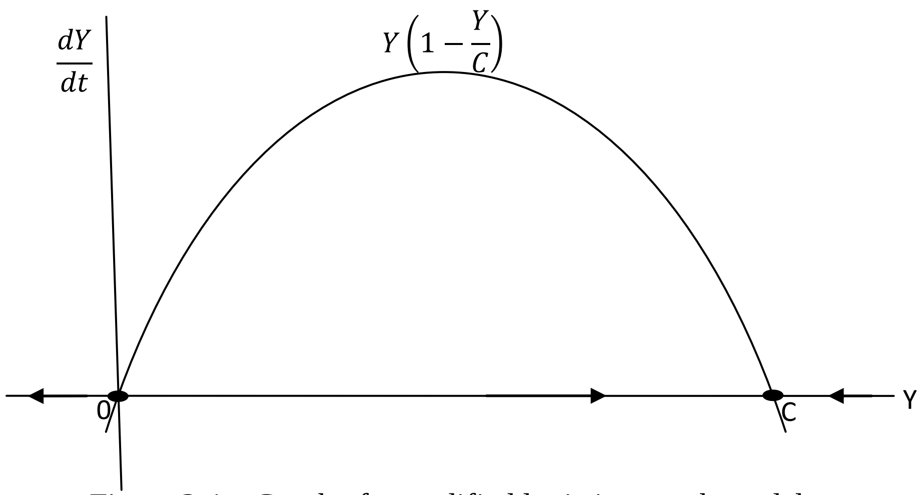

Figure 3.4:   Graph of a modified logistic growth model

The modified logistic growth graph, displayed in Figure 3.4, crosses the Y-axis at the two points Y = 0 and Y = C, and represent critical points.

For $0 < Y < C$, we have $\frac{dY}{dt} > 0$. Hence slopes are positive at any point and solutions must increase in this region. When $Y < 0 \; or \; Y > C$, we have $\frac{dY}{dt} < 0$ and so solutions must decrease.

One way we can analyse the predictions of modified logistic model thus, equation (3.03) is to solve it. The following section describes the solution process.

3.3.3 SOLUTION OF THE MODIFIED LOGISTIC GROWTH MODEL

The following is the process of solving equation (3.03)

Given

$$\frac{dY}{dt} = rY\left(1 - \frac{Y}{C}\right)$$

Let

$$a = \frac{Y}{C}$$

By the separation of variables, we have

$$\frac{dY}{Y(1-a)} = rdt$$

By partial fraction resolving, and integrating we have

$$ln\left(\frac{Y}{1-a}\right) = rt + K$$

Replacing a and simplifying, we have

$$Y(t) = \frac{Cke^{rt}}{C + k\, ?^{rt}}$$

Evaluating with the initial condition $Y(0) = y_0$, we found that

$$k = \frac{Cy_0}{C - y_0}$$

Using this, the solution of equation (3.03) is given as

$$Y(t) = \frac{Cy_0}{y_0 + (C - y_0)e^{-rt}} \tag{3.04}$$

$$y_0 > 0, \quad C > 0$$

3.3.3.1 ANALYSIS OF SOLUTION OF THE MODIFIED LOGISTIC GROWTH MODEL

In fishery production, if the initial population is less than the environmental carrying capacity, the fish will grow quickly to fill the living space (carrying capacity).

For y_0 if $< \ C$,

the population will monotonically increase toward the carrying capacity C and remain there (figure 3.4).

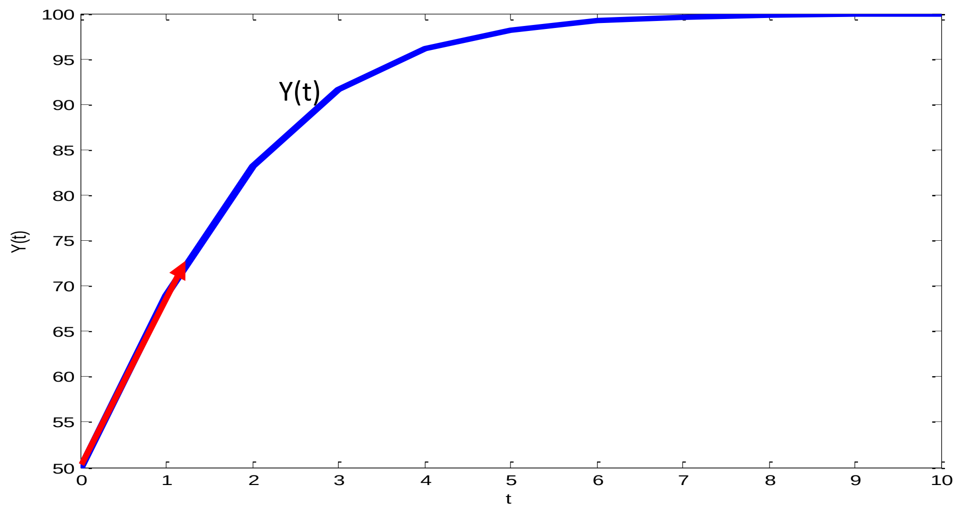

Figure 3.5: Logistic growth model with initial population less than carrying capacity ($y_0 < C$)

From the logistic curve in figure 3.4, we observed that as time increases from t =0 to t =7 months, fish population Y(t) increases to the carrying capacity C. The population remains there as time increases (t >7). This implies that in fishery production; if the pond surface area is very large and few f fingerlings were put in it, the fingerlings would grow and reproduce quickly to fill the pond.

For $y_0 > C$,

the population will monotonically decrease toward the carrying capacity C and remain there (figure 3.5).

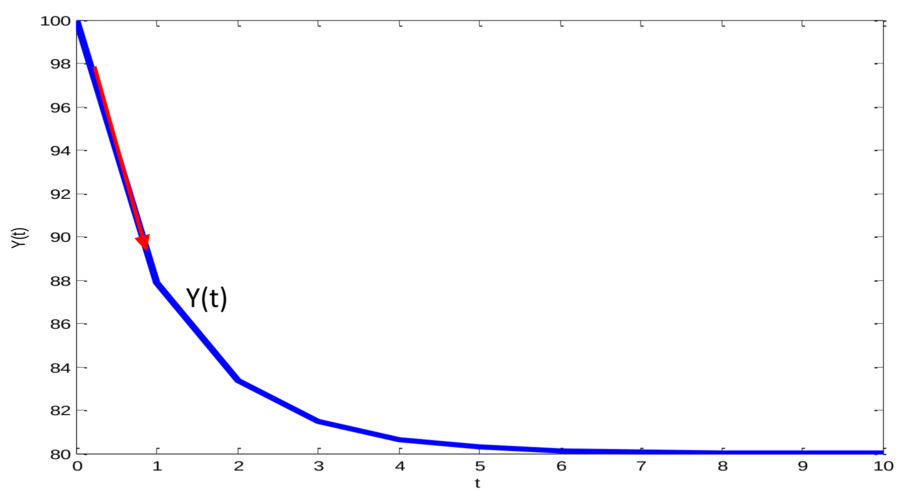

Figure 3.6: Logistic growth model with carrying capacity more than initial population ($y_0 > C$)

Similarly, figure 3.6 shows that at time t = 0, the fish population Y(t) = 100, but after six month the population Y(t), decreases to carrying capacity C = 80 and remains constant at that level as time increases. Hence, we can conclude that if fish population is more than the pond surface area, some will die off, until the population is at the level of the pond carrying capacity.

Another way, we could, analyse equation (3.03) is to find the equilibrium solutions. The next section presents the equilibrium solutions and their analyses.

3.3.3.2 EQUILIBRIUM SOLUTIONS OF THE MODIFIED LOGISTIC GROWTH MODEL

The equilibrium solution is also called critical point or stationary point. At this point fish population remains unchanged. The equilibrium solutions are given as follows:

$$\frac{dY}{dt} = rY\left(1 - \frac{Y}{C}\right) = 0$$

$$rY\left(1 - \frac{Y}{C}\right) = 0$$

$$rY = 0$$

$$\Rightarrow Y = 0$$

and

$$1 - \frac{Y}{C} = 0$$

$$\Rightarrow Y = C$$

That is $Y = 0$ and $Y = C$ are the equilibrium solutions. These equilibria solutions are represented in figure 3.6.

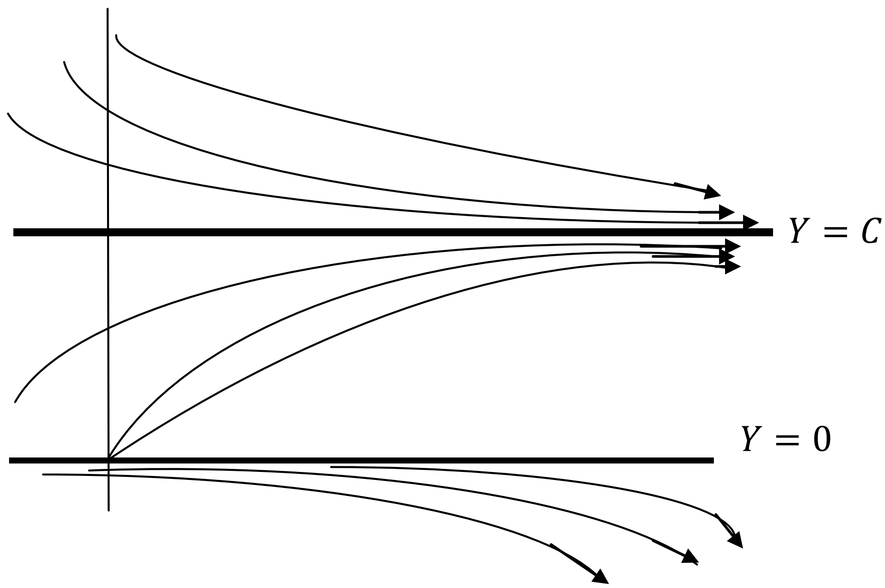

Figure 3.7: Equilibrium solutions of logistic growth model

The equilibrium solution $Y = 0$ is unstable equilibrium point because solutions move away from it and $Y = C$ is asymptotically stable equilibrium point because solutions move towards it, as shown in Figure 3.7. Thus, we can conclude that when we stock population anywhere between 0 and C, it increases to asymptotically to the level of C. But when the population is stock above C, it decreases towards C. The solutions below the axis, that is $Y < 0$, Do not have any physical significant.

The following section presents the modified logistic growth model with harvesting.

3.4 MODIFIED LOGISTIC GROWTH MODEL WITH HARVESTING

In this section, the modified logistic growth model would be adjusted to take into account harvesting of the population. This will enable fish harvesters to determine what frequency of harvesting rate their population can tolerate. These harvesting models are the constant harvesting and seasonal (periodic) harvesting.

3.4.1 MODIFIED LOGISTIC GROWTH MODEL WITH CONSTANT HARVESTING.

Constant harvesting is where a fixed number of fish were removed from the stock at constant time rate. We assume that a constant number, H, of tilapia fish are removed from the population. Hence, the model is given by

$$\frac{dY}{dt} = rY\left(1 - \frac{Y}{C}\right) - H \quad , \quad H > 0 \qquad (3.05)$$

where, the variable ***Y*** can be interpreted as the size of the fish population in tones,

r is called the rate of fish survival at maturity stage ,

C is referred to as the carrying capacity of the environment,

H is constant number of fish harvested each time.

The following section presents the equilibrium solutions and their analyses of the logistic growth with constant harvesting rate.

3.4.2 EQUILIBRIUM SOLUTIONS OF CONSTANT HARVESTING

B y setting equation (3.05) equal to zero, we have two equilibria solutions. These occurs when the growth rate of the fish population is equivalent to the harvest rate, thus,

$$rY\left(1-\frac{Y}{C}\right)-H=0$$

$$rY\left(1-\frac{Y}{C}\right)=H$$

$$rY-\frac{rY^2}{C}=H$$

$$rY^2-rCY+CH=0$$

By quadratic formula, we have the equilibrium solutions as follows:

$$Y_{1,2}=\frac{Cr\pm\sqrt{(Cr)^2-4r(CH)}}{2r} \qquad (3.06)$$

For maximum sustainable harvesting rate, we let the expression under the radical sign equal zero, as follows:

$$(Cr)^2-4r(CH)=0$$

$$C^2r - 4CH = 0$$

$$Cr - 4H = 0$$

$$H = \frac{rC}{4} \qquad (3.07)$$

Hence, equation (3.07) is the maximum rate of harvesting and this gives us the maximum sustainable yield (MSY). The value H = $\frac{rC}{4}$ is called the bifurcation point.

Figure 3.8, is the graph of functions $h(Y)$ in the three cases of the harvesting rate:

$H < \frac{rC}{4}$; $H = \frac{rC}{4}$ and $H > \frac{rC}{4}$.

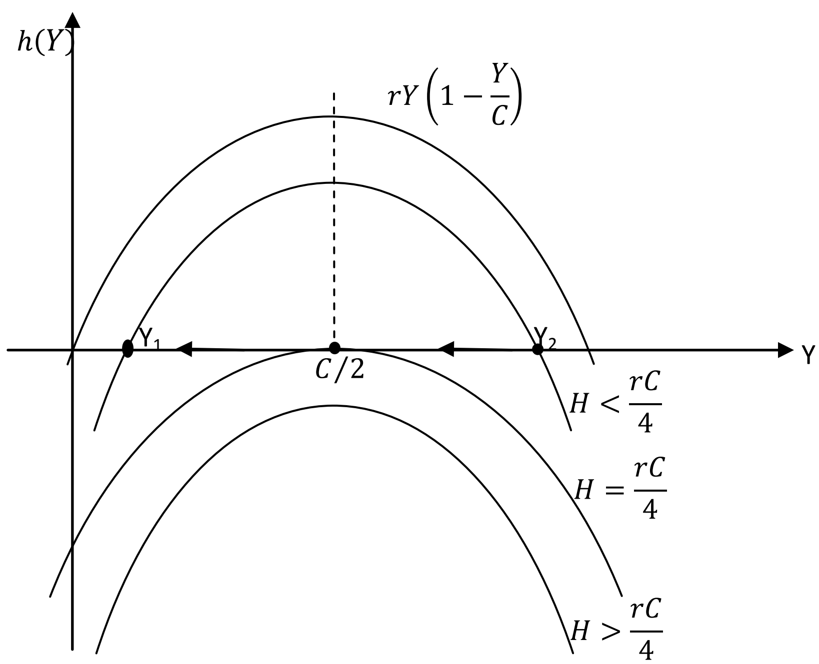

Figure 3.8: The graphs of the function $h(Y) = \left(1 - \frac{Y}{C}\right) - H$.

We can see that for a harvesting rate that is not too large $(0 < Y < \frac{rC}{4})$, there exist two equlilbria solutions (0 and C) as in Figure 3.8. The lower equilibrium solution $Y = 0$ is unstable. Thus, if for overfishing or disease outbreak, the size of the population Y drops below zero and the population eventually die out in a short time. The upper equilibrium solution $Y = C$ is stable. This is the steady state toward which the population approaches a constant harvesting C.

For $H > \frac{rC}{4}$,

there is no critical point or equilibrium solution, and the entire population will be harvested in a short time.

For $H = \frac{rC}{4}$,

there is one critical point or equilibrium state that is semi-stable. Thus, it is mathematically possible to continue harvesting indefinitely at such rate if the initial population is sufficiently large. However, any small change in the equilibrium size of the population will lead to a complete harvest of the population in a short time. This is shown in the figure 3.9 below.

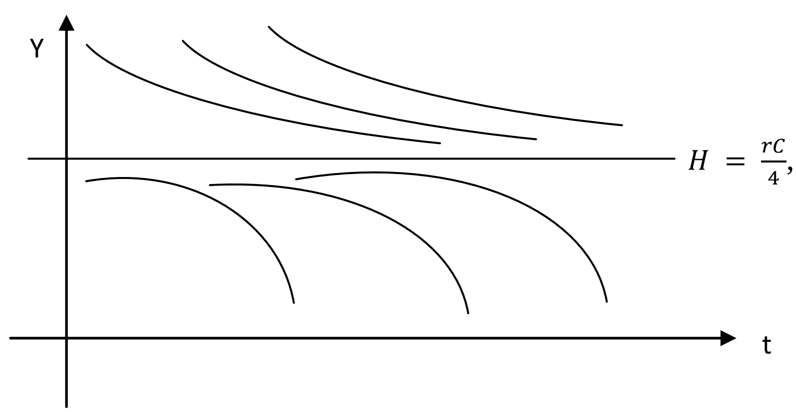

Figure 3.9: Solution curves for maximum sustainable harvest $H = \frac{rC}{4}$

It turns out however, that the harvest can be organized so as to obtain in a stable manner a harvest at the rate $\frac{rC}{4}$ for one unit time and more than this can be achieved since $\frac{rC}{4}$ is the maximal reproductive rate of the un harvested population.

For $H \; < \frac{rC}{4}$;

we have two critical points Y_1 and Y_2. Y_2 is the carrying capacity with human intervention and Y_1 is the extinction level that means if the population is less than Y_1, it will lead to extinction. From figure 3.8 above, as the harvesting rate H increases Y_1 increases and Y_2 decreases.

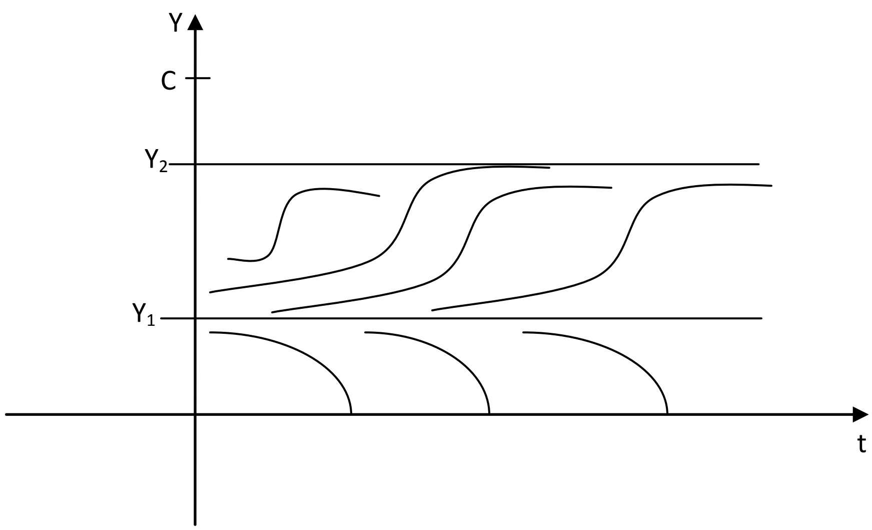

Figure 3.10: Solution curves $0 < Y < C$

In Ghana, there are two seasons, namely, colder (rainy season) and warmer (dry season). The colder season is from March to October, whiles the warmer season starts from November to February. Many fish species are harvested at a higher rate in warmer seasons than in colder months. In view of this, we would assume that the fish population is harvested at a periodic rate in the next session.

3.4.2 LOGISTIC GROWTH WITH PERIODIC HARVESTING

Functions or phenomena that repeat or are cyclic in nature are model with sine and cosine functions with a period of 2π. When fish population is harvested periodically, then the harvesting rate is models as

$$H(t) = h(1 + \sin(\omega t))$$

where h represents the coefficient that determines the total rate of periodic harvesting and additional 1 is included to insure a positive quantity for sine term.

Hence, we modify the logistic growth model (3.03) to include periodic harvesting, we have

$$\frac{dY}{dt} = rY\left(1 - \frac{Y}{C}\right) - h(1 + \sin(\omega t)) \quad (3.08)$$

Equation (3.08) might model extinction for stocks less than some threshold populationY_0, and a stable population that oscillates about carrying capacity C with period $T = \frac{2\pi}{\omega}$.

The variable ***Y*** can be interpreted as the size of the fish population in metric tonnes.

Its development over time, ***Y(t)*** depends on its initial value ***Y(0)*** and on the two parameters r and ***C***, where ***r*** is called the rate of fish survive at maturity stage, ***C*** is referred to as the carrying capacity of the environment.

3.4.2.1 PERIODIC SOLUTIONS OF PERIODIC HARVESTING

Equation (3.08) is a non autonomous nonlinear differential equation periodically forced with period $T = 1$ and depends explicitly on time. Also, this differential equation is no longer separable, so we cannot generate an analytic formula for its solution using the usual methods from calculus. Hence, we introduce the Poincaré Map a more qualitative approach to non autonomous nonlinear differential equations.

The following describes the process of solving equation (3.08) by Poincaré Map.

Given

$$\frac{dY}{dt} = f(t, Y) = rY\left(1 - \frac{Y}{C}\right) - h(1 + \sin(2\pi t))$$

We let

$$\emptyset(t, y_0)$$

be the particular solution $y(t)$ satisfying the initial condition $Y(0) = y_0$, then we introduce the Poincaré Map

$$P(Y) = \emptyset(1, Y)$$

The initial condition y_0 will correspond to a periodic solution if and only if y_0 is a fixed point of the Poincaré Map, $P(y_0) = y_0$.

For uniqueness of solution of the initial value problem, $\emptyset(t + 1, Y)$ satisfies $\frac{dY}{dt} = f(t, y_0)$ if

$$f(t+1, y_0) = f(t, y_0)$$

then

$$\emptyset(t+1, y_0) = \emptyset(t, y_0)$$

if only if at initial time $t = 0$

$$\emptyset(1, y_0) = \emptyset(0, y_0) = y_0$$

So we compute the derivative of $P(Y)$ as follows:

Let

$$\theta(t, y_0) = \frac{\partial}{\partial y_0}(t, y_0)$$

then

$$f(t,\ y_0) = r y_0 \left(1 - \frac{y_0}{C}\right) - h(1 + \sin(2\pi t))$$

differentiating with respect to y_0, we obtain

$$\frac{\partial}{\partial y_0}(t, y_0) = r - \frac{2r}{C} y_0$$

By separation of variables and integration, we compute the solution as follows:

$$(t, y_0) = e^{\int_0^t \left(1 - \frac{2r}{C} \emptyset(S,\ y_0)\right) dS}$$

If $t = 1$, we obtain

$$P'(y_0) = \quad e^{\left(1 - \frac{2r}{C} \int_0^1 \emptyset(S,\ y_0) dS\right)} \qquad (3.09)$$

Since $P'(y_0) > 0 \quad \Rightarrow Y(t)$ is increasing.

Differentiating equation (3.09) once more we have

$$P''(y_0) = \frac{-2r}{C}\left(\int_0^1 \emptyset\,(s, y_0)ds\right)P'(y_0) \qquad (3.10)$$

Since, $P''(y_0) < 0$, it shows that the graph of the Poincaré Map is concave down and the graph can cross the diagonal line $y = x$ at most two times. Therefore, the Poincaré Map has at most two fixed points. These fixed points give periodic solutions of the differential equation (3.08). Since the differential equation (3.08) also depends on the harvesting parameter h, then we differentiate

$$f(t, y_0)ry_0 = \left(1 - \frac{y_0}{C}\right) - h(1 + \sin(2\pi t))$$

with respect to the parameter h, and obtain

$$\frac{\partial f}{\partial h}(t, y_0) = -(1 + \sin(2\pi t)) \qquad (3.11)$$

From equation (3.11), we see that $\frac{\partial f}{\partial h} < 0$ for all values of t, except $t = \frac{3}{4}$. This implies that increasing the harvesting rate, decreases the population for all $t > 0$. Hence, there is a critical value h_c for which the Poincaré Map bifurcates.

For $h > h_c$, there are no fixed points for P and so $P(y_0) < y_0$ for all initial values. Intuitively, larger harvesting rate should lead to smaller populations and eventually the population becomes extinct in a short time.

For $h < h_c$, the Poincaré Map has two periodic solutions and For $h = h_c$, Poincaré Map has exactly one fixed point or periodic solution. Figure 3.8 shows Poincaré Map for different values of harvesting parameter h.

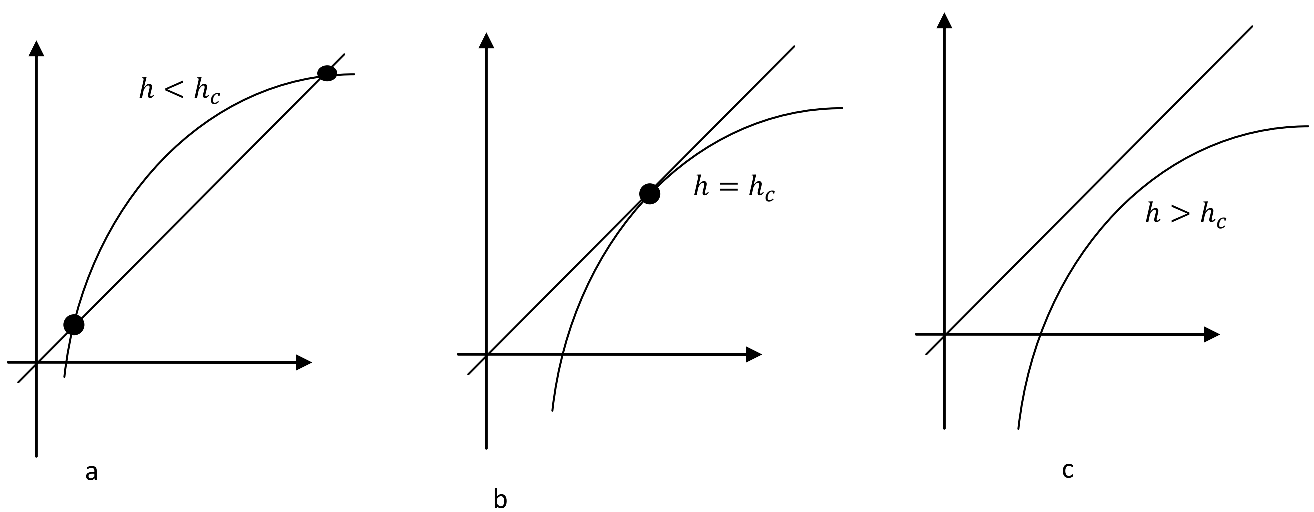

Figure 3.8: Poincaré Map for different values of harvesting parameter h.

The following chapter presents the data analysis and discussion of results obtained.

CHAPTER FOUR

DATA ANALYSIS AND DISCUSSION

4.1 INTRODUCTION

In this chapter, we shall deal with models implementation and discussion of results obtained from the models implemented. The data for this study has been obtained from the Commission of Fisheries of Ghana, Tropo Farms Limited and Crystal Lake Fish Limited, situated near the Volta Lake in the Eastern region of Ghana.

4.2 THE MODEL DATA

According to Ofori et al. 2009, the typical cage size used in the Volta Lake is about 6×4 metres on sides and 2 metres deep. The Commission of Fisheries of Ghana (2012) claimed that a fish pond can sustain 5 tilapia fish for every 1 m^2 surface area. Table 4.1 presents detail data on the selected pond and cage for the studies.

Table 4.1: Data on the Pond and Cage farming.

DESCRIPTION	**POND**	**CAGE**
SURFACE AREA OF THE POND	124100 m^2	
VOLUME OF THE CAGE		48 m^3
NUMBER OF FINGERLINGS PER $1m^2$	5 Fingerlings	
NUMBER OF FINGERLINGS PER $1m^3$		188 Fingerlings
CARRYING CAPACITY	620500 tonnes	9024 metric tones

The period of maturity for the tilapia fish is 6 months and about 80% will survive to maturity (Thomas and Michael, 1999 & Ofori et al. 2009).

4.2 LOGISTIC GROWTH MODEL WITHOUT FISHING.

The values of the parameters are $r = 0.8$ or 80%, the estimation of fingerlings that will survive at maturity stage and the value of carrying capacity, $C = 620500$ fingerlings in tonnes. The equilibrium point is also called critical point or stationary point. At this point fish population remains unchanged. The equilibrium points of the logistic growth model without fishing were obtained as follows: Given the logistic growth model without fishing, derived in chapter three,

$$\frac{dY}{dt} = rY\left(1 - \frac{Y}{C}\right) \tag{4.01}$$

At the equilibrium points of the model, we let

$$\frac{dY}{dt} = 0$$

$$\Rightarrow \qquad rY\left(1 - \frac{Y}{C}\right) = 0$$

Substituting the values of the parameters, we have

$$0.8Y\left(1 - \frac{Y}{620500}\right) = 0$$

by the zero property theorem

$$0.8Y = 0$$

$$\Rightarrow Y = 0$$

and

$$\left(1 - \frac{Y}{620500}\right) = 0$$

$$\frac{Y}{620500} = 1$$

$$\Rightarrow Y = 620500$$

Thus, $Y = 0$ and $Y = 620500$ are the equilibrium points. Hence, if the initial population of the fish started with $Y = 0$, thus no fingerlings were put into the pond, the population of the fish remains at $Y = 0$.

Similarly, If the initial population of the fish is started with $Y = 620500$, the population remains at the same level.

The stability of these equilibrium points can be seen from figure 4.1. These equilibrium points may be either stable or unstable. The equilibrium point $Y = 0$ is an unstable, because the solutions near this point are repelled or asymptotic. This means given an initial population of fish Y_0, just above $Y = 0$ and the Y_0 less than 0, the fish population grows away from $Y = 0$.

The equilibrium point at $Y_0 = 620500$ is stable, because solutions near this point are attracted to it. This means given an initial fish population in the interval $(0,620500)$, the population increases and reaches $Y = 620500$, and remains at the same level. But, if Y_0 is greater than the carrying capacity 620500, the fish population declines and approach a limiting value 620500 (figure 4.1).

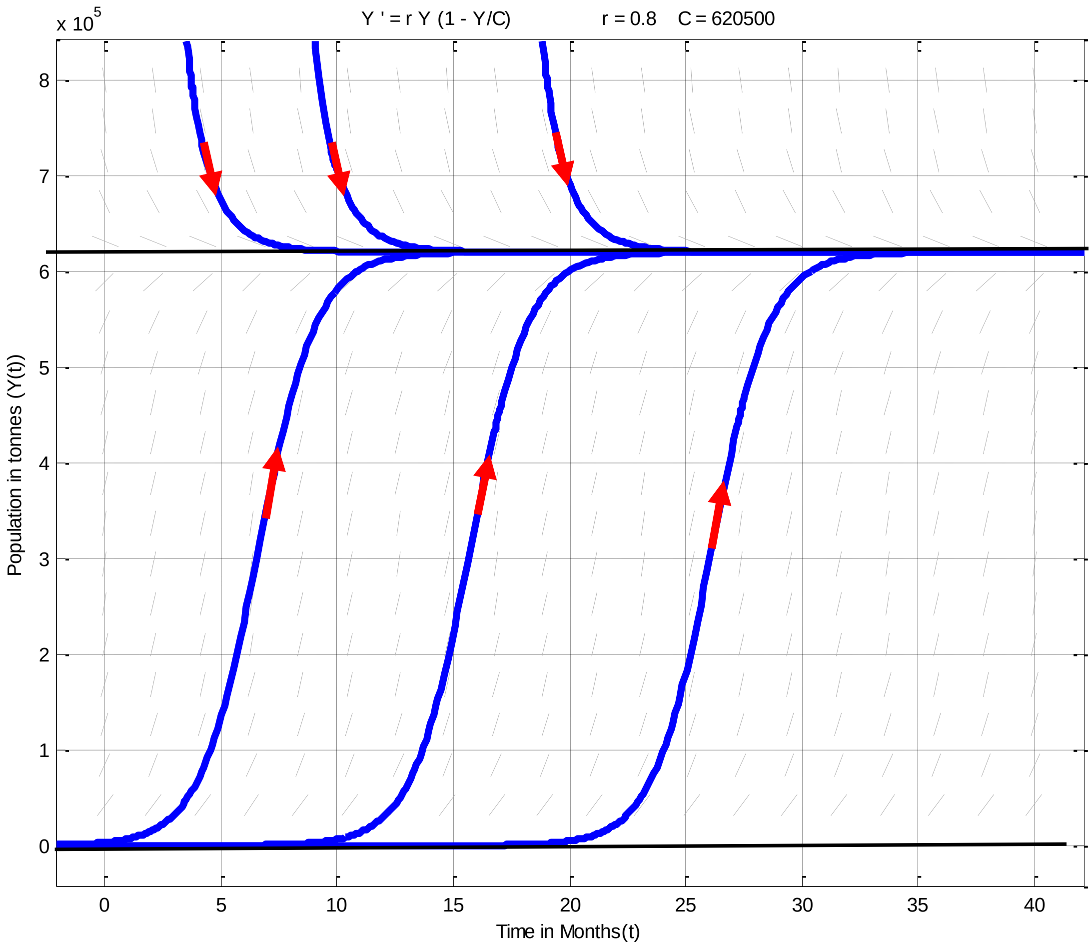

Figure 4.1: Direction field and solution curves of Logistic growth

From figure 4.1, we can see that population starts increasing exponentially from the first month and reaches the carrying capacity in the sixteenth month and remains there.

4.3 LOGISTIC GROWTH MODEL WITH CONSTANT HARVESTING

The Logistic Growth Model with constant harvesting derived in chapter 3 is as follows:

Given

$$\frac{dY}{dt} = rY\left(1 - \frac{Y}{C}\right) - H \qquad (4.02)$$

where the values of $r = 0.8,\ C = 620500$ and H is constant to be determine.

To determine the equilibrium points for constant H, we have:

$$0.8Y\left(1 - \frac{Y}{620500}\right) - H = 0,$$

$$0.8Y - \frac{0.8Y^2}{620500} - H = 0,$$

$$0.8Y - 0.00000128928283Y^2 - H = 0$$

By comparing with the general quadratic equation:

$$ax^2 + bx + c = 0,$$

$$a = 0.00000128928283$$

$$b = -0.8$$

$$c = H$$

then

$$Y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$Y = \frac{-(-0.8) \pm \sqrt{(-0.8)^2 - 4(0.00000128928283)H}}{2(0.00000128928283)}$$

We let the expression under the square root sign equal to zero, we have

$$(-0.8)^2 - 4(0.000001289283)H = 0$$

$$0.64 - 0.000005157132H = 0$$

$$H = \frac{0.64}{0.00000515713132}$$

$$H = 124100.0006 \approx 124100$$

The value $H = 124100$ is the maximum sustainable yield (MSY) or the total allowable catch that can be harvested from the stock or the biomass. The value H = 124100 is called the bifurcation point and at this point we consider three values of harvesting:

1. $H = 124100$
2. $H > 124100$
3. $H < 124100$

For $H = 124100$,

we have one equilibrium point, thus,

$$Y = \frac{-(-0.8) \pm \sqrt{(-0.8)^2 - 4(0.000001289282836)124100}}{2(0.000001289282836)}$$

$$Y = 310250$$

From figure 4.2, we can see that if the initial population of fish Y_0 is greater than 310250; the population of the fish will decrease and approach to 310250. Similarly, for initial population of fish Y_0 lower than 310250; the population of the fish will decrease and gets to extinction, if the harvesting rate is H= 124100.

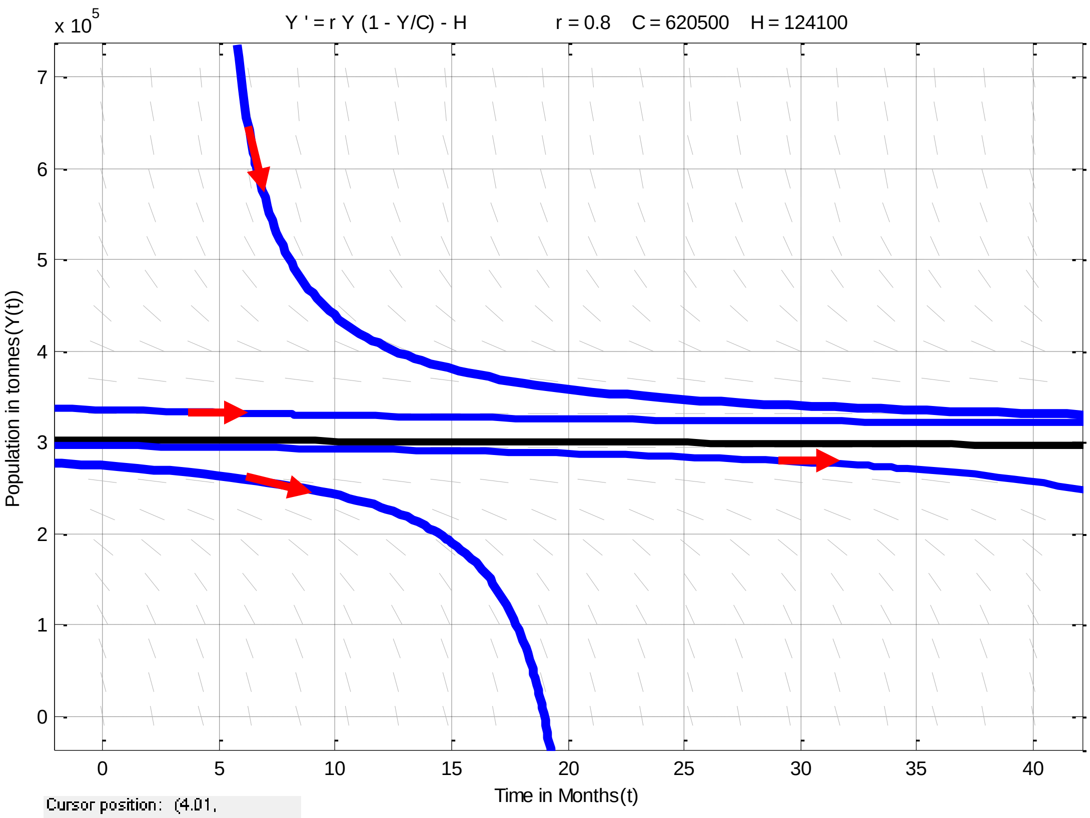

Figure 4.2: Direction field and solution curves of Constant harvesting H =124100

For $H > 124100$,

we assume the harvesting value, H = 130000 and this figure shows the decreasing trends of fish population. This means that if we continue to harvest this figure, the fish population will go to extinction regardless of the initial population size, as shown in figure 4.3.

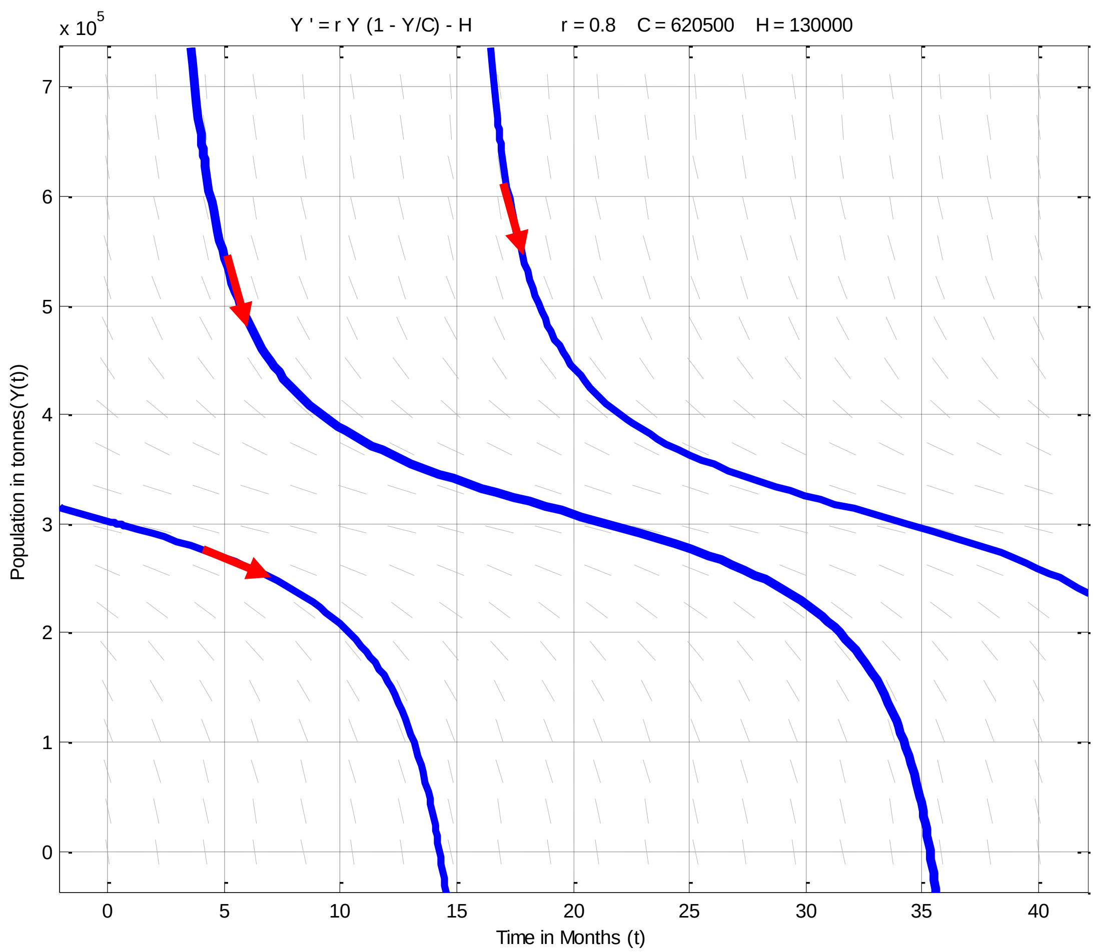

Figure 4.3: Direction field and solution curves of Constant harvesting H =130000

For $H < 124100$,

we assume the harvesting value, H = 110000. There are two equilibrium points exist when the value of harvesting is less than 124100. These are 414827 and 205673. The upper equilibrium point is stable and it shows the population of the fish is decreased. The lower equilibrium point is unstable because the solution near this point is repelled. Thus, we can conclude that the harvest cannot be too large without depleting the resource. However, we can see in figure 4.4 that in the interval (205673, 414827), the population of the fish is increased.

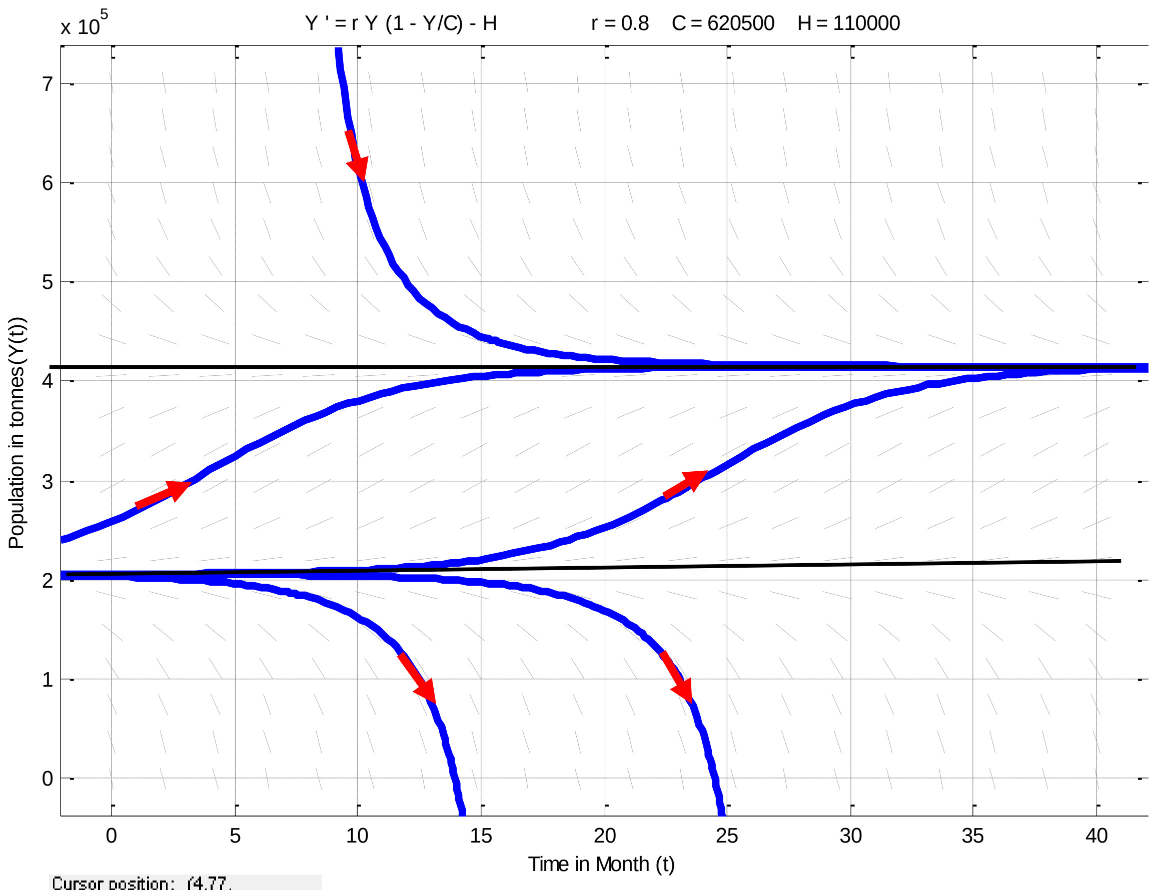

Figure 4.4: Direction field and solution curves of Constant harvesting H =110000

4.3.1 PREDICTIONS OF CONSTANT HARVESTING MODEL

Our constant harvesting model predicts the following:

(i). If the initial fish population is $205673 < Y_0 < 414827$, then the fish population will increase asymptotically to $Y = 414827$ as time t tends to infinite $(t \to \infty)$, figure 4.4. In fact, it will never approach $Y = 414827$, but for large times it will be close to it and will be increasing gradually.

(ii). If the initial fish population is $0 < Y_0 < 205673$, then the fish population is below a critical threshold and there are not much individual fishes in the pond to reproduce quickly enough to sustain the harvest rate. Hence, the population decreases, reaching extinction $Y = 0$ in a short period of time (figure 4.4)

(iii). If the initial population exceeds the maximum sustainable level, that is $Y > 414827$, then it decreases to 414827 from above, as time tends to infinite (figure 4.4).

4.3 LOGISTIC GROWTH MODEL WITH PERIODIC HARVESTING

The logistic growth model with periodic harvesting derived in chapter three is as follows

$$\frac{dY}{dt} = rY\left(1 - \frac{Y}{C}\right) - h(1 + \sin(\omega t)) \qquad (4.03)$$

where the value of $r = 0.8$, $C = 620500$, h = 124100 and $\omega = 2\pi$.

The fish population will not be able to extinct in fishing time since the harvesting rate is a periodic function and varies from season to season. The amount of fish might be able to increase again, if the fishing activity is stopped. The pond has full carrying capacity of C = 620500 tilapia fish in the pond as an initial population. For the first six months 124100 tilapia fish is assumed for harvesting until the population of tilapia remains 414827 and followed by no harvesting for the next six (6) months and this pattern repeats for several years, as seen in figure 4.5. In figure 4.5, we see that all of the solutions have a period of exactly 1.

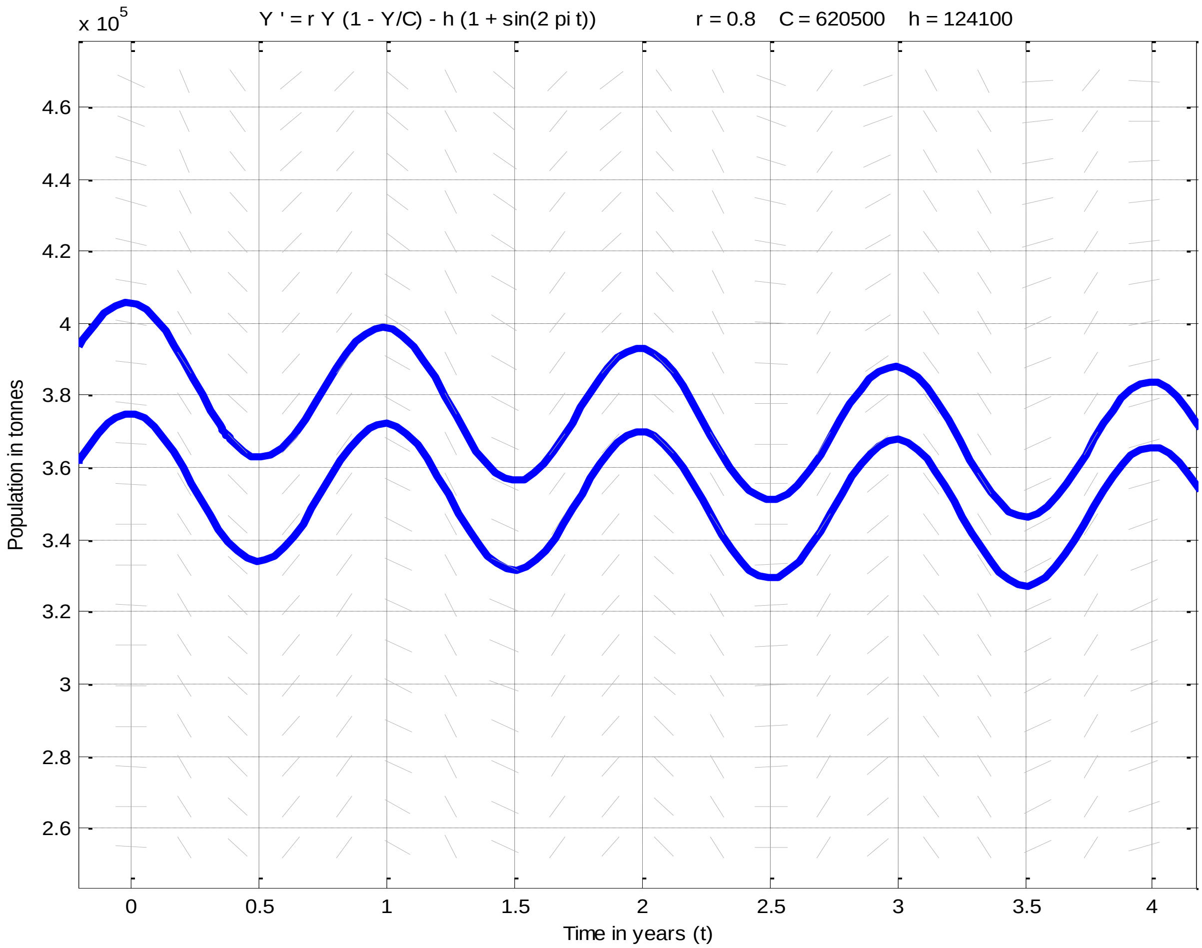

Figure 4.5: Logistic growth model with periodic harvesting

In order to ensure the population of tilapia fish is increasing, there are no harvesting in the next six (6) months and the population of tilapia fish will increase until it approaches the carrying capacity, C = 620500.

The next Chapter deals with conclusions and recommendations.

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1 CONCLUSIONS

In this work, we studied the sustainable harvesting strategies of the tilapia fish population in a pond. Management of fish populations to sustain catches and abundance levels can be based on several alternative means of strategic catch regulation. This thesis is intended to explore harvesting strategies that optimizes catch while still maintaining a sustainable tilapia fishing industry. We extend the modified logistic growth model developed by P.F Verhulst in 1838 by incorporating two types of harvesting strategies into the model and investigate how the demise (catch) of certain number of tilapia fish will affect the total population of tilapia fish in the pond. In particular we study the constant harvesting strategy model and periodic harvesting strategy model of tilapia fish population in a given pond over a period of time.

In the implementation of the logistic constant harvesting strategy of the selected fish pond with carrying capacity of 620500 metre square, the maximum sustainable yield (MSY) or the total allowable catch that can be harvested from the population is 124100 tonnes of fish. If this figure (MSY) is constantly removed from the population, the fish population does not have enough time to recover the fish population; hence, the fish population gets to extinction. However, for logistic periodic (seasonal) harvesting strategy, 124100 tonnes tilapia fish is assumed for harvesting for the first six months until the population of tilapia remains 414827 and this is followed by no harvesting for the next six (6) months to allow the tilapia fish to repopulate until it approaches the carrying capacity, 620500 metre square. This pattern is repeated for several years.

So, the harvesting strategy that optimizes the total allowable catch (harvest) while maintaining the stable population of tilapia fish is logistic periodic (seasonal) harvesting strategy. A harvesting strategy using logistic periodic (seasonal) harvesting strategy can be used to improve productivity, shorten investment return time and reduce risk from changes in sale price of tilapia fish and costs of productions of tilapia fish, particularly when comparatively short return periods are used. The development of fish harvesting strategy probably can supply the market demand throughout the year. It also can improve the commercial return to farmers before harvesting. This study can help in raising the fish such as tilapia fish in freshwater ponds for the farmer just like any other agricultural activity.

5.2 RECOMMENDATIONS

Based on the results of the study, we therefore recommend the following:

1. A periodic harvesting strategy for fish farmers, since it is a more sustainable technique in fishery management practices.
2. The Government, particularly Fishery Commission is encouraged to use this study as part of its guidance for training prospective fisher farmers.
3. Workshops and seminars should be regularly organized to educate fish harvesters and other stakeholders on a more sustainable harvesting strategy and be well monitored to ensure it usage.

Finally, we recommend for further study, the extension of our models to include issuing of Fishing Licenses and oxygen content of the body of water in the pond.

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